Cournot Competition Yields Spatial Avoiding Competition in Groups*

by

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Abstract. This paper characterizes the properties of equilibrium location patterns in an Anderson-Neven-Pal model and uses these characteristics to comprehensively find the subgame perfect Nash equilibria, most of which are not yet found in the literature. Since the external competition effect may be exactly canceled out, or internal competition strictly dominates external competition, or the internal competition effect is consistent with the external competition effect, therefore without any externality and prior collusion, a competitive group structure may form endogenously in equilibrium and firms tend to avoid competition inside each group. The analyses of an Anderson-Neven-Pal model are instructive in studying the conditions for a capacity to implement a "Nash combination."

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Abstract

This paper characterizes the properties of equilibrium location patterns in an Anderson-Neven-Pal model and uses these characteristics to comprehensively find the subgame perfect Nash equilibria, most of which are not yet found in the literature. Since the external competition effect may be exactly canceled out, or internal competition strictly dominates external competition, or the internal competition effect is consistent with the external competition effect, therefore without any externality and prior collusion, a competitive group structure may form endogenously in equilibrium and firms tend to avoid competition inside each group. The analyses of an Anderson-Neven-Pal model are instructive in studying the conditions for a capacity to implement a "Nash combination."

1 Introduction

Beginning with the original work of Hotelling (1929), spatial competition has been studied for over seven decades. Compared to other oligopoly models, the analyses of spatial competition not only present traditional oligopoly characteristics, but also display the equilibrium in a geometric way; thus, spatial competition models are often instructive so as to study the general rules about equilibrium competition patterns in a non-cooperative game. Hotelling (1929) analyzes a location-price non-cooperative game in a linear market with a linear transportation cost and he claims that both firms agglomerate at the market center in equilibrium. Hotelling's claim is termed as "the principle of minimum differentiation," and ever since then the analyses concerning locational agglomeration and dispersion have opened up.

Although the original intention in Hotelling (1929) is to try to relax Bertrand competition by a spatial or physical differentiation in products, d'Aspremont et al. (1979) show that the relaxing effect in competition is not strong enough when the two firms are too close. Hence, the second stage of Hotelling's location-price game is not well defined and the location equilibrium does not exist. To avoid the problem in Hotelling's framework, d'Aspremont et al. (1979) propose to apply a quadratic form of transportation costs and conclude that the two firms locate at the two ends of a linear market in equilibrium, which is termed as "the principle of maximum differentiation."

Instead of a linear market, Salop (1979) considers oligopoly competition in a one-dimensional bounded market without a boundary, i.e. a circular market. Kats (1995) follows Salop's settings and shows that the equal-distance dispersion is a location equilibrium in a location-price game. Kats attributes non-existence of the equilibrium in pure strategies under a Hotelling model to the fact that neither firm competes with rivals on both sides of their locations, and thus the locations of firms tend to agglomerate in a linear market, but disperse when a circular market is considered.

In contrast to a location-price game, Hamilton et al. (1989) and Anderson and Neven (1991) consider a location-quantity (Cournot competition) game in a linear market with spatial discrimination. Both papers state that the two firms agglomerate at the market center in the unique equilibrium. Anderson and Neven (1991) not only successfully explain the overlapping of duopolists' market shares, but also claim that the equilibrium location pattern is determined by the instruments in competition: Cournot competition yields a spatial agglomeration. Gupta et al. (1997) extend Anderson and Neven's framework to a non-uniform distribution of consumers and demonstrate that the agglomeration equilibrium is robust under a wide variety of consumer distributions. Mayer (2000) allows the production cost to vary in different market points and concludes that the agglomeration of firms is still a common result in a linear market.

In a recent paper, Pal and Sarkar (2002) extend a location-quantity game in a linear market to cases when multiple plants are owned by each firm. Pal and Sarkar (2002) provide a fascinating induction to demonstrate that the complex problem in determining the equilibrium locations of plants for competitive multi-plant firms can be approximated by a simple one whereby each firm behaves as a multi-plant monopolist in determining the locations of its plants. Pal and Sarkar (2002) verify that the inter-firm agglomeration equilibrium claimed in Anderson and Neven (1991) is robust in the multi-plant cases when both firms have an equal number of plants. When the number of plants owned by each firm is not the same, however, Cournot competition may give rise to complete spatial dispersion.

To verify the deterministic viewpoint in Anderson and Neven (1991), Pal (1998) proposes a location-quantity game to a circular market. The oligopoly model with Cournot competition in a circular market is called an Anderson-Neven-Pal model (or, in short, an A-N-P model) throughout this paper. Pal (1998) discovers a dispersed location equilibrium and hence he claims that the market structure (i.e. linear or circular market) is a more crucial factor than the competition device (i.e. price or quantity). Chamorro-Rivas (2000) further verifies Pal's dispersed location equilibrium even when multi-plant cases are considered. Matsushima (2001), however, offers a counter example to Pal's dispersed location pattern: When there is an even number of firms in a circular market engaging in Cournot competition, half of the firms agglomerate at one point and the rest of the firms agglomerate at the opposite point in equilibrium.

There are several questions which have not yet been answered in the literature. First, does a deterministic viewpoint exist to exhibit the most significant factor in determining whether the firms' locations are agglomerated or dispersed in equilibrium? Second, similar to the question asked in Pal (1998), does any general conclusion exist regarding the equilibrium location patterns in a circular Cournot competition? Due to the existence of a counter example in Matsushima (2001) against the intuition of the dispersed location equilibrium in Pal (1998) that equilibrium locations tend to minimize the aggregate transportation cost of all firms, the consistent intuition behind all equilibria in a circular Cournot competition needs to be re-examined.

The purpose of this paper is to show that there are many more symmetric and asymmetric equilibrium location patterns missing in the literature, which can be found by applying the characteristics of location equilibria. It is implied by the findings of equilibrium location patterns that the true intuition in an Anderson-Neven-Pal model is for each firm to "avoid competition in groups in equilibrium." In other words, without any externality and collusion in advance in a non-cooperative game, various competitive group structures may endogenously form, and firms tend to avoid competition with the other firms inside the same group in equilibrium.[1] The new findings of equilibria not only offer a great diversity of equilibrium location patterns in spatial competition, but also show that, unfortunately, whether equilibrium locations are agglomerated or dispersed cannot be systematically categorized by some principal determinants, such as market structures or competition devices.

Fortunately, this paper further presents a characteristic of an Anderson-Neven-Pal model whereby, under some conditions, an equilibrium profile with a large number of firms can be characterized by several sub-profiles, each corresponding to a Nash equilibrium with a small number of firms. These conditions may be applicable in other oligopoly models or non-cooperative games with a large number of players. Therefore, as with the varieties of equilibrium location patterns going beyond what many can imagine about, the value of an Anderson-Neven-Pal model in both the spatial competition and the group interaction topics exceeds our expectations. This value of the model can be strikingly highlighted only by examining the true intuitions behind the model itself.

Intuitively, the existence of a group of outside firms inevitably intensifies the competition faced by each firm inside a group. There are, however, some situations where the firm's best response for competition with the other firms inside the same group is not influenced by the existence of groups of outside firms. For instance, this occurs when the external competition effect from one outside firm is exactly canceled out by that from another outside firm; or the internal competition inside the group strictly dominates competition with outside firms; or the best response for the other inside firms' choices is consistent with that for the outside firms' choices. Given that the choices of outside firms match a specific pattern, when the best response of each firm inside a group for internal competition is not changed by the outside firms' choices, the existence of outside firms will not alter the Nash equilibrium status quo of competition inside the group. Suppose that the outside firms' best responses for their interior competition are not altered for the same reasons when the equilibrium locations inside the group is given; then a

[1] This is in contrast to the analyses of congestion games such as Rosenthal (1973), Milchtaich (1996), Konishi et al. (1997a), and group formation games in Konishi et al. (1997b) and Hollard (2000), whereby different groups are separated by different alternatives (i.e. the players in the same group choose the same alternative) and each player's action is equivalent to choosing which group he/she joins. In the present paper, each action has no apparent relationship with the group chosen by the player, and a group structure (which means a partition of the set of all players) is generated by the equilibrium competition patterns in locations. In other words, each firm forms a group with the most influential rivals in competition rather than with the other players choosing the same alternative.

combination of the equilibrium profile of the inside firms with that of the outside firms will of course corresponds to a Nash equilibrium profile of all firms of all groups. To the authors' best knowledge, this intuitive view is a novel aspect and will be examined henceforth.

The rest of this paper is organized as follows. Section 2 shows the settings of the model. Section 3 presents the characteristics of a location equilibrium. The analysis of SPNE location patterns is shown in Section 4. Finally, the conclusions are discussed in Section 5.

2 The Model

Suppose there are N firms engaging in spatial Cournot competition, $N \geq 2$, where consumers are uniformly distributed on a circular market with a perimeter normalized to 1. Denote q_i and x_i as the quantity and the location of firm i, respectively, $i \in \{1, \dots, N\}$. Hence, the strategy profiles $(q_i)_{i=1}^N$ and $(x_i)_{i=1}^N$ represent the quantity and the location choices of N firms, respectively.

