Advertising, Pricing & Market Structure in Competitive Matching Markets*

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Abstract

This paper develops a model of pricing and advertising in a matching environment with capacity constrained sellers. Sellers' expenditure on directly informative advertising attracts consumers only probabilistically. Consumers who happen to observe advertisements randomize over the advertised sellers using symmetric mixed strategies. Equilibrium prices and profit maximizing advertising levels are derived and their properties analyzed, including the interplay of prices and advertising with the market structure. The model generates a unimodal (inverted U-shape) relationship between both, individual and industry advertising level, and market structure. The relationship results from a trade off between a price effect and a market structure-matching effect. We find that the decentralized market has underprovision of advertising, both for individual sellers and industry wide, and that entry is excessive relative to the efficient level. We present a quantitative analysis to highlight properties of the models and to demonstrate the extent of inefficiency.

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1 Introduction

We analyze the equilibrium relationships between advertising, pricing and market concentration in a market in which there are frictions. The market has N identical consumers and M identical sellers. Each consumer wants to buy one unit and each seller wants to sell one unit of an indivisible good. Each seller *must* advertise in order to be known to consumers. Each advertisement contains location, capacity and price information.[1] Based on the ads observed, consumers select one and only one seller with whom they want to trade. Then all trades occur at the advertised prices. Frictions arise in this kind of market from the fact that sellers have a capacity constraint, and consumers are uncoordinated in decisions to select a particular seller. The implication is that a particular seller might be selected by more then one consumer, in which case all but one consumer faces rationing. Models of this sort, but without explicit costly advertising have become known as markets in which sellers choose their price and probability of service. Examples of such models include Peters (1991), Deneckere and Peck (1995), and more recently Burdett, Shi and Wright (2001)[2].

This paper uses the framework of Burdett, Shi and Wright (2001), but modified to include the more realistic feature of explicit advertising choices by sellers. It is assumed that adverting is costly and that ads sent by sellers are observed by each consumer probabilistically as in Butters (1977). Without advertising (and consumer search), sellers would not be able to sell, and the market would not exist. The general idea conveyed by Butters (1977) advertising technology is one of "hit and miss" feature of advertising. The interpretation is that each consumer may not truly pay attention to the advertisement. Common examples are consumers who trash mailbox fliers before reading them, or uses television advertising time during a program to do something else. Therefore, when sellers send advertisements, only a fraction of consumers will get to observe at least one of the ads, and hence the returns on advertising expenditure is probabilistic.

We investigate how this type of advertising is influenced by equilibrium prices and by the consumers to sellers' ratio, which we refer to as market concentration.[3] In models of price and probability of service, the extent of frictions is influenced by the consumers to sellers' ratio. We are interested in how advertising and pricing are influenced by the presence of frictions, as well as how advertising in this environment affects the matching rate.

The model is simple enough to allow for the derivation of the closed form for the equilibrium advertising level, prices and matching rate, as a function of market concentration. Since we assume homogeneity on both sides of the market, all sellers choose the same level of advertising and pricing, and all consumers use the same mixed strategy over sellers from whom they have observed at least one ad.

[1] This is a model of directly informative advertising. For a recent survey on different forms of advertising see Bagwell (2001).

[2] Some models exist assuming competition in auctions using reserve prices such as McAfee (1993) and Peters and Severinov (1997). There are also models applied to the labor market such as Ciao and Shi (2000) and Julien, Kennes and King (2000).

[3] Competitive intensity is measured by the number of sellers relative to the number of consumers. Since all sellers and buyers are homogeneous groups, it is reasonable to measure concentration this way.

We find that equilibrium advertising is influenced by equilibrium market price positively as intuition dictates. Higher price makes costly advertising more worthwhile. But the magnitude of the impact depends on the market concentration. Markets with substantially less sellers then consumers (concentrated), yields a larger impact of price on advertising level then markets with substantially more sellers then consumers (competitive). This occurs because the equilibrium price is concave in market concentration (convex in the number of sellers). We also find equilibrium advertising to have a unimodal relationship with market concentration. Advertising per seller, and at the industry level reaches its peak for intermediate values of the ratio of consumers to sellers. For extreme values of the ratio, advertising converges to zero. Therefore for very slack or tight markets, advertising is found to be minimal. In other words, it suggests that peak advertising would be observed in oligopolistic markets. This result is driven by the nature of the matching environment. The derived matching function implies that for a given number of consumers, the probability with which a seller is visited by one consumer is lower when either there are numerous or a small number of sellers. The former is obvious since numerous sellers implies a very low probability a consumer will select a particular seller. Hence, it makes advertising less worthwhile. In the latter case, when there are too few sellers, the probability with which a consumer will select a particular seller is high, meaning that each seller is more likely to be visited by several consumers. Hence, no need to advertise much. There exist a good body of empirical literature emphasizing the causality that concentration influences advertising showing significant non-linear relationships, inverted U-shape, between concentration and advertising, as reported in the extensive survey from Bagwell (2003). Buxton, Davies and Lyons (1984) and Uri (1987) also provide empirical support for an inverted U-shape relationship especially for industries where a greater proportion of sales go to final consumers. In our model, all sales go to final consumers.

This paper is closely related to Butters (1977). We use his advertising technology and also assume a unit demand for each consumer. However, Butters (1977) does not have capacity constraint and our consumers are strategic, using mixed strategies in selecting sellers. In fact, introducing limited capacity along with an endogenous matching technology to analyze equilibrium advertising intensity is what differentiates this paper from all the literature.[4]. Stegemen (1991) essentially extends Butters's (1977) model to the case of consumers heterogeneous reservation values and assume large number of buyers and sellers. Stahl (1994) considers finite number of buyers and sellers and allows for downward sloping individual demand curve and a general advertising technology. He considers the influence of consumers to sellers' ratio on the price and advertising distributions. He finds, as in this paper, that more sellers imply lower prices, but less advertising per seller, as opposed to a unimodal relationship. The models of Stegemen and Stahl do not have capacity constraint also. These three models suffer from the standard non-existence in pure strategy equilibrium in prices for sellers due to the fact that sellers's payoffs are discontinuous when prices are equal.

[4] Our model is also suitable for analyzing competition in sales (advertised price discounts) with limited advertised supply (e.g., sale until supply lasts). However here, capacity is exogenous.

Hence, they resort to mixed strategy equilibrium in prices, generating price dispersion as a consequence. The matching environment assumed in this paper alleviated this discontinuity problem and allows for unique symmetric equilibrium price to be derived. Finally, none of these papers show an unimodal influence of concentration on advertising, which is a main contribution of this paper.

Remarkably Butters (1977) has shown advertising to be efficient in his model. Stahl (1994) also finds advertising to be efficient when considering a unit demand as in our model. Here, as in Stegemen (1991), we find underadvertising relative to the efficient level. Moreover, we show how this result extends to the industry level of advertising. The main reason for inefficient advertising is well known in search models. On one hand the search cost (advertising cost) is born entirely by sellers while the benefits are shared by both consumers and sellers upon a match. This induce less than socially optimal advertising. On the other hand, there is a business stealing effect when a seller sets a lower price then others, and this externality generally tend to cause excessive advertising. Butters (1977) finds that these two effects cancel, while the search cost dominates in Stegemen (1991) and Stahl (1994). We find in this paper, with limited capacity and endogenous matching, that search cost also dominate to create underadvertising relative to the efficient level for any market concentration levels. Furthermore, we show that there is excessive entry relative to the efficient level. Finally, we show how an entry tax and an advertising rate subsidy can induce efficiency of advertising and entry within a balanced budget.

The following sections are organized as follows: Section two presents the basic model with consumers and sellers' choices. Section three assesses the efficiency of advertising and entry. Section four considers a numerical simulation of the model. A discussion and conclusion follows.

