

# Endogenous Timing in a Mixed Oligopoly with Foreign Competitors

Yuanzhu Lu<sup>\*†</sup>

Department of Economics, National University of Singapore

August 2005

## Abstract

Endogenous order of moves in quantity choice is analyzed in a mixed oligopoly with one public firm,  $n (\geq 1)$  domestic private firms and  $m (\geq 1)$  foreign private firms. We consider the observable delay game of Hamilton and Slutsky (1990) in the context of a quantity setting mixed oligopoly where firms first choose the timing of choosing their quantities before quantity choice and find subgame perfect Nash equilibria (SPNE). The main results are that the public firm chooses to be a follower of all domestic private firms and not to be a leader of all foreign private firms, and that the number of SPNE depends on the number of domestic private firms and that of foreign private firms.

*Keywords:* Mixed Oligopoly; Endogenous Timing; Foreign Competitors; Public Firm; Private Firm, Simultaneous, Sequential

*JEL classification:* C72; D43; H42; L13

## 1. Introduction

---

<sup>\*</sup> Correspondence: Yuanzhu Lu, Department of Economics, Faculty of Arts and Social Sciences, National University of Singapore, AS2 Level6, 1 Arts Link, Singapore, 117570. Tel: 65-93385758. Email: [yuanzhulu@nus.edu.sg](mailto:yuanzhulu@nus.edu.sg).

<sup>†</sup> I am grateful to John Wooders for his valuable comments on an earlier version of this paper. All errors are mine.

Studies of mixed markets, in which welfare-maximizing public firms compete against profit-maximizing private firms, have become increasingly popular in recent years.<sup>1</sup> In the literature there have been some papers discussing endogenous timing in a mixed oligopoly since alternate order of moves often produces significantly different results and thus leads to different welfare level. For example, Pal (1998) analyzed endogenous order of moves in a mixed oligopoly where the firms first choose the timing of choosing their quantities.<sup>2</sup> Matsumura (2003) considered endogenous roles of firms in Stackelberg mixed duopoly models where a state-owned public firm and a foreign private firm compete.

However, there is no paper discussing endogenous timing in a mixed oligopoly with both domestic and foreign private firms. There are no foreign private firms in Pal (1998) and no domestic private firms in Matsumura (2003). In reality, public firms, domestic private firms and foreign private firms coexist in many industries and in many countries. So the endogenous timing in such a mixed oligopoly is very important and it is surprising that there is no paper discussing such a question. The purpose of this paper is to fill in this gap and to address the issue of endogenous timing in a mixed oligopoly with public firm, domestic and foreign private firms. We consider the observable delay game of Hamilton and Slutsky (1990) in the context of a quantity setting mixed oligopoly where firms first choose the timing of choosing their quantities before quantity choice and find that in any equilibrium, the public firm chooses to be a follower of all the domestic

---

<sup>1</sup> See De Fraja and Delbono (1990) and Nett (1993) for general reviews of the mixed oligopoly model. For recent literature on mixed oligopoly (duopoly), see Fjell and Heywood (2002), Matsushima and Matsumura (2003a), Matsushima and Matsumura (2003b), Fjell and Heywood (2004), etc.

<sup>2</sup> Jacques (2004) slightly corrects Proposition 4.1 of Pal (1998).

private firms and not to be a leader of all the foreign private firms, and that the number of subgame perfect Nash equilibria (SPNE) depends on the number of domestic private firms and that of foreign private firms.

## 2. The model

Consider a mixed oligopoly model with one public firm,  $n$  ( $\geq 1$ ) domestic private firms and  $m$  ( $\geq 1$ ) foreign private firms, all producing a single homogenous product. Let  $q_0, q_i^d$  and  $q_j^f$  be the quantities of the public firm, of domestic private firm  $i$  and of foreign private firm  $j$ , respectively. Let  $Q = q_0 + \sum_{i=1}^n q_i^d + \sum_{j=1}^m q_j^f$  denote the aggregate quantity. The market price is determined by the inverse demand function  $p = a - Q$ . Assume that  $a$  is sufficiently large. All domestic and foreign private firms have constant and identical marginal costs of production, which are normalized to 0. The public firm also has constant marginal cost of production. However, it is assumed to be less efficient than the private firms.<sup>3,4</sup> Let  $c > 0$  be the marginal cost of the public firm. For the sake of simplification, fixed costs are assumed to be zero for all firms.