Following Anderson and Neven (1991) and Pal (1998), each firm's demand function on each point x in a circular market is set to be

$$p_i(x) = \alpha - Q(x) = \alpha - \sum_{i=1}^{N} q_i(x). \tag{1}$$

It is assumed that all firms have the same production technology and zero production cost. Following the same notations in Kats (1995) and Pal (1998), the transportation cost of goods from plant x_i to one point x is expressed as $t \cdot |x - x_i|$, $i = 1, ..., N$, where $|x - x_i|$ represents the shortest distance between x_i and x in a circular market. To ensure that each firm serves the whole market, $\alpha > Nt/2$ is assumed, and for simplicity t is assumed to be 1. Given the locations and quantities of all firms, the profit function of firm i in one market point x, where $x \in [0, 1)$, is expressed as

$$\pi_i(x_1, x_2, ..., x_N, x) = (p_i(x) - |x - x_i|)\, q_i(x), \quad i, = 1, ..., N. \tag{2}$$

The equilibrium concept adopted in this paper is that of a subgame perfect Nash equilibrium (Selten, 1975). The backward induction approach is applied to find the subgame perfect Nash equilibria of a two-stage non-cooperative game, where all firms choose their locations simultaneously in the first stage and then they simultaneously decide their quantities in the second stage. Assume that any arbitrage among the consumers is infeasible and production costs are irrelevant to quantities and locations. Hence, the firm's quantity choices and the competitions among firms in quantities are strategically independent across different market points. Therefore, the Cournot equilibrium in the second stage is a composite of the equilibrium quantities at all market points $x \in [0, 1)$. It can be checked that the equilibrium quantities and profits in the second stage are

$$q_i(x_1, x_2, ..., x_N, x) = \frac{1}{N+1}(\alpha + \sum_{j=1}^{N} |x - x_j| - (N+1)|x - x_i|), \tag{3}$$

$$\pi_i(x_1, x_2, ..., x_N, x) = q_i(x_1, x_2, ..., x_N, x)^2, \quad i = 1, ..., N. \tag{4}$$

In the first stage, given the other firms' locations, each firm chooses its plant location to maximize total profit over the circular market which is

$$\Pi_i(x_1, x_2, ..., x_N) = \int_0^1 \pi_i(x_1, x_2, ..., x_N, x)\, dx, \tag{5}$$

$$s.t. \;\; x_i \in [0, 1), \quad i = 1, ..., N. \tag{6}$$

To simplify the induction processes of finding the subgame perfect Nash equilibria, it is appropriate to analyze the properties characterizing the location equilibria, which are presented in the next section.

3 Characteristics of a Location Equilibrium

Following the symbolic notations in Osborne and Rubinstein (1994), for any strategy profile $s \equiv (x_i)_{i=1}^N$ and any $i = 1, .., N$, denote s_{-i} to be the vector of all firms' locations except the location of firm i, i.e. $s_{-i} \equiv (x_1, ..., x_{i-1}, x_{i+1}, ..., x_N)$. One can also denote (s_{-i}, x_i) to represent the profile $(x_i)_{i=1}^N$. Furthermore, the locations of a subset of all firms with n_l firms, where $1 \leq n_l \leq N$, can be expressed by a list $s^l = (x_1^l, \ldots, x_{n_l}^l)$, whereby every element $x_j^l \in s^l$ corresponds to a different firm's location $x_i \in s$. Suppose there are L lists s^l, $l = 1, \ldots, L$, whereby the location of each firm i corresponds to one (and only one) element of one and only one list for all $i = 1, \ldots, N$, then one can also denote $(s^l)_{l=1}^L$ to represent the profile $(x_i)_{i=1}^N$. The definition of s^l helps highlight the formation of competitive groups. It should be emphasized first that, however, the properties of the subgame perfect Nash equilibrium (SPNE) locations. These properties help to determine the SPNE location patterns in the next section.

Proposition 1. *Given s_{-i}^* and $x_i \in [0, \frac{1}{2}]$, the necessary condition of an optimal location for firm i is equivalent to the transportation-cost-median condition that*

$$\int_{x_i}^{x_i+\frac{1}{2}} \sum_{j \neq i}^{N} |x - x_j|\, dx = \int_0^{x_i} \sum_{j \neq i}^{N} |x - x_j|\, dx + \int_{x_i+\frac{1}{2}}^{1} \sum_{j \neq i}^{N} |x - x_j|\, dx. \tag{7}$$

Proof. From equation (5), the total profit of firm i can be expressed as

$$\Pi_i(x_1, \ldots, x_N) = \\ \int_0^{x_i} \pi_i(x_1, \ldots, x_N, x)dx + \int_{x_i}^{x_i+\frac{1}{2}} \pi_i(x_1, \ldots, x_N, x)dx + \int_{x_i+\frac{1}{2}}^{1} \pi_i(x_1, \ldots, x_N, x)dx. \tag{8}$$

Differentiating $\Pi_i(x_1, \ldots, x_N)$ with respect to those x_i which appear in the upper and lower bounds of each integrator yields[2]

$$q_i(x_1, ..., x_N, x)^2|_{x=x_i} + \left\{ q_i(x_1, ..., x_N, x)^2|_{x=x_i+\frac{1}{2}} - q_i(x_1, ..., x_N, x)^2|_{x=x_i} \right\}$$
$$-(q_i(x_1, ..., x_N, x)^2|_{x=x_i+\frac{1}{2}} = 0. \tag{9}$$

Since $|x - x_i|$ equals $(x_i - x)$, $(x - x_i)$, and $(1 - x + x_i)$, for all x belongs to $[0, x_i)$, $[x_i, x_i + \frac{1}{2})$, and $[x_i + \frac{1}{2}, 1)$, respectively, it can be checked that

$$\frac{\partial \pi_i(x_1, ..., x_N, x)}{\partial x_i} = 2q_i(x_1, ..., x_N, x) \frac{\partial q_i(x_1, ..., x_N, x)}{\partial x_i}, \tag{10}$$

where

$$\frac{\partial q_i(x_1, ..., x_N, x)}{\partial x_i} = -\frac{N}{N+1} \left(\frac{\partial |x - x_i|}{\partial x_i} \right) = \begin{cases} \frac{-N}{N+1}, & \forall\, x \in [0, x_i) \cup [x_i + \frac{1}{2}, 1), \\ \\ \frac{N}{N+1}, & \forall\, x \in [x_i, x_i + \frac{1}{2}). \end{cases} \tag{11}$$

Hence, the first-order derivative of $\Pi_i(x_1, \ldots, x_N)$ with respect to x_i is

$$\begin{aligned} \frac{\partial \Pi_i(x_1, ..., x_N)}{\partial x_i} &= \int_0^{x_i} \frac{\partial \pi_i(x_1, ..., x_N, x)}{\partial x_i} dx + \int_{x_i}^{x_i+\frac{1}{2}} \frac{\partial \pi_i(x_1, ..., x_N, x)}{\partial x_i} dx \\ &\quad + \int_{x_i+\frac{1}{2}}^{1} \frac{\partial \pi_i(x_1, ..., x_N, x)}{\partial x_i} dx \\ &= \frac{2N}{N+1} \left\{ - \int_0^{x_i} q_i(x_1, ..., x_N, x) dx + \int_{x_i}^{x_i+\frac{1}{2}} q_i(x_1, ..., x_N, x) dx \right. \\ &\quad \left. - \int_{x_i+\frac{1}{2}}^{1} q_i(x_1, ..., x_N, x) dx \right\}. \end{aligned} \tag{12}$$

Since N is positive, the first-order condition of optimization for firm i is equivalent to

$$\int_{x_i}^{x_i+\frac{1}{2}} q_i(x_1, \ldots, x_N, x) dx = \int_0^{x_i} q_i(x_1, \ldots, x_N, x) dx + \int_{x_i+\frac{1}{2}}^{1} q_i(x_1, \ldots, x_N, x) dx. \tag{13}$$

Equation (13) shows that the optimal location for firm i coincides with its quantity-median. For there is no boundary in a circular market, however, by contrast to the quantity-median condition in a linear market as claimed in Pal and Sarkar (2002), one extra item that is $\int_{x_i+\frac{1}{2}}^{1} q_i(x_1, \ldots, x_N, x)dx$ is added to the right-hand side of the quantity-median condition in a circular market (equation (13)). Furthermore, it is always true for all $x_i \in [0, \frac{1}{2}]$ in a circular market that

$$\int_{x_i}^{x_i+\frac{1}{2}} \alpha - N(|x - x_i|) dx = \int_0^{x_i} \alpha - N(|x - x_i|) dx + \int_{x_i+\frac{1}{2}}^{1} \alpha - N(|x - x_i|) dx. \tag{14}$$

[2]The intuition of equation (9) is explained as follows. Since each firm serves the whole market for all its locations, any adjustment of x_i affects firm i's aggregate profit through only a redrawing of the distribution of quantities supplied over the whole market rather than changing the market range served by firm i.

From equations (3), (13), and (14), the quantity-median condition can be further simplified to be[3]

$$\int_{x_i}^{x_i+\frac{1}{2}} \sum_{j\neq i}^{N} |x - x_j| dx = \int_{0}^{x_i} \sum_{j\neq i}^{N} |x - x_j| dx + \int_{x_i+\frac{1}{2}}^{1} \sum_{j\neq i}^{N} |x - x_j| dx. \tag{15}$$

Equation (15) is defined as the "transportation-cost-median (of other firms) condition" throughout this paper.[4] ||

Any location x_i is said to be at a transportation-cost-median if x_i coincides with equation (7). It should be noted that equation (7) is equivalent to the first-order condition of optimization, and furthermore, this equivalence is not affected by the order of locations x_i, $i = 1, \ldots, N$. In other words, given the locations of the other firms, the transportation-cost-median condition checks which candidate point is firm i's optimal location in the interval $[0, \frac{1}{2}]$.[5]

For firm i's arbitrary location, the diameter passing through x_i of course divides the whole market into two half-circles. The transportation-cost-median condition implies that, given the other firms' locations, the aggregate transportation cost to the consumers in one half-circle left (or right) to firm i's optimal location is the same as that in the other half-circle. The intuition is that, when x_i^* is not at a transportation-cost-median, it is not at a quantity-median either. In a spatial Cournot competition, the marginal profit from one half-circle (left or right to x_i^*) due to a tiny change in x_i^* is positively related to the aggregate quantity from that half-circle. Hence, once the aggregate quantity of one half-circle is not equal to that of the other half-circle, the aggregate profit for firm i can be raised by an infinitesimal increase or decrease in x_i^*. Thus, x_i^* cannot be an optimal location for firm i. To save words in the following statements, the right-hand side and the left-hand side of the transportation-cost-median condition should be denoted by some simple and clear notations.

Definition 1. *Denote LHS and RHS to represent the left-hand side and the right-hand side of the transportation-cost-median condition (equation (7)), respectively. That is, for all $x_i \in [0, 1/2]$,*

$$LHS = \int_{x_i}^{x_i+\frac{1}{2}} \sum_{j\neq i}^{N} |x - x_j| dx,$$

[3]There does not exist an equality similar to equation (14) for all x_i in a linear market; thus, the quantity-median condition in a linear Cournot competition such as Pal and Sarkar (2002) cannot be simplified to be a transportation-cost-median condition.

