Innovation, Market Structure and Economic Growth*

Nuno Pedro Gonçalves Palma†

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Abstract

This paper presents a dynamic partial equilibrium model that endogenizes firms' investment decision on innovation: product innovation causes horizontal expansion growth, and process innovation causes vertical expansion growth. Market structure in different markets emerges as a consequence of different investment on innovation opportunities. Main variables that constrain this structure in a given market are the rate of scientific (basic) discoveries that permit innovation productivity, and the degree of substitution between varieties, together with the possible existence of scope economies.

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†Technical University of Lisbon - Faculty of Economics and Management (I.S.E.G.)
E-mail address: nunopedropalma@hotmail.com

1. Introduction and Literature Review

There have been traditionally two different approaches to the relation between industrial organization and economic growth: The horizontal expansion or product innovation approach, taken by Dixit-Stiglitz (1977) and Romer (1990), and the vertical expansion or process innovation approach, sometimes also called quality ladders, as in Aghion and Howit (1992). In the first case, growth is caused by the increase in the number of existing varieties. In the latter case the cause is technological improvements on existing products.

In the real-world firms produce both types of innovations. They may choose to improve the quality of their products, lower production costs, or introduce new varieties. Their decision of how much and in which type to invest, like any other investment decision, is based on a cost-benefit analysis. Does market structure influence their decision or is it the other way around? In other words, is Microsoft efficient because it has market power, or does it have market power because it is efficient?

This paper presents a model that endogenizes firms' simultaneous decision about both types of innovation. This decision is not really a choice; it is a survival necessity for firms. In a high innovation environment, who doesn't follow, eventually dies. In this sense it may seem that the investment on innovation decision is constrained by market structure but this structure is really a consequence of different investment opportunities that exist in different markets. This is a different approach from the standard paradigm that relates growth to market structure (see Aghion and Howit 1998 for an overview).

One consequence of the above is that there is reverse causality in the Schumpeter hypothesis. This is, firms with large market shares produce more innovation because they have more access to capital markets, risk diversification and can benefit from scale economies in R&D. (Schumpeter 1947). The model presented in this paper implies that innovation, bounded by investment opportunities, determines market structure, and not the other way around. Having this in mind, it is not surprising the apparent contradiction of empirical work in search of confirmation for the Schumpeter hypothesis. For example, some studies find evidence that supports this hypothesis, as larger firms have higher patent rates. The larger a firm's market share, the more likely it is to file patents (see Liberman 1987). Others find evidence that contradicts Schumpeter: In some cases, small firms are relatively more likely to make major innovations (see Scherer 1984). Other studies report results that seem contradictory: Cohen, Levin and Mowery (1994) conclude that R&D intensity varies with firm size in some industries and not others, and where it does vary, it ay be negatively or positively related to size. We will see that all these results are consistent with the model presented in this paper.

It is now for a long time acknowledged by economists, at least since the work of Solow (Solow, 1957) that growth is not mainly caused by the accumulation of physical factors of production like capital and labour. For simplicity reasons, the model in this paper does not include capital or labour as factors of production. It uses only technology, because it is the main source of growth. This is certainly a shortcoming, but smaller that the one in textbook models that use these factors but do not endogenize innovation. Also this model can be easily expanded to include other factors, only at the cost of some analytical complication.

It has been recognized by others before me (see, for example, Baumol 2002) that knowledge is created in a process that follows two types of discovery. Basic research creates breakthrough ideas and is done primarily by universities and government agencies. I will call these ideas inventions. It is however the subsequent development of these ideas by private firms in search for profits - innovations - that is responsible for the lion's share of the growth. Statistical evidence of this is presented in Baumol (2002). However, inventions are a necessary condition for the innovations to exist. In other words, the R&D process can be divided in research (of inventions), and development of these inventions into innovations. This will become clear with the presentation of the model. Figure 1 represents a schematic view of what I just described.

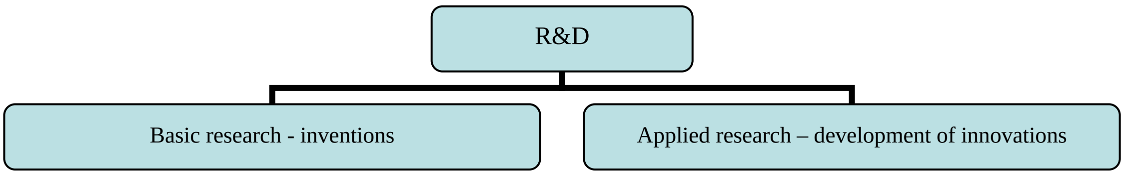

Figure 1: The R&D process

This model takes inventions as exogenous. These have existed since the beginning of mankind and were frequent in past civilizations before the industrial revolution. It is however the unique nature of capitalism that allows them to be developed into innovations, and cause modern economic growth.

No theory can be stated as useful if it cannot be tested empirically. In economics this often poses the question of observables. In case of the study of market structure, an important question is if when a given market was created, entry was sequential or simultaneous. I argue that in absence of non-natural barriers to entry, this is mainly a matter of information asymmetries. Empirically this is a question that

can only be addressed in an historical perspective, so I will not address it at all. Also because of this, this model does not consider strategic behaviour. The usage of game theory would also depend on the nature of competition: we can have Nash equilibrium on quantities (Cournot equilibrium) or in price (Bertrand equilibrium). Price (or quantity) competition is not, however, the main kind of competition in most markets today. Rather, in a dynamic environment it is innovation the main war that firms battle. (see Baumol 2002).
On part 2, the basic model is presented. Its solution is in part 3, as well as some crucial developments, such as dynamics. Economic policy and its consequences for growth also appear in this part. Part 4 presents some conclusions, and part 5 suggests possible future extensions to the model, as well as some of its limitations.