We consider the observable delay game of Hamilton and Slutsky (1990) in the context of a quantity setting mixed oligopoly where the firms first choose the timing of choosing their quantities. There are  $T \geq 3$  possible periods for quantity choice and each firm may choose its quantity in only one of those  $T$  periods. We consider a two stage game. In stage one, the firms simultaneously announce in which period they will choose

---

<sup>3</sup> This assumption also allows us to avoid a trivial solution. If the public firm is more or equally efficient than the private firms, it would produce a quantity such that the market price equals its marginal cost, resulting in a public monopoly.

<sup>4</sup> We don't consider the efficiency differential between the domestic private firms and the foreign private firms for the sake of simplification. See "conclusion remarks" section for discussion.

their quantities and are committed to this choice. In stage two, after the announcement, firms then select their quantities knowing when the other firms will make their quantity choices.

The public firm's objective is to maximize domestic social surplus defined as the sum of consumer surplus and profits of domestic firms (including itself and all domestic private firms), whereas each private firm's objective is to maximize its own profit. Thus, the objective functions of the public firm, of domestic private firm  $i$  and of foreign private firm  $j$  are given by

$$SS = \int_0^Q (a-x-p)dx + \sum_{i=1}^n pq_i^d + (p-c)q_0$$

$$= a(q_0 + \sum_{i=1}^n q_i^d + \sum_{j=1}^m q_j^f) - \frac{1}{2}(q_0 + \sum_{i=1}^n q_i^d + \sum_{j=1}^m q_j^f)^2 - (a - q_0 - \sum_{i=1}^n q_i^d - \sum_{j=1}^m q_j^f) \sum_{j=1}^m q_j^f - cq_0 \quad ,$$

$$\pi_i^d = pq_i^d = (a - q_0 - \sum_{i=1}^n q_i^d - \sum_{j=1}^m q_j^f)q_i^d \quad , \quad \text{and} \quad \pi_j^f = pq_j^f = (a - q_0 - \sum_{i=1}^n q_i^d - \sum_{j=1}^m q_j^f)q_j^f$$

respectively.

Our objective is to solve the SPNE of this extended quantity setting mixed oligopoly game. We restrict our attention to symmetric equilibria in which all firms of the same type choose to produce in the same period.<sup>5</sup>

### 3. Results for three periods ( $T = 3$ )

First, we derive the results for three periods ( $T = 3$ ). The results for more than three periods are presented in Section 4.

***Lemma 3.1.*** *The public firm and all domestic private firms producing simultaneously in*

---

<sup>5</sup> Though we restrict our attention to symmetric equilibria, we are sure that there is no asymmetric equilibrium because no domestic (foreign) private firm wants to be a follower of the other domestic (foreign) private firms if there are at least two domestic (foreign) private firms.

*the same period cannot be sustained as a SPNE outcome.*

**Proof.** We can show that either the public firm or a domestic private firm has incentive to deviate if the public firm and all domestic private firms produce simultaneously in the same period.

If the public firm and all domestic private firms produce simultaneously as leaders and all foreign private firms produce simultaneously as followers, then  $q_0^* = a - (n+1)(m+1)^2 c / (2m+1)$  ,  $q_d^* = (m+1)^2 c / (2m+1)$  ,  $q_f^* = (m+1)c / (2m+1)$  ,  $Q^* = a - (m+1)c / (2m+1)$  and  $p^* = (m+1)c / (2m+1)$ . If the public firm deviates to produce simultaneously with all foreign private firms instead, then  $q_0^* = 0$ , the total equilibrium output of the domestic private firms is  $a - c$  ,  $q_f^* = c / (m+1)$  ,  $Q^* = a - c / (m+1)$  and  $p^* = c / (m+1)$ . The total output is larger, the equilibrium price is smaller, the output of a foreign private firm is smaller, and more output is now being produced by more efficient domestic private firms. So the public firm has incentive to deviate.