[4]For all $x_i \in [0, \frac{1}{2}]$, it can be checked that $\int_{x_i}^{x_i+\frac{1}{2}} |x - x_i| dx = \int_0^{x_i} |x - x_i| dx + \int_{x_i+\frac{1}{2}}^{1} |x - x_i| dx = \frac{1}{8}$. Hence, the words in the parentheses "of other firms" can be replaced by the words "of all firms" and will be omitted hereafter.

[5]For the location problem in another half-circle $x_i \in [1/2, 1)$, the necessary condition of optimization for firm i can be analogized by the same way to be $\int_{x_i-\frac{1}{2}}^{x_i} \sum_{j\neq i} |x - x_j|\, dx = \int_0^{x_i+\frac{1}{2}} \sum_{j\neq i} |x - x_j|\, dx + \int_{x_i}^{1} \sum_{j\neq i} |x - x_j|\, dx$.

and

$$RHS = \int_0^{x_i} \sum_{j \neq i}^{N} |x - x_j| dx + \int_{x_i + \frac{1}{2}}^{1} \sum_{j \neq i}^{N} |x - x_j| dx.$$

There are several implications behind the transportation-cost-median condition which are useful to simplify subsequent inductions and instructive to explore an A-N-P model.

Remark 1. *In an A-N-P model with N firms, the transportation-cost-median condition is equivalent to*

$$LHS = \frac{N-1}{8}. \tag{16}$$

Proof. Since for each $j \neq i$, $\int_0^1 |x - x_j| dx = \frac{1}{4}$,[6] and the number of all firms except firm i is $N-1$, thus $LHS + RHS = (N-1)\int_0^1 |x - x_j| dx = (N-1)/4$. Therefore, the transportation-cost-median condition $LHS = RHS$ can be rewritten as $LHS = (N-1)/8$. ‖

Remark 1 implies that the information from one side of the transportation-cost-median condition is sufficient to find the locations satisfying the necessary condition of optimization, and thus attention can be focused on only LHS hereafter.[7] From the transportation-cost-median condition, the optimal location for firm i must balance the aggregate transportation costs of the two half-circles which are divided by the diameter passing through x_i^*. Hence, the following remark is consistent with normal intuition.

Remark 2. *Given s_{-i}^* and $x_i \in [0, \frac{1}{2}]$, if no firm is located in the interval $(x_i + \frac{1}{2}, 1) \cup [0, x_i)$, but there exists at least one firm located in the other interval $(x_i, x_i + \frac{1}{2})$, or vice versa, then x_i does not satisfy the transportation-cost-median condition for firm i.*

Proof. For each $x_j \in [x_i, x_i + \frac{1}{2}]$, $j \neq i$, since $x_j - x_i \geq 0$ and $x_j - (x_i + \frac{1}{2}) \leq 0$, $\forall j \in \{1, \ldots, N\} \backslash \{i\}$, hence, $\int_{x_i}^{x_i + \frac{1}{2}} |x - x_j| dx = \int_{x_i}^{x_j} (x_j - x) dx + \int_{x_j}^{x_i + \frac{1}{2}} (x - x_j) dx = (x_j - x_i)(x_j - (x_i + \frac{1}{2})) + \frac{1}{8} \leq \frac{1}{8}$. There exists, however, at least one firm with a location $x_j \in (x_i, x_i + \frac{1}{2})$ where for this firm, $\int_{x_i}^{x_i + \frac{1}{2}} |x - x_j| dx < \frac{1}{8}$. Thus, it is proved that $LHS < \frac{N-1}{8}$ and from Remark 1, the transportation-cost-median condition for firm i is not satisfied. ‖

In an A-N-P model, each firm offers its most quantities to those consumers living around its plant. The implication of Remark 2 shows that, if there exists a half-circle without any firm, then firm i can earn more profit by moving its major market to serve those consumers. The phenomenon shown in Remark 2 is due to the strategic location effect that the more competitor's

[6] For all $x_j \in [0, \frac{1}{2}]$, $\int_0^1 |x - x_j| dx = \int_0^{x_j} (x_j - x)\, dx + \int_{x_j}^{x_j + \frac{1}{2}} (x - x_j)\, dx + \int_{x_j + \frac{1}{2}}^{1} (1 - x + x_j)\, dx = \frac{1}{4}$. On the other side, for all $x_j \in [\frac{1}{2}, 1)$, $\int_0^1 |x - x_j| dx = \int_0^{x_j - \frac{1}{2}} (1 - x_j + x)\, dx + \int_{x_j - \frac{1}{2}}^{x_j} (x_j - x)\, dx + \int_{x_j}^{1} (x - x_j)\, dx = \frac{1}{4}$.

[7] The reason for choosing LHS rather than RHS is based on the convenience in calculations.

quantity there is, the less one firm's demand will be when the products of the firms are complete substitutes (refer to equation (1)). Under spatial Cournot competition, the area with the rival's highest quantity is the location of the rival's plant, and thus the best response for each firm is to move away from rivals' locations as far as possible.

Corollary 1. (Transportation-cost-median property) *In an A-N-P model with N firms, a profile of locations $s^* = (x_1^*, x_2^*, \ldots, x_N^*)$ constitutes a subgame perfect Nash equilibrium with N firms if and only if x_i^* is at a transportation-cost-median, for all $i = 1, \ldots, N$.*

The intuition of Corollary 1 is that, given the locations of other firms, the optimal location for the firm is at a transportation-cost-median. Hence, if in an SPNE there exists any firm whose location is not at a transportation-cost-median, then this firm has an incentive to deviate from its equilibrium location to a transportation-cost-median location, a contradiction to the definition of SPNE. The transportation-cost-median condition, however, is only a necessary condition of optimization. The candidate point satisfying the transportation-cost-median condition must be checked by the second-order condition to ensure that it is a maximizer rather than a minimizer for firm i's profit function.

Proposition 2. *In an A-N-P model with N firms, given s_{-i}^* and suppose $x_i \in [0, \frac{1}{2}]$, the sign of the second-order derivative of $\Pi_i(x_1, \ldots, x_N)$ with respect to x_i is the same as the sign of the first-order derivative of LHS with respect to x_i. That is,*

$$\frac{\partial^2 \Pi_i(x_1, \ldots, x_N)}{\partial x_i^2} \gtreqless 0 \quad \textit{iff} \quad h \overset{def}{=} \frac{\partial LHS}{\partial x_i} = \sum_{j \neq i}^{N} |x_i + \frac{1}{2} - x_j| - \sum_{j \neq i}^{N} |x_i - x_j| \gtreqless 0. \tag{17}$$

Proof. From equation (12), the second-order derivative of firm i's profit function is[8]

$$\begin{aligned} & \frac{\partial^2 \Pi_i(x_1, \ldots, x_N, x)}{\partial x_i^2} \\ &= \frac{2N}{N+1} \left\{ \frac{N}{N+1} + 2q_i(x_1, \ldots, x_N, x) \Big|_{x = x_i + \frac{1}{2}} - 2q_i(x_1, \ldots, x_N, x) \,|_{x = x_i} \right\} \\ &= \frac{4N}{(N+1)^2} \left(\sum_{j \neq i}^{N} |x_i + \frac{1}{2} - x_j| - \sum_{j \neq i}^{N} |x_i - x_j| \right). \end{aligned} \tag{18}$$

From Definition 1, it can be checked that

$$h(x_1, \ldots, x_N) \overset{def}{=} \frac{\partial LHS}{\partial x_i} = \sum_{j \neq i}^{N} |x_i + \frac{1}{2} - x_j| - \sum_{j \neq i}^{N} |x_i - x_j|. \tag{19}$$

[8] A very detailed calculation is shown here. $\frac{\partial^2 \Pi_i(x_1, \ldots, x_N, x)}{\partial x_i^2} = \frac{2N}{N+1} \{ \frac{Nx}{N+1}|_0^{x_i} + \frac{Nx}{N+1}|_{x_i}^{x_i+\frac{1}{2}} + \frac{Nx}{N+1}|_{x_i+\frac{1}{2}}^{1} + 2q_i(x_1, \ldots, x_N, x)|_{x=x_i+\frac{1}{2}} - 2q_i(x_1, \ldots, x_N, x)|_{x=x_i} \}$, where $\frac{Nx}{N+1}|_0^{x_i} + \frac{Nx}{N+1}|_{x_i}^{x_i+\frac{1}{2}} + \frac{Nx}{N+1}|_{x_i+\frac{1}{2}}^{1} = \frac{N}{N+1}$ and $q_i(x_1, \ldots, x_N, x)|_{x=x_i+\frac{1}{2}} - q_i(x_1, \ldots, x_N, x)|_{x=x_i} = \frac{1}{N+1}(\sum_{j \neq i}^{N} |x_i + \frac{1}{2} - x_j| - \frac{N}{2} - \sum_{j \neq i}^{N} |x_i - x_j|)$.

Since $N > 0$, the proposition can be proved by a comparison of equations (18) and (19). $\|$

Proposition 2 shows that LHS must be negatively sloped at x_i^* to ensure it being the optimal location for firm i. Otherwise, if $\partial LHS/\partial x_i = 0$, then the third-order derivative of the profit function must be checked. Instead of the complexity in calculating $\partial^2\Pi_i/\partial x_i^2$, Proposition 2 and Remark 1 show that the equilibrium locations can be found and verified by using only the information of LHS of the transportation-cost-median condition. Furthermore, the second-order condition of optimization can be simplified as follows.

Remark 3. *In an A-N-P model with N firms, the second-order condition of optimization for firm i is equivalent to*

$$\tilde{h} \overset{def}{=} \sum_{j\neq i}^{N} |x_i - x_j| > \frac{N-1}{4}, \tag{20}$$

at the optimal location x_i^*.

Proof. Since for each $j \neq i$, $|x_i + \frac{1}{2} - x_j| = \frac{1}{2} - |x_j - x_i|$, thus, $\sum_{j\neq i}^{N} |x_i + \frac{1}{2} - x_j| = \frac{N-1}{2} - \sum_{j\neq i}^{N} |x_i - x_j|$. Therefore, a sufficient condition $h < 0$ can be re-written as $\sum_{j\neq i}^{N} |x_i - x_j| > \frac{N-1}{4}$. $\|$

It is noted that once $\tilde{h} < (N-1)/4$, then the second-order condition of optimization is not satisfied. The simplified form of the second-order condition of optimization shown in Remark 3 is helpful in proving the following remark.

