Optimal Divisionalization for Selling Networks of Cable Television Services

JOÃO LEITÃO*

Universidade da Beira Interior (UBI)
Departamento de Gestão e Economia (DGE)
(Covilhã – Portugal)

Abstract

In this article, a condition for the optimal division's number is calculated, for a market with two cable operators who offer a network service. The rationale for justifying the partial covering of the national market from the cable operators is presented. Furthermore, a problem of moral hazard is revealed, which is able to appear through the implementation of franchising schemes with independent divisions. This is particularly interesting because it can be applied to several industries such as Cable Television and Entertainment, and other activities including Internet and Computer Games Centres, which offer Internet broadband access and network games.

Key Words: Cable Television, Divisionalization, Franchising.

JEL: L11, L20, M55.

1 Introduction

The industries, which operate the creation of, structuralized networks (as for example, the cable networks), and which use digital platforms of distribution, have come to assume an increasing importance in the development of the national economies.

These bi-directional networks, that allow upload and download of information flows, need further investigations, that, on the one hand, explore the producer' (or operator') strategy in the determination of the optimal number of selling divisions, and, on the other hand, the choices in terms of the type of legal and business relationship, to establish between the agents.

The present article conciliates two research lines, namely, the study of the Economic of Networks (which includes the systems whose constitution is based on products or services that present a complementary and interconectable nature), and the determination of the optimal dimension of the networks (that is, the number of selling divisions that guarantee the maximization of the operator profit).

---

* Mailing address: Universidade da Beira Interior, Pólo das Ciências Sociais e Humanas, Departamento de Gestão e Economia, 6200 209 Covilhã, Portugal. E-mail: jleitao@fenix.ubi.pt.

The author is grateful to Carlos Osório (UBI), José R. Pires Manso (UBI) and José R. Nogueira (UFPE-Brazil) for thought-provoking discussions and helpful comments on previous versions of this article. I would like to thank also to seminar participants: Steffen Hoernig (FEUNL) and Tiago Sequeira (UBI), at 8.ª Conferência da Sociedade Portuguesa de Investigação em Economia (SPiE), Universidade Nova de Lisboa, Faculdade de Economia, June 2003, for comments and suggestions. A special debt of gratitude is due to Carlos Osório for the positive incentives and constructive commentaries about the initial draft of this manuscript. The usual caveat applies.

The main innovation of the present analysis is the inclusion of the demand curve with realized expectations for network services, proposed by Economides and Himmelberg (1995) and Economides (1996), into the analysis of the optimal dimension of the networks. Moreover, the influence of the characteristics of the network services is explored, which may reveal some complementarities, expressed by network externalities (in production, and consumption) that may influence the strategic decision concerned with the (total or partial) covering of a national market of consumers (or subscribers).

Taking into consideration the installed network, the operators must proceed to the optimisation of network externalities in consumption, that are expressed by the service valuation attributed by the consumers, which increases with the number of consumers who subscribe this service.

In the present analysis, one considers a national market with two operators, who offer network services, under the form of integrated packages, a re-evaluation of the incentives for the creation of selling divisions networks, is made. Later, the results of a game that considers the implementation of distinct royalties' modalities are analysed.

This analysis aims to extend the scope of application of literature about the creation of selling divisions networks, to present a condition for the determination of the optimal number of selling divisions incorporating the demand curve with realized expectations, and to evaluate the impact of distinct royalties' modalities on the profit earned by the operator.

In the first section, a model for the determination of the optimal number of selling divisions is presented, based in a game with two phases, in which participate two cable television operators. In the second section, a game expanded with three phases is developed, taking into consideration the possibility of celebrating franchising contracts.

Finally, the main conclusions related with the development of the model are presented, and the reasons for regulating cable television industry are revealed.

2 The Model

In the works developed by Corchón (1991), Polaski (1992), Corchón e González-Maestre (2000), Baye, Crocker e Ju (1996), Yuan (1999) and Bru, Faulí-Oller e Haro (2001) is presented the problematic of the incentives for the companies to create selling divisions' networks which interact in the market. Considering that Cournot competition scheme leads to a perfectly competitive result, the costs of creation of these networks tend for zero, given the dissipation of the oligopoly results.

The strategic decision related with the creation of a selling divisions' network has usually two effects on the total profit obtained by a firm, namely, an expansion of the firm's market share and an increase of the competition between the existent selling divisions (Yuan, 1999).

Taking into consideration the transaction of homogeneous products or services, the results obtained in the works cited above, establish that the creation of a new selling division has two main effects. First, it reduces the aggregate profit because of the increased competition and, second, it increases a firm's share in the aggregate output and profit. If the goods or services offered by the firms are perfect substitutes, so the second effect always supplants the first.

Nevertheless, the operators can delegate to the administrators of the selling divisions, the production decisions, and modify its behaviour through the implementation of incentives schemes, although they still maintain for itself investment decisions regarding capacities or new selling divisions (Veendorp, 1991 and González-Maestre, 2000).