2 The Model

The model consists of a large number M of identical sellers each carrying only one unit to sell, and a large number N of identical potential consumers each with a unitary demand. We refer to the ratio of consumers to sellers $\phi = N/M$ as the market tightness or size-concentration[5]. The number of sellers M and consumers N is assumed common knowledge to all sellers and consumers. However, we assume that sellers know also each others's identity, but consumers do not know sellers' identities. Each seller must advertise in order to be known to consumers. They do so by sending a number of ads or messages, s, and each consumers observe the ads only probabilistically. Each consumer has interest in purchasing from a particular seller if and only if he has observed at least one ad from that seller. Consequently, the advertising

[5]Reference to ϕ as market tightness is more commonly used in search models of the labor market using a matching function. In this paper we refer to it as size-concentration. The reason is as follows. Let σ_i be each seller's market share. The Herfindahl-Hirschman Index of market concentration is $HHI = \sum_{i=1}^{M} \sigma_i^2$. This index has the property that if the firms are identical, in this model $\sigma_i = 1/M$ for all i, then the $HHI = 1/M$. Then we refer to $\phi = (HHI)N$ as a measure of market concentration. One can also use the $C_1 = 1/M$ concentration ratio to form $\phi = C_1 N$.

rate is defined as $a = s/N$. Each ad only contains information about location/identity and price for the seller. We assume that the product features are common knowledge and focus on search goods and directly informative advertising, abstracting from issues of advertising as a signal of quality, product related informational asymmetries and persuasive advertising.

The structure of the model is described by the following sequence of events:

1. Each seller i chooses *jointly* an advertsing rate $a_i = s_i/N$ and a price p_i to maximize expected profits, taking as given other sellers's choices.

2. Each seller i sends a number of ads s_i in order to reach and attract N consumers. Each ad contains the seller's identity, location and price information p_i.

3. Each consumer observes at least one of the ads from each particular seller with probability $h = h(a_i)$ given the advertising rate a_i chosen from that seller based on a given number of consumers N. This probability is common knowledge and will be explicitly derived in the next section.

4. Upon observing locations and prices from a set of sellers' ads, each consumer selects one and only one particular seller i from which to purchase with probability $\pi_i(\mathbf{p})$, where $(\mathbf{p})$ is the observed price vector from a set of sellers.

5. Whenever matches are formed, transactions are performed at the advertised prices.

In order to solve the model, we focus on symmetric equilibria in which all sellers choose the same price and advertising rate, and consumers select over sellers using mixed strategies. This particular structure implies a tradeoff: consumers try to minimize competition at a seller, but sellers try to maximize it. In other words, consumers select over sellers trading off a price and a probability of trade.

2.1 Advertising Technology and Consumer Choice

In this section we derive a typical consumer's choice. We assume that consumers know the total population of sellers, M, but do not know the identity or location of any particular sellers, and hence, the availability of a product and price, until they have seen ads from them. A consumer's choice is then a selection of a particular seller from which to purchase an item once it has observed *at least* one ad. However, we assume that not all sellers's ads are observed by consumers. In order to capture this feature we adopt a specification of an advertising technology based on the model of Butters (1977). Assuming that each consumer observes an ad or a signal with probability $1/N$, then $(1 - 1/N)$ is the probability to observe no ads. If a seller send a number of ads s, then $(1 - 1/N)^s$ is the probability to observe none of the s ads sent by a seller. Since we focus on large markets, taking the limit of $(1-1/N)^s$ as N and s go to infinity but keeping the ratio s/N fixed, the probability with which a consumer does not observe any of the s ads is $e^{-s/N}$. Therefore, the probability to observe at least one of the s ads from a particular seller is $(1-e^{-s/N})$. Using the advertising rate $a = s/N$, we write $h(a) = (1-e^{-a})$ and note that this probability is independent of advertising rates performed by other sellers. This seems natural in the context of directly informative advertising where it only conveys information about a particular seller's identity, location, price and possibly other informative features. This advertising technology is only one way to represent how sellers search for consumers. It also considers consumers as passive in the sense that they do not take any actions to increase the probability to observe a seller's advertisement. Although we could introduce costly consumers search, for example by allowing them to sample more communication media, or simply searching for prices as in Robert and Stahl (1993), we focus on search intensity on the sellers' side. This is equivalent to assuming that the net benefits of searching for sellers is negative for consumers. This may be explained by high opportunity cost of search such as work hours and/or home production.

The immediate implication of introducing an advertising technology is that each consumer j gets to observe at least one ad, and hence, prices from a set of sellers $m^j \subseteq \{0, 1, ..., M\}$. Let $\#m^j$ be the cardinality of the set m^j. If $\#m^j = 0$ consumer j recieved no ads at all and is *Uninformed*. If $\#m^j = 1$, consumer j has observed ads from only one seller so he is *captive*, implying $\pi^j(\mathbf{p}(m^j)) = 1$. When consumer j observes ads from several sellers, $\#m^j > 1$, he is *selective* and uses a mixed strategy. Since m^j is private information to consumer j, for any vector of advertising rates $\mathbf{a}$, using the probabilities $h(a_i)$ for each seller i, one obtains the probability with which consumer j observes m^j as

$$\omega(m^j, \mathbf{a}) = \binom{M}{m^j} \prod_{\tau=1}^{m^j} h(a_\tau) \prod_{\ell=m^j+1}^{M} (1 - h(a_\ell)). \tag{1}$$

Consumer j's strategy consists of selecting one and only one seller with probability $\pi^j(\mathbf{p}(m^j))$ where $\mathbf{p}(m^j) = (p_1, ..., p_{m^j})$ is the observed price vector from the m^j sellers, and $\sum_{i=1}^{m^j} \pi_i^j(\mathbf{p}(m^j)) = 1$. That is consumers use mixed strategies over their set of known prices. The probabilities $\omega(m^j, \mathbf{a})$ can be used to form a discrete probabiltity distribution over the possible mixed strategies $\pi_i^j(\mathbf{p}(m^j))\ \forall m^j \subseteq \{0, 1, ..., M\}$. From this distribution one can obtain the probability that any one of the M sellers be *observed* and *selected* by consumer j as

$$\theta^j(\mathbf{p}, \mathbf{a}) = \sum_{m^j=1}^{M} \omega^j(m^j, \mathbf{a})\pi^j(\mathbf{p}(m^j)) \tag{2}$$

where $(\mathbf{p}, \mathbf{a}) = (p_1, ..., p_M, a_1, ..., a_M)$. This probability simply represents the fact that $\pi^j(\mathbf{p}(m^j))$ used by each consumer j is private information, and hence, $\theta^j(\mathbf{p}, \mathbf{a})$ is the expected mixed strategy of consumer j. Symmetry on the consumers' side implies $\theta^j(\mathbf{p}, \mathbf{a}) = \theta(\mathbf{p}, \mathbf{a})$ for all j and any vectors $\mathbf{p}$ and $\mathbf{a}$.

When selecting a particular seller $i \in m^j$, a consumer needs to assess the probability with which other consumers select seller i. This probability depends on how many other sellers these consumers have observed as captured by the probabiltiy $\theta(\mathbf{p}, \mathbf{a})$. This probability also depends on the vectors of advertising and prices. However, consumers do not observe the vector of advertising rates $\mathbf{a}$ and may not get to observe the overall price vector $\mathbf{p}$. Letting $\mathbf{p}(M) = (p_1, ..., p_M)$ be the overall price vector, then $\mathbf{p}(M \backslash m^j)$ is the set of prices not observed by consumer j. Therefore, any other consumer who has observed a price $p_k \in \mathbf{p}(M \backslash m^j)$ will select seller k with positive probability and the actual price set by seller k will inflence the mixed strategy they use. Consumer j must form expectations about advertising rates, but also about unobserved prices based on his set of observed prices. That is, a consumer forms the following expectations $E(\mathbf{p}(M \backslash m^j)) = \tilde{\mathbf{p}}(M \backslash m^j)$ and $E(\mathbf{a}) = \tilde{\mathbf{a}}$ which he uses to evaluate the probability with which other consumers observe and select a particular seller i. For example, a consumer who has observed seller $i \in m^j$ and selects this seller with probability π_i^j will make the following calculations about other consumer ℓ's probability to observe and select seller i :

$$\tilde{\theta}_i^{\ell}(\mathbf{p}(m^j), \tilde{\mathbf{p}}(M \backslash m^j), \tilde{\mathbf{a}}) = \sum_{m^{\ell}=1}^{M} \omega_i^{\ell}(m^{\ell}, \tilde{\mathbf{a}}) \pi_i^{\ell}(\mathbf{p}(m^j \cap m^{\ell}), \tilde{\mathbf{p}}(m^{\ell} \backslash m^j)).$$

where $\omega_i(m^{\ell}, \tilde{\mathbf{a}}) = \binom{M}{m^{\ell}} \prod_{\tau=1}^{m^{\ell}} h(\tilde{a}_{\tau}) \prod_{\ell=m^{\ell}+1}^{M} (1 - h(\tilde{a}_{\ell}))$ and $\mathbf{p}(m^j \cap m^{\ell})$ is the price vector commonly observed by any two consumers j and ℓ, and $\tilde{\mathbf{p}}(m^{\ell} \backslash m^j)$ is the vector of prices observed by consumer ℓ and not by consumer j, but has formed expectations over it.