Remark 4. *In an A-N-P model with N firms ($N \geq 3$), given $s_{-1}^* = (x_2^*, x_3^*, \ldots, x_N^*) = (x_2^*, x_2^* + d, \ldots, x_2^* + (N-2)d)$ such that $d \geq 0$ and $|x_N^* - x_2^*| = (N-2)d < \frac{1}{2}$, there then exists a non-negative distance $a \equiv \frac{1}{4} - \frac{1}{2}|x_N^* - x_2^*|$ where the second-order condition of optimization for firm 1 is not satisfied for all $x_1 \in (x_2^* - a, x_N^* + a)$.*

Proof. See Appendix 1. $\|$

When $N = 3$ and $a \equiv \frac{1}{4} - \frac{1}{2}|x_3^* - x_2^*|$ is given, it should be noted that $(x_3^* + a) - (x_2^* - a) = \frac{1}{2}$. Thus, Remark 4 shows that, given two competitors' locations are not at the two ends of the same diameter, firm 1's optimal location is only possible in the half-circle farthest from its competitors on average. When $N > 3$, it is implied by Remark 4 that, compared to any location in a less competitive market area, to locate in an area with aggressive competition is not an optimal choice and further verifies the avoiding-competition gravity in an A-N-P model.

4 Analysis of the SPNE Location Patterns

4.1 Basic equilibrium location patterns

After examining several properties of the equilibrium locations, there are now enough instruments to induct the SPNE location patterns in an A-N-P model. It is appropriate to first investigate the basic equilibrium location patterns in the cases with a small number of firms. Some interesting implications can be pointed out by a comparison of these basic location patterns with those patterns when there is a generalized number of firms in the market.

Proposition 3 (Pal, 1998). *When there are two firms* ($N = 2$)*, the dispersed location pattern* (x_1^*, x_2^*) *such that* $|x_1^* - x_2^*| = \frac{1}{2}$ *constitutes the unique subgame perfect Nash equilibrium.*

Proof. Without loss of generality, given $x_2^* = 0$ and consider $x_1 \in [0, \frac{1}{2}]$, then from Remark 2 only two points $x_1 = 0$ and $x_1 = \frac{1}{2}$ coincide with the transportation-cost-median condition. Furthermore, when $N = 2$, $\tilde{h} = 0 < \frac{N-1}{4} = \frac{1}{4}$ at $x_1 = 0$ and $\tilde{h} = \frac{1}{2} > \frac{1}{4}$ at $x_1 = \frac{1}{2}$. Thus, from Remark 3, given $x_2^* = 0$, only $x_1^* = \frac{1}{2}$ is the unique optimal location for firm 1, and vice versa. Therefore, it is proved that $(x_1^*, x_2^*) = (0, \frac{1}{2})$ constitutes the unique SPNE. ‖

Based on the remarks in the previous section, the proof of the above proposition can be written in a very concise form. From Proposition 3, the uniqueness of the equilibrium is valid in an A-N-P model with two firms. In what follows, however, it will be shown that the variety of the equilibrium location patterns is not unique when there are more than two firms competing in a circular market with quantities and locations.

Lemma 1. *Suppose there are three firms in an A-N-P model* ($N = 3$)*. The profit is the same for firm 1 for all its available location choices when a dispersed location pattern of the other firms* $s_{-1} = (x_2, x_3)$ *is given such that* $|x_2 - x_3| = \frac{1}{2}$*.*

Proof. Without loss of generality, given $x_2 \in [0, \frac{1}{2}]$ and $x_3 = x_2 + \frac{1}{2}$, the profit for firm 1 can be checked to be

$$\Pi_1 = \frac{1}{64}(4\alpha^2 - 2\alpha + 1), \quad \forall\, x_1 \in [0, 1). \tag{21}$$

It is noted that Π_1 is independent of x_1, and thus the profit is the same for firm 1 for all locations. ‖

Starting from an arbitrary location in a circular market, given that the other two firms are located at two ends of the same diameter, when firm 1 moves toward one firm, its location at the same time moves away from the other firm in the same distance. Furthermore, when the firms engage in spatial Cournot competition, the shorter the distance is between firm 1 and any other firm, the more intensive the competition (and thus the smaller equilibrium quantities) will be between them (refer to equation (3)). Hence, given the locations of the other two firms being opposite to each other, an increase in the intensity of competition from firm 1 with one of the other two firms will be exactly canceled out by a decrease in that competition with the other firm. Therefore, all location choices yield the same intensity in competition and thus the profit is the same for firm 1 for all its locations. Lemma 1 highlights a characteristic of an A-N-P model with three firms whereby one firm's optimal location choice is independent of the choices of the other firms, given that the locations of the other firms match one specific pattern in that they are opposite to each other.

The property shown in Lemma 1 will be generalized to an N-firm version in Lemma 2. This property can help find asymmetric location equilibria in an A-N-P model with three firms - a finding that is missing in the literature.

Proposition 4. *When there are three firms* ($N = 3$). (i) *The dispersed location pattern* $(x_1^*, x_2^*, x_3^*) = (0, \frac{1}{3}, \frac{2}{3})$ *constitutes a symmetric subgame perfect Nash equilibrium;* (ii) *The semi-agglomerated-at-two-points location pattern where* $(x_1^*, x_2^*, x_3^*) = (0, 0, \frac{1}{2})$ *or* $(x_1^*, x_2^*, x_3^*) = (0, \frac{1}{2}, \frac{1}{2})$ *constitutes an asymmetric subgame perfect Nash equilibrium.*

Proof. See Appendix 2. ||

Besides Matsushima's (2001) agglomeration-at-two-points location pattern, Proposition 4 offers another counter example to Pal's (1998) intuition that equilibrium location patterns minimize the total transportation cost of serving the entire market. To minimize the total transportation cost, given $s_{-1}^* = (0, \frac{1}{2})$, the location $x_1 = \frac{1}{4}$ (or $x_1 = \frac{3}{4}$) should strictly dominate other location choices for firm 1. The statement is not true in an A-N-P model, however. From Lemma 1, when $s_{-1}^* = (0, \frac{1}{2})$ is given, the profit is the same for all firm 1's location choices. Hence, in the case with three firms, the minimum transportation cost principle is valid only in the dispersed location pattern. The real intuition behind all location equilibria in an A-N-P model can be strikingly revealed by examining the generalized N-firm cases, while the intuition of the second part of Proposition 4 will be explained after Proposition 6.

4.2 Equilibrium location patterns with N firms

When there are N firms engaging in Cournot competition in a circular market, all location equilibria can be categorized into five equilibrium location patterns which are shown subsequently in what follows.

Proposition 5. (Dispersed-by-pairs location pattern) *In an A-N-P model with N firms (N is even), suppose there are $\frac{N}{2}$ lists $s^{l*} = (x_1^{l*}, x_2^{l*})$, $l = 1, \ldots, \frac{N}{2}$, such that $|x_1^{l*} - x_2^{l*}| = \frac{1}{2}$, then the profile $s^* = (s^{l*})_{l=1}^{\frac{N}{2}}$ constitutes a symmetric subgame perfect Nash equilibrium.*

Proof. See Appendix 3. ||

The dispersed-by-pairs location pattern in Proposition 5 shows that all firms are located at two ends of $\frac{N}{2}$ diameters where each diameter may or may not overlap with another diameter. In other words, when there is an even number of firms in an A-N-P model, an equilibrium location pattern with N firms is just an *arbitrary* combination of $\frac{N}{2}$ lists such that each of them corresponds to an equilibrium location profile with two firms. Denote $\theta_l \in [0, \pi]$ to be the degree of angle between the diameter passing through the points in s^{l*} and the diameter of $s^{(l-1)*}$, $l = 2, \ldots, \frac{N}{2}$. The most surprising characteristic shown in Proposition 5 is that every $(\theta_l)_{l=2}^{\frac{N}{2}}$ such that $\theta_l \in [0, \pi]$, $l = 2, \ldots, \frac{N}{2}$, corresponds to a location equilibrium. Therefore, Matsushima's (2001) agglomerated-at-two-points location pattern is a special case with $\theta_l = 0$, $\forall\, l = 2, \ldots, \frac{N}{2}$, and Pal's (1998) dispersed location pattern is also a specific case with $\theta_l = 2\pi/N$, $\forall\, l = 2, \ldots, \frac{N}{2}$. In fact, in the aspect of $(\theta_l)_{l=2}^{\frac{N}{2}}$, there are infinite equilibrium location patterns.

Proposition 5 implies that, given $N-2$ firms' locations $(s^{l*})_{l=2}^{\frac{N}{2}}$ whereby each firm is opposite to another on the same diameter, the relative consideration among different location choices for the firm with location x_1^1 is affected only by the firm whose equilibrium location x_2^{1*} is not included in $(s^{l*})_{l=2}^{\frac{N}{2}}$. In other words, the firm with x_1^1 chooses to avoid competition (or say, to be paired) with the firm that is not paired with any other firms, as does the firm with x_2^1. Therefore, the existence of the other $N-2$ firms that are paired with each other is neutral to the competition of the firms with location variables in $s^1 = (x_1^1, x_2^1)$, and the same property is presented for all s^l. All firms can thus be viewed as several groups which are divided by the equilibrium competition patterns.

Proposition 5 is very helpful in revealing the intuition in an A-N-P model. In a circular Cournot competition, the consideration of each firm may not be consistent with that to minimize the transportation cost of all firms (as claimed in Pal (1998)), nor is it consistent to agglomerate with some other firms (as claimed in Matsushima (2001)).[9] As highlighted in Proposition 5 and the following propositions, the real intuition behind the competition in an A-N-P model is to "avoid competition with the other firm(s) in the same group (pair)." That is, in equilibrium, a competitive group structure is naturally formed where each firm tends to avoid competition with the other firms inside the same group. The reason for Proposition 5 being valid is closely related to several characteristics of an A-N-P model, which will be explained after Lemma 2.

