The Sport League's Dilemma: Competitive Balance versus Incentives to Win*

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Abstract

We analyze a dynamic model of strategic interaction between a professional sport league that organizes a tournament, the teams competing to win it, and the broadcasters paying for the rights to televise it. Teams and broadcasters maximize expected profits, while the league's objective may be either to maximize the demand for the sport or to maximize the teams' joint profits. Demand depends positively on symmetry among teams (competitive balance) and how aggressively teams try to win (incentives to win). Revenue sharing increases competitive balance but decreases incentives to win. Under demand maximization, a performance-based reward scheme (used by European sport leagues) may be optimal. Under joint profit maximization, full revenue sharing (used by many US leagues) is always optimal. These results reflect institutional differences among European and American sports leagues.

Keywords: Sport league, Revenue sharing, competitive balance, incentives to win.
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1 Introduction

Revenue sharing is a controversial topic in the organization of many professional sport leagues. In recent years, its importance has become even more evident given the large payments American and European professional sports leagues fetch from television broadcasters.[1] In this paper, we present a rigorous analysis of the opposing views in this controversy.

The argument in favor of revenue sharing in sports observes that there are large differences among revenues and wealth of teams. For example, Scully (1995) and Fort and Quirk (1995) provide evidence on large disparities of ticket sales and revenues from local TV deals among teams located in different cities. As a consequence, richer teams tend to be more successful since they can afford better players.[2] A mechanism which redistributes income from richer to poorer teams makes future competition more balanced, hence more enjoyable to the fans. A consequence of this argument is that revenue sharing increases future demand for the sport, hence increasing the revenues of the league. Furthermore, if teams are profit maximizers, revenue sharing also decreases the price teams pay for top players since their marginal value decreases. Hence, revenue sharing also has a positive impact on the profit of teams.

The case against revenue sharing is based on the idea that if there are no prizes for winning teams' profits are independent of a competition's outcome. Without a prize there are no monetary incentives for a team to win. In the end, this may have a negative effect on demand since the lack of incentives for team owners induces lack of incentives for players.[3] As noticed by Daly (1992) and Fort and Quirk (1995), if teams have nothing to compete for, fans may strongly doubt the integrity of the competition on the playing field with an obvious negative effect on demand. Hence, revenue sharing has a negative impact on current demand and team profits.

Unsurprisingly, professional economists have debated different revenue sharing arrangements, and their consequences for teams and players as well as for the demand for sports (see Fort and Quirk (1995) for a comprehensive review). In the end, the question is how to allocate revenues if the product is the result of a joint production of effort by several participating firms. Surprisingly, there are relatively few theoretical analyses of this question.

As seen above, revenue sharing has different consequences for current and future demand for the sport. Therefore we study a dynamic model where professional teams compete to win a tournament and the league decides how to allocate revenues generated by the competition among

[1] The latest reported television deals for NFL and NBA, for example, are \$17.6 billion over eight years and \$2.4 billion over four years respectively (see Araton 1998).

[2] See Scully (1995) for detailed evidence.

[3] An example of this effect is given by the higher TV ratings for playoff matches when compared to regular season ones.

winners and losers. In particular, each period has the following sequence of moves. First, the league's revenues are determined by the sale of broadcasting rights. Then, the league decides how to allocated these revenues among winners and losers. Finally, the teams compete to win the tournament organized by the league. In this framework, we can properly address the following question: how should a professional sport league allocate revenues among participating teams?

The starting point is a description of aggregate demand for a sporting competition. This determines how much money the league may obtain for selling the rights to broadcast the event. Aggregate demand for a sport is ultimately determined by how much the fans enjoy watching the tournament in which the teams compete. Following the literature surveyed in Fort and Quirk (1995), we assume it depends on three factors. These are quality of the league, how hard teams try to prevail, and competitive balance in the tournament.

Quality of the league reflects its ability to attract talented athletes. It is measured by the combined wealth of the teams. A wealthier league (i.e., a league with a larger total wealth of teams) attracts better players. Therefore, teams' combined wealth has a positive effect on demand. The size of this effect is influenced by the environment in which the league operates. For example, US sport leagues are essentially monopsonists in the market for players in a given sport.[4] In this case, only intra-league trades are observed and league-wide talent is roughly constant. European sport leagues, on the other hand, compete with each for top players. In this case, inter-league trades of top players are observed frequently.

Willingness to win by teams is reflected by the salaries teams pay to their athletes. If the effort players produce is observable, a higher salary is the consequence of a higher effort. If the effort is not observable, higher prize when winning the competition generates a higher effort.

Competitive balance is measured by uncertainty of the outcome. Fans enjoy more sporting events whose winners are not easy to predict. In other words, the more symmetric the winning chances of the competitors the more exciting the tournament is to watch. Since a team's probability of winning ultimately depends on the athletes playing for it, competitive balance also depends on a team's wealth and how much it pays its athletes.

The league chooses a monetary reward scheme conditional on the tournament's outcome. It knows this choice influences how teams compete in the event. Hence, the league knows more or less revenue sharing influence aggregate demand for the sport. In a dynamic setting, revenue sharing has two effects on demand. The first effect we call *competitive balance*: increased revenue sharing at time t increases demand at time $t+1$ by making the teams' future winning chances more equal. This effect has consequences for competitive balance at time $t+1$ even if teams are identical at time t: a large prize for today's winner introduces an asymmetry in

[4]There are very few cases of athletes being able to play more than one sport at professional level.

tomorrow's winning probabilities. The second effect we call *incentives to win*: increased revenue sharing decreases current demand by lowering the monetary value of winning and consequently diminishing teams' interest and effort toward winning. This effect lowers demand since fans enjoy more effort from players.

In this paper, we derive the optimal level of revenue sharing in a repeated tournament by analyzing the trade-off between competitive balance and incentives to win. Of course, the optimal level of revenue sharing depends on what the league wants to accomplish. We consider two natural possibilities for the objective function of professional sport leagues. First, we assume the league maximizes its revenues, given by the amount of money it obtains from television broadcasters. In our framework, this assumption is equivalent to maximizing demand for the sport. Second, we also consider the league as a cartel of profit maximizing firms and assume it maximizes the teams' joint profit (as assumed by Atkinson, Stanley and Tschirhart (1988)). The first case roughly corresponds to sport leagues more independent from teams, and it matches the European institutional organization of professional sports;[5] the second case resembles more the way in which American professional sport leagues are organized.

Under demand maximization, a performance-based reward scheme which depends on the tournament's outcome may be optimal. This implies that full revenue sharing is not optimal. Under joint profits maximization, the reward scheme does not depend on the tournament's outcome: full revenue sharing is always optimal. These results match observed behavior of some sports leagues. In European top soccer leagues, broadcasting revenues are distributed according to performance (see Tables 1 and 2).[6] Sharing of national TV deals, on the other hand, is more common in US sport leagues (see Scully, 1995). The main reason for the difference of results between the two objective functions is that under joint profit maximization the league internalizes players' salaries in its objective function while in the case of the demand maximization, it does not.

Our paper extends the existing literature in several ways. First, we consider a multi-period model. Therefore, we are able to capture the intertemporal trade-off generated by revenue sharing between demand and profits today and demand and profits tomorrow. Second, we consider the

[5] For example, in France, the organization of the Premiere Division championship (top national competition) is delegated by the Federation Francaise de Football (FFF) to the Ligue National de Football (LNF). All French teams (both professional and non professional) are affiliated to the FFF. All professional teams (both those in Premiere Division and those participating in minor championships) are affiliated to the LNF. A fraction of the money received by the LNF for the sale of Premiere Division broadcasting rights is transferred to the FFF and a fraction is allocated to minor championship teams. In such a context, the maximization of revenues from TV deals seems a reasonable assumption.

[6] See also Hamil, Michie and Oughton (1999) for more details about England.

possibility that a league faces competition from other leagues and that they compete for top players as is the case in Europe. Therefore, we can study the influence of revenue sharing at time t on league-wide talent at time $t+1$. Related papers in a competitive environment are those of Hoen and Szymanski (1999) and Palomino and Sakovics (2000). Hoen and Szymanski study the impact of the participation of top clubs in international competitions on the competitive balance of the domestic leagues. They do not address the issue of the optimal level of revenue sharing. Palomino and Sakovics (2000) consider a static model and compare the organization of markets for talent when a league is isolated and when it operates in a competitive environment. They show that full revenue sharing in US sports may result from monopsony power in the market for talent rather then from competitive balance considerations. Other papers (El Hodiri and Quirk (1971), Atkinson, Stanley and Tschirhart (1988), Fort and Quirk (1995), Vrooman (1999)) consider static models and focus on the case of US sport leagues that do not face competition.

The analysis carried out in this paper goes beyond the sports literature. Our model presents an example of a repeated moral-hazard problem between a principal and multiple agents in which the difference in output produced by the agents is detrimental to the principal.[7] An example of such a situation is the production and distribution of electricity.[8] In this setting, the principal faces a trade-off between "output balance" among agents and incentives to produce large quantities. The solution for the principal is to propose agent-specific contracts, the agent with the higher productivity rate receiving a lower marginal revenue in order to decrease his incentives to produce. The specific feature of a sport competition (or any other contest) is that the contract the principal proposes has to be the same for all agents. Rewards can only be based on relative performances. The reward for the winner cannot vary with his identity.

Our model is also related to Moldovanu and Sela (2000) who study the optimal allocation of prizes in contests. They consider a contest where the highest bidder (or agent making the highest effort) wins and the goal of the designer is to maximize the sum of all bids. In this framework, they show that if the contestants face linear or concave cost functions, then the allocation of the entire prize money to the winner is optimal. Conversely, if the contestants face convex cost functions, several prizes may be optimal. Our model differs from Moldovanu and Sela's in two crucial ways. First, we consider sequential contests and both the bids (the effort in our context) at time t and the outcome of the contest at time t influence bids made at time $t+1$. Hence, when deciding on the allocation of prizes at time t, the contest designer (i.e., the league in our case)

[7] For example, consider a situation such that there are two agents 1 and 2, the income of the principal at time t is $\text{Min}(q_{t,1}, q_{t,2})$, $q_{t,i}$ being the output of agent i at time t. Moreover, $q_{t,i}$ depends on agent i's (unobservable) effort, his productivity and some noise, and the productivity at time t depends on past income. (One can think of productivity as being the consequence of investment in more or less sophisticated machines.)

[8] We would like to thank Ines Macho-Stadler for suggesting this example.

must take into account the impact on future contests. Second, one of the specificity of sport events is that competitive balance is valuable. This reduces incentives for the contest designer to award all the prize money to the winner at time t since it may decrease competitive balance at time $t + 1$.

The organization of the paper is as follows. Section 2 introduces the basic model and Section 3 derives its equilibrium. Section 4 considers three possible extensions: multi-period TV contracts, sources of teams revenues that do not depend on the leagues' decisions, teams that cannot observe players effort (how hard they try to win). Section 5 concludes.

2 The Model

In this section, we present a simple two periods model of strategic interaction between professional sport teams, a professional sport league, and television broadcasters. Two teams compete in the tournament organized by the league and shown on television by a broadcaster. More specifically, the following sequence of moves occurs in each period. First, the broadcaster decides how much to pay for the exclusive right to televise the sporting event. Then, the league decides how to divide this money between the loser and the winner of the tournament. Finally, the teams simultaneously decide how much to spend on players' incentives. At the end of the period, the tournament is played, the winner is determined, and money is awarded accordingly.