Under symmetry and assuming rational expectations $\tilde{\theta}_i^{\ell}(\mathbf{p}(m^j), \tilde{\mathbf{p}}(M \backslash m^j), \tilde{\mathbf{a}}) = \theta_i^{\ell}(\mathbf{p}, \mathbf{a}) = \theta(\mathbf{p}, \mathbf{a})$ for all ℓ and for all i.

Assuming that each consumer extracts a utility value normalized to 1 from consuming the good, the consumer surplus from selecting seller i is $(1 - p_i)$. Since each seller carries only one unit of the good to be sold, the rationing rule is such that when several consumers select the same seller, each one get the good with equiprobability. Consumers' selections and the rationing rule translate into a probability Λ_i^j to get served which depends on the number of other consumers also selecting seller i.

A consumer j's expected utility from selecting seller i is

$$U_i^j(\mathbf{p}, \mathbf{a}) = (1 - p_i)\Lambda_i^j(\mathbf{p}, \mathbf{a}). \tag{3}$$

We focus on mixed-strategy equilibrium selection for consumers. (See Burdett, et al. (2001).) The mixed-strategy equilibrium selection is such that a consumer is indifferent between selecting any two observed sellers. For any given vectors of prices and advertising rates $(\mathbf{p}, \mathbf{a})$ set by sellers, a mixed-strategy equilibrium selection for any consumer $j = 1, ...N$ is a vector of probabilities $\boldsymbol{\pi}^j(\mathbf{p}) = (\pi_1^j(\mathbf{p}), ..., \pi_{m^j}^j(\mathbf{p}))$ solving

$$U_i^j(\mathbf{p}, \mathbf{a}) = U_k^j(\mathbf{p}, \mathbf{a}) \tag{4}$$

for all $i, k \in m^j$, such that $\sum_{i=1}^{m^j} \pi_i^j(\mathbf{p}) = 1$. Unlike the model of Burdett et *al.* (2001), advertising generates a distribution of these equilibrium mixed strategies.

The probability with which consumer j gets served when selecting seller i is

$$\Lambda_i^j(\mathbf{p},\mathbf{a}) = \sum_{n_i=1}^{N} \left(\frac{1}{n_i+1}\right) \binom{N-1}{n_i} \theta_i(\mathbf{p},\mathbf{a})^{n_i} (1-\theta_i(\mathbf{p},\mathbf{a}))^{N-1-n_i}. \tag{5}$$

where n_i is the number of consumers (excluding consumer j) who have selected seller i, and $\left(\frac{1}{n_i+1}\right)$ is the rationing rule.

To simplify this probability, consider the probability that a consumer is alone selecting seller i, which occurs when $n_i = 0$,

$$\Pr\{n_i = 0\} = (1-\theta_i(\mathbf{p},\mathbf{a}))^N$$

and the probability of not being alone selecting seller i is

$$\Pr\{n_i > 0\} = 1-(1-\theta_i(\mathbf{p},\mathbf{a}))^N$$

Using this simplification,[6]

$$\Lambda_i(p,a) = \frac{\left[1-(1-\theta_i(\mathbf{p},\mathbf{a}))^N\right]}{N\theta_i(\mathbf{p},\mathbf{a})}.$$

The probability to get served at seller i is the probability that at least someone has seen the ad and selects seller i, $(1-(1-\theta_i(\mathbf{p},\mathbf{a}))^N$, times the probability to "win" the item $\frac{1}{N\theta_i(\mathbf{p},\mathbf{a})}$,

[6] Since probabilities sum to one

$$\Pr\{n>0\} = \sum_{n=1}^{N-1} \binom{N-1}{n} \theta^n (1-\theta)^{N-1-n} = 1-(1-\theta)^{N-1}$$

is the probability that *at least* one other consumer observes the ad of a seller and selects that particular seller. Using this simplification along with the rationing rule, the probability to get served is transformed as follows:

$$\begin{aligned}
\Lambda(\mathbf{p},a) &= \left[\sum_{n=1}^{N-1} \left(\frac{1}{n+1}\right) \binom{N-1}{n} \theta^n (1-\theta)^{N-1-n}\right] \\
&= \frac{1}{N} \sum_{n=0}^{N-1} \binom{N}{n+1} \theta^n (1-\theta)^{N-1-n} \\
&= \frac{1}{N\theta} \sum_{n=1}^{N} \binom{N}{n} \theta^n (1-\theta)^{N-n} \\
&= \frac{(1-(1-\theta)^N)}{N\theta}.
\end{aligned}$$

where $N\theta_i(\mathbf{p}, \mathbf{a})$ is the expected number of consumers who have observed and selected that seller.

Since there are M sellers in the market from which advertisements can potentially be observed, we have

$$\sum_{i=1}^{M} \theta_i(\mathbf{p}, \mathbf{a}) = 1 - \prod_{i=1}^{M}(1 - h(a_i)) \tag{6}$$

where the term $\prod_{i=1}^{M}(1-h(a_i))$ simply reflects the possibility that a consumer observed no ads at all. If all sellers set the same advertising rate a then $\prod_{i=1}^{M}(1 - h(a_i)) = (1 - h(a))^M$.

Suppose now that a deviant seller sets price $\hat{p}$ while all other sellers set price p, but all sellers set the same advertising rate. This yields $\hat{\theta} + (M - 1)\theta = 1 - (1 - h(a))^M$ and the probability with which a consumer oberves and is expected to select any of the non-deviant sellers is $\theta = \frac{1-\hat{\theta}-(1-h(a))^M}{(M-1)}$.

The expected utility from observing and selecting the deviant seller is

$$\hat{U}(\hat{p}, \mathbf{p}_{-1}, \mathbf{a}) = (1 - \hat{p})\frac{\left[1 - \left(1 - \hat{\theta}\right)^N\right]}{N\hat{\theta}}. \tag{7}$$

Similarly, the expected utility from selecting a non deviant seller is

$$U(\hat{p}, \mathbf{p}_{-1}, \mathbf{a}) = (1 - p)\frac{\left[1 - \left(1 - \left(\frac{1-\hat{\theta}-(1-h(a))^M}{M-1}\right)\right)^N\right]}{N\left(\frac{1-\hat{\theta}-(1-h(a))^M}{M-1}\right)}. \tag{8}$$

A symmetric, mixed-strategy, equilibrium selection $\hat{\theta}(\hat{p}, \mathbf{p}_{-1}, \mathbf{a})$ solves:

$$U(\hat{p}, \mathbf{p}_{-1}, \mathbf{a}) = \hat{U}(\hat{p}, \mathbf{p}_{-1}, \mathbf{a}) \tag{9}$$

However, explicit solutions for $\theta(\mathbf{p}, \mathbf{a})$ is characterized by a polynomials of high degree and cumbersome to handle.[7] Fortunately, the explicit solutions are not needed for the equilibrium derivations. All we need is the conditions under which a mixed strategy equilibrium selection exist. It can be shown that as long as the price deviation is not too small or not too big, there is a unique $\hat{\theta} \in (0, 1)$ that makes consumers indifferent between the deviant seller and the non-deviant seller.[8]

[7] This property of $\theta(p, a)$ is particular to the use of a posted price mechanism. When using auctions or ex post bidding, these probabilities have explicit solutions (see Julien, Kennes, and King (2000, 2005).

[8] See Burdett, Shi and Wright (2001) for a demonstration of the conditions.