2. The Model

A. Firms and markets

There are s markets, and each has $v_{k,t}$ endogenously determined firms in moment t, $k=1,...,s$. For now let's define market as a strong break in the chain of substitution on the demand side. A more formal definition will be given later in this paper. Each firm is assumed to use a production function of the form:

$$x_{i,t} = A_{i,t} \qquad (1)$$

This is, the quantity produced of each variety i in each period t, $x_{i,t}$ is equal to the technological level in production, $A_{i,t}$, of that variety. All varieties are heterogeneous and each variety is produced by a single firm. The number of varieties produced by firm j are $n_{j,t}$.

The profit function for any firm j takes the following form, for each period t:

$$\Pi_{j,t} = \sum_{i=1}^{n_{j,t}} x_{i,t}\, p_{i,t} - \Psi c_{j,t} - \Psi \sum_{i=1}^{n_{j,t}} d_{i,t} \qquad (2)$$

Where $x_{i,t}$ and $p_{i,t}$ are respectively the quantity and the price of variety i. There are only two types of costs: $c_{j,t}$ is spending on product innovation and $\sum^{n_{j,t}} d_{i,t}$ total spending on process innovation. Marginal cost of both types of innovation is assumed to be constant and equal to Ψ.

Assumption 1. The total number of product innovations in each period t made by each individual firm is never very large.

This means that when optimizing, each firm will take as null the impact of an additional variety on its profits from the varieties that existed in previous periods.

This assumption is necessary for the sake of the analytical simplicity of the model. (this can be easily seen in the appendix).

B. Invention and innovation

Process innovation follows the process:

(3) $$A_{i,t+1} - A_{i,t} = \lambda_k . A_{i,t} . d_{i,t}$$

and product innovation follows the process:

(4) $$n_{i,t+1} - n_{i,t} = \lambda_k . n_{i,t} . c_{j,t}$$

Where λ_k is the (exogenous) invention rate which is assumed to be the same for both kinds of innovation processes. Note that this is a constant that would correspond to a Poisson arrival rate in a model with uncertainty. New varieties are created with initial technology A_0.

Assumption 2. There is a potentially different invention rate λ_k for each one of the $k = 1,\ldots,s$ markets.

At first this may seem an odd assumption. If the invention rate is related to scientific progress, it should differ between different scientific areas, and not between breaks in the chain of substitution that define different markets. This is true, but does not contradict assumption 2. What is assumed is that each market has its own invention rate because it corresponds to some scientific area. In markets that belong to close scientific areas, the invention rate may be similar, or even equal. Markets that are in-between two or more areas will have a rate that reflects the degree of closeness to each. Figure 2 presents an example.

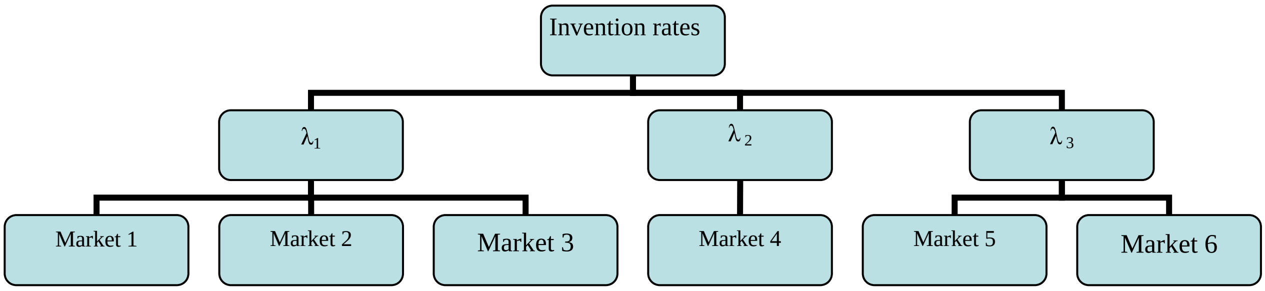

Figure 2: Inventions and Markets: An example

As was empathized in the introduction, in the real world different sectors of the economy suffer scientific shocks with different intensities and time lags. Later in this paper I will consider the effects on a change in the invention rate of a given market.

C. Demand

Following Dixit-Stiglitz (1977), the demand for variety *i* in period *t* is:

(5) $$x^d_{i,t} = (p_{i,t}/ P_t)^{\sigma_k} . [y_{k,t}/m_t]$$

The demand $y_{k,t}$ corresponds to the total demand that in period *t* is directed to the industry *k* to which variety *i* belongs to. As this is a partial equilibrium model, it is considered exogenous. P_t and $p_{i,t}$ are respectively the price level and the price of variety *i* in period *t* in market *k*. (Wherever clear from the context to which market they belong, these shall not be indexed by *k*.) The parameter σ_k measures the degree of linkage between submarkets of the same market, in the Sutton (1998) fashion. Lower σ_k means, on the demand side, less substitutability between varieties to the consumers, and on the supply side it may represent the existence of scope economies in production (or the existence of the so-called love for variety). The total number of varieties in the industry in period t are $m_t = \sum^{v_k} n_{j,t}$.

I will now follow a third assumption.

Assumption 3. There is independence between different markets. This means: a) there is a potentially different σ_k for each market. b) varieties are only substitutes between submarkets of the same market and never between markets. Any scope economies that may exist apply only inside the same market. c) there is a fixed and exogenous demand $y_{k,t}$, potentially different for each market $k = 1,...,s$.

