Adoption of Product and Process Innovations in Differentiated Markets: The Impact of Competition

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Abstract

The paper examines the effects of the degree of competition on the firms' decision to innovate in differentiated markets. We find that a low (high) degree of product differentiation (competition) weakly supports the introduction of new products. Firms' weakly favour a process innovation if the degree of product differentiation (competition) is high (low). In addition, assumptions on the strategic complementarity of product and process innovations and on the decreasing returns of a product innovation are found to be the critical assumptions in the sense of Milgrom and Roberts (1994).
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1 Introduction

Every year, the producers of basic fashion items have to decide whether they want to introduce a new collection, to sell the present one with no or only small

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changes, or to reorganise the production process for the present collection in the next year. This is an example for a familiar and reoccurring situation which every chief executive officer faces. Basically, it describes a decision problem: Does it pay for a firm to pursue a product innovation (new collection), to undertake a process innovation (reorganisation of production) or to do nothing? Clearly, by choosing one or the other option, the firm changes its position and prospectives over the next years' competition. It may also be suspected that firms' choice depends on the market structure, i.e. whether or not the competition is intense. On the one hand, one may argue that firms try to escape fierce competition by introducing new, differentiated products. On the other hand, firms may tend to a more aggressive strategy and try to reduce costs so that competitors are induced to exit the market altogether. The present paper is concerned with the question of whether or not intense competition supports the introduction of new products. In addition, the paper aims at discovering the critical assumption in the sense of Milgrom and Roberts (1994).

The interrelation of the market structure of an industry and the R&D investment or innovations has received much attention over decades.[1] Earlier works were mostly concerned with R&D investments in homogeneous product markets (for an excellent overview see e.g. Reinganum, 1992). Within this strand of literature, the effect of the degree of competition on the incentives to innovate were widely discussed. In his seminal contribution, Schumpeter (1942) argued in favour of the monopoly while Arrow (1962) established the reverse proposition. Only recently, this question has been addressed in a framework of a differentiated product market.[2]

Most contributions have focused on either a product or a process innovation or else have left unspecified which particular type of innovation is studied. However,

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1. There are many aspects of market structure. Here, it is used as a synonym for the degree of competition. Also, (R&D) investments and innovations are used interchangeably.
2. See e.g. Bester and Petrakis (1993) for cost–reducing innovations in a model of horizontal product differentiation. Among others, Bonanno and Haworth (1998) and Boone (2001) consider process innovations in a model of vertical product differentiation. See e.g. Greenstein and Ramey (1998), Shaked and Sutton (1990), and Rosenkranz (1995) for models of vertically differentiated products and product innovations.

often the firms have the choice at least between a product and a process innovation. Only a few contributions directed attention to this problem. Among them are Rosenkranz (1996), Bonanno and Haworth (1998) and Filippini and Martini (2002). Rosenkranz considers a duopoly in a horizontally differentiated market with Cournot competition. She demonstrates that firms will always invest into product and process innovations and that an increase in the market size shifts R&D investments towards product innovations.

Bonanno and Haworth rely on a model with vertical product differentiation. Only one of the duopolists has the chance to innovate and the respective firm has to decide between a product and a process innovation. They find that the high (low) quality firm opts for the product (process) innovation when the competition is intense (Bertrand competition). The reverse result occurs when the degree of competition is low (Cournot competition). Filippini and Martini extend the model of Bonanno and Haworth in that the duopolists simultaneously decide on the innovation to be carried out. They find two equilibria in which the low and high quality firm choose the same type of innovation and one asymmetric equilibrium. In the latter, the high (low) quality firm undertakes a product (process) innovation. In addition, they demonstrate that only the asymmetric case supports a situation in which the vertical intensity of competition is relaxed.

The present paper studies the innovation problem as a two–stage decision game. In the initial state there are two firms each producing a variant of a basic good. In the first stage, the firms simultaneously choose among three alternatives: a product innovation, a process innovation, or no innovation at all. In the second–stage they compete in the market. Deliberately, the nature of the product differentiation is left unspecified, so that the model may apply to a vertical differentiation, to a horizonal one or a mixture of both. Also, the nature of the second–stage market game is not defined, so that the firms' strategic variable may be the price as well as the quantity. Under these circumstances, a product innovation is a change in the degree of product differentiation and a process innovation is a reduction in the unit costs of production. We find that intense competition, i.e. an initially low degree of product differentiation weakly, supports the introduction of new products while a low degree of competition weakly promotes process innovations. As critical assumptions in the sense of Milgrom and Roberts (1994) assumptions concerning the strategic complementarity of product and process innovations of a firm and the concerning the diminishing returns of a product innovation are identified. Hence, the paper offers a conclusive guidline as to the situations under which we may expect firms to choose product or process innovations in order to compete successfully in the near future. As deciding for the product innovation in a highly competitive environment, the paper also provides an answer to the question under which circumstances firms react to fierce competiton with an aggressive strategy.

The paper is organised as follows: Section 2 introduces the basic framework. Section 3 describes the properties of the reduced–form profits. In section 4 the firms' innovation decision is studied and the main results are derived. Section 5 investigates the critical assumptions leading to the main results. Two examples are discussed in section 6 and section 7 concludes.