Instead of the Cournot scheme that is usually considered in the divisionalization literature, the companies can still enter in a competitive war (Huck, Konrade and Müller, 2001).

The network owner can have a predatory behaviour to its direct competitors, as far as the effect of property of the network generates inter-temporal incomes (Farrel and Katz, 2001).

In the literature concerning to the creation of selling divisions networks, the companies have a strategic incentive to create independent franchisee divisions, assuming that this procedure leads to a more aggressive behaviour in order to increase the market share of the Mother Company (Warren-Boulton, 1974; O'Brien and Shaffer, 1992; Baye et al, 1996; Bru et al, 2001 and Dana and Spier, 2001).

A monopolist who offers only highly differentiated products or services, may threat the entrant with the possibility of creating more selling divisions, in order to ensure the monopoly outcome. At monopoly, the credible threat of divisionalization in case of entry is enough for the incumbent firm to earn persistently and abnormally high profits in free-entry equilibrium, relative to the no-divisionalization case (Yuan, 1999).

In the game presented at the last section of this article, the main economic fact that characterizes the franchising contracts is that, the incentives of the contractual parts do not always coincide (Klein, 1995).

Taking as starting point the pioneering work of Baye et al (1996), a simple model of duopoly applied to a network market is presented, in which it is considered the co-existence of two upstream operators, who sell packages of cable television services, and for the purpose of distribution, use a downstream selling divisions network, or alternatively, an independent franchisee divisions network.

We consider a two-stage divisionalization game with a duopoly supplying homogeneous services, with perfect information[1], where, firstly, it is determined the optimal number of subscribers and, afterwards, is calculated the optimal number of selling divisions.

To determine the optimal number of selling divisions, we consider the establishment of competing independent units, as being the process of creation of the selling divisions network (in the company internal environment), even so the model is also applicable equally to the cases where the franchising option is followed (in the company external environment)[2].

For simplification, we consider two identical cable operators, which offer a homogeneous cable television service, and support a constant marginal cost (c). In this formalization is assumed that the selling divisions of the operator support the same marginal cost, in the distribution of the television service.

In this analysis, we consider the transactions of a network service, so the externalities generated by this kind of service allow to explore the possibility of offering complementary services, by each one of the operators, under the form of integrated packages, as far as the number of subscribers who subscribe the same cable television service increases (Economides and Himmelberg, 1995; Economides, 1996; Cabral, Salant, Woroch, 1997; Yang, 1997 and Yannelis, 2001).

[1] The operators have perfect information about the demand structure and technologies used in the cable television market.

[2] In the present model, like in other works in the divisionalization literature, such as, Corchón and González--Maestre (2000), Baye, Crocker and Ju (1996) and Yuan (1999), we assume that autonomous selling divisions will not further divide into more independent subdivisions.

2.1 Phase 1: Optimal Number of Subscribers

In phase 1, all the selling divisions behave *a la Cournot*, as independent players, in a simultaneous game, and each operator takes the decision about the number of subscribers to reach, but considering that the market price will depend on the number of subscribers obtained by both operators.

For a certain cable operator, when we consider that the number of subscribers of the potential base (N^e) is equal to the number of subscribers of the installed base (N), the following proposition is observed:

Proposition 1: Given the configuration in form of inverted U of the demand curve with realized expectations, the maximum profit of a cable operator is obtained in $N = 2/3$. For $2/3 < N < 1$, decreasing prices are observed that do not assure the profit maximization of the cable operator.

Proof: See the appendix ■

To proceed to the resolution of this game, we consider that competition is initiated in two phases. Having n_{ij}, as being the amount of subscriptions obtained by the ith division of operator j, where: $i = 1..., n_j$; and $j = 1, 2$. Additionally, $N_{\_ij}$ is considered as being the total amount of subscriptions obtained by all the selling divisions, except the number of subscriptions reached by the ith selling division of operator j.

The profit of the selling division (π_{ij}) can be enunciated as:

$$\pi_{ij}(N_{ij}, N_{\_ij}) = N^e(1-N)n_{ij} - cn_{ij} \tag{1}$$

Where: $\mathrm{N} = \sum_{i=1}^{n_j}\sum_{j=1}^{2} n_{ij}$ = Total number of subscribers in the cable television market.

We consider that the ith selling division of operator j chooses n_{ij}, in order to maximize its profit. This requires that, the condition of profit maximization be respected, given the equality between the marginal revenue (MR) and the marginal cost (MC). This implies that, for any selling division (n_{ij}^*) the optimal number of subscribers must satisfy the following condition:

$$N^e - N^e N_{\_ij} - 2N^e n_{ij}^* = c \tag{2}$$

To simplify the model, we consider that all the selling divisions are identical, and then all must choose, in equilibrium, the same optimal number of subscribers, that is, $\forall\, i,\ j\, ;\, n^* = n_{ij}^*$.

