Uncertainty over Demand and the Energy Market

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1 Introduction

The paper tries to give some insights on the economic reasons that can entail a blackout, and, as a consequence, on the extent to which there is room for a regualator in the sector.

A variety of countries have recently been hit by a blackout, including the United States and Italy.

If it is true that a large fraction of the blackout-related studies are performed by engineers, it is also true that there is some economics involved.

This works focuses on one economic aspect of the blackout that I think is relevant and worth analyzing.

One of the real problems that the energy industry faces is uncertainty over demand. It can show up in two ways. At first, energy is a just-in-time industry, in which the product is released under consumers' request. With respect to other just-in-time products, however, the energy market has a difference that, in my view, bears significant economic implications. That is, the energy is to be delivered promptly, contextually to the time consumer requires it. Demand for energy fluctuates at every moment, and every firm has to sustain positive costs, such as keeping multiple energy plants ready to produce, in order to make sure to fulfil the whole demand. Another problem is uncertainty over demand in the long run. However, long run uncertainty over demand is a common problem that many firms face, and that does not require a specific analysis.

Therefore, the present paper is centered on the specific issue wih energy firms, that is the impossibility of forecasting the demand in the short run, and the consequent trade-off between sure fulfillment of the whole demand, with likely dispersion of valuable resources, and chance of not serving the totality of consumers, being it less likely to waste resources.

The model is supposed to capture this short run uncertainty. It is supposed to do it in a stylized way, assuming a discrete lenght period, uncertainty over the demand, and decision on the quantity produced - and as a consequence of the cost paid by the firm - before the demand is revealed.

As a first approximation, each firm in an area is a monopolist, which can be justified by the presence of huge fixed costs in connecting users to the network. The option for local monopolists to buy a fraction of their supply from other producers is available, and will be considered in what follows.

To synthetize, the paper examines the three following issues connected to the energy market:

1. Uncertainty ex ante over demand, and the consequent choice of risking to either overproduce, or to underserve the market.

2. Capacity constraints for the energy firms, with the consequence that production is shared by a number of plants.

3. High fixed costs of setting up the distribution procedure, which make it likely to have regional monopolies in the retail market.

The questions that the paper tries to answer are the following:

1) Will the energy firms be willing to connect in order to avoid blackouts? The answer in the paper is yes, but we did not consider the fixed cost of establishing the connection, very important indeed

2) What is the optimal structure of the market, i.e., what would the outcome of an unregulated process be? Would this outcome avoid blackouts?

I believe the questions are important, as the energy market is subject to a lot of regulations that ASSUME to be inspired to a competitive outcome. But the question is: are we sure the competitive outcome regulators inspire to is the true competitive outcome?

And more, are we sure that the true competitive outcome wouldn't be welfare enhancing also for consumers with respect to the regulation outcome?

The model is by now far too complicated to answer these questions satisfactorily. I hope I'll manga to simplify it.

2 The model

2.1 Version A: the basic framework, static and without interconnection

The most basic setup has the following assumptions:

1. The market is monopolistic, and the same firm is produces and distributing the good.

2. All consumers attribute an identical value ν to the product, when they need it, whereas their value is 0 when they do not need the product.

3. The consumers are a represented by a set of measure one. Only a fraction of them needs the good in a given period. The demand $\overline{q}$ is uncertain

at the time in which the monopolist takes the production decision. That implies that the firm is paying the cost for the units it decides to produce, regardless of whether or not it will actually sell them.

4. The marginal cost for each produced unit is c

5. The environment is static. Equivalently, with a dynamic environment, consumers' behavior in the dynamic environment does not depend on the production decisions of the firm in the previous periods. In other words, no punishment is put in place by consumers when they are not served.

6. $v > c > 0$. $v > c$ basically guarantees the existence of the firm, that otherwise wouldn't even be willing to produce. $c > 0$ guarantees that the problem is not trivial. Indeed if $c = 0$, firms would always decide to serve the whole market, whatever the dimension of the market turns out to be.

7. The environment is static, i.e., firms make a one-period decision regarding their production level.

Example 1 *I start with an example that I think is useful to clarify the problem. It concerns the solution of the problem when the demand is uniformly distributed between 0 and 1. Denote q^* the quantity produced and $\overline{q}$ the quantity demanded. The firm then faces the following problem:*

$$\max_{q^*} E\left(v\overline{q} - cq^* | \overline{q} \leq q^*\right) \Pr\left(\overline{q} \leq q^*\right) + E\left(vq^* - cq^* | \overline{q} > q^*\right) \Pr\left(\overline{q} > q^*\right) =$$

$$= \max_{q*} \left(v\frac{q^*}{2} - cq^*\right) q^* + \left(vq^* - cq^*\right)\left(1 - q^*\right)$$

The first derivative yields:

$$v\frac{q^*}{2} - 2cq^* + v\frac{q^*}{2} - \left[vq^* - cq^* + \left(-v + c\right)\left(1 - q^*\right)\right] \Leftrightarrow -vq^* + v - c$$

The second order condition yields:

$$-v < 0$$

The problem is concave. Therefore, the solution to the first order condition yields the optimal point:

$$vq^* = v - c \Leftrightarrow q^* = \frac{v - c}{v}$$

Given the assumptions on v, c, the quantity is always positive. A bit of simple comparative statics may help make the result clearer:

$$\begin{aligned} \frac{dq^*}{dc} &= \frac{-1}{v} < 0 \\ \frac{dq^*}{dv} &= \frac{c}{v^2} > 0 \end{aligned}$$

As intuitively plausible, the quantity produced decreases with the marginal cost, while it increases with the value for consumers.

The profit is given by:

$$\left(v\frac{\frac{v-c}{v}}{2}-c\frac{v-c}{v}\right)\frac{v-c}{v}+\left(v\frac{v-c}{v}-c\frac{v-c}{v}\right)\left(1-\frac{v-c}{v}\right)=$$
$$\frac{v}{2}+\frac{c}{2}-vc+c^2+c+\frac{c^3}{v^2}-\frac{2c^2}{v}$$

The next step is to try and generalize a bit the result, not assuming any specific form for the utility function.

The result, albeit obvious, is worth to be stated and proved.

Proposition 2 *When a monopolist faces uncertainty over demand, his production decision is strictly increasing with the value of the product to the consumer, if we assume homogeneity of all consumers except for the decision of whether or not to buy, and strictly decreasing with its marginal production cost.*

Proof. (to be completed) It follows form the fact that we are simply performing a monotonic transformation.

