# Invention under uncertainty and the threat of ex post entry*

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**Abstract**

This paper proposes a theoretical framework for studying the invention of new products when demand is uncertain. In this framework, under general conditions, the threat of ex post entry by a competitor can deter invention ex ante. Asymmetric market power in the ex post market exacerbates the problem. The implications of these general results are examined in a series of examples that represent important markets in the computer industry. The first is a model that shows how an operating system monopolist, by its mere presence, can deter the invention of complements, to its own detriment as well as that of society. The implications of policies such as patent protection, price regulation, and mandatory divestiture are considered. Three additional examples consider the ability of a monopolist in one market to commit to bundling an unrelated product, a pair of horizontally differentiated firms that can add a new feature to their products, and a platform leader that can be challenged in its base market by the supplier of a complementary product. **JEL Classifications**: L12, L13, O31

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# 1 Introduction

Consider two firms, each interested in developing the same new product. Suppose that demand for the product is known, and the firms decide simultaneously whether or not to enter the market.[1] If monopoly profits are high, duopoly profits are low, and development costs are moderate, then there are two pure strategy equilibria, in each of which exactly one firm enters. If after the first firm enters, the firm that chose not to enter initially is given a second chance to enter, it will decline the opportunity.

This simple picture changes drastically when demand for the new product is unknown. In this paper I show that the introduction of demand uncertainty can lead both firms to forego entry even when development costs are moderate, leaving a promising market untapped. I develop a two-period framework for studying this problem in Section 2, and show that invention is foregone when a firm with moderately high entry costs can benefit ex post from free riding on the demand information revealed by the first mover. Because the first mover cannot internalize this informational externality, some invention that may be both privately and socially beneficial is foregone due to the threat of ex post entry by the free-riding firm. Though symmetric firms may also deter each other from investing, the problem is exacerbated when a firm with moderately high entry costs has an ex post selling advantage.

More precisely, if the first mover has entered the market and revealed high demand, a second mover may benefit from entering the market ex post, if its entry costs are outweighed by its prospective profits as a duopolist. Ex ante, the first mover recognizes it will earn either low monopoly profits if demand is low, or low duopoly profits if demand is high and the second mover enters ex post. Hence the first mover may not want to enter the market ex ante unless its entry costs are very low. As for the second mover, why does it not elect to become the first mover? Because demand is uncertain, so its expected profits are too low to justify incurring the entry costs, even though it would enter ex post if demand were known to be high. Though this reasoning was asymmetric, it can apply to symmetric situations, where the possibility that each firm could play the ex post role of the second mover deters the other from entering ex ante in its role as the first mover.

I apply these general results to four examples drawn from stylized facts in the computer software industry. The invention of software products is fraught with uncertainty, and complementarities among markets create asymmetries among firms. In Section 3, I examine a model with a base system monopolist that also competes in a complementary application market. The monopolist owns a selling advantage since consumers must purchase a base system in order to use the application. In this setting, the base system monopolist, by its mere presence, can depress the invention of complements to its own detriment. In Section 4,

[1] In this paper I use "development", "investment", and "entry" as synonyms; "invention" and "innovation" refer to the first act of investment in a particular product category.

I adapt the basic framework to three other examples of competition common in software markets: the bundling of software applications; the addition of new features to horizontally differentiated products; and an example inspired by the "divided technical leadership" hypothesis of Bresnahan and Greenstein (1999), in which the salient threats to a platform leader are complementors rather than new entrants. Section 5 discusses the results and reviews the related literature.

The main result—that invention may be foregone due to the threat of ex post entry—depends on two key forces. The first is the uncertainty inherent in the process of invention, which puts a wedge between ex ante and ex post incentives. The second is entry cost heterogeneity across firms, which implies that some firms may have an incentive to enter an empty product market while other firms will not enter until the level of demand has been revealed. A third force, asymmetry in the ex post selling stage, although it is not necessary for the result (as the example in Section 4.2 demonstrates), amplifies the problem because a high entry cost firm with a selling advantage has a greater incentive to free ride on the information externality that is generated when a low entry cost firm reveals demand.

In software markets, uncertainty takes two primary forms: technological uncertainty and demand uncertainty. Technological uncertainty stems directly from complexity. Although it may be simple to determine in theory whether a program can be written to accomplish a task, the amount of effort and coordination among programmers required to write the program is unknown. Demand uncertainty also has its source in the inherent complexity of computer software—potential consumers for a product that does not exist may have difficulty perceiving or articulating their needs for the product. And even after purchasing a software product, they may engage in what Bresnahan and Greenstein (1996) term "co-invention": the process of discovering how the software's capabilities interact with existing organizations and habits, and of applying the software to uses not foreseen by its inventors. In this paper all the uncertainty is on the demand side, but a similar result would obtain if demand were known but the technological uncertainty faced by an early entrant were more severe than the uncertainty faced by subsequent entrants.

Cost heterogeneity across firms is central to many of the results. In the model of Section 3, for example, if entry costs were identical across firms then there would be no reason for a base system monopolist to ever decline to enter a promising complementary application market, and as a consequence there would be no reason for an independent firm to ever contemplate entry. Firms might face substantially different entry costs for any of a host of reasons. There might be legal constraints, such as patents or antitrust concerns; there might be history-dependent effects, such as learning-by-doing or unique human capital retained by a particular firm; there might be consequences of past decisions regarding internal organization, financing, or employee compensation that are costly to change. Perhaps most importantly, firms of different sizes face different problems of providing their employees with incentives. However, invention can be foregone in contexts without cost heterogeneity, as in the model of Section 4.2.

Asymmetry in the ex post selling stage can arise due to complementarity: a firm with a monopoly over an existing product can enter a complementary market with a competitively low price, yet still profit by the accompanying increasing in demand for its monopoly product. In such cases, the monopolist's ex post advantage may be self-defeating ex ante, because it deters the invention of complements. Asymmetry can also arise as a consequence of substitutability: if a newly introduced product is complementary to a substitute for a firm's existing product, the firm may be forced to defend itself by developing a competing product (Section 4.3 gives an example).

The problem of foregone invention is fundamentally one of appropriability—when a firm cannot guarantee itself a monopoly over its invention, its incentives to invent are reduced. To solve the appropriability problem, governments commonly employ patent systems that guarantee a monopoly to each inventor. However, patents are socially desirable only when inventions would not otherwise be undertaken, and undesirable otherwise. In the models I present, under the same parameterizations for which there exist entry cost profiles that leave privately profitable opportunities unexploited, there are also cost profiles for which one or more firms undertake invention even in the absence of patents. More desirable would be a finely tuned patent system that awards a patent only when necessary to induce invention, but the information requirements of such a system are heavy: it would need to know the prior distribution of demand as well as the cost structures of all potential entrants. I discuss the applicability of patents briefly, but in the computer software industry patents are often ruled out as a matter of law (software application programs are usually not patentable, although particular aspects of an application program may be patentable) or because they conflict with patents for pre-existing technologies.

# 2 The basic framework

The framework consists of a two-period game, $t = 1, 2$, between two firms. There is an empty potential market with unknown demand, and either firm $j \in \{M, X\}$ ("M" for monopolist, "X" for alternative) may enter the market after expending an idiosyncratic entry cost $c_j \geq 0$. Firm $j$'s entry decision in period $t$ is written as $\text{in}_j^t$ or $\text{out}_j^t$. Demand in the market is revealed to both firms once any firm enters.[2] Entry decisions are made simultaneously at

[2] It is not necessary for the results that demand be revealed perfectly; it is enough that entry by the first firm reveals some information to the second firm. Indeed, if the model were expanded to allow each firm to privately observe a noisy signal of demand before period 1, then entry by the first firm would reveal that it had observed a signal of high demand. The second firm would then update its estimate of demand.

the start of each period; exit is not considered and entry costs cannot be recovered. In the basic framework, instead of modeling price and quantity decisions explicitly, I take per-period profits as the primitive element.[3] (The examples in later sections model price and quantity decisions in detail.) Let $\omega$ be the level of demand, which is initially unknown but whose probability distribution is common knowledge. The payoff for firm $j$ in period $t$ given demand $\omega$ is indicated by $\pi_j^t(s,\omega)$, where $s$ is the *market structure*, or the set of firms that have already entered. Rather than write $s$ in set notation, I adopt the shorthand $s \in \{\emptyset, M, X, MX\}$ for brevity. In the absence of entry ($s=\emptyset$), $\omega$ is irrelevant, so the payoff for firm $j$ is written simply as $\pi_j^t(\emptyset)$. All these details—including $c_M$ and $c_X$—are common knowledge.[4]

The extensive form of this game is shown in Figure 1. The probabilistic nature of $\omega$ is indicated by the choice of "Nature," which chooses at the nodes marked "$N$". Note that if neither firm enters in the first period, then neither of them is allowed to enter in the second period. I make this restriction to focus on the incentives for ex ante entry when there is a possibility of ex post entry. This restriction also allows the two-period framework to be mapped onto infinite horizon models, as in the later sections, in which each firm may enter at any time. In such models the incentives to enter are stationary; i.e., if invention is ever rational in any period, then it is rational in the first period.

Lemma 1, in the Appendix, derives the conditions under which neither firm enters the empty market in any subgame perfect equilibrium. Lemma 1 requires no structure on the profit functions, but to model realistic situations it helps to impose some assumptions on the structure of profits. The following assumptions are maintained for the remainder of this section and are satisfied by all the examples in later sections.

**Assumption 1** (Relevance). $\mathbb{E}\big[\pi_j^1(j,\omega)+\pi_j^2(j,\omega)\big] > \pi_j^1(\emptyset)+\pi_j^2(\emptyset)$ *for all* $j$.

**Assumption 2** (Common demand). $\omega \in [0,1]$ *and* $\pi_j^t(s,\omega)$ *is increasing in* $\omega$ *for all* $j \in s$.

**Assumption 3** (Competitiveness). $\pi_j^t(j,\omega) > \pi_j^t(MX,\omega)$ *for all* $j$*, all* $t$*, and all* $\omega > 0$.

**Assumption 4** (Positive profits). $\pi_j^t(MX,\omega) > \pi_j^t(-j,\omega)$ *for all* $j$*, all* $t$*, and all* $\omega > 0$.

[3] Differences in profits across time do not play an important role in the basic framework. However, expressing the profits as a function of time aids in mapping the framework onto applications.

[4] This framework includes, as a special case, the model of Jensen (2004), in which a large firm and a small firm can adopt a cost-reducing innovation of uncertain success, with asymmetric payoffs due to the presence of multiplant economies, but under the extreme assumption that any entrant in the first period earns zero profits in that period. Jensen (1992a, 2001) are also closely related, but allow entry in the second period if neither firm has entered in the first period. Both these papers focus on the identity of the first adopter, rather than on underinvestment. See Section 5 for a discussion.

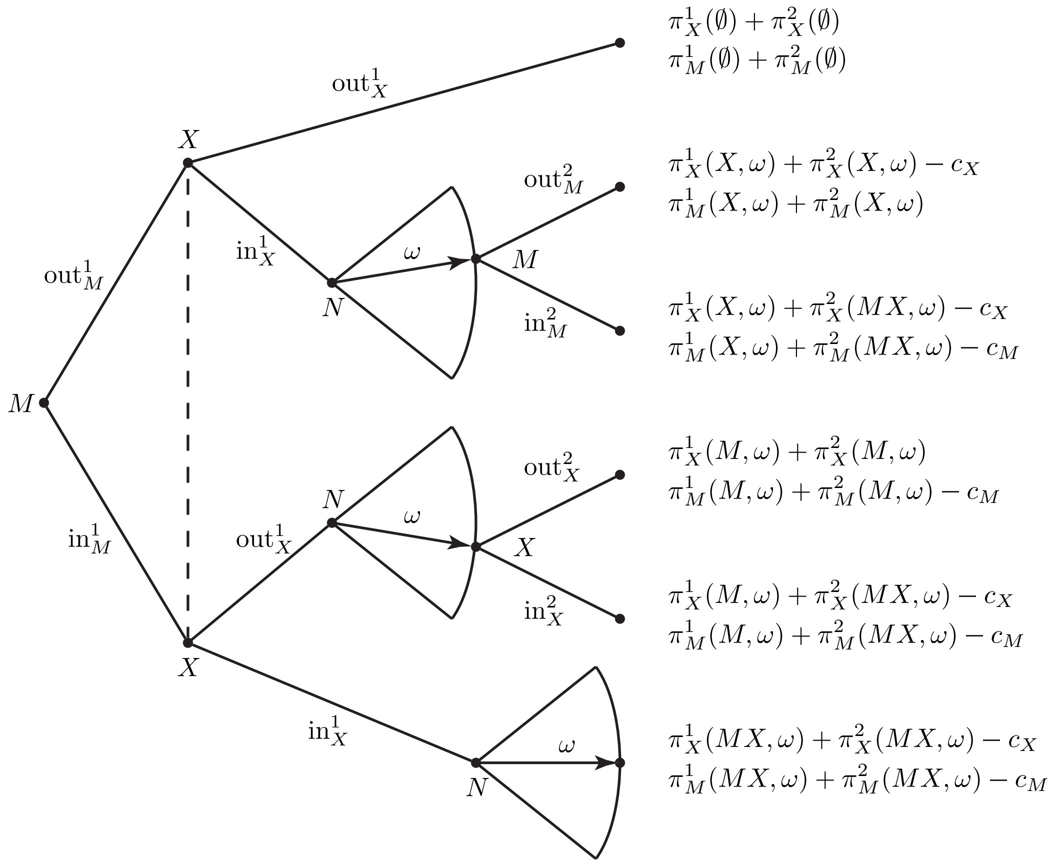


FIGURE 1: EXTENSIVE FORM OF THE BASIC FRAMEWORK

These assumptions define a class of interesting situations. If *relevance* holds, then the empty market offers enough profit potential that each firm would enter if its entry cost were sufficiently low and it knew that it would gain a monopoly. If *common demand* holds, then demand is characterized by a single parameter and an increase in that parameter leads to higher profits for every firm in the market. If *competitiveness* holds, then each firm prefers to monopolize the market rather than share it with a competitor. *Positive profits* means that revenues for any firm in the market will always cover its operating costs, so that the act of entry risks at most the entry costs.

