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2002/10

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Keele Economics Research Papers

KEELE UNIVERSITY

Keele ▪ September 2002

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ISSN 1352-8955

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Advantageous or Disadvantageous Semi-collusion

by

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Date September 2002

Abstract This paper compares profits and consumer surplus under non-cooperation and collusion in the product market when the firms have the option for R&D before production. We show that whether R&D investment would be higher under non-cooperation or product market collusion depends on the R&D productivity. If the market size is sufficiently small then firms are always better off under product market collusion. If the market size is moderate (relatively large) then the firms are better off under non-cooperation (semi-collusion) for sufficiently lower pre-innovation costs of production. We also show that in case of moderate (relatively large) market size, firms are better off under non-cooperation for relatively lower (higher) R&D productivity. However, we find that consumer welfare is always higher under non-cooperation in product market compared to collusion in product market.

Keywords Entry,Consumer surplus, Collusion, Profit, Uncertain R&D

J.E.L. Class L10, L13

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1 Introduction

The textbook view says that cooperation in the product market helps the firms to increase their profits compared to non-cooperation in the product market. But cooperation in the product market by firms reduces consumer welfare. However, this argument considers same market demand and cost structure of the firms and ignores the effects of other non-production activities such as R&D. Recently, this textbook view has been challenged by revisionists providing different conclusions on profit and consumer welfare for non-cooperation and collusion in the product market. For example, one may look at Matsui (1989), Sevy (1992), Mitchell (1993), Fershtman and Gandal (1994) and Brod and Shivakumar (1999).[1]

In this paper we re-examine this issue in a two-stage game of R&D competition where the success in R&D is uncertain. In particular, we consider a duopoly market where cost reduction from R&D is given but firms can increase their probability of success in R&D through their own investment in R&D. We assume that the firms produce homogeneous products in the product market. In this framework, we compare profits and consumer surplus for two types of production strategies: (i) where the firms choose their outputs like Cournot duopolists to maximize their own profits, and (ii) where the firms choose their outputs to maximize joint profit. Following the terminology used in the literature, we call the former regime as ‘non-cooperation’ and the latter regime as ‘semi-collusion’. Further, for simplicity, we focus on drastic innovation in our analysis. This implies that if only one of these firms is successful in R&D then only the successful firm produces in the product market like a monopolist.

This present paper is closely related to Fershtman and Gandal (1994) and Brod and Shivakumar (1999). In their paper Fershtman and Gandal (1994) considered a two-stage game of R&D and production in a homogeneous good duopoly market. They considered that, at the beginning, each firm would decide its R&D investment, which reduces the cost of production. Then, given the cost of production determined by R&D investment, each firm would take decision on production. In the product market the firms compete either like Cournot duopolists or they cooperate to maximize joint profit. They conclude that even if consumer welfare is lower when the firms cooperate in the product market, the possibility of R&D before production may generate lower optimal profit under product market cooperation compared to non-cooperation in the product market. Brod and Shivakumar (1999) extended the analysis of Fershtman and Gandal (1993) by incorporating horizontal product differentiation and the possibility of knowledge spillover under R&D. However, one common feature of these two papers is their R&D function. Both of these papers consider that success in R&D is certain and R&D investment reduces the cost of production.

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[1] In an oligopolistic market without R&D before production, Salant et al. (1983) showed that product market cooperation could reduce the profit of the firms making cooperation compared to product market competition if sufficiently large number of firms does not cooperate in the market.

In this paper we assume that success in R&D is uncertain and show the impacts of market size[2], R&D productivity and pre-innovation costs of production on profits and consumer welfare under non-cooperation and collusion in the product market. Further, like Fershtman and Gandal (1994) we abstract our analysis from knowledge spillover in R&D and product differentiation. This will help us to focus on the role played by the uncertainty in R&D. Hence our results could be compared with the situation of homogenous product with no knowledge spillover of Brod and Shivakumar (1999).

We show that when the success in R&D is uncertain then whether optimal R&D investment would be more under non-cooperative regime or cooperative regime is ambiguous and depends on the probability of success. If the productivity in R&D is sufficiently low then optimal R&D investment will be more under non-cooperation in the product market compared to product market collusion. But for sufficiently large R&D productivity, firms would invest more in R&D under cooperative regime compared to non-cooperative regime. This finding contradicts the findings of Fershtman and Gandal (1994) and Brod and Shivakumar (1999) where the firms always invested more under cooperative regime compared to non-cooperative regime.

If the R&D productivity of the competitor is sufficiently low then it increases a firm's possibility of being the sole innovator. The gain from being a sole innovator is lower under product market collusion compared to non-cooperation in the product market. So, each firm has lower incentive for R&D investment under semi-collusion compared to the non-cooperation, when R&D productivity is sufficiently low. But when the R&D productivity of the competitor is sufficiently large then each firm expects the market structure to be duopoly with new innovation. Higher expected payoff in a duopoly market under cooperation compared to non-cooperation induces these firms to invest more in R&D under cooperation compared to non-cooperation.

Comparison of profits under semi-collusion and non-cooperation shows the importance of market size, pre-innovation cost of production and R&D productivity. Assume that the pre-innovation cost of production is such that the firms earn positive profit only if they are successful in R&D. In this situation we find that the firms would always be benefited from cooperation in the product market compared to non-cooperation. In this situation, R&D invests are always higher under semi-collusion. Therefore, here the firms are better off from product market collusion ex-post R&D and also this collusion in the product market increases the possibility of successful innovation. Both factors help to increase the profits of these firms under cooperative regime compared to non-cooperative regime.

Now consider that the pre-innovation costs are such that the firms could earn positive profit when both of them are unsuccessful in R&D. In this situation, we show that market size, pre-innovation cost of production and R&D productivity have important implications for our analysis. If the market size is sufficiently small then the firms are always better off under semi-collusion. If the market size is moderate (relatively large) and the pre-innovation cost is

[2] In our analysis the demand parameter will be the proxy for the market size.

relatively small then the firms are better off under non-cooperation (semi-collusion). Further, we show that, in case of moderate market size, there are pre-innovation costs of production where firms are better off under non-cooperation for relatively lower R&D productivity. In contrary, for relatively large market size, we find that there are pre-innovation costs of production where the firms are better off under non-cooperation for relatively higher R&D productivity. Thus, we show that whether the firms would be better off under non-cooperation or semi-collusion depends on the market size, pre-innovation costs of production and R&D productivity. These findings provide new insights on this debate and also contradict previous results (see, e.g., Fershtman and Gandal, 1994).

