IS PERFECT PRICE DISCRIMINATION REALLY EFFICIENT? AN ANALYSIS OF FREE ENTRY EQUILIBRIA

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ABSTRACT. We analyze models of product differentiation with perfect price discrimination and free entry. Although perfect price discrimination ensures efficient output decisions given product characteristics, coordination failures may prevent efficiency in the choice of product characteristics. More fundamentally, even if we have efficient product choices for a fixed number of firms, one always has excessive entry in free entry equilibrium. Our results apply to a large class of models of product differentiation including location models as well as representative consumer models of the demand for variety. These results also apply to models of common agency or lobbying with free entry and imply that one has excessive entry into the ranks of lobbyists.

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1. Introduction

We investigate the efficiency of free-entry equilibria when firms have the ability to practice perfect price discrimination, constrained only by the competition they face from rival firms, and unconstrained by informational limitations about consumer characteristics. There is much work examining the efficiency consequences of imperfect price discrimination in the context of oligopoly or monopolistic competition—Armstrong and Vickers (1998), Borenstein (1985), Corts (1998), Katz (1984), Rochet and Stole (1999) and Stole (1995) are prominent examples.[1] However, apart from the early work of Spence (1976), previous work on *perfect* price discrimination has been limited, perhaps because of Spence's sweeping conclusion that "if sellers can price discriminate in an appropriate sense, the welfare aspects of the product choice problem are eliminated" (pp. 217–8). Spence's argument is simple and seems compelling. With perfect price discrimination, each seller will be able to capture her marginal contribution to consumer welfare and hence her profits coincide with her marginal contribution to social welfare. In consequence, a producer will choose her product variety so as to maximize her marginal contribution, i.e., to maximize social welfare. Finally, entry decisions will be efficient, since a firm will enter the market if and only if its marginal contribution exceeds the entry cost.

The implication of Spence's argument is that inefficiencies arise in models of oligopoly only because consumer characteristics are private information (or perhaps if there are legal restrictions on price discrimination) and not due to the exercise of market power *per se*. Spence's argument also finds application to models of common agency with perfect information (Berhneim and Whinston, 1986), which have been widely used to study the labor market as well as the lobbying process. Spence's argument suggests that if one can ensure that each principal or lobbyist can capture her marginal contribution to the agent's utility, then this would imply that investment/entry incentives for the principal are correctly specified and the overall outcome will be efficient, from the point of view of the principals and the agent.[2]

We provide a comprehensive analysis of the welfare consequences of perfect price discrimination. We find that Spence's argument is correct, but only in some respects. For example, if the number of firms is given, each firm will choose product variety so as to maximize its contribution to social welfare and hence variety choices correspond to a decentralized maximum of social welfare. Nevertheless, we find a general tendency to excessive entry, even if each firm captures its marginal contribution to social welfare. The key question to ask is, "what is the marginal contribution relative to?" We

[1] See also the survey by Varian, who discusses the welfare consequences of imperfect price discrimination extensively.

[2] This point has been emphasized by Bergemann and Valimaki (1999), who consequently focus on the conditions under which each principal gets her marginal contribution in equilibrium.

find that the marginal firm captures its marginal contribution relative to an *inefficient* allocation rather than an efficient one and this is the reason why there is excessive entry.

The basic argument is as follows. Assume that for any integer n, if n firms enter, they will choose their product characteristics and outputs so as to maximize social welfare. Denote this optimal choice by $\boldsymbol{\zeta} = (\boldsymbol{\zeta}_1, \boldsymbol{\zeta}_2, \ldots, \boldsymbol{\zeta}_n)$, where $\boldsymbol{\zeta}_i$ denotes the choice of firm i. On the other hand, if $n-1$ firms were to enter, the welfare maximizing choice would be the $n-1$ vector $\boldsymbol{\zeta}' = (\boldsymbol{\zeta}'_1, \boldsymbol{\zeta}'_2, \ldots, \boldsymbol{\zeta}'_{n-1})$. Hence the increase in welfare due to the entry of the marginal firm is $W(\boldsymbol{\zeta}) - W(\boldsymbol{\zeta}')$. However, the profits of the marginal firm, n, are given by its marginal contribution to social welfare *at the vector* $\boldsymbol{\zeta}$ so that profits equal $W(\boldsymbol{\zeta}) - W(\boldsymbol{\zeta}_1, \boldsymbol{\zeta}_2, \ldots, \boldsymbol{\zeta}_{n-1})$. Since $\boldsymbol{\zeta}'$ is welfare maximizing when there are $n-1$ firms, it follows that $W(\boldsymbol{\zeta}') \geq W(\boldsymbol{\zeta}_1, \boldsymbol{\zeta}_2, \ldots, \boldsymbol{\zeta}_{n-1})$ and hence the profits of the marginal firm are always greater than its contribution to social welfare. In consequence, there will always be too much entry.

This argument demonstrates excess entry, in a weak sense. In a wide variety of models of monopolistic competition, we find the optimal output/product variety choices are sensitive to the number of entrants. In all such models, the above inequality is strict, so that excess entry obtains in a strict sense. This is true, for example, in many models of "locational" competition, such as the Salop or Hotelling models and in models of vertical differentiation. It is also true in several representative consumer models such as the models due to Dixit and Stiglitz (1977) or Spence (1976).[3]

Our basic result, that there is excessive entry, holds in discrete choice models (where each consumer only consumes a single variety) as well as representative consumer models, where the consumer desires variety. We discuss the former class of model in Section 2 and the latter in Section 3. In each case, we begin with a concrete example before proceeding to a more general analysis. In Section 4, we discuss the implications of our analysis for common agency models and the final Section concludes.

2. Discrete Choice Models

We now analyze the efficiency of entry in models where consumers are heterogeneous and where each consumer consumes at most one type of product.

[3]Lederer and Hurter (1986) and MacLeod et al. (1988) also analyze spatial models where the firm's ability to absorb transport costs permits perfect price discrimination. Lederer and Hurter analyze a two-firm Hotelling style location model, while MacLeod et al. analyze a more general spatial model. These papers focus on the fact that discriminatory pricing allows the existence of pure strategy locational equilibria in contexts (such as the Hotelling model with linear transport costs) where non-discriminatory pricing does not permit existence. They also note that locational equilibria can have multiple equilibria, so that global efficiency is not ensured. While we discuss these arguments in greater detail in context, our basic point is different from theirs—we show that inefficiency arises in free entry even when such coordination failures are ruled out.

Such models are termed discrete choice models.[4] The extensive form we analyze has three stages, as follows. In the first stage, each potential firm (from an infinite or sufficiently large set) must decide whether to enter or not. Let $N = \{1, 2, \dots n\}$ be the set of entrants. In stage 2, each entrant firm observes n, the number of entrants in stage 1 and the entrants choose a product variety. In stage 3, firms compete by offering a consumer specific price to each consumer. Throughout, we shall focus attention on pure strategy equilibria. If we were to allow mixed equilibria, these would typically be inefficient and one could not expect price discrimination to ensure efficiency.

It will be useful to begin with an analysis of models of spatial competition, as in Hotelling or Salop's (1979), before proceeding to a more general analysis.