An additional definition formalizes the idea that a firm faces a disadvantage in the ex post market.

**Definition 1.** Investments are **strategic substitutes** for firm $j$ if, for all $t$,

$$\mathbb{E}\left[\pi_j^t\left(MX,\omega\right) - \pi_j^t(-j,\omega)\right] < \mathbb{E}\left[\pi_j^t(j,\omega)\right] - \pi_j^t\left(\emptyset\right). \tag{1}$$

That is, Firm X (for example) gains less from investing when Firm M also invests than it gains from investing when Firm M does not invest. If Firm X's profits when it does not enter the market do not depend on Firm M's action (i.e., $\pi_X^t(M,\omega) = \pi_X^t(\emptyset)$ for all $\omega$ and all $t$), then competitiveness implies that investments are strategic substitutes for Firm X. Otherwise, that investments are strategic substitutes for Firm X implies that Firm M's entry does not depress Firm X's profits (e.g., in some related market) too much when Firm X does not invest. Investments can be strategic substitutes for both firms, but in the results that follow they need only be strategic substitutes for at least one firm—which I name Firm X.

When Firm X perceives investments as strategic substitutes, there are situations in which Firm X does not enter a promising market. Define $\hat{c}_j$ as the entry cost at which firm $j$ is exactly indifferent between investing and not investing in the absence of firm $-j$:

$$\hat{c}_j \equiv \mathbb{E}\left[\pi_j^1(j,\omega) + \pi_j^2(j,\omega)\right] - \pi_j^1(\emptyset) - \pi_j^2(\emptyset)\,. \tag{2}$$

If Firm M were for some reason unable to enter the market, then at entry cost $\hat{c}_X$ Firm X would be indifferent between entering and not entering. Since Firm M is able to enter ex post, there exists a threshold entry cost $\tilde{c}_M > 0$ below which it will indeed enter with positive probability once demand is revealed. Then Firm X's strict best response given entry cost $\hat{c}_X$ is not to enter. That Firm X perceives investments as strategic substitutes means that Firm X has less incentive to enter when Firm M enters, so if Firm M were to enter ex ante then Firm X's strict best response given $\hat{c}_X$ also would be not to enter. Since these strict best responses also hold in a neighborhood around $\hat{c}_X$, there are costs $c_X < \hat{c}_X$ at which, given $c_M < \tilde{c}_M$, Firm X does not enter in equilibrium even though it would strictly prefer to enter given $c_M > \tilde{c}_M$. Lemma 2, in the Appendix, makes this argument formally.

Combining the results for the individual firms reveals the conditions under which neither firm enters a promising market. It is helpful to visualize each firm's optimal first period entry decision as a function of its entry cost profile, as is shown in Figure 2A for Firm X. The region in which Firm X does not enter the market solely due to the possibility of entry by Firm M covers intermediate values of $c_X$ and low values of $c_M$. Overlaying the regions in which Firm M and Firm X do not enter in the first period, as is shown in Figure 2B, reveals a region of concern in which neither firm enters the market even though at least one firm would enter if it could be assured of a monopoly. I call this the region of *foregone invention.*

**Definition 2.** A **region of foregone invention** is a set of cost vectors $\{(c_2, c_2)\}$ with positive measure[5] for which (i) no firm enters in any subgame perfect equilibrium, (ii) but at least one firm would enter in a subgame perfect equilibrium if it could be assured a monopoly.

[5]Lebesgue measure in $\mathcal{C}_A \times \mathcal{C}_B$, where $c_j \in \mathcal{C}_j = \mathbb{R}_+$.

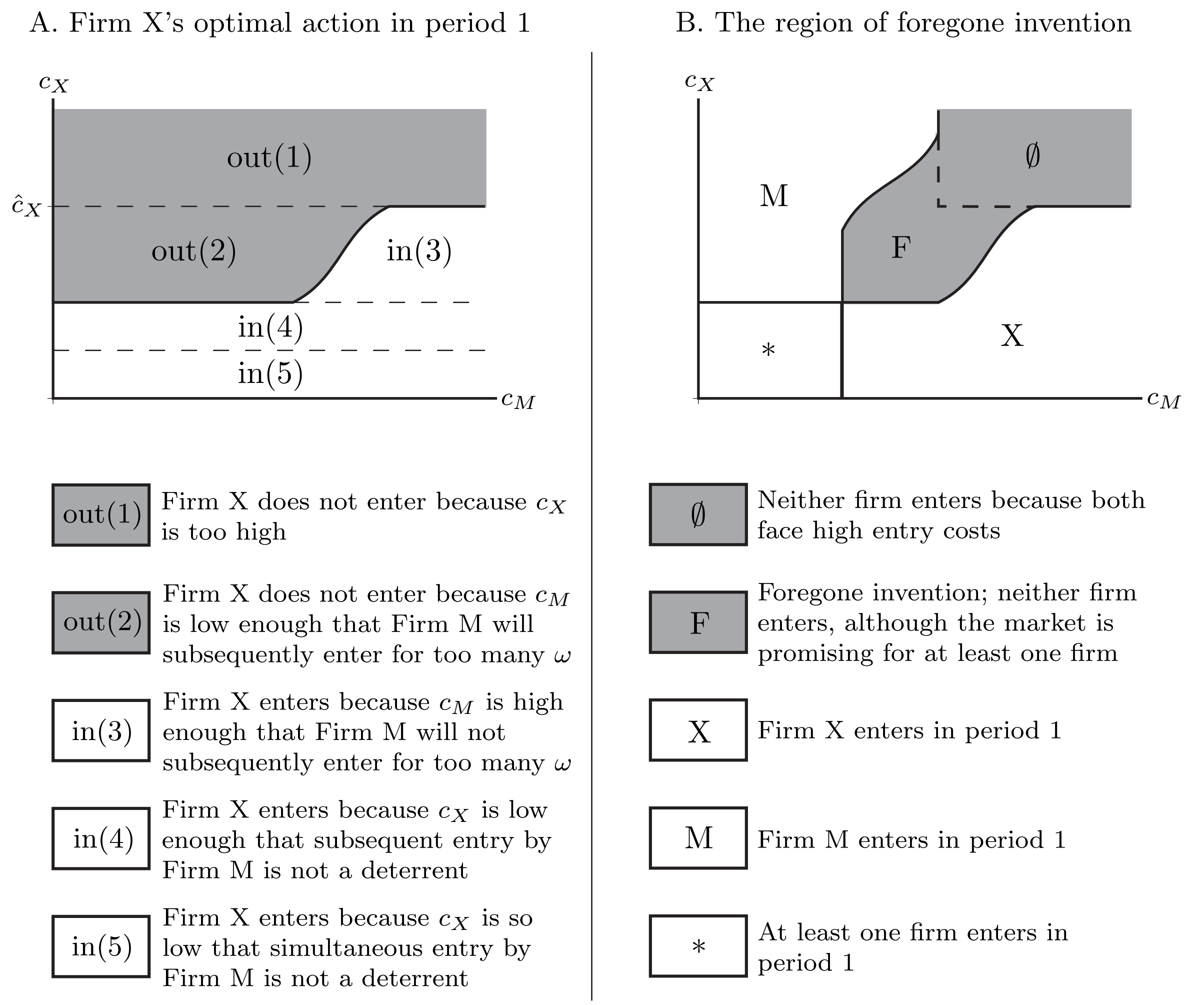


Figure 2: The region of foregone invention in the basic framework Each graph displays the cost space, with $c_M$ on the horizontal axis and $c_X$ on the vertical axis.

It is important to note that when the parameters of the game fall in the region of foregone invention, there does not exist any equilibrium in which entry occurs. This rules out not only pure strategy equilibria in which one or both firms enters for sure, but also mixed strategy equilibria in which one or both firms enters with probability less than one. Hence, in such situations, invention is foregone with certainty.

The region of foregone intervention covers an area of intermediate entry costs, because very high entry costs or very low entry costs make the deterrence of invention either difficult or moot. More specifically, if Firm X has very low entry costs it is not likely to be deterred by the prospect of Firm M's entry, while if Firm M has very high entry costs it is less likely to enter and thus poses less of a threat to Firm X. On the other hand, if Firm X has very high entry costs then it is unlikely to enter even without the threat of Firm M's subsequent entry, while if Firm M has low entry costs then it is likely to enter ex ante regardless of whether it deters Firm X. It is quite possible that these effects that operate against foregone invention may overlap, in which case the region of foregone invention does not exist. Figure 2B suggests that the region of foregone invention will have positive measure if the curves that divide entry from non-entry for each firm do not intersect at $(\hat{c}_M, \hat{c}_X)$. In fact this is the case, as is implied by the following proposition.

**Proposition 1.** *Let $s_j^*(\omega, c_j)$ indicate the market structure that results from Firm j's sequentially optimal choice after Firm $-j$ enters and reveals $\omega$:*

$$s_j^*(\omega, c_j) = \begin{cases} MX & \text{if } \pi_j^2(MX, \omega) - c_j > \pi_j^2(-j, \omega) \\ -j & \text{otherwise.} \end{cases} \tag{3}$$

*In the basic framework, if investments are strategic substitutes for Firm X and*

$$\mathbb{E}\left[\pi_X^2(X, \omega)\right] > \mathbb{E}\left[\pi_X^2\left(s_M^*(\omega, \hat{c}_M), \omega\right)\right], \tag{4}$$

*then the region of foregone invention has positive measure.*

Eq. 4 requires that Firm M has an ex post incentive to enter when its entry cost is high enough that it does not enter ex ante. The proof (in the Appendix) uses Assumptions 1–4 and Eq. 4 to establish that $(\text{out}_M^1, \text{out}_X^1)$ is an equilibrium at entry cost profiles near $(\hat{c}_M, \hat{c}_X)$, while Firm X's perception that investments are strategic substitutes rules out the possibility that $(\text{in}_M^1, \text{in}_X^1)$ is also an equilibrium.[6]

Under what conditions is Eq. 4 likely to be satisfied? The left hand side is high when Firm X's monopoly profits in the new product market are likely to be high; the right hand side is low when Firm X's duopoly profits are likely to be low and when Firm M is likely to enter ex post. Thus the condition is more likely to be satisfied if Firm M has an advantage over Firm X in the ex post market, since Firm M is more likely to enter ex post if its duopoly profits are likely to be high. Although there are symmetric conditions that can lead to foregone invention, symmetry increases or decreases both firms' duopoly profits in tandem, leading to counteracting effects on the right hand side of Eq. 4. But as Firm M grows stronger ex post at the expense of Firm X, ex post entry becomes more likely and Firm X's expected profits fall unambiguously. Hence foregone invention is a more serious problem when ex post market power is asymmetric. In subsequent sections I explore several

[6]These are sufficient, but not necessary, conditions for foregone invention. It is possible for invention to be foregone when Eq. 4 does not hold, such as when the curves that divide entry from non-entry for each firm intersect at $(\hat{c}_M, \hat{c}_X)$ but also intersect at other points.

examples of asymmetric market power, in which the asymmetry derives from one firm's market power in a related market.[7]

In sum, the basic framework provides a convenient characterization of situations in which invention may be foregone. The conditions that lead to foregone invention are two: one firm must perceive ex ante investments as strategic substitutes, and the other firm must have an ex post incentive to enter when it would not enter ex ante (Eq. 4).[8] The region (in entry cost space) of foregone invention, if it exists, occupies entry cost profiles with intermediate entry cost levels for both firms. There is more likely to be a significant region of foregone invention when ex post market power is asymmetric, because one firm is more likely to be deterred from investing if the other firm has an ex post advantage. Because they are based on reduced form payoff functions, these insights can be applied to a wide range of market situations, including infinite horizon problems, as shown in the sections that follow.