However, we find that consumer welfare is always higher under non-cooperation. Even if the chance of getting the innovation can be higher under semi-collusion, market concentration is higher under semi-collusion. The negative impact of the latter effect outweighs the benefit of the former effect on the consumer welfare. As a result, consumer welfare is more under non-cooperative regime compared to cooperative regime. While this finding is similar to Fershtman and Gandal (1994), this contradicts the conclusion of Brod and Shivakumar (1999).

Rest of the paper is organized as follows. We describe the model and prove the results in the next section. Section 3 concludes the paper.

2 Model

Consider an economy with two firms, firm 1 and 2, producing homogeneous products. Assume that the firms have similar technologies at the beginning and each of them faces constant average cost of production $\bar{c}$.

We consider a two-stage game. In stage 1, both firms invest in R&D and try to invent a technology corresponding to constant average cost of production c. However, the R&D process is uncertain and firm i, $i = 1,2$, can succeed with an unconditional probability p_i where the probability of success depends on firm i's R&D investment x_i. We consider that $p_i'(x_i) > 0$, $p_i''(x_i) < 0$, $p_i'(0) = \infty$ and $p_i'(\infty) = 0$ for $i = 1,2$. Since our purpose is to focus on the effects of product market competition, we will assume that both firms face same probability function, so that the results are not influenced by the asymmetry in probability functions. Hence, we do our analysis under the assumption that $p_i(x) = p_j(x) = p(x)$. Thus, our model of R&D competition is similar to Choi (1993). Further, for simplicity, we assume that there are no other costs of doing R&D. Then, in stage 2, these firms produce their outputs. We consider two different scenarios for the product market. First, we will consider the situation where these firms choose their outputs non-cooperatively like Cournot duopolists to maximize their own profit. Secondly, we will consider a situation where these firms choose their outputs to maximize joint profit in the product market, given their own cost of production. We will solve the game through backward induction. For simplicity, we will further assume that the new technology is drastic in nature.

Assume that the inverse market demand function is given by

$$P = a - q_1 - q_2, \qquad (1)$$

where q_1 and q_2 are the outputs of firm 1 and firm 2. If $a \leq \bar{c}$ then the optimal output and profit of these firms are 0 even if neither of them is successful in R&D. In our analysis we will assume that $a > \bar{c}$ if not specified otherwise. However, we will always assume that $a > c$.

2.1 *Non-cooperation in the product market*

In this subsection we consider that these firms choose their output like Cournot duopolists. We call this situation as non-cooperative regime. Due to symmetry of these firms, without loss of generality, we consider the problem of firm 1 only. Therefore, given the optimal profit in the product market, expected profit of firm 1 in the R&D stage is

$$p(x_1)p(x_2)\pi_1(c,c) + p(x_1)(1 - p(x_2)\pi_1(c) + (1 - p(x_1))(1 - p(x_2))\pi_1(\bar{c},\bar{c}) - x_1, \qquad (2)$$

where the profit levels of firm 1 are denoted by $\pi_1(c)$, $\pi_1(c,c)$ and $\pi_1(\bar{c},\bar{c})$ respectively for the situations where only firm 1 is successful in R&D, where both firms are successful in R&D and where neither firm is successful in R&D. Since we consider drastic innovation, the term $\pi_1(c)$ shows the monopoly profit of firm 1. The first (second) argument in the profit functions is showing the constant average cost of production for firm 1 (firm 2). We have similar profit functions for firm 2 also.

Therefore, firm 1 chooses its optimal R&D investment by maximizing the expression (2). So, the optimal R&D investment of firm 1 satisfies the following expression:

$$p'(x_1)p(x_2)\pi_1(c,c) + p'(x_1)(1 - p(x_2))\pi_1(c) - p_1'(x_1)(1 - p(x_2)\pi_1(\bar{c},\bar{c}) = 1. \qquad (3)$$

The second order condition for maximization is satisfied. The expression (3) gives the reaction function for firm 1's R&D investment, i.e., it shows the optimal R&D investment of firm 1 for any given R&D investment of firm 2. We can find a similar reaction function for the R&D investment of firm 2. Straightforward calculation will show that these reaction functions are negatively sloped. We solve these two reaction functions to get the optimal R&D investment of these firms. Suppose, x_{1nc}^* and x_{2nc}^* are the optimal R&D investments of these firms. Throughout the paper we assume that the probability functions are such that we have unique equilibrium in R&D investments. Further, the symmetry of the problem implies that the optimal R&D investments are same, i.e., $x_{1nc}^* = x_{2nc}^*$.

2.2 *Cooperation in the product market*

In this subsection we consider that in the product market these firms choose their outputs in a way that maximizes joint profit in the product market, given the cost of production of these firms. We call this situation as cooperative regime. So, the assumption of drastic R&D implies that in case of an unilateral success in R&D, the successful firm gets the same monopoly profit and the unsuccessful firm gets nothing even if there is cooperation in the product market. But for other cases, i.e., for both success and no success, these firms produce in a way so that the joint profit is the monopoly profit corresponding to the constant average cost of production and each firm gets the half of this monopoly profit.