Taking into consideration that the total number of selling divisions is $(\eta_1 + \eta_2)$, so N_ij must be equal to: $(\eta_1 + \eta_2 - 1) \cdot n^*$. Substituting the conditions presented in the Eq. (2), the following one is obtained:

Proposition 2: The optimal number of subscribers for each operator is given by:

$$n^* = \frac{N^e - c}{N^e(\eta_1 + \eta_2 + 1)} \tag{3}$$

Proof: See the appendix ■

Taking into consideration the result enunciated in Proposition 2, the total number of subscribers in the cable television market (*N*), and the price (*p*), are obtained in the following way:

Proposition 3: The total number of subscribers (*N*) is given by the product between the total of selling divisions of the two cable operators, and the optimal number of subscribers, and is expressed by:

$$N = (\eta_1 + \eta_2) \cdot n^* \Leftrightarrow N = \frac{(\eta_1 + \eta_2)}{(\eta_1 + \eta_2 + 1)} \cdot \frac{(N^e - c)}{N^e} \tag{4}$$

Proposition 4: The price (*p*) practised by each operator is given by:

$$p = N^e(1 - N) \Leftrightarrow p = \frac{N^e + (\eta_1 + \eta_2) \cdot c}{(\eta_1 + \eta_2 + 1)} \tag{5}$$

Proof: See the appendix ■

2.2 Phase 2: Optimal Number of Selling Divisions

In phase 2, each one of the two operators chooses the number of selling divisions in order to operate in the downstream market, having, η_1 *and* η_2, as being the number of downstream selling divisions, chosen by operators 1 and 2, respectively. Additionally, a company incurs into a sunk cost *K*, in the creation of the local network (in the specific localization of the selling division) for the distribution of the cable television service. In this phase of the game, each selling division, when establishes a price (*p*), earns a profit (π_{ij}) given by:

Proposition 5: The profit of each selling division (π_{ij}) is given by the following expression:

$$\pi_{ij} = p \cdot n^* - c \cdot n^* \Leftrightarrow \pi_{ij} = \frac{(N^e - c)^2}{N^e \cdot (\eta_1 + \eta_2 + 1)^2} \tag{6}$$

Proof: See the appendix ■

The two operators who anticipate the competition between selling divisions in phase 2, and that do not intend to cover the totality of the national market of cable television services[3], have to establish, in phase 1, the number of selling divisions, η_1 *and* η_2, that is going to implement.

The profit of operator 1 can be written in the following way:

$$\pi_1 = \sum_{i=1}^{\eta_1} \pi_{i1} - K \cdot \eta_1 \tag{7}$$

Where: π_{i1} = Profit of *i*th selling division of operator 1, in phase 2.

Reminding the Eq. (6), which expresses the profit earned by each selling division of operator 1, in phase 2, the total profit of operator 1 can be rewritten as follows:

$$\pi_1(\eta_1, \eta_2) = \eta_1 \cdot \frac{\left(N^e - c\right)^2}{N^e \cdot (\eta_1 + \eta_2 + 1)^2} - K \cdot \eta_1 \tag{8}$$

Therefore, operator 1, in order to maximize the profit, chooses the total number of selling divisions (η_1^*) that will constitute its downstream distribution network, considering that operator 2 has (η_2) selling divisions. Therefore, operator 1 chooses the best response function (η_1^*), taking into consideration the number of divisions owned by the operator 2 (η_2). By the calculation of the first-order condition $\left(\partial \pi_1 / \partial \eta_1 = 0\right)$, we obtain the following expression:

Proposition 6: The best response function of operator 1 is given by:

$$K = \frac{\left(N^e - c\right)^2}{N^e \cdot (\eta_1 + \eta_2 + 1)^2} \cdot \left[1 - \frac{2\eta_1}{(\eta_1 + \eta_2 + 1)}\right] \tag{9}$$

Proof: See the appendix ■

[3] Remind the result enunciated by proposition 1.

In this game, when we consider that operator 2 is identical to operator 1, a symmetrical condition for η_2^* can be met. For such, having that: $\eta^* = \eta_1^* = \eta_2^*$, it can be recognized that this symmetry implies that: $2\eta^* = \eta_1^* + \eta_2^*$. Solving the Eq. (9), in order to η^*, the following one is obtained:

Proposition 7: The optimal number of selling divisions is dependent of the differential between the number of subscribers of the potential base and the marginal cost, as well as of the sunk cost supported in the creation of cable television network, and is given by:

$$\eta^* = \frac{1}{2}\left[\left(\frac{\left(N^e - c\right)^2}{K \cdot N^e}\right)^{\frac{1}{3}} - 1\right] \tag{10}$$

Proof: See the appendix ■

3 Franchising Options

The theoretical framework related with franchising was developed from the seminal work of Coase (1937), which originated distinct research lines, such as, the theory of agency (Ross, 1973; Arrow, 1985 and Rees 1985a, 1985b), and the theory of specific assets and opportunism (Klein, Crawford and Alchian, 1978 and Williamson, 1985).

