The Value of a Reputation System

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Abstract

This paper explores the trade-off between the short-term benefits of false quality advertisements against the longer term costs of reputation damage. A directed search model is constructed in which submarkets are created by the advertisements and reputations of sellers. A reputation system links misleading advertisements in the present period to a lower reputation in the next period. We show that a reputation system always increases the prices of high quality products and directs search more accurately towards the sellers with such products. We also show that buyers are hurt by a reputation system if the market is thin – has few sellers – because the equilibrium increase in prices is greater than the equilibrium increase in the quality of trade. Finally, we show that a reputation system which screens for honesty increases social welfare by making sellers more truthful. However, we also show that a reputation for honesty is not always highly valued and that an alternative reputation system which screens for type can be more effective.

1 Introduction

Sticks and stones may break my bones, but words will never hurt me.

The pen is mightier than the sword.

A reputation system is a method of gathering and reporting information about past transactions. On-line reputation systems include eBay, which gathers customer reports about the behavior of traders in the auctions that

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it hosts, and Epinions, which collects expert reviews. Many types of reputation systems existed before the Internet. For example, credit and employment agencies are arguably reputation systems. However, these systems also generally enforce explicit punishments and/or are used by the same people that operate them. The experience of the Internet demonstrates that successful business models simply aggregate information. This phenomenon has come as something of a surprise. Old reputation systems, such as the Better Business Bureau, once relied on public subsidies. Now these systems, as evidenced by eBay, can operate at a profit.

In this paper, we show that the theory of directed search (see, for example, Moen 1997 and Julien, *et al.* 2000) can be used to explain the value of different reputation systems. A directed search equilibrium is one in which decentralized sellers adopt selling mechanisms strategically in an effort to coordinate trade with buyers. Directed search models emphasize two factors that are essential to modelling reputation systems: (i) infrequent interaction of traders, and (ii) the creation of a set of submarkets by similarly positioned sellers. Infrequent interaction of traders is essential to explaining why market coordination on the basis of past indiscretions is impossible without a third-party reputation system. Submarket creation is essential to understanding why a reputation system works.

In our model, a reputation system augments the creation of submarkets. Submarkets are defined not only by the advertisements of sellers but also by their reputations. The reputation system links misleading advertisements in the current period to lower reputations in the next period. Both sellers and buyers are aware of this process and act strategically. In order to make the model as simple as possible, we assume that sellers coordinate trade by advertising a set of competing auctions (Wolinsky (1988), McAfee (1991), Peters and Severinov (1997), Julien, *et al.* (2000, 2002a)). This assumption is useful for our purposes, because it is the closest institutionally to eBay, and makes the model easy to solve, because sellers simply direct the search of buyers by *ex ante* product quality advertisements with prices being determined *ex post.*[1]

We investigate the effects of a reputation system on the payoffs of buyers. We show that buyers' payoffs either increase or decrease depending on the number of sellers and the amount of information buyers obtain about their quality. A reputation system hurts buyers if it forces them to compete too intensely for high quality products. We find that buyers are always hurt by

[1] A number of other authors assume search is directed by price posting, however, these models are generally much more complicated to solve (see Peters (1984), Montgomery (1991), Moen (1997), Julien, *et al* (2000), and Burdett, *et al* (2001)). An important unifying result in this literature is the equivalence of auctions and posted prices when sellers and buyers are homogeneous and markets are large (Kultti (1999) and Julien *et al* (2002)). The existence of heterogenous valuations by buyers favours auctions (Kennes *et al* (2001)).

a reputation system if the market is sufficiently thin, that is, it has relatively few sellers compared to buyers. Buyers are also always hurt by a reputation system if the creation of a low quality submarket is not so informative as to warrant exclusion of these sellers. Therefore, small amounts of information are always harmful to buyers. In a thick market, buyers are always benefited by a reputation system because the price increases for high quality products are much smaller. Thick markets are also necessary for any seller to be excluded, thus there is a close connection between our two results concerning a reduction in the welfare of buyers.

We also investigate the effects a reputation system has on overall welfare. For many parameters the value added of a reputation system is small, in the order of one or two percent of the volume of trade. Thus, if a company like eBay oversees ten billion dollars in trade, the value added by the reputation system of eBay is one hundred or two hundred million dollars. The potentially small value added explains two things about this type of business. First, the cost of collecting information must be extremely small if such a business model is to be effective. This fact may explain why such businesses were not effective prior to the Internet. The second fact is that the volume of trade must be extremely large to pay for even moderate fixed costs. Thus eBay could be described as a natural monopoly. We also parameter discuss cases where the welfare benefit of a reputation system is quite large in the order of five or ten percent. Effects of a reputation system on overall welfare are large if many good and bad sellers coexist in the market and the product quality of bad sellers is significantly less than the product quality of good sellers.

Finally, we investigate the effects a reputation system has on sellers' payoffs and behaviour. We consider two types of reputation systems for two types of markets. First, if the product quality of the sellers does not change over time, a sensible reputation system functions as a detector of low quality and advertisements of low quality only serve to place a seller in a low quality submarket. In this case, the reputation system raises welfare by providing information about sellers, but it does not foster honesty. However, if good sellers face stochastic product quality, an effective reputation system functions as a detector of honesty. A seller may then be able to signal type by choosing to be honest in the event that product quality is low.

Our work is related to the economics of asymmetric information initiated by Akerlof (1970), Spence (1973) and Rothschild and Stiglitz (1976).[2] In our model, we extend this work by assuming that past indiscretions are revealed to traders only by a reputation system. This assumption is contrasted by a number of formal models of reputation that have the assumption of asymmetric information (Kreps and Wilson (1982) and Diamond (1989)) but do not address the problem of how information about indiscretions is gathered

[2]Other pioneering work in this area includes Stigler (1961) and Arrow (1963).

and reported. Random matching models by Ellison (1994) and Tirole (1996) address some of the issues related to our paper. However, these models do not have reputationally driven submarkets and thus do not fully explain how a reputation system coordinates trade.

The paper is organized as follows. In the next section, we give a preliminary discussion of model. This section gives an illustrative example of the model and some basic definitions of reputation systems and market structures without going into much formal detail about the nature of the players, strategies and payoffs. In the subsequent sections, we describe the formal model and then solve its equilibrium. In the final two sections, we give some directions for future research and our concluding remarks.

2 Preliminaries

To fix ideas, this section first gives a simple heuristic example of a market in which the introduction of a reputation system changes the behaviour of buyers and sellers. We then introduce formal definitions of two reputation systems and of two market structures in which such reputation systems can operate.

2.1 An illustrative example

Consider a large number of 'pick your own' orchards and a large number of potential customers – fruit-eaters/pickers – who live in the city. The orchard owners auction their fruit based on local market demand, that is, there is competitive bidding. If they get many customers they get a high price, if they get few or no customers, they get a low or zero price. The customers are uncoordinated over which orchard they visit. Furthermore, the distances are sufficiently remote that they must choose only one orchard to visit.[3] This coordination problem yields a random number of customers at each orchard. In the absence of any perceived differences in orchard quality, each orchard owner earns equal expected profits although there are differences in realized profit due to the coordination friction.

Now suppose that there is a quality difference between the orchards. For example, suppose that half of the orchard owners are endowed with ladders which make picking fruit easier and more enjoyable. If these orchard owners advertise that they have ladders then they can create a submarket in which more fruit-pickers are willing to enter. Customers would arrive at the orchard and enjoy greater utility from picking and consequently pay a higher price for the fruit, other things equal. Thus the high quality fruit orchards (orchards with ladders) will obtain a higher probability of trade and a higher price relative to those orchards without ladders. The problem

[3] Other models of spatially separated sellers include Phelps (1968) and Lucas (1973).

is that orchard owners without ladders also wish to advertise easy pickings. The reason is that if they do, they can enjoy the higher probability of trade associated with such advertisements. Of course, they won't get the greatest possible price in the event of high demand since customers can determine whether the orchard has ladders when they turn up and before they pay for the fruit. However, they ensure that the high demand state is most likely. Consequently, the decentralized outcome is for all orchard owners to advertise that they have ladders.

Now suppose that a reputation system is introduced. This system monitors customer reports from year to year. The reputation system compiles a list of good quality orchards and a list of bad quality orchards based on customer reports, or some other form of monitoring, from the previous year. This system ensures that good quality orchards enjoy a higher probability of trade in the following year, because they have now earned good reputations. In other words, a reputation system fosters the creation of submarkets. Notice that the reputation system in this case does not foster honesty, that is, all bad orchards continue to lie.