The firm now solves an analogous problem with respect to the previous one:

$$\max_{q^*} E\left(v\overline{q}-cq^*|\overline{q}\leq q^*\right)\Pr\left(\overline{q}\leq q^*\right)+E\left(vq^*-cq^*|\overline{q}>q^*\right)\Pr\left(\overline{q}>q^*\right)$$

In this case,

$$\Pr\left(\overline{q}\leq q^*\right)=\int_{\overline{q}_{\min}}^{q^*} f\left(\overline{q}\right)d\overline{q}=F\left(q^*\right)$$
$$\Pr\left(\overline{q}>q^*\right)=\int_{q^*}^{\overline{q}_{\max}} f\left(\overline{q}\right)d\overline{q}=1-F\left(q^*\right)$$
$$E\left(\overline{q}|\overline{q}\leq q^*\right)=\int_{\overline{q}_{\min}}^{q^*}\overline{q}\left(f\left(\overline{q}\right)|\overline{q}\leq q^*\right)d\overline{q}=\int_{\overline{q}_{\min}}^{q^*}\overline{q}dF\left(\overline{q}|\overline{q}\leq q^*\right)$$
$$E\left(\overline{q}|\overline{q}>q^*\right)=\int_{q^*}^{\overline{q}_{\max}}\overline{q}\left(f\left(\overline{q}\right)|\overline{q}>q^*\right)d\overline{q}=\int_{q^*}^{\overline{q}_{\max}}\overline{q}dF\left(\overline{q}|\overline{q}>q^*\right)$$

It therefore follows that:

$$\max_{q^*} E\left(v\overline{q} - cq^* | \overline{q} \leq q^*\right) \Pr\left(\overline{q} \leq q^*\right) + E\left(vq^* - cq^* | \overline{q} > q^*\right) \Pr\left(\overline{q} > q^*\right) =$$

$$= \left(v \left(\int_{\overline{q}_{\min}}^{q^*} \overline{q} dF\left(\overline{q} | \overline{q} \leq q^*\right) \right) - cq^* \right) F\left(q^*\right) + \left(vq^* - cq^*\right)\left(1 - F\left(q^*\right)\right) =$$

$$= \left(v \left(\int_{\overline{q}_{\min}}^{q^*} \overline{q} dF\left(\overline{q} | \overline{q} \leq q^*\right) - q^* \right) \right) F\left(q^*\right) + \left(vq^* - cq^*\right) =$$

At the optimum, we have:

$$v\left(q^*\left(f\left(q^* | \overline{q} \leq q^*\right)\right) - 1\right) F\left(q^*\right) + F'\left(q^*\right) \left(v \left(\int_{\overline{q}_{\min}}^{q^*} \overline{q} dF\left(\overline{q} | \overline{q} \leq q^*\right) - q^* \right) \right) + (v - c) = 0$$

Now, we can proceed in two ways, either to prove that the first order condition is supermodular in q^* and v, and also in q^* and $-c$, which I have not been able to do, or, on the other hand, to use the brute force approach, and totally differentiate. The latter is the chosen.

$$\left[v\left(f\left(q^* | \overline{q} \leq q^*\right)\right) F\left(q^*\right) + vf'\left(q^* | \overline{q} \leq q^*\right) q^* F\left(q^*\right) + F'\left(q^*\right) v \left(q^*\left(f\left(q^* | \overline{q} \leq q^*\right)\right) - 1\right)\right] dq^* +$$
$$\left[f'\left(q^*\right) \left(v \left(\int_{\overline{q}_{\min}}^{q^*} \overline{q} dF\left(\overline{q} | \overline{q} \leq q^*\right) \right) - q^* \right) + F'\left(q^*\right)\left(vq^*\left(f\left(q^* | \overline{q} \leq q^*\right)\right) - 1\right) \right] dq^* - 1dc = 0$$

The results are the following:

$$v\left(f\left(q^* | \overline{q} \leq q^*\right)\right) F\left(q^*\right) + vf'\left(q^* | \overline{q} \leq q^*\right) q^* F\left(q^*\right) + F'\left(q^*\right) v \left(q^*\left(f\left(q^* | \overline{q} \leq q^*\right)\right) - 1\right) +$$
$$f'\left(q^*\right) \left(v \left(\int_{\overline{q}_{\min}}^{q^*} \overline{q} dF\left(\overline{q} | \overline{q} \leq q^*\right) \right) - q^* \right) + F'\left(q^*\right)\left(vq^*\left(f\left(q^* | \overline{q} \leq q^*\right)\right) - 1\right) = \frac{dc}{dq^*}$$

■

The result is fairly simple and intuitive, and it tells us that in the presence of uncertainty over demand, the firm will bear a positive probability of leaving some potential consumers unserved.

This first result can be significant, as it may somehow illustrate the economic dynamics of a blackout.

The next attempt is to make the problem dynamic, through the introduction of multiple periods. I will here establish the framework, without actually solving the problem.

It is clear that, as stated before, if consumers do not react to the monopolist's decisions, the string of decisions in a dynamic context is simply the replication of the one period decision, with the average per period payoff for the firm that approaches the expected per period profit, invoking the the law of large numbers.

Something changes, on the other hand, if we assume that consumers are reacting to the decisions of the monopolist. For example, we can say that if a consumer is left unserved, i.e., if a blackout happens, he punishes the monopolist, and stop being supplied by him. The consumer left unserved can, for example, construct his own energy generation system, which, in spite of being more costly, will guarantee him from any future blackout. Consumers are drastic, and the monopolist has to react to this drastic form of punishment. The company has indeed the option of blocking the supply of energy for a fraction of customers. In this reduced form model, it is hard to establish whether the punishment by the consumer is actually an equilibrium punishment. Therefore, I will just assume so, and stick to the assumption throughout the paper.

2.2 Version B: dynamic aspects without interconnection

Besides assumptions 1 through 6, a set of additional assumptions has to be made:

7b. Consumers do not know ex ante the firm' cost, and ex ante they think they will be served with reasonably high probability by the energy supplier

8. Ex post, consumers who are not served once stop buying from the supplier. They construct their own system of energy generation, and operate it.

9. Demand is uniformly distributed between 0 and 1 in the first period, then the upper bound of demand declines according to the above specified rule in the following periods. In any period, demand is uniformly distributed between 0 and its upper bound. This assumption has already been used in example 1.

Assumption 7b, which replaces assumption 7, is purposedly capturing the punishment that consumers give to the firm. In otder for the punishment to be reasonable, it has to be an equilibrium punishment which cannot be checked in the present framework, owing to the reduced form representation of consumers' preferences. Future extensions, hopefully, will take care of that problem.