2 The Framework

Consider a differentiated product market. Deliberately, it is left unspecified whether the product differentiation is of a horizontal or vertical nature so that the model applies models of horizontal and vertical product differentiation. There are two firms, $i = 1,2$, producing different variants of a basic good. Which particular variant a firm supplies is considered to be the result of history.

The firm's decision problem is modelled as a two–stage, non–cooperative game. In the first stage, the firms simultaneously decide on the type of innovation they wish to carry out: a product (*PI*) *or* process innovation (*CI*) *or* no innovation (*NI*). In the second stage, after the innovations are completed, the firms compete in the market. Their strategic variable may be prices (Bertrand competition) or quantities (Cournot competition). As usual, the equilibrium concept applied is that of sub–game perfection.

For expositional reasons and, hence, without loss of generality, it is assumed that both firms initially operate with identical and constant unit cost of production c. There are no fixed costs of production.[3] Therefore, producing variant 1 and 2 is equally expensive in the pre–innovation state. Moreover, it is posited that

[3] As the considered game is a one-shot one and with strictly positive cost of an innovation (see below), the innovation costs may be interpreted including the fixed costs.

only the initial degree of product differentiation d, $d \in [0,1]$, and the scale of the product innovation affect the firms' decision variables.[4] The variants are called a homogeneous product if $d = 0$, and they are completely independent if $d = 1$. Since firms receive maximal (monopoly) profits when they produce independent products and obtain minimal ones if there is only one homogeneous good, the degree of product differentiation is inversely related to the degree of competition.

In absence of any fixed costs, a process innovation is assumed to reduce the variable costs by a fixed amount γ, $\gamma \in (0,c)$. A firm pursuing a product innovation supplies a new variant in the second stage and abandons the 'old' one. By doing so, the degree of product differentiation is increased by a fixed amount δ_i, $\delta_i > 0$, where the subscript indicates the initiator of the product innovation, i.e. $\delta_i = \delta_j$, $i, j = 1,2$. As the degree of product differentiation is restricted to values from the interval $[0,1]$, δ_i has to be such that $\delta_i \in (0,(1-d)/2]$. Preferably, the scale of a product and a process innovation should be regarded to be sufficiently small. Each innovation requires a fixed amount of investment costs. For simplicity the innovation costs I are regarded to be identical for both types of innovation. As the scale of the innovations may be different, this assumption imposes no restriction on the generality of the model. In addition, it serves to exclude differences in the innovation costs as a determinant for or against one type of innovation.

3 The Market Stage

After the firms learned about the rival's first–stage actions, they compete in the market. The second–stage decision variables may be prices as well as quantities. Given the firms behave rationally, the second–stage reduced form profits are functions of the own costs, the rival's costs and the degree or product differentiation, i.e. $\pi_i = \pi_i(c_i, c_j, d)$, where $\pi_i(\cdot)$ is the ith firm's profit net of investment costs.

Rather than specifying the firm's demand and cost functions for a particular

[4] In non–spatial models of product differentiation this assumption is automatically satisfied. Yet, in spatial models, as e.g. the Hotellings–model, the firms' demand function usually depends also on the firms' position on the product line. However, once one considers only symmetric pre–innovation positions on the product line, the demand function becomes independent of the initial position.

model and deducing the properties of the appropriate reduced–form profits, it is posited that the demand and cost functions are such that the reduced form profits have the following properties:

Assumption 1 (A1). *The first derivatives of a firm's reduced form profits obey:*

1. $\Delta_{Ci}\pi_i(c_i, c_{j1}, d) \equiv \pi_i(c_{i2}, c_j, d) - \pi_i(c_{i1}, c_j, d) > 0$,

2. $\Delta_{Cj}\pi_i(c_i, c_{j1}, d) \equiv \pi_i(c_i, c_{j2}, d) - \pi_i(c_i, c_{j1}, d) < 0$,

3. $\pi i(c_i, c_j, d_2) \equiv \pi_i(c_i, c_j, d_1) - \pi_i(c_i, c_j, d_2) > 0$,

where $c_{i1} > c_{i2}$, $d_1 > d_2$ *and* $i, j = 1, 2$.

In accordance with all models of product differentiation, a firm's profit is the higher the lower (higher) the own (rival's) unit costs of production. The last derivative in A1 shows that the profits are supposed to increase with the degree of product differentiation. In other words, the profits increase as the degree of competition becomes smaller.

In addition, the following presumptions are imposed:

Assumption 2 (A2). *The second derivatives of the reduced form profits obey:*

1. $\Delta_{Ci}\pi_i(\cdot)$ *(*$\Delta_{Pi}\pi_i(\cdot)$*) is monotonically increasing in d (*c_i*),*

2. $\Delta_{Ci}\pi_i(\cdot)$ *(*$\Delta_{Cj}\pi_i(\cdot)$*) is monotonically decreasing in* c_j *(*c_i*),*

3. $\Delta_{Cj}\pi_i(\cdot)$ *(*$\Delta_{Pi}\pi_i(\cdot)$*) is monotonically increasing in d (*c_j*),*

4. $\Delta_{Pi}\pi_i(\cdot)$ *is monotonically decreasing in* d.