Therefore, in case of cooperation in the product market, the expected optimal profit of firm 1 in the R&D stage is

$$p(x_1)\,p(x_2)\frac{p_1(c)}{2}+p(x_1)(1-p(x_2)p_1(c)+(1-p(x_1))(1-p(x_2))\frac{p_1(\bar{c})}{2}-x_1\,. \tag{4}$$

So, firm 1 maximizes expression (4) for its optimal R&D investment. Therefore, optimal R&D investment of firm 1 will satisfy the following expression when these firms produce their outputs cooperatively:

$$p'(x_1)\,p(x_2)\frac{p_1(c)}{2}+p'(x_1)(1-p(x_2))p_1(c)-p_1'(x_1)(1-p(x_2)\frac{p_1(\bar{c})}{2}=1\,. \tag{5}$$

The second order condition for maximization is satisfied. The expression (5) gives the optimal R&D investment of firm 1 for any given R&D investment of firm 2. We get the similar reaction function for firm 2 also. Straightforward calculation shows that these reaction functions are negatively sloped. Solving these two reaction functions, we can get the optimal R&D investments of these firms when these firms produce cooperatively. Suppose, x_{1c}^* and x_{2c}^* show the optimal R&D investments of firm 1 and firm 2 respectively under this cooperative regime. Symmetry of the solution implies that $x_{1c}^* = x_{2c}^*$.

2.3 *Comparison of R&D investments under non-cooperation and cooperation*

In this subsection we compare the optimal R&D investments under non-cooperative and cooperative regime. Consider the optimal R&D investments under non-cooperative regime. From the expressions (3) and (5), it is easy to check that, given the value of x_{1nc}^* and x_{2nc}^*, the left hand side of (3) is greater than or less than the left hand side of (5) provided

$$\left[\frac{p_1(\bar{c})}{2}-p_1(\bar{c},\bar{c})\right]\underset{<}{\geq} p_2(x_{2nc}^*)\left[\frac{p_1(c)}{2}+\frac{p_1(\bar{c})}{2}-p_1(\bar{c},\bar{c})-p_1(c,c)\right] \tag{6}$$

or, $p_2(x^*_{2nc}) \stackrel{\leq}{>} \frac{[\frac{p_1(\bar{c})}{2} - p_1(\bar{c},\bar{c})]}{[\frac{p_1(c)}{2} + \frac{p_1(\bar{c})}{2} - p_1(c,c) - p_1(\bar{c},\bar{c})]} = \bar{p}$. (6')

If the equilibrium probability of firm 2 under non-cooperation is less than $\bar{p}$ then the left hand side of (3) is greater than the left hand side of (5). Therefore, in this situation, given the R&D investment of firm 2 as x^*_{2nc}, the optimal R&D investment of firm 1 under cooperative regime will be lower than x^*_{1nc}. Since, the firms are symmetric, we can have a condition similar to (6') for firm 2 also. Thus, we can say that if the equilibrium probability under non-cooperative regime is less than $\bar{p}$ then the optimal R&D investments of these firms are lower under cooperative regime compared to non-cooperative regime. Since, the probability of success is increasing in the R&D investment, we can conclude that equilibrium probability of success is lower under cooperative regime compared to non-cooperative regime when optimal probability of success is lower than a critical level.

Similarly, we can find that if the equilibrium probability is greater than $\bar{p}$ then the equilibrium R&D investments and equilibrium probability of success is higher under cooperative regime compared to non-cooperative regime.

The following proposition summarizes the above discussion.

Proposition 1: *If the equilibrium probability is less (greater) than* $\bar{p}$ *then the equilibrium R&D investments are lower (greater) under cooperative regime compared to non-cooperative regime.*

Consider that each firm knows that the probability of success is sufficiently low for the competitor. Hence, in this situation, each firm expects a higher chance of being the sole innovator. Since, the gain from sole innovator is lower under cooperative regime, each firm has lower incentive for R&D under cooperative regime. So, in this situation, equilibrium R&D investments of these firms are lower under cooperative regime compared to non-cooperative regime.

Next, consider the situation for sufficiently higher probability of success. Here, each firm expects that the competitor will be successful in R&D. So, each firm expects that the chance of being the sole innovator is very small and it is more likely that both firms will get the innovation. Since, the gain from cooperative regime is higher when both firms succeed in R&D, these firms have higher incentive for R&D under cooperative regime. Therefore, when probability of success is sufficiently high, equilibrium R&D investments are more under cooperative regime compared to non-cooperative regime.

The above proposition contradicts the results of Fershtman and Gandal (1994) and Brod and Shivakumar (1999). While those papers show that the equilibrium R&D investments of the firms are always higher under the cooperative regime, we show that this result does not hold if the probability of success in R&D is sufficiently low.

2.3.1 An example

In this subsection we provide an example for the Proposition 1 with a specific probability function. We consider that the each firm faces the probability function $p(x_i) = mx_i^{\frac{1}{2}}$, where $i = 1,2$. Given this probability function, the equilibrium investment and probability of success under non-cooperative and cooperative regime are given by

$$x_{1nc}^{*} = \left[\frac{m[p_1(c) - p_1(\bar{c},\bar{c})]}{2 + m^2[p_1(c) - p_1(\bar{c},\bar{c}) - p_1(c,c)]}\right]^2, \qquad x_{1c}^{*} = \left[\frac{m[p_1(c) - \frac{p_1(\bar{c})}{2}]}{2 + m^2[\frac{p_1(c)}{2} - \frac{p_1(\bar{c})}{2}]}\right]^2, \quad (7)$$

and

$$p_{1nc}^{*} = \left[\frac{m^2[p_1(c) - p_1(\bar{c},\bar{c})]}{2 + m^2[p_1(c) - p_1(\bar{c},\bar{c}) - p_1(c,c)]}\right], \qquad p_{1c}^{*} = \left[\frac{m^2[p_1(c) - \frac{p_1(\bar{c})}{2}]}{2 + m^2[\frac{p_1(c)}{2} - \frac{p_1(\bar{c})}{2}]}\right]. \quad (8)$$

It is easy to check that the second order condition for profit maximization with respect to the R&D investment is always satisfied at the optimal level. Straightforward calculation shows that these equilibrium probabilities are greater than or less than the critical value $\bar{p}$ provided

$$m^2\left[\frac{p_1^2(c) - p_1(c)p_1(\bar{c},\bar{c}) + p_1(\bar{c})p_1(c,c)}{2} - p_1(c)p_1(c,c)\right] \underset{<}{\geq} 2\left[\frac{p_1(\bar{c})}{2} - p_1(\bar{c},\bar{c})\right]. \quad (9)$$

It is clear from (9) that the chance of having the equilibrium probabilities higher than $\bar{p}$ increases as the profit from pre-innovation technology reduces. At the extreme situation, if the profit from pre-innovation technology is 0 then the equilibrium probabilities are higher than $\bar{p}$. On the other hand, lower value of m increases the possibility of lower equilibrium probabilities compared to $\bar{p}$.