Franchising can be considered as a form of economic relationship that is amongst the extremes of the hierarchies and the markets (Williamson, 1985).

This relationship signals the principal that grants and the agent who receives the license or the concession for using a product, service, technology or trade mark.

This organizational form is usually analysed in the existing literature from the theory of agency perspective, stressing that monitoring costs might explain the use of franchising contracts to create a mechanism for an optimal coordination of the company activities, and the decentralization of the decision taking process. It also allows the creation of downstream distribution networks, which guarantee a greater dissemination of independent selling divisions (Brickley and Dark, 1987; Norton, 1988; Brickley, Dark and Weisbach, 1991; and González-Maestre, 2000).

To modelling this situation, after deducting the optimal number of selling divisions that guarantees the profit maximization for the cable operator, an extension of the game presented in section 2 is made, that equates the implementation of two distinct franchising options, which can be implemented by each one of the operators.

To extend the model presented in section 2, a game with the following phases is considered:

- Phase 1: The operator chooses the optimal number of divisions, in franchising;
- Phase 2: The operator selects the royalty modality;
- Phase 3: The selling divisions (or the franchisees) take the decision about the number of subscribers to reach (according to the royalty modality selected by the operator).

3.1 Optimal Number of Divisions in Franchising

In phase 1, and taking into consideration the result founded in section 2.2. (Eq. (10)), each operator chooses the number of selling divisions to operate in the downstream market.

Lemma 1: Considering that, operators 1 and 2 are identical ($\eta_1^* = \eta_2^*$), the optimal number of selling divisions, in franchising, is given by: $\eta^* = \frac{1}{2}\left[\left(\frac{(N^e - c)^2}{K \cdot N^e}\right)^{\frac{1}{3}} - 1\right]$.

3.2 Royalties Modalities

In phase 2, the operator can apply two alternative royalties' modalities, to the franchisee.

3.2.1 Modality 1

In modality 1, we assume that the contract celebrated between the operator and the franchisee, establishes a price for the cable television service equal to a cost *c*, as well as a royalty payment, which corresponds to a fraction (α) of the total revenue.

Celebrating this kind of contract, the profit (π_1^f) [4] will be equal to total revenue after--royalty, less the corresponding cost, and is enunciated as follows:

$$\pi_1^f = (1-\alpha)\cdot P \cdot n_{ij} - c \cdot n_{ij} \tag{11}$$

Lemma 2: For any positive royalty $(\alpha > 0)$, the number of subscribers obtained through the application of modality 1, is smaller than the number of subscribers desired by the upstream operator.

Proof: See the appendix ■

[4] Consider Franchisee Profit (*f*), with modality 1.

3.2.2 Modality 2

In modality 2, the contract between the operator and the franchisee establishes a price for the cable television service equal to a cost *c*, and the royalty payment corresponds to a fraction (α) of the total profit obtained by the franchisee.

Therefore, the profit of the franchisee (π_2^f) [5] can be enunciated in the following way:

$$\pi_2^f = (1-\alpha)\cdot\left[P\cdot n_{ij} - c\cdot n_{ij}\right] \tag{12}$$

In an analogous way to modality 1, this payment system causes a reduction on the MR of the franchisee. However, such as it is easily observable in the Eq. (12), the modality now presented also provokes a reduction of the MC supported by the franchisee one.

3.3 Optimal Number of Subscribers

3.3.1 Modality 1

In phase 3, applying the modality 1, and considering the equality between the net MR of the franchisee, and the MC supported by the operator, we can derive the optimal level of the franchisee subscribers, which is given by:

$$n_{ij} = \frac{N^e(1-\alpha) - c}{N^e(\eta_1 + \eta_2 + 1)(1-\alpha)} \tag{13}$$

Nevertheless, the operator can try to increase the profit level, through the application of a higher royalty.

Observing the Eq. (13), we find out that this procedure would result in a decrease of the number of subscribers, that is, in practice; the royalty would be charged over a smaller volume of subscribers.

Alternatively, the operator could establish one royalty equal to zero, however this option would result in selling the cable television service to the franchisee, at the marginal cost. This situation would cause an optimal joint profit, but the totality of this profit would be earned by the downstream franchisee.

The imposition of a royalty different from zero is the only way for the operator to get a positive profit. However, any increase observed in α, would cause a reduction of the number of subscribers, which would not guarantees the maximization of the two companies joint profit.

Under the franchisee point of view, the imposition of a royalty over the total revenue is similar to the application of a rate over the total revenue. Therefore, this provides the reduction of its MR, as well as the reduction of the global incentives to reach new subscribers.