2.1. **The Salop Model.** A unit measure of consumers is uniformly distributed around the unit circle (as in Salop) or the interval $[0, 1]$ (Hotelling). Consumers have unit demands (with reservation values v) and incur a transport cost—if the consumer located at x purchases from the firm located at y, the transport cost incurred is $T(x - y)$. We assume that $T(z) = T(-z)$ for all z and that $T(z)$ is strictly increasing if $z > 0$ and differentiable except possibly at zero. Normalize $T(0)$ to equal zero. An example is where $T(x - y) = t|x - y|^{\alpha}$; $\alpha = 1$ corresponds to the case of linear transportation costs, originally used by Salop. Marginal costs are constant and normalized to zero and firms incur a fixed cost F if they enter the market. We assume that there are infinitely many potential firms, i.e., the number of potential entrants is greater than the number which actually enter in any equilibrium.

2.1.1. *Competitive Price Discrimination.* Consider the consumer who is located at point x and purchases the product at price $p_i(x)$ from firm i located at point y_i. The utility from this purchase is given by

$$v - T(x - y_i) - p_i(x).$$

With perfect price discrimination, firms compete separately for each consumer. Hence one has Bertrand competition for each consumer and the firm (i) which is nearest to the consumer will limit price the firm which is the next nearest. Let $g(x) = \arg\min_{i \in N} |x - y_i|$ be the index of the closest firm, which is located at $y_{g(x)}$ and let $h(x) = \arg\min_{i \in N \setminus \{g(x)\}} |x - y_i|$ be the index of the next closest firm, which is located at $y_{h(x)}$. Two possibilities arise: if $v - T(x - y_{h(x)}) \geq 0$, firm $h(x)$ can effectively compete for the customer and firm $g(x)$ will sell to consumer x at a price

$$p_{g(x)}(x) = T\left(x - y_{h(x)}\right) - T\left(x - y_{g(x)}\right).$$

[4]Note however that consumers may purchase variable amounts of the single good that they choose to consume. Anderson et al. (1992) provide an excellent overview of discrete choice models.

Alternatively, if $v - T\left(x - y_{h(x)}\right) < 0$, firm $g(x)$ only competes with the consumer's reservation utility and will sell to the consumer at a price

$$p_{g(x)}(x) = v - T\left(x - y_{g(x)}\right).$$

In either case, the pricing stage ensures efficient provision—the consumer buys if and only if it is efficient to buy and always buys from the right firm.

For the remainder of this section we shall assume that consumer preferences and firm locations are such that firms effectively compete for each consumer. This is purely for expositional ease. Hence, each consumer buys from the nearest firm and pays a price which equals the difference between her utility from buying from this firm and her utility from buying from the next nearest firm at price zero.

2.1.2. *Location Choice.* Assume that $n \geq 3$ firms have entered the market and must simultaneously choose locations in stage 2 of the game defined above. We now show that given the equilibrium locations of the $n-1$ other firm, the firm concerned will choose its location so as to maximize it contribution to social welfare. In other words, there is an equivalence between equilibrium locations and locations which are *decentralized maximizers* of social welfare. A locational configuration is a decentralized maximizer of social welfare if no single firm can increase welfare by altering its location. Note that a decentralized maximizer need not maximize social welfare globally. However, in the case of the Salop and Hotelling models, it turns out that any decentralized maximizer also maximizes social welfare globally.

Consider the location choice of a firm which is considering locating at a point y between the equilibrium locations of two other firms. Normalize the locations of these firms to 0 and a respectively, so that $a > y > 0$ (if $y = 0$ or $y = a$, the firm makes zero revenue since there is Bertrand competition for each consumer). We shall show that $y^* = a/2$ is the optimal location choice for this firm.

If the firm locates at y, its revenues are given by

$$R(y) = \int_{\frac{y}{2}}^{\frac{a}{2}} [T(x) - T(y-x)]dx + \int_{\frac{a}{2}}^{\frac{a+y}{2}} [T(a-x) - T(x-y)]dx. \tag{1}$$

On the other hand, the total welfare of consumers located in $[0, a]$, as a function of y, is given by the negative of total transport costs, as follows

$$-W(y) = \int_0^{\frac{y}{2}} T(x)dx + \int_{\frac{y}{2}}^{\frac{a+y}{2}} T(y-x)dx + \int_{\frac{a+y}{2}}^{a} T(a-x)dx. \tag{2}$$

After some manipulation, we find

$$R(y) = W(y) + \int_0^{\frac{a}{2}} T(x)dx + \int_{\frac{a}{2}}^{a} T(a-x)dx. \tag{3}$$

Hence $R(y)$ is identical to $W(y)$ except for a constant term which is independent of y. Hence a firm which locates between any two other firms will choose a welfare optimal location.

The welfare optimal location in the interval $[0, a]$ can be obtained from the first order condition

$$W'(y) = -\int_{\frac{y}{2}}^{\frac{a}{2}} T'(y-x)dx + \int_{\frac{a}{2}}^{\frac{a+y}{2}} T'(x-y)dx = 0. \tag{4}$$

This yields the optimum, y^*, which satisfies

$$T\left(\frac{a-y^*}{2}\right) - T\left(\frac{y^*}{2}\right). \tag{5}$$

Hence $y^* = a/2$, which is the unique optimum since $T(\,\cdot\,)$ is strictly increasing.[5]

Finally we note from the expressions for revenue and welfare that both are greatest when the length of the integral, a, is largest. Therefore any firm will pick a location such that a) the distance between its neighbors, a, is largest and b) locate half-way between these two neighbors. These two conditions must be satisfied for every firm if we have a pure strategy Nash equilibrium in locations. It follows, that in any such Nash equilibrium, firms must be equally spaced around the circle since otherwise, for at least one firm, its distance to its left-hand neighbor must differ from its distance to its right-hand neighbor.

Similarly, in the case of the Hotelling model of competition between two firms on the line, we find that Nash equilibrium locations are welfare optimal. Such optimality is obtained uniquely at 1/4 and 3/4 for any strictly increasing transport cost function. This contrasts with the sub-optimality of equilibrium locations in the absence of price discrimination—e.g., with quadratic transport costs, one has maximal differentiation.

To summarize, we have found that price-discriminating firms will choose locations in a welfare optimal way, and have also explicitly characterized such locations.[6] Hence we find that price discrimination ensures efficiency at pricing as well as location stages.

2.1.3. *Free Entry.* We now analyze the first stage of the game, where firms enter. If n firms enter the market, they will space themselves equally in stage 2. Hence the profits earned by each firm, $\pi(n)$, is given by

$$\pi(n) = 2\int_0^{\frac{1}{2n}} \left[T\left(\frac{1}{n} - x\right) - T(x)\right] dx - F. \tag{6}$$

[5] It is easily checked that the second order condition is also satisfied at this optimum.

[6] If $n = 2$ so that there are only two firms, it is easy to show that Nash equilibrium/welfare optimal locations must lie on a diameter of the circle.

This can be re-written as

$$\pi(n) = 2\int_{\frac{1}{2n}}^{\frac{1}{n}} T(x)dx - 2\int_{0}^{\frac{1}{2n}} T(x)dx - F. \tag{7}$$

Since T is strictly increasing, $T(1/n-x)$ is decreasing in n and hence $\pi(n)$ is a strictly decreasing function, which tends to $-F$ as $n \to \infty$. If we assume that the market is sufficiently large so that a monopolist makes positive profits, this establishes that there is a unique value of n at which profits are zero—call this value n^*. If one requires n to only take integer values, there is similarly a unique value, n^*, such that $\pi(n^*) \geq 0$ and $\pi(n^*+1) < 0$. This establishes that there is a unique equilibrium outcome under free entry.