Can a reputation system encourage honesty? To answer this question, we need to identify the factors that might lead the owner of an orchard without ladders to be honest if there exists a reputation system. Suppose that good orchards cannot ensure the availability of ladders. For example, the orchard owner might have a 'restricted use' contract for ladders such that they are unavailable if the precious grape harvest is under way. In that case, the needs of grape growers take priority. The customers cannot observe whether this event is true since grapes are harvested under great secrecy. The question now to the owner of an orchard with this type of contract is whether to be honest in the event that the ladders are in use in the vineyard and so are unavailable in the orchard. If the orchard owners are honest they can keep their good reputation and signal that they will probably have ladders available next year. Alternatively, they could simply advertise that they have ladders, enjoy the higher probability of trade in the current period, and suffer the possible loss of reputation in the future.

2.2 Reputation systems and market structure

The orchard example and its permutations suggest that it is possible to describe how a reputation system can change the strategic behaviour of buyers and sellers. The purpose of this section is to provide some formal definitions of reputation systems and to make some preliminary comments about how market structures and reputation systems are related.

In many markets, the intrinsic qualities of sellers and their products do not change over time. For example, some orchards might have beautiful views while others might be located next to toxic waste dumps. In these cases, a reputation system is useful only because it identifies a seller's type through the identification of the quality of the product sold in the previous period. Honesty is clearly not an important criteria for establishing a good reputation in such cases. In particular, if the orchard owner is honest about toxic waste in the present period, a sensible reputation system should not allocate the seller to a high quality submarket in the next period. More cogent examples include human capital. A reputation system might observe that someone is honest about being not intelligent this period, but that does not imply that the reputation system should assign the seller a 'good' reputation in the subsequent period. In such markets where quality is immutable, an effective reputation system screens for type:

Definition 1 *A reputation system **screens for type** if advertisements in the previous period are reported and 'asterisks' are assigned to those sellers that reportedly lied about product quality.*

This reputation system is complicated because it attempts to record and report two types of information. The first type of information is whether a seller is honest and the second type of information is whether the seller achieved this reputation for honesty by selling low or high quality products in the past. To get a good reputation the seller is graded on both measures. This reputation system is effective if product quality never changes, because it rules out a strategy of bad sellers attempting to get a good reputation simply by being honest

In many other markets, a good seller might often have high quality products for sale, but occasionally the seller gets a lemon that he wishes to unload. If good sellers have variable product quality, honesty is potentially the most valued aspect of a reputation. A reputation system that screens predominantly for honesty is defined as follows.

Definition 2 *A reputation system **screens for honesty** if sellers that reportedly lied about their product quality are assigned a bad reputation in the subsequent period.*

This type of reputation system has an number of important advantages over the more complicated reputation system that screens for type. First, the amount of information that is gathered and reported by a reputation system that screens for type might be overwhelming. Second, the two measures of a reputation will have to be aggregated by some meaningful algorithm. Third, practical problems of such a system might occur if the product market falls into a wide set of categories such that product qualities are hard to detect by a third party who records the set of advertisements. A reputation system that screens for honesty has none of these disadvantages. For example, it seems reasonable to assume that buyers are generally expert enough to ascertain quality, such that upon negotiating with the seller, the buyer can report whether the seller misrepresented the quality of products for sale.

3 The model

A market is assumed to operate for two periods, denoted $t = 1, 2$. Let M denote the set and the number of buyer and let N denote the set and the number of sellers in the market.

3.1 Sellers and products

Sellers offer products for sale that can take one of two quality levels. Let $Q = \{\underline{q}, \overline{q}\}$ denote the set of product quality levels. A product of quality $\underline{q} \geq 0$ is of *low* quality and a product of quality $\overline{q} > \underline{q}$ is of *high* quality.

Each seller is one of two quality types: *good* or *bad*. Let η denote the fraction of sellers that are of the good type and let $1 - \eta$ denote the fraction of sellers that are the bad type.

At the start of each period, each seller draws one product from a probability distribution defined over Q. It is assumed that these distributions are such that bad type sellers always have a low quality product for sale, while good type sellers have a high quality product with some strictly positive probability $0 < \gamma \leq 1$ in each period. We let $\widehat{q} \equiv \gamma \overline{q} + (1 - \gamma)\, \underline{q}$ denote the expected quality of a good type seller's product. The average quality of the products of all sellers is given by

$$\widetilde{q} \equiv \eta \widehat{q} + (1 - \eta)\, \underline{q}.$$

In each period sellers advertise, possibly untruthfully, whether they have a high quality or low quality product. All advertisements are seen by all buyers. Following the advertisements, every seller sells their product using an ascending bid auction. For simplicity, the reserve price at every auction is assumed to be zero.

3.2 Buyers and bidding

Each buyer $i \in M$ seeks to buy one unit of the product in each period. Buyers are identical in their willingness to pay for quality. In particular, the net utility function of buyers over all price-quality outcomes of bidding at an auction of seller j in period t is given by

$$u_i\left(q_{jt}, p_{ijt}\right) = \begin{cases} q_{jt} - p_{ijt} & \text{if } p_{ijt} \text{ is the winning bid} \\ 0 & \text{otherwise} \end{cases},$$

where q_{jt} is the quality of product of seller j and p_{ijt} is the bid of buyer i at seller j's auction.

A buyer can purchase the product only by going to a seller's location and participating in that seller's auction. Upon visiting the seller, the buyer becomes perfectly informed of the good's quality, before bidding commences. The bidding at seller j's auction depends on the number of buyers visiting seller j. Under an ascending bid auction, if m_{jt} is the number of buyers choosing to visit seller j in period t, buyer i maximizes utility by the bidding strategy

$$p_{ijt}^{*} = \begin{cases} 0 & \text{if } m_{jt} = 1 \\ q_{jt} & \text{if } m_{jt} > 1 \end{cases} .$$

Thus, a seller receives a non-zero price for his or her product if and only if more than one buyer turns up to the auction.

3.3 Frictional assignment with submarkets

A search friction exists in each period because buyers are uncoordinated and can choose to visit the location of only one seller. This location decision is interpreted broadly as representing the buyer's maximum sphere of attention in the period.[4] We also assume that buyers never purchase from the same seller twice, that is, we rule out long-term relationships by assumption.

The search of each buyer is directed by a common set of submarkets, which are created by the advertisements and reputations of sellers. In the absence of a reputation system, sellers are only distinguished by their advertisements, and hence a submarket consists of sellers who are advertising the same quality level. Thus, without a reputation system, there are up to two different submarkets in each period, consisting of sellers who advertise high and low quality. With a reputation system, there are also up to two submarkets in the first period as sellers do not have reputations in the first period. However in the second period there can be up to four submarkets, since sellers are now distinguished by both advertisements (high quality and low quality) and by two reputations (good and bad). However, in the second period, because the game ends, we will show in proposition 2 below that sellers' advertisements will convey no useful information to buyers. Thus in the second period, sellers are only distinguished by their reputations.

Let $\Phi \equiv M/N$ denote market tightness, that is, the buyer-seller ratio for the whole market and let ϕ denote the market tightness of a particular submarket. We use ϕ_l^1 and ϕ_h^1 to denote the measures of market tightness in the first-period submarkets defined by sellers who advertise low and high quality respectively. Similarly, we use ϕ_l^2 and ϕ_h^2 to denote the market tightnesses in the second-period submarkets defined by sellers who have bad and good reputations respectively. At this point, readers who are unfamilar with the directed search literature are referred to our appendix, in which we describe the matching process and derive some important functional forms.

Wolinsky (1988) shows (also in our appendix) that the probability distribution that b buyers turn up to a seller's auction in a (sub)market with

[4]Lucas (1973) suggests that models of trade between the inhabitants of 'islands' capture essential economic frictions that extend well beyond their 'nautical interpretation'.

tightness ϕ is given by

$$\Pr(x=b)=\begin{cases} e^{-\phi} & \text{for } b=0 \\ \phi e^{-\phi} & \text{for } b=1 \\ 1-e^{-\phi}-\phi e^{-\phi} & \text{for } b>1 \end{cases}.$$

The most important of these probabilities for the seller is that for $b > 1$, since only in this case does the seller receives a non-zero price from his or her auction. Accordingly, we define the function $p(\phi) = 1 - e^{-\phi} - \phi e^{-\phi}$. The expected profit of a seller with product quality q_j in a submarket of tightness ϕ is then given by

$$V_j = p(\phi)\, q_j. \tag{1}$$

From a buyer's point of view, what matters is whether or not they are alone at a seller's auction, since if they are alone they will get a strictly positive surplus from the auction while if they are not alone their surplus will be zero. The probability that a buyer is alone at any given seller in a submarket with market tightness ϕ is given by $e^{-\phi}$, while the probability a buyer is not alone at a seller is given by $1 - e^{-\phi}$. The expected utility of a representative buyer i of visiting a seller in a submarket with tightness ϕ is given by

$$U_i = e^{-\phi} q_z, \tag{2}$$

where q_z is the average quality of sellers' products in this submarket.