## 3 Invention to complement a base system

In this section, I show that a computing platform with a base system monopolist is liable to experience foregone invention. The problem is that the base system monopolist possesses an ex post advantage in markets for complementary products since it can enter a complementary market with a low price yet still benefit from the increased demand for its base system. In this "Base System" model, Firm M has a pre-existing and exogenously protected monopoly in the market for software operating systems, while the empty market is for a software application that requires Firm M's base system in order to run. Firm X is a startup firm that can enter the empty market. The asymmetric complementarity between the base system and the application amplifies the problem of foregone invention, to the potential detriment of both firms as well as to social welfare.

Although designed to represent aspects of the computer software industry, the model applies to any set of markets in which one product (a "base system") is required for the operation of others ("applications") and in which there is potential for the invention of new applications. Gawer and Henderson (2002) and Gawer and Cusumano (2002) pose an example in the computer hardware industry: Intel, given its dominant position in the market for microprocessors, has an incentive to enter complementary markets because it can internalize

[7]There are other possible sources of asymmetry, such as asymmetric operating costs. For example, in a two period model with Cournot competition the following parameterization yields a region of foregone invention with positive measure: demand $Q = \omega (10 - p)$, uncertainty $\omega = 1$ with probability $\gamma$ and $\omega = 0$ otherwise, and marginal costs of 0 for Firm M and 3 for Firm X.

[8]This is why no invention is foregone in the absence of demand uncertainty: without demand uncertainty any firm that would enter ex post would also enter ex ante. Note that if entry decisions are made literally simultaneously, then there can be a mixed strategy equilibrium, but this equilibrium is not robust to the possibility that one firm or the other might be able to make its entry decision just a little bit earlier.

the complementarity. But Intel also benefits when other firms innovate in these areas, because such innovations increase the market for Intel's core products. When both Intel and another firm are aware of an opportunity to open a new market, foregone invention may be a problem if either demand in this market is unknown (as in the model), or, equivalently, if the technological viability of the product is unknown. Intel has developed organizational structures that may help it commit not to "trample all over everybody's business" (Gawer and Henderson), and it has succeeded in keeping most of its efforts focused on its core business. However, Intel has entered a number of already-occupied complementary markets, including videoconferencing, digital cameras, chipsets, motherboards, home networking, and network adapters. The model in this section gives a theoretical rationale for the contrast between Intel's stated concern for complementors and its apparent inability to fully commit not to behave aggressively toward them.

Naturally, another application of the model is to Microsoft's dominant position in the market for personal computer operating systems and the many applications that can run on top of its Windows operating systems. Microsoft's ex post success in the markets for Windows applications—internet browsers, e-mail clients, media players, spreadsheets, word processors, etc.—illustrates the strength of its ex post advantage. Since the foregone invention result predicts the non-existence of certain products, this variety of observed products raises the question of whether the model's predictions are falsifiable. However, the steadily falling development costs that characterize computer technology imply that the development costs for any given application will eventually leave of the region of foregone invention. Accordingly, the results in this section suggest that a market structure with a base system monopolist may pose hazards that retard innovation if, among the various potential products that can work with the base system, there are some for which independent firms have lower development costs than the monopolist.

### 3.1 The model

**Technology and firms** Let $\{b, a\}$ be the set of possible products—$b$ for "base system" and $a$ for "application"—neither a durable good.[9] The base system has already been developed and is produced by Firm M, while Firm X is exogenously barred from the base system market. Either Firm may enter the application market by incurring a fixed cost $c_j > 0$. The application requires the base system in order to provide value, but the base system can provide value without the application; i.e., the base system is "essential." One way to interpret this assumption is that it represents the potential for users to program their own

[9]Durable goods considerations in a dynamic oligopoly setting, as well as other similar concerns such as network effects, are beyond the scope of this paper. But the results in Section 2 indicate that the phenomenon of foregone invention is general in the set of all models that satisfy the conditions that Proposition 1 places on the profit functions, and this set can accommodate these concerns.

software to run on the base system. More broadly, it reflects the ability of the base system to serve as a platform upon which any number of other software products can run.[10]

**Markets and consumers** Once firms have had the opportunity to develop the application, they simultaneously set prices, produce at zero cost any products that they have developed, and sell to consumers. The set of branded products available for sale to consumers is $s \subset \{b, a\} \times \{M, X\}$, where, for shorthand, $k_j \in s$ is product $k$ sold by firm $j$ at price $p_j^k$. The set $s$ also represents the "industry structure"—the combination of firms present in each market. For brevity, $s$ will often be written in shorthand as $\emptyset$, $M$, $X$, or $MX$ to indicate which which firms have entered the application market.

There is a continuum of consumers with unit total mass, and each consumer $i$ values product $k$ by the amount $\theta_i^k \in [0, 1]$.[11] Each consumer demands at most one of each product in any period. In each period, each consumer $i$ purchases a basket of branded products $s_i \subset s$ to maximize quasilinear utility: $\sum_{k_j \in s_i} (\theta_i^k - p_j^k)$.[12] Since a consumer must own the base system in order to make use of an application, utility maximization is subject to the constraint that if $s_i$ is non-empty then it must contain $b_M$. Consumer values $\theta_i^b$ are distributed uniformly on $[0, 1]$, while, prior to the first period, demand in the application market is unknown. With probability $\gamma$, $\theta_i^a$ are distributed uniformly on $[0, 1]$; otherwise $\theta_i^a = 0$ for all $i$.

**Dynamics** The profits for each firm in each type of market constitute a payoff function that can be plugged into the basic framework. This is accomplished by looking at the problem through the lens of an infinite period model[13] in which the firms share a discount factor $\delta \in (0, 1)$, and letting the second period of the basic framework summarize the discounted profits in periods $2, \ldots, \infty$. Since $s = MX$ is an absorbing state, the equilibrium can still be found by backward induction. As in the basic framework, each period $t$ consists of two stages, an investment stage and a selling stage. Once any firm enters the application

[10]This assumption contrasts with several earlier models in the literature which assume that both products are used in fixed proportions, as in Economides and Salop (1992), Matutes and Regibeau (1998), Farrell and Katz (2000), and Choi and Stefanadis (2001) However, the asymmetry that is inherent when applications require a base system in order to run provides Firm M with an ex post advantage that cannot be derived under a fixed proportions assumption. Heeb (2003) is a recent theoretical work that shares the assumption of asymmetric complementarity.

[11]This implies that applications $a_M$ and $a_X$ are perfect substitutes. Adding a measure of product differentiation could quantitatively change the problem of foregone invention, but would not alter the qualitative conclusions.

[12]If software is durable, then the measure of consumers in the market can be thought of as those who are ready to upgrade. Since this Base System model does not allow for incremental improvement, the idea is that upgrading is motivated by an exogenous baseline rate of improvement.

[13]The infinite horizon provides stationary payoffs, whereas a two period model would force payoffs in the second period to be perceived differently from the perspectives of the first period and the second period.

market, demand is revealed to all. Though the context is dynamic, I assume that market pricing in each period reflects the static Nash equilibrium of a simultaneous pricing game; collusive pricing is considered in Section 3.4.1.

The state vector at the start of each period consists of the sets of products that have been developed by the firms together with the common prior distribution over demand for the application. The state can be summarized in shorthand (and with some abuse of notation) by the pair $\langle s, \omega \rangle$, where $s \in \{\emptyset, M, X, MX\}$, $\omega = 1$ if demand for the application is known to be high, $\omega = 0$ if demand for the application is known to be low, and $\omega = \gamma$ if demand for the application is yet unknown.

## 3.2 Static prices and profits

This section characterizes the static market equilibrium of the Base System model under the various industry structures that can arise. These static outcomes enable us to analyze the dynamic equilibrium in Section 3.3. The paragraphs that follow display the static equilibrium prices and profits for each firm under each industry structure in turn. The derivations of these static equilibria are straightforward and thus omitted. Consumer choices are shown in Figure 3, where the space of consumer valuations $[0, 1]^2$ is divided into regions according to which consumers purchase which baskets, with $\theta^b$ on the horizontal axis and $\theta^a$ on the vertical axis. Observe that whenever Firm M offers the application, assuming that it prices its two products separately is without loss of generality, since offering a bundle would be equivalent to setting a price of zero for the application. I call this "trivial bundling" when it arises in equilibrium.

**Base system monopoly** When there is no application (i.e., $\omega$ is either $\gamma$ or 0), Firm M faces a simple monopoly problem. It sets its price for the base system at $p_M^b = 0.5$, yielding per-period profits of $\pi_M(\emptyset) = 0.25$. Consumer choices are displayed in Figure 3A.

**Integrated monopoly** An integrated monopoly arises when Firm M monopolizes both markets (with high demand for the application, i.e., $\omega = 1$). In this case Firm M sets prices $p_M^b = 0.667$ and $p_M^a = 0.167$, resulting in per-period profits of $\pi_M(M, 1) = 0.546$.[14] Consumer choices are displayed in Figure 3B.

The price for the application is relatively low because the application makes the base system more valuable, allowing Firm M to price the base system well above the price it would set as merely a base system monopolist. This illustrates how Firm M internalizes the complementarity, yielding benefits for consumers as well as itself.[15]

[14] For simplicity, I use notation as if three-digit approximations are exact.

[15] Note that consumer surplus also increases over the base system monopoly, although it is not a Pareto

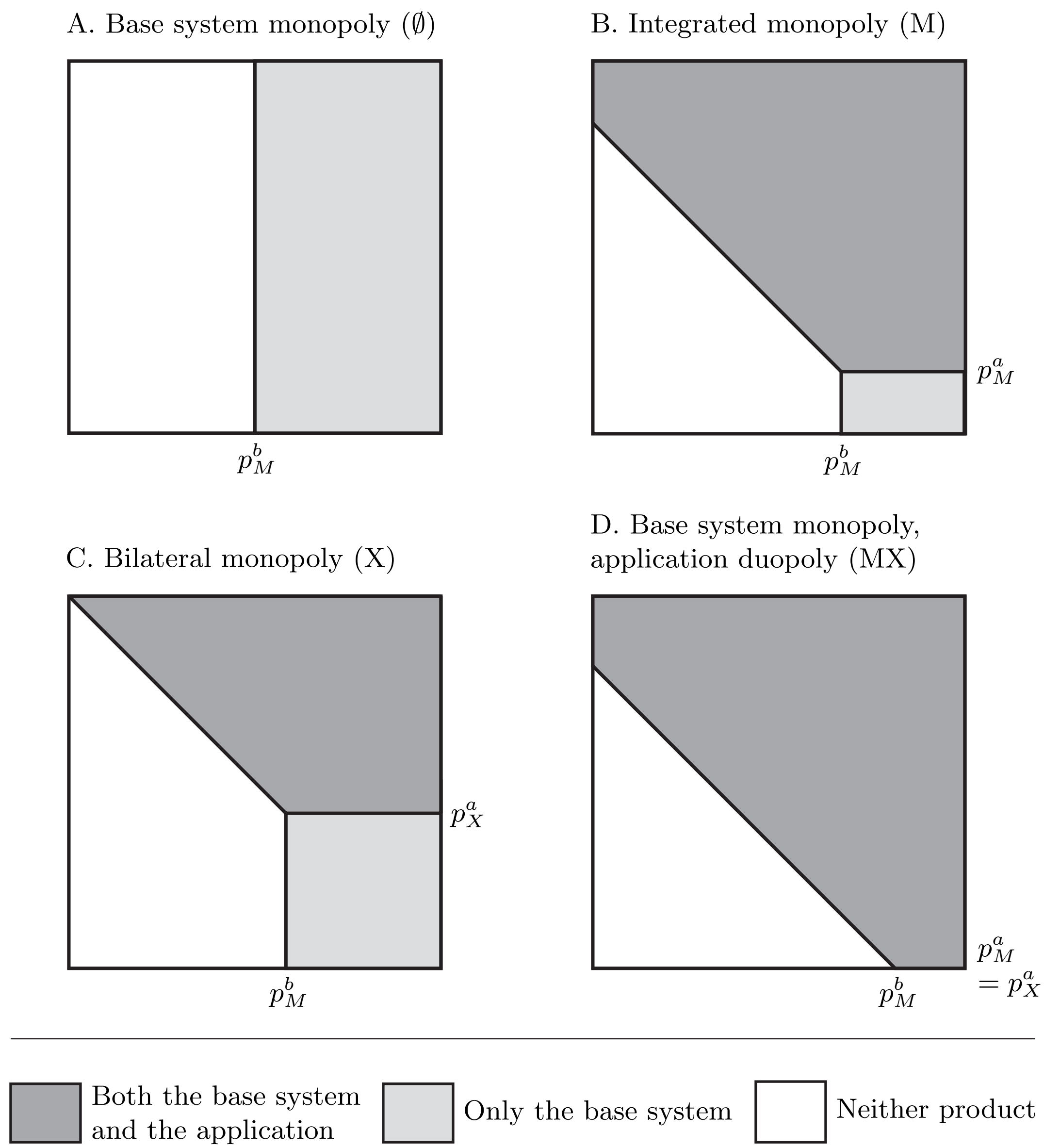


FIGURE 3: CONSUMER PURCHASES AS A FUNCTION OF TASTES IN THE BASE SYSTEM MODEL Each graph displays the space of consumer tastes, with $\theta^b$ on the horizontal axis and $\theta^a$ on the vertical axis. Regions are shaded according to which consumers purchase which products.