How does this equilibrium compare with the social optimum? Let social welfare, W, be measured by the sum of consumer surplus and profits. Clearly, with n firms, maximizing welfare requires that these firms are equally spaced. Maximizing W is hence equivalent to minimizing the sum of consumer transport costs and total fixed costs.

$$W(n) = -2n\int_{0}^{\frac{1}{2n}} T(x)dx - nF. \tag{8}$$

Let $\Delta W(n) = W(n) - W(n-1)$ be the change in social welfare due to the entry of the n^{th} firm.

$$\Delta W(n) = 2(n-1)\int_{\frac{1}{2n}}^{\frac{1}{2(n-1)}} T(x)dx - 2\int_{0}^{\frac{1}{2n}} T(x)dx - F. \tag{9}$$

We now compare the profits of the additional firm with its contribution to social welfare and show that profits are always larger. The intuition for this result comes from comparing (7) and (9). Note that the last two terms in both these equations are identical—i.e., the fixed cost and the transport cost involved for consumers purchasing from this firm enter both expressions in the same way. Hence the comparison only depends upon the first term. Note that profits depend upon the integral of $T(x)$ over the range $1/2n$ to $1/n$—these capture the opportunity cost for the consumers of buying from the nearest competitor. On the other hand, the contribution to welfare depends upon $(n-1)$ times the integral over the range $1/2n$ to $1/2(n-1)$, a range which is $n-1$ times smaller. Breaking the first term in the profit function into a sum over $n-1$ integrals over intervals of length $1/(2n(n-1))$ yields:

$$\pi(n) = 2\sum_{l=1}^{n-1}\int_{\frac{1}{2n}+\frac{l-1}{2n(n-1)}}^{\frac{1}{2n}+\frac{l}{2n(n-1)}} T(x)dx - 2\int_{0}^{\frac{1}{2n}} T(x)dx - F. \tag{10}$$

Since T is an increasing function,

$$\int_{\frac{1}{2n}+\frac{l-1}{2n(n-1)}}^{\frac{1}{2n}+\frac{l}{2n(n-1)}} T(x)dx \geq \int_{\frac{1}{2n}}^{\frac{1}{2(n-1)}} T(x)dx \tag{11}$$

with equality for $l=1$ and strict inequality for $l>1$. This implies that

$$\begin{aligned}\pi(n) &> 2(n-1)\int_{\frac{1}{2n}}^{\frac{1}{2(n-1)}} T(x)dx - 2\int_0^{\frac{1}{2n}} T(x)dx - F \\ &= \Delta W(n).\end{aligned} \tag{12}$$

This verifies that the profit of the n^{th} firm exceeds its contribution to social welfare. Hence there must be excessive entry.

Why does free entry not produce the efficient number of firms? To understand this, consider optimal locations with $n-1$ firms in the market. These locations are equally spaced around the circle, so that each firm is at distance $1/(n-1)$ from its neighbor. If n firms enter the market, they will locate equi-distantly and hence at distance $1/n$ from each other. Hence the welfare contribution of the marginal (n^{th}) firm is the difference in welfare between the latter and the former situation. On the other hand, the profits of the marginal firm equal its contribution to welfare when the other $n-1$ firms are unequally spaced, so that the distance between firms 1 and firm $n-1$ is $2/n$, while the distances between all other adjacent pairs of firms is $1/n$. Hence the profits of the marginal firm are given by its marginal contribution to welfare relative to an *inefficient* configuration of $n-1$ firms, whereas its contribution to welfare is its marginal contribution relative to an efficient configuration of $n-1$ firms. Hence profits are excessive and there is excess entry.

One can explicitly compute equilibrium outcomes and the social optimum for simple specifications of the transport cost function. Consider the case of transport costs of the form ($T(z)=t|z|^{\alpha}$). Ignoring integer constraints, the free entry equilibrium number of firms is given by

$$n^* = \left(\frac{2t(1-2^{-\alpha})}{F(1+\alpha)}\right)^{\frac{1}{1+\alpha}}.$$

In the case of linear transport costs considered by Salop ($\alpha=1$), this reduces to

$$n^* = \sqrt{\frac{t}{2F}}.$$

The welfare optimal number of firms is given by

$$\hat{n} = \left(\frac{t\alpha}{2^{\alpha}F(1+\alpha)}\right)^{\frac{1}{1+\alpha}}.$$

In the case of linear transport costs, this reduces to the expression

$$\hat{n} = \sqrt{\frac{t}{4F}}.$$

In this case $n^* = \sqrt{2}\hat{n}$. In the case of quadratic transport costs ($\alpha = 2$), we find that $n^* = \sqrt[3]{3}\hat{n}$. In general,

$$n^* = 2\left(\frac{1-2^{-\alpha}}{\alpha}\right)^{\frac{1}{1+\alpha}} \times \hat{n}.$$

Finally, it is worth comparing outcomes in the linear transport cost case with the equilibrium number of firms without price discrimination, $\tilde{n}$, which equals

$$\tilde{n} = \sqrt{\frac{t}{F}}.$$

Hence price discrimination reduces excessive entry to some extent, but does not ensure full efficiency.

2.2. **A General Discrete Choice Model.** We now consider a general discrete model of product differentiation and explore the conditions under which efficiency is ensured with perfect price discrimination. We use our canonical model, where entry takes place in stage 1, product characteristics ("location") are chosen in stage 2 and prices are chosen in stage 3. In order to ensure efficiency, we must have three features. First, in the pricing stage it must be the case that each firm can capture exactly its marginal contribution to consumer welfare. Second, we must have efficiency in the location stage. Finally, given efficiency on these counts, we must have efficient entry. We shall show that efficiency can be obtained on the first two counts, under certain assumptions, but is never obtained on the third count.

Let us assume that n firms have entered, indexed by $i \in \{1, 2, \ldots, n\} = N$. Assume that each firm i must choose $\theta_i \in \Theta$ where Θ is a compact metric space. We interpret θ_i as the characteristic of the product or "location" for short. Let $\boldsymbol{\theta} = (\theta_1, \theta_2, \ldots, \theta_n) \in \Theta^n$ denote the vector of location choices. We now analyze the pricing stage of the game where each firm i makes a take-it-or-leave-it offer to each consumer j.

We assume that each consumer purchases only one of the n varieties offered. Let $\lambda^j \in \Lambda$ parameterize the utility of consumer j and let this be distributed with a density $f(j)$. Let $u(x_i^j, \theta_i, \lambda^j) + Z$ denote the utility of the consumer with characteristic λ^j when she consumes x_i^j units of the product with characteristic θ_i and Z units of an outside numeraire. Let $c(\theta_i)$ denote the (constant) marginal cost of firm i when it produces the product with characteristic θ_i. Let $\hat{x}_i^j$ be the value of x_i^j that maximizes $u(x_i^j, \theta_i, \lambda^j) - c(\theta_i)x_i^j$ and let $b(\theta_i, \lambda^j)$ denote the maximized value. Let $N' = N \cup \{0\}$ be the set of firms, augmented by the outside option of non-consumption. The net benefit of the outside option, which we denote by $b(\theta_0, \lambda^j)$, is zero for every λ^j. Let $\phi(\boldsymbol{\theta}, \lambda^j) = \arg\max_{i \in N'} b^j(\theta_i, \lambda^j)$ be the index of the firm which provides the greatest net benefit to consumer j.