Buyers are uncoordinated over which seller to visit in each submarket. Therefore, the total number of x sellers obtaining at least one bidder in period t in the two submarkets is given by

$$x = \alpha^t N\left(1 - e^{-\phi_h^t}\right) + \left(1 - \alpha^t\right) N\left(1 - e^{-\phi_l^t}\right)$$

where α^t is the fraction of sellers that are in the high quality submarket - see the appendix for the derivation of this function.[5] This matching function has all the usual properties commonly assumed about matching functions, including constant returns to scale, differentiability and concavity.

3.4 The reputation system

At the end of the first period, the reputation system collects information on seller behaviour and assigns reputations to sellers according to the appropriate algorithm. At the beginning of the second period, buyers observe sellers' reputations, as well as their new advertisements, before choosing which seller's auction to visit.

[5]Butters (1977) first derives this matching technology.

We model the reputation system by introducing a parameter $k \in [0, 1]$ that represents the probability that a seller who is dishonest in period 1 is 'caught' and is assigned a bad reputation in the second period. For simplicity it will be assumed that all sellers who are not caught get a good reputation and thus cannot be distinguished by buyers from sellers who were honest.

If the reputation system is based on monitoring by the system operator itself, the value of k reflects the effectiveness of this monitoring and is taken as given. If reputations are assigned on the basis of buyer reports, as occurs on eBay, then k becomes endogenous. Under such a system we assume that the winning bidder at every auction submits a report as to whether the seller told the truth or not, and that all buyers are honest in their reports. Under such a 'customer-report reputation system', the value of k is endogenous, and is equal to the probability that at least one buyer turns up to a seller's auction in the first-period submarket of sellers who advertise high quality.

3.5 The timing of the reputation system game

The timing of the game with a reputation system is summarized in figure 1. At the start of period 1, each seller observes their type (good or bad) and their product quality (high or low). They then choose to advertise either high or low product quality. Buyers observe the sellers' advertisements and choose one seller to visit. Frictional assignment implies that buyers in any particular submarket of similarly advertised sellers are randomly assigned to the sellers. After this assignment, buyers bid on the seller's product and the good is sold to the highest bidder. At the start of period 2, a reputation system assigns a reputation to each seller based on events in the previous period and the properties of the reputation system under investigation. Sellers draw a new product according to their type. Buyers choose a reputational submarket and are frictionally assigned to a seller in this submarket. Bidding then takes place, the good is sold to the highest bidder, and finally the game ends.

4 Directed search and quality differentiated submarkets

In this section, we first derive the equilibrium 'reaction functions' of buyers to any arbitrary partition of sellers into two quality differentiated submarkets. We then consider the effect on buyers' welfare of any partition of sellers into two quality differentiated submarkets and the effect on overall welfare of any such partition.

Each period has either (i) two submarkets or (ii) a single market in which all buyers are randomly assigned to all sellers. The expected payoff of any

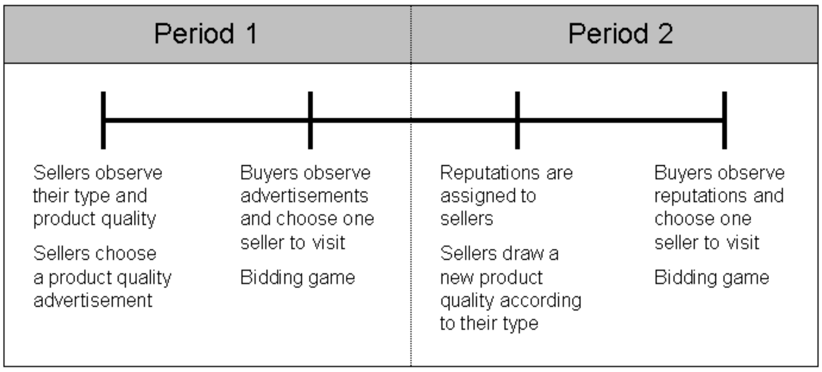

Figure 1: Timing of the reputation game.

given buyer in this case is

$$U_{NR} \equiv \widetilde{q}e^{-\Phi},$$

where time superscripts are dropped for notational convenience from here on whereever possible.

Suppose now that sellers are allocated by some mechanism into two quality differentiated submarkets. Let q_l and q_h denote the expected quality levels of sellers in the two submarkets, and let α denote the fraction of sellers that are allocated to the submarket with expected quality q_h. Without loss of generality we assume $q_h > q_l$. If sellers are allocated into two quality differentiated submarkets, the average quality of sellers across submarkets cannot change, thus

$$\widetilde{q} = \alpha q_h + (1 - \alpha)\, q_l. \tag{3}$$

Also, the number of buyers is fixed, and so market tightness for each submarket is related to overall market tightness as follows:

$$\Phi = \alpha \phi_h + (1 - \alpha)\, \phi_l, \tag{4}$$

where ϕ_l and ϕ_h denote the buyer-seller ratios of the two submarkets.

The division of sellers into submarkets leads to two basic types of equilibrium allocations of buyers. These allocations depend upon the average quality of sellers in each submarket, their relative numbers, and the overall ratio of buyers to sellers. These conditions are summarized by what we call the *exclusion constraint*:

$$q_h e^{-\Phi/\alpha} \geq q_l. \tag{EC}$$

The left hand side of the exclusion constraint is the expected utility of buyers if all buyers locate in the high quality submarket. The right hand side of this constraint is the average quality of sellers products in the low quality submarket. If the partition of sellers into submarkets satisfies the exclusion constraint, all buyers locate in the high quality submarket. Therefore, if EC is satisfied,

$$\phi_h = \Phi/\alpha \text{ and } \phi_l = 0. \tag{5}$$

A partition of sellers into the low submarket which satisfies EC is the maximum punishment a reputation system can impose on these sellers. In this case, sellers in the low quality submarket are not allocated any buyers. If the partition of sellers into submarkets does not satisfy EC, buyers locate in both the high quality and low quality submarkets. Therefore, in a mixed strategy equilibrium where EC is not satisfied, we must have

$$q_h e^{-\phi_h} = q_l e^{-\phi_l}. \tag{6}$$

In this case, buyers locate in both the high quality and low quality submarkets such that expected utility to buyers is the same in each submarket.

The behavior of buyers can be expressed as a function of the distribution of sellers across each submarket. This function depends crucially on EC. From (4), (5) and (6), market tightness in the high submarket is given by

$$\phi_h = \begin{cases} \Phi/\alpha & \text{if EC} \\ \Phi + (1-\alpha)\ln(q_h/q_l) & \text{otherwise} \end{cases}. \tag{7}$$

Similarly, market tightness in the low submarket is given by

$$\phi_l = \begin{cases} 0 & \text{if EC} \\ \Phi - \alpha \ln(q_h/q_l) & \text{otherwise} \end{cases} \tag{8}$$

Table 1 gives some numerical examples of the equilibrium market tightness in each submarket for different partitions of sellers. This example illustrates that the exclusion constraint is satisfied if the market is thick – a small Φ such that there are many sellers – or if the difference in the average quality of the sellers in the two submarkets is large. In these cases, low quality sellers receive the maximum punishment of a reputation system and are excluded from the market. This example also shows that, for a large number of parameters, the partition of sellers into quality differentiated submarkets is not so informative as to exclude sellers in the low quality submarket.