Denote γ as the measure of consumers that have always been served in the previous periods, whenever they have requested the product. In other words, γ_t represents the upper bound of the distribution of potential consumer in period t

Proposition 3 *A firm facing uncertainty over demand and the punishment rule put in place by unserved consumers described above solves the following dynamic problem:*

$$\max_{q_1, E\{q_t\}t=2,\ldots,\infty} E \sum_{t=1}^{\infty} \delta^t \left[\left(v\frac{q_t^*}{2} - cq_t^* \right) \frac{q_t^*}{\gamma_t} + (vq_t^* - cq_t^*) \left(1 - \frac{q_t^*}{\gamma_t} \right) \right]$$

subject to the following law of motion for γ_t :

$$\gamma_1 = 1$$
$$E(\gamma_t) = E(\gamma_{t-1}) - \left(\frac{E(\gamma_{t-1}) - q_{t-1}^*}{2}\right)(1 - q_{t-1}^*)$$

Proof. The problem can be expressed in the following form:

$$\max_{q_1, E\{q_t\} t=2,\ldots,\infty} \sum_{t=1}^{\infty} \delta^t \left[E\left(v\overline{q}_t - cq_t^* | \overline{q}_t \leq q_t^*\right) \Pr\left(\overline{q}_t \leq q_t^*\right) + E\left(vq_t^* - cq_t^* | \overline{q}_t > q_t^*\right) \Pr\left(\overline{q}_t > q_t^*\right)\right]$$

Solving term by term, we have:

$$\Pr\left(\overline{q}_t \leq q_t^*\right) = \frac{q_t^*}{\gamma}$$
$$E\left(v\overline{q}_t - cq_t^* | \overline{q}_t \leq q_t^*\right) = v\frac{q_t^*}{2} - cq_t^*$$
$$\Pr\left(\overline{q}_t > q_t^*\right) = 1 - \frac{q_t^*}{\gamma}$$
$$E\left(vq_t^* - cq_t^* | \overline{q}_t > q_t^*\right) = vq_t^* - cq_t^*$$

The value of γ - representing the expected measure of consumers that the firm never 'disappointed', and thus the upper bound of the density of potential buyers, follows the evolution rule now presented:

$$\gamma_1 = 1$$
$$E(\gamma_t) = E(\gamma_{t-1}) - E\left(\overline{q}_{t-1} - q_{t-1}^*\right) =$$
$$= E(\gamma_{t-1}) - \left(E\left(E\left(\overline{q}_{t-1}\right) - q_{t-1}^*\right) | q_{t-1}^* < \overline{q}_{t-1}\right) \Pr\left(q_{t-1}^* < \overline{q}_{t-1}\right) =$$
$$= E(\gamma_{t-1}) - \left(\frac{E(\gamma_{t-1}) - q_{t-1}^*}{2}\right)(1 - q_{t-1}^*) =$$
$$= E(\gamma_{t-1}) \Pr\left(q_{t-1}^* > \overline{q}_{t-1}\right) + \left(\frac{E(\gamma_{t-1}) + q_{t-1}^*}{2}\right) \Pr\left(q_{t-1}^* < \overline{q}_{t-1}\right) =$$
$$E(\gamma_{t-1})\, q_{t-1}^* + \left(\frac{E(\gamma_{t-1}) + q_{t-1}^*}{2}\right)(1 - q_{t-1}^*)$$

It is natural that the firm will have an incentive to update its production at every period, after uncertainty over the demand in the previous period has been resolved ■

The solution to the previous problem requires a huge computational burden, and I am not sure if it is worth performing it by now. I followed the old rule 'in dubio abstine', and to be safe I did not do it!

A couple of interesting results can still be stated.

Claim 4 *As the discount factor approaches 0, the solution to the maximization problem approaches the solution to the single period problem*

Proof. . When $\delta = 0$, the firm does not value the future, and therefore the dynamic problem converges to the static one-shot game. Intuitively, as the value of the future for the firm disappears, the incentive to raise production with respect to the one shot game in order to gain future profits becomes weaker

Claim 5 *As the discount factor approaches 1, the firm will serve the whole market with probability one if in the single shot game the expected value of profits from producing 1 exceeds its cost, i.e., if $v > 2c$*

Proof. The firm is now facing an infinite stream of equally valuable profit. Therefore, the benefit of serving the whole market forever offsets any other finite-periods gain. The only point to check is whether this strategy entails a positive average payoff per period, that implies an infinte payoff in the dynamic game. This happens if

$$v\frac{q^*}{2} > cq^* \Leftrightarrow v > 2c$$

Suppose $v > 2c$, and the firm decides to leave some consumers unserved in the first period. Then, $(1 - q^*)$ is positive. Suppose also $(1 - q^*)$ is very small - a conservative assumption. Then, the firm's strategy entails a positive expected one-period earning equal to

$$cq^* - \frac{(1 - q^*)^2}{2}$$

On the other hand, the firm's losses are given by the infinite stream of losses. Therefore, the deviation is not profitable. ∎ ∎

I will solve, however, a two period model, satisfying assumptions 1 through 6, and 7b through 9. The discount factor is assumed to be1.

Proposition 6 *In a two periods model satisfying assumptions 1 through 9, in which profits in the two periods have equal weight in the firm's decision, the monopolist produces in the first period a higher quantity than in the one-shot game*

Proof. *The monopolist solves the following problem*

$$\max_{q_i^*,\ i=1,2,3} E\left(v\overline{q}_1 - cq_1^* | \overline{q}_1 \leq q_1^*\right) \Pr\left(\overline{q}_1 \leq q_1^*\right) + E\left(vq_1^* - cq_1^* | \overline{q}_1 > q_1^*\right) \Pr\left(\overline{q}_1 > q_1^*\right) +$$
$$+E\left(v\overline{q}_2 - cq_2^* | \overline{q}_2 \leq q_2^*\right) \Pr\left(\overline{q}_2 \leq q_2^*\right) + E\left(vq_2^* - cq_2^* | \overline{q}_2 > q_2^*\right) \Pr\left(\overline{q}_2 > q_2^*\right)$$

It can be rewritten as

$$\max_{q_i^*, i=1,2} \left(v\frac{q_1^*}{2} - cq_1^* \right) q_1^* + \left(vq_1^* - cq_1^* \right) \left(1 - q_1^* \right) +$$

$$+ \left(v\frac{q_2^*}{2} - cq_2^* \right) \frac{q_2^*}{\Pr\left(\overline{q}_1 < q_1^*\right) + E\left(\overline{q}_1 - q_1^* | \overline{q}_1 > q_1^*\right) \Pr\left(\overline{q}_1 > q_1^*\right)} +$$

$$+ \left(vq_2^* - cq_2^* \right) \left(1 - \frac{q_2^*}{\Pr\left(\overline{q}_1 < q_1^*\right) + E\left(\overline{q}_1 - q_1^* | \overline{q}_1 > q_1^*\right) \Pr\left(\overline{q}_1 > q_1^*\right)} \right) =$$

$$= \max_{q_1^*, E\left(q_2^*\right)} \left(v\frac{q_1^*}{2} - cq_1^* \right) q_1^* + \left(vq_1^* - cq_1^* \right) \left(1 - q_1^* \right) +$$

$$+ \left(v\frac{q_2^*}{2} - cq_2^* \right) \frac{q_2^*}{q_1^* + \frac{1}{2} - \frac{q_1^{*2}}{2}} +$$

$$+ \left(vq_2^* - cq_2^* \right) \left(1 - \frac{q_2^*}{q_1^* + \frac{1}{2} - \frac{q_1^{*2}}{2}} \right)$$