If we think of firms competing Bertrand fashion in offering benefits to the consumer then in equilibrium each firm $i \neq \phi(\boldsymbol{\theta}, \lambda^j)$ offers the consumer $b(\theta_i, \lambda^j)$ whereas firm $\phi(\boldsymbol{\theta}, \lambda^j) \in N$ offers $\max_{i \in N' \setminus \{\phi(\boldsymbol{\theta}, \lambda^j)\}} b(\theta_i, \lambda^j)$. Let

$\Phi_i(\boldsymbol{\theta})$ be the set of consumers j such that firm $i = \phi(\boldsymbol{\theta}, \lambda^j)$. The expression for total welfare corresponding to the characteristic vector $\boldsymbol{\theta}$ is given by

$$W(\boldsymbol{\theta}) = \sum_{k \in N} \int_{j \in \Phi_k(\boldsymbol{\theta})} b(\theta_k, \lambda^j) f(j) dj - nF. \tag{13}$$

On the other hand, the profits of firm i are given by

(14)
$$\begin{aligned}\pi_i(\boldsymbol{\theta}) &= \int_{j \in \Phi_i(\boldsymbol{\theta})} \left[b(\theta_i, \lambda^j) - \max_{k \in N' \setminus \{i\}} b(\theta_k, \lambda^j) \right] f(j) dj - F \\ &= \int_{j \in \Phi_i(\boldsymbol{\theta})} b(\theta_i, \lambda^j) f(j) dj - \sum_{k \neq i} \int_{j \in \Phi_k(\boldsymbol{\theta}_{-i}) \cap \Phi_i(\boldsymbol{\theta})} b(\theta_k, \lambda^j) f(j) dj - F.\end{aligned}$$

The second line of equation (14) follows from our assumption that $b(\theta_0, \lambda^j) = 0$ and the fact that $\arg\max_{k \in N' \setminus \{i\}} b(\theta_k, \lambda^j)$ is the variety that consumer j would purchase in the absence of variety i. It is straightforward to see that $\Phi_k(\boldsymbol{\theta}_{-i}) \cap \Phi_i(\boldsymbol{\theta})$ is the set of consumers firm k loses to firm i.

We now claim that

$$\pi_i(\boldsymbol{\theta}) = W(\boldsymbol{\theta}) - W(\boldsymbol{\theta}_{-i}). \tag{15}$$

To derive this, let us add $\sum_{k \neq i} \int_{j \in \Phi_k(\boldsymbol{\theta})} b(\theta_k, \lambda^j) f(j) dj - (n-1)F$ to the first term and also to the second term. The first term now becomes $W(\boldsymbol{\theta})$. The second term becomes $W(\boldsymbol{\theta}_{-i})$, since adding $\Phi_k(\boldsymbol{\theta})$, the set of k's remaining consumers, yields $\Phi_k(\boldsymbol{\theta}_{-i})$.

Hence we see that each firm's profits, as a function of its chosen characteristic θ_i, are equal to welfare as a function of θ_i less a constant which does not depend on θ_i. Therefore the profit maximizing characteristic θ_i^* also maximizes welfare and vice versa. Indeed, it follows that any decentralized maximizer of welfare must be a Nash equilibrium and conversely every Nash equilibrium must be a decentralized welfare maximizer.

The following example shows that one may have several characteristic vectors which are decentralized maximizers of welfare, whereas only one of these maximizes welfare globally. Since any decentralized maximizer is a Nash equilibrium, this shows that one can have coordination failures which prevent efficiency. One cannot expect price discrimination to prevent such coordination failures.

Example 1. Let there be two firms, $N = \{1, 2\}$ and let the set of possible locations equal $\Theta = \{1, 2, 3, 4, 5\}$. A unit mass of consumers is distributed across these locations, with a mass α_i at location i. Each consumer has inelastic unit demand for the product and incurs a transport cost—if the consumer at i purchases from the firm at location j, the transport cost is $T(\min\{\mid i - j \mid, \mid i + 5 - j \mid\})$. If the two firms locate at different places, then the maximum distance that any consumer has to travel is 2. Let $0 = T(0) < T(1) < T(2)$. The welfare criterion is therefore the sum of transport costs incurred by all the consumers in this market. Furthermore, assume that $T(2)$ is sufficiently large relative to $T(1)$, so that $\max \alpha_i T(1) < \min_j \alpha_j T(2)$—this ensures that it is never optimal the two firms to locate adjacent to each other. For example, if the firms locate at $(1,2)$, total transport costs are $(\alpha_3+\alpha_5)T(1)+\alpha_4 T(2)$, whereas if locations are $(1,3)$, transport costs are $(\alpha_2+\alpha_4+\alpha_5)T(1)$, and the latter is strictly lower by our assumption. Let $\alpha_2 > \alpha_1 > \alpha_3 > \alpha_4 > \alpha_5$. It follows that $(1,3)$ is a decentralized maximizer of the welfare function, with total transport cost $(\alpha_2+\alpha_4+\alpha_5)T(1)$. To verify this, note that neither firm can raise welfare by moving to location 2 and nor can the firm at 1 raise welfare by moving to 5 or the firm at 3 by moving to 4, by our assumption that $T(2)$ is large relative to $T(1)$. If the firm at 1 moves to location 5, the increase in transport costs is $(\alpha_1-\alpha_5)T(1)$ which is strictly positive. Similarly, if firm at 3 moves to location 4, the increase in transport costs is $(\alpha_3-\alpha_4)T(1)$, which is strictly positive. One can also verify that $(2,4)$ is a decentralized maximizer of the welfare function. Both these decentralized maximizers are Nash equilibria in the game where firms choose locations. The global maximum of the welfare function depends upon the relative sizes of $(\alpha_1+\alpha_3)$ and $(\alpha_2+\alpha_4)$: if the former is larger, $(1,3)$ is globally optimal, whereas if the latter is larger, $(2,4)$ is globally optimal. Hence Nash equilibria need not globally maximize welfare.

It is also instructive to consider the profits of the firms. In the equilibrium $(1,3)$, the firm at 1 earns $\alpha_5[T(2)-T(1)]$, while the firm at 3 earns $\alpha_4[T(2)-T(1)]$. In the equilibrium $(2,4)$, the firm at 2 earns $\alpha_1[T(2)-T(1)]$, while the firm at 4 earns $\alpha_5[T(2)-T(1)]$. Hence the profit vector at $(2,4)$ weakly dominates (in sense of a vector inequality) the profit vector at $(1,3)$, even though welfare may well be greater at $(1,3)$. In consequence firms may not have any incentives to coordinate their decisions in order to achieve a welfare optimum.

In Example 1, the distribution of consumer valuations is asymmetric, which gives rise to multiple decentralized maximizers with distinct welfare levels. Lederer and Hurter (1986) have also provided a similar example. They analyze price discrimination in the context of a two-firm Hotelling type model and show that equilibrium locations need not be globally optimal. Conversely, in the standard symmetric Hotelling and Salop models, every locational configuration which is a decentralized maximizer of welfare also maximizes welfare globally.[7] For example, given any locational configuration with $n-1$ firms located arbitrarily around the circle, the n^{th} firm will choose to locate between the two firms which are maximally far apart and will also locate equidistant between them. Therefore, in any Nash equilibrium, firms are equally spaced, since otherwise some firm could do better. Decentralized location decisions lead to a global optimum given the symmetry of this model.