According to A2 (1), a process and a product innovation are strategic complements from firm *i*'s perspective. This assumption displays the idea that a firm would like to carry out both types of innovation if thy had not to decide for one or the other. A2 (2) specifies that a reduction in a firm's own cost and that of the rival are are strategic substitutes. Hence, leaving the option of a product innovation aside, a firm will try to reduce its costs in response to the competitor's process innovation. A2 (3) shows that firm *i*'s profit decreases less as the rival's efficiency is increased the higher the degree of product differentiation. Therefore, the lower the degree of competition the less cares a firm about the rival's efficiency.[5] Finally, A2 (4) formalises the idea that the benefits from a product innovation exhibits decreasing returns.

Clearly, reduced–form profits depend on the firms' first–stage actions. Consider e.g. the case in which firm i undertakes a process innovation and it's rival neglects either investment option. Given the firms' pre–innovation costs, c, are identical, firm i's second–stage technology can be described by $c-\gamma$. Firm i receives a net profit of $\pi_i(c-\gamma,c,d)$, while firm j's profit is $\pi_j(c,c-\gamma,d)$. Similarly, $\pi_i(c,c,d+\delta_j)$ marks firm i's net payoff, when i declines to innovate and firm j supplies a new variant. Then, the functions $\Delta_{Pi}\pi_i(c,c,d)$ and $\Delta_{Ci}\pi_i(c,c,d)$ can be rewritten to

$$\Delta_{Pi}\pi_i(c,c,d) = \pi_i(c,c,d+\delta_i) - \pi_i(c,c,d), \tag{1}$$

$$\Delta_{Ci}\pi_i(c,c,d) = \pi_i(c-\gamma,c,d) - \pi_i(c,c,d). \tag{2}$$

$\Delta_{Pi}\pi_i(c,c,d)$ denotes firm i's return on a product innovation, given the rival does not invest at all. By analogy, $\Delta_{Ci}\pi_i(c,c,d)$ is firm i's incentive for undertaking a process innovation when the competitor declines to innovate. We find the following:

Lemma 1. *Under Assumption A1, the incentives to introduce a process (product) innovation is strictly positive, i.e.*

$$\Delta_{Pi}\pi_i(\cdot) > 0, \quad \Delta_{Ci}\pi_i(\cdot) > 0, \quad i = 1,2.$$

Proof. This is a direct consequence of A1. □

Independent of the opponent's first–stage action, the incentives to innovate are always positive.

A necessary condition for a product innovation to be attractive is that the benefits from introducing a new product are higher than the one from implementing a cost–reducing technology. Let $\chi_i(c,c,d)$ be defined as

$$\chi_i(c,c,d) \equiv \Delta_{Pi}\pi_i(c,c,d) - \Delta_{Ci}\pi_i(c,c,d). \tag{3}$$

[5] For the case of $d = 1$, i.e. when firms are monopolies, a firm's profit function should be independent of the opponents costs.

Then, firm i prefers a product over a process innovation whenever $\chi_i(\cdot)$ is positive. Here, $\chi_i(\cdot)$ is the relative profitability of a product innovation and has the following property:

Lemma 2. *Under A1 and A2, the relative profitability to introduce a product innovation decreases in the degree of product differentiation, i.e.*

$$\Delta\chi_i(\cdot, d_2) \equiv \chi_i(\cdot, d_1) - \chi(\cdot, d_2) < 0, \quad d_1 > d_2.$$

Proof. According to A2 (4), ${}_d\Delta_{Pi}\pi_i(c,c,d) \equiv \Delta_{Pi}\pi_i(c,c,d+\delta_k) - \Delta_{Pi}\pi_i(c,c,d) < 0$, $k = 1,2$, and A2 (1) implies ${}_d\Delta_{Ci}\pi_i(c,c,d) \equiv \Delta_{Ci}\pi_i(c,c,d+\delta_k) - \Delta_{Ci}\pi_i(c,c,d) > 0$. Given the definition of $\chi_i(\cdot)$ in (3), $\Delta\chi_i(\cdot,d_2) = {}_d\Delta_{Pi}\pi_i(\cdot,d_2) - {}_d\Delta_{Ci}\pi_i(\cdot,d_2) < 0$, with $d_1 > d_2$. □

The relative attractiveness of a product innovation decreases with the initial existing degree of product differentiation. This result arises for all possible first–stage actions of the rival. Hence, the relative benefits to carry out a product innovation is larger when the variants are initially close substitutes and the competition is dense as opposed to the situation where the products are initially nearly independent and the degree of competition is low.

The next Lemma describes how a firm's incentives to innovate depend on the rival's actions.

Lemma 3. *Under A1 and A2, the following inequalities hold true:*

$$\Delta_{Ci}\pi_i(c,c,d+\delta_j) > \Delta_{Ci}\pi_i(c,c,d) > \Delta_{Ci}\pi_i(c,c-\gamma,d), \tag{4}$$

$$\Delta_{Pi}\pi_i(c,c-\gamma,d) > \Delta_{Pi}\pi_i(c,c,d) > \Delta_{Pi}\pi_i(c,c,d+\delta_j). \tag{5}$$

Proof. Part (1): According to A2 (1), $\Delta_{Ci}\pi_i(c,c,d+\delta_j) > \Delta_{Ci}\pi_i(c,c,d)$. From A2 (2) follows $\Delta_{Ci}\pi_i(c,c,d) > \Delta_{Ci}\pi_i(c,c-\gamma,d) > 0$. The first inequality follows immediately.