With the demand function specified in (1), we find that the condition (9) reduces to

$$m \underset{<}{\geq} \frac{2\sqrt{2}(a - \bar{c})}{(a - c)^2}. \quad (10)$$

The same conclusions could be obtained from the direct comparison of the probabilities given in (8).

2.4 Comparison of profits under non-cooperation and semi-collusion

In this subsection we will compare expected profits under these two regimes. In particular, we will show the importance of market size, pre-innovation cost of production and R&D productivity on the expected profits.

The possibility of product market collusion helps the firms to increase profits by reducing industry outputs. Thus, product market collusion provides the benefit to the firms. However, as we have seen in the previous section, firms invest more in R&D when R&D productivity is sufficiently low. Therefore, if R&D productivity is sufficiently low then the probability of getting the innovation is higher under non-cooperation compared to semi-collusion. If the benefit from higher probability of success under non-cooperation is sufficiently large to outweigh the benefit of product market collusion then the firms are better off under non-cooperation compared to semi-collusion.

To get a clear exposition of results, in the following analysis we will consider that each firm faces the probability function $p(x_i) = mx_i^{\frac{1}{2}}$, where $i = 1,2$.

From the analysis of the previous section, it is clear that if the firms do not earn positive profit without the success in R&D, i.e., $p_1(\bar{c}) = p_1(\bar{c},\bar{c}) = 0$, the probability of success in R&D is always higher under semi-collusion compared to non-cooperation. Hence, the profits of these firms in the product market and the probability of success in R&D are higher under semi-collusion compared to non-cooperation. Therefore, we can have the following proposition when $p_1(\bar{c}) = p_1(\bar{c},\bar{c}) = 0$.

Proposition 2: *Assume that* $a \le \bar{c}$ *and* $p(x_i) = mx_i^{\frac{1}{2}}$, *where* $i = 1,2$. *Expected profit of these firms will be higher under semi-collusion compared to non-cooperation.*

Proof: Expected profit of firm 1 (and similarly for firm 2) will be higher under semi-collusion compared to non-cooperation provided

$$(p_{1c}^*)^2 \frac{p_1(c)}{2} + p_{1c}^*(1-p_{1c}^*)p_1(c) - \frac{(p_{1c}^*)^2}{m^2} \gtrless (p_{1nc}^*)^2 p_1(c,c) + p_{1nc}^*(1-p_{1nc}^*)p_1(c) - \frac{(p_{1nc}^*)^2}{m^2}, \tag{11}$$

since the optimal R&D investments and the equilibrium probabilities are same and $x_{1m}^* = \frac{(p_{1m}^*)^2}{m^2}$, where $m = c, nc$.

Given the assumptions of this proposition and the demand function specified in (1), the condition (11) reduces to

$$\left[\frac{72+5m^2(a-c)^2}{32+2m^2(a-c)^2}\right]^2 \begin{matrix}\geq\\<\end{matrix} \frac{324}{64}. \qquad (12)$$

Left hand side (LHS) of (12) is continuous and increasing in $m(a-c)$ and is equal to the right hand side (RHS) of (12) for $m(a-c)=0$. Hence, this implies that $\forall m(a-c)>0$ the LHS of (12) is greater than the RHS of (12) and expected profit of these firms are higher under semi-collusion compared to non-cooperation. Q.E.D.

The above proposition has considered that the pre-innovation cost of production is sufficiently higher such that the firms do not earn positive profit with successful R&D. This finding supports the textbook view and contradicts revisionists view (e.g., Fershtman and Gandal, 1994 and Brod and Shivakumar, 1999). Now, we will relax this assumption and will consider the situation where each firm earns positive profit when both of them are unsuccessful in R&D. This situation implies that now the probability of success in R&D could be higher under non-cooperation compared to semi-collusion and might generate higher expected profit under non-cooperation compared to semi-collusion.

Given the demand function specified in (1) and the probability function $p(x_i)=mx_i^{\frac{1}{2}}$, where $i=1,2$, expected profits of firm 1 (and also for firm 2 due to symmetry) under non-cooperation and semi-collusion, i.e., the optimal values of (2) and (4), are given by

$$p^*_{1nc}=\frac{\left[\begin{array}{l} m^2(9(a-c)^2-4(a-\bar{c})^2)[4m^2(a-c)^2(9(a-c)^2-4(a-\bar{c})^2) \\ +9(a-c)^2(72-4m^2(a-c)^2)-36(9(a-c)^2-4(a-\bar{c})^2)] \\ +4(a-\bar{c})^2(72-4m^2(a-c)^2)^2 \end{array}\right]}{36[72+m^2(5(a-c)^2-4(a-\bar{c})^2)]^2} \qquad (13)$$

and

$$p^*_{1c}=\frac{\left[\begin{array}{l} m^2(2(a-c)^2-(a-\bar{c})^2)[m^2(a-c)^2(2(a-c)^2-(a-\bar{c})^2) \\ +2(a-c)^2(16-m^2(a-c)^2)-8(2(a-c)^2-(a-\bar{c})^2)] \\ +(a-\bar{c})^2(16-m^2(a-c)^2)^2 \end{array}\right]}{8[16+m^2((a-c)^2-(a-\bar{c})^2)]^2}. \qquad (14)$$

It is difficult to compare the optimal profits under non-cooperation and semi-collusion from the expressions (13) and (14). In the following analysis we will take the help of numerical examples to compare the optimal profits under these two types of product market characterizations.

In Figures 1, 2(a, b) and 3(a, b) we plot the difference in expected profits under semi-collusion and non-cooperation for firm 1, i.e., $(p^*_{1c} - p^*_{1nc})$.[3] In all these figures, we assume that the post-innovation cost of production, if successful, is 0, i.e., $c = 0$. In these figures we plot the difference $(p^*_{1c} - p^*_{1nc})$ for different market size with all feasible values of m and $\bar{c}$, given the particular market size. The restriction on the minimum permissible value of $\bar{c}$ is given by the condition for drastic innovation[4] and the maximum value of m is given by the condition that the probability of success under semi-collusion does exceed 1[5].