As a result of this, the royalties' scheme shows an imperfection revealed by the fact that franchisee reach a smaller number of subscribers, using a higher price. In this situation, we expect that, the franchisee is more interested in the profit maximization, in detriment of the maximization of the number of subscribers.

[5] Consider Franchisee Profit (*f*), with modality 2.

This situation describes the coordination problems originated by the implementation of this royalty modality, which may arise between the interests of the upstream companies, and the downstream ones.

3.3.2 Modality 2

In phase 3, applying the modality 2, and considering the equality between the MR and the MC, we can derive the optimal level of subscribers for the franchisee, expressed by the following:

$$n^{*} = n_{ij} = \frac{N^{e} - c}{N^{e} \cdot (\eta_1 + \eta_2 + 1)} \tag{14}$$

Lemma 3: For any positive royalty $(\alpha > 0)$, the number of subscribers obtained through the application of modality 2, is equal to the optimal number of subscribers desired by an integrated company.

Proof: See the appendix ■

3.4 Information Asymmetry

The joint profit maximization is provided by the Eq. (12), which reveals the optimal level of subscribers that is obtained. However, this option, sometimes is not successful in the real world, because contrarily to the assumptions used in this analysis, the upstream operator can have an insufficient level of information about the net profit obtained by the franchisee, due to lack of information concerning the formation of the costs supported by the franchisee.

The existence of this information asymmetry means that, the upstream operator faces a problem of moral hazard in the achievement of its commercial relations with the downstream franchisee.

For example, considering that a franchisee supports a fixed cost (*F*), in the development of its activity, known by the franchisee, but not by the operator. For such, the net profit obtained by the franchisee ($\pi_{2'}^{f}$) [6] would be equal to:

$$\pi_{2'}^{f} = (1 - \alpha) \cdot \left[p \cdot n_{ij} - c \cdot n_{ij} - F \right] \tag{15}$$

In this situation, the franchisee can communicate a higher value of *F*, or transmit an inflated value of the total costs, in order to appropriate the totality of the profit.

In short, the liquidation of royalties based on the total profit (or, simply on the total revenue) does not provide a satisfactory result for the upstream operators, since harmonization of interests between the operators and the franchisees, can effectively fail.

[6] Consider Franchisee Profit (*f*), incorporating a fixed cost, with Modality 2.

4 Conclusions

The main contribution of the model is revealed in Eq. (10), which shows that the bigger the difference between the number of subscribers of the potential base (N^e) and the marginal cost (c), and the smaller the sunk cost of the cable network installation (K), the bigger will be the number of selling divisions chosen by the two operators, in phase 1 of the game. The denominator of the Eq. (10) is also affected by N^e, and then if N^eK increased, the number of selling divisions chosen by the operators would decrease.

The operators' decision about the decreasing of the number of selling divisions, on the one hand, aims to prevent the total profit dissipation and, on the other hand, reveals the unilateral incentive to restrict their divisions from further dividing.

This result diverges from the conclusions of Baye et al (1996), where is argued that a higher price-cost differential, will generate incentives for the creation of selling divisions, and, for such reason, the optimal number of selling divisions (η^*) will be higher.

It must be stressed that the present analysis incorporates the demand curve with realized expectations for a network service, in form of inverted U, having this procedure as main implication the determination of a new η^*, that guarantees the maximization of the operators profit, using selling division networks that guarantee a partial market covering, and taking into consideration the sunk cost supported in the creation of the cable network.

The determination of η^* allows the two operators (duopolists, in the national market) not to come close to the competitive balance.

This analysis can help to explain the reason for the existence of a partial covering of the national territory, in terms of the cable television service.

The situation described above suggests that the vertical relations established with the creation of an integrated selling divisions network, or alternatively, a franchisees network can be harmful to total welfare.

This type of strategic conduct by the operators and the franchisees needs special attention from regulatory agencies. The cable television industry presents, furthermore, a complex technical composition that must be regulated, in a competitive sense, since it congregates the broadcasting and the circuits' areas, which make possible the access to integrated packages tie-in sales of different services (Basic, Premium, Internet, Fixed Telephone, Interactive Television, Data Transmission, etc).

The regulatory agencies should also take into consideration the effect of this type of vertical relations on the total welfare, as well as on defining the relevant markets, inquiring the conditions of entry, and controlling the existence of dominant positions, in order to guarantee the inexistence of some kind of foreclosure, against the entrants in the cable television industry.

5 References

ARROW, K. (1985), "The Economics of Agency", In PRATT, J.; ZECKHAUSER, R., *Principals and Agents: The Structure of Business*, Harvard Business School Press, Boston.

BAYE, M.; CROCKER, K.; JU, J. (1996), "Divisionalization, Franchising, and Divestiture Incentives in Oligopoly", *American Economic Review*, March 1996: 223 – 236.