Part (2): A2 (3) implies $\Delta_{Pi}\pi_i(c,c-\gamma,d) > \Delta_{Pi}\pi_i(c,c,d)$. From A2 (4) we know that $\Delta_{Pi}\pi_i(c,c,d) > \Delta_{Pi}\pi_i(c,c,d+\delta_j)$. □

The first relation in (4) shows that a firm's incentive to implement a cost–reducing technology is higher when the rival chooses a product innovation instead of declining to innovate at all. Clearly, a firm always prefers the competitor to introduce a new product as an increase in the degree of product differentiation reduces the impact of the rival's price or quantity decisions on the firm's demand

and profit — the degree of competition becomes less intense. In addition, according to A2 (1), a reduction of the own costs and an increase of the degree of product differentiation are strategic complements, so that the benefits from a process innovation are the higher the more distinct the variants. The second relation in (4) reveals that the incentive to implement a new technology are higher when the rival does nothing as compared to a situation in which he adopts a process innovation as well. The degree of product differentiation is identical in both cases. Hence, a firm always receives a higher return when interacting with a less efficient competitor.

Similarly, the first relation in (5) describes the fact that the incentives to introduce a new product are higher when the competitor undertakes a process innovation instead of declining the innovations. This is a consequence of A2 (3) showing that a product innovation and an increase in the rival's costs are strategic substitutes. Hence, by undertaking a product innovation, a firms counterbalances the negative effects of the competitor's cost reduction. Finally, the second relation in (5) evinces that a firm's incentive to carry out a product innovation is higher when the rival does nothing rather than choosing a product innovation as well. The notation $\pi_i(c,c,d+\delta_j)$ reveals that it is unimportant which firm initiates the product innovation for the firms' profits *net* of innovation costs. Only the result matters, i.e. the increase (decrease) of the degree of product differentiation (competition). Therefore, the second part of the inequality in (5) reflects the assumption of a product innovation's diminishing returns. The notation also implies that a firm offering a new product always generates a positive external effect.

The next Lemma establishes a comparable connection between the relative profitability to introduce a product innovation for the different options of the competitor.

Lemma 4. *Under A1 and A2, the following holds true:*

$$\chi_i(c,c-\gamma,d) > \chi_i(c,c,d) > \chi_i(c,c,d+\delta_j).$$

Proof. Given (3), $\chi_i(c,c-\gamma,d) - \chi_i(c,c,d) = [\Delta_{Pi}\pi_i(c,c-\gamma,d) - \Delta_{Pi}\pi_i(c,c,d)] - [\Delta_{Ci}\pi_i(c,c-\gamma,d) - \Delta_{Ci}\pi_i(c,c,d)]$. According to Lemma 3, the first bracket term is positive, while the second one is negative. This proves the first part of the inequality. Similarly, $\chi_i(c,c,d) - \chi_i(c,c,d+\delta_j) = [\Delta_{Pi}\pi_i(c,c,d) - \Delta_{Pi}\pi_i(c,c,d+\delta_j)] - [\Delta_{Ci}\pi_i(c,c,d) -$

$\Delta_{Ci}\pi_i(c,c,d+\delta_j)]$. Again, from Lemma 3 follows that the first bracket term is positive and the second one negative. This proves the second part of the inequality. □

The incentive and the relative attractiveness of introducing a new variant have identical properties: The relative return on a product innovation is highest when the rival pursues a process innovation, and lowest when the competitor decides in favour of a product innovation.

4 The Innovation Decisions

In the first stage of the game, the firms simultaneously decide whether to pursue a process or a product innovation or to decline innovations altogether. Given the firms behave rationally, they anticipate the second stage actions.

An industry configuration is represented by a tupel indicating the firms' first–stage actions. Hence (PI,NI) stands for a situation in which firm 1 pursues a product innovation (PI) and firm 2 does not innovate (NI). By analogy, (CI,CI) shows that both firms undertake a process innovation (CI).

Since a firm has three options, there are nine different industry configurations: three symmetric and six asymmetric ones. Due to the symmetry of the game, we know that (NI,PI) is a Nash equilibrium whenever (PI,NI) proves to be an equilibrium. Consequently, we need only to consider the follow six potential equilibria: (NI,NI), (PI,PI), (CI,CI), (PI,NI), (CI,NI), and (PI,CI).

The configuration (PI,NI) constitutes an equilibrium if

$$\begin{aligned} \pi_i(c,c,d+\delta_i)-I &> \pi_i(c,c,d), \\ \pi_i(c,c,d+\delta_i)-I &> \pi_i(c-\gamma,c,d)-I, \\ \pi_j(c,c,d) &> \pi_j(c,c,d+\delta_j)-I, \\ \pi_j(c,c,d) &> \pi_j(c-\gamma,c,d)-I. \end{aligned} \tag{6}$$

By analogy, one can establish the other 5 equilibrium conditions.

To establish one of the main results, it is useful to introduce two additional variables. Let $d_{\delta i}$ and $d_{\gamma i}$ be defined as $d_{\delta i} \equiv \{d \in [0,1] : \chi_i(c,c,d+\delta_j)=0\}$ and $d_{\gamma i} \equiv \{d \in [0,1] : \chi_i(c,c-\gamma,d)=0\}$. Then, $d_{\delta i}$ ($d_{\gamma i}$) is the degree of product differentiation for which firm i is indifferent between introducing a new product and a new process given the rival undertakes a product (process) innovation. By examining the equilibrium conditions closely, the subsequent result ensues.