**Bilateral monopoly** A bilateral monopoly arises when Firm M monopolizes the base system market and Firm X monopolizes the application market (with $\omega = 1$). The firms set equilibrium prices of $p_M^b = 0.586$ and $p_X^a = 1 - p_M^b$. The per-period profits are $\pi_M(X, 1) = 0.343$ and $\pi_X(X, 1) = 0.172$. Consumer choices are displayed in Figure 3C.

The price for the application is higher than under an integrated monopoly because Firm X cannot reap the positive externality that its application provides to Firm M. Thanks to this externality, Firm M can raise the price of its base system relative to the price it would set in the absence of the application, but not to the level it would set as an integrated monopolist. Both total industry profits and consumer surplus are lower than under integrated monopoly, illustrating the social consequences of Firm X's inability to internalize the complementarity. Despite the essentiality assumption maintained here, the contrast between bilateral monopoly and integrated monopoly in this model follows the same logic as in Cournot (1838) and other static models such as Economides and Salop (1992) and Chen and Ross (1998).

**Application duopoly** When Firm M monopolizes the base system market but competes with Firm X in the application market (with $\omega = 1$), both firms set a price of zero for the application, while Firm M sets a price of $p_M^b = 0.816$ for the base system. The resulting per-period profits are $\pi_M(MX, 1) = 0.544$ and $\pi_X(MX, 1) = 0$. Consumer choices are displayed in Figure 3D.

Firm M trivially bundles its application with the base system, forcing Firm X to sell its application at a zero price. By this strategy, Firm M achieves profits nearly as high as if it enjoyed an integrated monopoly. It is akin to the strategy of bundling demonstrated by Nalebuff (2000), except that here the bundling arises as a result of competitive pricing in the application market and monopoly pricing in the base system market. This illustrates the power of a base system monopoly. Ordinarily, no firm would enter a pre-existing market with undifferentiated Bertrand competition if in order to do so it would have to pay a positive entry cost. But Firm M benefits from selling the application even at a zero price because by doing so it increases the price of its base system. As with integrated monopoly, both total industry profits and consumer surplus are higher than under bilateral monopoly, so from an ex post perspective Firm M's entry into the application market is both privately and socially desirable so long as its investment costs are not too high.

improvement because some consumers who would buy the base system under a base system monopoly are priced out of the market under integrated monopoly.

### 3.3 Dynamic equilibrium

Although the game is constructed with an infinite horizon, it can be solved by backward induction from the absorbing states. The analysis is equivalent to that of the basic framework. Recall that Proposition 1 gave the conditions for foregone invention in the two period basic framework. Proposition 2 applies that result to this infinite horizon model.

**Proposition 2.** *In the Base System model, if the prior distribution over demand is sufficiently pessimistic ($\gamma < 0.679$) then the region of forgone invention has positive measure.*

To gain a more economic interpretation of this result, it is helpful to look a little more closely at the conditions that define the region of foregone invention. I first identify three effects that drive Firm M's investment incentives, and then analyze the equilibrium outcome as a function of the entry cost profile. This helps visualize the region of foregone invention and identify two effects that summarize how Firm X responds to Firm M's incentives. Begin with Firm M's entry rules. When demand for the application is high, Firm M chooses to invest ex post if the gain from investing more than offsets the fixed investment cost:

$$c_M < \frac{1}{1-\delta}\big(\pi_M(MX,1) - \pi_M(X,1)\big). \tag{5}$$

Note that the right hand side may be large if Firm M can profit in its base system market by trivially bundling the application. Compare this to Firm M's ex ante entry rule when Firm X is unable to invest: if $c_X = \infty$ then Firm M enters if

$$c_M < \frac{\gamma}{1-\delta}\big(\pi_M(M,1) - \pi_M(\emptyset)\big) = \hat{c}_M. \tag{6}$$

Most of the difference between Eq. 5 and Eq. 6 is due to two effects: First, if Firm X has already invested and revealed high demand, Firm M receives a higher payoff even if it does not invest because the availability of Firm X's application makes Firm M's base system more valuable; i.e., $\pi_M(X,1) > \pi_M(\emptyset)$. This *complementarity effect* reduces the incentive for Firm M to invest. Second, if Firm X has already revealed that there is high demand for the application, then Firm M's expected payoff from investing is higher, increasing its incentive to invest. This *demand revelation effect* is reflected in the absence of the uncertainty term $\gamma$ from from Firm M's ex post entry rule (Eq. 5). A potential third effect, the *competitive effect*, stems from the prospect of competition in the application market. It exerts almost no influence on Firm M's decision because Firm M can achieve almost as much profit by trivial bundling against Firm X as it can by monopolizing both market. The demand revelation effect swamps the competitive effect except when $\gamma$ is extremely close to 1. The demand revelation effect outweighs both the complementarity effect and the competitive effect when

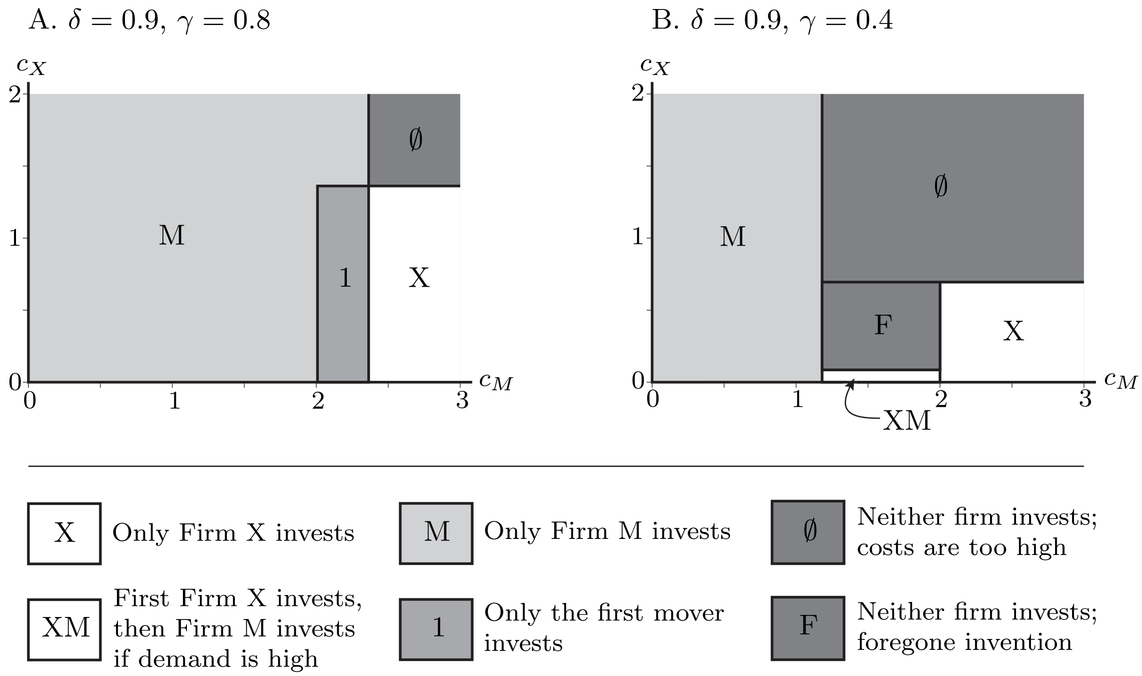


Figure 4: Equilibrium outcome as a function of costs in the Base System model Each graph displays the cost space, with $c_M$ on the horizontal axis and $c_X$ on the vertical axis. Regions are shaded according to which firm invests first, and labeled according to the type of equilibrium outcome.

$\gamma < 0.679$.

To graph the equilibrium outcome, suppose for a moment that $\delta = 0.9$, and consider the two cases $\gamma = 0.8$ and $\gamma = 0.4$. When $\gamma = 0.8$, equilibria in which Firm X and Firm M invest ex ante share a region of overlap. This is illustrated in Figure 4A, where the region of overlap is labeled as the region of first mover advantage.[16] Since $\gamma = 0.8 > 0.679$, there is no region of foregone invention. Note that even if the firm with lower costs is the first mover, there is still productive inefficiency (conditional on some firm investing, it is the high–entry cost firm that invests) over a significant range of entry costs. This is a natural consequence of entry cost heterogeneity combined with Firm M's internalization of the complementarity.

When $\gamma = 0.4$, the regions in which Firm X and Firm M may invest ex ante do not overlap, and a region of foregone invention fills the gap: in equilibrium, neither firm invests even in situations in which investment could be privately profitable under non-equilibrium

[16]There are some regions of the parameters in which either Firm M or Firm X, but not both, could invest in pure strategy equilibrium. This implies the existence of an unstable mixed strategy equilibrium. However, this mixed strategy equilibrium is not robust to a small first-mover advantage for either firm. Accordingly, I rule out mixed strategy equilibria and merely indicate that only the first mover will invest, while remaining agnostic regarding the identity of the first mover.

strategies. This is illustrated in Figure 4B. There are three factors that contribute to the region of foregone invention, all of which are necessary in the Base System model. First, the threat of encroachment: Firm X's entry costs are high enough that it doesn't want to invest if Firm M will subsequently invest (when demand for the application is high). Second, demand uncertainty: Firm M's entry costs are high enough that it invests only if it knows that demand for the application is high. Third, for $\gamma = 0.4$ the demand revelation effect outweighs the complementarity and competitive effects, leading Firm M to invest if Firm X has revealed that demand for the application is high.

In general, the region of foregone invention reaches its maximum area at an intermediate level of uncertainty, $\gamma = 0.340$. This occurs for two reasons: First, if demand is less likely to be high, then entry would be relatively unlikely even if foregone invention were not a problem, so foregone invention can occur over only a small range of entry costs. Second, if demand is more likely to be high (but still $\gamma < 0.679$), then ex post incentives are more in line with ex ante incentives. For any fixed $\gamma \in (0, 0.679)$, the region of foregone invention grows with increasing $\delta$. Although greater patience gives firms a greater incentive to invent new products, it does not help align ex ante incentives with ex post incentives, so the range of entry costs over which invention is foregone increases.

## 3.4 Strategies to encourage invention

The region of foregone invention is troubling for social welfare: opportunities to expand social welfare by inventing new products may go unexploited even though they offer potential for private profit. It is also troubling for Firm M, because it means that opportunities to expand its core business may go unexploited. Under the assumption that, of the various application development opportunities that may arise, each poses its own characteristic profile of entry costs for Firm M and any potential Firm X's, both Firm M and a social regulator might be interested in policy strategies for trying to overcome the problem of foregone invention. In this section I first consider how Firm M might try to encourage the invention of complements. I then consider the effects of several public policy interventions that a social regulator might consider. I interpret these policies informally, both within the model and in the context of some considerations outside the model. In general, I do not find any policy that yields an unambiguous improvement, unless the policy maker is assumed to have unrealistically detailed information about market conditions.

### 3.4.1 Private strategies for encouraging invention

There are several strategies that Firm M might pursue in order to attempt to overcome the problem of foregone invention. For example, it can merge with outside inventors like Firm X, it can sponsor a research tournament, it can attempt to collude in the ex post market, or it can attempt to coordinate on a long run equilibrium in which it commits not to enter certain types of markets.

**Mergers** From both a social standpoint and from the firms' perspective, a situation in which the application is first invented by the firm with lower costs and then marketed by Firm M is usually preferable to a situation in which Firm M invests to enter an application market already occupied by Firm X. When Firm X has lower entry costs, there are two classes of simple contractual remedies: one, Firm M buys Firm X (or the rights to sell its application) ex post; two, Firm M buys Firm X ex ante. I consider each in turn.

Suppose that Firm M can offer to buy Firm X ex post. For the moment, restrict attention to situations in which $c_M$ is in an intermediate range for which invention may be foregone. Once Firm X has revealed that demand for the application is high, Firm M prefers to buy Firm X rather than pay the investment cost to compete in the application market. However, since Firm M's entry costs are relatively low, it can credibly threaten to enter the application market on its own, giving Firm X an outside option of $\pi_X(MX, 1) = 0$. Firm M's influence over Firm X's outside option leads to a low buyout price, making it difficult for ex post merger to solve the ex ante incentive problem.