D. National Product

The national product is given by:

(6) $$Y_t = \sum_{j=1}^{s} \left(\sum_{i=1}^{m_t} x_{i,t}\right)$$

E. Optimization

Each firm's problem is to maximize the intertemporal present discounted value of profits:

$$\sup \left(\sum_{t=0}^{\infty} \beta^{t} \Pi_{j,t} \right) \tag{7}$$

The state variables for each firm are the technology level for each one of the existing varieties $A_{i,t}$, $i=1,...,n_{j,t}$ and the total number of varieties $n_{j,t}$. The control variables are the amount of spending on innovation on each one of the varieties i on period t (process innovation), $d_{i,t}$, $i=1,...,n_{j,t}$ and the amount of spending on the development of new products, $c_{j,t}$.

3. Solution and policy

A. Closed-form solution

It turns out that a unique closed-form solution exists for the optimal spending on each type of investment on innovation in each period t. (the proof can be found in the appendix).

$$d_{i,t} = \frac{[(1-1/\sigma_k)\beta P_{t+1} \; \lambda_k \; A_{i,t}{}^{1-1/\sigma k}]^{\sigma_k} \; (y_{k,t+1} \; m_{t+1}{}^{\square 1}) - \Psi^{\sigma k}}{\lambda_k \; \Psi^{\sigma k}} \tag{8}$$

and

$$c_{j,t} = \frac{[\beta P_{t+1} \; \lambda_k \; A_0{}^{1-1/\sigma k}]^{\sigma_k} \; (y_{k,t+1} \; n_{j,t}{}^{\sigma k \square 1}) - [(m_{t+1}+1)/n_{j,t}]}{\lambda_k \; \Psi^{\sigma k}} \tag{9}$$

B. The long run

In the short run the number of firms is fixed. In the long run there is free entry in all markets. Because of this in each market there will be entry until the next firm could not achieve positive discounted profits. The theory does not exclude the possibility of different structures of profits coexisting in the same market. However, without imposing any restriction relative to the entry process, nothing can be said about what kind of structure will emerge. This requires an additional assumption.

Assumption 4. There is symmetric equilibrium in all markets, but not between different markets. This means that in a given market k, all firms produce the same output and invest the same on innovation. However, in different markets, these quantities obviously vary.

This assumption is not essential for the model. In fact, if instead an assumption about the nature of the entry process is taken, the model is valid and can be used, only to reach different conclusion about the final market structure.

In each market k, the long run condition is:

$$(10) \qquad v_k : \left(\sum_{t=0}^{\infty} \beta^t \Pi_{j,t}\right) = 0 \quad \text{for all } j.$$

C. Economic growth

Growth can occur for two reasons: New varieties or improvements on the technology of the existing ones. (quality improvements). Investment on innovation is the cause for both. Having in mind the equation for output (6) it is possible to write:

$$(11) \qquad \frac{\Delta Y_{t+1}}{Y_{t+1}} = \sum_{k=1}^{s} \left(\sum_{j=1}^{v_{k,t}} A_0 \frac{\Delta n_{j,t}}{n_{j,t}} + \sum_{i=1}^{m_t} \frac{\Delta A_{i,t}}{A_{i,t}} \right)$$

using equations (3) and (4) we get:

$$(12) \qquad \frac{\Delta Y_{t+1}}{Y_{t+1}} = \sum_{k=1}^{s} \left[\lambda_k \left(\sum_{j=1}^{v_{k,t}} A_0\, c_{j,t} + \sum_{i=1}^{m_t} d_{i,t} \right) \right]$$

Where the optimal level of investment in each type of innovation can be taken from equations (7) and (8).

D. Fiscal policy

The fiscal multiplier is given by:

$$(13) \qquad \Delta Y_{t+1} = \Delta G_{k,t} \left(\frac{\partial Y_{t+1}}{\partial n_{j,t}} \frac{\partial n_{j,t}}{\partial c_{j,t}} \frac{\partial c_{j,t}}{\partial G_{k,t}} + \sum_{i=1}^{n_{j,t+1}} \frac{\partial Y_{t+1}}{\partial A_{i,t+1}} \frac{\partial A_{i,t+1}}{\partial d_{i,t}} \frac{\partial d_{i,t}}{\partial G_{k,t}} + \sum_{k=1}^{s} \frac{\partial Y_{t+1}}{\partial \Pi_{k,t}} \frac{\partial \Pi_{k,t}}{\partial v_{k,t+1}} \frac{\partial v_{k,t+1}}{\partial G_{k,t}} \right)$$

Note that:

$$(14)\qquad \frac{\partial\, d_{i,t}}{\partial\, G_{k,t}} = \frac{\partial\, d_{i,t}}{\partial\, y_{k,t}} = \frac{[(1-1/\sigma_k)\beta P_{t+1}\, \lambda_k\, A_{i,t}{}^{1-1/\sigma k}]^{\sigma_k}}{m_{t+1}\, \lambda_k\, \Psi^{\sigma k}}$$

$$(15)\qquad \frac{\partial\, c_{j,t}}{\partial\, G_{k,t}} = \frac{\partial\, c_{j,t}}{\partial\, y_{k,t}} = \frac{[n_{j,t}\, \beta P_{t+1}\, \lambda_k\, A_{i,t}{}^{1-1/\sigma k}]^{\sigma_k}}{n_{j,t}\, \lambda_k\, \Psi^{\sigma k}}$$

Both derivatives are obviously positive.
The part of the multiplier associated with the change of the number of firms in the market that received the shock will only have temporary effects if the shock is temporary. However, regarding the two parts of the multiplier associated with innovations, temporary shocks will have permanent effects.
As an example, let us consider a positive temporary government demand shock. It may seem that after the demand returns to its original value the optimal amount of innovation, and thus growth, returns to the same amount as before. There are however permanent changes in the supply side. The shock induces innovation that otherwise would not have been done: both the creation of new products and the improvement of previous. These remain after the shock is gone. This means that the trajectory of the economy is altered in a permanent way: hysteresis result. The degree of change will be however small unless the public expenditure is very large in proportion to the original output. This whole process can be easily seen with a numerical example.