Similarly, if all foreign private firms produce simultaneously as leaders and the public firm and all domestic private firms produce simultaneously as followers, a domestic private firm can increase its profit by producing with all foreign private firms simultaneously.

We can also show that either the public firm or a domestic private firm has incentive to deviate if all the firms produce simultaneously in the same period. ■

Comparing this lemma with Proposition 3.1 in Pal (1998), we find that, in the presence of foreign private firms in the market, the public firm and all domestic private

firms choosing quantities simultaneously cannot be sustained as a SPNE outcome either.

**Lemma 3.2.** *The public firm acting as a leader of all domestic private firms cannot be sustained as a SPNE outcome.*

**Proof.** We can show that either the public firm, or a domestic private firm, or a foreign private firm has incentive to deviate from the cases in which the public firm acts as a leader of all domestic private firms. Here, we give an example. If the public firm acts as a leader in period 1 and all domestic and foreign private firms produce simultaneously as followers in period 2, then  $q_0^* = a - (n+m+1)^2 c / (2m+1)$ ,  $q_d^* = q_f^* = (n+m+1)c / (2m+1)$ ,  $Q^* = a - (n+m+1)c / (2m+1)$  and  $p^* = (n+m+1)c / (2m+1)$ . If the public firm produces in period 3 instead, then  $q_0^* = 0$ , the total equilibrium output of the domestic private firms is  $a - c$ ,  $q_f^* = c / (m+1)$ ,  $Q^* = a - c / (m+1)$  and  $p^* = c / (m+1)$ . We can easily check that the public firm has incentive to deviate. ■

**Proposition 3.1.** *In any SPNE, the public firm acts as a follower of all domestic private firms.*

**Proof.** This follows from Lemma 3.1 and 3.2. ■

Intuitively, a domestic private firm has incentive to produce as a leader of the public firm because a domestic private firm can produce a large amount of output so that the public firm does not produce at all. Since the domestic private firms produce more efficiently than the public firm, the public firm wants to make the domestic private firms produce more by acting as a follower.

Note that the result is independent of the number of domestic private firms, which is different from Proposition 3.4 in Pal (1998). In Pal (1998), there is a second SPNE in which the public firm produces in period 1 and the domestic private firms produce in period 2 if there are two periods to be chosen. Even if there are more than two periods to be chosen, Jacques (2004) demonstrated that the public firm producing in period 1 and the domestic private firm producing in the last period can be sustained as SPNE when there is only one domestic private firm. Why does the different result arise? It is because of the presence of foreign private firms. If the public firm chooses to be the leader of domestic private firms, foreign private firms will also choose to be leaders of them.

***Claim.** The public firm chooses to produce in period 2 only when the number of foreign private firms is at least two ( $m \geq 2$ ) and in this case all domestic and foreign private firms choose to produce in period 1.*

**Proof.** With such an order of moves, we can easily show that a foreign private firm has incentive to deviate to produce in period 3 if  $m = 1$ . And we can show that the public firm producing in period 2 cannot be sustained as a SPNE outcome except in the case stipulated in the claim. ■

***Proposition 3.2.** The public firm acting as a leader of all foreign private firms cannot be sustained as a SPNE outcome. That is, in any SPNE, the public firm produces with all foreign private firms simultaneously or as a follower of them.*

**Proof.** This follows from Proposition 3.1 and Claim. ■

The result that the public firm chooses to produce with all foreign private firms simultaneously or as a follower of them in any SPNE is sharply different from the result in Matsumura (2003) that the public firm becomes the leader in the endogenous role game.<sup>6</sup> So we can see that the presence of domestic private firms has a great impact on the role of the public firm.