When instead Firm M's entry costs are high enough that it will not enter ex post, it offers Firm X a buyout price at least as high as Firm X's outside option, $\frac{1}{1-\delta}\pi_X(X, 1)$. Though this has the helpful effect of increasing the range of $c_X$ over which Firm X will invest when Firm M poses no threat to enter the application market, the prospect of ex post merger actually increases the region of foregone invention, because it increases the maximum entry cost at which Firm X could profit from a monopoly more than it increases the maximum entry cost at which Firm X would invest when Firm M can threaten to enter.

If the firms could merge ex ante, then they could jointly internalize the complementarity and eradicate the problem of foregone invention. There are two important reasons to think that merging ex ante would be problematic, however. First, if entry costs were private information then Firm M would face an adverse selection problem in identifying which firms to choose as merger partners. In return for merging with Firm M, many firms might offer their development services even if their development costs were not low, since after the merger they would be able to "hold up" Firm M and compel it to pay higher development costs than it had anticipated. Second, even if the correct Firm X could be identified, the transaction costs of the merger might outweigh the difference in entry costs between the two independent firms. Such transaction costs could include not only the accounting costs of the merger, but also the effect on the incentives of the software engineers of moving from a small firm to a larger firm.

**Research tournaments** Firm M could attempt to solve the adverse selection problem posed by ex ante mergers by offering a tournament prize that independent firms could win by developing the application. The prize, of course, would need to be large enough that the desired contestants enter the tournament even though there is a strong likelihood of losing. There is also the difficulty of identifying the correct application to invent, and furthermore writing an appropriate contract for such a tournament would be difficult. Since software applications are complex objects, any contract that does not completely specify the desired product will give the contestants distorted incentives that may lower the quality of the winning application (which can be interpreted as raising the costs that Firm M must incur to perfect the application after paying out the prize). For the specific setting of computer software, the mere act of writing a substantive tournament contract could require accomplishing a significant portion of the development work.

**Collusion** Firm M and Firm X, having both introduced competing applications, could collude to keep the prices of their applications above zero. Joint profit maximization implies that $p_M^b = 0.667$ and $p_M^a = p_X^a = 0.167$, yielding profits per period of 0.495 for Firm M and 0.051 for Firm X if they split the application market equally. However, Firm M can actually earn higher per period profits ($\pi_M(MX, 1) = 0.544$) by driving the application price to zero. Collusion on positive application prices is possible only if Firm X restricts its output to much less than half of the total number of applications. This would leavs Firm X with little in the way of profits, and offers little inducement to innovate in situations under which Firm M will subsequently enter and collude.

**Building a reputation for enabling complements** The Base System model considers a single application, but a fundamental characteristic of software operating systems is that their functionality is open-ended. If a new potential application market arises in each period, we might expect there to be an equilibrium in which Firm M does not enter certain complementary markets already occupied by independent firms, because the independent firms will cease innovating if they ever observe Firm M enter a forbidden complementary market. Gawer and Cusumano (2002) theorize that Intel has pursued the strategy of attempting to coordinate on such an equilibrium, characterizing its strategy as "communicating commitment to third parties." However, extending the model to allow for new applications each period is problematic because it generates a dynamic game with an infinite state space, for which there is no folk theorem to invoke and for which static prices and profits are difficult to compute, even numerically.[17]

[17] It can be shown that if total expenditure in the markets for applications and base systems is bounded, then Firm M cannot commit to enabling complements even if the number of applications is unbounded.

### 3.4.2 Public policies for inducing invention

This section considers several potential policy interventions: a property rights approach that gives Firm X a patent if it innovates first, a structural regulation approach that forces Firm M to divest from any applications it invents, and a price regulation approach that prevents Firm M from driving application prices down. Throughout, I assume that the policy planner cannot observe firms' investment costs.

**Patent protection** The appropriability problem facing Firm X can be completely eliminated by a simple patent policy. Unfortunately, though patent protection eliminates the problem of foregone invention, it poses other problems. In general, since patent protection prevents competition, it can reduce social surplus whenever it is not necessary to induce investment. This phenomenon manifests in at least three specific ways: First, the costs of reduced competition are evident in the Base System model whenever Firm X's entry costs can be so low that Firm M's subsequent entry does not dissuade Firm X from innovating ex ante. In these situations, it is socially desirable ex ante for Firm M to enter a market occupied by Firm X, but a patent policy prevents it from doing so. Second, though the Base System model does not allow for vertical differentiation, a lack of competition can be particularly problematic in markets in which incremental vertical (quality) improvement is important because competition can induce firms to invest more in improving quality. Heeb (2003) shows that Firm M is actually the stronger incremental innovator in such a situation, so long as Firm X is not induced to exit. Third, the Base System model does not allow for horizontal differentiation, and in markets in which consumers have varying horizontal tastes it may be beneficial to have multiple vendors to satisfy different types of consumers. Whether a patent policy is socially beneficial in any given market is subject to the balance between these three effects on the one hand and the value of protecting innovators on the other hand.

In the case of computer software, the issue of patent protection is often moot, because an attempt to patent an entire software application could be blocked by any firm that holds a patent on some aspect of the underlying technologies included in the application. For example, many contemporary applications make use of patented compression, encryption, and networking technologies. In particular, base system vendors like Microsoft and Intel tend to hold many of these sorts of patents—enough to potentially head off attempts by

If instead the useful lifetimes of applications are uniformly bounded, then the folk theorem of Dutta (1995) applies. However, the idea that applications have uniformly bounded lifetimes should be applied with caution. Although a particular version of a particular application (e.g., Microsoft Word 5.0) may have a short useful lifetime, in the model an "application" should be interpreted as a proxy for an application category, which is likely to have a long or even indefinite lifetime. Once the first particular application in any category has been invented and demand has been revealed, subsequent applications in that category are subject to a qualitatively different level of demand uncertainty.

complementors to patent application categories.[18] In addition, entire software applications are generally not patentable.

**Mandatory divestiture** Because the model features undifferentiated Bertrand competition, requiring Firm M to divest itself of any applications that it develops makes little sense: the spun-off firm (call it Firm Y) would drive the application price to zero if $s = XY$, while if $s = X$ it would lead to a bilateral monopoly, which is inferior to an integrated monopoly. However, a mandatory divestiture rule could potentially be useful in a model with horizontal or vertical product differentiation. Horizontal product differentiation would allow both Firm X and Firm Y to earn modest profits when they compete, partially alleviating the problem of foregone innovation. Not only would mandatory divestiture offer greater incentives for Firm X to innovate ex ante, it would also give Firm X a greater opportunity to enter a market already occupied by Firm Y to provide a horizontally differentiated alternative.[19] Vertical product differentiation combined with horizontal differentiation would give Firms X and Y incentives for incremental quality improvement. On one hand, Heeb (2003) shows that Firm Y would be a stronger incremental innovator if it were integrated with Firm M, as long as Firm X is not induced to exit. On the other hand, Firm X is more likely to be induced to exit when facing an integrated competitor than when facing a symmetric competitor.

Unlike patent protection, mandatory divestiture cannot eliminate the problem of foregone invention: Firm M still has some ex post incentive to develop the application in order to induce Firm X and Firm Y to drive down application prices and increase demand for the base system. Another downside of mandatory divestiture is that Firm M's static pricing policy no longer internalizes the complementarity between the base system and complementary applications, weakening Firm M's ex ante incentives to invest. Whether a mandatory divestiture policy would be socially beneficial depends on the balance among the social benefits of radical innovation, incremental improvement, product variety, and static efficiency, as well as the probability distribution over entry costs.

**Price regulation** There are also more and less attractive policies of price regulation that can be used to reduce Firm M's threat to innovation without reducing its incentives to innovate. A first policy would be to prevent Firm M from excluding rivals by means other than pricing, as in the U.S. Justice Department's 2001 settlement with Microsoft. A more severe second policy would be to regulate Firm M's pricing so that, when it enters an already-occupied market ex post, it cannot undercut its rival or bundle its application with

[18] See Bessen (2003) for a closer examination of this idea.

[19] Such entry is not necessarily socially beneficial; whether it increases or decreases social surplus depends on the specification of demand and entry costs.

any other products. In the Base System model presented here, pegging Firm M's price to Firm X's price would restore some of the Firm X's ex post profits, alleviating but not completely solving the foregone invention problem. Because it prevents trivial bundling, it also reduces the incentive for Firm M to enter ex post. Outside the model, such a policy would not harm incremental quality improvement—in fact, it gives Firm M an incentive to create excellent products because it cannot compete on price. On the down side, such a policy invites possibly inefficient ex post entry by low-cost third parties. It also begs the question of how to define the boundaries between application markets.

# 4 Additional examples

In this section I examine three additional examples that complement the Base System model of Section 3. The purpose is to demonstrate how the ideas of the basic framework and the Base System model can be applied to additional contexts. Each example is motivated by an interesting phenomenon in the computer software market, each leading to foregone invention in a qualitatively different way. What all the examples share is a vertical relationship between a potential new product an established product over which some firm has market power. The nature of the vertical relationship determines the conditions under which invention is foregone.

All the examples here rely on a demand system similar to that in the Base System model. The derivation of period profits under various market structures is accordingly analogous, so I do not present the details in the text.

## 4.1 Bundling unrelated products

The first example is bundling with commitment, by which a firm can leverage market power even from an unrelated market. The ability to commit to bundling creates an artificial vertical relationship where one does not arise naturally. In the Base System model, Firm M was able to leverage its market power through trivial bundling because it controlled the base system upon which the application depended. This "Bundling" example instead assumes that products $b$ and $a$ are completely unrelated. This implies that the monopolist in the market for product $b$ can extend its market power to the market for product $a$ only if it can find a device to create a relationship between the two products. The device I consider is an ability to commit to offer unrelated products only as a bundle.

This example is motivated in part by Microsoft's dominance in the market for business productivity software, and its practice of bundling a word processor, spreadsheet, personal information manager, and presentation editor together under the brand "Microsoft Office." Nalebuff (2004), Carlton and Waldman (2002), and Bakos and Brynjolfsson (2000) show that bundling can serve as an entry deterrent, and Bakos and Brynjolfsson speculate informally that innovation incentives in an empty market will be distorted by the presence of a potential bundler. Here, I show that Microsoft's ex post option to bundle future products into Office may deter the ex ante invention of these products. In the works cited above, the monopolist initially monopolizes multiple markets and tries to prevent entry into one of them. Here, the incumbent monopolizes only one market, while another market lies unoccupied, with unknown demand.

As in Section 3.1, $\theta_i^b$ are distributed uniformly on $[0,1]$, while $\theta_i^a$ are either distributed uniformly on $[0,1]$, with probability $\gamma$, or otherwise $\theta_i^a = 0$ for all consumers. There are two firms, M and X, and four relevant market structures, $s \in \{\emptyset, M, X, MX\}$. In each static market, Firm M first chooses which bundles to offer from among the products it has developed, and then both firms set prices simultaneously. Since the two products are unrelated, independent pricing is no longer without loss of generality. Accordingly, I allow non-trivial bundling, including "mixed" bundling (in which a firm offers a bundle alongside independent products).The static prices and profits are as follows:[20]

- When $s = \emptyset$, Firm M monopolizes its existing market, setting price $p_M^b(\emptyset) = 0.5$ and earning profits $\pi_M(\emptyset) = 0.25$, as in the Base System model.
- When $s = M$ and demand for product $a$ is high, Firm M monopolizes both markets, and it offers a bundle $ba$ at price $p_M^{ba}(M) = 0.862$ as well as each product independently at prices $p_M^b(M) = p_M^a(M) = 0.667$, giving it profits of $\pi_M(M,1) = 0.549$.
- When $s = X$ and demand for product $a$ is high, the two markets are independent and each firm sets $p_M^b(X) = p_X^a(X) = 0.5$ and earns monopoly profits $\pi_M(X,1) = \pi_X(X,1) = 0.25$.
- When $s = MX$ and demand for product $a$ is high, Firm M offers a bundle at price $p_M^{ba}(MX) = 0.607$, while Firm X offers its independent product at price $p_X^a(MX) = 0.245$; this combination yields profits of $\pi_M(MX,1) = 0.369$ and $\pi_X(MX,1) = 0.067$. Firm M does not offer either of its products independently: if it offered product $a$ independently then Bertrand competition would drive its price to zero, while it cannot gain from offering product $b$ independently because the bundle price is less than 0.667.

Proposition 3 demonstrates that the period profits in this model can lead to foregone invention.

**Proposition 3.** *In the Bundling example, if the prior distribution over demand is sufficiently pessimistic ($\gamma < 0.398$) then the region of foregone invention has positive measure.*

[20] Though this example shares the demand setup of Nalebuff's model, these prices and profits differ slightly due to differences in timing and because Nalebuff does not consider mixed bundling.