At the optimum, we have

$$q_1^* = \frac{3v^2 + c^2 - 4cv}{-2cv + 3v^2 + c^2}, \; q_2^* = \frac{-21v^4c + 20c^2v^3 - 12c^3v^2 + 5vc^4 + 9v^5 - c^5}{v\left(-2cv + 3v^2 + c^2\right)^2}$$

The function at the optimum attains

$$\left\{ \frac{1}{2v} \frac{6v^4 - 16v^3c + 15c^2v^2 - 6c^3v + c^4}{-2cv + 3v^2 + c^2} \right\}$$

It is easily possible to prove that the optimal quantity is positive for any values satisfying our initial assumption, that $v > c > 0$.To do that, it is sufficient to just solve the following inequality and check that it holds for all values that satisfy our assumptions.

$$\frac{3v^2 + c^2 - 4cv}{-2cv + 3v^2 + c^2} > 0$$

Also, it is possible to show that $0 < q_1^* < 1$.

Now, we compare q_1^* in the two cases, i.e., when the monopolist has a 1 period horizon, and when it has a two period horizon. In order to do that, we normalize the cost to 1, in order to have the results more easily understandable.

$$q_1^* \text{ two period} > q_1^* \text{ one period} \iff$$

$$\frac{3v^2 + c^2 - 4cv}{-2cv + 3v^2 + c^2} > \frac{v-1}{v} \iff \text{(given that } c = 1\text{)}$$

$$\frac{3v^2 + 1 - 4v}{-2v + 3v^2 + 1} - \frac{v-1}{v} > 0$$

The inequality always holds. ■

Once more, the result is intuitively plausible; if the firm produces in two periods, its production in the first period will be higher, due to the threat of consumers not buying from he firm in the next period.

Essentially, the presence of multiple periods enhances the consumer welfare, in the sense that a greater production is carried out with respect to the static game. No meaningful comparison can be performed on the firm's profit, since in this last case, the firm's profit is spread across multiple periods. The first periodprofit for the firm, however, is lower under this circumstance than in the basic one.

2.3 Version C: static game with interconnection

Consider now an alternative situation, in which the regional monopolist can buy some energy from another firm, with respect to which the monopolist is a price-taker. There is no limit to the quantity the other firm offers, so the monopolist can always choose ex post to buy from its supplier, of which he knows the price ex ante. This is a first simplified form of interconnection between firms, just to get introduced to the -maybe- more interesting next scenarios.

Formally, the new assumptions are:

10. In order to satisfy its demand, the monopolist can buy an unlimited amount of energy from a supplier.

11. The monopolist is a price-taker with respect to the supplier, and the price p charged by the supplier is known by the monopolist ex ante, i.e., before the monopolist takes its production decisions.

12. $v > p > c$

Proposition 7 *If assumptions 1 through 12 hold, the quanity produced by the monopolist is the following:the market is completely served, and the quantity produced by the monopolist is decreasing with respect to the situation of unavailability of the supplier, and its expected profit is increasing*

Proof. The monopolist solves now the following problem:

$$\max_{q^*} E\left(\pi|\overline{q} \leq q^*\right)\Pr\left(\overline{q} \leq q^*\right) + E\left(\pi|\overline{q} > q^*\right)\Pr\left(\overline{q} > q^*\right) =$$

$$= E\left(v\overline{q} - cq^*|\overline{q} \leq q^*\right)\Pr\left(\overline{q} \leq q^*\right) + E\left(vq^* - cq^* + (v-p)\left(\overline{q} - q^*\right)|\overline{q} > q^*\right)\Pr\left(\overline{q} > q^*\right) =$$

$$\max_{q^*} \ (vq^* - cq^*)(1-q^*) + \left(\frac{vq^*}{2} - cq^*\right)q^* + \left((v-p)\left(\frac{1-q^*}{2}\right)\right)(1-q^*)$$

The first order conditions are the following:

$$-q^*v + v - c - (1-q^*)(v-p) = 0 \Leftrightarrow -q^*v + v - c - v + p + q^*v - q^*p = 0$$

They imply

$$q^* = \frac{p-c}{p}$$

The previous results show that the quantity produced is decreasing if the option of buying it is available.

To check it formally,

$$q^* \text{ without supplier } > q^* \text{ with supplier} \Longleftrightarrow$$
$$\frac{v-c}{v} < \frac{p-c}{p} \Leftrightarrow p < v$$

As usual, we verify that the second order conditions hold:

$$\begin{aligned} \frac{\partial \pi}{\partial q^*} &= -q^* v + v - c - v + p + q^* v - q^* p \\ \frac{\partial \pi}{\partial q^* q^*} &= -p < 0 \end{aligned}$$

Now, we provide some comparative statics:

$$\frac{dq^*}{dp} = \frac{c}{p^2} > 0$$

It arises because the firm fears the high prices of its alternative supplier

$$\frac{dq^*}{dc} = -\frac{1}{p} < 0$$

As intuitively plausible, the quantity increases with the cost. ■

Now, let us consider the problem of the selling firm. The selling firm sets prices, in order to maximize profits. If we are willing to assume that risks are uncorrelated, i.e., that the firm sells to many local monopolist, so something similar to the law of large number applies, the energy supplier will be maximizing its price without aggregate uncertainty, i.e., without dispersion of unused resources.

$$\begin{aligned} \max_p E\pi &= E(\pi | \pi > 0) \Pr(\pi > 0) \\ & \max_p (p-c)\left(\frac{c}{2p}\right)\left(\frac{c}{p}\right) \end{aligned}$$

From the first order condition, we get the solution to the problem, actually concave:

$$p = 2c$$

For the firm to be willing to produce at this prices, it has to make positive profit, so $p = 2c < v$. Otherwise, the constraint $p = 2v$ becomes binding and the firm sets $p = v$.

Overall, the pricing structure is the following:

$$q^* = \begin{cases} \frac{1}{2} \; if \; v > 2c \\ \frac{v-c}{v} \; if \; 2c > v > c \\ 0 \; if \; c > v \end{cases}$$

Now, there are two firms, both vertically integrated. Each of them still has the exclusive for serving the consumers in its area, due to the extremely high costs of duplicating the retail network. However, now, the firms can trade energy with each ther, being interconnected. That means that, if one is in excess supply, and the other is in excess demand, a gainful trade will take place.