BRICKLEY, J.; DARK, F. (1987), "The Choice of Organizational Form: The Case of Franchising, *Journal of Financial Economics*, 18: 401 – 420.

BRICKLEY, J.; DARK, F.; WEISBACH, M. (1991), "An Agency Perspective on Franchising", *Financial Management*, 20: 27 – 35.

BRU, L.; FAULÍ-OLLER, R.; HARO, J-M. (2001), "Divisionalization in Vertical Structures", *CEPR Working Papers*, DP3011, October 2001.

CABRAL, L.; SALANT, D.; WOROCH, G. (1997), "Monopoly Pricing with Network Externalities", *International Journal of Industrial Organization*, 17 (1999): 199 – 214.

COASE, R. (1937), "The nature of the firm", *Economica*, Vol. 4: 386 – 405.

CORCHÓN, L. (1991), "Oligopolistic Competition among groups", *Economic Letters*, 36: 1 – 3.

CORCHÓN, L.; GONZÁLEZ-MAESTRE, M. (2000), "On the competitive effects of Divisionalization", *Mathematical Social Sciences*, 39: 71 – 79.

DANA, J.; SPIER, K. (2001), "Revenue sharing and vertical control in the video rental industry", *The Journal of Industrial Economics*, Vol. XLIX, September 2001, N.º 3: 223 – 245.

ECONOMIDES, N.; HIMMELBERG, C. (1995), "Critical Mass and Network Evolution in Telecommunications", In BROCK, G. (ed.), *Toward a competitive Telecommunications Industry: Selected Papers from the 1994 Telecommunication Policy Research Conference*, University of Maryland, College Park, MD, July 1995: 31 – 42.

ECONOMIDES, N. (1996), "The Economics of Networks", *International Journal of Industrial Organization*, October, 1996, vol. 16, n.º 4: 673 – 699.

FARREL, J.; KATZ, M. (2001), "Competition or Predation? Schumpeterian Rivalry in Network Markets", *Working Papers of University of California, Department of Economics*, WP E01-306, August 2001, Berkeley.

GONZÁLEZ-MAESTRE, M. (2000), "Divisionalization and Delegation in Oligopoly", *Journal of Economics and Management Strategy*, 9: 321 – 338.

HUCK, S.; KONRAD, K.; MÜLLER, W. (2001), "Divisionalization in Contests", *Economic Letters*, 70: 89 – 93.

KLEIN, B.; CRAWFORD, R.; ALCHIAN, A. (1978), "Vertical integration appropriable rents and the competitive contracting process", *Journal of Law and Economics*, Vol. 21: 297 – 326.

KLEIN, B. (1995), "The economics of franchise contracts", *Journal of Corporate Finance*, 2: 9 – 37.

NORTON, S. (1988), "An Empirical Look at Franchising as an Organizational Form", *Journal of Business*, 61: 197 – 217.

O'BRIEN, D.; SHAFFER, G. (1992), "Vertical Control with Bilateral Contracts'", *Rand Journal of Economics*, 23: 299 – 308.

POLASKI, S. (1992), "Divide and Conquer: On the Profitability of Forming Independent Rival Divisions", *Economic Letters*, 40(3): 365 – 371.

REES, R. (1985a), "The theory of principal and agent", *Bulletin of Economic Research*, Vol. 37: 3 – 26.

REES, R. (1985b), "The theory of principal and agent", *Bulletin of Economic Research*, Vol. 37: 75 – 95.

ROSS, S. (1973), "The economic theory of agency: The principal's problem", *American Economic Review*, Vol. 63, n.º 2: 134 – 139.

VEENDORP, E. (1991), "Entry Deterrence, Divisionalization and Investment Decisions", *Quarterly Journal of Economics*, 106(1): 291 – 307.

WARREN-BOULTON, F. (1974), "Vertical Control with Variable Proportions", *Journal of Political Economy*, 82: 773 – 802.

WILLIAMSON, O. (1985), *The Economics Institutions of Capitalism*, Free Press, New York.

YANG, Y-N. (1997), *Essays on Networks Effects*, PhD Thesis in Economics, Utah State University, Utah.

YANNELIS, D. (2001), "On the simple Welfare economics of network externalities", *International Journal of Social Economics*, Vol. 28, n.º 4 (2001): 344 – 348.

YUAN, L. (1999), "Product Differentiation, Strategic Divisionalization, and Persistence of Monopoly", *Journal of Economics & Management Strategy*, Volume 8, Number 4, Winter 1999: 581 – 602.