Proposition 1. *Given $d_{\delta i} > 0$, $d_{\gamma i} < 1$ and A1 and A2 are satisfied, the following industry configurations constitute Nash equilibria:*

1. (NI,NI) *for* $I \in [\max\{\Delta_{Pi}\pi_i(c,c,d),\Delta_{Ci}\pi_i(c,c,d)\},\infty)$,

2. (PI,NI) *for* $I \in [\max\{\Delta_{Pi}\pi_i(c,c,d+\delta_j),\Delta_{Ci}\pi_i(c,c,d+\delta_j)\},\Delta_{Pi}\pi_i(c,c,d))$ *and* $\chi_i(c,c,d) > 0$,

3. (CI,NI) *for* $I \in [\max\{\Delta_{Pi}\pi_i(c,c-\gamma,d),\Delta_{Ci}\pi_i(c,c-\gamma,d)\},\Delta_{Ci}\pi_i(c,c,d))$ *and* $\chi_i(c,c,d) < 0$,

4. (PI,CI) *for* $I \in [0,\min\{\Delta_{Pi}\pi_i(c,c-\gamma,d),\Delta_{Ci}\pi_i(c,c,d+\delta_j)\})$, $\chi_i(c,c,d+\delta_j) < 0$ *and* $\chi_i(c,c-\gamma,d) > 0$,

5. (PI,PI) *for* $I \in [0,\Delta_{Pi}\pi_i(c,c,d+\delta_j))$ *and* $\chi_i(c,c,d+\delta_j) > 0$,

6. (CI,ic) *for* $I \in [0,\Delta_{Ci}\pi_i(c,c-\gamma,d))$ *and* $\chi_i(c,c-\gamma,d) < 0$.

Proof. The proof is in the Appendix. □

Figure 1 represents the first–stage equilibria in an I/d–diagram. It can be seen that the number of innovations observed decreases with higher innovation costs I independent of the degree of product differentiation. Clearly, for very high levels of the innovation costs, neither firm will find it profitable to innovate at all as the innovation costs exceed the incentives for the innovations. On the other hand, if the innovation costs are zero, both firms will innovate as long as the incentives are strictly positive. The latter is ensured by Lemma 1. By analogy, we find that both firms undertake innovations when the innovation costs are low. For intermediate levels of the innovation costs, only one firm innovates. The other firm prefers to do nothing.

In addition, the following result can be verified in Figure 1.

Corollary 1. *Given $d_{\delta i} \in (0,1)$ and $d_{\gamma i} \in (0,1)$ and A1 and A2 are satisfied we find that the firms have a weakly prefer product over process innovations for similar products. For distinct products, the firms weakly favour process to product innovations.*

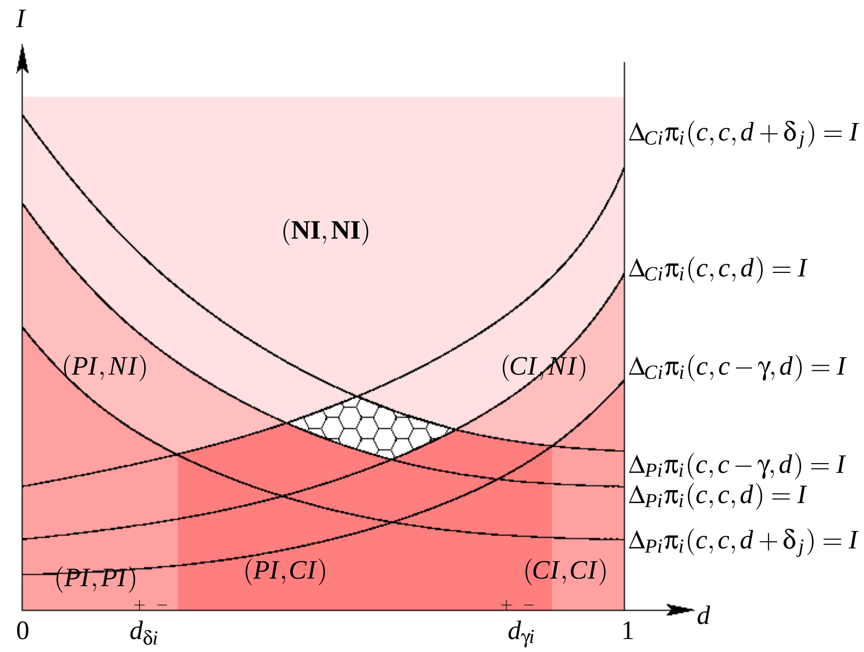

Figure 1: The equilibria

Proof. This is a direct consequence of Proposition 1. □

To reveal the forces leading to Corollary 1, the extreme cases of a homogeneous product, $d = 0$, and the one of completely independent products, $d = 1$ and zero innovation costs are considered. Clearly, when products are completely independent, the firms are monopolies and earn the appropriate profit. Introducing a new product cannot increase the degree of product differentiation and, hence, the profit. Accordingly, the only measure by which the profits can still be increased is by trying to produce more efficiently, i.e. to undertake a process innovation. When there is only one homogeneous product, $d = 0$, the degree of competition takes its maximum. Together with the assumption that $d_{\delta i} > 0$, this implies that to introduce a new product becomes a dominant strategy independent of the rival's choice. Firms benefit less from a process than from a product innovation, they try to escape competition.