Lemma 2. (Neutral property) *In an A-N-P model with $N+1$ firms (N is even), the profile of all firms' locations can be viewed as $s = (x_1, (s^l)_{l=1}^{\frac{N}{2}})$ such that $s^l = (x_1^l, x_2^l)$. Suppose x_1^{l*} and x_2^{l*} are given that $|x_1^{l*} - x_2^{l*}| = \frac{1}{2}$, $l = 1, \ldots, \frac{N}{2}$, then when $(s^{l*})_{l=1}^{\frac{N}{2}}$ is given, the profit is the*

[9] For example, when $N = 4$, $s^* = (x_1^*, x_2^*, x_3^*, x_4^*) = (0, 0, \frac{1}{2}, \frac{1}{2})$ constitutes a Nash equilibrium. Moreover, given $(x_2^*, x_4^*) = (0, \frac{1}{2})$ and for arbitrary $x_3' \in [0, 1)$, the optimal location for firm 1 is always x_1^* such that $|x_1^* - x_3'| = \frac{1}{2}$. Given $(x_3^*, x_4^*) = (\frac{1}{2}, \frac{1}{2})$ and arbitrary $x_2' \neq 0$, however, $x_1 = x_2'$ is not an optimal location for firm 1. Therefore, when $s_{-1}^* = (0, \frac{1}{2}, \frac{1}{2})$ is given, the consideration of firm 1 in choosing $x_1^* = 0$ is to avoid competition with firm 3 (or firm 4), rather than to agglomerate with firm 2.

same for firm 1 for its all available location choices $x_1 \in [0, 1)$. At this time, $(s^{l*})_{l=1}^{\frac{N}{2}}$ is said to present the neutral property.

Proof. Since all firms except firm 1 are located at the two ends of $\frac{N}{2}$ diameters (one diameter may or may not coincide with another diameter), then without loss of generality, one can denote x_1^{l*} and x_2^{l*} to be locations in the intervals $[x_1^1, x_1^1 + \frac{1}{2})$ and $[x_1^1 + \frac{1}{2}, 1) \cup [0, x_1^1)$, respectively, such that $|x_1^{l*} - x_2^{l*}| = \frac{1}{2}$.

As shown in the proof of Proposition 5, for each $x \in [x_1^1, x_1^1 + \frac{1}{2})$, there exists one and only one point $x' \in [x_1^1 + \frac{1}{2}, 1) \cup [0, x_1^1)$ and $|x' - x| = \frac{1}{2}$ such that $|x - x_1^{j*}| = |x' - x_2^{j*}|$ and $|x - x_2^{j*}| = |x' - x_1^{j*}|$. Hence, the transportation-cost-median condition is satisfied for all $x_1 \in [0, 1)$, which is equivalent to stating that $\partial \Pi_1 / \partial x_1 = 0$, $\forall\, x_1 \in [0, 1)$. Since Π_1 is continuous in x_1 when $(s^{l*})_{l=1}^{\frac{N}{2}}$ is given, Π_1 is a constant with respect to x_1. Thus, it is proved that the profit is the same for all firm 1's available locations. ||

Lemma 2 is a generalized version of Lemma 1 with more than three competitive firms. The intuition behind Lemma 2 is similar to that of Lemma 1: Given that the locations of the other firms match the dispersed-by-pairs location pattern, for any diameter dividing the whole market into two half-circles, then the location pattern in one half-circle will be the same as that in the other half-circle after rotating it in a counter-clock direction of 180 degrees, and vice versa. Therefore, for all diameters, the location pattern in one half-circle is effectively symmetric to that in the other half-circle. This symmetry of the two half-circles ensures that the intensity of the competition from firm 1 with other firms is the same for all its location choices, and thus the profit is the same for all $x_1 \in [0, 1)$.

Intuitively, under what situations is the existence of another group of outside firms neutral to the competition inside a group of firms? One trial answer is the situation where there is no competition between the firms in different groups. The current analysis, however, offers another potential but interesting explanation for a neutral phenomenon between competitive groups in an A-N-P model: Given a group of outside firms achieving an equilibrium, for each inside firm, the external competition effects from all outside firms are exactly canceled out with each other. Therefore, the comparison among different choices only relates to the firms in the same group, and the existence of a group of outsiders is neutral to the competition of the firms inside the group.

The value of understanding the intuition and the neutral phenomenon in an A-N-P model is in that, under some circumstances, a Nash equilibrium of a large number of players is composed of several sub-profiles, each corresponding to a Nash equilibrium of a smaller group of players. From the analysis of Proposition 5, the sufficient conditions to construct such a circumstance are related to two characteristics of an A-N-P model. The first characteristic is the neutral property as shown in Lemma 2 that, given a group of players achieving an equilibrium, the payoff is the same for any outside player for all its available strategies. When one player makes a comparison among different strategy alternatives, given an equilibrium profile of outside players presenting the neutral property, the player is just like facing a market without any outside player, and thus the competition with outside players is neutral to the internal competition with the other players inside the same group. The second characteristic is the linearity of each player's best-response function in every other player's alternative (as shown in the transportation-cost-median condition). This linearity isolates the impact for one player's best response caused by each other player and thus the mutual reinforcement of the internal competition effect and the external competition effect is evaded.

The following proposition verifies whether the intuition of avoiding competition in groups is valid when there is an odd number of firms in an A-N-P model.

Proposition 6. (Semi-agglomerated-at-two-points location pattern) *For any odd number $N > 2$, suppose there are $\frac{N-1}{2}$ lists s^{l*}, $l = 1, \ldots, \frac{N-1}{2}$, such that $s^{1*} = (x_1^{1*}, x_2^{1*}, x_3^{1*}) = (0, 0, \frac{1}{2})$ or $(0, \frac{1}{2}, \frac{1}{2})$ and $s^{l*} = (x_1^{l*}, x_2^{l*}) = (0, \frac{1}{2})$, $l = 2, \ldots, \frac{N-1}{2}$. The semi-agglomerated-at-two-points location pattern $s^* = (s^{1*}, s^{2*}, ..., s^{\frac{N-1}{2}*})$ constitutes an asymmetric subgame perfect Nash equilibrium.*

Proof. See Appendix 4. ‖

Proposition 6 offers an equilibrium location pattern, which has not yet been found in the literature, when there is an odd number of firms in an A-N-P model.[10] Intuitively, for any firm with an equilibrium location at 0, since the locations of the other firms just match the dispersed-by-pairs location pattern, from the analyses of Lemma 2, the external competition effect from any firm is exactly canceled out by another firm. Therefore, the profit is the same for all its available locations and the firm has no incentive to deviate from the equilibrium location 0.

On the other hand, for an arbitrary firm with an equilibrium location at $\frac{1}{2}$, since the locations of $N-3$ of the other firms form a dispersed-by-pairs location pattern, the external competition effect from these $N-3$ firms is canceled out with each other. Hence, the optimal location is determined only by the other two remaining firms whose equilibrium locations are both at 0. From Proposition 4, the internal competition effect from these two firms induces the firm to have no incentive to deviate from the equilibrium location $\frac{1}{2}$.

The semi-agglomerated-at-two-points location pattern shown in Proposition 6 can be viewed as a composite of $\frac{N-1}{2}$ location sub-profiles such that $(\frac{N-1}{2}-1)$ of them each fits the equilibrium location pattern with two firms (as shown in Proposition 3) and one of them fits the equilibrium location pattern with three firms (as shown Proposition 4). Hence, the intuition of avoiding competition in groups is still valid in the semi-agglomerated-at-two-points location pattern. Unlike the implications in Proposition 5 whereby any *arbitrary* combination of the location equilibria (each with two firms) corresponds to a location equilibrium with N firms, Proposition 6

[10]In fact, the reference point which is the point "0" in Proposition 6 can be generalized to represent any point in a circular market.

implies that these $\frac{N-1}{2}$ location sub-profiles can be combined to constitute a location equilibrium only with some *specific* directions. It is noted that the firms in each location sub-profile, which matches the equilibrium location pattern with two or three firms, are on the two ends of the same diameter. Therefore, the specific directions require that all these $\frac{N-1}{2}$ diameters must overlap with each other.

Even in an A-N-P model, there exist some location equilibria which cannot be decomposed into several equilibria of different and exclusive groups. In these location equilibria, only one group which contains all firms in the market is formed, just as shown in Proposition 7 with an odd number of firms.

Proposition 7. (Dispersed-completely location pattern) *For any odd or even number $N \geq 2$, the dispersed location pattern at an equal distance from the nearest neighboring firms such that $s^* = (x_1^*, x_2^*, \dots, x_N^*) = (0, \frac{1}{N}, \dots, \frac{N-1}{N})$ constitutes a symmetric subgame perfect Nash equilibrium.*

Proof. See Appendix 5. ‖

When there is an odd number of firms, as shown in Proposition 7, no competition effect from any firm is canceled out by that from another firm, and only one group containing all firms is formed in equilibrium. The internal competition effect disperses all firms as evenly as possible, and as conjured in Pal (1998), the dispersed-completely location pattern minimizes the total transportation cost of all firms. The following lemma plays a crucial role in the proof of Proposition 8 and Proposition 9.

Lemma 3. *In an A-N-P model with $N+1$ firms (N is even), the profile containing all firms' locations can be viewed as $s = (x_1, (x_l)_{l=2}^{N+1})$. When $(x_l^*)_{l=2}^{N+1}$ is given to follow the dispersed-completely location pattern, then point x_1^* is firm 1's optimal location if and only if there exists x_l^*, $l = 2, \dots, N+1$, such that $|x_1^* - x_l^*| = \frac{1}{2}$.*

Proof. Given $x_l = \frac{l-2}{N}$, $l = 2, \dots, N+1$, and without loss of generality, consider $x_1 \in [0, \frac{1}{2N}]$. It suffices to check whether $x_1^* = \frac{1}{2N}$ is an optimal location for firm 1.