Lemma 1 *The unique symmetric mixed-strategy equilibrium when $p_i = p^*$ and $a_i = a^*$ for all i is $\pi_i^{j*} = 1/m^j$ for all j and for all $m^j \in \{1, ..., M\}$.*[9]

In a symmetric mixed-strategy equilibrium in which all sellers choose the same advertising rate a^* and price p^*, the unique expected selection strategy is $\theta = \hat{\theta} = \theta^* = \frac{1-(1-h(a^*))^M}{M}$.

In this model, we focus on mixed strategy equilibrium in order to capture the feature that consumers' decisions are uncoordinated. While coordination is a more probable behavior in small markets, in large markets this is unlikely and harder to implement. This is equivalent to assuming a communication technology constraint preventing consumers from coordinating on their selection strategies. In this model there are pure selection strategy equilibria. But the implementation of these equilibria would require a coordination technology that we assume away here.

2.2 Equilibrium Prices and Advertising Rates

Equilibrium market price

Each seller jointly chooses its price and advertising rate simultaneously to maximize expected profits, taking as given other sellers's choices, and *expected* consumers' behavior.[10] In order to be active on the market, each seller must incur a fixed cost $F \in [0, 1]$, which may or may not be associated with advertising as in the cost of setting up a selling location.[11] Each seller faces a variable cost $c(a)$ for an advertising rate of a, with constant marginal cost, that is $c' > 0$ and $c'' = 0$. The expected profit for a seller is

$$\Pi(\mathbf{p}, \mathbf{a}) = pq(\mathbf{p}, \mathbf{a}) - c(a) - F \tag{10}$$

where $q(\mathbf{p}, \mathbf{a}) = \left[1 - (1 - \theta(\mathbf{p}, \mathbf{a}))^N\right]$ is the probability of sale. Sellers use the probability $\theta(\mathbf{p}, \mathbf{a})$ to derive the probability of sale since they do not know consumers' exact mixed strategies. Finding the symmetric equilibrium market price involves solving a set of M reaction functions. Instead we use the technique in Burdett, Shi and Wright (2001) which

[9] For a proof of this Lemma see Burdett et *al.* (2001).

[10] McAfee (1994) considers a two–stage game theoretic model where firms choose advertising first and prices in the second stage. However, for a model considering simultaneous choices see Robert and Stahl (1993).

[11] Fixed costs associated with advertising are sunk costs. Advertisements are costly to set up, or more generally a marketing campaign needs preparation. However, even for industries where fixed costs associated with advertising may not be present or even minimal, the fixed cost we introduce can be taken as fixed cost for a seller to be on the market, such as cost of setting up the shop/location. Given the constant marginal cost assumption, there are economies of scale in advertising. This is consistent with existing communication technologies.

is to assume as above that all sellers but one set a price p and advertising rate a, while a deviant seller sets price $\hat{p}$. The objective of a deviant seller is

$$\max_{<\hat{p}>} \hat{\Pi}\left(\hat{p}, \mathbf{p}_{-1}, \mathbf{a}\right) = \max_{<\hat{p},a>} \left\{\hat{p}q\left(\hat{p}, \mathbf{p}_{-1}, \mathbf{a}\right) - c(a) - F\right\}, \tag{11}$$

where now $q\left(\hat{p}, \mathbf{p}_{-1}, \mathbf{a}\right) = \left(1 - \left(1 - \theta\left(\hat{p}, \mathbf{p}_{-1}, \mathbf{a}\right)\right)^N\right)$. Consider first the pricing decision, taking the price p and advertising rate a of all other sellers as given. The profit maximizing deviation satisfies

$$\frac{\partial \hat{\Pi}}{\partial \hat{p}} = q\left(\hat{p}, \mathbf{p}_{-1}, \mathbf{a}\right) + \hat{p}\frac{\partial q\left(\hat{p}, \mathbf{p}_{-1}, \mathbf{a}\right)}{\partial \hat{p}} = 0. \tag{12}$$

or

$$\left(1 - \left(1 - \hat{\theta}\right)^N\right) + \hat{p}N\left(1 - \hat{\theta}\right)^{N-1}\frac{\partial \hat{\theta}}{\partial \hat{p}} = 0$$

where $\frac{\partial \hat{\theta}}{\partial \hat{p}} = \sum_{m=1}^{M} \omega(m, \mathbf{a})\frac{\partial \pi(\hat{p}, \mathbf{p}_{-1}(m-1)}{\partial \hat{p}}$. The second-order condition is satisfied since $q(\hat{p}, \mathbf{p}_{-1}, \mathbf{a})$ is concave in $\hat{p}$. Assuming $\hat{\theta} \in (0,1)$, differentiating (9) with respect to $\hat{p}$ and inserting the symmetric equilibrium conditions $\hat{p} = p$, and a for all sellers, and $\hat{\theta} = \theta = \frac{1-(1-h(a))^M}{M}$ yields

$$\frac{\partial \hat{\theta}}{\partial \hat{p}} = \frac{-\left(M-1\right)\left(1-(1-h(a))^M\right)\left(M-(1-(1-h(a))^M)\right)\left[1 - \left(\frac{M-(1-(1-h(a))^M)}{M}\right)^N\right]}{M^2\left(1-p\right)\left[M - 1 - \left(\frac{M-(1-(1-h(a))^M)}{M}\right)^N\left(M + \left(N-1\right)\left(1-(1-h(a))^M\right)\right)\right]} < 0. \tag{13}$$

Inserting this into (12), yields

$$p^*(M, N, \mathbf{a}) = \frac{M - M\left(\frac{M+(N-1)(1-(1-h(a))^M)}{M-(1-(1-h(a))^M)}\right)\left(\frac{M-(1-(1-h(a))^M)}{M}\right)^N}{M - \left(\frac{M^2-(M-N)(1-(1-h(a))^M)}{M-(1-(1-h(a))^M)}\right)\left(\frac{M-(1-(1-h(a))^M)}{M}\right)^N}. \tag{14}$$

The equilibrium price in finite markets depends on the advertising rates. It is easily shown that $p^*(M, N, \mathbf{a})$ is strictly increasing in $\mathbf{a}$, meaning that higher advertising rates by all sellers translates into a higher equilibrium market price. This simply means that if all sellers choose more advertising, they choose a higher price in equilibrium to generate the expected revenue to compensate for the extra advertising expenditure. In large markets the equilibrium price $p^*\left(M, N\right)$ converges very quickly to

$$p^*\left(\phi\right) = \lim_{M,N\to\infty} p^*\left(M, N\right) = 1 - \frac{\phi}{e^{\phi} - 1}. \tag{15}$$

which is the same limit price found by Burdett, Shi and Wright (2001). Therefore, in large markets, the influence of advertising on equilibrium prices becomes insignificant as one would expect.

Properties of the equilibrium market price

The equilibrium market price $p^*(\phi)$ is strictly increasing and strictly concave in ϕ with limits

$$\lim_{\phi\longrightarrow 0} p^*(\phi) = 0 \quad \text{and} \quad \lim_{\phi\longrightarrow\infty} p^*(\phi) = 1. \tag{16}$$

Furthermore, $p^*(\phi)$ converges quickly to its limit value of 1. For instance a value $\phi = 10$ is enough to have $p^*(\phi)$ very close to 1. Since we have an equilibrium market price determined by concentration, we can define the elasticity of market price with respect to concentration as $\xi(\phi) = \frac{\partial p^*(\phi)}{\partial \phi}\frac{p^*(\phi)}{\phi}$, which is always positive, along with properties that $\xi(\phi)$ is strictly decreasing in ϕ, with $\lim_{\phi\longrightarrow 0}\xi(\phi) = 1$ and $\lim_{\phi\longrightarrow\infty}\xi(\phi) = 0$. These properties indicate that as the market becomes less competitive, higher ϕ, the equilibrium market price increases and this price becomes less sensitive to a further decline in competition. In other words, with a large consumer to seller ratio, an exit by one seller will not have much of an impact on the equilibrium price. However, when such a ratio is small, an exit by one seller creates a bigger impact on the equilibrium price.