6 Appendix

Proof of Proposition 1:

Taking into consideration the inverse demand curve with realized expectations, we find that:

$$\pi = p \cdot N \Leftrightarrow \pi = N^e(1-N)N \Leftrightarrow \pi = N(1-N)N \Leftrightarrow \pi = N^2 - N^3$$

From the first order condition, we obtain that:

$$\frac{\partial \pi}{\partial N} = 0 \Leftrightarrow 2N - 3N^2 = 0 \Leftrightarrow N(2-3N) = 0 \Leftrightarrow N = 0 \vee N = \frac{2}{3} \blacksquare$$

Proof of Proposition 2:

Considering that: $p = N^e(1-N)$

Having: $N = N_{\_ij} + n_{ij}$

$$N_{\_ij} = (\eta_1 + \eta_2 - 1)n_{ij} \Leftrightarrow N_{\_ij} = an_{ij}$$

The total profit is given by:

$$\pi_{ij} = N^e(1-N)n_{ij} - cn_{ij} \Leftrightarrow \pi_{ij} = N^e(1 - N_{-ij} - n_{ij})n_{ij} - cn_{ij} \Leftrightarrow$$

$$\Leftrightarrow \pi_{ij} = N^e n_{ij} - N^e N_{\_ij} n_{ij} - N^e n_{ij}^{\ 2} - cn_{ij}$$

The marginal revenue (MR) is given by the following expression:

$$MR = \frac{\partial TR}{\partial n_{ij}} = N^e - N^e N_{\_ij} - 2N^e n_{ij}$$

From the maximization profit condition, we derive the following:

$$MR = MC \Leftrightarrow N^e - N^e N_{\_ij} - 2N^e n_{ij} = c \Leftrightarrow N^e - N^e a n_{ij} - c = 2N^e n_{ij} \Leftrightarrow$$

$$\Leftrightarrow N^e - c = (2N^e + N^e a)n_{ij} \Leftrightarrow n_{ij} = \frac{N^e - c}{N^e(2+a)} \Leftrightarrow n_{ij} = \frac{N^e - c}{N^e(\eta_1 + \eta_2 + 1)} \blacksquare$$

Proof of Proposition 4:

The price of the service offered by each cable operator is given by:

$$p = N^e(1-N) \Leftrightarrow p = N^e \cdot \left[1 - \frac{(\eta_1+\eta_2)}{(\eta_1+\eta_2+1)} \cdot \frac{(N^e - c)}{N^e}\right] \Leftrightarrow$$

$$\Leftrightarrow p = N^e \cdot \left[\frac{(\eta_1+\eta_2+1)\cdot N^e - (\eta_1+\eta_2)\cdot(N^e - c)}{(\eta_1+\eta_2+1)\cdot N^e}\right] \Leftrightarrow$$

$$\Leftrightarrow p = N^e \cdot \left[\frac{N^e + (\eta_1+\eta_2)\cdot c}{(\eta_1+\eta_2+1)}\right] \blacksquare$$

Proof of Proposition 5:

The profit obtained by each selling division is given by:

$$\pi_{ij} = p\cdot n_{ij} - c\cdot n_{ij} \Leftrightarrow \pi_{ij} = \left[\frac{N^e + (\eta_1+\eta_2)\cdot c}{(\eta_1+\eta_2+1)}\right]\cdot\left[\frac{N^e - c}{N^e(\eta_1+\eta_2+1)}\right] - c\cdot\left[\frac{N^e - c}{N^e(\eta_1+\eta_2+1)}\right] \Leftrightarrow$$

$$\Leftrightarrow \pi_{ij} = \left[\frac{N^e + (\eta_1+\eta_2)\cdot c}{(\eta_1+\eta_2+1)} - c\right]\cdot\left[\frac{N^e - c}{N^e(\eta_1+\eta_2+1)}\right] \Leftrightarrow$$

$$\Leftrightarrow \pi_{ij} = \left[\frac{(N^e - c)^2}{N^e(\eta_1+\eta_2+1)^2}\right] \blacksquare$$

Proof of Proposition 6:

The total profit function of operator 1 is given by:

$$\pi_1(\eta_1, \eta_2) = \eta_1 \cdot \frac{(N^e - c)^2}{N^e\cdot(\eta_1+\eta_2+1)^2} - K\cdot\eta_1$$

From the first order condition, we get the following:

$$\frac{\partial \pi_1}{\partial \eta_1} = 0 \Leftrightarrow \frac{(N^e - c)^2}{N^e\cdot(\eta_1+\eta_2+1)^2} - \frac{2\eta_1\cdot(N^e - c)^2}{N^e\cdot(\eta_1+\eta_2+1)^3} - K = 0 \Leftrightarrow$$

$$\Leftrightarrow K = \frac{(N^e - c)^2}{N^e\cdot(\eta_1+\eta_2+1)^2}\cdot\left[1 - \frac{2\eta_1}{(\eta_1+\eta_2+1)}\right] \blacksquare$$

Proof of Proposition 7:

Taking into consideration the result enunciated by the Proposition 6, that is:

$$K = \frac{\left(N^e - c\right)^2}{N^e \cdot \left(\eta_1 + \eta_2 + 1\right)^2} \cdot \left[1 - \frac{2\eta_1}{N^e \cdot \left(\eta_1 + \eta_2 + 1\right)}\right]$$