E. Monetary Policy

A similar process happens with monetary shocks. As long as the adjustment of the price level has any lag, money is nonneutral even in the long run, because it changes the economy's trajectory permanently. These changes should be however quite modest.
Also if the discount β rate is affected, this will be another channel of influence that will have both temporary and permanent effects.

F. Dynamics

It is useful to determine how does total innovation in a given market varies with changes in some of the fundamental variables that vary between markets, an possibly trough time, notably the degree of linkage between submarkets, the invention rate, and the demand. This is given by:

$$\frac{\partial(d_{i,t} + c_{j,t})}{\partial \sigma_k} > 0 \qquad (16)$$

The proof can be found in the appendix.

The intuition for this result is what follows: Note that the linkage between submarkets σ_k has a positive influence in both process and product spending. This makes sense, because this parameter measures the degree of market power of firms in a given market. But its weight is not the same in both cases. The bigger is σ_k, more will firms invest in product innovation relatively to process innovation because they will more easily "steal" demand from other trajectories (see Sutton 1998). Then, all else constant, total innovation depends positively on the linkage between submarkets σ_k.

This can be expressed in graphical terms.

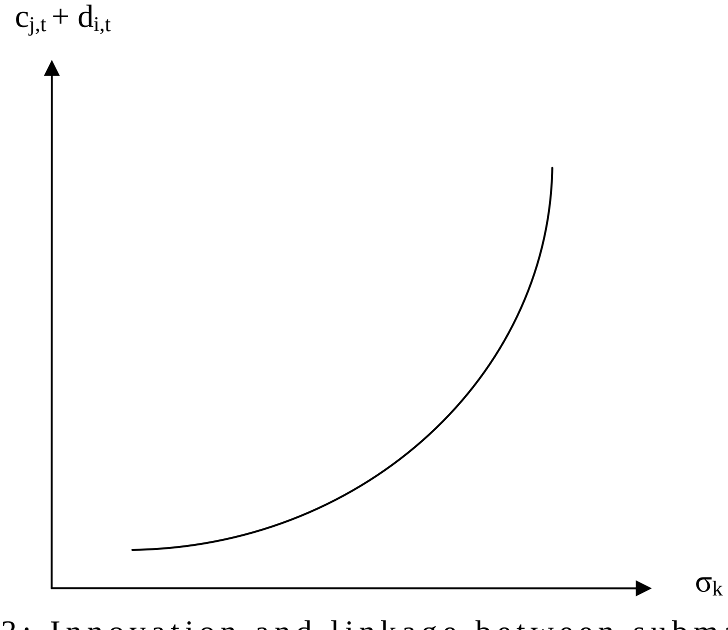

Figure 3: Innovation and linkage between submarkets

Now remember that σ_k is just parameter, potentially different in each market. Then, an equilibrium can be found.

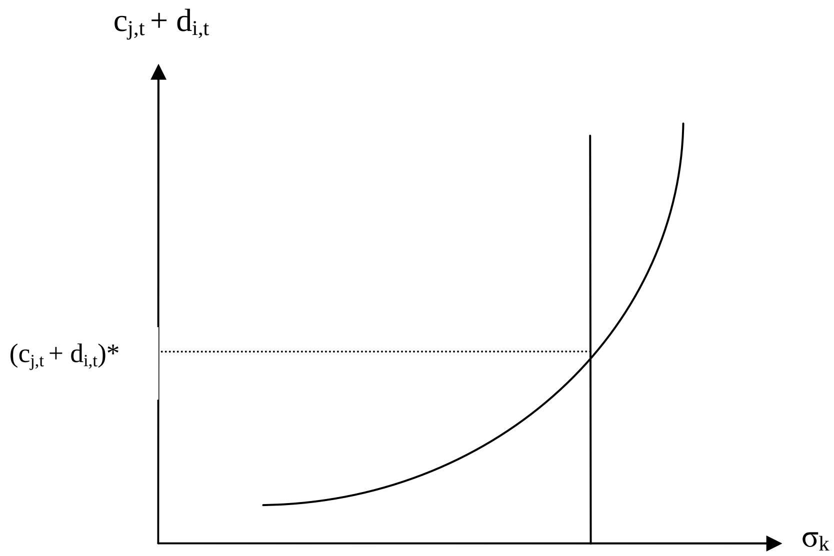

Figure 4: Optimal Innovation

The next figure will illustrate an increase in the linkage between submarkets σ_k. This can represent what happens in two different markets or a change in consumer tastes (more elasticity of substitution between varieties) in the same market.