Before presenting the model in detail, two remarks are in order. First, we focus on the sale of rights to national TV networks and on the allocation of the corresponding revenues among teams. Of course, teams have other sources of revenues (for example, ticket sales, sponsoring, merchandising, local TV deals). In our model, these are captured by differences in initial wealth among teams; in other words, we assume these other sources of profits are constant over the two periods. Second, the model does not explicitly include a market for talent.[9,10] Although both these aspects deserve attention, our focus is on a model simple enough to capture the basic trade-off between competitive balance and incentives.

Demand

Demand for the sport depends on three sets of variables: the talent of the athletes playing the sport (league quality), their attempt to prevail in the competition (willingness to win),

[9] See Palomino and Sakovics (2000) for a market for talent with competing leagues.

[10] Implicitly, we assume that talent is linear in price and that teams maximize their expected profit from talent under the constraint that they cannot borrow. In this case, if for a cost of talent equal to the total wealth of a team, the marginal profit is larger than the marginal cost, teams invest their entire wealth.

and competitive balance of the tournament. The league's quality is measured by the wealth of the participating teams; this reflects their ability to attract skilled athletes away from other sports. Willingness to win is measured by incentives to induce players' effort; it is important because fans enjoy athletes playing hard.[11] Competitive balance is measured by uncertainty of the tournament's outcome; fans enjoy sporting events more if the winner is not easy to predict. In other words, the more symmetric are the teams' winning chances, the more exciting is the tournament.

In each period t, D_t denotes demand in monetary terms, $e_{t,i}$ denotes team i's effort, $p_{t,i}$ its probability of winning, and $W_{t,i}$ its wealth; similarly for team j. Then, demand is given by

$$D_t = \gamma\,(e_{t,i} + e_{t,j}) + \delta[1 - (p_{t,i} - p_{t,j})^2] + \nu\,(W_{t,i} + W_{t,j})\,, \tag{1}$$

with $\gamma \in (0,1)$, $\nu \in (0,1)$, and $\delta > 0$.[12] Note that δ represents the monetary value of one unit of competitive balance while γ represents the monetary value of one unit of willingness to win. Equation (1) can loosely be interpreted as measuring fans welfare from watching the tournament. In this respect, the first term measures the importance of watching athletes 'give their best'. The second term measures the importance of watching an evenly matched tournament. The third term measures the importance of watching good athletes. Roughly, this last effect allows for several competing leagues where each league talent level depends on the total wealth of its teams.[13]

Broadcasters

The market for TV rights is perfectly competitive. Therefore, a broadcaster expects zero profits in equilibrium. Since demand is expressed in monetary terms, we assume that broadcasting of the games generates income from advertising and this income increases with the audience that watches them. In particular, we let K_t denote the amount paid by the broadcaster and impose $K_t = D_t$ in each period. In other words, at this stage we only consider one-period deals between broadcasters and the league. Later in the paper (see Section 4.1), we extend the framework to include multi-period deals.

[11] A possible fourth set of variables may measure fans' attachment to a team. Since we model demand for the sport, we assume that these "individual team" effects wash out in the aggregate.

[12] Many simplifying assumptions, like the functional form chosen for the demand for the sport, are made for simplicity and are not necessary for our qualitative results.

[13] This assumption corresponds to the case of European sport leagues who organize domestic competitions and sign TV deals with national broadcasters. Top players often switch from one league to another, hence changing league-wide talents. US sport leagues, on the other hand, are in an isolated environment where league-wide talent is given and only intra-league trades occur (with few exceptions at the draft level).

League

After receiving K_t from the broadcaster, the league decides how to allocate this sum between the two teams. We assume this allocation can only be contingent on the outcome of the tournament. In particular, this rules out allocations which explicitly depend on the teams' wealth.[14]

The league's interest is to maximize the demand for the sport. This does not necessarily imply the league cares exclusively about fans' welfare, since it may have long run objectives to maximize demand.[15] Given the assumption of perfect competition in the broadcasting industry, this is equivalent to assuming that the league maximizes the revenues from the sale of TV rights.

Because of the perfect competition in broadcasting assumption, maximizing demand and maximizing revenues from TV rights are equivalent objectives. Let $K_{t,w}$ and $K_{t,l}$ denote the amounts allocated to period t winner and loser respectively (obviously, $K_{t,w} + K_{t,l} = K_t$). Then, in period 2 the league chooses $K_{2,w}$ and $K_{2,l}$ to maximize D_2, while in period 1 it chooses $K_{1,w}$ and $K_{1,l}$ to maximize $D_1 + D_2$.

Probability of winning the tournament

The tournament's outcome depends on the quality of each team and on their choices of incentives for players. The first aspect represents teams' initial ability; the second represents the effort spent towards winning. A richer team can buy better players, hence having an initial advantage. However, a poorer team can compensate this initial disadvantage by producing a higher effort level. In order to make players produce a higher effort level, teams must reward them. Here, the effort level is measured in monetary terms. We capture these ideas modelling the probability to win.

The probability that team i wins period t tournament depends on its players' talent and how hard they play. Talent can be thought of as a team's ability to sign players at the beginning of the season; therefore, it is measured by the team's wealth $W_{t,i}$. How hard players try to win can be thought of as effort, and is measured by the incentives necessary for players to perform during the season $e_{t,i}$. Formally, the probability that team i wins in period t is

$$p_{t,i} = \begin{cases} \alpha \frac{e_{t,i}}{e_{t,i}+e_{t,j}} + \beta \frac{W_{t,i}}{W_{t,i}+W_{t,j}} & \text{if} \quad e_{t,i} + e_{t,j} > 0 \\ \frac{W_{t,i}}{W_{t,i}+W_{t,j}} & \text{if} \quad e_{t,i} + e_{t,j} = 0 \end{cases}$$

[14] This does not seem an unrealistic assumption since we do not see tournament prizes of the sort "since the richer team has won, amount x goes to the winner and amount y goes to the loser".

[15] We also solve the equivalent model where the league maximizes teams' joint profits. There, the league internalizes the costs faced by the teams and it represents a cartel as assumed by Atkinson, Stanley and Tschirhart (1988). Unsuprisingly, full revenue sharing is always optimal in that case since incentives to players by teams cancel out with each other and represent a waste of resources from the cartel's point of view.

with $\alpha + \beta = 1$. Quite obviously, $p_{t,j} = (1 - p_{t,i})$ since there are only two teams. The probability of winning increases with the difference in effort and the difference in wealth. When the two teams are equally wealthy and produce the same effort level, their probability of winning is $\frac{1}{2}$. One can think of $\frac{\alpha}{\beta}$ as a measure of how winning depends on incentives relative to initial quality. If $\frac{\alpha}{\beta} > 1$ the marginal return to effort is higher than the marginal return to wealth; loosely speaking, trying hard is more important than being better.

Teams

Each team's objective is to maximize current expected profits with an appropriate choice of incentives for its players. This choice is made knowing the prizes the league will award to winner and loser of the current tournament. Formally, team i's profits are:

$$\Pi_{t,i} = p_{t,i}K_{t,w} + (1 - p_{t,i})K_{t,l} - e_{t,i}$$

Implicitly, here we assume that the effort produced by players is observable. Later we relax this assumption and explicitly consider the case where it is not (see Section 4.3).

Team i has an initial wealth equal to $W_{1,i}$. If it wins the tournament, its wealth in the second period is

$$W_{2,i} = W_{1,i} + K_{1,w} - e_{1,i}.$$

If it looses the tournament, its wealth in the second period is

$$W_{2,i} = W_{1,i} + K_{1,l} - e_{1,i}.$$

Therefore, the prizes awarded to the winner and loser of the first period have an effect on competitive balance in the second period.[16] In other words, the probability of winning in period 2 depends on the outcome of period 1 because of the effect prizes have on the teams' wealth, hence on teams' talent. Here, talent should be interpreted as a durable good. Team i invests $W_{i,1}$ at the beginning of period 1 and $\Pi_{i,1}$ at the beginning of period 2, and the amount of talent accumulated at the beginning of period 2 is $W_{i,2} = W_{i,1} + \Pi_{i,1}$.

Finally, we assume that in each period, teams cannot borrow and face a solvency constraint: a losing team has enough cash to compensate players for their effort. This implies that for all t, $e_{t,i} \leq K_{t,l}$.[17]

[16] Hence, a fully rational team should consider the influence of its strategy in period 1 on the game that will be played in period 2. At this point, we consider the behavior of myopic teams. This seems realistic since a league is usually made of a relatively large number of teams and the strategic influence of a specific team on the revenue of the league in the following period is small. In any case, the behavior of farsighted fully rational teams is analyzed in Appendix A.2.

[17] We could derive this restriction as the result of a much more complicated model. Suppose, for example, that

3 The Equilibrium

In this section, we characterize the equilibrium of the game described previously. We begin by analyzing the subgame starting at the beginning of period 2. The solution concept we use is subgame perfect Nash equilibrium. We start with period 2 subgame and look at three optimization problems. First, the teams' optimal effort choices given their wealth, the prizes decided by the league, and the TV rights; second, the league's optimal prize choice, given the TV rights, and the teams' equilibrium play that follows; finally, the broadcaster's optimal TV rights choice, given teams' and league equilibrium plays. Then, we repeat a similar procedure for period 1, considering equilibrium play in period 2.

3.1 The Effort Game

Period 2 ends with the teams playing a simultaneous move game in which they choose incentives for players. Each team i chooses the level of incentives to maximize period 2 profits. Formally, team i solves $\max_{e_{i,t} \leq K_{t,l}} \Pi_{t,i}$ where

$$\Pi_{t,i} = p_{t,i} K_{t,w} + (1 - p_{t,i}) K_{t,l} - e_{t,i}. \tag{2}$$

In other words, in every period t we have a simultaneous moves game in effort.[18] The equilibrium of this game is characterized by the following proposition.

Proposition 1 *In the equilibrium of the effort game, the optimal strategies are*

$$e_{t,i} = e_{t,j} = min\left(K_{t,l}, \frac{\alpha}{4}\Delta K_t\right) \tag{3}$$

where $\Delta K_t \equiv K_{t,w} - K_{t,l}$.

Proof: See Appendix.

This proposition says that the effort produced by teams increases with the difference between the prize money going to the winner and the loser. For any given K_t, we can interpret the size as ΔK_t as a measure of (lack of) revenue sharing; in particular, full revenue sharing corresponds to $\Delta K_t = 0$ (and therefore $K_{t,w} = K_{t,l} = \frac{1}{2} K_t$) while no revenue sharing corresponds to $\Delta K_t = K_t$. Then, Proposition 1 gives a relationship between the level of revenue sharing and the equilibrium

the league pays $K_{t,l}$ upfront (before the tournament takes place) to both teams while ΔK_t is paid to the winner after the tournament. In this case, wealth evolves over time along the lines described previously and, because of the no borrowing condition, teams cannot pay more than $K_{t,l}$ as incentives to players.

[18] As noticed previously, we assumer teams are myopic; hence, the game they play in period 1 is identical to the one in period 2.

choices of incentives by the teams. The larger the amount of revenue sharing (i.e., the smaller ΔK_t), the smaller the effort level produced by teams. Intuitively, players need incentives to produce effort; the size of the prize for winning the tournament determines the incentives teams are willing to pay to their players.