Formally, I introduce three new assumptions, bound to replace 10, 11, and 12. They are the following:

13. There are two regional monopolist

14. Each of them can buy some energy from the other, if that energy is available.Specifically, as a simplifying assumption, a firm will buy some energy if and only if its excess demand is lower than the excess supply of the other firm. In other words, firm i will buy energy from firm j if and only if the following conditions are satisfied:

- Firm i has underproduced with respect to the demand
- Firm j has overproduced with respect to the demand
- The overproduction of firm j outweighs the underproduction of firm i, i.e., firm i serves all of its consumers after buying from firm j

15. The two regional monopolists simultaneously set prices and quantites

16. Quantities are assumed to be equal for the two firms, i.e., we are restricting attention to pure strategy symeetric Bayesian nash equilibria.

Proposition 8 *In the game described by assumption 1-6, 7b-9, 13-16, the quantity produced and the price charged in any symmetric Bayesian Nash equilibrium in quantities and prices are the following:*

$$p = -\frac{87}{16} + \frac{67}{16}v, x = \frac{1}{2}$$

Proof. At first, I sketch the maximization problem, which takes the following form:

$$\max_{p_1,q_1^*} E\pi =$$

$$\begin{aligned}
&1) E\left(\pi | q_1^* > \overline{q}_1,\ q_2^* > \overline{q}_2,\ q_1^* - \overline{q}_1 < \overline{q}_2 - q_2^*\right) 2) \Pr\left(\ q_1^* > \overline{q}_1,\ q_2^* > \overline{q}_2,\ q_1^* - \overline{q}_1 < \overline{q}_2 - q_2^*\right) + \\
&3) E\left(\pi | q_1^* > \overline{q}_1,\ q_2^* < \overline{q}_2,\ q_1^* - \overline{q}_1 < \overline{q}_2 - q_2^*\right) 4) \Pr\left(\ q_1^* > \overline{q}_1,\ q_2^* < \overline{q}_2,\ q_1^* - \overline{q}_1 < \overline{q}_2 - q_2^*\right) + \\
&5) E\left(\pi | q_1^* > \overline{q}_1,\ q_2^* > \overline{q}_2,\ q_1^* - \overline{q}_1 > \overline{q}_2 - q_2^*\right) 6) \Pr\left(\ q_1^* > \overline{q}_1,\ q_2^* > \overline{q}_2,\ q_1^* - \overline{q}_1 > \overline{q}_2 - q_2^*\right) + \\
&7) E\left(\pi | q_1^* > \overline{q}_1,\ q_2^* < \overline{q}_2,\ q_1^* - \overline{q}_1 > \overline{q}_2 - q_2^*\right) 8) \Pr\left(\ q_1^* > \overline{q}_1,\ q_2^* < \overline{q}_2,\ q_1^* - \overline{q}_1 > \overline{q}_2 - q_2^*\right) + \\
&9) E\left(\pi | q_1^* < \overline{q}_1,\ q_2^* > \overline{q}_2,\ \overline{q}_1 - q_1^* < q_2^* - \overline{q}_2\right) 10) \Pr\left(q_1^* < \overline{q}_1,\ q_2^* > \overline{q}_2,\ \overline{q}_1 - q_1^* < q_2^* - \overline{q}_2\right) + \\
&11) E\left(\pi | q_1^* < \overline{q}_1,\ q_2^* < \overline{q}_2,\ \overline{q}_1 - q_1^* < q_2^* - \overline{q}_2\right) 12) \Pr\left(q_1^* < \overline{q}_1,\ q_2^* < \overline{q}_2,\ \overline{q}_1 - q_1^* < q_2^* - \overline{q}_2\right) + \\
&13) E\left(\pi | q_1^* < \overline{q}_1,\ q_2^* > \overline{q}_2,\ \overline{q}_1 - q_1^* > q_2^* - \overline{q}_2\right) 14) \Pr\left(q_1^* < \overline{q}_1,\ q_2^* > \overline{q}_2,\ \overline{q}_1 - q_1^* > q_2^* - \overline{q}_2\right) + \\
&15) E\left(\pi | q_1^* < \overline{q}_1,\ q_2^* < \overline{q}_2,\ \overline{q}_1 - q_1^* > q_2^* - \overline{q}_2\right) 16) \Pr\left(q_1^* < \overline{q}_1,\ q_2^* < \overline{q}_2,\ \overline{q}_1 - q_1^* > q_2^* - \overline{q}_2\right)
\end{aligned}$$

Now, we are giving the expressions for the terms in the previous big equation:

$$\begin{gathered}
1) E\left(v\overline{q}_1 - cq_1^*\right) | \left(q_1^* > \overline{q}_1,\ q_2^* > \overline{q}_2,\ q_1^* - \overline{q}_1 < \overline{q}_2 - q_2^*\right) \\
3) E\left(v\overline{q}_1 - cq_1^*\right) | \left(q_1^* > \overline{q}_1,\ q_2^* < \overline{q}_2,\ q_1^* - \overline{q}_1 < \overline{q}_2 - q_2^*\right) \\
5) E\left(v\overline{q}_1 - cq_1^*\right) | \left(q_1^* > \overline{q}_1,\ q_2^* > \overline{q}_2,\ q_1^* - \overline{q}_1 > \overline{q}_2 - q_2^*\right) \\
7) E\left(v\overline{q}_1 - cq_1^*\right) + p_1 \frac{E\left(\overline{q}_2\right) - q_2^*}{2} | \left(q_1^* > \overline{q}_1,\ q_2^* < \overline{q}_2,\ q_1^* - \overline{q}_1 > \overline{q}_2 - q_2^*\right) \\
9) \left(vq_1^* - cq_1^*\right) + \left(v - p_2\right) E\left(\overline{q}_1 - q_1^*\right) | \left(q_1^* < \overline{q}_1,\ q_2^* > \overline{q}_2,\ \overline{q}_1 - q_1^* < q_2^* - \overline{q}_2\right) \\
11) \left(vq_1^* - cq_1^*\right) \\
13) \left(vq_1^* - cq_1^*\right) \\
15) \left(vq_1^* - cq_1^*\right)
\end{gathered}$$

Now, it is clear that

$$\begin{gathered}
\Pr\left(q_1^* > \overline{q}_1,\ q_2^* > \overline{q}_2,\ q_1^* - \overline{q}_1 < \overline{q}_2 - q_2^*\right) = \\
= \Pr\left(q_1^* > \overline{q}_1 > q_1^* - \overline{q}_2 + q_2^*,\ q_2^* > \overline{q}_2\right) \\
= \Pr\left(\overline{q}_2 - q_2^* > 0,\ q_2^* > \overline{q}_2\right) = 0
\end{gathered}$$

It therefore follows that

$$2) = 0$$

Also,

$$\begin{gathered}
\Pr\left(q_1^* < \overline{q}_1,\ q_2^* < \overline{q}_2,\ \overline{q}_1 - q_1^* < q_2^* - \overline{q}_2\right) = \\
\Pr\left(q_1^* < \overline{q}_1 < q_2^* - \overline{q}_2 + q_1^*,\ q_2^* < \overline{q}_2\right) = \\
\Pr\left(\ q_2^* - \overline{q}_2 > 0,\ q_2^* < \overline{q}_2\right) = 0
\end{gathered}$$