If EC holds, sellers in the low quality submarket are excluded, and from (2), a buyer's utility in any period is simply $U = e^{-\Phi/\alpha} q_h$. If EC does not hold, buyers visit both submarkets with strictly positive probability. From (2) and (6), a buyer's utility in any period is $U = e^{-\phi_h} q_h$. Substituting in (7), we obtain $U = e^{-\Phi} q_h^{\alpha} q_l^{1-\alpha}$. Summarising, for any distribution of sellers across submarkets, the expected payoff of a buyer, U in the period under

(q_h, q_l, α, Φ)	ϕ_h	ϕ_l
$\left(1, \frac{1}{8}, \frac{1}{2}, 1\right)$	2	0
$\left(1, \frac{1}{2}, \frac{1}{2}, 1\right)$	1.35	0.65
$\left(1, \frac{3}{4}, \frac{1}{2}, 1\right)$	1.14	0.86
$\left(1, \frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right)$	0.5	0
$\left(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{4}\right)$	0.5	0
$\left(1, \frac{3}{4}, \frac{1}{2}, \frac{1}{4}\right)$	0.39	0.11

Table 1: Numerical examples of equilibrium market tightnesses

consideration is given by

$$U = \begin{cases} e^{-\Phi/\alpha} q_h & \text{if EC} \\ e^{-\Phi} q_h^{\alpha} q_l^{1-\alpha} & \text{otherwise} \end{cases} . \tag{9}$$

The utility of buyers is a linear function of the average quality of sellers in the high submarket if sellers in the low submarket are excluded. If no sellers are excluded, the utility of buyers is a Cobb-Douglas function of the average quality of the sellers' products in each submarket with the weights being the fraction of sellers in each submarket.

Theorem 1 *A partition of sellers into two quality differentiated submarkets deceases the welfare of buyers if (i) the partition does not satisfy EC or (ii) the market is sufficiently 'thin'. The welfare of buyers increases otherwise.*

Proof. If not EC,

$$\begin{aligned} U - U_{NR} &= e^{-\Phi} q_h^{\alpha} q_l^{1-\alpha} - e^{-\Phi} \widetilde{q} \\ &= e^{-\Phi} \left[q_h^{\alpha} q_l^{1-\alpha} - \left(\alpha q_h + (1-\alpha) q_l \right) \right] \\ &< 0 \text{ for all } q_h \neq q_l \text{ and } \alpha \in (0,1). \end{aligned}$$

To see the inequality, note that in general, $x^{\alpha} y^{1-\alpha} < \alpha x + (1-\alpha) y$ for $\alpha \in (0,1)$ and $x \neq y$. Taking the log of the left-hand side, $\log\left(x^{\alpha} y^{1-\alpha}\right) = \alpha \log x + (1-\alpha) \log y < \log\left(\alpha x + (1-\alpha) y\right)$ since log is a concave function, and log monotone implies $x^{\alpha} y^{1-\alpha} < \alpha x + (1-\alpha) y$.

If EC,

$$\begin{aligned} U - U_{NR} &= e^{-\Phi/\alpha} q_h - e^{-\Phi} \widetilde{q} \\ < 0 & \quad \text{if } \Phi > \left[\alpha / (1-\alpha)\right] \ln\left(q_h / \widetilde{q}\right) \\ > 0 & \quad \text{otherwise.} \end{aligned}$$

■

A reputation system hurts buyers if it forces them to compete more intensely for high quality products. Theorem 1 demonstrates that buyers are always hurt by a reputation system if the creation of the low quality submarket is not so informative as to warrant exclusion of these sellers. In the case where low quality sellers are excluded, theorem 1 demonstrates that buyers are always hurt by a reputation system if the market is sufficiently thin – if it has only a few sellers. In both cases, buyers are hurt because they pay much higher prices in equilibrium even though they obtain high quality products with greater frequency. In a thick market, buyers are benefited by the reputation system because the associated price increases for high quality products are much smaller.

The decentralized actions of buyers in response to the information supplied by the creation of submarkets raises a question over whether submarket creation is socially efficient. For example, if EC is satisfied, then there is increased competition between buyers for the remaining high quality sellers. Likewise, if low quality sellers are included, then it is not clear that too many buyers will locate in this submarket. However, it is possible to show that the creation of submarkets always raises social welfare. If there is only one market, social welfare in a period is given by

$$W_{NR} \equiv N\left(1-e^{-\Phi}\right)\widetilde{q},$$

where $\widetilde{q}$ is the average quality of sellers products and $N(1-e^{-\Phi})$ is the total number of trades between sellers and buyers in this purely random assignment. If there are two submarkets, total welfare is given by

$$W = \begin{cases} \alpha N\left(1-e^{-\Phi/\alpha}\right)q_h & \text{if EC} \\ \alpha N\left(1-e^{-\phi_h}\right)q_h + (1-\alpha)N\left(1-e^{-\phi_l}\right)q_l & \text{otherwise} \end{cases}. \tag{10}$$

The welfare in a period with submarkets can be directed compared to welfare without submarkets.

Theorem 2 *A partition of sellers into two quality differentiated submarkets always increases social welfare.*

Proof. If not EC,

$$\begin{aligned} W - W_{NR} &= \alpha N\left(1-e^{-\phi_h}\right)q_h + (1-\alpha)N\left(1-e^{-\phi_l}\right)q_l - N\left(1-e^{-\Phi}\right)\widetilde{q} \\ &= Ne^{-\Phi}\left[\alpha q_h + (1-\alpha)q_l - q_h^{\alpha}q_l^{(1-\alpha)}\right] \text{ by eqs (3), (7) and (8)} \\ &> 0 \text{ for all } q_h \neq q_l \text{ and } \alpha \in (0,1). \end{aligned}$$

If EC,

$$W - W_{NR} = \alpha N\left(1-e^{-\Phi/\alpha}\right)q_h - N\left(1-e^{-\Phi}\right)\widetilde{q}.$$

Note that $W - W_{NR}$ is strictly decreasing in q_l, hence the worst case is when q_l is as large as possible that satisfies EC, that is when $q_l = q_h e^{-\Phi/\alpha}$. Thus we only need to show that $W - W_{NR} \geq 0$ for EC satisfied with equality. In this case,

$$\begin{aligned} W - W_{NR} &= \alpha N \left(1 - e^{-\Phi/\alpha}\right) q_h - N \left(1 - e^{-\Phi}\right) \left(\alpha q_h + (1-\alpha) q_h e^{-\Phi/\alpha}\right) \\ &= N q_h \left[\alpha e^{-\Phi} - e^{-\Phi/\alpha} + (1-\alpha) e^{-\Phi \frac{1+\alpha}{\alpha}}\right] \end{aligned}$$

The term in the square brackets can be written as

$$e^{-\Phi \frac{1+\alpha}{\alpha}} (1-\alpha) - e^{-\frac{\Phi}{\alpha}} \left(1 - \alpha e^{\Phi \frac{1-\alpha}{\alpha}}\right)$$

This is positive since $e^{-\Phi \frac{1+\alpha}{\alpha}} > e^{-\frac{\Phi}{\alpha}}$ and $\alpha < \alpha e^{\Phi \frac{1-\alpha}{\alpha}}$ for all $\Phi > 0$ and $\alpha \in (0, 1)$. ■

Welfare is improved by submarket creation because buyers' search is directed more accurately and thus the number of quality adjusted matches increases.

5 Strategic equilibrium and reputation systems

In the previous section, we considered the effects on buyers and welfare of an arbitrary separation of sellers into quality differentiated submarkets in the general case, without saying what creates or maintains the separation of sellers. In this section we consider strategic behaviour of sellers in the presence of a reputation system and show how this can lead to the creation, in equilibrium, of quality differentiated submarkets. In particular, we derive the equilibrium strategies of sellers given the 'reaction functions' of buyers that were derived in the previous section. We investigate the equilibrium predictions of the model with and without a reputation system. We first consider the case where good sellers always have high quality products and the reputation system screens for type. We then consider the case where good sellers sometimes have low quality products and the reputation system screens for honesty. Finally, we compare the equilibria for the two types of reputation system when good sellers always have high quality products.

5.1 Behaviour of sellers without a reputation system

In the absence of a reputation system, there is nothing to link the two periods for either buyers of sellers, and we can thus treat the two periods as independent. In this case, it is straightforward to see the following result.

Proposition 1 *In the absence of a reputation system, a seller's equilibrium advertisement carries no information to buyers about the quality of the seller's product.*

Proof. Suppose to the contrary that sellers' advertisements were informative. Then by definition there would exist two quality differentiated submarkets. Since by (6) there is a higher buyer-seller ratio in a submarket with higher expcted quality, and since by (1) a seller's expected utility is strictly increasing in the buyer-seller ratio, all sellers would instead choose their advertisement so as to locate in the higher quality submarket, a contradiction. ∎

In practical terms, proposition 1 implies that there are three possible equilibria in each period without a reputation system: all sellers advertise high quality; all sellers advertise low quality; or sellers randomise over advertising high and low quality such that there are two submarkets having identical expected qualities. All of these three cases are essentially identical, and in particular all will have the same levels of welfare and payoffs to buyers and sellers. Accordingly, for the purposes of our exposition, we shall maintain that all sellers advertise high quality in the absence of a reputation system.