The equilibrium of the effort game has some consequences on the demand for the sport in each period: substituting Equation (3) in Equation (1), we obtain

$$D_t = \gamma \frac{\alpha \Delta K_t}{2} + \delta \left[1 - \left(\beta \frac{W_{t,i} - W_{t,j}}{W_{t,i} + W_{t,j}} \right)^2 \right] + \nu \left(W_{t,i} + W_{t,j} \right) \tag{4}$$

Since the effort level is symmetric, the competitive balance part of the demand is unaffected by effort: it only depends on the wealth differential. The willingness to win part of the demand, instead, is determined by the prize difference between winning and losing. Equilibrium in the effort game implies that the strategically relevant variable from the league point of view is ΔK_t.

3.2 Play in the Second Period

While the effort game is, by assumption, the same in both periods, the analysis of league and broadcasters behavior differs. Therefore, we start with the subgame in which the league has received some money through the sale of the TV rights.

League Behavior in Period 2

Given the solution of the effort game, the league maximizes demand for the sport maximizing the teams aggregate effort by making the prize for winning as large as possible. This can be seen by substituting the teams' equilibrium effort choices in period 2 demand function:

$$D_2 = \gamma \frac{\alpha \Delta K_2}{2} + \delta \left[1 - \left(\beta \frac{W_{2,i} - W_{2,j}}{W_{2,i} + W_{2,j}} \right)^2 \right] + \nu \left(W_{2,i} + W_{2,j} \right).$$

The league's optimal allocation for a given amount of funds available is described by the following proposition.

Proposition 2 *A league that maximizes demand for the sport chooses*

$$\Delta K_2 = K_{2,w} - K_{2,l} = \frac{2}{2+\alpha} K_2 \tag{5}$$

Proof: see Appendix.

In the last period of the game full revenue sharing cannot be optimal. Since this is the end of the game, the effect prizes have on future wealth is irrelevant to the league decision. Furthermore, current wealth does not depend on the outcome of this period's tournament. In other words, the competitive balance in the second period cannot be changed. Therefore, only the incentives to win effect matters to the league. In these conditions, the optimal choice is to make the effort chosen by teams as large as possible. When the demand for sport depends on the effort produced by teams, and competitive balance is irrelevant, full revenue sharing does not lead to the maximization of demand for the sport.

Broadcasters' Behavior in Period 2

Using Proposition 2, we can write demand in period 2 as a function of K_2:

$$D_2 = \gamma \frac{\alpha K_2}{2+\alpha} + \delta \left[1 - \left(\beta \frac{W_{2,i} - W_{2,j}}{W_{2,i} + W_{2,j}} \right)^2 \right] + \nu \left(W_{2,i} + W_{2,j} \right). \tag{6}$$

Demand for the sport depends positively on K_2, the amount of money made available by TV rights. This is due to the willingness to win effect: the more money the broadcaster pays, the larger the prize awarded to the winner, the more incentives teams offer to players. In the end, this represents a positive effect on demand because spectators enjoy players who try hard.

To conclude the analysis of play in period 2, we determine K_2, the revenues paid to the league by broadcasters. Since there is perfect competition in the broadcasting industry, broadcaster are willing to pay the full monetary value of demand. Formally, $D_2 = K_2$. Substituting this in Equation (6), we find that the equilibrium amount broadcaster pay in period 2 is given by

$$K_2 = \frac{2+\alpha}{2+\alpha\left(1-\gamma\right)} \left\{ \delta \left[1 - \left(\beta \frac{W_{2,i} - W_{2,j}}{W_{2,i} + W_{2,j}} \right)^2 \right] + \nu \left(W_{2,i} + W_{2,j} \right) \right\}. \tag{7}$$

The Equilibrium in Period 2 **and the Link Between Periods**

The following corollary to Proposition 1 and Proposition 2 describes the equilibrium in Period 2 subgame.

Corollary 3 *The equilibrium in period* 2 *is characterized as follows: the amount paid for television rights equals demand in that period and is given by*

$$K_2^* = D_2^* = \frac{2+\alpha}{2+\alpha\left(1-\gamma\right)} \left\{ \delta \left[1 - \left(\beta \frac{W_{2,i} - W_{2,j}}{W_{2,i} + W_{2,j}} \right)^2 \right] + \nu \left(W_{2,i} + W_{2,j} \right) \right\};$$

the prize received by the team which wins the tournament is given by

$$\Delta K_2^* = \frac{2}{2+\alpha(1-\gamma)}\left\{\delta\left[1-\left(\beta\frac{W_{2,i}-W_{2,j}}{W_{2,i}+W_{2,j}}\right)^2\right]+\nu\left(W_{2,i}+W_{2,j}\right)\right\};$$

and the incentives teams pay are given by

$$e_{2,1}^* = e_{2,2}^* = \frac{\alpha}{4+2\alpha(1-\gamma)}\left\{\delta\left[1-\left(\beta\frac{W_{2,i}-W_{2,j}}{W_{2,i}+W_{2,j}}\right)^2\right]+\nu\left(W_{2,i}+W_{2,j}\right)\right\}.$$

Proof: substitute Equation (7) in Equations (5) and (3). □

The two periods are linked through the decisions players make in the first period and through the outcome of the tournament in the first period. In particular, when winner and loser in period 1 perceive different prizes, this reflects on their wealth in period 2. Therefore, broadcasters and league behavior in period 1 could influence the equilibrium in period 2 through the teams' wealth. In particular, two period 2 variables are relevant: aggregate wealth and the wealth difference.

Period 2 aggregate wealth does not depend on the outcome of the tournament which took place in the first period:

$$\begin{aligned} W_{2,i}+W_{2,j} &= W_{1,i}+W_{1,j}+K_1-\left(e_{1,i}+e_{1,j}\right) \\ &= W_{1,i}+W_{1,j}+K_1-\frac{\alpha\Delta K_1}{2}. \end{aligned}$$

Period 2 difference in wealth, instead, depends on the outcome of this tournament. If team i has won, the difference is given by:

$$\begin{aligned} W_{2,i}-W_{2,j} &= W_{1,i}-W_{1,j}+\Delta K_1-e_{1,i}+e_{1,j} \\ &= W_{1,i}-W_{1,j}+\Delta K_1 \end{aligned}$$

while the same difference when team i has lost is

$$\begin{aligned} W_{2,i}-W_{2,j} &= W_{1,i}-W_{1,j}-\Delta K_1-e_{1,i}+e_{1,j} \\ &= W_{1,i}-W_{1,j}-\Delta K_1 \end{aligned}$$

The demand (i.e., the amount the broadcaster pays), and the effort chosen by the teams both depend on the outcome of period 1's tournament. More specifically, let D_2^i denote the demand in period 2 when team i has won the previous tournament, and let and D_2^j denote the demand in period 2 when team j ($j \neq i$) has won. Using Equation (7) we can compute these demands explicitly, obtaining:

$$D_2^i = \frac{2+\alpha}{2+\alpha(1-\gamma)} \left\{ \begin{array}{c} \delta \left[1 - \left(\beta \frac{W_{1,i} - W_{1,j} + \Delta K_1}{W_{1,i} + W_{1,j} + K_1 - \frac{\alpha \Delta K_1}{2}} \right)^2 \right] \\ + \nu \left(W_{1,i} + W_{1,j} + K_1 - \frac{\alpha \Delta K_1}{2} \right) \end{array} \right\} \tag{8}$$

and

$$D_2^j = \frac{2+\alpha}{2+\alpha(1-\gamma)} \left\{ \begin{array}{c} \delta \left[1 - \left(\beta \frac{W_{1,i} - W_{1,j} - \Delta K_1}{W_{1,i} + W_{1,j} + K_1 - \frac{\alpha \Delta K_1}{2}} \right)^2 \right] \\ + \nu \left(W_{1,i} + W_{1,j} + K_1 - \frac{\alpha \Delta K_1}{2} \right) \end{array} \right\}. \tag{9}$$

3.3 League Behavior in the First Period

Since the game teams play in period 1 is identical to the one in period 2, Proposition 1 holds: in equilibrium we have $e_{1,i} = \frac{\alpha(K_{1,w} - K_{1,l})}{4}$. Therefore, we can study the league's optimization problem in period 1. The league takes into account the influence of period 1 decisions on the games teams play in periods 1 *and* 2. Moreover, there is uncertainty about the game that will take place in period 2 since $W_{2,i} - W_{2,j}$ depends on which team has won the first period tournament (if revenue sharing is not full).

The league needs to compute the expected revenues for period 2, considering the possible outcomes of period 1's tournament. That is, in the first period, the league maximizes

$$D = D_1 + p_{1,i} D_2^i + p_{1,j} D_2^j \tag{10}$$

The individual components of this objective function are then easily specified. First, D_2^i and D_2^i are given by equations and (8), (9). Second, Proposition 1 applies and therefore we know that

$$p_{1,i} = \frac{\alpha}{2} + \beta \frac{W_{1,i}}{W_{1,i} + W_{1,j}} \tag{11}$$

and

$$D_1 = \gamma \frac{\alpha \Delta K_1}{2} + \delta \left[1 - \left(\beta \frac{W_{1,i} - W_{1,j}}{W_{1,i} + W_{1,j}} \right)^2 \right] + \nu \left(W_{1,i} + W_{1,j} \right). \tag{12}$$

From equations (8) to (12), we deduce that the level of revenue sharing chosen by the league in period 1, represented by ΔK_1, influences demand for the sport in three ways. For example, suppose the level of revenue sharing increases (ΔK_1 decreases). The first effect is to decrease period 1 demand since the equilibrium level of effort in period 1 decreases. The second effect is to increase period 2 demand through the larger total wealth of the teams after they receive the tournament prizes. The third effect is to increase period 2 demand because more revenue sharing implies the league is more balanced, i.e. $|p_{2,1} - p_{2,2}|$ decreases. Disregarding the role of league's wealth in attracting talent, revenue sharing is chosen on the basis of the trade off between *incentives to win*, represented by how sensitive demand is to effort, i.e., players trying hard to win (the first effect above), and *competitive balance*, represented by how sensitive demand is to uncertainty of the tournament's outcome (the third effect above).

In general, a closed form solution for the optimal value of revenue sharing is difficult to obtain even in our simple setting. The following propositions, though, characterizes the two most interesting extreme cases.

Proposition 4 *In the first period, revenue sharing is determined as follows:*
(i) *if* $\gamma\frac{2+\alpha(1-\gamma)}{(2+\alpha)} > \nu$ *there exists some* $\underline{\delta}$ *such that for any* $\delta < \underline{\delta}$ *minimal revenue sharing is optimal, that is,* $\Delta K_1 = \frac{2}{2+\alpha}K_1$*;*
(ii) *if* $\gamma\frac{2+\alpha(1-\gamma)}{(2+\alpha)} < \nu$ *full revenue sharing is optimal, that is,* $\Delta K_1 = 0$*.*

Proof: see Appendix.

Part (i) states that when the incentives to win effect is large (so that the sensitivity of demand to effort is large relative to the sensitivity of demand to total wealth) *and* the competitive balance effect is small enough, the league chooses the level of revenue sharing so as to maximize the effort level produced by teams (as in period 2). In this case, the league chooses minimal revenue sharing ($\Delta K_1 = 2K_1/(2+\alpha)$). Intuitively, if effort is a relatively important factor for the fans of the sport revenue sharing is not optimal.