Equilibrium advertising rates

Each seller now chooses an advertising rate $\hat{a}$, given their choices of price p, and taking as given all other sellers choices of advertising rates $\mathbf{a}_{-1}$ and prices $\mathbf{p}$, to solve:

$$\max_{\hat{a}} \Pi(\mathbf{p}, \hat{a}, \mathbf{a}_{-1}) = \max_{\hat{a}} \{p(\phi, \mathbf{a}) q(\mathbf{p}, \hat{a}, \mathbf{a}_{-1}) - c(\hat{a}) - F\}$$

where $q(\mathbf{p}, \hat{a}, \mathbf{a}_{-1}) = \left[1 - (1 - \theta(\mathbf{p}, \hat{a}, \mathbf{a}_{-1}))^N\right]$. The first-order condition is

$$\frac{\partial \hat{\Pi}}{\partial \hat{a}} = \left(\frac{\partial p(M, N, \mathbf{a})}{\partial \hat{a}} q(\mathbf{p}, \hat{a}, \mathbf{a}_{-1}) + p\frac{\partial q(\mathbf{p}, \hat{a}, \mathbf{a}_{-1})}{\partial \hat{a}} - \frac{\partial c(\hat{a})}{\partial \hat{a}}\right) = 0, \tag{17}$$

which is the standard marginal revenue equals marginal cost of advertising rate. Note that $\frac{\partial q(\mathbf{p},\hat{a},\mathbf{a}_{-1})}{\partial \hat{a}} = N(1 - \theta(\mathbf{p}, \hat{a}, \mathbf{a}_{-1}))^{N-1}\frac{\partial \theta(\mathbf{p},\hat{a},\mathbf{a}_{-1})}{\partial \hat{a}}$, and $\frac{\partial \theta(\mathbf{p},\hat{a},\mathbf{a}_{-1})}{\partial \hat{a}} = \sum_{m=1}^{M}\frac{\partial \omega(m,\hat{a},\mathbf{a}_{-1})}{\partial \hat{a}}\pi(\mathbf{p}(m)) = \frac{\theta(\mathbf{p},\mathbf{a})h'(\hat{a})}{h(a)} > 0$, meaning that a higher advertising rate by a seller increases the probability to be observed and selected, but *not* the conditional probability to be selected once the consumer has observed at least one ad, that is $\boldsymbol{\pi}(\mathbf{p})$. This is the nature of informative advertising. Here, once a consumer has observed at least one ad from a seller, a higher advertising rate cannot induced consumers to change their mixed strategy $\boldsymbol{\pi}(\mathbf{p})$ in his favor. If we were to allow for this effect, advertising would need to have a persuasive element.

The first-order condition yields the reaction function

$$\frac{\partial p(M, N, \mathbf{a})}{\partial \hat{a}} q(\mathbf{p}, \hat{a}, \mathbf{a}_{-1}) + p(M, N, a) N (1 - \theta(\mathbf{p}, \hat{a}, \mathbf{a}_{-1}))^{N-1}\frac{\partial \theta(\mathbf{p}, \hat{a}, \mathbf{a}_{-1})}{\partial \hat{a}} = c'(\hat{a}). \tag{18}$$

The second-order condition is satisfied since $h(a)$, and hence $q(\mathbf{p}, \mathbf{a})$, are concave in a. Under symmetric equilibrium $p = p^*$ and $\hat{a}^* = a^*$ for all M with $\theta(\mathbf{p}^*, \mathbf{a}^*) = \frac{1-(1-h(a^*))}{M}$, where p^* is defined in (15). Assuming a total cost of advertising of $c(a) = \beta a$, and using $h'(a) = e^{-a}$, the first-order condition becomes

$$\frac{\partial p(M, N, \mathbf{a})}{\partial \hat{a}} q(\mathbf{p}, \hat{a}, \mathbf{a}_{-1}) + \frac{p^*(M, N, \mathbf{a}) N \left(1 - \frac{1-(1-h(a^*))}{M}\right)^{N-1} e^{-a^*}(1-(1-h(a^*))^M)}{M(1-e^{-a^*})} = \beta, \tag{19}$$

and in large markets,

$$\frac{p^*(\phi)\,\phi e^{-\phi} e^{-a^*}}{(1-e^{-a^*})} = \beta. \tag{20}$$

This is the standard condition equalizing marginal private benefit (MPB_a) and marginal private cost (MPC_a) for optimal advertising rate. The profit maximizing advertising rate for each seller for all market concentration values ϕ is

$$a^*(\phi) = \ln\left(p^*(\phi)\,\phi e^{-\phi} + \beta\right) - \ln \beta. \tag{21}$$

The profit maximizing individual advertising rate is driven by the equilibrium market price, p^*, the probability with which a consumer will be *captive*, $\phi e^{-\phi}$, and the marginal cost of advertising, β.

Properties of the equilibrium advertising rate

Higher marginal cost of advertising induces less advertising

$$\frac{\partial a^*(\phi)}{\partial \beta} = \left[\frac{1}{p^*(\phi)\phi e^{-\phi} + \beta} - \frac{1}{\beta}\right] < 0, \quad \text{for all } \phi. \tag{22}$$

Sellers choose advertising to maximize expected profits being cognizant of the equilibrium price to result in the market as a function of concentration ϕ. Hence, $a^*(\phi)$ is strictly increasing and strictly concave in $p^*(\phi)$:

$$\frac{\partial a^*(\phi)}{\partial p^*(\phi)} = \frac{e^{-\phi}\phi}{p^*(\phi)\,\phi e^{-\phi} + \beta} > 0 \text{ and } \frac{\partial^2 a^*(\phi)}{\partial p^*(\phi)^2} = -\frac{(e^{-\phi}\phi)^2}{(p^*(\phi)\,\phi e^{-\phi} + \beta)^2} < 0. \tag{23}$$

All else constant, a higher equilibrium price is associated with higher advertising rate. As one would expect, a higher price makes it more worthwhile for a seller to spend more on advertising. The strict concavity of a^* in p^* implies that continual increases in price brought about by other factors than market concentration would require less and less increment in individual advertising rates. The profit maximizing advertising rate has the following limits:

$$\lim_{\phi \longrightarrow 0} a^*(\phi) = \lim_{\phi \longrightarrow \infty} a^*(\phi) = 0 \tag{24}$$

As the market looks like the perfectly competitive one ($\phi \to 0$), the equilibrium price converges to zero which calls for zero advertising. Sellers in this model are required to perform advertising in order to have a chance to make a sale. But the model's outcomes do converges to the standard perfectly competitive outcome with no advertising and equilibrium price equals marginal cost which is zero in the model. On the other hand, as the market looks like a monopoly ($\phi \to \infty$), advertising converges to zero.

In order to demonstrate further properties of $a^*(\phi)$, note that in large markets the equilibrium expected revenue for each seller is

$$R(\phi) = p^*(\phi)(1 - e^{-\phi}) = \left(\frac{e^{\phi} - 1 - \phi}{e^{\phi}}\right) \tag{25}$$

with

$$R'(\phi) = \frac{\partial R(\phi)}{\partial \phi} = e^{-\phi}\phi > 0 \text{ and } R''(\phi) = \frac{\partial R'(\phi)}{\partial \phi} = e^{-\phi}(1-\phi) \gtreqless 0, \tag{26}$$

and

$$\lim_{\phi \longrightarrow 0} R(\phi) = 0 \text{ and } \lim_{\phi \longrightarrow \infty} R(\phi) = 1. \tag{27}$$

These properties imply that the equilibrium expected revenue for each seller is convex for all $\phi \leq 1$ and concave for all $\phi \geq 1$.[12] Using the expected revenue properties, the profit maximizing advertising rate becomes:

$$a^*(\phi) = \ln\left(p^*(\phi) R'(\phi) + \beta\right) - \ln \beta. \tag{28}$$

where $R'(\phi)$ is the marginal expected revenue from a change in concentration which turns out to be equal to the probability with which a seller faces only one consumer after choosing a price and advertising rate. The following proposition summarizes the final property of advertising rate.