It therefore follows that

$$12) = 0$$

Now, we can go a step further and establish another result. We know that 2) = 0, hence it follows that

$$\Pr\left(\ q_1^* > \overline{q}_1,\ q_2^* > \overline{q}_2,\ q_1^* - \overline{q}_1 > \overline{q}_2 - q_2^*\right) = \Pr\left(\ q_1^* > \overline{q}_1,\ q_2^* > \overline{q}_2\right) = \ q_1^*\ q_2^*$$

Hence, it follows that

$$6) = \ q_1^*\ q_2^*$$

Analogously, since we know that 12) = 0, it follows that

$$\Pr\left(q_1^* < \overline{q}_1,\ q_2^* < \overline{q}_2,\ \overline{q}_1 - q_1^* > q_2^* - \overline{q}_2\right) = \Pr\left(q_1^* < \overline{q}_1,\ q_2^* < \overline{q}_2\right) = \left(1 - q_1^*\right)\left(1 - \ q_2^*\right)$$

It follows that

$$16) = \left(1 - q_1^*\right)\left(1 - \ q_2^*\right)$$

The computation of 6) and 8) becomes slightly more complicated. Basically, we have to establish $\Pr\left(\overline{q}_1 - q_1^* > q_2^* - \overline{q}_2\right)$. In order to do that, it is necessary to remeber that the joint probability of

$$\Pr\left(\overline{q}_1 + \overline{q}_2 > q_2^* + q_1^*\right)$$

is given by the triangular distribution.

Now, it is clear that there is not an unique expression that defines the probability; instead, the definition crucially depends on the relation between $q_1^* + q_2^*$ and $\frac{1}{2}$. We need at this point a guess about the value of $q_1^* + q_2^*$, solve the problem assuming the guess is correct, then verify the guess eventually. Our guess is that $q_1^* + q_2^* \geq \frac{1}{2}$.

Looking at the picture, and computing areas (the result can be more formally derived through the standard integration procedure, but this simplification helps, maybe, to give more intuition for the result). The result is the following - note that from now, I'll always behave as such my guess is correct, even when this is not clearly stated in the paper.

$$\Pr\left(\overline{q}_1 + \overline{q}_2 < q_2^* + q_1^*\right) = \frac{1}{2} + \frac{\left(q_2^* + q_1^* - 1\right)}{2}\left(3 - q_2^* - q_1^*\right)$$

It then follows that

$$\Pr\left(\overline{q}_1 + \overline{q}_2 < q_2^* + q_1^*\right) = \frac{1}{2} + \frac{\left(q_2^* + q_1^* - 1\right)}{2}\left(3 - q_2^* - q_1^*\right)$$

At this point, it follows that

$$\Pr\left(\ \overline{q}_1 + \overline{q}_2 < q_1^* + q_2^* | q_1^* > \overline{q}_1,\ q_2^* < \overline{q}_2\right) = = \frac{\left(\frac{1-q_2^*}{2}\right)\left(1 + q_2^*\right) + \frac{\left(q_2^* + q_1^* - 1\right)}{2}\left(3 - q_2^* - q_1^*\right)}{\left(\frac{1-q_2^*}{2}\right)\left(1 + q_2^*\right) + \left(\frac{q_1^*}{2}\right)\left(2 - q_1^*\right)}$$

As a consequence,

$$\Pr\left(\, q_1^* > \overline{q}_1,\ q_2^* < \overline{q}_2,\ q_1^* - \overline{q}_1 > \overline{q}_2 - q_2^*\right) =$$
$$\Pr\left(\, q_1^* > \overline{q}_1,\ q_2^* < \overline{q}_2,\ q_1^* + q_2^* > \overline{q}_2 + \overline{q}_1\right) =$$
$$= \frac{\left(\frac{1-q_2^*}{2}\right)(1+q_2^*) + \frac{(q_2^*+q_1^*-1)}{2}(3-q_2^*-q_1^*)}{\left(\frac{1-q_2^*}{2}\right)(1+q_2^*) + \left(\frac{q_1^*}{2}\right)(2-q_1^*)} q_1^*\,(1-q_2^*)$$

This implies that

$$8) = \frac{\left(\frac{1-q_2^*}{2}\right)(1+q_2^*) + \frac{(q_2^*+q_1^*-1)}{2}(3-q_2^*-q_1^*)}{\left(\frac{1-q_2^*}{2}\right)(1+q_2^*) + \left(\frac{q_1^*}{2}\right)(2-q_1^*)} q_1^*\,(1-q_2^*)$$

It also follows that:

$$\Pr\left(\, q_1^* > \overline{q}_1,\ q_2^* < \overline{q}_2,\ q_1^* - \overline{q}_1 < \overline{q}_2 - q_2^*\right) =$$
$$= \Pr\left(\, q_1^* > \overline{q}_1,\ q_2^* < \overline{q}_2,\ q_1^* + q_2^* < \overline{q}_2 + \overline{q}_1\right) =$$
$$= \frac{\left(1-\frac{q_2^*}{2}\right)(3-2q_1^*-q_2^*)}{\left(\frac{1-q_2^*}{2}\right)(1+q_2^*) + \left(\frac{q_1^*}{2}\right)(2-q_1^*)} q_1^*\,(1-q_2^*)$$

Therefore,

$$4) = \frac{\left(1-\frac{q_2^*}{2}\right)(3-2q_1^*-q_2^*)}{\left(\frac{1-q_2^*}{2}\right)(1+q_2^*) + \left(\frac{q_1^*}{2}\right)(2-q_1^*)} q_1^*\,(1-q_2^*)$$

Now, we compute:

$$\Pr\left(q_1^* < \overline{q}_1,\ q_2^* > \overline{q}_2,\ \overline{q}_1 - q_1^* < q_2^* - \overline{q}_2\right) =$$
$$= \Pr\left(q_1^* < \overline{q}_1,\ q_2^* > \overline{q}_2,\ \overline{q}_1 + \overline{q}_2 < q_2^* + q_1^*\right) =$$
$$= \frac{\frac{(q_2^*+q_1^*-1)}{2}(3-q_2^*-q_1^*) + \left(1-\frac{q_1^*}{2}\right)(1+q_1^*)}{\left(\frac{1-q_1^*}{2}\right)(1+q_1^*) + \left(\frac{q_2^*}{2}\right)(2-q_2^*)} (1-q_1^*)\, q_2^*$$

That implies that

$$10) = \frac{\frac{(q_2^*+q_1^*-1)}{2}(3-q_2^*-q_1^*) + \left(1-\frac{q_1^*}{2}\right)(1+q_1^*)}{\left(\frac{1-q_1^*}{2}\right)(1+q_1^*) + \left(\frac{q_2^*}{2}\right)(2-q_2^*)} (1-q_1^*)\, q_2^*$$