[7]The decentralized maximizer is unique in Hotelling, while in the Salop model, there is a continuum of decentralized maximizers, all of which yield identical levels of welfare.

We now investigate the efficiency of free entry equilibrium. Since free entry can only ensure efficiency if there is efficiency in the post entry game, let us now assume:

Assumption 1. Given n, if the vector $\boldsymbol{\theta}$ is a decentralized maximizer of welfare, $\boldsymbol{\theta}$ also maximizes welfare globally.

Let $\boldsymbol{\theta}^*(n) \in \Theta^n$ denote a welfare maximizing configuration of firm characteristics when n firms have entered and let $W(\boldsymbol{\theta}^*(n))$ denote the corresponding level of welfare, gross of any fixed costs. If each firm incurs a fixed cost F of entering, the associated net welfare is $W(\boldsymbol{\theta}^*(n)) - nF$. Hence the increase in net welfare associated with the entry of the n^{th} firm is

$$\Delta W(n) = W(\boldsymbol{\theta}^*(n)) - W(\boldsymbol{\theta}^*(n-1)). \tag{16}$$

Consider now the profits of firm i when there are n firms which have entered. Let $\boldsymbol{\theta}^*_{-i}(n)$ be the $n-1$ dimensional characteristic vector $(\theta^*_1(n), \ldots, \theta^*_{i-1}(n), \theta^*_{i+1}(n), \ldots, \theta^*_n(n))$. $\boldsymbol{\theta}^*_{-i}(n)$ is the optimal location of the remaining $n-1$ firms when n firms have entered the market and $W(\boldsymbol{\theta}^*_{-i}(n))$ is the contribution to total welfare of these $n-1$ firms at this locational configuration. From equation (14) and the logic following it, we can rewrite profits as:

$$\pi(n) = W(\boldsymbol{\theta}^*(n)) - W(\boldsymbol{\theta}^*_{-i}(n)). \tag{17}$$

Hence the difference between profits and the contribution to welfare of the marginal firm is given by

$$\pi(n) - \Delta W(n) = W(\boldsymbol{\theta}^*(n-1)) - W(\boldsymbol{\theta}^*_{-i}(n)). \tag{18}$$

This expression (18) is always non-negative. To see this, note that $\boldsymbol{\theta}^*(n-1)$ maximizes welfare when there are $n-1$ firms and hence $W(\boldsymbol{\theta}^*(n-1)) \geq W(\boldsymbol{\theta}^*_{-i}(n))$. Furthermore, in general, $\boldsymbol{\theta}^*_{-i}(n)$, the optimal locational configuration of $n-1$ firms when n firms enter the market, will in general be different from $\boldsymbol{\theta}^*(n-1)$, the optimal locational configuration when $n-1$ firms enter and hence (18) will in general be strictly positive. It follows that we will have too much entry under perfect price discrimination, even when we have optimal provision of product characteristics with a fixed number of firms.

We summarize our results in the following proposition.

PROPOSITION 1. *Suppose that preferences are such that each consumer consumes at most one variety of the product and Assumption 1 holds.*

i) A vector of product variety choices, $\boldsymbol{\theta} \in \Theta^n$, is an equilibrium when n firms enter the market if and only if $\boldsymbol{\theta}$ is a decentralized maximizer of total welfare.

ii) If product variety choices are always efficient for any number of entrants, then the profits of the marginal entrant are always greater than its contribution to social welfare.

Our analysis applies to a wide variety of models of product differentiation. Indeed, it is particularly easy to check in applications whether one has efficiency or not. Most standard models in the literature are sufficiently symmetric that Assumption 1 is satisfied, i.e., for a fixed number of firms, locational equilibria are efficient. Free entry ensures efficiency if and only if it is the case that the set of locations in an efficient locational configuration with $n-1$ firms is a subset of the set of efficient locations when there are n firms. Using this insight, one can verify the following.

(1) Consider the model of vertical differentiation, as in Gabszewicz and Thisse (1980) or Shaked and Sutton (1983), where consumers are heterogeneous in terms of their willingness to pay for quality and where providing higher quality is also costly. The set of optimal qualities when there are $n-1$ entrants is not, in general, a subset of the set of optimal qualities when there are n entrants. For example, it is easily verified that with one firm, the optimal quality is intermediate between the optimal quality pair when there are two firms. Hence there will be excess entry.

(2) Consider the model of Deneckere and Rothchild (1992),[8] where there are K possible brands and a each consumer has a ranking of these brands, so that there are $K!$ types of consumer and where each consumer's cardinal utility from a product depends only upon its rank. The distribution of consumer types is symmetric if each of these $K!$ types is equally numerous. In the symmetric case, it is clear that every brand of product is equally efficient and hence the entry decisions will be efficient. However, if the distribution of consumer types is not symmetric, then the optimal set of brands with $n-1$ consumers will not in general be a subset of the set of optimal set of brands with n consumers. In this case, entry decisions will not be efficient. It is easy to construct such asymmetric examples.

2.3. **Simultaneous Entry and Location Choice.** Our results are very different from Spence, who argued that with perfect price discrimination, one has optimal product variety. His result depends crucially on the assumption that firms must simultaneously make their entry and location decisions. To understand Spence's argument, let us return to the Salop model and consider a situation where $n-1$ firms are already in the market, at fixed locations which are equally spaced around the circle. Suppose that an additional firm, n, is now given the option to enter the market and consider its entry decision. If this firm enters the market, it will be able to capture, as revenues, the increase in consumer surplus that such entry causes. If these revenues are greater than its costs, the firm will enter, but since such revenues are the firm's marginal contribution to social welfare, the firm's entry decision will be efficient. Furthermore, if the firm does enter, it will choose a location

[8] This model is closely related to that of Perloff and Salop (1985), so our comments apply equally to their model.

which results in the largest marginal contribution to social welfare. In other words, we see that the firm's entry/location decision must be a decentralized maximizer of social welfare.[9]

In our model, free entry does not result in social efficiency precisely because the entry of an additional firm does not leave the locations of the other n firms unchanged, since entry takes place in the first stage, before location decisions are made. In effect, if a firm enters "between" two other firms, all the firms rearrange themselves so that they are now equidistant from each other in the new situation. This implies that the revenues of the entrant are greater as compared to the fixed location case, since its two neighbors make room for it. Hence the equivalence between the marginal contribution to consumers' utility and the entrant's revenues no longer holds and entry is no longer efficient.

This discussion suggests that the inefficient entry only takes place because entry decisions affect subsequent location choices. Hence we consider a game where entry and location choices are made simultaneously, prior to pricing decisions being taken. In other words, in stage 1 firms have to decide whether to enter or not. If they enter, they also have to choose location. In stage 2, firms observe the entry/location decisions and choose consumer specific prices.