Without a reputation system, the expected payoffs to good and bad sellers in each period are respectively given by

$$\begin{aligned} V_{NR}^{g} &\equiv p(\Phi)\widehat{q} \\ V_{NR}^{b} &\equiv p(\Phi)\underline{q}, \end{aligned}$$

where $V_{NR}^{g} > V_{NR}^{b}$, because $\widehat{q} > \underline{q}$.

5.2 Second period behavior of sellers

Since our model has only two periods, there is nothing to link the actions of sellers in the second period with any future payoff. It is therefore straightforward to see that sellers' advertisements will not convey any information to buyers in the second period, even in the presence of a reputation system.

Proposition 2 *With a reputation system, sellers' second-period equilibrium advertisements carry no information to buyers.*

Proof. Similar to the proof of proposition 1 except that in the second period with a reputation system, quality differentiated submarkets defined by reputations may exist. Advertisements, however, cannot create additional quality differentiated submarkets by the same argument as in the proof of proposition 1. ∎

As in the case without a reputation system, for the purposes of exposition we will maintain that all sellers advertise high quality in the second period.

The key implication of proposition 2 is that sellers in the second period are distinguished to buyers only by their reputation. That is, there are at most two submarkets in the second period, one with sellers who have good reputations, and one with sellers who have bad reputations. Since sellers do not yet have reputations in the first period, there is also a maximum of two submarkets in the first period. Thus theorems 1 and 2 are sufficiently general to apply to every possible equilibrium of our model.

5.3 Reputations for type

In many markets, the intrinsic qualities of sellers and their products do not change over time. For example, some orchards might have beautiful views while others are next to toxic waste dumps. In these cases, a reputation system is useful only because it identifies a seller's type by identifying the quality of the product sold in the previous period. Honesty is clearly not an important criteria for establishing a good reputation in such cases. We can analyse this type of market using our model by assuming that sellers are immutable in type and product (ITP). In this case, we assume that good sellers always have quality products, that is, that $\gamma = 1$.

Given the second period behaviour implied by proposition 2, the equilibrium when $\gamma = 1$ is characterized by the following proposition.

Proposition 3 *Under a reputation system that screens for type, if all good sellers have high quality products and all bad sellers have low quality products, all sellers advertise high quality in the first period.*

Proof. All good sellers have a dominant strategy of advertising high quality in the first period, because this strategy always places them in the high submarket each period. A bad seller also advertises good quality in the first period, because this strategy places the seller in the high submarket in period one and in the low submarket with only some probability next period. The alternative strategy of honesty by a bad seller is dominated because a bad seller is placed in the low quality submarket both periods if he or she is honest in the first period. ■

In this equilibrium, in the first period, there is only one 'submarket', since all sellers are apparently identical from a buyer's point of view. The first period payoff to a buyer is U_{NR}, to a good seller is V^g_{NR}, to a bad seller is V^b_{NR} and the contribution to social welfare is W_{NR}.

All good type sellers will have a good reputation in the second period, while some bad sellers will have a bad reputation and some will have a good reputation, depending on the value of k. Note that under a customer report system, $k = 1 - e^{-\Phi}$.[6]

[6] To benchmark the efficiency of a customer report reputation system in this case, suppose that $\Phi = 1$. Then the probability that at least one buyer turns up to a seller's auction in the first period is $1 - e^{-1} = 0.63$. Thus when good type sellers always have a high quality product for sale, approximately 63% of bad type sellers are 'caught' under a customer report system when the overall ratio of buyers to sellers is one to one.

With a reputation system and $\gamma = 1$, the distribution of sellers and their average product quality in each of the second period submarkets is given by

$$q_h = \frac{\eta\overline{q} + (1-k)(1-\eta)\underline{q}}{\eta + (1-k)(1-\eta)}$$

$$q_l = \underline{q},$$

and

$$\alpha = \eta + (1-k)(1-\eta).$$

From these equations we can use the reaction functions of buyers described in the previous section to compute the equilibrium values of ϕ_h and ϕ_l. The sellers' payoffs in the second period are given by

$$V_g = p(\phi_h)\overline{q}$$

and

$$V_b = ((1-k)p(\phi_h) + kp(\phi_l))\underline{q}.$$

Payoffs of buyers and total welfare are given by (9) and (10) respectively.

To illustrate the effects of a reputation system that screens for type on the welfare of agents for different measures of market tightness, consider the following numerical example. Suppose that $\overline{q} = 1$ and $\underline{q} = \frac{1}{2}$ and that the reputation system is based on customer reports. Figure 2 depicts the effects of a reputation system that screens for type on (i) the welfare of buyers, (ii) the welfare of good sellers, (iii) the welfare of bad sellers and (iv) total welfare for different market tightness conditions. From figure 2, as would be expected, we can see that a reputation system only has value if there exist both types of sellers. That is, the value of a reputation system that screens for type is zero if $\eta = 0$ or $\eta = 1$. We can also see that the results of theorems 1 and 2 are confirmed for these parameters.

5.4 Reputations for honesty

In this section, we analyse how a reputation system that screens for honesty can foster equilibrium signalling by good sellers who occasionally have low quality products. This type of market follows from our earlier discussion of an orchard in which the supply of ladders is stochastic. In this case, we assume that $0 < \gamma < 1$. That is, good type sellers do not have a high quality product with certainty in each period, although their expected product quality, $\widehat{q}$, is always strictly greater than that of a bad type seller. Given this assumption we can investigate the strategy of sellers which we call

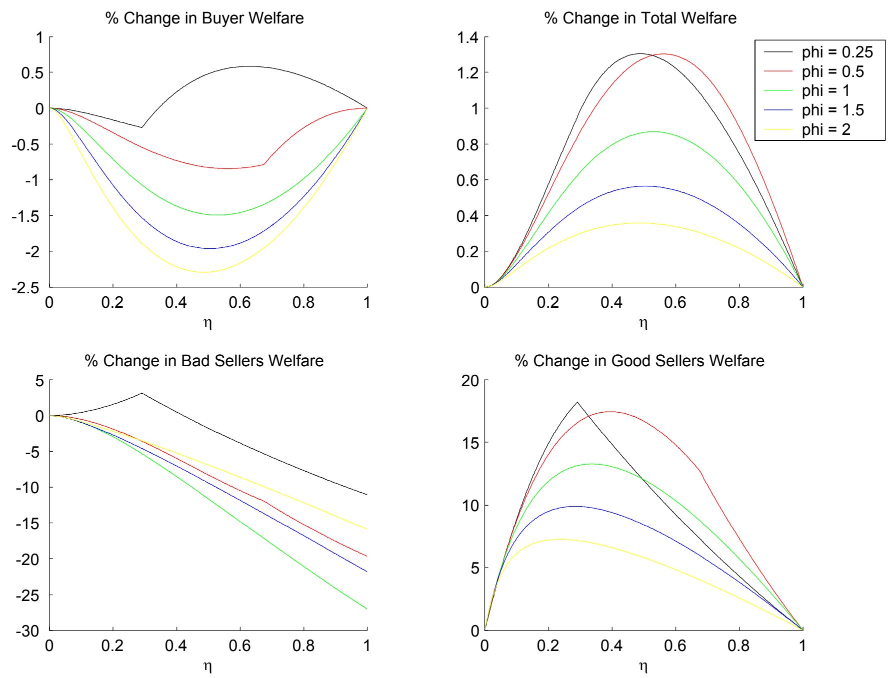

Figure 2: Effects of a reputation that screens for type when type is immutable. Parameter values: $\overline{q} = 1$, $\underline{q} = \frac{1}{2}$, customer report system.

'being honest'. To keep the analysis of this strategy as simple as possible, we assume that the reputation system screens for honesty (see definition 2).

A seller who is *honest* truthfully advertises their product quality in period 1. Note that by proposition 2, all sellers will advertise high quality in the second period, thus in the second period the only thing that distinguishes sellers from a buyer's point of view is their reputation.

For obvious reasons, a good seller with a high quality product realization will always advertise high quality and thus always tells the truth. Therefore we only need to solve the decision problems for the bad type sellers and the good type sellers with a low quality realization in period 1. We allow for symmetric mixed strategies by sellers and we let ξ_b and ξ_g respectively denote the probabilities that a bad seller and a good seller with a low quality realization are honest in period 1.