Clearly, the assumptions of $d_{\delta i} > 0$ and $d_{\gamma i} < 1$ play a role. Removing them leads to the following result:

Corollary 2. *When* $d_{\delta i} < 0$ *the symmetric industry configuration (2,0) is not an NE. Similarly, if* $d_{\gamma i} > 1$, *the configuration (0,2) fails to be an NE.*

Proof. This is also an immediate consequence of Proposition 1 (5) and (6). □

This can directly be verified in Figure 1. According to the definition of the variable $d_{\delta i}$, $d_{\delta i} < 0$ indicates that firm *i* prefers a process to a product innovation even if the good good is homogeneous, given the rival introduces a new product. Similarly, $d_{\gamma i} > 1$ signifies that firm *i* opts for the new product even if the products are completely independent, whenever the rival chooses the new process.

Finally, the almost black area in Figure 1 displays combinations of the innovation costs and the degree of product differentiation where multiple equilibria occur. From Proposition 1 follows that (NI, NI) and (PI, CI) are the industry configurations possibly observed.

5 The Robustness of the Results

The main result in Corollary 1 reveals that firms tend to undertake product innovations when the degree of product differentiation is low in order to escape the intense competition. In contrast, firms are inclined to pursue a process innovation thereby improving their efficiency if the degree of competition is low. The present section deals with the critical assumption in the sense of Milgrom and Roberts (1994) leading to the results of Corollary 1.

Inspecting point (4) through (6) of Proposition 1 closely reveals that the properties of the relative profitability of a product innovation $\chi_i(\cdot)$ determine whether firms bend towards a product or process innovation when competition is fierce. Lemma 2 and 4 state the properties of $\chi_i(\cdot)$ associated with A1 and A2. According to Lemma 2, $\chi_i(\cdot)$ is negatively correlated to the degree of product differentiation. In Figure 1 the plus and the minus sign indicate the sign of the appropriate $\chi_i(\cdot)$ function. From (5) of Proposition 1 it becomes apparent that the results stated in Corollary 1 are reversed if $\chi_i(\cdot)$ is positively instead of negatively correlated to the degree of product differentiation. Whether or not $\chi_i(\cdot)$ decreases with d depends on A2 (2) and A2 (4). Hence, A2 (2) and A2 (4) are the critical assumptions. More formally, we find:

Lemma 5. *The relative attractiveness of a product innovation* $\chi_i(\cdot)$ *obeys*

$$\Delta\chi_i(\cdot,d2) \lesseqqgtr 0 \quad \Longleftrightarrow \quad {}_d\Delta_{Pi}\pi_i(\cdot,d_2) \lesseqqgtr {}_d\Delta_{Ci}\pi_i(\cdot,d_2), \quad d_1 > d_2.$$

Proof. This follows directly from the definition of $\chi_i(\cdot)$. □

According to the Lemma, similar products support the introduction of new products whenever a small increase in d has a larger effect on the incentive to create a new product than on the one to implement a new process. By analogy, intense competition favours the adoption of cost–reducing technologies when a small rise in the product differentiation leads to a greater change in the return on a process innovation as compared to the one on a product innovation. Clearly, when ${}_d\Delta_{Pi}\pi_i(\cdot)$ is positive (negative), a product innovation exhibits increasing (decreasing) returns. Similarly, if ${}_d\Delta_{Ci}\pi_i(\cdot)$ is positive (negative), a cost reduction and a new product are complements (substitutes) from firm *i*'s point of view. Accordingly, the combination of assumptions on the complementarity of *CI* and *PI* and on the diminishing returns of a π determines whether or not firms tend to introduce new products when the degree of competition is high. The following Proposition gives an overview on all possible combinations:

Proposition 2. *(a) Corollary 1 holds true whenever*

1. *PI and CI are complements for firm i and d exhibits decreasing returns or*

2. *PI and CI are complements, d exhibits increasing returns and* $|{}_d\Delta_{Pi}\pi_i(\cdot)| < |{}_d\Delta_{Ci}\pi_i(\cdot)|$ *or*

3. *PI and CI are substitutes, d exhibits decreasing returns and* $|{}_d\Delta_{Pi}\pi_i(\cdot)| > |{}_d\Delta_{Ci}\pi_i(\cdot)|$.

(b) The results of Corollary 1 are reversed whenever

1. *PI and CI are substitutes for firm i and d exhibits increasing returns or*

2. *PI and CI are complements, d exhibits increasing returns and* $|{}_d\Delta_{Pi}\pi_i(\cdot)| > |{}_d\Delta_{Ci}\pi_i(\cdot)|$ *or*

3. *PI and CI are substitutes, d exhibits decreasing returns and* $|{}_d\Delta_{Pi}\pi_i(\cdot)| < |{}_d\Delta_{Ci}\pi_i(\cdot)|$.