Proposition 1 *The profit maximizing advertising rate $a^*(\phi)$ is unimodal (inverted U-shape) in ϕ, and reaches a maximum at $\tilde{\phi} > 1$.*

Proof. Taking the derivative of $a^*(\phi)$ with respect to ϕ,

$$\frac{\partial a^*(\phi)}{\partial \phi} = \frac{\left[\frac{\partial p^*(\phi)}{\partial \phi} R'(\phi) + p^*(\phi) R''(\phi)\right]}{\left(p^*(\phi) R'(\phi) + \beta\right)} \gtreqless 0 \tag{29}$$

and rearranging as

$$\frac{\partial a^*(\phi)}{\partial \phi} = \frac{p^*(\phi) R'(\phi)}{\phi\left(p^*(\phi) R'(\phi) + \beta\right)} \left[\frac{\partial p^*(\phi)}{\partial \phi}\frac{p^*(\phi)}{\phi} + \phi \frac{R''(\phi)}{R'(\phi)}\right]$$

[12] The marginal equilibrium revenue from a change in market structure can decomposed as $\frac{\partial R(\phi)}{\partial \phi} = \frac{\partial p^*(\phi)}{\partial \phi}(1 - e^{-\phi}) + p^*(\phi)e^{-\phi}$, where the first term is the increase in equilibrium price each seller will get if they make a sale when ϕ increases (lower M). The second term is the equilibrium price lost if a seller does not make a sale.

and letting $\rho(\phi) = \phi \frac{R''(\phi)}{R'(\phi)}$,

$$\frac{\partial a^*(\phi)}{\partial \phi} = \frac{p^*(\phi)R'(\phi)}{\phi\left(p^*\left(\phi\right)R'(\phi) + \beta\right)}[\xi(\phi) + \rho(\phi)] \gtreqless 0. \tag{30}$$

It follows that since $\frac{p^*(\phi)R'(\phi)}{\phi(p^*(\phi)R'(\phi)+\beta)} > 0$ for all ϕ, the sign of $\frac{\partial a^*(\phi)}{\partial \phi}$ has the sign of $[\xi(\phi)+\rho(\phi)]$. There exist a $\tilde{\phi} > 1$ such that $\xi(\tilde{\phi}) + \rho(\tilde{\phi}) = 0$ and hence $\frac{\partial a^*(\phi)}{\partial \phi} = 0$. For all $\phi \in (0, \tilde{\phi})$, it implies that $\frac{\partial a^*(\phi)}{\partial \phi} > 0$ and $\phi > \tilde{\phi}$, $\frac{\partial a^*(\phi)}{\partial \phi} < 0$. Note that $\xi(\phi) > 0$ for all ϕ. For $\phi < 1$, $R(\phi)$ is convex and $\rho(\phi)$ is positive, hence $\frac{\partial a^*(\phi)}{\partial \phi} > 0$. For $\phi = 1$, $R(\phi)$ is at an inflexion point and $\rho(\phi) = 0$, so $\frac{\partial a^*(\phi)}{\partial \phi} > 0$. For $\phi > 1$, $R(\phi)$ is concave and $\rho(\phi) < 0$. However, ρ needs to be negative enough to change the sign of $\alpha^*(\phi)$. The advertising rate reaches a maximum, at $a^*(\tilde{\phi})$, which can only happen when $R(\phi)$ is concave, or when $\rho(\phi)$ is negative, then $\tilde{\phi} > 1$. Therefore, for all $\phi \in (1, \tilde{\phi}]$, $\frac{\partial a^*(\phi)}{\partial \phi} > 0$. ■

What explains the unimodal form of $a^*(\phi)$ in ϕ is the relative magnitude of how the equilibrium market price and the expected marginal revenue change with concentration. The price effect is always positive as shown by the elasticity $\xi(\phi)$. The expected marginal revenue effect however is unimodal in ϕ as measure by $\rho(\phi)$. Basically, the expected marginal revenue effect is determined by the probability to face one consumer only by a seller. As the market is relatively fragmented, an increase in concentration yields a higher probability to face only *one* consumer by a seller relative to the probability to face many. But the probability to make a sale is driven by the probability to face at least one consumer. In such case, it is worthwhile for a seller to increase advertising intensity to maximize expected profits. For relatively concentrated markets, an increase in concentration yields a lower probability of facing only *one* relative to the probability of facing several consumers. In such case, their is less need for advertising and advertising intensity decreases in concentration.

It is informative to decompose further the marginal revenue effect which depends on ϕ and $e^{-\phi}$. Clearly an increase in ϕ induce a higher marginal revenue effect. However, the probability with which a seller does not make a sale, $e^{-\phi}$, decreases with ϕ. For concentration values $\phi \in (0, 1]$, both effect work in the same direction to increase $a^*(\phi)$. The important impact of reduction in competition on equilibrium market price and marginal revenue makes it worthwhile for each seller to increase advertising intensity. For concentration values of $\phi \in (1, \tilde{\phi}]$, the marginal revenue effect is negative but not enough to warrant a decrease in advertising intensity. For concentration values below $\tilde{\phi}$, the reduction in probability of no sale for each seller from a reduction in competition is not important enough, inducing each seller to increase advertising intensity in order to boost the probability of sale. However, for values of $\phi > \tilde{\phi}$, the marginal revenue effect is more important, meaning that reduced competition warrants a decrease in advertising intensity. Essentially, when competition is less intense, as measured by a smaller number of sellers relative to consumers, the probability to not make a sale as the number of seller decreases becomes important. This happens because there are already a relatively small number of seller. Since the decrease in probability of not making a sale becomes less important, and the probability to be visited by several consumers get higher, there is less need for each seller to advertise, and hence a lower advertising intensity is performed.

Basically, there is a trade off between an incentive to increase advertising driven by equilibrium price increase, but an incentive to advertise less since there are less sellers and the probability of no sale decreases. For low concentration markets, the price effect is more important. Sellers have an incentive to advertise more to compensate the marginal reduction in probability of no sale. Advertising rate increases in concentration. Eventually, the number of seller gets relatively small and the equilibrium price increases but not by much, while the probability of no sale becomes more important. In highly concentrated markets, the reduction in probability of no sale dominates, hence less need to advertise. Advertising rate decreases in concentration.

There is an interesting alternative way to highlight the unimodal aspect of advertising intensity relative to concentration which could be more suitable for empirical testing, and is also useful to demonstrate the properties of the industry advertising rate. This is summarized by the following corollary.

Corollary 1 *Let* $\sigma(\phi) = \frac{\partial a^*(\phi)}{\partial p^*(\phi)} \frac{p^*(\phi)}{a^*(\phi)}$ *be the price elasticity of advertising rate,* $\alpha^*(\phi) = \frac{\partial a^*(\phi)}{\partial \phi} \frac{\phi}{a^*(\phi)}$ *be the market concentration elasticity of advertising rate, and* $\rho(\phi) = \phi \frac{R''(\phi)}{R'(\phi)}$ *be a relative measure of the curvature of* $R(\phi)$*. Then*

$$\alpha^*(\phi) = \sigma(\phi)[\xi(\phi) + \rho(\phi)] \gtreqless 0 \text{ if and only if } \phi \lesseqgtr \tilde{\phi}$$

where $\tilde{\phi}$ *solves* $\xi(\tilde{\phi}) + \rho(\tilde{\phi}) = 0$*.*

Dividing both sides of 30 by $a^*(\phi)$ yields

$$\frac{\partial a^*(\phi)}{\partial \phi} \frac{\phi}{a^*(\phi)} = \frac{p^*(\phi) R'(\phi) \left[\xi(\phi) + \rho(\phi)\right]}{a^*(\phi) \left(p^*(\phi) R'(\phi) + \beta\right)}.$$

Recognizing that $\frac{\partial a^*(\phi)}{\partial p^*(\phi)} = \frac{e^{-\phi}\phi}{p^*(\phi)\phi e^{-\phi} + \beta} = \frac{R'(\phi)}{p^*(\phi)R'(\phi) + \beta}$,

$$\alpha^*(\phi) = \sigma(\phi)[\xi(\phi) + \rho(\phi)].$$

Estimates of $\alpha^*(\phi)$ becomes possible. First, one can estimate the number of consumers N. Then, given that firms are symmetric, their respective market shares are $\sigma = \frac{1}{M}$, and the Herfindahl-Hirschman Index (HHI) is also $\frac{1}{M}$. Therefore $\phi = N(HHI)$. For very small HHI, large ϕ, the market looks like a monopoly while for very large HHI, small ϕ, it looks like perfect competition. Estimates of $\sigma(\phi)$ and $\xi(\phi)$ from historical data can be obtained for a particular industry characterized by capacity constraint and product homogeneity. Using the derived properties of expected revenue, we find that $\rho(\phi) = 1 - \phi$, allowing an estimate for the marginal revenue effect of a change in market concentration.