Let us analyze the Salop model, using this extensive form. From our previous analysis of location choice, it follows that any equilibrium must have the firms equally spaced, since otherwise a firm could increase its profits by changing location. Let $\pi(n)$ be firm profits given that n firms have entered at equally spaced locations. For $\bar{n}$ to be an equilibrium, we must have $\pi(\bar{n}) \geq 0$. We must also have that the profit of an additional entrant must be less than zero, given the entry and location choices of these n firms. Any optimal location for an additional entrant is halfway between two other firms. The profits at this configuration are given by $\pi(2\bar{n})$, i.e., the profits of a typical firm when $2\bar{n}$ firms are equally spaced around the circle. Hence we must have that the equilibrium satisfies $\{\bar{n} \mid \pi(\bar{n}) \geq 0 \text{ and } \pi(2\bar{n}) \leq 0\}$. Clearly there is a great multiplicity of equilibria here. For example, in the case of linear transport costs, we have

$$\sqrt{\frac{t}{8F}} \leq \bar{n} \leq \sqrt{\frac{t}{2F}}.$$

Recall that the social optimum, $\hat{n} = \sqrt{t/4F}$, which lies in this range. Hence the social optimum is an equilibrium, but there is also a continuum of inefficient equilibria where one has both too many firms and too few firms. This point has been noted by MacLeod et al. (1988), who analyze such a two stage game with perfect price discrimination. Note also that the excess entry

[9]Formally, if X is the set of possible locations and we augment this to $X \cup \{0\}$ where 0 denotes the decision to stay out, the argument of Proposition 1 shows that such augmented location decisions must be decentralized maximizers of social welfare.

equilibrium which obtains in our three stage game is always an equilibrium in this model.

To summarize, the two stage game has a large number of equilibria, and in many of these, the firms make positive profits. These equilibria have the flavor of entry-deterrence—the firms that enter choose locations in such a way as to restrict the number of entrants. However, as is argued by Judd (1985), if the cost of brand re-positioning is relatively low, such equilibria are not credible. This is because when an unanticipated rival chooses to enter, the remaining firms will prefer to re-brand their product. Therefore if the cost of re-branding is sufficiently low, the efficient equilibrium when entry and variety decisions are simultaneous is not credible.

Finally, we should note that the model implicit in the original Spence (1976) article is not completely clear. An alternative interpretation suggests a "citizen-candidate" model of product differentiation, where each potential entrant has the technology to produce one variety and one variety only.[10] In the Hotelling version of such a model, we would have each potential entrant located on the unit interval, and each potential entrant would have to decide whether to enter or not. Such a model has a plethora of inefficient equilibria—for example, if the firms located at 1/3 and 2/3 enter, then the firm located at 1/4 will not find it profitable to enter. Note that each of these equilibria correspond to decentralized maximizers of social welfare, since the entrant at 1/3 increases social welfare by entering rather than staying out, given the decisions of all other firms.

3. Representative Consumer Models

We now consider representative consumer models. Somewhat different issues arise in such a model, since the consumer's benefit from one product depends also on her consumption of other products and hence one needs additional assumptions to ensure that a firm will capture its marginal contribution to welfare. Nevertheless, it remains the case that even if a firm were to so capture its marginal contribution, we still have excessive entry. We begin with a simple example, a version of the Dixit-Stiglitz model and then consider a more general framework.

3.1. **A Dixit-Stiglitz Model.** We suppose that a consumer has utility function

$$U = \left(\sum_{i=1}^{n} {x_i}^{\rho} \right)^{\alpha/\rho} + Z \tag{19}$$

where $\rho \in (0,1)$ and $\alpha \in (0,1)$. Z denotes the outside good. We assume also that firms have constant marginal cost of production c and fixed cost F. The game is as follows: in the first stage, each firm decides whether to

[10]Such a model has the flavor of models of political competition used by Besley and Coate (1997) and Osborne and Slivinski (1996).

enter or stay out and in stage 2, firms make their take it or leave it offers to the consumer after having observed the number of entrants in stage 1.

If firm i's $n-1$ rivals produce output, x then i's profits under perfect price discrimination is given by

$$\pi_i(x_i, x, n) = ((n-1)x^\rho + {x_i}^\rho)^{\alpha/\rho} - ((n-1)x^\rho)^{\alpha/\rho} - cx_i - F. \tag{20}$$

In a symmetric equilibrium, each firm will produce $x_i = x$ which solves:

$$\alpha x^{\alpha-1} n^{(\alpha-\rho)/\rho} = c. \tag{21}$$

Call this symmetric equilibrium output x_n^*. It is straightforward to see that x_n^* also maximizes welfare when there are n firms.

Notice x_n^* is decreasing in n if $\rho > \alpha$—this can be interpreted as the case where there is diminishing marginal utility of the composite good of the monopolistic competitive sector, i.e., when the products of the sector are substitutes. Conversely, x_n^* is increasing in n if $\rho < \alpha$, so that $\alpha/\rho > 1$. In this case, we see that the products are complements. We assume that $\alpha < \rho$ so that x_n^* is decreasing in n.

Now consider the optimal entry decision under perfect price discrimination. Given some n, firms will still choose the optimal level of output to produce, x_n^*. However, consider the expression for welfare:

$$W(n) = (n{x_n^*}^\rho)^{\alpha/\rho} - n(cx_n^* + F). \tag{22}$$

The change in welfare when the n^{th} firm enters is therefore:

(23)

$$\begin{aligned}\triangle W(n) &= (n{x_n^*}^\rho)^{\alpha/\rho} - ((n-1){x_{n-1}^*}^\rho)^{\alpha/\rho} - ncx_n^* + (n-1)cx_{n-1}^* - F \\ &= \pi_i(x_n^*, x_n^*, n) + [((n-1){x_n^*}^\rho)^{\alpha/\rho} - (n-1)cx_n^*] - \\ &\quad [((n-1){x_{n-1}^*}^\rho)^{\alpha/\rho} - (n-1)cx_{n-1}^*].\end{aligned}$$

Adding and subtracting $((n-1){x_n^*}^\rho)^{\alpha/\rho}$ yields the second line. The bracketed terms following $\pi_i(x_n^*, x_n^*, n)$ are total welfare, gross of fixed costs, with $n-1$ varieties and when firms produce x_n^* and x_{n-1}^* respectively. Notice that the second bracketed term is strictly greater than the first since x_{n-1}^* is chosen optimally when there are $n-1$ varieties. Therefore the private value of entry is strictly greater than the social value of entry.

The rationale for excess entry is similar to the rationale in the discrete choice model. Given n entrants, equilibrium output for each equals the welfare maximizing level of output, x_n^*. At this output configuration, each firm captures as revenues, its marginal contribution to consumer welfare, given that every other firm is producing x_n^*. However, the contribution to consumer welfare of the marginal entrant is given by the difference in consumer utility between the n vector where each firm produces x_n^* and the $n-1$ vector where each firm produces x_{n-1}^*. Since x_{n-1}^* is welfare optimal when there are $n-1$ firms, the marginal entrant's profits are greater than its contribution to social welfare. Put differently, the act of entry alters rival firms' optimal output choices so that (20) overstates the social benefit from entry. This basic rationale extends to a general representative consumer model.

3.2. **A General Representative Consumer Model.** Consider now the case where the representative consumer consumes all varieties of the product, supplied by n firms. Let $\boldsymbol{x} = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n_+$ denote the vector of choices of these n firms. Let $U(\boldsymbol{x}, \boldsymbol{\theta}) + Z$ be the consumer utility corresponding to the chosen characteristics vector $\boldsymbol{\theta}$, the output profile $\boldsymbol{x}$ and consumption of the outside good Z. Let $\boldsymbol{e}_i$ denote the n-vector with a 1 in its i^{th} component and a 0 everywhere else and for any set $S \subset N$, let $\boldsymbol{e}_S = \sum_{i \in S} \boldsymbol{e}_i$. To model non-linear pricing, we assume that firms can make take-it-or-leave-it offers to consumers. Firm i's marginal contribution to consumer utility at the pair $(\boldsymbol{x}, \boldsymbol{\theta})$ is given by

$$M_i(\boldsymbol{x}, \boldsymbol{\theta}) = U(\boldsymbol{x}, \boldsymbol{\theta}) - U((\boldsymbol{x} - \boldsymbol{e}_i \boldsymbol{x}), \boldsymbol{\theta}). \tag{24}$$

Now suppose that each firm i makes a take-it-or-leave-it offer at the vector $(\boldsymbol{x}, \boldsymbol{\theta})$ where it demands $M_i(\boldsymbol{x}, \boldsymbol{\theta})$. Clearly the consumer is indifferent between accepting and refusing the offer of firm i, given that she accepts all the offers of the remaining $n-1$ firms. In addition, the consumer must not be better off by refusing the offers of two or more firms.