The nine different configurations that the equilibrium can take are characterized in table 2. We define the 'honesty valuation', θ_g, of a good seller with a bad realization to be the difference between the expected payoff to that seller from being honest and lying, that is,

$$\begin{aligned}\theta_g &= p\left(\phi_l^1\right)\underline{q} + p\left(\phi_h^2\right)\widehat{q} \\ &\quad - \left[p\left(\phi_h^1\right)\underline{q} + \left(kp\left(\phi_l^2\right) + (1-k)\,p\left(\phi_h^2\right)\right)\widehat{q}\right]\end{aligned}$$

Case	Behaviour of sellers with low quality products	θ_g	θ_b
1	All good sellers are honest, all bad sellers lie.	$+$	$-$
2	All good sellers are honest, some bad sellers honest.	$+$	0
3	Some good sellers are honest, all bad sellers lie.	0	$-$
4	All sellers lie.	$-$	$-$
5	All sellers are honest.	$+$	$+$
6*	All good sellers lie, all bad sellers are honest.	$-$	$+$
7*	Some good sellers and all bad sellers are honest.	0	$+$
8*	All good sellers lie, some bad sellers are honest.	$-$	0
9*	Some good and bad sellers are honest.	0	0

Table 2: Relationship between 'honesty valuation' and signalling equilibrium. A '*' indicates a case that is ruled out in proposition 4.

which simplifies to

$$\theta_g = \left[p\left(\phi_l^1\right) - p\left(\phi_h^1\right)\right] \underline{q} + k\left[p\left(\phi_h^2\right) - p\left(\phi_l^2\right)\right] \widehat{q}. \tag{11}$$

Similarly, the honesty valuation of a bad seller is

$$\theta_b = \left[p\left(\phi_l^1\right) - p\left(\phi_h^1\right)\right] \underline{q} + k\left[p\left(\phi_h^2\right) - p\left(\phi_l^2\right)\right] \underline{q}. \tag{12}$$

Proposition 4 *The only possible equilibria are cases 1 – 5.*

Proof. Since $\widehat{q} > \underline{q}$, from (11) and (12), $\theta_g > \theta_b$. That is, the 'honesty valuation' of a good seller with a bad realization in period 1 is always greater than that of a bad seller. This fact allows us to rule out all of cases 6 – 9 since an equilibrium in any of these cases requires $\theta_b \geq \theta_g$. ■

We are left with cases 1 to 5 to consider. In each of the cases, we used a numerical algorithm programmed in *Matlab* to test which type(s) of equilibrium occurred for any given parameter values. To see how this works, let us consider case 1 as an example. Case 1 is a pure strategy separating equilibrium in which all good sellers are honest and all bad sellers lie.

In this case, in the first period we have

$$\alpha^1 = \gamma\eta + (1-\eta),$$

$$q_h^1 = \frac{\gamma\eta\overline{q} + (1-\eta)\underline{q}}{\gamma\eta + (1-\eta)},$$

and

$$q_l^1 = \underline{q}.$$

From (5), if the EC holds, we have $\phi_l^1 = 0$ and $\phi_h^1 = \Phi/\alpha^1$ while if EC does not hold from (6) we have $\phi_l^1 = \Phi - \alpha^1 \ln\left(q_h^1/q_l^1\right)$ and $\phi_h^1 = \Phi + \left(1-\alpha^1\right)\ln\left(q_h^1/q_l^1\right)$.

In the second period, we have

$$\alpha^2 = \eta + (1-k)(1-\eta),$$

$$q_h^2 = \frac{\eta\widehat{q} + (1-k)(1-\eta)\underline{q}}{\eta + (1-k)(1-\eta)},$$

and

$$q_l^2 = \underline{q}.$$

Similarly with the first period, checking the (EC) constraint determines the values of ϕ_l^2 and ϕ_h^2.

Having found the values of ϕ_l^1, ϕ_h^1, ϕ_l^2, and ϕ_h^2 for a given set of parameters, we then substitute these into the expressions for θ_g and θ_b. If $\theta_g > 0$ and $\theta_b < 0$, we conclude that an equilibrium of this type exists for the parameter values given.

The solution method for cases 2 – 5 is similar, hence we do not give all the details here. The only differences being in the specification of the values of α, q_l and q_h in each period, and the conditions on θ_g and θ_b that must be satisfied.

As a benchmark, we chose parameter values $M = N = 1$ (and hence $\Phi = 1$), $\overline{q} = 1$, and $\underline{q} = 0.5$. Figure 3 shows the existence of the 5 different possible types of equilibria varying η (the proportion of good type sellers in the population) on the horizontal axis and γ (the probability that a good type seller has a high quality product) on the vertical axis, for a reputation system based on customer reports.

From figure 3, we first notice that for these parameter values, there is never an equilibrium in which all sellers are honest (case 5). This is a feature of the results from many different sets of parameters that we have tried. From figure 3 we can also see that for some parameter values there are multiple equilibria. Accordingly, figure 4 shows the equilibrium at each point that gives the highest overall social welfare out of all the possible equilibria at that point.

We can see from figures 3 and 4 that good sellers with bad products are encouraged to be honest if there are few good sellers in the population or if the probability that a good seller obtains a bad product is sufficiently small.

Figure 5 shows the effects on total welfare and the welfare of buyers and sellers of introducing a reputation system that screens for honesty. For parameter cases where there are multiple equilibria under a reputation system, the equilibrium with the highest overall welfare was chosen, corresponding

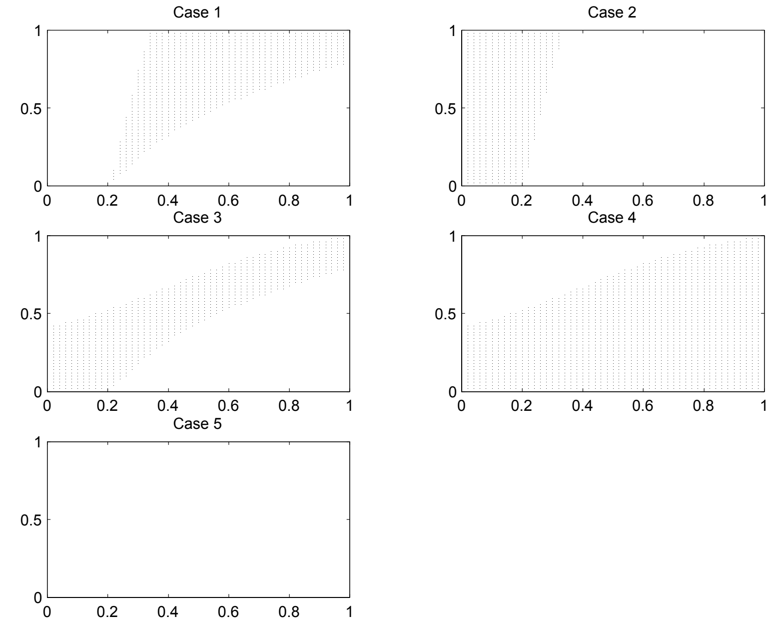

Figure 3: Regions where the different types of signaling equilibria exist. In all cases η is on the horizontal axis and γ is on the vertical axis.

to the equilibrium shown in figure 4. Hence figure 5 gives an upper bound on the welfare effects of introducing a reputation system that screens for honesty, for these parameter values.

5.5 Comparing reputation systems

We argued earlier in the preliminaries that a reputation that screens for honesty might not be the most effective reputation system if good sellers offer nice views and bad sellers have toxic waste dumps – i.e. product qualities are invariant over time. The problem with a reputation system that screens for honesty in this case is that a strategy of 'honesty' by a bad seller gives that seller a good reputation in the second period. This strategy has a high payoff in the second period if buyer search is higly directed towards sellers in the second period with high quality reputations.

Accordingly, figure 6 shows the difference in welfare effects of a reputation system that screens for honesty and a reputation system that screens for type, when type is immutable (i.e., $\gamma = 1$). The figure shows the effect of a reputation system that screens for honesty less the effect of one that screens for type. We can see that the increase in total welfare of a reputation system that screens for honesty is always less than or equal to the simpler system that screens for type. On the other hand, buyers and bad sellers

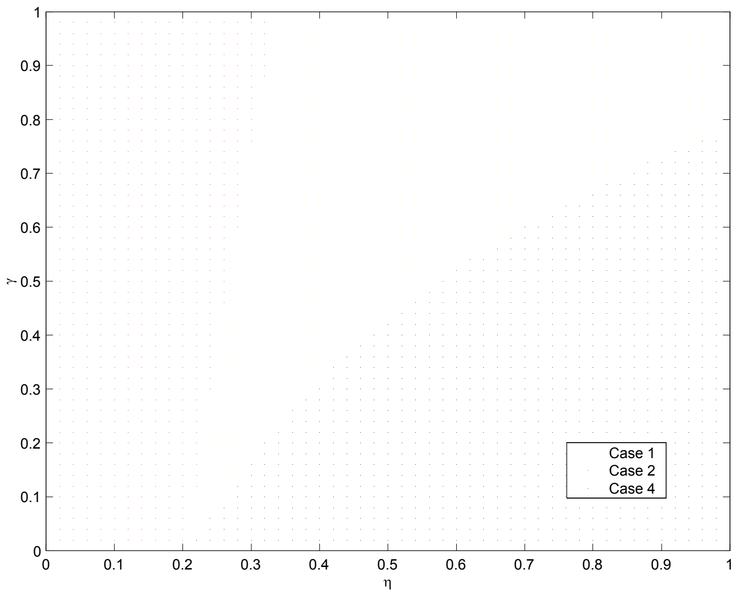

Figure 4: The equilibrium with the honest welfare level out of all the signaling cases.

are made (weakly) better off by the honesty reputation system, while good sellers are made worse off.