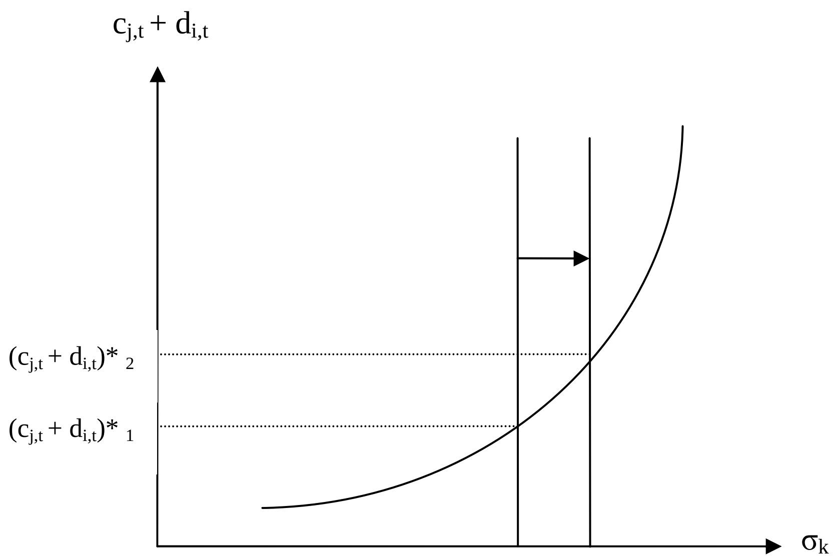

Figure 5: An increase in the linkage between submarkets

Note that there is an effect that can not be seen in the figure: as total innovation increases, product innovation increases relatively more than process innovation: the higher linkage between submarkets (more elasticity of substitution to the consumer) means a higher payoff to the firms on investing in process rather than product innovation. (because they can more easily capture demand from alternative varieties that exist in the same market). Because of this firms will concentrate their efforts and spend relatively more on process rather than product innovation in comparison with before.

An increase in the invention rate λ_k will obviously have a positive impact in both kinds of innovation. The next figure shows this effect. This can represent what happens in two different markets, which are equal in all except in that parameter, or an exogenous positive evolution of the rate on a market, for example because of a scientific breakthrough.

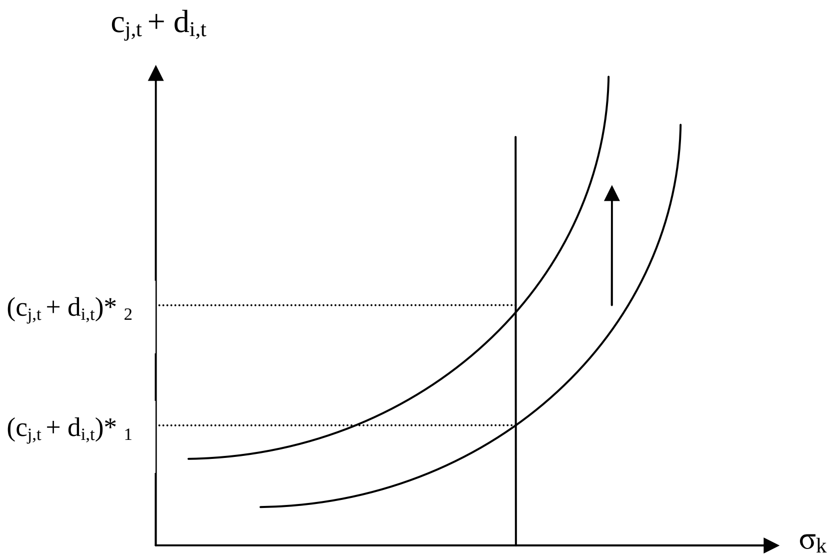

Figure 6: An increase in the invention rate

Total investment on innovation is positively related to market share. It is possible to prove this analytically, but I will concentrate in the intuition. Following assumption 4, the level of concentration in each market can be measured simply by the number of firms that the market can support. For example, if the market can only support one firm, we have a monopoly, and thus maximum concentration.

Suppose now that on period *t-1*, all firms invest a given amount on innovation. For some reason, on period *t* that amount increases. Note that in the next periods firms will have more capacity, i.e. will have products of superior quality and a larger number of varieties that otherwise would have existed if the investment in innovation had stayed in the original level. Because of this, if all other variables remain

constant (notably demand) the market is capable of holding an inferior number of firms that otherwise would have been. Thus, concentration on that market rises.
This relation can be represented graphically:

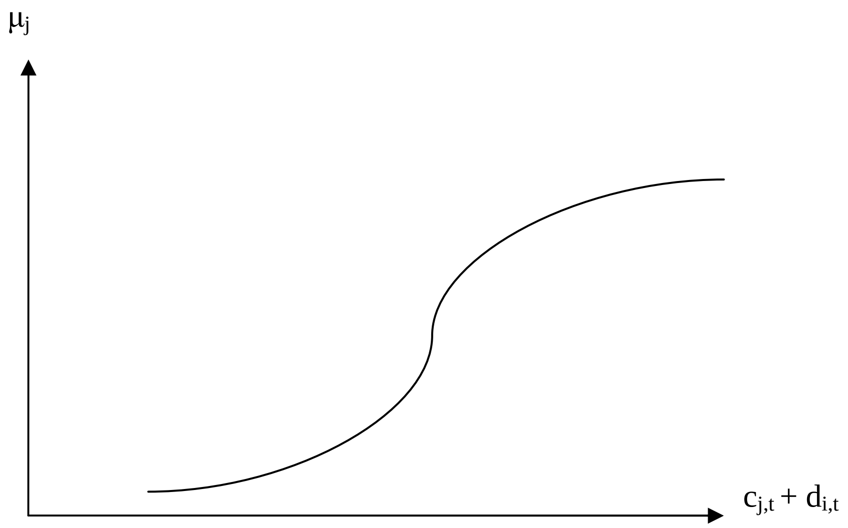

Figure 7: Innovation and Market Concentration

Where μ_j represents the expected market power of firm *j* in the long run. This is a subtle point that will be made more clear when the equilibrium analysis is complete.
We can find the equilibrium value of concentration if we join the optimal value of investment in innovation found in figure 4 to the last figure:

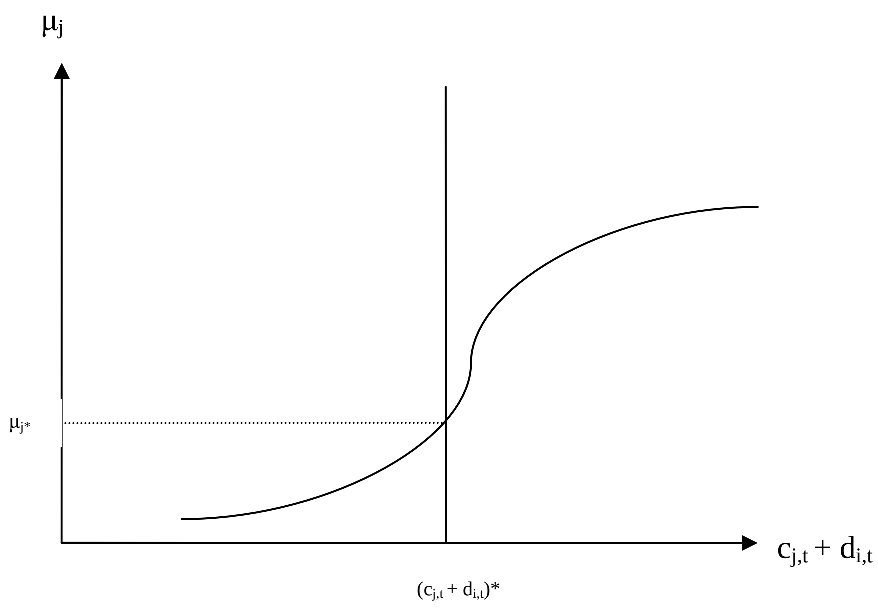

Figure 8: Equilibrium Concentration

Now about the equilibrium value for concentration μ_j*. *If firm j does not exit the market, it will end up with market power* μ_j*. This is, after entry and exit occur, in the long run, the firm(s) that are still in business will have that market power. These will supposedly be the more efficient ones. If assumption 4 holds, then market concentration can determined based only on the decision of one firm. It is now possible to build a figure to see what happens in different markets where somewhat extreme values of the linkage between submarkets σ_k and the invention rate λ_k exist, and take the necessary conclusions on how these parameters influence market structure.

Let us then consider the existence of four hypothetical markets that have parameter values according to the following table, but are in all else equal. By definition, σ□>σ□ and λ□>λ□.

	σ□	σ□
λ□	Market B	Market D
λ□	Market A	Market C

Table 1: Characteristics of four markets

We can now build the two basic figures based on this table.

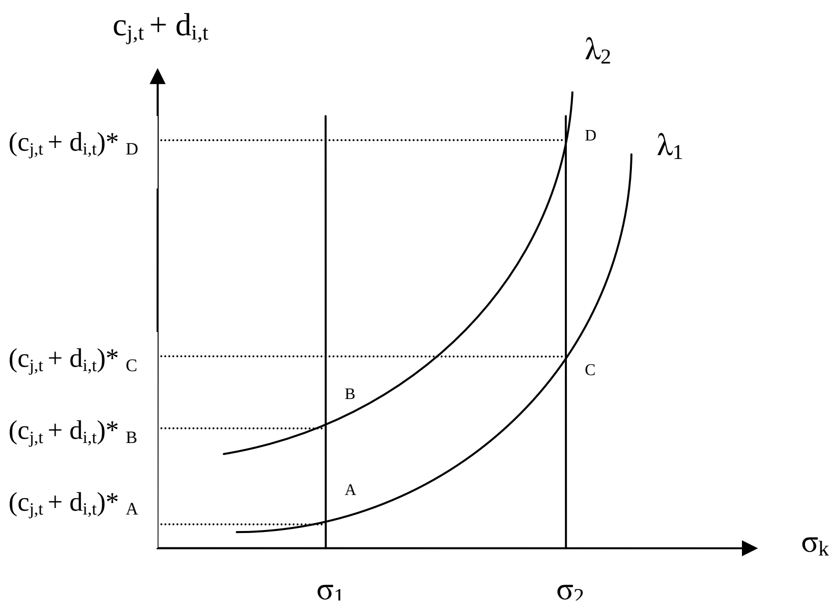

Figure 8: Several Markets: Innovation levels

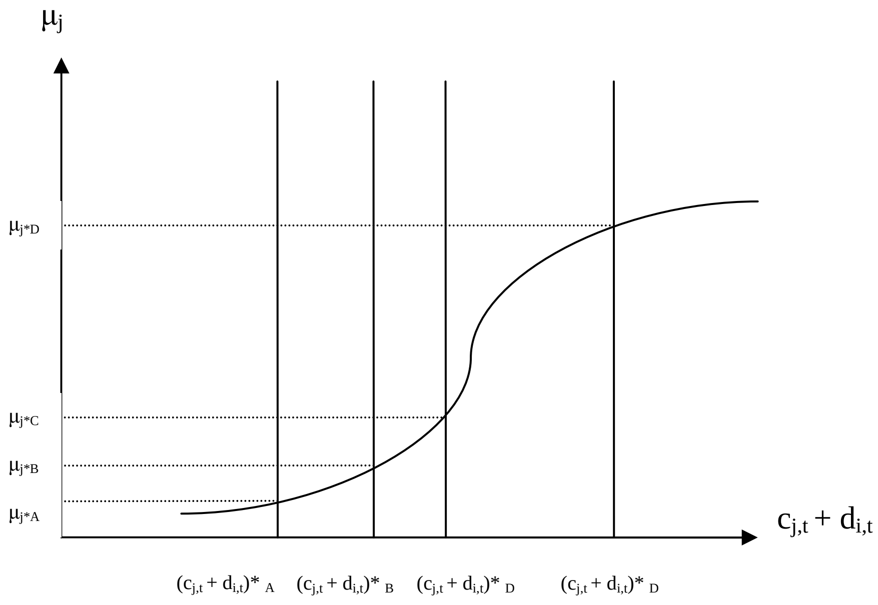

Figure 9: Several Markets: Concentration levels

As it is possible to see in the above figures, maximum concentration will emerge on market D whereas minimum concentration corresponds to market A. Markets B and C will surely have concentration in-between those two. Note that it is not necessary that $\mu_C^* > \mu_B^*$ as depicted in the graph. The contrary is possible depending on the relative sizes of the parameters σ_k and λ_k (remember that the elasticity of the curves in figure 8 also depend on σ_k).