Proof. From A2 (1) follows that *CI* and *PI* are complements (substitutes) if ${}_d\Delta_{Ci}\pi_i(\cdot)$ is positive (negative). By A2 (4), d exhibits decreasing (increasing) returns if ${}_d\Delta_{Pi}\pi_i(\cdot)$ is negative (positive).

Part (a): 1. If *CI* and *PI* are complements and d exhibits decreasing returns it is true that ${}_d\Delta_{Pi}\pi_i(\cdot) < 0 < {}_d\Delta_{Ci}\pi_i(\cdot)$, so that $\Delta\chi_i < 0$ by Lemma 5.

2. If *CI* and *PI* are complements and d yields increasing returns $0 < {}_d\Delta_{Pi}\pi_i(\cdot) < {}_d\Delta_{Ci}\pi_i(\cdot)$ implies $\Delta\chi_i(\cdot) < 0$ by Lemma 5.

3. If *CI* and *PI* are substitutes and d exhibits increasing returns ${}_d\Delta_{Pi}\pi_i(\cdot) < {}_d\Delta_{Ci}\pi_i(\cdot) < 0$ yields $\Delta\chi_i(\cdot) < 0$ by Lemma 5.

Hence, for cases (1) through (3), $\Delta\chi_i(\cdot)$ is negative. Given the functions $\chi_i(\cdot)$ possess a zero in $(0,1)$, the configurations (PI,NI) and (PI,PI) are equilibria for small values of the degree of product differentiation d by Proposition 1 (2) and (5). In contrast, the configurations (CI,NI) and (CI,CI) will be equilibria for high value of product differentiation according to Proposition 1 (3) and (6).

Part (b): By analogy, it can be established that $\Delta\chi_i(\cdot)$ is positive for cases (1) through (3). Given the functions $\chi_i(\cdot)$ possess a zero in $(0,1)$, the conditions on $\chi_i(\cdot)$ stated in Proposition 1 (2) and (5) are satisfied for high values of d so that (PI,NI) and (PI,PI) are equilibria when the products are distinct. Similarly, the appropriate functions of $\chi_i(\cdot)$ are negative for small values of d, so that (CI,NI) and (CI,CI) are equilibria for similar products according to Proposition 1 (3) and (6). □

6 Applications

So far, neither the nature of the product differentiation, i.e. horizontal, vertical or both, nor the one of the product market competition, i.e. Bertrand or Cournot, has been specified. In addition, the results have been derived without resting on a specific demand function. Consequently, any combination of a specific demand function, strategic–second stage variable and nature of product differentiation will yield the results stated in Proposition 1, Corollary 1 and 2 as long as the reduced–form profits obey assumptions A1 and 2. Clearly, depending on the precise specification of the model, the above results may be valid for certain parameter values only. We now present two examples, a Bertrand and a Cournot model of non–spatial product differentiation with linear demand functions, to demonstrate how these models fit into the analysis presented above.

Assume that the inverse demand functions are given by

$$p_i = a - q_i - \theta q_j, \quad i = 1,2, \tag{7}$$

where θ, $\theta \equiv 1-d$, is also a measure of the degree of product differentiation. Then, the corresponding direct demand schedule reads (cf. Vives, 2000, pp. 146)

$$q_i = \alpha - \beta p_i + \gamma p_j, \quad i = 1,2, \tag{8}$$

with $\alpha \equiv a/(1+\theta)$, $\beta \equiv 1/(1-\theta^2)$, and $\gamma \equiv \theta/(1-\theta^2)$. The reduced–form profits are derived with

$$\pi_i^C = \left(\frac{2(a-c_1)-(1-d)(a-c_2)}{(1+d)(3-d)}\right)^2, \tag{9}$$

$$\pi_i^B = \frac{1}{d(2-d)}\left(\frac{(1+2d-d^2)a(a-c_1)-(1-d)(a-c_2)}{(1+d)(3-d)}\right)^2, \tag{10}$$

where $\theta = 1-d$ has been used and the superscript C (B) stands for Cournot (Bertrand).

In the case of Cournot competition on the second stage, the second–order derivatives read

$$\frac{\partial^2 \pi_i^C(c,c,d)}{\partial c_i \partial d} = \frac{4(a-c)(1-3d)}{(3-d)^3(1+d)^2},$$
$$\frac{\partial^2 \pi_i^C(c,c,d)}{\partial d^2} = \frac{6(a-c)^2}{(3-d)^4} > 0$$

for the situation in which neither firm innovates. The lower row shows that a product innovation exhibits increasing returns under Cournot competition for all values of the degree of product differentiation. According to the first row, a product and a process innovation are substitutes whenever $d < 1/3$, i.e. for very similar products. If $d \geq 1/3$ a product and a process innovation are strategic complements. Thus, for $d \in [0,1/3)$, Cournot competition satisfies condition (b)1 of Proposition 2. For values of the degree of product differentiation in the range of $d \in [1/3,1]$, either condition (a)2 or (b)2 is satisfied. Which of the cases proves to be relevant depends on the magnitude of $\partial \Delta_{Ci}\pi_i(c,c,d)/\partial d$ relative to $\partial \Delta_{Ci}\pi_i(c,c,d)/\partial d$. Per definition, both expressions depend on the scale of a process innovation, γ, and a product innovation δ. Hence, for suitable restrictions on γ and δ and all degrees of product differentiation, the reverse results of Corollary 1 are applicable even though a process and a product innovation are not strategic substitutes for all values of d.