This simple comparison offers only an upper bound on the extent to which the two types of reputation systems have different welfare effects. A more general comparison requires a solution to the equilibrium in which a reputation system screens for type and good sellers sometimes have bad products. This analysis lies slightly outside the scope of the present paper, because the analysis would have to incorporate buyer reaction functions for markets with more than two submarkets. We will develop this extension in future analysis where we intend to give a more comprehensive analysis of alternative reputation systems.

6 Future Research

The present paper represents a first attempt to model reputation systems in a directed search equilibrium. As such, we have chosen to focus on a very simple two period model with competing auctions, homogeneous buyers and two types of sellers all separated by location. The purpose of this section is to briefly outline a set of possible directions for future research.

Loyalty. Long-term relationships are a potential substitute for a reputation system. In particular, long-term relationships internalise the returns

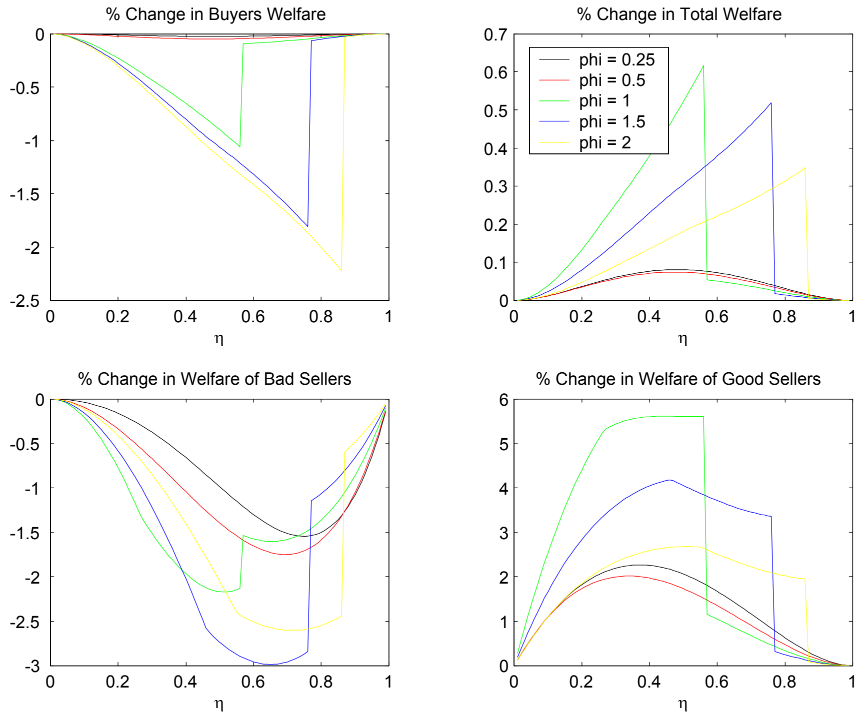

Figure 5: Effects of a reputation system that screens for honesty when $\gamma = \frac{1}{2}$.

to any information that is gathered by sellers and buyers from past transactions. Long-term relationships of endogenous duration are analysed by Julien, Kennes and King (2001) in a directed search model with on-the-job search. It would be interesting to see how the introduction of a reputation system changes (i) the duration of these long-term buyer-seller relationships and (ii) the price (wage) profiles of agents in such relationships.

Parasitic customers. We have assumed that all buyers are sincere in their efforts to buy high quality products. This assumption begs the question as to what would happen if some buyers were not sincere. An interesting extension of the model would include buyers who act as parasites by threatening sellers with bad reports. It would then be interesting to analyse a reputation system that reports information about buyers in addition to sellers. In particular, the opinions of chronic complainers might be weighed less than the opinions of other buyers.

Endogenous selling mechanisms. In many frictional markets, the valuations of buyers are idiosyncratic over locations. For example, in Jovanovic (1979), the valuations of buyers over locations are homogeneous *ex ante* but heterogeneous *ex post.* Auctions are extremely effective in such markets, be-

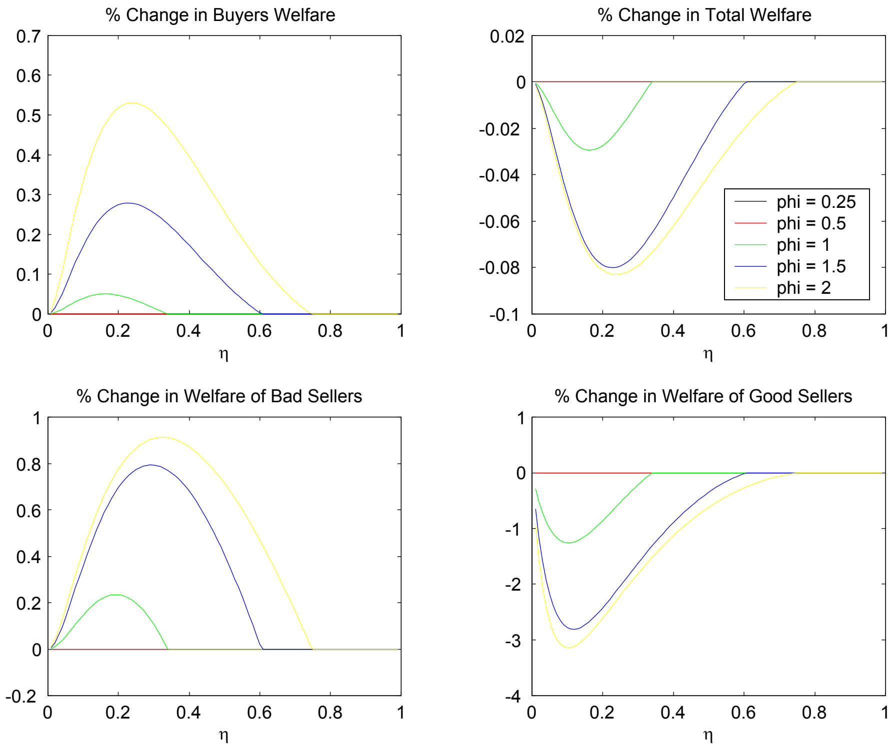

Figure 6: A comparison of the changes brought about by a reputation systems that screen for honesty and for type when $\gamma = 1$.

cause the auctions ensure all viable trades are executed.[7] Posted prices are disadvantaged in this setting because low quality trades are ruled out owing to the commitment to the price posted. However, there is a potential trade-off if reputations for quality are important. In particular, a commitment to a high posted price is a potential signal of quality because low quality trades are ruled out by the commitment to the price posted. That is to say, price signals quality. Given this trade-off, it would be interesting to see how a reputation system affects the choice of whether to auction or post a price.[8]

Middlemen. Another strategy of sellers is the sale of multiple goods at a location. The cost of maintaining large stocks by middlemen are associated with diseconomies of scale concerning the application of specialized knowledge and economies of scale associated with 'one-stop shopping' (ref: Alchian 1977). It would be interesting to see how a reputation system affects the number of products offered by each seller and whether a reputation

[7]Wolinsky (1988) allows for this type of *ex post* uncertainty in his model of competing auctions.

[8]Coles and Eeckhout (2000) discuss a broader set of alternative selling mechanisms.

system affects the creation of dealer networks, i.e. middlemen.