The last two figures, in association with equation (10) have an immediate consequence. It is a well known fact that a limited number of sectors in the economy are responsible for most of the growth. This is what the model predicts for markets with high linkage σ_k and invention rate λ_k.

4. Conclusion

The consequences of modern economic growth to human welfare cannot be overstressed. Growth is the ultimate reason why the standards of living in today's developed countries are overwhelmingly superior to what they were during the most part of human existence and in developing countries today. Understanding the mechanism that underlies beneath this phenomenon may give us tools to control it, extend it to developing countries, and maintain it in a sustained way. Thus, explaining growth is a, if not the, fundamental challenge to human knowledge. The model presented in this paper attempts this task relating growth to its ultimate cause: The search from profits from private firms.

This model predicts that market structure is ultimately a consequence of different opportunities that exist in different markets. Given the existing conditions of a market, firms will have no other choice but to comply to a specific investment strategy. Because of this markets characterised by different degrees of linkage between submarkets and invention rates will have different market structures and will contribute in a different way to economic growth. The next figure provides an example.

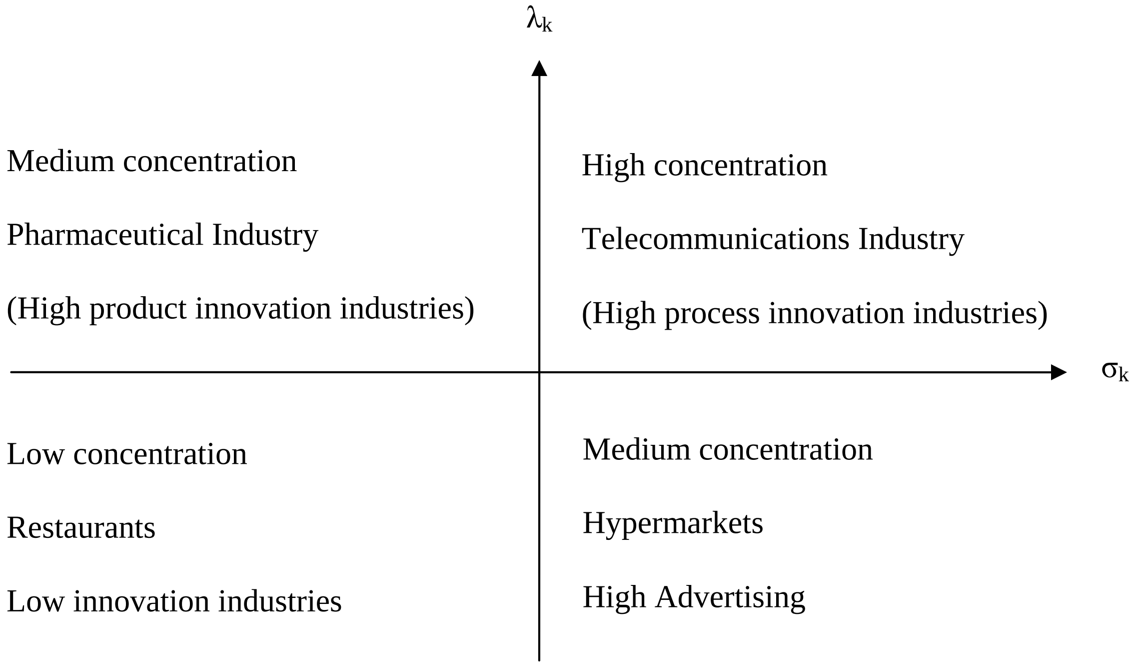

Figure 10: Theoretical prediction for market structure

In markets where both the linkage between submarkets and the invention rate are high, process innovation is dominant. On one side, it is profitable for firms to invest on only one or a few trajectories because of the high degree of linkage between submarkets. The few high quality products that result will capture all demand (in a creative destruction process, they will drive lower quality varieties out of the market), so markets will be characterised by a limited number of high quality products. These will be provided by a small number of firms, so that investment of each can be high to achieve high-quality. Examples are the cell phone manufacturers (and operators) industry, and the automobile industry. The first example given is pragmatic: The substitutability between two different cell phones is certainly high. Inventions in this area are frequent, and product innovations follow: smaller and cheaper phones, with camera, internet, and so on.

If however the linkage between submarkets is weak, the payoff to a firm of investing in a single trajectory is not that great. Because each variety is produced by a monopolist, to invest much in a single trajectory, you need one big firm. However, to invest much in a large number of varieties as in this case, you may have several firms investing in several trajectories. Because of this, these markets are less concentrated that those with high linkage between submarkets. An example is the pharmaceutical industry. In this industry the substitution between varieties is small, and scope economies are likely to exist; Therefore, the linkage between submarkets of the market for drugs is weak. On the other hand this is a sector where science is active and new discoveries are frequent. Because of this the invention rate is high. What follows is the proliferation of product innovations.

Now regarding the markets where the linkage between submarkets is high but the invention rate is low. The innovation process in these sectors in unproductive, however investment in a limited number of trajectories is the right choice. Because of this, firms will probably be characterised by high investment in advertisement of those. A lot of products that are strongly advertised effectively fall in this category: shampoos, industrial food industry and soft drinks, hypermarkets.