In Figure 1 we consider that $a = 1$ and plot the difference $(p^*_{1c} - p^*_{1nc})$ for different combinations of m and $\bar{c}$, where $m \in [0,4]$ and $\bar{c} \in [.5,1]$. We find that here the difference is positive for all combinations of m and $\bar{c}$. Therefore, in this situation, firms would always be better off under semi-collusion.

In Figure 2a we consider that $a = 5$ and plot the difference $(p^*_{1c} - p^*_{1nc})$ for different combinations of m and $\bar{c}$, where $m \in [0,.8]$ and $\bar{c} \in [2.5,5]$. We find that here the difference is negative for all values of m when $\bar{c}$ is sufficiently small, e.g., for $\bar{c} < 2.8$. We also find that there are values of $\bar{c}$ for which the difference $(p^*_{1c} - p^*_{1nc})$ is negative for sufficiently lower values of m. For example, it can be found that if $\bar{c} = 3.5$ then the difference $(p^*_{1c} - p^*_{1nc})$ is negative for $m < .66$ (see Figure 2b). Thus, we find that, in case of moderate market size, firms could be better off under non-cooperation when the pre-innovation costs are sufficiently small or R&D productivities are sufficiently small.

If the market size is not sufficiently small (as in Figure 2a) then these firms may be better off under non-cooperation compared to semi-collusion when the pre-innovation cost of production is sufficiently small. Given the R&D productivity and market size, R&D investment is likely to be higher under non-cooperation compared to semi-collusion if the pre-innovation cost is not sufficiently large. Further, the difference in R&D investments between non-cooperation and semi-collusion tends to increase with higher market size. Therefore, if the market size is not very small (as in Figure 2a) and the pre-innovation cost is not sufficiently large then the probability of getting the innovation is sufficiently higher under non-cooperation compared to semi-collusion. This higher probability of getting the innovation outweighs the benefit of product market collusion and makes the firms better off under non-cooperation compared to semi-collusion.

If the R&D productivity is sufficiently small then we know that the probability of success in R&D is higher under non-cooperation compared to semi-collusion (see subsection

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[3] We use 'The Mathematica 4' for all the figures of this subsection and next subsection.

[4] Given the demand function specified in (1), the condition for drastic innovation implies that $\bar{c} \geq \frac{a+c}{2}$.

[5] Given the demand function specified in (1), the probability of success in R&D under semi-collusion will be less than 1 for $m < \frac{4}{(a-c)}$.

2.3.1). Therefore, the possibility of getting the innovation under non-cooperation is higher with relatively lower R&D productivity. This benefit from higher probability of success in R&D outweighs the benefit from product market collusion when the R&D productivity is sufficiently low and makes non-cooperation more profitable compared to semi-collusion for a given pre-innovation cost of production. But, when the R&D productivity is sufficiently high then the probability of success in R&D is higher under semi-collusion. Therefore, in this situation, both the probability of getting the innovation and ex-post R&D profit are higher under semi-collusion and here the firms are better off under semi-collusion compared to non-cooperation (see Figure 2b).

In their paper Fershtman and Gandal (1994) showed that the firms were better off under non-cooperation when cost of R&D investment was sufficiently small. In terms of our framework, one may reinterpret this result as a situation where the firms are better off under non-cooperation compared to semi-collusion when the R&D productivity is sufficiently high. In contrary to them, we show that the firms are better off under non-cooperation compared to semi-collusion for sufficiently lower R&D productivity (see Figure 2b).

Let us now consider Figure 3(a, b) where $a = 50$ and plot the difference $(p^*_{1c} - p^*_{1nc})$ for different combinations of m and $\bar{c}$, where $m \in [0,.08]$ and $\bar{c} \in [25,50]$. We find that here the difference is positive for all values of m when $\bar{c}$ is sufficiently small, e.g., for $\bar{c} < 40$. Also we find that there are values of $\bar{c}$ for which the difference $(p^*_{1c} - p^*_{1nc})$ is negative for sufficiently higher values of m. For example, it can be found that if $\bar{c} = 42$ then the difference $(p^*_{1c} - p^*_{1nc})$ is negative for $m > .069$ (see Figure 3b). Hence, contrary to the situation for moderate market size, here we show that firms could be better off under non-cooperation for higher R&D productivity. We also find from Figure 3a that, for relatively higher R&D productivity, firms are better off under semi-collusion for relatively lower and higher values of pre-innovation cost of production while they are better off under non-cooperation for moderate pre-innovation costs of production. Thus, these figures show the importance of the market size on the profitability under semi-collusion and non-cooperation.

If market size and R&D productivity are sufficiently large then the firms may invest sufficiently large amount under semi-collusion compared to non-cooperation. Over-investment in R&D under semi-collusion may be large enough to outweigh the positive effects of product market cooperation and higher probability of success in R&D under semi-collusion compared to non-cooperation. Therefore, for relatively large market size, the firms are better off under non-cooperation when R&D productivity is sufficiently high (see Figure 3b). This finding is similar to Fershtman and Gandal (1994), where they have shown that over-investment under semi-collusion may be responsible for higher profit under non-cooperation compared to semi-collusion.

In the following proposition we summarize the above discussions.

Proposition 3: *Assume that* $c = 0$, $a > \bar{c}$ *and* $p(x_i) = m x_i^{\frac{1}{2}}$, *where* $i = 1,2$.

(i) If the market is sufficiently small, i.e., $a=1$, then the firms are always better off under semi-collusion compared to non-cooperation.
(ii) Suppose the market size is not very small, i.e., $a=5$.
(a) Firms earn higher payoff under non-cooperation when the pre-innovation costs are sufficiently small.
(b) There are pre-innovation costs for which the firms are better of under non-cooperation (semi-collusion) if the R&D productivity is sufficiently small (high).
(iii) Suppose the market is relatively large, i.e., $a=50$.
(a) Firms may earn higher payoff under non-cooperation when the pre-innovation costs are moderate.
(b) There are pre-innovation costs for which the firms are better of under non-cooperation (semi-collusion) if the R&D productivity is sufficiently high (small).