From the proof of Proposition 7 (equation (**??**) in page **??**), when only the locations of firms $l \in \{3, \dots, N+1\}$ are considered (i.e. the last $N-1$ firms), it is shown that

$$\int_{x_1}^{x_1+\frac{1}{2}} \sum_{x_j \in (x_l^*)_{l=3}^{N+1}} |x - x_j| dx = \frac{(N-1)(N-4x_1)}{8N}. \tag{22}$$

From the proof of Proposition 6, equation (**??**), when $x_2^* = 0$ is given,

$$\int_{x_1}^{x_1+\frac{1}{2}} |x - x_2^*| dx = -(x_1)^2 + \frac{x_1}{2} + \frac{1}{8}. \tag{23}$$

Hence, a summation of the above two equations shows that

$$LHS = \int_{x_1}^{x_1+\frac{1}{2}} \sum_{x_j \in (x_l^*)_{l=2}^{N+1}} |x - x_j| dx = -(x_1)^2 + \frac{x_1}{2N} + \frac{N}{8}. \tag{24}$$

Since there are $N+1$ firms in the market, from Remark 1, the transportation-cost-median condition requires that $LHS = \frac{N}{8}$, and thus only two points $x_1 = 0$ and $x_1 = \frac{1}{2N}$ are candidates for optimization.

From Proposition 2, it is shown that

$$h = \frac{\partial LHS}{\partial x_1} = -2x_1 + \frac{1}{2N}. \tag{25}$$

Therefore, $h = \frac{1}{2N} > 0$ at $x_1 = 0$ and $h = -\frac{1}{2N} < 0$ at $x_1 = \frac{1}{2N}$. It is proved that, given $(x_l^*)_{l=2}^{N+1} = (0, \frac{1}{N}, \ldots, \frac{N-1}{N})$, $x_1^* = \frac{1}{2N}$ is an optimal location for firm 1. ||

Lemma 3 shows that every dispersed-completely location pattern with an odd number of firms does not present the neutral property defined in Lemma 2. That is, the profit is not the same for any outside firm for all its locations when an odd number of the other firms' locations are given to match the dispersed-completely location pattern. The following proposition, however, shows that a combination of two or more dispersed-completely location profiles corresponds to a location equilibrium with a larger number of players, given that the number of overlapping stacks is not larger than the number of the elements in each stack.

Proposition 8. (Intergroup-agglomeration-and-intragroup-dispersion location pattern) *In an A-N-P model with N firms, suppose $N = n\,J$, where $2 \leq J \leq n$. There exists a location equilibrium $s^* = (s^{1*}, \ldots, s^{J*})$ where $s^{l*} = (0, \frac{1}{n}, \frac{2}{n}, \ldots, \frac{n-1}{n})$, for all $l = 1, \ldots, J$.*

Proof. See Appendix 6. ||

Given the other firms' locations, since the location problem is homogeneous for all firms, it suffices to verify whether any firm chosen arbitrarily has an incentive to deviate from its equilibrium location, i.e. the firm with $x_1^{1*} = 0$. When only the firms in s_{-1}^{1*} are considered, then from Proposition 7 the internal competition effect indicates that the optimal location is indeed at $x_1^1 = 0$. When only the firms in $(s^{l*})_{l=2}^{J}$ are considered, then from Lemma 3 the external competition effect indicates that the optimal location is at $x_1^1 = \frac{1}{2n}$. When the number of outside groups is smaller than the number of firms in each group, however, the competition with other inside firms strictly dominates the competition with all outside firms. Therefore, even when $(s_{-1}^{1*}, (s^{l*})_{l=2}^{J})$ is given, the optimal location $x_1^{1*} = 0$ is the same as the optimal location when only s_{-1}^{1*} is given.

Proposition 8 displays an equilibrium location pattern whereby a competitive group structure with J groups is formed in equilibrium. Here, firms tend to avoid competition with the other firms inside the same group, and at the same time each of them will agglomerate with one and only one firm in each other group. In other words, a combination of J sub-profiles each matching with the dispersed-completely location pattern with n firms exactly constitutes SPNE locations with nJ firms, when the locations of these groups exactly overlap with each other and the number of outside groups $(J-1)$ is not larger than the number of the other inside firms (n-1). The intergroup-agglomeration-and-intragroup-dispersion location pattern shows that, in fact, the locational agglomeration or locational dispersion cannot be clearly classified.

Suppose that each group with n firms is viewed as a multi-plant firm with n plants. The intergroup-agglomeration-and-intragroup-dispersion location pattern is very similar to the equilibrium location pattern in the multi-plant cases in a linear market as shown in Pal and Sarkar (2002). In a linear market with multiple plants analyzed in Pal and Sarkar (2002), it is the intra-brand effect that disperses all of one firm's plants and the natural location effect that dominates the strategic location effect to agglomerate some of the plants of different firms. In a circular market with a single plant as shown in Proposition 8, however, it is the strategic location effect (coming from the substitution in demand) that makes all firms in the same group avoid competition with each other, while the inter-group agglomeration comes from the internal competition effect in each group strictly dominating the external competition effect with outside firms.

After comparing the dispersed-by-pairs location pattern with the intergroup-agglomeration-and-intragroup-dispersion location pattern, it is shown that the neutral property is only a sufficient, rather than a necessary, condition for a capacity to implement a "Nash combination": A combination of multiple profiles, whereby each of them corresponds to a Nash equilibrium of a smaller group of players, constitutes a profile corresponding to a Nash equilibrium of all players in all groups. Proposition 9 offers an equilibrium location pattern composed of several sub-profiles whereby some of them present the neutral property, while the others do not.

Proposition 9. (Intergroup-partial-agglomeration-and-intragroup-dispersion location pattern) *In an A-N-P model with N firms, suppose there are L lists $s^{l*} = (x_1^{l*}, x_2^{l*})$, $l = 1, \ldots, L$, and $J-L$ lists $s^{g*} = (x_1^{g*}, x_2^{g*}, \ldots, x_n^{g*})$, $g = L+1, \ldots, J$, such that*
(i) $|x_1^{l*} - x_2^{l*}| = \frac{1}{2}$, $l = 1, \ldots, L$, $L \geq 1$,
(ii) $s^{g*} = (0, \frac{1}{n}, \ldots, \frac{n-1}{n})$, $g = L+1, \ldots, J$ where $1 \leq J-L \leq n$, $n \geq 3$, and
(iii) $\forall\, l = 1, \ldots, L$, there exists x_i^{l*}, x_k^{g*}, $i = 1, 2$, $k = 1, \ldots, n$, such that $x_i^{l*} = x_k^{g*}$;
then $s^* = ((s^{l*})_{l=1}^{L}, (s^{g*})_{g=L+1}^{J})$ constitutes a subgame perfect Nash equilibrium with N firms whereby $N = 2L + n(J-L)$.

Proof. See Appendix 7. ||

Proposition 9 captures the characteristics of a new equilibrium location pattern which has never been seen in the literature. Suppose there are J lists where (i) L of them follow the dispersed-by-pairs location pattern; (ii) $J-L$ of them follow the intergroup-agglomeration-and-intragroup-dispersion location pattern; and (iii) each paired group has at least one firm agglomerating with one (and only one) firm of every dispersed-completely group; then a combination of all these J lists corresponds to a Nash equilibrium in locations of all firms in all groups.

The intuition of Proposition 9 can be discussed in three parts. First, take a look at any arbitrary firm in arbitrary group g; for example, firm 1 in group J (with a location x_1^J). From the analyses of Lemma 2, the external competition effect from one firm in each group l, $l = 1, \ldots, L$, is canceled out by the other firm in that group, and thus the external competition effect coming from each group l is nil for all $l = 1, \ldots, L$. From the analyses of Proposition 9, when the number of outside groups $(J - L - 1)$ is not larger than the number of the other inside firms $(n - 1)$, the internal competition effect from the other firms in the same group J strictly dominates the external competition effect from outside firms in groups $g \in \{L + 1, \ldots, J - 1\}$. Thus, the optimal location x_1^{J*} is eventually determined by the other inside firms' locations in the same group J.

Second, in each group l, $l = 1, \ldots, L$, for any firm whose equilibrium location is agglomerated with at least one firm in every dispersed-completely group g, $g = L + 1, \ldots, J$. For example, the firm with an equilibrium location x_1^{1*} agglomerates with another firm with a location x_1^{J*}. Since the equilibrium profile of all firms' locations except x_1^{1*} is exactly the same as that except x_1^{J*}, then the reasons for x_1^{J*} being an optimal location can also be applied to x_1^{1*}.

Finally, in each group l, $l = 1, \ldots, L$, for any arbitrary firm whose equilibrium location does not agglomerate with any firm in any dispersed-completely group; for instance, the firm with a location variable x_2^1. Again, from Lemma 2, the external competition effect from the other paired groups $l \in \{2, \ldots, L\}$ is nil. Moreover, from Lemma 3, the best response for internal competition with the firm with x_1^{1*} is consistent for external competition with the dispersed-completely groups. Therefore, the optimal location x_2^{1*} is the same as the location to avoid competition with the other firm inside the same group $l = 1$.

Proposition 9 shows that, since the external competition effect may be exactly canceled out, or the external competition effect may be consistent with the internal competition effect, or the internal competition effect strictly dominates the external competition effect, an optimal location for local competition may be consistent with that for global competition. Thus, an equilibrium location pattern under global competition seems the same as a composite of several location patterns each corresponding to an equilibrium for local competition of a few firms. It is now suitable to summarize the equilibrium location patterns in an A-N-P model from the aspect of an odd or an even number of firms.