Finally, there are some markets in which both the linkage between submarkets and the invention rate are low. These will be sectors of low growth and possibly high turbulence (a lot of entry and exit) as they have trouble adapting to changing conditions. The restaurant business is a good example. Why doesn’t this happen in other sectors? One reason is that assumption 4 does not hold in most other sectors but it probably does in this one. Remember that this assumption was only taken so we could ignore the nature of the entry process. In the real-world only marginal firms have null profits. In most markets more efficient firms will have positive profits that can be adjusted preventing entry or exit. However in this sector most, if not all, firms are truly marginal. Because of this, adjustment can only be made by entry and exit, causing economic turbulence.

Note that an artificial change of the market structure, for example anti-concentration legislation, will certainly have consequences to the firms' investment decisions, and thus growth.

It is common sense that some (maybe most) sectors are not truly responsible for economic growth, while others are the main driving forces. The usual explanation is demand oriented, i.e. new demand is for sectors with high elasticity of income. (Engel’s law is an example). This model provides an alternative supply-side result. (note that all the analysis took market demand, as well as demand growth, as taken).

5. Extensions and limitations

Several extensions can be made to the model: uncertainty, non constant returns do innovation, explicit consideration of scope and scale economies, love for variety, other factors of production and strategic interactions between firms. Principal-agent problems can also be considered. An especially interesting extension would be to turn the model into general equilibrium.

One limitation of the model is that it does not consider the imitation issue. Firms innovate by imitating as well as doing their own research. Thus, this is an alternative that should be considered. However the results subtly depend on the nature of assumptions regarding the nature of the patenting process.

The model can be tested empirically using observables as proxies for the main parameters of the model. A homogeneity index can be used in relation to the linkage between submarkets (see Sutton 1998). About the invention rate, several measures can be used. Industry usage of human capital of specific scientific areas is an example. Public investment in research is supposed to increase the rate, possibly with decreasing marginal productivity. Specific events, for example, an arms race, will affect primarily certain sectors. As this is a theoretical paper, I will leave these details to later research.

Appendix

Proof of equations (7) and (8):

The solution can be obtained using dynamic programming. The objective function is (6). The constraints are (1), (2), (3), (4) and (5). The state variables are $n_{j,t}$ and $A_{i,t}$ for $i=1,\ldots, n_{j,t}$. The control variables are $c_{j,t}$ and $d_{i,t}$ for $i=1,\ldots, n_{j,t}$.

We are now able to set up the Bellman equation:

(A1) $$V(A_{i,t}, n_{j,t}) = sup\,[\,\Pi_{j,t} + \beta V(A_{i,t+1},, n_{j,t+1} \mid A_{i,t}, n_{j,t})\,]$$

The first-order conditions are:

(A2) $$\Psi = \beta \frac{\partial V(A_{i,t+1},, n_{j,t+1} \mid A_{i,t}, n_{j,t})}{\partial d_{1,t}}$$

...

$$\text{(A3)} \qquad \Psi = \beta \; \frac{\partial \; V(A_{i,t+1},, n_{j,t+1} \square A_{i,t}, n_{j,t})}{\partial \; d_{n_{j,t},t}}$$

$$\text{(A4)} \qquad \Psi = \beta \; \frac{\partial \; V(A_{i,t+1},, n_{j,t+1} \square A_{i,t}, n_{j,t})}{\partial \; c_{j,t}}$$

Note that these correspond to the usual marginal cost equals marginal revenue conditions.

Solving the first order conditions (A2) to (A3) is straightforward. After using the chain rule on the right hand side and substituting restriction (3), when solving in order to $d_{i,t}$ we get equation (8).

For solving the right hand side of (A4) it is necessary to use assumption 1.
Using the chain rule:

$$\text{(A5)} \qquad \Psi = \beta \; \frac{\partial \; V(A_{i,t+1},, n_{j,t+1} \square A_{i,t}, n_{j,t})}{\partial \; n_{j,t}} \; \frac{\partial \; n_{j,t}}{\partial \; c_{j,t}}$$

development of the second term:

$$\text{(A6)} \qquad \frac{\partial \; V(A_{i,t+1},, n_{j,t+1} \square A_{i,t}, n_{j,t})}{\partial \; n_{j,t}} =$$

$$= A_0^{(1-1/\sigma_k)} P_{t+1} [y_{k,t+1}/(m_{t+1}+1)]^{1/\sigma_k} \underbrace{- \Big[\sum_{i=1}^{n_{j,t+1}} A_{i,t}^{(1-1/\sigma_k)} P_{t+1} (y_{k,t+1}/m_{t+1})^{1/\sigma_k} - \sum_{i=1}^{n_{j,t+1}} A_{i,t}^{(1-1/\sigma_k)} P_{t+1} (y_{k,t+1}/m_{t+1}+1)^{1/\sigma_k} \Big]}_{\text{equal to zero by assumption}}$$

Substituting the above and restriction (4) on (A5), the result is equation (9).

Proof of preposition (16):

$$\frac{y_{k,t+1}\{n_{j,t}^{\sigma k}\square^{1}\varphi^{\sigma_k}(\ln\varphi+\ln n_{j,t}+\ln A_0/\sigma_k-\ln\Psi)+\zeta^{\sigma_k}/m_{t+1}[\ln\zeta+(1/\sigma_k-1)+(\ln A_{i,t})/\sigma_k-\ln\Psi]+\ln\Psi(1/n_{j,t}[m_{t+1}+1])\}}{\lambda_k\Psi^{\sigma k}}$$

where $\varphi = \beta\ \lambda_k\ P_{t+1}\ A_0^{(1-1/\sigma_k)}$ and $\zeta = (1-1/\sigma_k)\ \beta\ \lambda_k\ P_{t+1}\ A_{i,t}^{(1-1/\sigma_k)}$. The expression is positive assuming the marginal cost Ψ is not disproportionately large in comparison to the other variables (in which case there will be no research at all).

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