Part (ii) of the proposition states that when the sensitivity of demand to effort is small relative to the sensitivity of demand to total wealth the league chooses full revenue sharing. Notice that this is only a sufficient condition for optimality of full revenue sharing since there may be values of δ such that $\Delta K_1 = 0$ even when $\gamma\frac{2+\alpha(1-\gamma)}{(2+\alpha)} > \nu$.

Finally two remarks should be made. First, although a full characterization of the equilibrium is not provided by Proposition 4, we have an answer to the optimality of not sharing revenues. If the incentives to win effect is large enough while the competitive balance effect is not too large, then sharing revenues does not maximize demand for the sport.

Second, as mentioned previously, the equilibrium of the second period suffers from the absence of a competitive balance effect, and is thus biased against revenue sharing. The incentives to win effect dominates. What is more interesting, is that revenue sharing may not be optimal even if the competitive balance effect is taken into account. Intuitively, the optimal prize for winning depends on the trade-off between current incentives and future wealth asymmetries.

3.4 Equilibrium When the League Maximizes Teams' Profits

In this section we assume the league is a cartel that maximizes the joint profits of the two teams. In this case, full revenue sharing is optimal. Intuitively, the value of an increase of television revenues (via demand) through higher incentives for the teams' players is exactly offset by the cost of these incentives. Hence, the league chooses ΔK_t so that teams give no incentives to players. In a cartel, competing hard to win is wasteful.

Let Π_2 denote the joint profits of the teams in period 2; formally, we have

$$\Pi_t = \Pi_{t,i} + \Pi_{t,j} = K_{t,w} + K_{t,l} - (e_{t,i} + e_{t,j}) = K_t - (e_{t,i} + e_{t,j}) \, .$$

Using the equilibrium condition for the effort game, we obtain

$$\Pi_t = K_t - \frac{\alpha}{2} \Delta K_t.$$

One can see that Π_2 is decreasing in the overall effort level. Hence, $\Delta K2 = 0$. In period 1, things are not so simple because effort produces an increase in demand which is then reflected in higher payment for TV rights and hence higher revenues for teams. It runs out this increase is exactly offset by the corresponding cost of providing effort. Hence, the first effect dominates and full revenue sharing is optimal. This result, is stated formally as follows.

Proposition 5 *If the league maximizes teams' joint profit, full revenue sharing is optimal in both periods; that is,* $\Delta K_t = 0$ *and* $K_{t,w} = K_{t,l} = \frac{1}{2} K_t$.

Proof: see Appendix.

4 Extensions of the Model

In this section, we propose three extensions of the basic model. First, we examine the situation in which the league and the broadcaster sign a long term contract. Then, we analyze the case were teams have sources of revenues which are performance based but independent from the prizes awarded by the league. Finally, we analyze the case in which players effort is unobservable and hence teams makes players' compensation contingent of the outcome of the tournament.

All these extensions contribute to the realism of the model without changing the basic conclusion. Full revenue sharing is not always the way to maximize demand for the sport.

4.1 Multi-period TV Contracts

In the previous section, TV deals were negotiated for one period at a time. In reality, broadcasting contracts span many years. To account for this, we consider a modification of the model where at the beginning of period 1 the league sells the right to broadcast games for both seasons. In this case, the league decides two things: the allocation of total prizes between periods and the division of the total prize between winner and loser in each period.

This change in assumptions has two main implications. First, at the beginning of period 1, teams know prizes to be awarded in the second period. This was not the case before since K_2 was a function of the team that won in the first period. Second, we do not have $D_t = K_t$, $(t = 1, 2)$. If we denote K the revenue of the league at the beginning of period 1, then perfect competition in the broadcasting industry yields $K = D_1 + E(D_2^i)$.

Note that the problem faced by teams in each period remains unchanged. Hence, Proposition 1 holds and the effort level of each team in period t equals $\frac{\alpha}{4}\Delta K_t$. Also, the league can still set the second period prizes after observing the outcome of the first period tournament. Therefore, Proposition 2 applies and $\Delta K_2 = \frac{2K_2}{2+\alpha}$. Hence, demand in period 2 depends on the winner of period 1 tournament in the way described by Equation (6). More specifically, demand in period 2 is given by

$$D_2^i = \gamma\frac{\alpha K_2}{2+\alpha} + \delta\left[1 - \left(\beta\frac{W_{1,i} - W_{1,j} + \Delta K_1}{W_{1,i} + W_{1,j} + K_1 - \frac{\alpha\Delta K_1}{2}}\right)^2\right] + \nu\left(W_{1,i} + W_{1,j} + K_1 - \frac{\alpha\Delta K_1}{2}\right)$$

when team i has won, and by

$$D_2^j = \gamma\frac{\alpha K_2}{2+\alpha} + \delta\left[1 - \left(\beta\frac{W_{1,i} - W_{1,j} - \Delta K_1}{W_{1,i} + W_{1,j} + K_1 - \frac{\alpha\Delta K_1}{2}}\right)^2\right] + \nu\left(W_{1,i} + W_{1,j} + K_1 - \frac{\alpha\Delta K_1}{2}\right)$$

when team j has won. Demand in the first period, instead, is computed according to Equation (12). These observation imply the league chooses K_1, K_2, and ΔK_1 to maximize

$$D = D_1 + p_{1,i}D_2^i + p_{1,j}D_2^j$$

or equivalently,

$$\begin{aligned} D \;=\; & \gamma\frac{\alpha\Delta K_1}{2} + \delta\left[1 - \left(\beta\frac{W_{1,i}-W_{1,j}}{W_{1,i}+W_{1,j}}\right)^2\right] + \nu\left(W_{1,i} + W_{1,j}\right) + \gamma\frac{\alpha(K-K_1)}{2+\alpha} \\ & +\nu\left(W_{1,i} + W_{1,j} + K_1 - \frac{\alpha\Delta K_1}{2}\right) + \delta\left\{1 - \beta^2\frac{\left[(W_{1,i}-W_{1,j})^2 + (\Delta K_1)^2 + 2\beta\frac{\left(W_{1,i}-W_{1,j}\right)^2}{W_{1,i}+W_{1,j}}\Delta K_1\right]}{\left(W_{1,i}+W_{1,j}+K_1-\frac{\alpha\Delta K_1}{2}\right)^2}\right\} \end{aligned} \tag{13}$$

The league prefers to concentrate incentives provided by the teams in the first period since they increase total wealth in the next period. Also, for any given prize structure, the larger the amount distributed in the first period the smaller the difference in relative wealth between the two teams in the second period; hence the more balanced the competition. For these two reasons, the league distributes all the prize money in the first period. From Equation (13), we have the following result about revenue sharing.

Proposition 6 *The equilibrium when there are two-period tv contracts has the following properties.*
(i) *If* $1 \geq \alpha > 0$ *and* $\gamma > \nu$ *there exists some* $\underline{\underline{\delta}}$ *such that for any* $\delta < \underline{\underline{\delta}}$ *minimal revenue sharing is optimal, that is,* $\Delta K_1 = \frac{2}{2+\alpha} K_1$*; in this case,* $K_2 = 0$ *and* $K_1 = K$*.*
(ii) *If* $\alpha = 1$ *and* $\gamma > \nu$*, then* $\Delta K_1 = 2K/3$*,* $K_1 = K$*, and* $K_2 = 0$*.*
(iii) *If* $\alpha = 0$ *or* $\gamma < \nu$*, then* $\Delta K_1 = 0$*,* $K_1 = K$*, and* $K_2 = 0$*.*

Proof: see Appendix.

As already mentioned, the league prefers to concentrate teams' efforts in period 1 for two reasons: first, they generate an increase in total wealth in period 2; second, for a given ΔK_1 the larger K_1 the smaller the difference in relative wealth between the two teams in period 2 and the more balanced the competition.

When deciding how to allocate money between teams and between periods, the league has to take into account two effects. First, the sensitivity of the probability of winning to effort (measured by α) relative to its sensitivity to wealth (measured by β). In particular, a large α implies large incentives for players. Second, the sensitivity of demand to effort (measured by γ) relative to the sensitivity of demand to wealth (measured by ν).

When α is strictly positive, the choice of ΔK_1 for a given K_1 depends on the values of γ and ν. The league faces a trade-off between demand in the two periods. If ΔK_1 is large, teams produce a high effort level in period 1 and generate a high demand in that period. In this case, teams' total profits are small since they face a high cost for such an effort level. Therefore, total wealth in period 2 is small and so is the demand induced by period 2 total wealth. Given this trade-off between demand in the first period and demand in the second, the level of revenue sharing chosen by the league depends of the sensitivity of demand to quality (ν) relative to the sensitivity of demand to effort (γ). When $\nu > \gamma$, the league's objective is equivalent to maximizing total wealth in period 2. This is achieved by full revenue sharing in period 1 since it implies that total profits in that period are maximized, and so is total wealth in period 2. Conversely, when $\nu < \gamma$ the league's objective is equivalent to maximizing total effort in period 1. This is achieved by setting ΔK_1 positive.

Multi period contracts when the league maximizes teams' profits

Matters are much simpler when the league maximizes the teams joint profits. As the following proposition shows, then full revenue sharing is still optimal and multi-period contracts make no difference

Proposition 7 *If the league maximizes teams' joint profits. Then,* $K_1 = K$*,* $K_2 = 0$ *and* $\Delta K_1 = 0$*.*

Proof: see Appendix.

4.2 Teams Have Multiple Sources of Revenues

In this section, we assume that teams have revenues that are not submitted to possible revenue sharing by the league. For example, these revenues may come from local TV deal or from merchandising. However, we assume that these revenues are dependent of past performances, the idea being that the better a team is performing, the more attractive it is, hence the higher its revenue. Formally, we assume that the winner of the competition in period t receives $K_{t,w} + A$ with A strictly positive and independent of the degree of revenue sharing chosen by the league. As before, the loser receives $K_{t,l}$. Under such an assumption we have the following results about revenue sharing in period 2.

Proposition 8 *Let*

$$A^* = \frac{2\delta}{\alpha(1-\gamma)}\left[1 - \beta^2\frac{(W_{2,i} - W_{2,j})^2}{(W_{2,i} + W_{2,j})^2}\right] + \frac{2\nu}{\alpha(1-\gamma)}(W_{2,i} + W_{2,j})$$

If $A > A^*$ *then* $\Delta K_2 = 0$*,* $e_{2,m} = \frac{\alpha}{4}A$ $(m = i, j)$ *and*

$$K_2 = \frac{\gamma\alpha A}{2} + \delta\left[1 - \beta^2\frac{(W_{2,i} - W_{2,j})^2}{(W_{2,i} + W_{2,j})^2}\right] + \nu(W_{2,i} + W_{2,j}) \tag{14}$$

If $A \leq A^*$ *then* $\Delta K_2 = \frac{2K_2 - \alpha A}{2+\alpha}$ *and* $e_{2,i} = e_{2,j} = \frac{\alpha}{4}(\Delta K_2 + A)$ *and*

$$K_2 = \frac{\gamma\alpha A}{2 + \alpha(1-\gamma)} + \frac{\delta(2+\alpha)}{2 + \alpha(1-\gamma)}\left[1 - \beta^2\frac{(W_{2,i} - W_{2,j})^2}{(W_{2,i} + W_{2,j})^2}\right] + \frac{\nu(2+\alpha)}{2 + \alpha(1-\gamma)}(W_{2,i} + W_{2,j}) \tag{15}$$

Proof: See Appendix.