Since $\eta_1^* = \eta_2^* = \eta^*$, we can make the correspondent substitution, and get the following:

$$K = \frac{\left(N^e - c\right)^2}{N^e \cdot \left(1 + 2\eta^*\right)^2} \cdot \left[1 - \frac{2\eta^*}{\left(1 + 2\eta^*\right)}\right] \Leftrightarrow$$

$$\Leftrightarrow K \cdot N^e \cdot \left(1 + 2\eta^*\right)^2 = \left(N^e - c\right)^2 \cdot \left[1 - \frac{2\eta^*}{\left(1 + 2\eta^*\right)}\right] \Leftrightarrow$$

$$\Leftrightarrow K \cdot N^e \cdot \left(1 + 2\eta^*\right)^2 = \left(N^e - c\right)^2 \cdot \left[\frac{1}{\left(1 + 2\eta^*\right)}\right] \Leftrightarrow K \cdot N^e \cdot \left(1 + 2\eta^*\right)^3 = \left(N^e - c\right)^2 \Leftrightarrow$$

$$\Leftrightarrow \left(1 + 2\eta^*\right)^3 = \frac{\left(N^e - c\right)^2}{K \cdot N^e} \Leftrightarrow 1 + 2\eta^* = \left[\frac{\left(N^e - c\right)^2}{K \cdot N^e}\right]^{\frac{1}{3}} \Leftrightarrow 2\eta^* = \left[\frac{\left(N^e - c\right)^2}{K \cdot N^e}\right]^{\frac{1}{3}} - 1 \Leftrightarrow$$

$$\Leftrightarrow \eta^* = \frac{1}{2}\left[\left(\frac{\left(N^e - c\right)^2}{K \cdot N^e}\right)^{\frac{1}{3}} - 1\right] \blacksquare$$

Proof of Lemma 2:

With modality 1 (that is, payment of a royalty over the total revenue), the franchisee profit is the following:

$$\pi_1^f = (1-\alpha)\cdot P \cdot n_{ij} - c \cdot n_{ij} \Leftrightarrow \pi_1^f = (1-\alpha)\cdot \left(N^e n_{ij} - N^e N_{\_ij} n_{ij} - N^e n_{ij}{}^2\right) - c \cdot n_{ij}$$

From the maximization profit condition (MR = MC), we obtain the optimal level of subscribers with the application of modality 1:

$$MR = MC \Leftrightarrow \left[N^e - N^e N_{\_ij} - 2N^e n_{ij}\right]\cdot(1-\alpha) = c \Leftrightarrow$$

$$\Leftrightarrow \left[N^e - N^e(\eta_1 + \eta_2 - 1)n_{ij} - 2N^e n_{ij}\right]\cdot(1-\alpha) = c \Leftrightarrow \left[N^e - N^e a n_{ij} - 2N^e n_{ij}\right]\cdot(1-\alpha) = c \Leftrightarrow$$

$$\Leftrightarrow N^e - N^e a n_{ij} - 2N^e n_{ij} - \alpha N^e + \alpha N^e a n_{ij} + 2\alpha N^e n_{ij} = c \Leftrightarrow$$

$$\Leftrightarrow N^e - \alpha N^e - c = \left(N^e a + 2N^e - \alpha N^e a - 2\alpha N^e\right) n_{ij} \Leftrightarrow$$

$$\Leftrightarrow N^e(1-\alpha) - c = n_{ij} N^e(a+2))(1-\alpha) \Leftrightarrow n_{ij} = \frac{N^e(1-\alpha) - c}{N^e(\eta_1 + \eta_2 + 1)(1-\alpha)} \quad \blacksquare$$

Proof of Lemma 3:

Under the modality 2 (that is, payment of a royalty over the total profit), the franchisee profit is the following:

$$\pi_2^f = (1-\alpha)\cdot\left[P \cdot n_{ij} - c \cdot n_{ij}\right] \Leftrightarrow \pi_2^f = (1-\alpha)\cdot\left[N^e n_{ij} - N^e N_{\_ij} n_{ij} - N^e n_{ij}{}^2 - c \cdot n_{ij}\right]$$

From the maximization profit condition (MR = MC), we obtain the optimal level of subscribers with the application of modality 2:

$$MR = MC \Leftrightarrow \left[N^e - N^e N_{\_ij} - 2N^e n_{ij}\right]\cdot(1-\alpha) = c \cdot (1-\alpha) \Leftrightarrow$$

$$\Leftrightarrow \left[N^e - N^e a n_{ij} - 2N^e n_{ij}\right] = c \Leftrightarrow N^e - c = \left(2N^e + aN^e\right) n_{ij} \Leftrightarrow$$

$$\Leftrightarrow n_{ij} = \frac{N^e - c}{N^e(\eta_1 + \eta_2 + 1)} \quad \blacksquare$$