The relationship between concentration and advertising has long been the focus of empirical work. The attempt was to explain a positive linear relationship between concentration and advertising by positing a causation that advertising intensity influences concentration. After the 70s, empirical work also focused on reverse causation, that concentration influences advertising suggesting a non-linear relationship, quadratic inverted U-shape, between concentration and advertising intensity. There are many studies reporting significant non-linear such relationship as reported in the extensive survey of the advertising literature by Bagwell (2003). Although some studies do provide little support for a non-linear relationship, more recent work, Buxton, Davies and Lyon (1984) and Uri (1987), emphasize industries in which a large share of sales go to final consumers. They show how the significance of the inverted U-shape pattern increases across industries with the increasing importance of sales to final consumers. Our model provides a theoretical implications leading to a causation that concentration influences advertising intensity and *all* final sales go to consumers. Hence, the model may provide some theoretical underpinning for these recent empirical studies by providing a possible interpretation of the inverted U-shape relationship.

2.3 Industry Advertising Rate

This section derives the industry advertising rate and its properties. Since a is the individual advertising rate, the industry advertising rate is determined by

$$A^*(\phi) = Ma^*(\phi) = \frac{Na^*(\phi)}{\phi} \tag{31}$$

with the following limits:

$$\lim_{\phi \to 0} A^*(\phi) = \lim_{\phi \to \infty} A^*(\phi) = 0. \tag{32}$$

These limits have similar interpretations as the ones for individual advertising rates found previously. The impact of concentration on the industry advertising rate is summarized by the following proposition.

Proposition 2 *The industry advertising rate is unimodal in ϕ reaching a peak at $\bar{\phi} < \tilde{\phi}$. Let $\bar{\alpha}^*(\phi) = \frac{\partial A^*(\phi)}{\partial \phi} \frac{\phi}{A^*(\phi)}$ be the concentration elasticity of industry advertising rate, then*

$$\bar{\alpha}^*(\phi) = (\alpha^*(\phi) - 1) \gtreqless 0 \text{ if and only if } \alpha^*(\phi) \gtreqless 1 \tag{33}$$

Proof. Taking the derivative of (31) with respect to ϕ,

$$\frac{\partial A^*(\phi)}{\partial \phi} = \frac{N \frac{\partial a^*(\phi)}{\partial \phi} \phi - Na^*(\phi)}{\phi^2} = \frac{Na^*(\phi)}{\phi^2} \left(\frac{\partial a^*(\phi)}{\partial \phi} \frac{\phi}{a^*(\phi)} - 1 \right).$$

yields

$$\bar{\alpha}^*(\phi) = (\alpha^*(\phi) - 1) \gtreqless 0 \tag{34}$$

Therefore, $\bar{\alpha}^*(\phi)$ reaches a maximum for a value of $\bar{\phi}$ such that $\bar{\alpha}^*(\bar{\phi}) = 0$. This happens when $\alpha^* \left(\bar{\phi}\right) = 1$. Since $\alpha^*(\phi)$ reaches a maximum at $\tilde{\phi} > 1$, it implies that $\bar{\phi} < \tilde{\phi}$. The industry advertising rate reaches a peak at a lower value of ϕ then the individual advertising rate. ■

The direction of a change in concentration on the industry advertising rate depends on the concentration elasticity of individual advertising rate. The additional effect via ϕ in the denominator of $A^*(\phi)$ affects the properties. For any values of ϕ either lower then $\bar{\phi}$ or higher then $\tilde{\phi}$, both individual and industry advertising rates move in the same direction following a change in ϕ. However, for all values of $\phi \in (\bar{\phi}, \tilde{\phi})$, they move in opposite directions. This means that for these values of ϕ, the industry advertising rate is negatively related to a change in ϕ, while the individual advertising rate is positively related to a change in ϕ. For instance, this implies that as competition increases, more sellers lower ϕ, leads each seller to reduce their advertising rate. But more sellers doing slightly less advertising rates still yields higher industry advertising rate. This finding suggests that one must be careful in aggregating individual advertising rate properties to the industry level, or to infer individual firms's behavior with respect to advertising from industry level data.

2.4 Equilibrium Advertising Rates and Prices under Free Entry

Until now, we have not allowed new sellers to enter or exit in response to positive or negative net expected profits being realized in the industry. Assume that sellers can enter or exit the market freely until expected profits are driven down to zero. However, in order to enter, a seller must advertise and incur set up and variable costs of advertising. Under free entry, profit maximizing advertising rates become equilibrium advertising, because upon entry, a seller knows that it must choose a^* such that market concentration including himself is determined by ϕ. Using the equilibrium market price $p^*(\phi)$ and the profit maximizing advertising rate $a^*(\phi)$, the equilibrium expected profit upon entry is:

$$\Pi^*(\phi) = R(\phi) - c(a^*(\phi)) - F$$

Under free entry, $\Pi^*(\phi^*) = 0$ defines the value of market concentration, ϕ^*. Using $c'(a^*(\phi)) = \beta$,

$$R(\phi^*) = \beta a^*(\phi^*) + F \tag{35}$$

yields the free entry individual equilibrium advertising rate $a^*(\phi^*)$ and equilibrium market price , $p^*(\phi^*)$. With appropriate substitution, the industry equilibrium advertising rate is $A^*(\phi^*)$. In Section 3 we compare these equilibrium values with the efficient ones.

An increase in marginal cost reduce the free entry individual and industry advertising rate as shown by 22 and **??**. Totally differentiating (35) evaluated at ϕ^* it is easily shown that $\frac{d\phi^*}{dF}|_{\phi^*} > 0$.A higher fixed cost increases ϕ^*, and hence, reduces equilibrium entry. Since marginal cost does not affect expected revenue we also find that $\frac{\partial\Pi^*(\phi^*)}{\partial\beta} < 0$.For all values of $\phi^* > 0$, an increase in β increases total cost and reduces profit, increasing ϕ^* and inducing less entry. Finally, from the comparative statics, as $F \to \infty$, $\phi^* \to \infty$, the market shuts down, and hence no advertising. When $F \to 0$, $\phi^* \to 0$, the market converges to the perfectly competitive outcome with no advertising.

2.5 Interplay of Advertising Rates and Prices

Given that each seller choose ads and price simultaneously, there is an interesting synergy between the two variables. Basically, a seller chooses ads to grow consumers attention as shown by $\frac{\partial\theta(\mathbf{p},\hat{a},\mathbf{a}_{-1})}{\partial\hat{a}} > 0$, but once consumers have sellers attention, sellers choose prices to fight for selection, as shown by $\frac{\partial\hat{\boldsymbol{\pi}}}{\partial\hat{p}} < 0$. Equilibrium advertising and prices are directly and indirectly related in this model. We have found the profit maximizing advertising rate to be directly increasing in equilibrium market price. But also, advertising rates and prices are indirectly related via market concentration. In this model, advertising intensifies price competition when considering an increase in the number of sellers, each advertising a positive amount. This is determined by the extensive margin of advertising rather then the intensive margin. When more sellers advertise, consumers are more likely to observe ads by more sellers increasing the number of sellers over which consumers randomize. More sellers on the market doing advertising, at whatever levels, implies that the equilibrium market price declines due to intensified price competition.

Since equilibrium market price is strictly increasing in ϕ and individual advertising rate is unimodal in ϕ, as market concentration changes, equilibrium market price and advertising rate do not necessarily move in the same direction. This is summarized by the following proposition.