The additional source of revenue A affects the effort level produced by teams in two ways. The first effect is a direct one. If the amount earned by the winning team increases, it provides incentives for teams to increase their effort level. This generates an indirect effect: the no loss constraint implies that the league increases the amount awarded to the loser, hence increasing the level of revenue sharing. When A is not too large (i.e., smaller than A^*), the aggregate effect is an increase of the equilibrium effort level: For a given K_2, the effort level is increasing in A. The consequence is an increase of the demand and therefore an increase of K_2.

When the additional source of revenue is large (i.e., larger than A^*), the league chooses full revenue sharing and teams' effort level is only determined by A. Furthermore, the losing team makes a loss.

In period 1, the problem teams face is the same as in period 2. Therefore,

$$e_{1,i} = e_{1,j} = \frac{\alpha(\Delta K_1 + A)}{4} \tag{16}$$

From Proposition 8 and equation (16), we deduce the following result.

Proposition 9 *If* $\gamma\frac{2+\alpha(1-\gamma)}{(2+\alpha)} > \nu$ *there exists* $\bar{A} > 0$ *and* $\underline{\underline{\delta}}$ *such that for any* $A < \bar{A}$ *and* $\delta < \underline{\underline{\delta}}$ *then* $\Delta K_1 = \frac{2}{2+\alpha}K_1$.

This result suggests that in a league in which revenues from TV deals represent a fraction not too large of team revenues, full revenue sharing is not damaging to effort since other source of revenues provide incentives for teams to produce effort. Conversely, in a league in which revenues from TV represent a large fraction of teams' revenues, then the league chooses a performance-based prize allocation.

4.3 Unobservable Effort

So far, we have implicitly assumed that effort produced by team players was observable, hence teams could offer effort-based compensation to players. In this section, we relax this assumption. A direct consequence is that teams can only offer performance-based contracts to players. Let $\mu_{t,i}(w)$ and $\mu_{t,i}(l)$ the fraction of the gain paid to players when team i earns $K_{t,w}$ and $K_{t,l}$, respectively. The objective of team i is to maximize

$$\Pi_{t,i} = p_{t,i}\left(1 - \mu_{t,i}(w)\right)K_{t,w} + \left(1 - p_{t,i}\right)\left(1 - \mu_{t,i}(l)\right)K_{t,l}$$

subject to $\mu_{t,i}(w) \geq 0$, $\mu_{t,i}(l) \geq 0$, and

$$e^*_{t,i} \in \arg\max\ p_{t,i}\mu_{t,i}(w)K_{t,w} + (1 - p_{t,i})\mu_{t,i}(l)K_{t,l} \tag{17}$$

This last equation represents the incentive compatibility constraint for the players.

Let $\Delta K_{t,i} = \mu_{t,i}(w)K_{t,w} - \mu_{t,i}(l)K_{t,l}$. Then, proceeding as in the proof of Proposition 1, one shows that the equilibrium of the effort game is such that

$$e^*_{t,i} = \sup\left\{0, \frac{\alpha(\Delta K_{t,i})^2 \Delta K_{t,j}}{(\Delta K_{t,1} + \Delta K_{t,2})^2}\right\} \tag{18}$$

with $i \neq j$. It follows that if $e^*_{t,i} > 0$, then

$$p_{t,i} = \alpha \frac{\Delta K_{t,i}}{\Delta K_{t,1} + \Delta K_{t,2}} + \beta \frac{W_{t,i}}{W_{t,1} + W_{t,2}} \tag{19}$$

From these results, we derive the following proposition about the compensation of players by teams.

Proposition 10 *Assume that $\Delta K_t > 0$. There exists an equilibrium such that*
(i) $\mu_{t,i}(l) = 0$ *($i = 1, 2$)*
(ii) *If $W_{t,i} > W_{t,j}$, then $0 < \mu_{t,i}(w) < \mu_{t,j}(w)$ and $p_{t,i} > p_{t,j}$.*
(iii) $\mu_{t,i}$ *($i = 1, 2$) is an increasing function of $K_{t,w}$.*

We deduce that

$$e^*_{t,i} = \frac{\alpha \mu_{t,i}(w)^2 \mu_{t,j}(w) K_{t,w}}{(\mu_{t,1}(w) + \mu_{t,2}(w))^2}$$

and

$$p_{t,i} = \alpha \frac{\mu_{t,i}(w)}{\mu_{t,1}(w) + \mu_{t,2}(w)} + \beta \frac{W_{t,i}}{W_{t,1} + W_{t,2}}$$

The proposition says that players are only compensated in case of success and the incentives are more important for the team with the smaller wealth. It follows that players from the wealthier team exert a lower effort. However, in equilibrium, the wealthier team has a higher probability of winning the competition. A direct consequence of *(iii)* is that the level of revenue sharing influences the effort level produced by teams in two ways: directly through the difference of gains between the winner and the loser, and indirectly through the compensation scheme of the players ($\mu_{t,i}(w)$).

We turn now to the problem of the league. A main differences with the case of observable effort is that teams never make losses. Hence, in period 1, the league does not have to take into account the possibility that a team will have a negative wealth if it loses in period 1. From the previous proposition we derive the following results about the level of revenue sharing in period 2.

Proposition 11 *Assume that the league maximizes the demand for sport. Then:*
(i) $\Delta K_2 = K_2$.
(ii) *There exists $\underline{\alpha} < 1$ such that if $\alpha > \underline{\alpha}$ and $\gamma(6 - \gamma) > 36\nu$ then $\Delta K_1 > 0$.*

The proposition states that, qualitatively, the results obtained in the case of observable effort still hold if this assumption is relaxed. That is, the league minimizes the level of revenue sharing in the second period and if the influence of effort on demand is large enough with respect to the influence of total wealth, then the league does not choose full revenue sharing in the first period.

5 Conclusions

We presented a theoretical model of revenue sharing in sport leagues. Our main results derive explicit conditions under which full revenue sharing is optimal. These can be summarized by looking at the relative importance of (current) incentives to win relative to (future) competitive balance. Higher revenues sharing increases future demand through a better competitive balance, but decreases current demand through a lower effort to win from teams. If the league maximizes the demand for sport, then a performance-based reward scheme (as used by European top soccer leagues for national TV deals) may be optimal. Conversely, if the league act as a cartel and maximizes joint profits, then full revenue sharing (as used by US team sport leagues for national TV deals) is always optimal.

Our results contribute to the moral-hazard and contest design literatures. In a moral hazard context, our model is an example of a repeated agency problem between a principal and multiple agents in which the difference in output produced by the agents is detrimental to the principal. In this setting, the principal faces a trade-off between 'output balance' among agents and incentives to produce large quantities. Our results show that the principal may have incentive to 'invest' in output balance; that is, lower the output today in order to get a lower difference in outputs tomorrow.

For the contest design literature, our model describes a situation in which a winner-takes-all prize allocation may be optimal for the one-contest case but not optimal in the case of repeated contests. Multiple-prize allocations may be optimal for repeated contests if the outcome of the contest at time t influences bids (or effort) in the following contests.

A Appendix

A.1 Proofs

Proof of Proposition 1.

Firm i expected profits are:

$$\Pi_{t,i}\left(e_{t,i},e_{t,j}\right)=\begin{cases}\left(\alpha\frac{e_{t,i}}{e_{t,i}+e_{t,j}}+\beta\frac{W_{t,i}}{W_{t,i}+W_{t,j}}\right)\Delta K_t+K_{t,l}-e_{t,i} & \text{if} \quad e_{t,i}+e_{t,j}>0\\ \frac{W_{t,i}}{W_{t,i}+W_{t,j}}\Delta K_t+K_{t,l} & \text{if} \quad e_{t,i}+e_{t,j}=0\end{cases}$$

Assume that $\Delta K_t>0$. Notice that

$$\Pi_{t,i}\left(0,0\right)=\frac{W_{t,i}}{W_{t,i}+W_{t,j}}\Delta K_t+K_{t,l}$$

and

$$\Pi_{t,i}\left(e,0\right)=\left(\alpha+\beta\frac{W_{t,i}}{W_{t,i}+W_{t,j}}\right)\Delta K_t+K_{t,l}-e;$$

hence

$$\begin{aligned}\Pi_{t,i}\left(e,0\right)-\Pi_{t,i}\left(0,0\right) &= \left(\alpha+\left(\beta-1\right)\frac{W_{t,i}}{W_{t,i}+W_{t,j}}\right)\Delta K_t-e\\ &= \left(\alpha-\alpha\frac{W_{t,i}}{W_{t,i}+W_{t,j}}\right)\Delta K_t-e\\ &= \alpha\frac{W_{t,j}}{W_{t,i}+W_{t,j}}\Delta K_t-e.\end{aligned}$$

Therefore, $e_{t,i}=0$ is not a best reply to $e_{t,j}=0$; this means aggregate effort equal to zero cannot be an equilibrium. Setting the derivative of $\Pi_{t,i}$ with respect to effort equal to zero one gets:

$$\frac{e_{t,j}}{\left(e_{t,i}+e_{t,j}\right)^2}\alpha\Delta K_t-1=0$$

Therefore, given the constraint $e_{t,i}\leq K_{t,l}$, team i's best response is:

$$e_{t,i}=\max\left\{0,\min\left(K_{t,l},\sqrt{\alpha e_{t,j}\Delta K_t}-e_{t,j}\right)\right\}$$

and similarly for team j. Therefore, the Nash equilibrium of the game is

$$e_{t,i}=e_{t,j}=\min\left(K_{t,l},\frac{\alpha}{4}\Delta K_t\right).$$

Assume $\Delta K_2=0$. In this case, effort is costly and does not increase expected revenues. Hence, they choose $e_{t,i}=e_{t,j}=0$.

□

Proof of Proposition 2.

The demand for the sport in period 2 is given by:

$$D_2^i = \frac{\gamma\alpha}{2}\Delta K_2 + \delta\left[1 - \left(\beta\frac{W_{2,i} - W_{2,j}}{W_{2,i} + W_{2,j}}\right)^2\right] + \nu\left(W_{2,i} + W_{2,j}\right).$$

Therefore, given any K_2 (the amount available for prizes to the winner and the loser) the demand for the sport is maximized by choosing ΔK_2 as large as possible. The only limit to the size of ΔK_2 is the constraint that teams make no losses. This constraint implies

$$K_{2,l} = e_{2,i} = \frac{\alpha\left(K_{2,w} - K_{2,l}\right)}{4}$$

or

$$\frac{\alpha}{2}\left(K_{2,w} - K_{2,l}\right) = 2K_{2,l}.$$

This can be rewritten as

$$\frac{\alpha}{2}\left(K_{2,w} - K_{2,l}\right) = K_{2,l} + K_{2,w} + K_{2,l} - K_{2,w};$$

since $K_{2,w} + K_{2,l} = K_2$, we can write

$$\left(\frac{\alpha}{2} + 1\right)\left(K_{2,w} - K_{2,l}\right) = K_2$$

or

$$\Delta K_2 = \frac{1}{1 + \frac{\alpha}{2}}K_2$$

yielding the result.

□

Proof of Proposition 4.