From the results of the above proposition, it is clear that the relative profitability under semi-collusion and non-cooperation is non-monotonic with respect to the market size. If market size is sufficiently small or sufficiently large then the firms may be better off under semi-collusion compared to non-cooperation (see Propositions 2(i) and 2(iii(a))). For moderate market size the firms are better off under non-cooperation compared to semi-collusion (see Proposition 2(ii(a))). We found these results under the assumption that the firms produce homogeneous products. Thus, we show that firms may have higher payoff under semi-collusion compared to non-cooperation for minimal product differentiation when the market size is sufficiently small or sufficiently large. This result contradicts the finding of Brod and Shivakumar (1999) where the firms earn lower (higher) payoff under semi-collusion for sufficiently lower (higher) degree of product differentiation in absence of knowledge spillover.

2.5 Comparing consumer surplus under non-cooperation and semi-collusion

In contrary to profits, industry outputs are lower under semi-collusion compared to non-cooperation for a given cost of production. However, probability of getting the innovation may be higher under semi-collusion. In this subsection, we will examine the effect of this trade-off on expected consumer surplus.

The discussion in subsection 2.3 shows that the probability of success in R&D is always higher under semi-collusion when neither firm earns positive profit without successful R&D i.e., $p_1(\bar{c}) = p_1(\bar{c},\bar{c}) = 0$. Therefore, in this situation, probability of success is always higher under semi-collusion compared to non-cooperation. We have the following proposition for this situation.

Proposition 4: *Assume that $a \le \bar{c}$ and $p(x_i) = mx_i^{\frac{1}{2}}$, where $i = 1,2$. Expected consumer surplus will always higher under non-cooperation compared to semi-collusion.*

Proof: Given the assumptions of this proposition and due to symmetry of these firms, we find that that expected consumer surplus is higher under non-cooperation compared to semi-collusion provided

$$\frac{(72+5m^2(a-c)^2)}{(16+m^2(a-c)^2))\sqrt{162+9m^2(a-c)^2}} \begin{matrix}\geq \\ <\end{matrix} \frac{1}{2\sqrt{2}}. \tag{15}$$

LHS of (15) is equal to the RHS for $m(a-c)=0$ and LHS of (15) is continuous and decreasing in $m(a-c)$. This proves the result. Q.E.D.

The above proposition considers a situation where probability of success in R&D is always higher under semi-collusion. However, the negative impact of product market collusion on consumer surplus outweighs the positive impact of product market collusion on the probability of success in R&D. As a result, consumer surplus is always higher under non-cooperation compared to semi-collusion.

If we consider a situation where the firms earn positive profit without successful R&D, we find that expected consumer surplus under non-cooperation and semi-collusion are given by

$$CS_{nc} = \frac{\begin{bmatrix} 8m^4(a-c)^2(9(a-c)^2-4(a-\bar{c})^2)^2 \\ +9m^2(a-c)^2(9(a-c)^2-4(a-\bar{c})^2)(72-4m^2(a-c)^2) \\ +8(a-\bar{c})^2(72-4m^2(a-c)^2)^2 \end{bmatrix}}{36[72+m^2(5(a-c)^2-4(a-\bar{c})^2)]^2} \tag{16}$$

and

$$CS_c = \frac{\begin{bmatrix} m^4(a-c)^2(2(a-c)^2-(a-\bar{c})^2)^2 \\ +2m^2(a-c)^2(2(a-c)^2-(a-\bar{c})^2)(16-m^2(a-c)^2) \\ +(a-\bar{c})^2(16-m^2(a-c)^2)^2 \end{bmatrix}}{8[16+m^2((a-c)^2-(a-\bar{c})^2)]^2}. \tag{17}$$

Proposition 4 considered a situation where the firms do not earn positive profit without success in R&D. It is easy to check that probability of success in R&D under semi-collusion and non-cooperation increases with lower $\bar{c}$. Also we found that $(p_{1c}^* - p_{1nc}^*)$ increases with lower $\bar{c}$. Therefore, the positive effect of higher probability of success in R&D under semi-collusion increases with higher profit under unsuccessful R&D by both firms. However, in the next proposition we show that still the negative impact of product market collusion outweighs the positive impact of higher probability of success in R&D and consumer surplus is higher under non-cooperation compared to semi-collusion. Generally speaking, it is difficult to compare the expressions (16) and (17). Like Proposition 3 we provide numerical examples for three different values of market size, which has been considered in that proposition.

Proposition 5: *Assume that* $c = 0$, $a > \bar{c}$ *and* $p(x_i) = mx_i^{\frac{1}{2}}$, *where* $i = 1,2$.
(i) Consider that $a = 1$, $\bar{c} \in [.5,1]$ *and* $m \in [0,4]$.
(ii) Consider that $a = 5$, $\bar{c} \in [2.5,5]$ *and* $m \in [0, \frac{4}{5}]$.
(iii) Consider that $a = 50$, $\bar{c} \in [25,50]$ *and* $m \in [0, \frac{4}{50}]$.
In all these situations, expected consumer surplus is always higher under non-cooperation compared to semi-collusion.

Proof: See Figures 4 (a, b, c). In these figures, we plot the difference in expected consumer surplus under semi-collusion and non-cooperation, i.e., $(CS_c - CS_{nc})$. Like the previous section, we define the range for m and $\bar{c}$ in the similar fashion. Q.E.D.

In the previous subsection we found that whether the profits of these firms will be higher under semi-collusion or non-cooperation depends on the market size, R&D productivity and pre-innovation cost of production. But, in this subsection we find that consumer welfare is always higher under non-cooperation compared to semi-collusion. While the analysis on profits contradicts the results of Fershtman and Gandal (1994), the analysis on consumer surplus shows that consumer surplus is higher under non-cooperation compared to semi-collusion. However, our findings contradict the result of Brod and Shivakumar (1999) where consumer welfare is higher under semi-collusion compared to non-cooperation for homogenous products and no knowledge spillover.