**Proposition 3.3.** *The number of SPNE depends on the number of domestic private firms and that of foreign private firms. Specifically,*

- (1) *There are 6 SPNE when  $n=1$  and  $m=1$ , that is, when there are only one domestic private firm and one foreign private firm in the market. In equilibrium, the public firm chooses to produce in period 3, the domestic private firm chooses to produce in period 1 or 2, and the foreign private firm chooses to produce in period 1, 2 or 3.*
- (2) *There are 3 SPNE when  $n=1$  and  $m \geq 2$ , that is, when there are only one domestic private firm and at least two foreign private firms in the market. In any equilibrium, all foreign private firms produce in period 1. The domestic private firm and the public firm produce in period 1 and 2, or period 1 and 3, or period 2 and 3, respectively.*
- (3) *There are 3 SPNE when  $n \geq 2$  and  $m=1$ , that is, when there are at least two domestic private firms and only one foreign private firm in the market. All domestic private firms produce in period 1, the public firm produces in period 3, and the foreign private firm produces in period 1, 2 or 3.*

---

<sup>6</sup> Here we want to point out that proposition 3 in Matsumura (2003) might be incorrect. Besides the equilibrium identified by proposition 3, there is another possible equilibrium in which the foreign private firm acts as a leader while the public firm acts as a follower. If we use the same framework as we use in this paper, we can find this equilibrium. However, this equilibrium is dominated by the one identified by proposition 3 in Matsumura (2003).



(4) *There are 2 SPNE when  $n \geq 2$  and  $m \geq 2$ , that is, when there are at least two domestic private firms and at least two foreign private firms in the market. All domestic and foreign private firms produce in period 1, and the public firm produces in period 2 or 3.*

**Proof.** We prove this proposition by checking that no firm has incentive to deviate in each SPNE of each case.

(1) When  $n = 1$  and  $m = 1$ : Firstly, note that the public firm chooses to produce in period 3 when  $m = 1$  (which follows from Claim). Secondly, the domestic private firm has no incentive to deviate since it is a leader of the public firm and cannot increase its output because there is only one domestic private firm. Thirdly, the first order condition of the public firm's domestic social welfare maximization problem is  $\frac{\partial SS}{\partial q_0} = a - (q_0 + q_1^d) - c = 0$ , that is,  $p = c - q_1^f$ . If the foreign private firm produces in

period 1 or 2, then its profit-maximization output is  $q_1^f = \frac{c}{2}$ . Meanwhile, when the

foreign private firm produces in period 3, the first order condition of its profit maximizing problem is  $\frac{\partial SS}{\partial q_1^f} = a - (q_0 + q_1^d) - 2q_1^f = 0$  and its profit-maximizing output

is the same. Hence, the foreign private firm does not care about in which period to produce.

(2) When  $n = 1$  and  $m \geq 2$ : Firstly, the public firm has no incentive to deviate. Secondly, the domestic private firm has no incentive to deviate for the same reason as in the first case. Thirdly, the first order condition of the public firm's domestic social welfare

maximization problem is  $\frac{\partial SS}{\partial q_0} = a - (q_0 + q_1^d) - c = 0$ , that is,  $p = c - \sum_{j=1}^m q_j^f$ . No foreign

private firm has incentive to deviate because its output decreases if it deviates.

- (3) When  $n \geq 2$  and  $m = 1$ : Firstly, the public firm has no incentive to deviate. Secondly, all domestic private firms have no incentive to deviate since they want to produce more in period 1 and cannot increase its output by deviating. Thirdly, the only foreign private firm has no incentive to deviate for the same reason as in the first case.
- (4) When  $n \geq 2$  and  $m \geq 2$ : Firstly, the public firm has no incentive to deviate. Secondly, all domestic and foreign private firms have no incentive to deviate since they want to produce more in period 1 and cannot increase its output by deviating. ■

#### 4. Main Results for more than three periods ( $T > 3$ )

**Proposition 4.1.** *If  $T > 3$ , there are the following SPNE:*

- (1) *When  $n = 1$  and  $m = 1$ , the public firm produces in the last period, the domestic private firm produces in any period except the last period, and the foreign private firm produces in any period;*
- (2) *When  $n = 1$  and  $m \geq 2$ , all foreign private firms produce in period 1, the domestic private firm produces in any period except the last period, and the public firm produces in any subsequent period after the period in which the domestic private firm produces;*
- (3) *When  $n \geq 2$  and  $m = 1$ , all domestic private firms produce in period 1 and the public firm produces in the last period, and the foreign private firm produces in any period;*