From Equations (8), (9), (12), and (11), we deduce that Equation (10) can be rewritten as

$$D = \begin{array}{l} \gamma\frac{\alpha\Delta K_1}{2} + \delta\left[1 - \left(\beta\frac{W_{1,i} - W_{1,j}}{W_{1,i} + W_{1,j}}\right)^2\right] + \frac{4+\alpha(2-\gamma)}{2+\alpha(1-\gamma)}\nu\left(W_{1,i} + W_{1,j}\right) \\ + \frac{2+\alpha}{2+\alpha(1-\gamma)}\nu\left(K_1 - \frac{\alpha\Delta K_1}{2}\right) + \frac{2+\alpha}{2+\alpha(1-\gamma)}\delta \\ - \frac{2+\alpha}{2+\alpha(1-\gamma)}\delta\beta^2\frac{\left[(W_{1,i} - W_{1,j})^2 + (\Delta K_1)^2 + 2\beta\frac{\left(W_{1,i} - W_{1,j}\right)^2}{W_{1,i} + W_{1,j}}\Delta K_1\right]}{\left(W_{1,i} + W_{1,j} + K_1 - \frac{\alpha\Delta K_1}{2}\right)^2} \end{array}$$

Let

$$H = \frac{\Delta K_1 \left((W_{1,1} + W_{1,2}) + \frac{\alpha\beta(W_{1,1}-W_{1,2})^2}{2(W_{1,1}+W_{1,2})} \right) + (\beta + \alpha/2)(W_{1,1} - W_{1,2})^2}{(W_{1,1} + W_{1,2} + K_1 - \alpha \Delta K_1/2)^3}$$

then

$$\frac{\partial D}{\partial \Delta K_1} = -\frac{2\beta^2 \delta (2+\alpha)}{2+\alpha(1-\gamma)} H + \frac{\alpha}{2}\left(\gamma - \frac{\nu(2+\alpha)}{2+\alpha(1-\gamma)} \right)$$

Note that $H > 0$. Therefore, if $\left(\gamma \frac{2+\alpha(1-\gamma)}{(2+\alpha)} - \nu\right) < 0$ then $\frac{\partial D}{\partial K_1}$ is always negative and the optimal ΔK_1 is zero. If $\left(\gamma \frac{2+\alpha(1-\gamma)}{(2+\alpha)} - \nu\right) > 0$ and δ is small enough, then $\frac{\partial D}{\partial K_1}$ is always positive and the optimal ΔK_1 is the largest feasible value assuming teams make no current losses. In the proof of Proposition 2 this value was shown to be $\frac{\alpha}{2+\alpha} K_1$. Finally, since $\frac{\partial D}{\partial K_1}$ is continuous in δ, there are values such that ΔK_1 is given by the solution(s) to $\frac{\partial D}{\partial K_1} = 0$.

□

Proof of Proposition 5.

Demand for the sport in period 2 is given by

$$D_2 = \delta \left[1 - \beta^2 \frac{(W_{2,i} - W_{2,j})^2}{(W_{2,i} + W_{2,j})^2} \right] + \nu \left(W_{2,i} + W_{2,j} \right) \tag{20}$$

and this equals the payment the league receives from the broadcasters; that is $D_2 = K_2$. We can now use this equation to compute the actual aggregate profits of the two teams in period 2 depending on who won the outcome of period 1 tournament. Let Π_2^i and Π_2^j indicate profits if team i won or lost respectively; then:

$$\begin{aligned}
\Pi_2^i &= \delta \left[1 - \beta^2 \frac{(W_{1,i} - W_{1,j} + \Delta K_1)^2}{\left(W_{1,i} + W_{1,j} + K_1 - \frac{\alpha}{2}\Delta K_1\right)^2} \right] + \nu \left(W_{1,i} + W_{1,j} + K_1 - \frac{\alpha}{2}\Delta K_1 \right) \\
\Pi_2^j &= \delta \left[1 - \beta^2 \frac{(W_{1,i} - W_{1,j} - \Delta K_1)^2}{\left(W_{1,i} + W_{1,j} + K_1 - \frac{\alpha}{2}\Delta K_1\right)^2} \right] + \nu \left(W_{1,i} + W_{1,j} + K_1 - \frac{\alpha}{2}\Delta K_1 \right)
\end{aligned}$$

Teams aggregate profits in period 1, defined Π_1, are given by:

$$\Pi_1 = \Pi_{1,i} + \Pi_{1,j} = K_1 - \frac{\alpha}{2}\Delta K_1.$$

Therefore, we can compute the objective function of the league as follows:

$$\Pi = K_1 - \frac{\alpha}{2}\Delta K_1 + p_{1,1}\Pi_2^i + (1 - p_{1,1})\,\Pi_2^i$$

which, after the appropriate substitutions, becomes:

$$\Pi = \begin{array}{c} K_1 - \frac{\alpha}{2}\Delta K_1 + \nu\left(W_{1,i} + W_{1,j} + K_1 - \frac{\alpha}{2}\Delta K_1\right) \\ \delta\left\{1 - \beta^2 \frac{(W_{1,i}-W_{1,j})^2+(\Delta K_1)^2+2\beta\left(\frac{W_{1,i}-W_{1,j}}{W_{1,i}+W_{1,j}}\right)(W_{1,i}-W_{1,j})\Delta K_1}{\left(W_{1,i}+W_{1,j}+K_1-\frac{\alpha}{2}\Delta K_1\right)^2}\right\} \end{array}$$

Differentiating this expression with respect to ΔK_1 we have:

$$\frac{\partial \Pi}{\partial \Delta K_1} = \begin{array}{c} -\frac{\alpha}{2}(1-\nu) - \delta\beta^2 \frac{\left[2\Delta K_1 + 2\beta\frac{\left(W_{1,i}-W_{1,j}\right)^2}{W_{1,i}+W_{1,j}}\right](W_{1,i}+W_{1,j}+K_1)}{\left(W_{1,i}+W_{1,j}+K_1-\frac{\alpha\Delta K_1}{2}\right)^3} \\ -\delta\beta_2 \frac{\alpha(W_{1,i}-W_{1,j})^2+\alpha\beta\frac{\left(W_{1,i}-W_{1,j}\right)^2}{W_{1,i}+W_{1,j}}\Delta K_1}{\left(W_{1,i}+W_{1,j}+K_1-\frac{\alpha\Delta K_1}{2}\right)^3} \end{array}$$

This expression is negative for any value of ΔK_1, hence $\Delta K_1 = 0$ at the optimum.

□

Proof of Proposition 6.

The league's objective is to maximize

$$D = \begin{array}{l} \gamma\frac{\alpha\Delta K_1}{2}\delta\left[1 - \left(\beta\frac{W_{1,i}-W_{1,j}}{W_{1,i}+W_{1,j}}\right)^2\right] + \nu\left(W_{1,i}+W_{1,j}\right) + \gamma\frac{\alpha(K-K_1)}{2+\alpha} + \nu\left(W_{1,i}+W_{1,j}+K_1-\frac{\alpha\Delta K_1}{2}\right) \\ +\delta\left\{1 - \beta^2 \frac{\left[(W_{1,i}-W_{1,j})^2+(\Delta K_1)^2+2\beta\frac{\left(W_{1,i}-W_{1,j}\right)^2}{W_{1,i}+W_{1,j}}\Delta K_1\right]}{\left(W_{1,i}+W_{1,j}+K_1-\frac{\alpha\Delta K_1}{2}\right)^2}\right\} \end{array} \tag{21}$$

Differentiating this expression with respect to ΔK_1 one obtains:

$$\frac{\partial D}{\partial \Delta K_1} = \begin{array}{l} \frac{\alpha}{2}(\gamma-\nu) \\ -\delta\beta^2 \frac{\left[2\Delta K_1+2\beta\frac{\left(W_{1,i}-W_{1,j}\right)^2}{W_{1,i}+W_{1,j}}\right](W_{1,i}+W_{1,j}+K_1)+\alpha(W_{1,i}-W_{1,j})^2\left(1+\beta\frac{\Delta K_1}{W_{1,i}+W_{1,j}}\right)}{\left(W_{1,i}+W_{1,j}+K_1-\frac{\alpha\Delta K_1}{2}\right)^3} \end{array} \tag{22}$$

Proof of *(iii)*: let $\alpha = 0$. In this case, $\frac{\partial D}{\partial \Delta K_1}$ is negative for any value of ΔK_1 since the second term in the equation above is always negative. Hence, at the optimum we must have $\Delta K_1 = 0$. Then D reduces to

$$D = \delta\left[1 - \left(\frac{W_{1,i}-W_{1,j}}{W_{1,i}+W_{1,j}}\right)^2\right] + \nu\left(2(W_{1,i}+W_{1,j}) + (K-K_2)\right) + \delta\left[1 - \frac{\left(W_{1,i}-W_{1,j}\right)^2}{\left(W_{1,i}+W_{1,j}+(K-K_2)\right)^2}\right].$$

This can be differentiated with respect to K_2 obtaining

$$\frac{\partial D}{\partial K_2} = -\nu - 2\delta \frac{(W_{1,i} - W_{1,j})^2}{(W_{1,i} + W_{1,j} + (K - K_2))^3};$$

this is always negative, hence $K_1 = K$, and $K_2 = 0$.

Proof of *(ii)*: let $\alpha = 1$ and $\gamma > \nu$. In this case, $\frac{\partial D}{\partial \Delta K_1}$ is positive for any value of ΔK_1 since the second term of Equation (22) is zero because $\beta = 1 - \alpha$. Therefore, ΔK_1 is as large as feasible (revenue sharing is minimal) and, following Proposition 2, we have

$$\Delta K_1 = \frac{2K_1}{2+\alpha} = \frac{2}{3} K_1.$$

Substituting this in Equation (21) we get

$$D = \frac{\gamma}{3} K_1 + 2\delta + \nu (W_{1,i} + W_{1,j}) + \gamma \frac{(K - K_1)}{3} + \nu \left(W_{1,i} + W_{1,j} + \frac{K_1}{3} \right)$$

which is strictly increasing in K_1. Therefore, $K_2 = 0$ and $K_1 = K$.

Proof of *(i)*: let $1 > \alpha > 0$ and $\gamma > \nu$. Inspecting Equation (22) we note that $\frac{\partial D}{\partial \Delta K_1}$ is continuous in δ. Therefore, there exists a δ small enough such that $\frac{\partial D}{\partial \Delta K_1}$ is positive for any value of ΔK_1 (say $\delta = 0$). As noted above, this implies $\Delta K_1 = \frac{2}{2+\alpha} K_1$. Substituting this in Equation (21) we get

$$D = \begin{array}{l} \gamma \frac{\alpha}{2+\alpha} K_1 + \delta \left[1 - \left(\beta \frac{W_{1,i} - W_{1,j}}{W_{1,i} + W_{1,j}} \right)^2 \right] + \nu (W_{1,i} + W_{1,j}) + \gamma \frac{\alpha (K - K_1)}{2+\alpha} + \nu \left(W_{1,i} + W_{1,j} + K_1 \frac{2}{2+\alpha} \right) \\ \qquad + \delta \left\{ 1 - \beta^2 \frac{\left[(W_{1,i} - W_{1,j})^2 + \left(\frac{2}{2+\alpha} K_1 \right)^2 + 2 \frac{2}{2+\alpha} K_1 (W_{1,i} - W_{1,j}) \beta \left(\frac{W_{1,i} - W_{1,j}}{W_{1,i} + W_{1,j}} \right) \right]}{\left(W_{1,i} + W_{1,j} + K_1 \frac{2}{2+\alpha} \right)^2} \right\} \end{array}$$

which can be differentiated with respect to K_1 to obtain

$$\frac{\partial D}{\partial K_1} = \nu \frac{\alpha}{2+\alpha} - 2 \frac{2}{2+\alpha} \delta \beta^2 \frac{\frac{2}{2+\alpha} K_1 (W_{1,i} + W_{1,j}) - \frac{2}{2+\alpha} K_1 \beta \frac{(W_{1,i} - W_{1,j})^2}{W_{1,i} + W_{1,j}} - \alpha (W_{1,i} - W_{1,j})^2}{\left(W_{1,i} + W_{1,j} + K_1 \frac{2}{2+\alpha} \right)^3}.$$

This is continuous in δ and therefore there exists some small δ such that $\frac{\partial D}{\partial K_1}$ is positive for any value of K_1. Hence $K_2 = 0$ and $K_1 = K$, concluding the proof.