3 Conclusion

The textbook view says that while the firms benefit from product market collusion, consumer welfare is higher under non-cooperation in the product market. However, this view does not take into account of the non-production activities of the firms such as R&D. Researchers have shown that if the firms have the option for R&D before production then textbook view may not true. Previous works showed that whether producers and consumers would be better of under product market cooperation depends on the cost of R&D, product differentiation and the extent of knowledge spillover under R&D. However, while considering R&D, the previous works assumed that success in R&D is certain.

In this paper we consider a model of R&D competition and production where the success in R&D is uncertain. We show that this possibility of failure in R&D has important implications on R&D investments, profits and consumer welfare. Unlike previous works, we show that the firms may invest more in R&D under product market competition compared to product market cooperation. Whether R&D investment would be higher under product market cooperation or product market competition depends on the R&D productivity.

We find that if the firms earn positive profit only if they are successful in R&D, then these firms would always be benefited from cooperation in the product market compared to non-cooperation. But, if the firms earn positive profit when both of them are unsuccessful in R&D, then we show that market size, pre-innovation cost and R&D productivity have important implications for our analysis. Firms could be better off under non-cooperation if the market size is not sufficiently small. If the market size is moderate (relatively large) and the pre-innovation cost is relatively small then the firms are better off under non-cooperation (semi-collusion). We also find that, in case of moderate market size, there are pre-innovation costs of production where firms are better off under non-cooperation for relatively lower R&D productivity. Contrary to this, if the market is relatively large then there are pre-innovation costs of production where the firms can be better off under non-cooperation for relatively higher R&D productivity. However, we find that the consumer welfare is always higher under non-cooperation.

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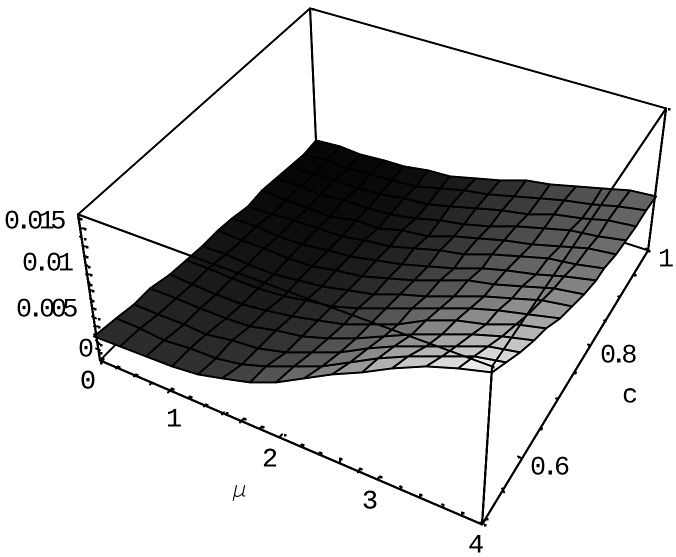

Figure 1: Difference in expected profits under semi-collusion and non-cooperation for $c = 0$, $a = 1$, $m \in [0,4]$ and $\bar{c} \in [.5,1]$.

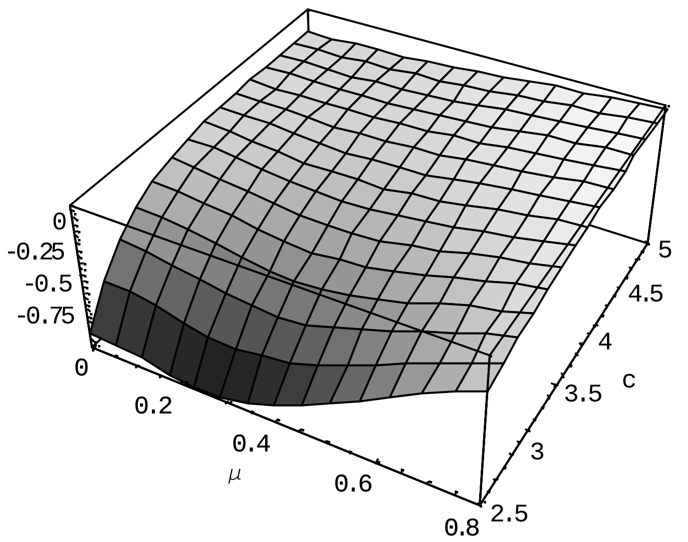

Figure 2a: Difference in expected profits under semi-collusion and non-cooperation for $c = 0$, $a = 5$, $m \in [0,.8]$ and $\bar{c} \in [2.5,5]$.

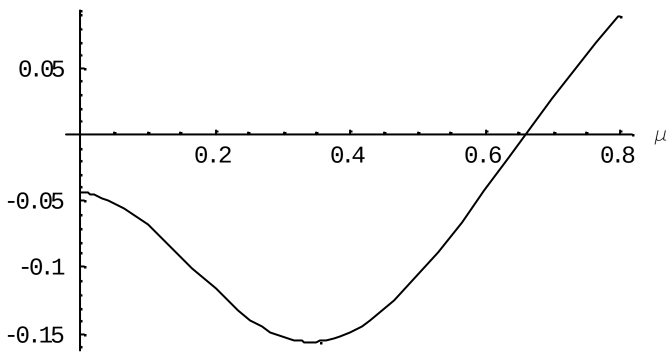

Figure 2b: Difference in expected profits under semi-collusion and non-cooperation for $c = 0$, $a = 5$, $\bar{c} = 3.5$ and $m \in [0,.8]$.