The condition on $\gamma$ is more restrictive than in the Base System model, because Firm X can earn some profits competing against Firm M's bundle. In this sense, a new product that is not dependent on a base system is better insulated from ex post entry and therefore is more likely to be invented.

Commitment is necessary for invention to be foregone. When $s = MX$, Firm M would want to undercut Firm X's price in the application market if it could renege on its bundling commitment, so the price of product $a$ would fall to zero under Bertrand competition. Thus Firm M would not enter the market for product $a$ ex post if it could not commit to bundling. This implies a simple public policy remedy: do not allow Firm M to bundle any unrelated products together with a product over which it has market power.

In this example, the two products are completely unrelated. But the results carry over to the case of partial complementarity. If the new product $a$ were complementary to Firm M's existing product $b$, then Firm M might enter in order to be able to increase the price on product $b$. The intuition is similar to that of the Base System model: Firm M prefers that complements to its own monopoly product should have low prices, and entering the complementary markets in order to increase price competition is one way to accomplish that goal. However, in the Base System model bundling was trivially implied when Firm M set a zero price for its application because consumers needed to purchase Firm M's base system in order to use the application. Here, although complementarity makes price competition in the new market valuable to Firm M in its monopoly market, Firm M would prefer to commit to a bundle whenever the complementarity between the two products is less than perfect. Firm X is able to earn positive profits when Firm M enters with bundle pricing, but Firm X earns zero profits if Firm M enters without bundling. Thus when complementarity is strong enough to induce Firm M to enter even if it could not commit to bundling, an ability to commit to bundling has a beneficial effect on Firm X's ex ante invention incentives. In this way, bundling as an entry strategy is less destructive to social surplus when complementarity is high.

### 4.2 Product features

The second example considers the addition of the same new feature to two horizontally differentiated products: the "Product Features" example. The necessity of bundling the new feature with an existing product is assumed as a technological constraint. In this example, neither firm is privileged in the ex post market; instead, it is the fact that the new feature brings them into closer competition that leads to an excessive incentive to invest ex post. This model illustrates that foregone invention may occur even in a symmetric situation. It also shows that market power need not take the form of monopoly in order to pose a deterrent to innovation in related markets.

In the example, there are initially two existing products, which are sufficiently differentiated horizontally that their vendors' desired market shares do not come into conflict. In the world of software, one example of such a pair might be a visual web page editor and an HTML code editor. Though the products differ, there is an opportunity to add a particular feature (such as the ability to integrate with a new type of back-end data source) to either or both of them. The demand for this feature, though unknown, is orthogonal to the horizontal differentiation between the existing products. If only one of the two products offers the new feature, some customers may switch to the one with the new feature. Thus the invention of the new feature brings the firms into closer competition.

The example is constructed as follows. Consumer $i$'s valuation of Firm M's product is $\theta_i^0$, while her valuation of Firm X's product is $1-\theta_i^0$. Consumer valuations $\theta_i^0$ are distributed uniformly on $[0, 1]$. Consumer $i$'s valuation of the new feature is $\theta_i^1$, which with probability $\gamma$ is distributed uniformly on $[0, 1]$, and is 0 for all consumers otherwise. There are four possible market structures, $s \in \{\emptyset, M, X, MX\}$, and their static prices and profits are as follows:

- When $s = \emptyset$, each firm sets $p_j(\emptyset) = 0.5$ and earns profits $\pi_j(\emptyset) = 0.25$. In this example, the horizontal differentiation is calibrated so that all consumers are served in equilibrium and each firm is able to earn monopoly profits.

- When $s = M$ and demand for the new feature is high, both firms set higher prices, $p_M(M) = 0.832$ and $p_X(M) = 0.534$, than when $s = \emptyset$, even though only Firm M has the new feature. This is because for any fixed $\theta^1$ it is the consumers with intermediate values of $\theta_i^0$ who are most willing to switch from Firm X to Firm M. As a consequence, Firm X now serves a population more heavily weighted toward having high values for its basic product (i.e., low $\theta^0$), so it raises its price to extract more surplus from them. Firm M, of course, now sees higher demand for its product, and raises its price accordingly. Since both firms raise their prices, some customers with low values for the new feature and intermediate values for each basic product now go unserved. The resulting profits are $\pi_M(M, 1) = 0.472$, and $\pi_X(M, 1) = 0.195$.

- When $s = X$ and demand for the new feature is high, $p_X(X) = 0.832$, $p_M(X) = 0.534$, $\pi_X(X, 1) = 0.472$, and $\pi_M(X, 1) = 0.195$.

- When $s = MX$ and demand for the new feature is high, $p_M(MX) = p_X(MX) = 0.75$ and $\pi_M(MX, 1) = \pi_X(MX, 1) = 0.352$.

Proposition 4 shows that the profit functions in this model lead to foregone invention.

**Proposition 4.** *In the Product Features example, if the prior distribution over demand is sufficiently pessimistic ($\gamma < 0.707$) then the region of foregone invention has positive measure.*

This conclusion indicates that firms may be deterred from developing new features for their products by the possibility that adding those features will bring them into closer competition with currently distant competitors. In this specification of demand, the new feature is quite valuable relative to the original products, so a firm will suffer badly if only its competitor offers the new feature. Thus the ex post incentive to develop the new feature is strong, and invention is foregone over a large range of $\gamma$. Thus, in contrast to the Base System model and the Bundling example, it is the prospect of losing market power, rather than gaining market power, that drives ex post entry and thus reduces ex ante investment.

### 4.3 Divided Technical Leadership

The third example presents a situation in which Firm M's existing power in a related market is so threatened by Firm X's introduction of a new product that Firm M is induced to develop a new competing product ex post in order to protect its existing product. This leads to a qualitatively different set of conditions for foregone invention. The example exemplifies some aspects of the "divided technical leadership" (DTL) hypothesis of Bresnahan and Greenstein (1996), as applied to the recent history of the market for internet browsers. It is also closely related to Carlton and Waldman (2002), which examines the ability of a monopolist to extend its monopoly to the market for a newly emerging substitute. My purpose is not to construct the definitive model of DTL, but rather to propose that DTL suggests that platform leaders may seek to discourage the invention of complementary platform layers. The example also illustrates how the conditions for foregone invention are qualitatively different when a dominant firm perceives its competitors' investments as strategic complements.

The scenario is based on a stylized conception of Microsoft's entry into the internet browser market once Netscape's browser became popular. Microsoft feared that the combination of Netscape's browser and Sun's Java Virtual Machine (JVM), as a middleware layer on top of Windows, could potentially "commoditize" the underlying operating system.[21] If Netscape and Sun succeeded, Microsoft might be left to compete on price against other operating system vendors while Netscape and Sun assumed the mantle of platform leadership. The DTL hypothesis claims that such transitions are the usual course of change in the computer industry: a new entrant develops a complementary computing layer to serve a market segment distinct from that of the current platform leader, but once it achieves success in its own market it exploits changing trends in the industry to wrest platform leadership from

[21] See Bresnahan (2001) for a discussion of Microsoft's motivations based on internal strategy documents.

the incumbent. Before Netscape popularized the browser, Microsoft faced only moderate incentives to enter the browser market because it could gain at most monopoly profits in that market. Microsoft judged, perhaps rightly, that it could better allocate its development resources to other projects ex ante. But once the browser-JVM combination threatened to become successful, Microsoft entered the browser market in order to counter the challenge to its leadership role. From Microsoft's perspective, invesments were strategic complements: its incentives to invest were greater once Netscape and Sun also invested, because if the Netscape-Sun middleware product had succeeded then Microsoft could have faced severe consequences in its existing markets.

In this example, Firm X can invent a complementary application (product $a$, and if it turns out to be popular then Firm X can invent a rival base system (product $d$) two periods later by investing $c_d$ in the second period. Firm X's application works with either base system. Firm M can also develop the complementary application, but Firm M's application works only with Firm M's base system; Firm M cannot invent a new base system. The only uncertainty is over the demand for the application, which, as in previous models, is distributed uniformly on $[0, 1]$ with probability $\gamma$ and is zero for all consumers otherwise. I assume that each consumer buys only one application when Firm M is the only base system vendor (in which case the cross-platform compatibility of Firm X's application's offers no advantage), so if Firm M bundles its application then no consumers buy Firm X's application, regardless of its price. Now there are six relevant market structures, $s \in \{\emptyset, M, X, MX, XX, MXX\}$, where $XX$ indicates that Firm X has developed both an application and a base system. Consumers' values for Firm M's base system $\theta_i^b$ are distributed uniformly on $[1, 2]$ and each consumer's value for Firm X's base system is $3 - \theta_i^b$ (which is also uniformly distributed on $[1, 2]$).[22]

**Proposition 5.** *In the DTL example, if $c_d < \frac{1}{1-\delta} 0.5$ and $\delta > 0.113$ then the region of foregone invention has positive measure, regardless of the prior disribution over demand for the application.*

Because there may be up to three investments in this model, it does not fit directly into the basic framework. Instead, it is necessary to solve backward from the absorbing state, $MXX$. This is accomplished for $c_d < \frac{1}{1-\delta} 0.5$ in Lemma 3 (in the Appendix).

In contrast to the previous examples, the conditions for foregone invention in the DTL example do not involve the level of uncertainty, $\gamma$. This is because Firm M's ex post incentive to use trivial bundling as an entry barrier to its base system market is higher than its ex ante incentive to invent a complementary application of its own, even if $\gamma = 1$. The static

[22]Consumers' base system valuations are higher by 1 than in the product features model. This is so that under symmetric conditions Firm X's new base system can capture some of Firm M's potential base system customers.

prices and profits in each market structure, below, generate this effect because the invention of a rival base system is disastrous for Firm M, compared to its fortunes when it shares the application market with Firm X:

- When $s = \emptyset$, Firm M sets $p^b_M(\emptyset) = 1$ and earns $\pi_M(\emptyset) = 1$.

- When $s = M$ and demand for the application is high, $p^b_M(M) = 1.272$, $p^a_M(M) = 0.340$, and $\pi_M(M, 1) = 1.319$.

- When $s = X$ and demand for the application is high, $p^b_M(X) = 1.222$, $p^a_X(X) = 0.488$, $\pi_M(X, 1) = 1.060$, and $\pi_X(X, 1) = 0.238$. Firm X's application is popular.

- When $s = MX$ and demand for the application is high, $p^a_M(MX) = p^a_X(MX) = 0$, $p^b_M(MX, 1) = 1.549$, $\pi_M(MX, 1) = 1.316$, and $\pi_X(MX, 1) = 0$. Competition drives the application price to zero. Since Firm M trivially bundles its application with its base system, no consumers purchase Firm X's application.

- When $s = XX$ and demand for the application is high, $p^b_M(XX) = 0.928$, $p^d_X(XX) = p^{da}_X(XX) = 1.297$, and $p^a_X(XX) = 0.655$. Firm X trivially bundles the application with its base system, but those who buy Firm M's base system must pay a high price for the application. Profits are $\pi_M(XX, 1) = 0.431$ and $\pi_X(XX, 1) = 0.776$.[23]

- When $s = MXX$ and demand for the application is high, $p^a_M(MXX) = p^a_X(MXX) = 0$, $p^b_M(MXX) = p^d_X(MXX) = 1$, and $\pi_M(MXX, 1) = \pi_X(MXX, 1) = 0.5$.

These static market outcomes give Firm M a large incentive to enter the application market once Firm X has entered and revealed high demand, because failure of Firm M to enter would allow Firm X to develop a rival base system, cutting Firm M's profits roughly in half. In contrast, when Firm X does not enter, Firm M has little interest in the application market because even if demand is sure to be high it can increase its profits by only about 30%. Hence Firm M perceives entry as strategic complements, even while Firm X perceives entry as strategic substitutes. In such a situation, uncertainty is not required to put a wedge between Firm M's ex ante and ex post incentives, and hence Firm X can be deterred from entering even when it is common knowledge that demand is high.[24]

[23]There is actually a superior, but quantitatively insignificant solution in which Firm X also offers its base system sans application at a slightly lower price. The change in profits for each firm is less than $10^{-4}$.

[24]In reality, Netscape did develop a browser and Sun did develop its JVM. In the context of the model, this could mean that their development costs were sufficiently low that Microsoft's ability to force the browser price to zero and to exclude them from the base system market was not sufficient to deter their initial investment. Outside the model, it is also possible that Netscape and Sun misjudged Microsoft's ability to overcome the network externalities that they had built up through wide distribution of the browser and JVM. On the other hand, Netscape and Sun may have judged the situation properly and merely experienced a bad realization from among a range of possible outcomes.