Assumption 2. For any subset S of firms, the sum of marginal contributions is less than contribution of the subset S:

$$\sum_{i \in S} M_i(\boldsymbol{x}, \boldsymbol{\theta}) \leq M_S(\boldsymbol{x}, \boldsymbol{\theta}) = U(\boldsymbol{x}, \boldsymbol{\theta}) - U(\boldsymbol{x} - \boldsymbol{e}_S \boldsymbol{x}, \boldsymbol{\theta}). \tag{25}$$

If this condition is satisfied for every subset S of N, then clearly the revenues of each firm $i \in N$ will equal its marginal contribution, $M_i(\boldsymbol{x}, \boldsymbol{\theta})$. If this condition is not satisfied, then it follows that each firm cannot capture its marginal contribution as revenues—if S is a subset of N such that Assumption 2 is not satisfied, and if firm revenues equal marginal contributions, then the consumer will be better off rejecting the offers of all firms in S.[11]

The general condition for equilibrium revenues, $(R_i^*(\boldsymbol{x}, \boldsymbol{\theta}))_{i \in N}$, is that for each i, $R_i^*(\boldsymbol{x}, \boldsymbol{\theta})$ is maximal subject to the constraints:

$$R_i^*(\boldsymbol{x}, \boldsymbol{\theta}) \leq M_i(\boldsymbol{x}, \boldsymbol{\theta}) \tag{26}$$

$$\sum_{i \in S} R_i^*(\boldsymbol{x}, \boldsymbol{\theta}) \leq M_S(\boldsymbol{x}, \boldsymbol{\theta}), \forall S \subset N. \tag{27}$$

In general, if Assumption 2 is not satisfied and firms cannot capture their marginal contributions, it follows that equilibrium revenues are not unique

[11] These conditions are analogous to those set out by Bergemann and Valimaki (1999) and Laussel and Le Breton (1996), in the context of common agency models. We discuss the relation between models of price discrimination and common agency models in greater detail in the following section.

and that we may have inefficient equilibria. To make this clear, consider the following example.

Example 2. There are two firms, $N = \{1, 2\}$. The consumer can consume $x_i \in \{0, 1\}$. Hence at the vector $(1, 1)$, firm 1's marginal contribution is $U(1, 1) - U(0, 1)$, while firm 2's marginal contribution is $U(1, 1) - U(1, 0)$. The condition (2) reduces to

$$U(1, 1) + U(0, 0) \leq U(0, 1) + U(1, 0). \tag{28}$$

This condition says that the utility function U must be sub-modular. If instead $U(1, 1) + U(0, 0) > U(0, 1) + U(1, 0)$, then equilibrium revenues satisfy $R_1^*(1, 1) + R_2^*(1, 1) = U(1, 1) - U(0, 0)$, with $R_1^*(1, 1) \leq U(1, 1) - U(0, 1)$ and $R_2^*(1, 1) \leq U(1, 1) - U(1, 0)$. Clearly there is a continuum of equilibria satisfying these conditions. Suppose now that marginal costs of production are zero and that the entry fixed costs are F_1 and F_2. If $F_i \leq M_i(1, 1)$ and $F_1 + F_2 \leq U(1, 1) - U(0, 0)$, then it is efficient for both firms to enter. However, Nash equilibrium revenues need not cover the fixed cost of one firm and hence there exist equilibria which are not efficient. For example, let $U(1, 1) = 3$, $U(0, 0) = 0$, $U(1, 0) = U(0, 1) = 1$. Hence $R_1^* \in [1, 2]$, $R_2^* = 3 - R_1^*$. If $F_1 + F_2 < 3$, $F_i < 2$ for $i \in \{1, 2\}$, then efficiency requires that both firms enter. However if $F_i > 1$ for some i, then there exists an equilibrium where firm i gets revenue less than F_i in the pricing stage and hence will not enter. Note that there always exists an equilibrium which is efficient, as well as inefficient equilibria.

In short, we cannot expect that firms will be able to capture their marginal contributions unless the submodularity condition is satisfied, which ensures that products are substitutes.

We shall henceforth assume that Assumption 2 is satisfied, so that each firm's revenue always equals its marginal contribution to consumer welfare. In this case, we may write the firm's profits as

$$\pi_i(\boldsymbol{x}, \boldsymbol{\theta}) = U(\boldsymbol{x}, \boldsymbol{\theta}) - U(\boldsymbol{x} - \boldsymbol{e}_i \boldsymbol{x}, \boldsymbol{\theta}) - c(\theta_i)x_i - F. \tag{29}$$

Now examine the choice of output of the firm. Since the firm chooses x_i to maximize $\pi_i(x_i, \boldsymbol{x}_{-i}, \boldsymbol{\theta})$, this is equivalent to maximizing $U(x_i, \boldsymbol{x}_{-i}, \boldsymbol{\theta}) - c(\theta_i)x_i$ save for a constant term, $U(\boldsymbol{x} - \boldsymbol{e}_i \boldsymbol{x}, \boldsymbol{\theta}) + F$, which does not depend upon x_i. Since this is the firm's contribution to social welfare, this output choice is decentralized welfare optimal. We therefore conclude that output choices will be optimal under Assumption 2.

Since product characteristic choice and free entry can be efficient only if production decisions are efficient, we make the following assumption:

Assumption 3. Given n and $\boldsymbol{\theta}$, if $\boldsymbol{x}$ is a decentralized maximizer of welfare, $\boldsymbol{x}$ also maximizes welfare globally.

Consider now the choice of product characteristics. Given $\boldsymbol{\theta}$, let $\boldsymbol{x}(\boldsymbol{\theta})$ denote the welfare maximizing output configuration. Hence we require that

$\boldsymbol{\theta}$ maximize

$$W(\boldsymbol{x}(\boldsymbol{\theta}), \boldsymbol{\theta}) = U(\boldsymbol{x}(\boldsymbol{\theta}), \boldsymbol{\theta}) - \sum_{i \in N} c(\theta_i) x_i(\boldsymbol{\theta}) - F. \tag{30}$$

On the other hand, the firm's profits at the profile $\boldsymbol{\theta}$ are given by

$$\tilde{\pi}_i(\boldsymbol{\theta}) = U(\boldsymbol{x}(\boldsymbol{\theta}), \boldsymbol{\theta}) - U(\boldsymbol{x}(\boldsymbol{\theta}) - \boldsymbol{e}_i \boldsymbol{x}(\boldsymbol{\theta}), \boldsymbol{\theta}) - c(\theta_i) x_i(\boldsymbol{\theta}) - F. \tag{31}$$

Note that $U(\boldsymbol{x}(\boldsymbol{\theta}) - \boldsymbol{e}_i \boldsymbol{x}(\boldsymbol{\theta}), \boldsymbol{\theta})$ does not depend upon θ_i, the choice of firm i. Intuitively, this says that consumer utility when firm i produces zero output does not depend upon the type of product of this firm. Hence we see that if each firm chooses θ_i to maximize $\tilde{\pi}_i(\boldsymbol{\theta})$, then $W(\boldsymbol{\theta})$ cannot be increased by altering any single component θ_i, so that $\boldsymbol{\theta}$ is a decentralized maximizer of social welfare. Our results here are identical to those obtained previously in the context of the model where each consumer only purchased one type of product. Every decentralized maximum is a Nash equilibrium and hence a globally optimal characteristic vector is always a Nash equilibrium. Again, global efficiency is not necessarily ensured.[12]

We now assume that each firm can capture its marginal contribution at the pricing stage (Assumption 2). In conjunction with Assumptions 1 and 3, this implies that the profile of characteristics and output choices are always globally optimal. We are now in a position where we can examine the properties of free entry equilibria.