Corollary 2. *In an A-N-P model, when the number of firms $N \geq 2$ is even, then the strategy profile $s^* = (s^{1*}, s^{2*}, \ldots, s^{J*})$ constitutes a subgame perfect Nash equilibrium, whereby*

$$s^{l*} = (x_1^{l*}, x_2^{l*}),\ \text{such that}\ |x_1^{l*} - x_2^{l*}| = \frac{1}{2},\ l = 1, \ldots, J,\ \text{and}\ J = \frac{N}{2},$$

or

$$s^{l*} = (0, \frac{1}{n}, \ldots, \frac{n-1}{n}),\ l = 1, \ldots, J,\ n \geq \sqrt{N} \textit{ is odd,} \quad \textit{and} \quad J = \frac{N}{n} \geq 2 \textit{ is even,}$$

or

$$s^{l*} = (x_1^{l*}, x_2^{l*}),\ \textit{such that } |x_1^{l*} - x_2^{l*}| = \frac{1}{2},\ l = 1, \ldots, L, L \geq 1,$$

$$s^{g*} = (0, \frac{1}{n}, \ldots, \frac{n-1}{n}),\ g = L+1, \ldots, J,\ n \geq \sqrt{N-2L} \textit{ is odd,} 1 \leq J - L \leq n \textit{ is even,}$$

$$\textit{and } \forall\, l = 1, \ldots, L, \textit{there exists } x_i^{l*}, x_k^{g*}, i = 1, 2,\ k = 1, \ldots, n, \textit{such that } x_i^{l*} = x_k^{g*}.$$

Proof. The first equilibrium location pattern comes from Proposition 5. Since the constraint $\frac{N}{n} \leq n$ is equivalent to $n \geq \sqrt{N}$, the second equilibrium location pattern is implied by Proposition 8. The third equilibrium location pattern comes from Proposition 9 and a conversion of the constraint $J - L = (N - 2L)/n \leq n$. ||

Corollary 3. *In an A-N-P model, when the number of firms $N > 2$ is odd, then the strategy profile $s^* = (s^{1*}, s^{2*}, \ldots, s^{J*})$ constitutes a subgame perfect Nash equilibrium, whereby*

$$s^{1*} = (0, \frac{1}{2}, 0) \quad \textit{or } (0, \frac{1}{2}, \frac{1}{2}),\ \textit{and} \ \ s^{l*} = (0, \frac{1}{2}),\ l = 2, \ldots, J, \quad \textit{where } J = \frac{N-1}{2},$$

or

$$s^{l*} = (0, \frac{1}{n}, \ldots, \frac{n-1}{n}),\ l = 1, \ldots, J,\ n > 2 \textit{ is odd,} \quad \textit{and} \quad J = \frac{N}{n} \geq 1 \textit{ is an odd number,}$$

or

$$s^{l*} = (x_1^{l*}, x_2^{l*}),\ \textit{such that } |x_1^{l*} - x_2^{l*}| = \frac{1}{2},\ l = 1, \ldots, L, L \geq 1,$$

$$s^{g*} = (0, \frac{1}{n}, \ldots, \frac{n-1}{n}),\ g = L+1, \ldots, J,\ n \geq \sqrt{N-2L} \textit{ is odd,} 1 \leq J - L \leq n \textit{ is odd,}$$

$$\textit{and } \forall\, l = 1, \ldots, L, \textit{there exists } x_i^{l*}, x_k^{g*}, i = 1, 2,\ k = 1, \ldots, n, \textit{such that } x_i^{l*} = x_k^{g*}.$$

Proof. The first equilibrium location pattern comes from Proposition 6, and the second one is a summary of Proposition 7 and Proposition 8. The third equilibrium location pattern comes from Proposition 9 where $J - L$ being odd is implied by an odd number N. ||

The equilibrium location patterns in an A-N-P model have now been systematically categorized in the above two corollaries. Although the possibility of other equilibrium location patterns is hard to be ruled out, the results of this paper so far are much more comprehensive than those in the literatue. The following numerical examples can help to strengthen the conviction of this paper.

Table 1. Numerical location equilibria in Anderson-Neven-Pal models.

Number of firms	Location equilibria
$N=2$	$(0,\frac{1}{2})$
$N=3$	$(0,\frac{1}{3},\frac{2}{3})$, and $(0,\frac{1}{2},0)$
$N=4$	$\{(0,\frac{1}{2},x_1^2,x_1^2+\frac{1}{2}) \mid x_1^2\in[0,\frac{1}{2}]\}$
$N=5$	$(0,\frac{1}{5},\frac{2}{5},\frac{3}{5},\frac{4}{5})$, $(0,0,0,\frac{1}{2},\frac{1}{2})$, and $(0,\frac{1}{3},\frac{2}{3},0,\frac{1}{2})$
$N=6$	$\{(0,\frac{1}{2},x_1^2,x_1^2+\frac{1}{2},x_1^3,x_1^3+\frac{1}{2}) \mid x_1^2,x_1^3\in[0,\frac{1}{2}]\}$, and $(0,\frac{1}{3},\frac{2}{3},0,\frac{1}{3},\frac{2}{3})$
$N=7$	$(0,\frac{1}{7},\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7},\frac{6}{7})$, $(0,0,0,0,\frac{1}{2},\frac{1}{2},\frac{1}{2})$, $(0,\frac{1}{5},\frac{2}{5},\frac{3}{5},\frac{4}{5},0,\frac{1}{2})$, $(0,\frac{1}{3},\frac{2}{3},0,\frac{1}{2},0,\frac{1}{2})$, and $(0,\frac{1}{3},\frac{2}{3},0,\frac{1}{2},\frac{1}{3},\frac{5}{6})$
$N=8$	$\{(0,\frac{1}{2},x_1^2,x_1^2+\frac{1}{2},x_1^3,x_1^3+\frac{1}{2},x_1^4,x_1^4+\frac{1}{2}) \mid x_1^2,x_1^3,x_1^4\in[0,\frac{1}{2}]\}$, and $(0,\frac{1}{2},0,\frac{1}{3},\frac{2}{3},0,\frac{1}{3},\frac{2}{3})$
$N=9$	$(0,\frac{1}{9},\frac{2}{9},\frac{3}{9},\frac{4}{9},\frac{5}{9},\frac{6}{9},\frac{7}{9},\frac{8}{9})$, $(0,0,0,0,0,\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2})$, $(0,\frac{1}{7},\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7},\frac{6}{7},0,\frac{1}{2})$, $(0,\frac{1}{5},\frac{2}{5},\frac{3}{5},\frac{4}{5},0,\frac{1}{2},0,\frac{1}{2})$, $(0,\frac{1}{3},\frac{2}{3},0,\frac{1}{3},\frac{2}{3},0,\frac{1}{3},\frac{2}{3})$, $(0,\frac{1}{2},\frac{1}{3},\frac{5}{6},\frac{2}{3},\frac{1}{6},0,\frac{1}{3},\frac{2}{3})$, and $(0,\frac{1}{5},\frac{2}{5},\frac{3}{5},\frac{4}{5},0,\frac{1}{2},\frac{1}{5},\frac{7}{10})$

* For simplicity, given that at least one firm is located at point 0.

** When $N=3$, the proof is shown in Proposition 4. When $N=5$ or $N=7$ or $N=9$, the first location equilibrium is proved by Proposition 7 and the second one is implied by Proposition 6. The details in verifying other location equilibria with an odd number of firms are shown in Proposition 9, while the listed equilibria with an even number of firms are all covered in Proposition 6 and Proposition 9.

The above numerical results (shown in Figure 1 and Figure 2) can also be checked individually and the complexity in calculations rapidly rises with the number of firms increasing in the market. In other words, the traditional methodology seems difficult to solve the equilibria in an A-N-P model with an arbitrary number of firms.[11] From these examples, the intuition that firms tend to avoid competition in groups in equilibrium in an A-N-P model is always true. For instance, given $s^{1*}=(0,\frac{1}{3},\frac{2}{3})$ and $s^{2*}=(0,\frac{1}{2})$, then the profile $s^*=(s^{1*},s^{2*})$ constitutes an SPNE with $N=5$, where s^{1*} and s^{2*} are just the location equilibria with two and three firms, respectively. When $N=7$, the location equilibria can be viewed as $(0,\frac{1}{7},\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7},\frac{6}{7})$, $((0,\frac{1}{5},\frac{2}{5},\frac{3}{5},\frac{4}{5}),(0,\frac{1}{2}))$, $((0,\frac{1}{3},\frac{2}{3}),(0,\frac{1}{2}),(0,\frac{1}{2}))$, and $((0,\frac{1}{3},\frac{2}{3}),(0,\frac{1}{2}),(\frac{1}{3},\frac{5}{6}))$, respectively. It is noted that in an A-N-P model, contrary to the analyses of congestion games such as Rosenthal (1s973), Milchtaich (1996), and Konishi et al. (1997a), and group formation games in Konishi et al. (1997b) and Hollard (2000), the group joined by one firm is not only determined by its own location alternative, but also by its rivals' choices. The results of this paper show that although the competitive group structures and the equilibrium competition patterns are much more varied, however, the

[11] This is the reason why this paper solves the equilibria by using the characteristics of the equilibria rather than applying a traditional methodology.

intuition behind all location equilibria is consistent in that firms tend to avoid competition with the other firms inside the same group which is endogenously formed in equilibrium in a circular market with Cournot competition.

5 Concluding Remarks

Instead of verifying the equilibrium location patterns in tedious ways, this paper first characterizes the properties of location equilibria in an Anderson-Neven-Pal model, and then uses these properties to solve the equilibrium location patterns, most of which are not yet found in the literature. The results show that equilibrium location patterns in a circular market with Cournot competition include the dispersed-by-pairs, semi-agglomerated-at-two-points, dispersed-completely, intergroup-agglomeration-and-intragroup-dispersion, and intergroup-partial-agglomeration-and-intragroup-dispersion location patterns. The puzzle about the conflict between Pal's (1998) dispersed location pattern and Matsushima's (2001) agglomeration-at-two-points equilibrium is solved by this paper, showing that both of the equilibria are special cases of the dispersed-by-pairs location pattern. These new findings of equilibrium location patterns imply that, like the shapes of amoebas, the equilibrium competition patterns and the competitive group structures of an economic society are much more manifold than what one can imagine about spatial competition.

It is also implied from the diversity of equilibrium location patterns that locational agglomeration or dispersion cannot be categorized simply by the market structure (linear or circular market) or the competition device (Bertrand or Cournot competition). What can be said is, at most, the intuition behind the location patterns in equilibrium under different structures and competition devices. In a circular market with Cournot competition, the consistent intuition of all equilibrium location patterns is that firms tend to avoid competition by groups. When the external competition effect is exactly canceled out, or the external competition effect is consistent with the internal competition effect, or the internal competition effect strictly dominates the external competition effect, the existence of a group of outside firms has no effective impact on one firm's best response for the competition with other inside firms. In other words, an optimal location under local competition is consistent with an optimal response for global competition. Therefore, without any externality and prior collusion, a competitive group structure is endogenously formed by the equilibrium location patterns, and inside each group, firms tend to avoid competition with the other insiders. In contrast to the implication of Pal and Sarkar (2002) where the location choices for one duopolist's multiple plants can be neutral to that of the other firm *in equilibrium*, this paper shows that, even though competition among all firms of all groups does exist in an Anderson-Neven-Pal model, the equilibrium locations of a group of firms may be neutral to that of another group of firms.