(4) *When  $n \geq 2$  and  $m \geq 2$ , all domestic and foreign private firms produce in period 1, and the public firm produces in any subsequent period.*

**Proof.** We need to note that a foreign private firm has incentive to deviate to be a follower of the public firm if  $m = 1$ . So the public firm must produce in the last period when  $m = 1$ . We also need to note that a domestic (foreign) private firm has incentive to produce in period 1 if there are at least two domestic (foreign) private firms. Therefore, all domestic (foreign) private firms must produce in period 1 if  $n \geq 2$  ( $m \geq 2$ ). ■

## 5. Concluding Remarks

In this paper, we investigate endogenous timing in a mixed oligopoly with one public firm,  $n (\geq 1)$  domestic private firms and  $m (\geq 1)$  foreign private firms by considering the observable delay game of Hamilton and Slutsky (1990) in the context of a quantity setting mixed oligopoly where firms first choose the timing of choosing their quantities before quantity choice. We find that in any equilibrium, the public firm chooses to be a follower of the domestic private firm and not to be a leader of all foreign private firms, and that the number of SPNE depends on the number of domestic private firms and that of foreign private firms.

The results in this paper question the exogenous timing in the research of a mixed oligopoly with both domestic and foreign private firms. If the firms could choose the timing of their quantity choices, the timing should be endogenous. The contribution of this paper is to extend Pal (1998) by introducing foreign private firms into a mixed oligopoly. Also a mixed oligopoly consisting of public firm, domestic and foreign private firms is more realistic.

One extension of this paper is to consider the efficiency differential between

domestic private firms and foreign private firms. Generally, foreign ones are more efficient. So we can assume foreign private firms' marginal costs are zero, while domestic ones' marginal costs are positive but less than public firms'. Most of the results are expected to hold in this extension provided that demand is sufficiently high. The public firm has incentive to be a follower of domestic private firms and has little incentive to be a leader of foreign private firms. Actually, we have considered such a setting when there are only one public firm, one domestic private firm and one foreign private firm and found that all the results hold true when demand is sufficiently high and the foreign private firms' cost advantage over the domestic ones is not very big.

## References

- De Fraja, G and F. Delbono, 1990, Game Theoretic Models of Mixed Oligopoly, *Journal of Economic Surveys* 4, 1-17.
- Fjell, K. and J. S. Heywood, 2002, Public Stackelberg Leadership in a Mixed Oligopoly with Foreign Firms, *Australian Economic Papers* 41, 267-81.
- Fjell, K. and J. S. Heywood, 2004, Mixed Oligopoly, Subsidization and the Order of Firm's Moves: the Relevance of Privatization, *Economics Letters* 83, 411-416.
- Hamilton, J. and S. Slutsky, 1990, Endogenous Timing in Duopoly Games: Stackelberg or Cournot Equilibria. *Games and Economic Behavior* 2, 29-46.
- Jacques, A., 2004, Endogenous Timing in a Mixed Oligopoly: a Forgotten Equilibrium, *Economics Letters* 83, 147-148.
- Matsumura, T., 2003, Stackelberg Mixed Duopoly with a Foreign Competitor, *Bulletin of Economic Research* 55, 275-287.

Matsushima, N. and T. Matsumura, 2003a, Mixed Duopoly with Product Differentiation: Sequential Choice of Location, *Australian Economic Papers* 42, 18-34.

Matsushima, N. and T. Matsumura, 2003b, Mixed Oligopoly and Spatial Agglomeration, *Canadian Journal of Economics* 36, 62-87.

Nett, L., 1993, Mixed Oligopoly with Homogenous Goods, *Annals of Public and Cooperative Economics* 64, 367–394.

Pal, D., 1998, Endogenous Timing in a Mixed Oligopoly, *Economics Letters*, 61, 181-185.