Suppose that n firms enter the market and will choose product characteristics $\boldsymbol{\theta}_n^*$ and produce outputs $\boldsymbol{x}_n^*$. We assume that these choices are socially optimal. Hence the change in welfare associated with the entry of the n^{th} firm is

$$\Delta W(n) = W(\boldsymbol{x}_n^*, \boldsymbol{\theta}_n^*) - W(\boldsymbol{x}_{n-1}^*, \boldsymbol{\theta}_{n-1}^*) - F. \tag{32}$$

On the other hand, the profits of the marginal firm n are given by its marginal contribution:

$$\hat{\pi}(n) = W(\boldsymbol{x}_n^*, \boldsymbol{\theta}_n^*) - W(\boldsymbol{x}_n^*, \boldsymbol{\theta}_n^* - e_n \boldsymbol{x}_n^*, \boldsymbol{\theta}_n^*) - F. \tag{33}$$

It follows that the difference between profits and contribution to welfare is given by

$$\pi(n) - \Delta W(n) = W(\boldsymbol{x}_{n-1}^*, \boldsymbol{\theta}_{n-1}^*) - W(\boldsymbol{x}_n^*, \boldsymbol{\theta}_n^* - e_n \boldsymbol{x}_n^*, \boldsymbol{\theta}_n^*). \tag{34}$$

Recall that by Assumption 1, for any integer m, the profile $(\boldsymbol{x}_m^*, \boldsymbol{\theta}_m^*)$ maximizes the function $W(\boldsymbol{x}, \boldsymbol{\theta})$. Since this true when $m = n-1$, it follows that $\pi(n) - \Delta W(n) \geq 0$ and hence one has excessive entry.

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[12] It is easy to provide an example with complementary goods. Let there be two firms and suppose that each firm $i \in \{1, 2\}$ must choose $\theta_i \in [0, 1]$, where θ_i is the quality of the product. The consumer consumes one or zero units of each product and obtains positive utility only by consuming both products. If she consumes both products, her utility given by $U(\theta_1, \theta_2) = V(\min(\theta_1, \theta_2))$. There will be a multiplicity of pairs $\theta_1 = \theta_2$ which are decentralized welfare maximizers even though there is only one which is globally optimal. With heterogenous consumers, we have seen that one can have such coordination failures even with substitute products.

We summarize the results of this section in the following proposition, which mirrors our previous result for the discrete choice model:

PROPOSITION 2. *Suppose that the representative consumer consumes many varieties of the product and that Assumptions 1, 2 and 3 are satisfied.*

i) Output and product variety choice pair, $(\boldsymbol{x}, \boldsymbol{\theta}) \in \mathbb{R}_+^n \times \Theta^n$*, is an equilibrium when* n *firms enter the market if and only if* $\boldsymbol{x}$ *and* $\boldsymbol{\theta}$ *are decentralized maximizers of total welfare.*

ii) If output and product variety choices are always efficient for any number of entrants, then the profits of the marginal entrant are always greater than its contribution to social welfare.

4. Common Agency

Our analysis is directly applicable to the model of common agency, introduced by Berhneim and Whinston (1986). This has found extensive applications, especially in the context of lobbying. The canonical model of common agency with complete information has a single agent who takes an action a in some set A, with n principals who offer transfers to the agent contingent on the action chosen. Berhneim and Whinston introduced the notion of *truthful equilibrium*—such truthful equilibria always induce efficient outcomes in this game between the principals. This model has many commonalities with our analysis of price discrimination—we may think of the single consumer (or the coalition of all consumers) as a single agent, while the firms are analogous to the principals. The action consists of the consumer's consumption bundle, with the difference that the transfers are from the principal to the agent. Since each firm only cares about what the consumer buys from it and is indifferent about the consumer's other purchases, one does not need the truthful refinement to ensure efficiency in this context.

The question is, do we have efficiency in common agency when the principals have to undertake prior non-contractible investments which affect payoffs in the common agency game? This question has been raised by Bergemann and Valimaki (1999). They argue that such efficiency can be ensured if and only if each principal can secure her *marginal contribution* in the truthful equilibrium. If this is the case, they argue that investment incentives are correctly specified.

Our analysis in the present paper shows that marginal contribution equilibria, in which each principal earns her marginal contribution to social welfare, is not sufficient to ensure efficiency if one considers the prior investment decisions of the principal and even more if one considers entry into the ranks of principals. To see this, let us reformulate our models of spatial competition—the Salop model and the five location model of Example 1—as common agency model. Suppose that there is a single agent, whose location will be randomly determined with probability as corresponding to this spatial model. That is, in the Salop model, the agent's location will be randomly determined by the uniform distribution on the circle and in Example 1, the probability that the agent is at location i is α_i. Suppose that the principal's have to choose locations before the realization of the agent's type, and that the agent can work for at most one principal and the value of this work to the principal concerned is a constant minus the transport cost incurred. These models are formally identical to the ones we have analyzed and since there are no externalities between the principals, all equilibria at the final stage, when the principals compete for the agent, are truthful and are efficient. Furthermore, each principal's expected return is her expected marginal contribution. However, as Example 1 demonstrates, the equilibria of the location stage need not maximize social welfare globally. More fundamentally, Propositions 1 and 2 demonstrate that even if one has efficiency at the stage where the principals choose their investment/location decisions, one will not have efficiency in the entry decision. In particular, one always has excessive entry into the ranks of the principals or lobbyists.

5. Concluding Comments

Does unfettered entry provide the socially optimal number of firms and products? While models of monopolistic competition agree that the answer is no, when firms cannot price discriminate, they differ on the direction of the bias. The Salop model shows that there will be overprovision of product variety, while in the Dixit-Stiglitz model, one may have either overprovision or underprovision. The literature agrees that there are two conflicting effects at work here. The business stealing effect makes for excessive entry.[13] On the other hand, with linear prices, firms are unable to appropriate their contribution to consumer surplus and hence there is a tendency towards insufficient entry.

Our main result is that quite generally, one has excessive entry if there is perfect price discrimination. In the light of the discussion above, one way of thinking about this is that with discriminatory pricing, firms can fully appropriate their contribution to consumer surplus and hence only the business stealing effect remains. An alternative to this intuition is that the marginal firm does appropriate its marginal contribution, but relative to an inefficient allocation rather than an efficient one, as argued by Spence. Hence profits are greater than its contribution to welfare.

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