Finally,

$$\Pr\left(q_1^* < \overline{q}_1,\ q_2^* > \overline{q}_2,\ \overline{q}_1 - q_1^* > q_2^* - \overline{q}_2\right) =$$
$$= \Pr\left(q_1^* < \overline{q}_1,\ q_2^* > \overline{q}_2,\ \overline{q}_1 + \overline{q}_2 > q_1^* + q_2^*\right) =$$
$$= \frac{\left(1-\frac{q_1^*}{2}\right)(3-2q_2^*-q_1^*)}{\left(\frac{1-q_1^*}{2}\right)(1+q_1^*) + \left(\frac{q_2^*}{2}\right)(2-q_2^*)} q_2^*\,(1-q_1^*)$$

It thus follows that:

$$14) = \frac{\left(1-\frac{q_1^*}{2}\right)(3-2q_2^*-q_1^*)}{\left(\frac{1-q_1^*}{2}\right)(1+q_1^*)+\left(\frac{q_2^*}{2}\right)(2-q_2^*)} q_2^*(1-q_1^*)$$

Now, we compute

$$E(v\overline{q}_1 - cq_1^*) \,|\, (q_1^* > \overline{q}_1,\ q_2^* < \overline{q}_2,\ q_1^* - \overline{q}_1 < \overline{q}_2 - q_2^*) =$$
$$E(v\overline{q}_1 - cq_1^*) \,|\, (q_1^* \ > \overline{q}_1 > q_1^* + q_2^* - \overline{q}_2,\ \overline{q}_2 > q_1^* - \overline{q}_1 + q_2^*)$$

This implies that

$$E(\overline{q}_1) \,|\, (q_1^* > \overline{q}_1,\ q_2^* < \overline{q}_2,\ q_1^* - \overline{q}_1 < \overline{q}_2 - q_2^*) =$$
$$= \frac{2q_1^* + q_2^* - E(\overline{q}_2)}{2}$$

$$E(\overline{q}_2) \,|\, (q_1^* > \overline{q}_1,\ q_2^* < \overline{q}_2,\ q_1^* - \overline{q}_1 < \overline{q}_2 - q_2^*) =$$
$$= \frac{q_1^* + \ q_2^* - E(\overline{q}_1) + 1}{2}$$

Finally, we obtain:

$$E(\overline{q}_1) \,|\, (q_1^* > \overline{q}_1,\ q_2^* < \overline{q}_2,\ q_1^* - \overline{q}_1 < \overline{q}_2 - q_2^*) = q_1^* + \frac{1}{3}q_2^* - \frac{1}{3}$$
$$E(\overline{q}_2) \,|\, (q_1^* > \overline{q}_1,\ q_2^* < \overline{q}_2,\ q_1^* - \overline{q}_1 < \overline{q}_2 - q_2^*) = \frac{1}{3}q_2^* + \frac{2}{3}$$

Finally, we have that

$$E(v\overline{q}_1 - cq_1^*) \,|\, (q_1^* > \overline{q}_1,\ q_2^* < \overline{q}_2,\ q_1^* - \overline{q}_1 < \overline{q}_2 - q_2^*) =$$
$$v\left(q_1^* + \frac{1}{3}q_2^* - \frac{1}{3}\right) - cq_1^*$$

As a result

$$3) = v\left(q_1^* + \frac{1}{3}q_2^* - \frac{1}{3}\right) - cq_1^*$$

Finally, let us compute

$$E(\pi | q_1^* > \overline{q}_1,\ q_2^* > \overline{q}_2,\ q_1^* - \overline{q}_1 > \overline{q}_2 - q_2^*) =$$
$$E((v\overline{q}_1 - cq_1^*) | q_1^* > \overline{q}_1,\ q_2^* > \overline{q}_2,\ q_1^* - \overline{q}_1 > \overline{q}_2 - q_2^*) = v\frac{q_1^*}{2} - cq_1^*$$

Thus, we have

$$5) = v\frac{q_1^*}{2} - cq_1^*$$

We have by now established the following results:

$$1)\ irrelevant$$
$$2) = 0$$
$$3) = v\left(q_1^* + \frac{1}{3}q_2^* - \frac{1}{3}\right) - cq_1^*$$
$$4) = \frac{\left(1 - \frac{q_2^*}{2}\right)(3 - 2q_1^* - q_2^*)}{\left(\frac{1-q_2^*}{2}\right)(1 + q_2^*) + \left(\frac{q_1^*}{2}\right)(2 - q_1^*)} q_1^* (1 - q_2^*)$$
$$5) = v\frac{q_1^*}{2} - cq_1^*$$
$$6) = q_1^*\ q_2^*$$
$$7) = \frac{vq_1^*}{3} - cq_1^* + p_1\frac{q_1^*}{3}$$
$$8) = \frac{\left(\frac{1-q_2^*}{2}\right)(1 + q_2^*) + \frac{(q_2^*+q_1^*-1)}{2}(3 - q_2^* - q_1^*)}{\left(\frac{1-q_2^*}{2}\right)(1 + q_2^*) + \left(\frac{q_1^*}{2}\right)(2 - q_1^*)} q_1^* (1 - q_2^*)$$
$$9)\ (vq_1^* - cq_1^*) + (v - p_2)\frac{q_2^*}{3}$$
$$10) = \frac{\frac{(q_2^*+q_1^*-1)}{2}(3 - q_2^* - q_1^*) + \left(1 - \frac{q_1^*}{2}\right)(1 + q_1^*)}{\left(\frac{1-q_1^*}{2}\right)(1 + q_1^*) + \left(\frac{q_2^*}{2}\right)(2 - q_2^*)} (1 - q_1^*)\, q_2^*$$
$$11)\ irrelevant$$
$$12) = 0$$
$$13)\ (vq_1^* - cq_1^*)$$
$$14) = \frac{\left(1 - \frac{q_1^*}{2}\right)(3 - 2q_2^* - q_1^*)}{\left(\frac{1-q_1^*}{2}\right)(1 + q_1^*) + \left(\frac{q_2^*}{2}\right)(2 - q_2^*)} q_2^* (1 - q_1^*)$$
$$15)\ (vq_1^* - cq_1^*)$$
$$16) = (1 - q_1^*)(1 - \ q_2^*)$$

Now, we are ready to state and solve the maximization problem that each firm faces, when it has choose the quantity to produce and the price to charge to the other firm, taking the price it is charged by the other firm, and the quantity produced by the rival, as given. The two firms are ex ante equal, and the only difference is introduced ex post, when the demand is revealed.