Alternative Reputation Systems. Although we have used two types of reputation systems in our analysis, we have chosen not to devote much attention to a comparison of these systems in the present paper. Instead, we have used a reputation that screens for type if good sellers always have high quality products and a reputation that screens for honesty if good sellers sometimes have low quality products. These choices allowed us to maintain a very simple analysis in which there always exists a maximum of two possible reputational submarkets. In future work, we expect to do a more systematic comparison of different types of reputation systems. It will be interesting to determine whether a reputation system that screens for honesty is a close substitute for a reputation system that screens for type.[9]

Fees and subsidies. In our model, we have assumed that the reputation system is exogenous and that no fees are required for its operation. The introduction of a fee structure for a reputation system that governs a market with diverse products raises a number of questions about the types of fees that can be charged. Should fees be charged to both sellers and buyers? Should selling fees be related to reputation? Are subsidies required to pay for buyers' reports?

Platform competition. The collection of fees by a reputation system raises questions about the viability of competing reputation systems. What are the advantages of the incumbent over the entrant. Is entry by a competing reputation system ever feasible? Should an industry with commercial reputation systems be regulated?

Experiments. The endogenous frictions described in this paper could easily be recreated in a laboratory. It would be interesting to develop a set of experiments that would translate the present framework into a reputation system "wind tunnel". Experimental testing could aid the design and understanding of reputation systems. In particular, our understanding of the effects of different reputation systems would be benefited by analysing the behaviour of real life players instead of the idealized agents which typically populate economic models.

7 Conclusion

Search models emphasize the value of time by assuming that buyers are restricted in the number of locations they can visit in a period. The implication of these models is that buyers economize on time when they search for new purchases. In this paper, we assumed that sellers are aware of these objectives and act strategically. The equilibrium strategic behaviour of sellers in a directed search model with asymmetric information about product

[9]Dellarocas (2002) offers a helpful survey of alternative online reputation systems/ mechanisms.

quality is to oversell product quality in order to maximize the probability of trade. Thus a seller with a low quality car advertises high quality. A reputation system imposes a punishment on these excessive advertisements. We showed that this punishment encourages honesty and reveals deception. We also showed that both of these factors add value to a reputation system.

The simplicity and, hopefully, transparency of our model of a reputation system is aided by parallel developments in the theory of directed search (Burdett, Shi and Wright (2001) and Julien, Kennes and King (2000)). Our particular version of a directed search model assumes decentralised competing auctions – an assumption that is quite close to the actual institutional structure of eBay. The main innovation of our model is to show how search is directed by a set of reputationally enhanced submarkets and to describe how such markets are created by a reputation system. The creation of these submarkets is a dynamic process in which misleading advertisements in one period lead to lower reputations in the next period.

Our model demonstrates that a reputation system increases not only the probability that a buyer finds high quality products, but also the equilibrium price of such products. Consequently, the effect of a reputation system on the welfare of buyers is ambiguous. We gave two conditions in which buyers are made worse off by a reputation system. First, we showed that buyers are made worse off by a reputation system in a thin market (few sellers), because prices then rise disproportionately more than the quality of products traded. Second, we showed that buyer welfare is a function of the amount of information provided by a reputation system for any number of sellers. In particular, if the amount of extra information supplied by a reputation system is sufficiently small such that no sellers are excluded from the market, buyers are always made worse off.

The simplicity of our framework suggests that it should be possible to extend it in a number of different directions. For example, it would be interesting to see how a reputation system affects customer loyalty, reports by chronic complainers, and the choice of selling mechanisms by competing sellers. It would also be interesting to look more deeply into the design, operation and regulation of a reputation system. These investigations would include the design of fee and subsidies structures, a detailed comparison of alternative types of reputation systems, the role of competition between the operators of rival reputation systems, and the possible benefits of regulation. Experimental research could also prove useful in this context. All of these topics await further research.

8 Appendix

Frictional assignment is the outcome of buyers being uncoordinated over which seller to visit in each submarket. An equilibrium with frictional assignment is a mixed strategy equilibrium in the location of buyers over sellers (ref: Julien, Kennes and King 2000 and Burdett, Shi and Wright 2001). A mixed strategy of a buyer $i = 1, 2, \ldots, M$ is a vector $\sigma^i = \left(\sigma_1^i, \sigma_2^i, \ldots, \sigma_N^i\right)$ where σ_j^i is the probability that buyer i visits seller j's auction. We will focus on symmetric equilibria in which $\sigma^i = \sigma^j$ for all $i \neq j = 1, 2, \ldots, M$. Suppose that the sellers are divided into an arbitrary number of mutually exclusive and completely exhaustive groups $G \in [2, N]$ according to some characteristic(s). Let N_g denote the number of sellers in group $g = 1, 2, \ldots, G$ and note that $\sum_{g=1}^{G} N_g = N$. Assume that the payoff to a buyer of winning a seller's auction is the same for every seller within a given group and denote this payoff by u_g.

Without loss of generality we can decompose the buyers' decision into a two-step process by which they first choose the group of sellers to visit and then choose which seller in the group to visit according to a probability distribution over that group. Let m_g be the probability that a buyer visits a seller in group g and let $\sigma_{g,i}$ denote the probability that a buyer visits seller i who is a member of group g, given that the buyer has chosen to visit a seller in group g. Note that $\sum_{i \in g} \sigma_{g,i} = 1$ and that $\sum_{g=1}^{G} m_g = 1$.

Within any given group g, in a mixed strategy equilibrium where all sellers are visited with strictly positive probability, the expected payoff to a buyer from visiting any seller must the same. Since a buyer gets u_g if alone at a seller in group g and 0 otherwise, we must have

$$(1 - \sigma_{g,i})^{m_g M - 1} u_g = (1 - \sigma_{g,j})^{m_g M - 1} u_g \text{ for all } i, j \in g,\ i \neq j.$$

This implies that $\sigma_{g,i} = \sigma_{g,j} = \sigma_g$ for all $i, j \in g$. Since $\sum_{i \in g} \sigma_g = 1$, we have $\sigma_g = 1/N_g$.

Now let us work out the probability distribution of buyers over sellers in a given group g. First, the probability that some subset of buyers of size $k \leq M$ all choose to visit group g is given by $m_g^k (1 - m_g)^{M-k}$. Then, given that all members of the subset of size k visit group g, the probability that exactly x of them visits a given seller is given by a binomial distribution with probability σ_g, that is,

$$\frac{k!}{x!\,(k-x)!} \cdot \sigma_g^x (1 - \sigma_g)^{k-x}.$$

To find the probability that $x \geq 0$ buyers visit a particular seller, we need to consider all possible subsets of buyers. For each k, there are $M!/\left(k!\,(M-k)!\right)$ subsets of size k. Summing over all possible k, we thus get the probability that exactly x buyers visit a particular seller as

$$\Pr(x) = \sum_{k=0}^{M} \left[\left(\frac{M!}{k!\,(M-k)!} \right) m_g^k (1 - m_g)^{M-k} \left(\frac{k!}{x!\,(k-x)!} \right) \sigma_g^x (1 - \sigma_g)^{k-x} \right]. \tag{13}$$

For auctions, we will be most interested in the probabilities that $x = 0$, $x = 1$, and $x > 1$. Let us define $\phi_g \equiv m_g M / N_g$, that is, ϕ_g is the ratio of buyers to sellers in the submarket consisting of sellers in group g. Thus, $M = \phi_g N_g / m_g$ for all g, and recalling that $\sigma_g = 1/N_g$, equation (13) gives the probability distribution of buyers over sellers as shown in table 3.

For readers who are unfamiliar with our matching technology, we now cover the basic results relevant for this paper. For further details, the reader is referred to Julien, Kennes and King (2000).

# Buyers	**Probability**
0	$\left(1 - \frac{m_g}{N_g}\right)^{\frac{\phi_g N_g}{m_g}}$
1	$\phi_g \left(1 - \frac{m_g}{N_g}\right)^{\frac{\phi_g N_g}{m_g} - 1}$
> 1	$1 - \left(1 - \frac{m_g}{N_g}\right)^{\frac{\phi_g N_g}{m_g}} - \phi_g \left(1 - \frac{m_g}{N_g}\right)^{\frac{\phi_g N_g}{m_g} - 1}$

Table 3: Probability distribution of buyers over sellers.

In this paper, for simplicity, we will work exclusively in the 'large market' case. To find the limiting probability distribution of buyers over sellers in a given group g we take the limits of the above probabilities as $N_g \to \infty$ while holding the group g buyer-seller ratio, ϕ_g, constant. This gives the probability distribution of buyers over sellers shown in table 4.

# Buyers	**Probability**
0	$e^{-\phi_g}$
1	$\phi_g e^{-\phi_g}$
> 1	$1 - e^{-\phi_g} - \phi_g e^{-\phi_g}$

Table 4: Probability distribution of buyers over sellers in a large market.

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