Faced with an n-person non-cooperative game problem where the number of players joining the game may be a large or an unknown number, an ideal treatment is to first divide all players into several different and exclusive groups and solve the equilibrium of each group; then a combination of all equilibrium profiles will correspond to an equilibrium of all players of all groups. When this ideal treatment is applicable, the model is said to have a capacity to implement a "Nash combination". In a model structure with a capacity to implement a Nash combination, all characteristics of the equilibrium of a 2-person game (or non-cooperative game with few players) can be maintained with that of an n-person game.

From the analyses of the dispersed-by-pairs location pattern, the sufficient conditions include the neutral property that, given a group of players achieving a Nash equilibrium, the payoff is the same for any outside player for all available strategies and the linearity of each player's reaction function in every other player's alternative. When one player makes a comparison among different strategy choices, given an equilibrium profile of outside players presenting the neutral property, the player is just like facing a market without any outsider and thus the external competition is neutral for the internal competition, while the linearity of the reaction function isolates the influence for one player's best response caused by each of the other players. It can be inferred that, not only the Anderson-Neven-Pal model, there is a capacity to implement a Nash combination in all symmetric non-cooperative games satisfying these two conditions. In these models with a large number of players, a Nash equilibrium under global competition can be found by first looking for Nash equilibria each for a group of few players and then a combination of these equilibrium profiles will yield the exact result. Further serious work, however, is needed to verify these observations.

The analyses of an Anderson-Neven-Pal model are not only instructive in spatial competition, but may also have great implications in studies about topics such as anti-trust policy,[12] international trade, and the competition between different schools,[13] organizations, industries, or networks.

[12]The sufficient conditions to implement a Nash combination can also be viewed as the theoretical conditions to isolate the competition of a group of firms from the competition of another group of firms in equilibrium, which may be applicable in analyzing industrial policies.

[13]A professor in one university competes in research not only with colleagues belonging to the same school, but also with other researchers in other universities, even though they may not be in the same country.

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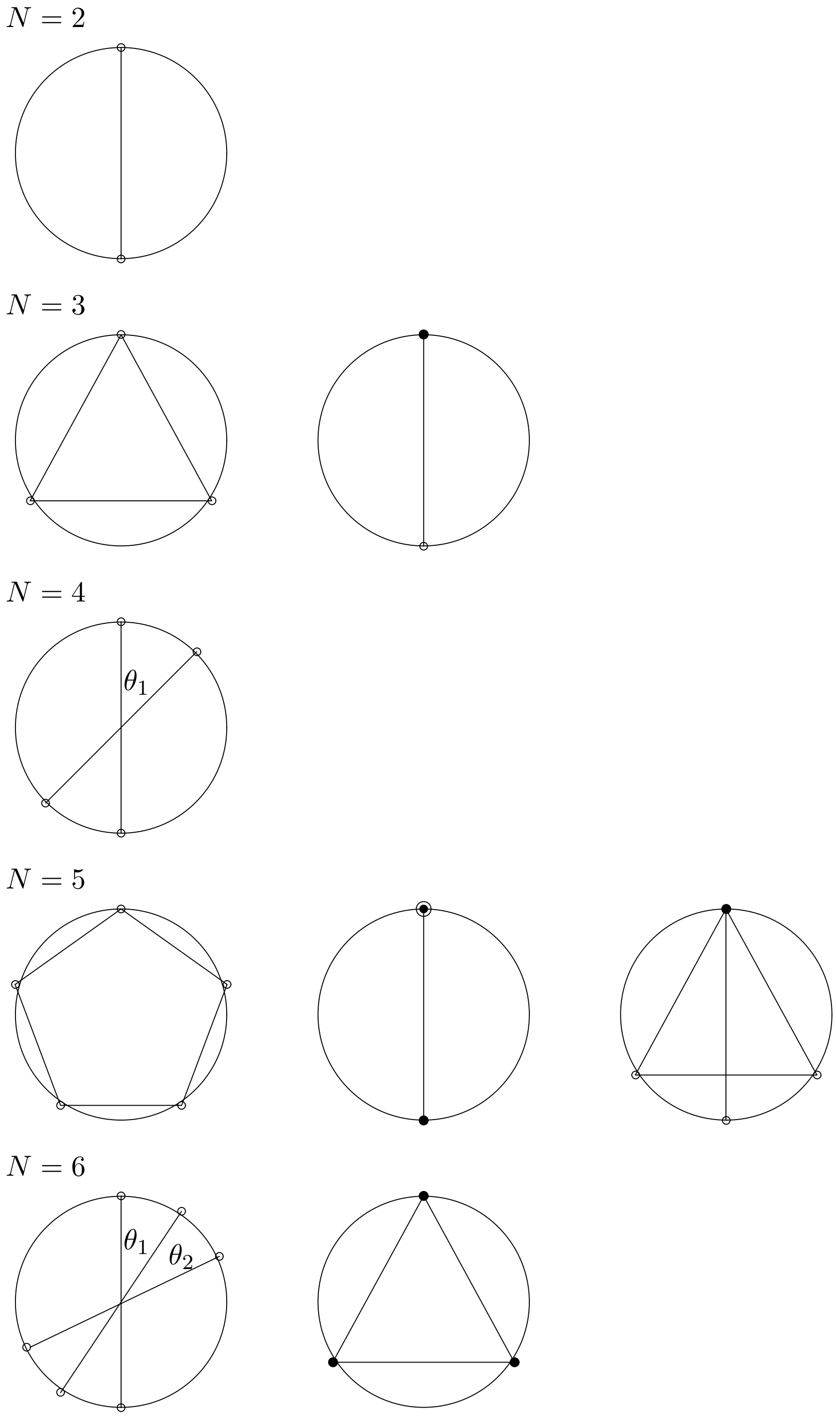


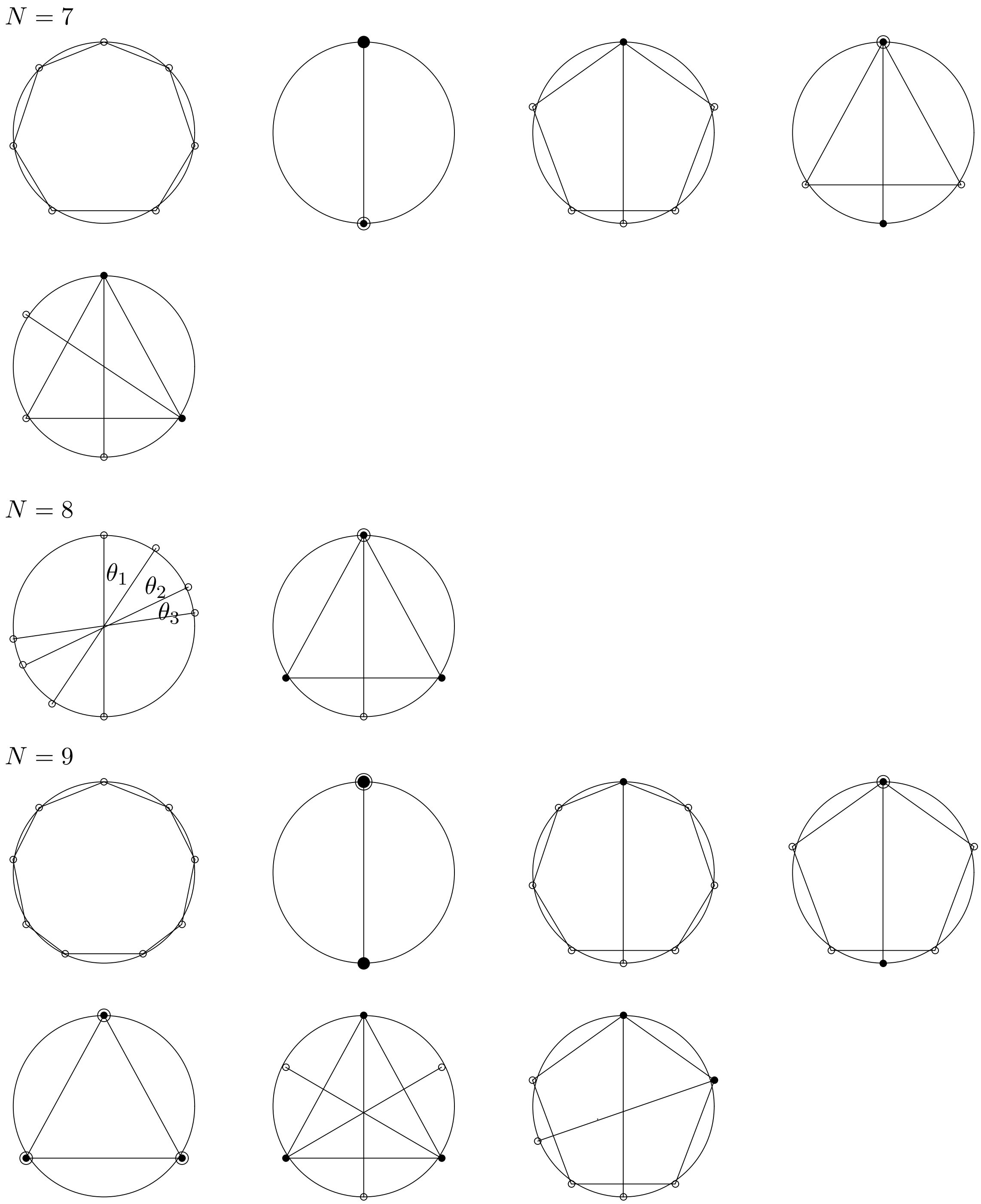
∘ represents one firm's location

• represents two firms agglomerating at the point

⦿ represents three firms agglomerating at the point

θ_i represents arbitrary degrees of the angle

Figure 1: The equilibrium location patterns with N firms, $N = 2, 3, 4, 5, 6$, in Anderson-Neven-Pal models.



- ∘ represents one firm's location
- • represents two firms agglomerating at the point
- ⊙ represents three firms agglomerating at the point
- ● represents four firms agglomerating at the point
- ◉ represents five firms agglomerating at the point
- θ_i represents arbitrary degrees of the angle

Figure 2: The equilibrium location patterns with N firms, $N = 7, 8, 9$, in Anderson-Neven-Pal models.