□

Proof of Proposition 7.

Given that $e_{t,1} = e_{t,2} = \alpha \Delta K_t / 4$, the league maximizes

$$\begin{array}{l} \Pi = 2(\gamma - \nu - 1) e_{1,1} + \delta \left(\frac{\alpha}{2} + \beta \frac{W_{1,1}}{W_{1,i} + W_{1,j}} \right) \left[1 - \beta^2 \frac{(W_{1,i} - W_{1,j} + \Delta K_1)^2}{(W_{1,i} + W_{1,j} + K_1 - 2e_{1,1})^2} \right] \\ + \delta \left(\frac{\alpha}{2} + \beta \frac{W_{1,2}}{W_{1,i} + W_{1,j}} \right) \left[1 - \beta^2 \frac{(W_{1,i} - W_{1,j} - \Delta K_1)^2}{(W_{1,i} + W_{1,j} + K_1 - 2e_{1,1})^2} \right] + 2(\gamma - 1) e_{2,1} + \nu K_1 \end{array} \qquad (23)$$

subject to $K = K_1 + K_2$, $\Delta K_t \leq K_{t,l}$ $(t = 1, 2)$.

It is straightforward that Π is decreasing in the effort in period 2. Hence the league sets $\Delta K_2 = 0$. Now,

$$\begin{aligned}\frac{\partial \Pi}{\partial \Delta K_1} = \frac{\alpha}{2}(\gamma - \nu - 1) - \frac{\beta^2 \left\{ \frac{\alpha}{2}\left(\alpha(W_{1,i}-W_{1,j})^2 + 2\Delta K_1(W_{1,i}+W_{1,j}+K_1)\right)\right\}}{(W_{1,i}+W_{1,j}+K_1-2e_{1,1})^3} \\ - \frac{\frac{\beta^2 \Delta K_1}{W_{1,i}+W_{1,j}}\left(K_1(W_{1,i}+W_{1,j}) + (W_{1,i}+W_{1,j})^2 + \frac{\alpha}{2}(W_{1,i}-W_{1,j})^2\right)}{(W_{1,i}+W_{1,j}+K_1-2e_{1,1})^3} \\ - \frac{\beta^2 (W_{1,i}-W_{1,j})^2 \left(\frac{K_1}{W_{1,i}+W_{1,j}} + 1 + \frac{\alpha}{2}\right)}{(W_{1,i}+W_{1,j}+K_1-2e_{1,1})^3} < 0\end{aligned} \tag{24}$$

Hence, $\Delta K_1 = 0$. Furthermore, it is straightforward that at $\Delta K_1 = 0$, $\partial \Pi / \partial K_1 > 0$ while $\partial \Pi / \partial K_2 = 0$. Hence, we have the desired result.

□

Proof of Proposition 8.

Proceeding as in the proof of Proposition 1, one shows that

$$e_{2,i} = e_{2,j} = \frac{\alpha(\Delta K_2 + A)}{4}.$$

Then, proceeding as in the proof of Proposition 2, we obtain that the league chooses

$$K_{2,w} = Max\left\{\frac{4K_2 + \alpha(K_2 - A)}{2(2+\alpha)}, \frac{K_2}{2}\right\}$$

which implies

$$\Delta K_2 = Max\left\{\frac{2K_2 - \alpha A}{2+\alpha}, 0\right\}.$$

If $\Delta K_2 > 0$, then K_2 is given by (14) and $K_2 > \frac{\alpha}{2}A$ is equivalent to $A < A^*$. If $\Delta K_2 = 0$ then $K_2 = \frac{\alpha}{2}A^*$, then $K_2 < \frac{\alpha}{2}A$ is equivalent to $A > A^*$.

□

Proof of Proposition 9.

Let

$$\begin{aligned}A_1^*(A) = \frac{2\delta}{\alpha(1-\gamma)}\left[1 - \beta^2 \frac{(W_{1,i}-W_{1,j}+\Delta K_1 + A)^2}{(W_{1,i}+W_{1,j}+K_1(A)+A-\alpha(\Delta K_1+A)/2)^2}\right] \\ + \frac{2\nu}{\alpha(1-\gamma)}(W_{1,i} + W_{1,j} + K_1(A) + A - \alpha(\Delta K_1 + A)/2)\end{aligned}$$

$$\begin{aligned}A_2^*(A) = \frac{2\delta}{\alpha(1-\gamma)}\left[1 - \beta^2 \frac{(W_{1,i}-W_{1,j}-\Delta K_1 - A)^2}{(W_{1,i}+W_{1,j}+K_1(A)+A-\alpha(\Delta K_1+A)/2)^2}\right] \\ + \frac{2\nu}{\alpha(1-\gamma)}(W_{1,i} + W_{1,j} + K_1(A) + A - \alpha(\Delta K_1 + A)/2)\end{aligned}$$

with

$$K_1(A) = \frac{\gamma\alpha A}{2+\alpha(1-\gamma)} + \frac{\delta(2+\alpha)}{2+\alpha(1-\gamma)}\left[1-\beta^2\frac{(W_{1,i}-W_{1,j})^2}{(W_{1,i}+W_{1,j})^2}\right] + \frac{\nu(2+\alpha)}{2+\alpha(1-\gamma)}(W_{1,i}+W_{1,j}) \quad (25)$$

Define the functions $F_1(A)$ and $F_2(A)$ as follows

$$F_1(A) = \begin{cases} F_{1,s}(A) & \text{if} \quad A \leq A_1^* \\ F_{1,l}(A) & \text{if} \quad A > A_1^* \end{cases}$$

$$F_2(A) = \begin{cases} F_{2,s}(A) & \text{if} \quad A \leq A_2^* \\ F_{2,l}(A) & \text{if} \quad A > A_2^* \end{cases}$$

where

$$\begin{aligned} F_{1,l}(A) &= \tfrac{\gamma\alpha A}{2} + \delta\left[1-\beta^2\tfrac{(W_{1,i}-W_{1,j}+\Delta K_1+A)^2}{(W_{1,i}+W_{1,j}+K_1(A)+A-\alpha(\Delta K_1+A)/2)^2}\right] \\ &\quad +\nu(W_{2,i}+W_{2,j}+K_1(A)+A-\alpha(\Delta K_1+A)/2) \\ F_{1,s}(A) &= \tfrac{\gamma\alpha A}{2+\alpha(1-\gamma)} + \tfrac{\delta(2+\alpha)}{2+\alpha(1-\gamma)}\left[1-\beta^2\tfrac{(W_{1,i}-W_{1,j}+\Delta K_1+A)^2}{(W_{1,i}+W_{1,j}+K_1(A)+A-\alpha(\Delta K_1+A)/2)^2}\right] \\ &\quad +\tfrac{\nu(2+\alpha)}{2+\alpha(1-\gamma)}(W_{1,i}+W_{1,j}+K_1(A)+A-\alpha(\Delta K_1+A)/2) \\ F_{2,l}(A) &= \tfrac{\gamma\alpha A}{2} + \delta\left[1-\beta^2\tfrac{(W_{1,i}-W_{1,j}-\Delta K_1-A)^2}{(W_{1,i}+W_{1,j}+K_1(A)+A-\alpha(\Delta K_1+A)/2)^2}\right] \\ &\quad +\nu(W_{2,i}+W_{2,j}+K_1(A)+A-\alpha(\Delta K_1+A)/2) \\ F_{2,s}(A) &= \tfrac{\gamma\alpha A}{2+\alpha(1-\gamma)} + \tfrac{\delta(2+\alpha)}{2+\alpha(1-\gamma)}\left[1-\beta^2\tfrac{(W_{1,i}-W_{1,j}-\Delta K_1-A)^2}{(W_{1,i}+W_{1,j}+K_1(A)+A-\alpha(\Delta K_1+A)/2)^2}\right] \\ &\quad +\tfrac{\nu(2+\alpha)}{2+\alpha(1-\gamma)}(W_{1,i}+W_{1,j}+K_1(A)+A-\alpha(\Delta K_1+A)/2) \end{aligned}$$

Let

$$D = D_1 + p_{1,1}F_{1,s}(A) + (1-p_{1,1})F_{2,s}(A)$$

where D_1 and $p_{1,i}$ are given by (12) and (11), respectively. Now, it is straightforward that there exists $\bar{A}_2$ such that if $A < \bar{A}_2$, then $F_1(A) = F_{1,s}(A)$ and $F_2(A) = F_{2,s}(A)$. Therefore, if $A < \bar{A}_2$, then proceeding as in the proof of Proposition 4, one shows that if $\gamma\frac{2+\alpha(1-\gamma)}{(2+\alpha)} > \nu$, there exists $\underline{\underline{\delta}}$ such that $\frac{\partial D}{\partial \Delta K_1} > 0$. Let

$$\bar{A}_1 = \frac{2\delta}{\alpha(1-\gamma)}\left[1-\beta^2\frac{(W_{1,i}-W_{1,j})^2}{(W_{1,i}+W_{1,j})^2}\right] + \frac{2\nu}{\alpha(1-\gamma)}(W_{1,i}+W_{1,j})$$

Proceeding as in the proof of Proposition 8, one shows that if $A < \bar{A}_1$, then

$$\Delta K_1 = \frac{2K_1(A) - \alpha A}{2+\alpha} \quad (26)$$

Then, then assumption of perfect competition in the broadcasting industry in period 1 (i.e., $D_1 = K_1$) implies that K_1 is given by (25). Hence, taking $\bar{A} = Min(\bar{A}_1, \bar{A}_2)$, we have the desired result.

□

Proof of Proposition 10.