In the case of Bertrand competition, the second–order derivatives are given by

$$\frac{\partial^2 \pi_i^B(c,c,d)}{\partial c_i \partial d} = -\frac{2(a-c)\phi_c}{(3-d)^2(2-d)^2(1+d)^3} < 0,$$
$$\frac{\partial^2 \pi_i^B(c,c,d)}{\partial d^2} = \frac{6(a-c)^2\phi_d}{(2-d)^3(1+d)^4},$$

with $\phi_c \equiv 2(1-d)^4 + 3(1-d)(1+d^2) + 4d^2 > 0$ and $\phi_d \equiv d^3 - 2d^2 + 4d - 2$. Again, both equations display the situation in which both firms prefer not to innovate. According to the first equation, a process and a product innovation are strategic complements for all values of d under Bertrand competition. However, a product innovation may exhibit decreasing as well as increasing returns. When there is only one homogeneous product, i.e. $d = 0$, the expression in the second row becomes negative, so that condition (a)1 of Proposition 2 applies. For independent products, i.e. $d = 1$, the derivative become positive, so that a production innovation yields increasing returns. Despite the possibility of increasing returns of a product differentiation, the results of Corollary 1 may hold for all values of d given suitable restrictions on the scale of the process and product differentiation.

The considered examples of a linear demand schedule reveal a slight tendency for second–stage Cournot competition to support process innovations for similar products. By analogy we find that the second–stage Bertrand competition promotes the product innovation when products are similar. However, for demand function different from the linear ones, this pattern need not to carry over.

7 Conclusions

In a general framework, we demonstrate that firms bend towards product innovations when the degree of competition is intense and the innovation costs are low. The rationale behind is simply to escape competition by introducing distinct products. As the degree of competition decreases, the returns on a product innovation decrease as well. As a consequence, firms favour process innovations when the degree of competition attains its minimum, i.e. when firms produce distinct products. In this situation they are monopolies and introducing an alternative product cannot increase profits any further. For intermediate levels of competition and low or intermediate levels of the innovation costs, we find that firms choose different alternatives. In case of low innovation costs, both firms innovate but choose different types.

The results apply for a wide variety of models including spatial and non-spatial product differentiation, Bertrand or Cournot competition and any type of demand functions. The only restrictions under which the above cited results are obtained are that the relative profitability of a product innovation is negatively correlated to the degree of product differentiation. If a particular model yields a positive correlation between the relative attractiveness of a product innovation and the degree of product differentiation, the results are reversed. The critical assumptions determining whether or not the relative profitability of a product differentiation negatively depends on the degree of product differentiation negative have been identified as the strategic complementarity of product and process innovations for a firm and the diminishing returns of a product innovation.

It would be desirable to generalise the results several respects. On the one hand, the analysis postulated that innovations are carried out under perfect certainty. This assumption may be justified if the firm buys the innovation from a third party rather engaging an own research department. However, even in this case there may be a certain amount of uncertainty left. In this respect, the derived results may serve as a benchmark case. On the other hand, we considered a situation in which the new product replaces the old one. Clearly, the above cited example of the fashion industry satisfies this requirement. Nevertheless, there are numerous instances in which a new product is added. We can buy a Walkman, a portable CD–player or a portable MD–player although all of them serve the same basic function and the last one is much younger than the previous two products.

Appendix

Proof of Proposition 1

The conditions listed in parts 1 through 6 are the conditions for a Nash equilibrium (NE), where the incentive notation has been employed rather than the original one using the profit functions. Hence, to prove Proposition 1, it suffices to report the equilibrium conditions.

Part 1: (0,0) is an NE if (1) $I \geq \Delta_{Ci}\pi_i(c,c,d)$ and (2) $I \geq \Delta_{Pi}\pi_i(c,c,d)$.

Part 2: (1,0) is an NE if (1) $\chi_i(c,c,d) \geq 0$, (2) $I < \Delta_{Pi}\pi_i(c,c,d)$, (3) $I \geq \Delta_{Ci}\pi_i(c,c,d+\delta_j)$, (4) $I \geq \Delta_{Pi}\pi_i(c,c,d+\delta_j)$.

Part 3: (0,1) is an NE if (1) $\chi_i(c,c,d) < 0$, (2) $I < \Delta_{Ci}\pi_i(c,c,d)$, (3) $I \geq \Delta_{Ci}\pi_i(c,c-\gamma,d)$, (4) $I \geq \Delta_{Pi}\pi_i(c,c-\gamma,d)$.

Part 4: (1,1) is an NE if (1) $\chi_i(c,c-\gamma,d) \geq 0$, (2) $I < \Delta_{Ci}\pi_i(c,c-\gamma,d)$, (3) $\chi_i(c,c,d+\delta_j) < 0$, (4) $I < \Delta_{Pi}\pi_i(c,c,d+\delta_j)$.

Part 5: (2,0) is an NE if (1) $\chi_i(c,c,d+\delta_j) \geq 0$, (2) $I < \Delta_{Pi}\pi_i(c,c,d+\delta_j)$.

Part 6: (0,2) is an NE if (1) $\chi_i(c,c-\gamma) < 0$, (2) $I < \Delta_{Ci}\pi_i(c,c-\gamma,d)$.

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