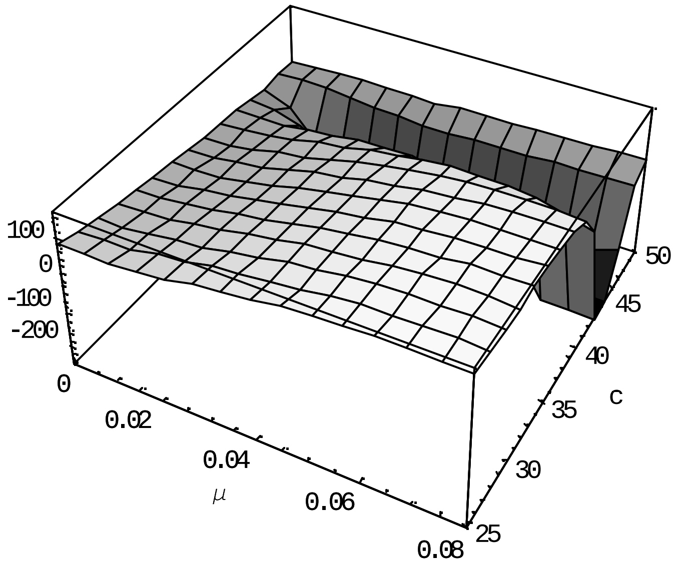

Figure 3a: Difference in expected profits under semi-collusion and non-cooperation for $c = 0$, $a = 50$, $m \in [0,.08]$ and $\bar{c} \in [25,50]$.

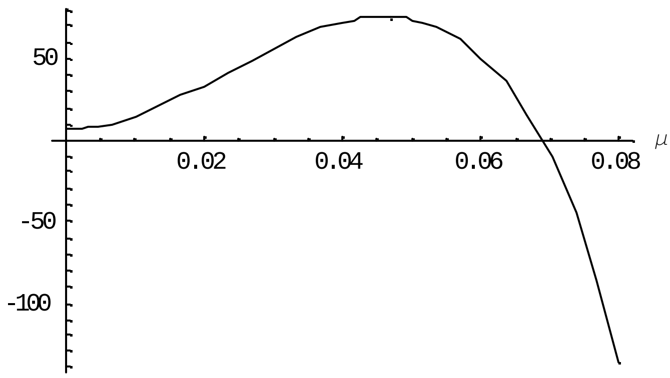

Figure 3b: Difference in expected profits under semi-collusion and non-cooperation for $c = 0$, $a = 50$, $\bar{c} = 42$ and $m \in [0,.08]$.

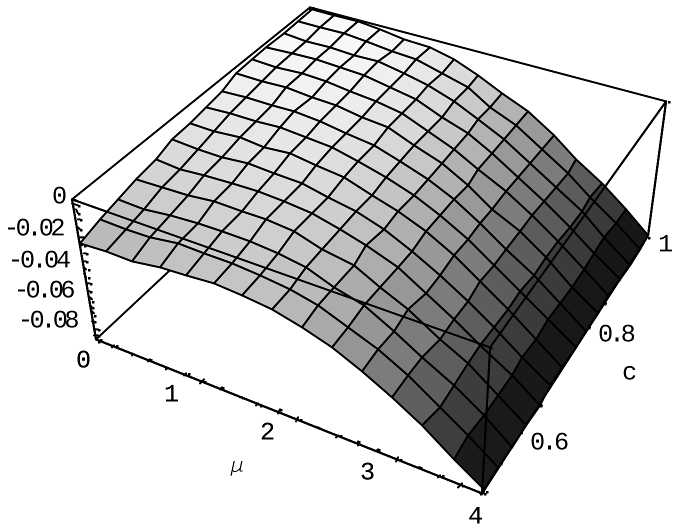

Figure 4a: Difference in expected consumer surplus under semi-collusion and non-cooperation for $a = 1$, $\bar{c} \in [.5,1]$ and $m \in [0,4]$.

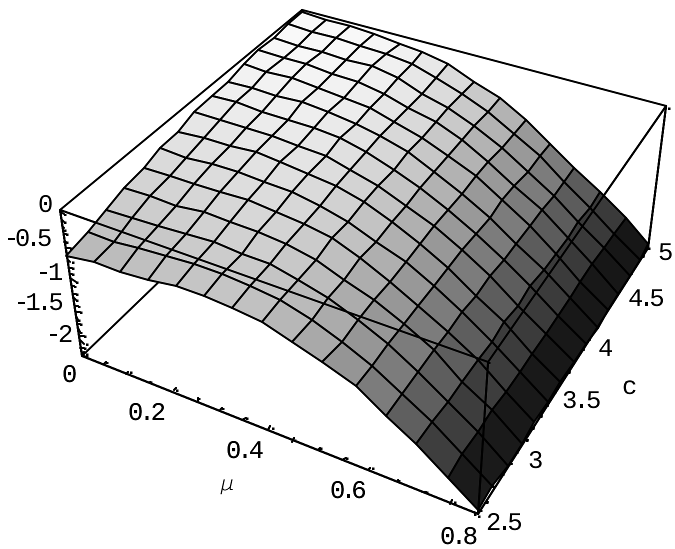

Figure 4b: Difference in expected consumer surplus under semi-collusion and non-cooperation for $a = 5$, $\bar{c} \in [2.5,5]$ and $m \in [0, \frac{4}{5}]$.

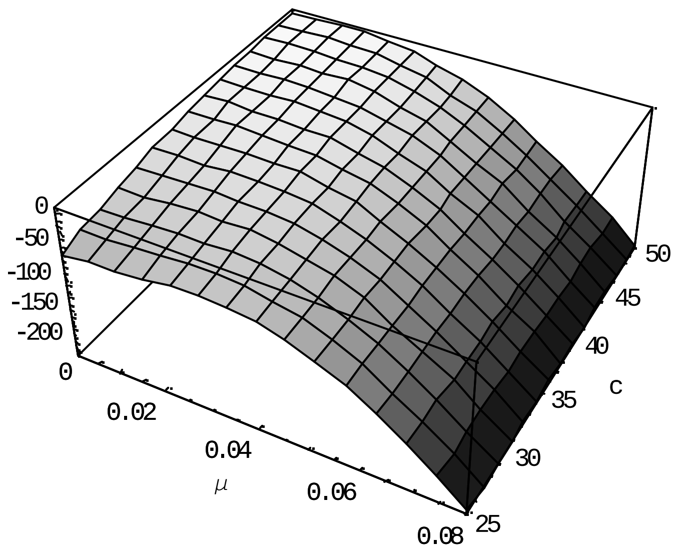

Figure 4c: Difference in expected consumer surplus under semi-collusion and non-cooperation for $a = 50$, $\bar{c} \in [25,50]$ and $m \in [0, \frac{4}{50}]$.

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