Proof of part *(i).* From equations (18) and (19), we derive that

$$\frac{\partial \pi_{t,i}}{\partial \mu_{t,i}(w)} = \frac{\alpha \Delta K_{t,j} K_{t,w}}{(\Delta K_{t,1} + \Delta K_{t,2})^2} [(1 - \mu_{t,i}(w))K_{t,w} - (1 - \mu_{t,i}(l))K_{t,l}] - p_{t,i} K_{t,w} \tag{27}$$

$$\frac{\partial \pi_{t,i}}{\partial \mu_{t,i}(l)} = -\frac{\alpha \Delta K_{t,j} K_{t,l}}{(\Delta K_{t,1} + \Delta K_{t,2})^2} [(1 - \mu_{t,i}(w))K_{t,w} - (1 - \mu_{t,i}(l))K_{t,l}] + (p_{t,i} - 1)K_{t,l} \tag{28}$$

Assume that there exists an equilibrium with $\mu_{t,i}(w) > 0$. This implies that

$$(1 - \mu_{t,i}(w))K_{t,w} - (1 - \mu_{t,i}(l))K_{t,l} > 0$$

In turn, this implies that $\partial \pi / \partial \mu_{t,i}(l) < 0$ in equilibrium. Hence, $\mu_{t,i}(l) = 0$. Now, we need to show that the system of equations

$$\frac{\alpha \mu_{t,j}(w)}{(\mu_{t,1}(w) + \mu_{t,2}(w))^2} [(1 - \mu_{t,i}(w))K_{t,w} - K_{t,l}] - p_{t,i} K_{t,w} = 0 \qquad i = 1, 2 \qquad i \neq j \tag{29}$$

has a solution in $(0,1) \times (0,1)$ which satisfies the second order conditions of profit maximization.

From equation (27), it is straightforward that if $\mu_{t,i}(l) = 0$ then $\partial^2 \pi_{t,i} / (\partial \mu_{t,i}(w))^2 < 0$. Now, when $\mu_{t,1}(w)$ and $\mu_{t,2}(w)$ converge to 0 at the same speed (so that there exists $H > 0$ such that $H < \mu_{t,i}(w)/\mu_{t,j(w)}$ ($i = 1, 2$ and $i \neq j$) as when $\mu_{t,1}(w)$ and $\mu_{t,2}(w)$ converge to 0), then the LHS of (29) goes to infinity. Furthermore, for any given $\mu_{t,i}(w) > 0$, $\alpha \mu_{t,j}(w)/(\mu_{t,1}(w) + \mu_{t,2}(w))^2$ converges to 0 as $\mu_{t,j}(w)$ converges to zero. Hence, we deduce that by continuity, there exist $\mu_{t,1}(w)$ and $\mu_{t,2}(w)$ such that the system of equations (29) has a solution in $(0,1) \times (0,1)$.

Proof of part *(ii)*: We use a contradiction argument. Assume that $W_{t,i} > W_{t,j}$ and $\mu_{t,i}(w) \geq \mu_{t,j}(w)$. This implies that $p_{t,i} > p_{t,j}$. From (29), it follows that

$$\frac{\mu_{t,j}(w)}{\mu_{t,i}(w)} > \frac{(1 - \mu_{t,j}(w))K_{t,w} - K_{t,l}}{(1 - \mu_{t,i}(w))K_{t,w} - K_{t,l}}$$

The LHS of this inequality is smaller than 1 while the RHS is larger than 1. Hence, the inequality does not hold and if $W_{t,i} > W_{t,j}$ then $\mu_{t,i}(w) < \mu_{t,j}(w)$. Now, $\mu_{t,i}(w) > \mu_{t,j}(w)$ implies $\pi_{t,i} > \pi_{t,j}$ follows directly from (29).

Proof of part *(iii)*: Let $R_t = K_t / K_{t,w}$. From (29), we deduce that

$$\frac{\partial \mu_{t,i}(w)}{\partial R_t} = -\left[\mu_{t,j}(w)(2 - R_t)(\mu_{t,1}(w) + \mu_{t,2}(w))\right]^{-1} \tag{30}$$

Hence, $\mu_{t,1}(w)$ and $\mu_{t,2}(w)$ are increasing functions of $K_{t,w}$.

□

Proof of Proposition 11.

Proof of *(i)*: From Proposition 10, we know that in each period the effort level is increasing in K_w. Hence, we only need to show that $(p_{2,1} - p_{2,2})^2$ is not increasing in ΔK_2.

$$p_{2,1} - p_{2,2} = \alpha \frac{\mu_{2,1}(w) - \mu_{2,2}(w)}{\mu_{2,1}(w) + \mu_{2,2}(w)} + \beta \frac{W_{2,i} - W_{2,j}}{W_{2,i} + W_{2,j}}$$

Let $R_t = K_t / K_{t,w}$.

$$\frac{d(p_{2,1} - p_{2,2})}{dR_2} = \frac{2(\mu_{2,2}(w)\frac{d\mu_{2,1}(w)}{dR_2} - \mu_{2,1}(w)\frac{d\mu_{2,2}(w)}{dR_2})}{(\mu_{2,1}(w) + \mu_{2,2}(w))^2} \tag{31}$$

From equation (30), we derive that $d(p_{2,1} - p_{2,2})/dR_2 = 0$. It follows that D_2^i is increasing in ΔK_2 and so the leagues sets $\Delta K_2 = K_2$.

Proof of *(ii)*. Assume that $\alpha = 1$. In such a case,

$$\mu_{2,1}(w) = \mu_{2,2}(w) = \frac{2K_{2,w} - K_2}{3K_{2,w}}$$

and

$$e_{2,1} = e_{2,2} = \frac{2K_{2,w} - K_1}{12}$$

From part *(i)*, we know that $K_{2,w} = K_2$. We deduce that

$$K_2 = \frac{\delta + \nu(W_{2,i} + W_{2,j})}{1 - \alpha\gamma/6} \tag{32}$$

Now, consider the problem of the league in period 1. Teams face the same problem as in period 2. Hence,

$$\mu_{1,1}(w) = \mu_{1,2}(w) = \frac{2K_{1,w} - K_1}{3K_{1,w}}$$

Therefore, if team i wins in period 1, then

$$W_{2,i} = W_{1,i} + \left(1 - \frac{2K_{1,w} - K_1}{3K_{1,w}}\right) K_{1,w}$$

while if it looses,

$$W_{2,i} = W_{1,i} + K_{1,l}$$

We deduce that, in period 1, the league maximizes

$$D = (\delta + \nu(W_{1,i} + W_{1,j})(1 + \frac{6}{6-\gamma}) + (2K_{1,w} - K_1)(\frac{\gamma}{6} - \frac{6\nu}{6-\gamma})$$

Hence, if $\gamma(6-\gamma) > 36\nu$ then $dD/dK_{1,w} > 0$. By continuity, we derive that there exists $\underline{\alpha}$ such that if $\alpha > \underline{\alpha}$ $dD/dK_{1,w} > 0$, $\Delta K_1 > 0$.

□

A.2 Fully Rational Teams

Fully rational teams take into account the impact of their action at time 1 on their wealth in period 2. Since probabilities of winning in period 2 and the revenue of the league in period 2 depend on teams' wealth, it follows that they take into the influence of their action in period 1 on their probability of winning in period 2 and on K_2. In period 2, the problem of the fully rational team is identical to that of a myopic team.

Formally, in period 1, fully rational team i solves the following problem

$$\begin{aligned} Max_{e_{1,i}} p_{1,i} \left[K_{1,w} + p_{2,i}(i)K^*_{2,w}(k,i) + (1-p_{2,i}(i))K^*_{2,l}(i) - e_{2,i}(i)\right] \\ +(1-p_{1,i}) \left[K_{1,l} + p_{2,i}(j)K^*_{2,w}(j) + (1-p_{2,i}(j))K^*_{2,l}(j) - e_{2,i}(j)\right] - e_{1,i} \end{aligned} \quad (33)$$

with $i \neq j$. $e_{2,i}(m)$, $K^*_{2,w}(m)$ and $K^*_{2,l}(m)$ represent the effort produced by team i in period 2, the amount awarded to the winner in period 2, the amount awarded to the loser in period 2, respectively, if team m wins in period 1 ($m = i, j$).

In the corner cases $\alpha = 0$ and $\alpha = 1$, we are able to derive closed form solution in the effort game played by teams. First, if $\alpha = 0$, it is straightforward that teams do not produce any effort . Hence, the problem faced by the league is identical to the case with myopic teams. Therefore, the league sets $\Delta K_1 = 0$. If $\alpha = 1$ we have the following result.

Proposition 12 *Assume that the league maximize the demand for sport and $\alpha = 1$. Then*

$$e_{1,1} = e_{1,2} = \frac{(3-\gamma)\Delta K_1}{12}$$

If $\gamma(3-\gamma) \geq 3\nu$ the league sets $\Delta K_1 = 5K_1/6$. If $\gamma(3-\gamma) < 3\nu$ the league sets $\Delta K_1 = 0$.

Proof: If $\alpha = 1$, then

$$K^*_2(1) = K^*_2(2) = \frac{3}{3-\gamma}\left(\delta + \nu(W_{1,i} + W_{1,j} + K_1 - e_{1,1} - e_{1,2})\right)$$

and $p_{2,j}(i) = 1/2$ $(i, j = 1, 2)$. Proceeding as in the proof of Proposition 1, we obtain that the equilibrium effort produced by teams in period 1 is

$$e_{1,i} = e_1 = \frac{(3-\gamma)\Delta K_1}{2[2(3-\gamma) + 3\nu]}$$

The objective of the league in period 1 is to maximize

$$D = 2\left(\gamma - \frac{3\nu}{3-\gamma}\right)e_1 + \left(1 + \frac{3}{3-\gamma}\right)\left(\delta + \nu(W_{1,i} + W_{1,j})\right)$$

Hence, if $\gamma(3-\gamma) < 3\nu$, the league sets ΔK_1 so as to minimize the effort level produced by teams, i.e., $\Delta K_1 = 0$. Conversely, if $\gamma(3-\gamma) > 3\nu$ the leagues sets ΔK_1 so as to maximize the level of effort by teams, i.e.,

$$\frac{(3-\gamma)\Delta K_1}{2[2(3-\gamma)+3\nu]} = K_{1,l}$$

Given that $\Delta K_1 = 2K_{1,w} - K_1$ and $K_{1,l} = K_1 - K_{1,l}$, we obtain

$$K_{1,w} = \frac{5(3-\gamma)+6\nu}{6[(3-\gamma)+\nu]}$$

We deduce that $\Delta K_1 = 5K_1/6$. By continuity, it implies that if $\gamma(3-\gamma) > 3\nu$, there exists $\underline{\alpha}$ such that if $\alpha > \underline{\alpha}$, then $\Delta K_1 > 0$.

□

From Proposition 12 we deduce that fully rational teams choose a lower effort level than myopic teams. The reason is that they take into account the influence of their effort in period 1 on the demand of period 2 through their wealth. It follows that by decreasing their effort level, they increase their future wealth, hence increasing the revenue of the league in period 2 and their expected gain in that period.

A.3 Tables

Table 1. Revenue allocation in the LNF (season 1999-2000, in Million FF)

Ranking	Fixed Amount	Variable Amount	Total
1	54.5	45.5	100
2	54.5	40.25	94.75
3	54.5	36.75	91.25
4	54.5	31.5	86
5	54.5	29.75	84.25
6	54.5	28	82.5
7	54.5	24.5	79
8	54.5	21	75.5
9	54.5	19.25	73.75
10	54.5	17.5	72
11	54.5	14	68.5
12	54.5	10.5	65
13	54.5	8.75	63.25
14	54.5	7	61.5
15	54.5	5.25	59.75
16	54.5	2	56.5
17	54.5	2	56.5
18	54.5	2	56.5

Table 2. Ratio of revenues for the season 1999-2000 is some top European soccer leagues. Source: L'Equipe.

Country	Best/Worst
England	2.2
France	1.8
Germany	1.7
Italy	3.4

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