## 5 Discussion

This paper deals with the dynamic consequences of ex post competition when heterogeneous inventors face uncertain demand. Demand uncertainty leads only low–entry cost firms to invent ex ante, but high–entry cost firms may wish to enter ex post if demand is revealed to be high. The prospect of ex post competition from high–entry cost firms dampens the ex ante incentives of low–entry cost firms, so that invention is foregone over an intermediate region of entry cost profiles. Proposition 1 gave sufficient conditions in a simple two-stage game for the region of foregone invention to have positive measure: at least one firm perceives investments as strategic substitutes; and expected ex post competition (Eq. 4), such that with positive probability demand is high enough that the other firm enters ex post when its entry costs are high enough that it would not enter ex ante. Although foregone invention can occur in symmetric situations, the problem is more likely to occur when one firm earns high monopoly profits and low duopoly profits, while the other firm earns high duopoly profits.

Throughout, I have taken an agnostic stance regarding the entry costs of each potential inventor. That is, I have expressed the results either as contingent on a particular cost profile, or as statements about regions of cost profiles. The idea behind this stance is that the development of any particular invention poses a unique set of challenges, and that each potential inventor will approach these challenges in its own way. For example, in the context of the Base System model, where Firm M is interpreted as Microsoft, for one kind of application one may believe that Microsoft has a low entry cost due to its knowledge of the inner workings of the Windows operating system. For another kind of application, one may believe that a startup funded by venture capital has a low entry cost, due to the strong incentives that its financial structure provides to its software engineers. These kinds of beliefs, aggregated across possible inventions, can be interpreted as a probability distribution over cost profiles, which in principle could be combined with knowledge about the region of foregone invention to yield the expected proportion of inventions that would be foregone. Empirically, such aggregation is hardly feasible, but the thought experiment gives a qualitative feel for how the problem of foregone invention can be viewed.

Of the three main factors that contribute to foregone invention—demand uncertainty, entry cost heterogeneity, and asymmetric market power—demand uncertainty has perhaps been the least studied. In the literature on invention, there are two main types of models. The first type gives several firms the opportunity to invest in developing a product for which demand is known, but the success or failure of their development efforts is dependent on their investments, often with a stochastic shock. Typically a patent system is assumed, so that competition takes the form of a race, with a single winner. Loury (1979), Dasgupta and Stiglitz (1980), and Reinganum (1981, 1982) are early works in this literature; notable recent contributions include d'Aspremont, Bhattacharya, and Gérard-Varet (2000), Denicolò (2000), Weeds (2002), and Doraszelski (2003). Adding demand uncertainty to such models adds no new insights, since the prize for winning the race is simply the expected monopoly profits.[25] The second type of model gives just one firm the opportunity to invent a product for which demand is known, but a second firm may imitate or otherwise exploit the invention ex post. Such models are designed to help determine optimal patent policy given the tradeoffs between incentives for the initial invention and the benefits of subsequent competition or sequential innovations. Notable examples include Gilbert and Shapiro (1990), Klemperer (1990), Gallini (1992), Chang (1995), Green and Scotchmer (1995), Schankerman and Scotchmer (2001), and Anton and Yao (2003, 2004). But in these kinds of models, one firm is given an exogenous monopoly over the ability to develop the primary innovation; the question arises why other firms cannot also develop the primary innovation. My basic framework can address such questions by varying the ability of each firm to invest in the primary innovation, i.e., by taking development costs as a parameter. The key new insight is that under these circumstances imitation poses no threat to innovation unless there is uncertainty over the demand for the invention.

A notable exception to the generalizations above is work by Jensen (1992a, 2001, 2004), which examines two firms that can invest in an innovation of uncertain success. As in the present paper, after one firm invests the success or failure of the innovation is revealed to all, and in the case of success the second firm can then invest it in the second period.[26] Jensen (1992a) and Jensen (2001) differ somewhat from the basic framework of Section 2, but would fit if modified such that neither firm could invest in the second period if neither invested in the first period. Jensen (1992a) focuses on the incentives of a third party inventor to license an innovation, but as a baseline considers a model with symmetric firms that can adopt an innovation of unknown success at a fixed cost, and identifies a threshold for the probability of success below which neither firm adopts. Both Jensen's 2001 and 2004 papers focus on the identity of the first mover rather than on the prospect of foregone invention. Jensen (2001) considers a multiproduct firm competing against a startup, and both can introduce a new product. For the multiproduct firm, success or failure will affect its reputation and thus its profit flows from its existing products. Under the assumption that investment costs are equal across firms, Jensen identifies a threshold probability of success below which neither firm invests. Jensen (2004), which fits into the basic framework, considers a large firm with

[25] Weeds (2002) models demand as a stochastic process, leading firms to want to delay innovation in order to see if demand is likely to be high. However, unlike in the present paper, no new information is revealed by the actions of the firms.

[26] In addition, Reinganum (1983) and Jensen (1992b) consider one-period, two-firm problems with invention under uncertainty, but since no investment is allowed after the uncertainty is resolved, there is no threat of ex post entry or imitation. The public resolution of uncertainty serves only to simplify the computation of equilibrium.

two plants competing against a small firm with one plant, and both can invest in a cost-reducing innovation of uncertain success. Under the assumptions that (i) any adopter in the first period earns zero profits in that period from the plant in which it adopts, and (ii) the fixed cost of adoption is zero, Jensen identifies a threshold probability of success below which neither firm invests. The results in the present paper indicate that for probabilities near the thresholds identified in Jensen (1992a, 2001, 2004), invention is foregone because at least one firm would prefer to invest if it could be assured that the other firm would not subsequently invest. Furthermore, the basic framework herein generalizes the foregone invention content of these analyses by allowing the costs and benefits of entry to vary across firms, so as to enable identification of the region of foregone invention as a function of the cost parameters.

Benveniste, Busaba, and Wilhelm (2002) look at a related problem in which there may be several private startup firms operating under similar circumstances, and the first one to make its initial public offering reveals information about capital market conditions that is useful to its followers. They find that firms go public too late relative to the social optimum, unless investment banks can force several of the firms to go public at the same time, or enforce a transfer payment from the followers to the leader. Although the Benveniste, Busaba, and Wilhelm model shares an informational externality effect with the model considered here, their model does not allow for competitive effects among the firms. So they say that the first firm goes public too late, even though the it does so at a time that is optimal from its own perspective, and the presence or absence of the follower firms has no effect on its decision. In the present paper, each firm's optimum in the absence of the other firm, rather than the social optimum, is the standard of comparison for determining when invention is foregone.

The structure of the basic framework is also related to the game theoretic literature on limit pricing and signaling, beginning with Milgrom and Roberts (1982). This literature considers a situation that begins after one firm has already entered to become the incumbent, and possesses private information about the market that a potential entrant would want to know when it makes its entry decision. This information could concern production costs, as in Milgrom and Roberts examples, or it could concern demand. If demand is low, the incumbent may wish to signal this to deter potential entrants, and this signaling can be effective if there exists a reasonable equilibrium with some separation. A separating equilibrium in this signaling game between incumbent and entrant can be seen as the second stage of the basic framework considered here, since separation implies that the second-stage entrant learns about demand before entering. Gertner, Gibbons, and Scharfstein (1988) consider the problem of a firm that wants to release financial information to capital markets, but is worried that rivals may also make use of that information. However, Gertner et al. find that all reasonable equilibria in their model are pooling, so that information is not revealed.

In contrast to demand uncertainty, asymmetric market power has received significant attention in the literature on innovation. When it is not the simple consequence of an inherent entry cost advantage or demand advantage, asymmetric market power typically stems from some sort of complementarity. A number of static results, including Chen and Ross (1998), Economides and Salop (1992), and stemming back to Cournot (1838), demonstrate that complementarity between markets makes a vertically integrated monopoly the socially preferred market structure. A number of authors have investigated dynamic consequences of these static results. Whinston (1990), Bakos and Brynjolfsson (2000), and Nalebuff (2000, 2004) indicate that a firm with products in multiple markets can tie its products together to deter entry or induce exit, to the detriment of society. Stefanadis (1997) and Choi and Stefanadis (2001) consider investment in cost-reducing improvements with uncertain technological success, and show that the threat of bundling or downstream foreclosure can reduce ex ante investment. Unlike my basic framework, their models do not allow firms to take advantage of information revealed by their competitors. Carlton and Waldman (2002) demonstrate how a monopolist in one market can extend its monopoly to another market. Farrell and Katz (2000) show that a monopolist in one market may invest too much in uncertain quality improvement for its complement while alternative suppliers invest too little; their analysis assumes ex ante symmetric investment costs and ascribes the uncertainty to technology rather than demand. Heeb (2003) investigates incremental quality improvement in a series of models similar to my Base System model, and finds that the base system monopolist faces greater incentives to invest in incrementally improving the application than an independent supplier. Adding heterogeneous entry costs and demand uncertainty to these types of models can lead to foregone invention.

Patents are the usual answer to problems of insufficient ex ante invention incentives. But for some entry cost profiles invention occurs without patents, and in such cases patents are harmful because they reduce ex post competition. Such competition is particularly important when there is scope for incremental quality improvement, because a firm that faces no competition ex post has less incentive to improve on its invention. Bessen and Maskin (2002) examine a sequential innovation model with hidden entry costs, uncertain technology, and free imitation. They find that ex post competition (when patents are not imposed) increases the incentive to innovate, because inventors look forward to being able to imitate subsequent improvements invented by others.[27] One final problem with patents

[27] In their model, each firm that invests in developing a sequential improvement draws its probability of success independently, so it always has some incentive to invest even when its competitor also invests. I conjecture that putting the uncertainty on the demand side and allowing asymmetric ex post market power would lead to the foregone invention problem, although ex post competition could still drive innovation in cost regions outside the region of foregone invention.

is that, even if they could be helpful in concept, it is difficult to apply the legal framework of the patent system to categories of software applications, both because software applications typically make use of previously patented technologies (see Bessen 2003), and because reverse engineering is legal and patent law does not necessarily threaten penalties severe enough to prevent imitation (see Anton and Yao 2004).

As for Microsoft, the results in this paper suggest that its monopoly over operating systems can pose a deterrent to innovation that has the potential to harm both society and Microsoft, but also that many policies that could eliminate or reduce this threat could have other negative effects on innovation. One commonly considered policy is to reduce Microsoft's ability to exclude its rivals other than by pricing. But the models show that such a policy is likely to fall short of solving the problem. Perhaps it is helpful to think of Microsoft as merely a dominant firm, rather than a true monopolist in the operating system market. The presence of multiple operating system vendors can help independent application developers in the ex post market, because they can market their applications for several platforms, while each operating system vendor can trivially bundle only with its own operating system. Competition may also drive incremental improvements in operating systems. In a static framework, on the other hand, if there are benefits of standardization on Windows then the continued presence of alternate operating systems means that some of these benefits are foregone. Likewise, development costs are likely to be higher for products that run on multiple operating systems. In the end, the results in this paper suggest that—although no policy option offers a sure solution—any policy that might be implemented should be made explicit ex ante so as to encourage complementors and competitors to innovate. A reliance on ex post litigation to enforce restrictions could increase both the uncertainty and the expense of ex ante entry, undermining the goal of improving ex ante incentives.

## Appendix: Proofs and supporting results

**Lemma 1.** *Let $s_j^*(\omega, c_j)$ and $z_j^*(\omega, c_j)$ indicate the market structure and Firm j's investment cost, respectively, that result from Firm j's sequentially optimal choice after Firm $-j$ has entered and revealed $\omega$:*[28]

$$s_j^*(\omega, c_j) = \begin{cases} MX & \text{if } \pi_j^2(MX, \omega) - c_j > \pi_j^2(-j, \omega) \\ -j & \text{otherwise,} \end{cases} \tag{7}$$

[28] This specification assumes that a firm indifferent to entry will not enter. This assumption plays no role in the analysis since firms almost always (taking their investment costs as a parameter) strictly prefer to either enter or not enter.

$$z_j^*(\omega, c_j) = \begin{cases} c_j & \text{if } \pi_j^2(MX,\omega) - c_j > \pi_j^2(-j,\omega) \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

*There exists a subgame perfect equilibrium in which neither firm enters the market if and only if*

$$\mathbb{E}\big[\pi_j^1(j,\omega) + \pi_j^2\left(s_{-j}^*(\omega, c_{-j}), \omega\right)\big] - c_j \le \pi_j^1(\emptyset) + \pi_j^2(\emptyset) \tag{9}$$

*for both $j$. Furthermore, this is the unique subgame perfect outcome if and only if*

$$\mathbb{E}\big[\pi_j^1(MX,\omega) + \pi_j^2(MX,\omega)\big] - c_j < \mathbb{E}\big[\pi_j^1(-j,\omega) + \pi_j^2\left(s_j^*(\omega, c_j), \omega\right) - z_j^*(\omega, c_j)\big], \tag{10}$$