Firm 1 solves the following problem:

$$\underset{q_1^*,\ p_1}{Max}$$

$$v\left(q_1^* + \frac{1}{3}q_2^* - \frac{1}{3}\right) - cq_1^*$$

$$+\frac{\left(1-\frac{q_2^*}{2}\right)(3-2q_1^*-q_2^*)}{\left(\frac{1-q_2^*}{2}\right)(1+q_2^*)+\left(\frac{q_1^*}{2}\right)(2-q_1^*)}q_1^*(1-q_2^*)$$

$$+v\frac{q_1^*}{2} - cq_1^*$$

$$+q_1^*\ q_2^*$$

$$+\frac{vq_1^*}{3} - cq_1^* + p_1\frac{q_1^*}{3}$$

$$+\frac{\left(\frac{1-q_2^*}{2}\right)(1+q_2^*)+\frac{(q_2^*+q_1^*-1)}{2}(3-q_2^*-q_1^*)}{\left(\frac{1-q_2^*}{2}\right)(1+q_2^*)+\left(\frac{q_1^*}{2}\right)(2-q_1^*)}q_1^*(1-q_2^*)$$

$$+(vq_1^*-cq_1^*)+(v-p_2)\frac{q_2^*}{3}$$

$$+\frac{\frac{(q_2^*+q_1^*-1)}{2}(3-q_2^*-q_1^*)+\left(1-\frac{q_1^*}{2}\right)(1+q_1^*)}{\left(\frac{1-q_1^*}{2}\right)(1+q_1^*)+\left(\frac{q_2^*}{2}\right)(2-q_2^*)}(1-q_1^*)\,q_2^*$$

$$(vq_1^*-cq_1^*)$$

$$+\frac{\left(1-\frac{q_1^*}{2}\right)(3-2q_2^*-q_1^*)}{\left(\frac{1-q_1^*}{2}\right)(1+q_1^*)+\left(\frac{q_2^*}{2}\right)(2-q_2^*)}q_2^*(1-q_1^*)$$

$$+(vq_1^*-cq_1^*)$$

$$+(1-q_1^*)(1-\ q_2^*)$$

To make the previous equation a bit more tractable, assume $c = 1$. The optimal point is the following:

$$p = -\frac{87}{16} + \frac{67}{16}v, x = \frac{1}{2}$$

The quantity produced in this case is higher than in the case of lack of interconnession, and so is the profit for each firm. Indeed, the following inequality

$$\frac{v}{2} + \frac{1}{2} - v + 1 + 1 + \frac{1}{v^2} - \frac{2}{v} < \frac{91}{192}v - \frac{37}{64}$$

is verified for all values of v. The previous thread of argument is valid only for $\frac{29}{17} < v < \frac{103}{67}$, given our assumptions on v and c ■

3 Version D: interconnection, intermediation, dynamic (incomplete)

Now, let us see the final case. The two firms sell their energy to a monopolistic retailer of energy, that is, an intermediary between the two firms and the consumer. Suppose that the intermediary does not have to compete with anyone, i.e., the firms are selling their products to him. The intermediary takes the risk of uncertainty over quantity. We have now two possible cases. In the first case, the intermediary takes the whole burden of risk on himself

Formally, there are new assumptions besides 1-6, 7b-9, and they are:

14. There is a monopolistic intermediary to which the two energy firms sell their product

15. The intermediary bears the aggregate uncertainty, and maximizes the expected ask-bid spread

16. In a two stage game, at first the intermediary sets a price, as a function of the demand, and then the two monopolists solve their problem with certainty

17. The two regional monopolists still behave monopolistically (it would make more sense to assume they play Cournot with capacity constraints. I'll do it later)

Proposition 9 *The retailer sells* $2\left(\frac{28}{17}+\frac{1}{51}\sqrt{6}\left(\frac{17}{2}\sqrt{6}\left(\frac{1}{102}c\sqrt{6}-\frac{14}{17}\right)\right)\right)$

Proof. Now, the intermediary does the following:

$$\max E\left(vq^*-cq^*|q^*<\overline{q}\right)\Pr\left(q^*<\overline{q}\right)+E\left(v\overline{q}-cq^*|q^*>\overline{q}\right)\Pr\left(q^*>\overline{q}\right)$$

Now, as we did previously, we try to solve th previous equation term by term, and get the following:

$$\Pr\left(q^*<\overline{q}\right)=\frac{\left(2-q^*\right)^2}{2}$$

$$\Pr\left(q^*>\overline{q}\right)=1-\frac{\left(2-q^*\right)^2}{2}$$

$$E\left(\overline{q}\right)=\sqrt[2]{2q^*-\frac{q^{*2}}{2}-1}$$

$$E\left(v\overline{q}-wq^*\right)=v\left(\sqrt[2]{2q^*-\frac{q^{*2}}{2}-1}\right)-cq^*$$

Now, the problem is the following:

$$\left(x-wx\right)\frac{\left(2-x\right)^2}{2}+\left(\left(\sqrt[2]{2x-\frac{x^2}{2}-1}\right)-wx\right)\left(1-\frac{\left(2-x\right)^2}{2}\right)$$

At the optimum, $x = \frac{28}{17} + \frac{1}{51}\sqrt{6}w$.

The monopolists maximize their production knowing the level of demand by the intermediary, who assumes on himself the whole risk. Therefore, they solve a standard optimization problem, not involving any uncertainty. Each monopolist maximizes:

$$\max_{w} \frac{q(w)}{2}(w-c)$$

$$\left(\frac{28}{34} + \frac{1}{102}\sqrt{6}w\right)(w-c)$$

At the optimum, the profit of the intermediary is given by $\left\{\left(\frac{1}{204}c\sqrt{6} + \frac{7}{17}\right)\left(\frac{17}{2}\sqrt{6}\left(\frac{1}{102}c\sqrt{6} - \frac{14}{17}\right) - c\right)\right\}$, at $[w =]$, supposing the two firms can collude to a monopoly price.

The total quantity produced is then

$$2\left(\frac{28}{17} + \frac{1}{51}\sqrt{6}\left(\frac{17}{2}\sqrt{6}\left(\frac{1}{102}c\sqrt{6} - \frac{14}{17}\right)\right)\right) > 2$$

■

In conclusion, the intermediary bears the risk, increases the firms' profit, and

4 Conclusion

It is obviously better for consumers to be served always, and not to be hit by a blackout. However, blackouts arise. In the model, and proably in the industry too, they arise because of uncertainty in demand. Connecting wires can share the risk. The question that is in my mind, and that I hope this paper will address when it will be completed, are the following:

1) Will the energy firms be willing to connect in order to avoid blackouts? The answer in the paper is yes, but we did not consider the fixed cost of establishing the connection, very important indeed

2) What is the optimal structure of the market, i.e., what would the outcome of an unregulated process be? Would this outcome avoid blackouts?

I believe the questions are important, as the energy market is subject to a lot of regulations that ASSUME to be inspired to a competitive outcome. But the question is: are we sure the competitive outcome regulators inspire to is the true competitive outcome?

And more, are we sure that the true competitive outcome wouldn't be welfare enhancing also for consumers with respect to the regulation outcome?

5 References

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