*for some firm $j$, and also the inequality in Eq. 9 is strict for firm $-j$.*

*Proof.* The proof operates by backward induction. In the second period, if only firm $-j$ has already entered, the market structure resulting from firm $j$'s sequentially optimal action is given by $s_j^*(\omega, c_j)$. Hence the first period reduced game, taking sequentially optimal actions into account, is represented by the following normal form game:

| | $\text{in}_X^1$ | $\text{out}_X^1$ |
|---|---|---|
| $\text{in}_M^1$ | $\mathbb{E}\big[\pi_M^1(MX,\omega) + \pi_M^2(MX,\omega)\big] - c_M$, $\mathbb{E}\big[\pi_X^1(MX,\omega) + \pi_X^2(MX,\omega)\big] - c_X$ | $\mathbb{E}\big[\pi_M^1(M,\omega) + \pi_M^2(s_X^*(\omega, c_X),\omega)\big] - c_M$, $\mathbb{E}\big[\pi_X^1(M,\omega) + \pi_X^2(s_X^*(\omega, c_X),\omega) - z_X^*(\omega)\big]$ |
| $\text{out}_M^1$ | $\mathbb{E}\big[\pi_M^1(X,\omega) + \pi_M^2(s_M^*(\omega, c_M),\omega) - z_M^*(\omega)\big]$, $\mathbb{E}\big[\pi_X^1(X,\omega) + \pi_X^2(s_M^*(\omega, c_M),\omega)\big] - c_X$ | $\pi_M^1(\emptyset) + \pi_M^2(\emptyset)$, $\pi_X^1(\emptyset) + \pi_X^2(\emptyset)$ |

Then Eq. 9 (for both firms) is the necessary and sufficient for $(\text{out}_M^1, \text{out}_X^1)$ to be a Nash equilibrium of this normal form game. Also, Eq. 9 with strict inequality for firm $-j$ and weak inequality for firm $j$, combined with Eq. 10 for firm $j$, are necessary and sufficient conditions for there to be no other Nash equilibrium. ■

**Lemma 2.** *In the basic framework under Assumptions 1–4, let*

$$\tilde{c}_M \equiv \sup\big\{c_M : \Pr\big[s_M^*(\omega, c_M) = MX\big] > 0\big\}. \tag{11}$$

*Suppose that investments are strategic substitutes for Firm X; then there exists $\varepsilon > 0$ such that, given entry cost $\hat{c}_X - \varepsilon$,*

1. *If $c_M < \tilde{c}_M$ then Firm X does not enter the empty market in the first period in any subgame perfect equilibrium*
2. *If $c_M > \tilde{c}_M$ then Firm X enters the empty market in the first period in every subgame perfect equilibrium*

*Proof.* Positive profits and common demand imply that $\tilde{c}_M > 0$. This fact combined with common demand and competitiveness implies that

$$\mathbb{E}\big[\pi_X^2(X,\omega)\big] > \mathbb{E}\big[\pi_X^2(s_M^*(\omega,c_M),\omega)\big] \tag{12}$$

for all $c_M < \tilde{c}_M$. Hence, given an entry cost profile $(c_M, \hat{c}_X)$ with $c_M < \tilde{c}_M$, Firm X's strict best response (in the first period reduced game) to $\text{out}_M^1$ is $\text{out}_X^1$:

$$\begin{aligned} \hat{c}_X &\equiv \mathbb{E}\big[\pi_X^1(X,\omega) + \pi_X^2(X,\omega)\big] - \pi_X^1(\emptyset) - \pi_X^2(\emptyset) \\ &> \mathbb{E}\big[\pi_X^1(X,\omega) + \pi_X^2(s_M^*(\omega,c_M),\omega)\big] - \pi_X^1(\emptyset) - \pi_X^2(\emptyset). \end{aligned} \tag{13}$$

That investments are strategic substitutes for Firm X further implies that Firm X's strict best response (in the first period reduced game) to $\text{in}_M^1$ is $\text{out}_X^1$:

$$\begin{aligned} \hat{c}_X &= \mathbb{E}\big[\pi_X^1(X,\omega) + \pi_X^2(X,\omega)\big] - \pi_X^1(\emptyset) - \pi_X^2(\emptyset) \\ &> \mathbb{E}\big[\pi_X^1(MX,\omega) + \pi_X^2(MX,\omega) - \pi_X^1(M,\omega) - \pi_X^2(M,\omega)\big] \\ &\geq \mathbb{E}\big[\pi_X^1(MX,\omega) + \pi_X^2(MX,\omega) - \pi_X^1(M,\omega) - \big(\pi_X^2(s_X^*(\omega,c_X),\omega) - z_X^*(\omega,c_X)\big)\big], \end{aligned} \tag{14}$$

where the strict inequality is by strategic substitutes and the weak inequality is because Firm X will enter ex post only if by doing so it improves its profits. Choose $\varepsilon > 0$ sufficiently small that both of these strict best responses continue to hold at $c_X = \hat{c}_X - \varepsilon$. Then, given an entry cost profile $(c_M, \hat{c}_X - \varepsilon)$ with $c_M > \tilde{c}_M$, Firm X's optimal first period action is $\text{in}_X^1$. The argument for $c_M > \tilde{c}_M$ is similar. ■

*Proof of Proposition 1 (page 9).* Eq. 4 and competitiveness imply that $\hat{c}_M < \tilde{c}_M$, so by Lemma 2, given entry cost profile $(\hat{c}_M, \hat{c}_X - \varepsilon_X)$ with $\varepsilon_X > 0$ sufficiently small, Firm X does not enter in any subgame perfect equilibrium even though it would strictly prefer to enter given an entry cost profile $(c_M, \hat{c}_X - \varepsilon_X)$ with $c_M > \tilde{c}_M$. This is also true at $(\hat{c}_M + \varepsilon_M, \hat{c}_X - \varepsilon_X)$, for $\varepsilon_M > 0$ sufficiently small, where Firm M also does not enter in any subgame perfect equilibrium. ■

*Proof of Proposition 2 (page 16).* First,

$$\hat{c}_M = \frac{\gamma}{1-\delta}\,0.546 + \frac{1-\gamma}{1-\delta}\,0.25 - \frac{1}{1-\delta}\,0.25 = \frac{\gamma}{1-\delta}\,0.296. \tag{15}$$

Second,

$$s_M^\star(1,\hat{c}_M) = \begin{cases} MX & \text{if } c_M < \frac{1}{1-\delta}\big(0.544 - 0.343\big) = \frac{1}{1-\delta}\,0.201 \\ X & \text{otherwise,} \end{cases} \tag{16}$$

while $s_M^\star(0,\hat{c}_M) = X$. Third, by assumption $\hat{c}_M < \frac{1}{1-\delta}\,0.201$, so

$$\mathbb{E}\big[\pi_X^2(X,\omega)\big] = \frac{\gamma\delta}{1-\delta}\,0.172 > \mathbb{E}\big[\pi_X^2(s_M^\star(\omega,\hat{c}_M),\omega)\big] = 0. \tag{17}$$

Finally, Firm X perceives investments as strategic substitutes because Firm X always earns zero profits when it does not invest. Thus the conditions of Proposition 1 are satisfied. ■

*Proof of Proposition 3 (page 24).* First,

$$\hat{c}_M = \frac{\gamma}{1-\delta}\,0.549 + \frac{1-\gamma}{1-\delta}\,0.25 - \frac{1}{1-\delta}\,0.25 = \frac{\gamma}{1-\delta}\,0.299. \tag{18}$$

Second,

$$s^\star_M(1, c_M) = \begin{cases} MX & \text{if } c_M < \frac{1}{1-\delta}\big(0.369 - 0.25\big) = \frac{1}{1-\delta}\,0.119 \\ X & \text{otherwise,} \end{cases} \tag{19}$$

while $s^\star_M(0, c_M) = X$. Third, by assumption $\hat{c}_M < \frac{1}{1-\delta}\,0.119$, so

$$\mathbb{E}\big[\pi^2_X(X,\omega)\big] = \frac{\gamma\delta}{1-\delta}\,0.25 > \mathbb{E}\big[\pi^2_X(s^\star_M(\omega,\hat{c}_M),\omega)\big] = \frac{\gamma\delta}{1-\delta}\,0.067. \tag{20}$$

Finally, Firm X perceives investments as strategic substitutes because Firm X earns zero profits whenever it does not invest. Thus the conditions of Proposition 1 are satisfied. ■

**Lemma 3.** *When $c_d < \frac{1}{1-\delta}\,0.5$, the Divided Technical Leadership model is equivalent to the basic framework with*

$$\pi^2_M(X,1) = \delta\,1.060 + \frac{\delta^2}{1-\delta}\begin{cases} 0.431 & \textit{if } c_M > \frac{\delta}{1-\delta}\,0.069 \\ 0.5 & \textit{if } c_M < \frac{\delta}{1-\delta}\,0.069, \end{cases} \tag{21}$$

$$\pi^2_X(X,1) = \delta\,0.238 + \frac{\delta^2}{1-\delta}\begin{cases} 0.776 & \textit{if } c_M > \frac{\delta}{1-\delta}\,0.069 \\ 0.5 & \textit{if } c_M < \frac{\delta}{1-\delta}\,0.069, \end{cases} \tag{22}$$

*and for all states $(s,\omega) \neq (X,1)$ with $s \in \{\emptyset, M, X, MX\}$, $\pi^1_j(s,\omega) = \pi_j(s,\omega)$ and $\pi^2_j(s,\omega) = \frac{\delta}{1-\delta}\pi_j(s,\omega)$.*

*Proof.* Solve backward from the absorbing state, $MXX$, to states $X$, $M$, and $MX$. If $s = XX$, then Firm M invests if $c_M < \frac{1}{1-\delta}\,0.069$. If $s = MX$ then Firm X cannot invest, because its application is not popular. If $s = M$, then Firm X does not invest. If $s = X$ and one period has already elapsed since Firm X invented the application, then Firm X's best response to no investment by Firm M is to invest if $c_d < \frac{1}{1-\delta}\,0.538$; Firm M's mutual best response is indeed not to invest if $c_M > \frac{1}{1-\delta}\,0.069$. Firm X's best response to investment by Firm M in this situation is to invest if $c_d < \frac{1}{1-\delta}\,0.5$; Firm M's mutual best response is indeed to invest if $c_M < \frac{1}{1-\delta}\,0.069$. If $s = X$ and Firm X has just invented the application, Firm M can switch to state $MX$ by investing, the incentives for which are accounted for in the basic framework. ■

**Lemma 4.** *Assuming that investments are strategic substitutes for Firm X, suppose that second period profits are much more important than first period profits and*

$$\mathbb{E}\big[\pi_M^2(MX,\omega) - \pi_M^2(X,\omega)\big] > \mathbb{E}\big[\pi_M^2(M,\omega)\big] - \pi_M^2(\emptyset). \tag{23}$$

*Then the region of foregone invention has positive measure.*

*Proof.* Since $\mathbb{E}\big[\pi_M^2(\cdot,\omega)\big]$ is supermodular in investment, the measure of $\omega$ for which

$$\pi_M^2(MX,\omega) - \pi_M^2(X,\omega) > \mathbb{E}[\pi_M^2(M,\omega)] - \pi_M^2(\emptyset) \tag{24}$$

is positive, and since first period profits are small, the measure of $\omega$ for which

$$\pi_M^2(MX,\omega) - \pi_M^2(X,\omega) > \mathbb{E}\big[\pi_M^1(M,\omega) + \pi_M^2(M,\omega)\big] - \big(\pi_M^1(\emptyset) + \pi_M^2(\emptyset)\big) \tag{25}$$

is also positive. Thus Firm M enters ex post with positive probability given $\hat{c}_M$:

$$\begin{aligned} 0 &< \Pr\big[\pi_M^2(MX,\omega) - \pi_M^2(X,\omega) > \mathbb{E}\big[\pi_M^1(M,\omega) + \pi_M^2(M,\omega)\big] - \big(\pi_M^1(\emptyset) + \pi_M^2(\emptyset)\big)\big] \\ &= \Pr\big[\pi_M^2(MX,\omega) - \hat{c}_M > \pi_M^2(X,\omega)\big] = \Pr\big[s_M^*(\omega,\hat{c}_M) = MX\big]. \end{aligned} \tag{26}$$

This implies the condition in Proposition 1. ■

*Proof of Proposition 5 (page 28).* Suppose that $c_d < \frac{1}{1-\delta}\,0.5$. If $c_M > \frac{1}{1-\delta}\,0.069$ then satisfying Eq. 23 requires

$$\frac{\gamma\delta}{1-\delta}\,1.316 - \gamma\left(\delta\,1.060 + \frac{\delta^2}{1-\delta}\,0.431\right) > \frac{\gamma\delta}{1-\delta}\big(1.319 - 1\big), \tag{27}$$

while if $c_M < \frac{1}{1-\delta}\,0.069$ then this requires

$$\frac{\gamma\delta}{1-\delta}\,1.316 - \gamma\left(\delta\,1.060 + \frac{\delta^2}{1-\delta}0.5\right) > \frac{\gamma\delta}{1-\delta}\big(1.319 - 1\big). \tag{28}$$

Both these conditions are satisfied when $\delta > 0.113$, so by Lemma 4 invention is foregone. ■

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