Proposition 3 *In relatively low size-concentration markets as measured by $\phi < \tilde{\phi}$, equilibrium market price and advertising rate move in the* same direction *following a change in market concentration. However, in relatively high size-concentrated markets as measured by $\phi > \tilde{\phi}$, equilibrium market price and advertising rate move in* opposite directions *following a change in market concentration.*

This proposition suggests that to establish an association between simultaneous changes in equilibrium market price and profit maximizing advertising rate, one must consider the existing market concentration, and whether the price and advertising movements were triggered by changes in market concentration.[13]

This implication may shed some light on prior empirical research and findings investigating the association between price and advertising. Prior findings that price and advertising are moving in opposite directions when concentration changes holds in this model for markets where the number of consumers is relatively high compared to the number of sellers, that is for relatively high concentration markets.[14] Similar implications carries over to the

[13] Note that in reality changes in market price may also result from collusive behavior.

[14] For such studies see See Benham (1972), Cady (1976), Comanor and Wilson (1979), Haas-Wilson (1986) and Steiner (1973).

industry rate of advertising in relation to the equilibrium market price with the exception that the threshold is now $\bar{\phi}$ instead of $\tilde{\phi}$.

2.6 Equilibrium Matching

In this model, consumers and sellers are matched by equilibrium choices that they make. Because of the advertising technology, there two ways to look at the equilibrium matching rates. One is to consider the matching at the beginning of the game, that is the matching rate between the total number of sellers M and consumers N given the equilibrium price and advertising rates. The equilibrium number of matches in finite markets given advertising rate and equilibrium price decisions are made is:

$$\mu^* (M, N) = M \left(1 - (1 - \theta^*(p^*, a^*))^N\right) \tag{36}$$

Since $\theta^*(p^*, a^*)$ is increasing in a^*, it implies that more advertising yields a higher expected number of matches. However, in large markets, the equilibrium matching rate per seller is:[15]

$$\frac{\mu^* (\phi)}{M} = \left(1 - e^{-\phi}\right) \tag{37}$$

and the equilibrium matching rate per consumer is

$$\frac{\mu^* (\phi)}{N} = \frac{\left(1 - e^{-\phi}\right)}{\phi}.$$

Interestingly, the large market equilibrium price and matching rates are the same wether or not advertising is introduced in the model (see Burdett, Shi and Wright (2001)). Hence, results of competitive matching models are robust to the implicit introduction of advertising, or more specifically, to a form of informational asymmetry, where the number of oberved sellers is private information to each consumer.

Under free entry, the equilibrium number of matches is simply expressed as $\mu^* (\phi^*)$. In the next sections, we derive the efficient advertising rates, with and witout free entry, as well as the efficient level of entry which we call ϕ°. As a consequence, the efficient number of matches without free entry, $\mu^{\circ} (\phi)$, and the efficient number of matches associated with efficient entry, $\mu^{\circ} \left(\phi^{\circ}\right)$, are obtained and compared to the equilibrium levels.

3 Efficiency of Advertising Rates and Entry

The efficiency of advertising rate is assessed by considering the social planner's problem. We assume that the social planner chooses the individual and industry advertising rate without

[15] For properties of this matching function but without advertising see Julien, Kennes and King (2000) and Burdett, Shi and Wright (2001).

being able to resolve the matching coordination problem and take as given the matching function derived above and the advertising technology. This yields a constrained efficiency measure. In this model, the planner's objective is to maximize the total surplus, that is the difference between the expected number of matches and the total cost of producing those matches.

3.1 Efficient Advertising Rates

Without free entry, the efficient individual advertising rate for each seller maximizes the total surplus as follows:

$$\max_{a} \left\{ M \left(1 - (1 - \theta^{*}(\mathbf{p}^{*}, \mathbf{a}^{*}))^{N} \right) - Mc(a) - MF \right\} \tag{38}$$

The first-order condition yields

$$MN \left(1 - h(a) \pi^{*}\right)^{N-1} h'(a) \boldsymbol{\pi}^{*}(\mathbf{p}, \mathbf{a}_{-1}) = Mc'(a) . \tag{39}$$

In large markets,

$$\frac{\phi e^{-\phi} e^{-a^{\circ}}}{\left(1 - e^{-a^{\circ}}\right)} = \beta, \tag{40}$$

which equates marginal social benefit (MSB_a) and marginal social cost (MSC_a).The efficient individual advertising rate is

$$a^{\circ}(\phi) = \left(\ln \left(\phi e^{-\phi} + \beta \right) - \ln \beta \right), \tag{41}$$

with the following properties:

$$\lim_{\phi \to 0} a^{\circ}(\phi) = \lim_{\phi \to \infty} a^{\circ}(\phi) = 0.$$

The efficient advertising rate as the same limit properties as the equilibrium one. However it differs as expressed by the following proposition.

Proposition 4 *The efficient individual advertising rate is unimodal reaching a maximum at $\phi = 1$.*

Proof. Differentiating the efficient advertising rate with respect to ϕ yields:

$$\frac{\partial a^{\circ}(\phi)}{\partial \phi} = \frac{e^{-\phi}}{\phi e^{-\phi} + \beta} (1 - \phi) \gtreqless 0 \text{ if and only if } \phi \lesseqgtr 1.$$

Therefore, the socially efficient individual advertising rate is also unimodal in ϕ and reaches a maximum when $\phi = 1$. ■

Since it has been established earlier that $\tilde{\phi} > 1$, then $a^{\circ}(\phi)$ reaches a maximum at a lower value of ϕ then $a^{*}(\phi)$. The explanation for the unimodal feature of efficient individual advertising rate is similar to the profit maximizing one, with the exception of no price effect.

At the industry level, the socially chosen advertising rate is $A^{\circ}(\phi) = Ma^{\circ}(\phi) = \frac{N}{\phi}a^{\circ}(\phi)$ with the following limits:

$$\lim_{\phi \to 0} A^{\circ}(\phi) = \frac{N}{\beta} > 0 \text{ and } \lim_{\phi \to \infty} A^{\circ}(\phi) = 0.$$

Once again, as the market shuts down, $\phi \to \infty$, efficiency calls for no advertising to be performed. However, when the market converges to a perfectly competitive one, efficiency calls for a maximum industry advertising rate. The reason lies fact that even though the market looks like a perfectly competitive one, each firm must advertising, even a small amount, to be known by consumers, with the implication of having a huge number of firms each performing minimal advertising rates. The relationship between efficient industry advertising rate and market concentration is summarized in the following proposition.

Proposition 5 *Let* $\alpha^{\circ}(\phi) = \frac{\partial a^{\circ}(\phi)}{\partial \phi}\frac{\phi}{a^{\circ}}$ *be the market concentration elasticity of efficient individual advertising rate and* $\bar{\alpha}^{\circ}(\phi) = \frac{\partial A^{\circ}(\phi)}{\partial \phi}\frac{\phi}{A^{\circ}}$ *the market concentration elasticity of efficient industry advertising rate. Then* $A^{\circ}(\phi)$ *is strictly decreasing in* ϕ *as shown by:*

$$\bar{\alpha}^{\circ}(\phi) = \left(\alpha^{\circ}(\phi) - 1\right) < 0.$$

Proof. Differentiating $A^{\circ}(\phi)$ with respect to ϕ yields

$$\frac{\partial A^{\circ}(\phi)}{\partial \phi} = \frac{Na^{\circ}(\phi)}{\phi^2}\left(\frac{\partial a^{\circ}(\phi)}{\partial \phi}\frac{\phi}{a^{\circ}(\phi)} - 1\right)$$

or

$$\frac{\partial A^{\circ}(\phi)}{\partial \phi} = \frac{Na^{\circ}}{\phi^2}\left(\alpha^{\circ}(\phi) - 1\right).$$

Hence

$$\bar{\alpha}^{\circ}(\phi) = \left(\alpha^{\circ}(\phi) - 1\right) < 0$$

It remains to show that $\alpha^{\circ}(\phi) < 1$ for all ϕ. Taking the second derivative of $a^{\circ}(\phi)$ with respect to ϕ yields

$$\frac{\partial^2 a^{\circ}(\phi)}{\partial \phi^2} = \frac{e^{-\phi}}{\left(\phi e^{-\phi} + \beta\right)^2}\left[(\phi - 2)\left(\phi e^{-\phi} + \beta\right) - (1-\phi)^2 e^{-\phi}\right] < 0$$

for $\phi \in [0,1]$. This implies that for $\phi \in [0,1]$, $a^{\circ}(\phi)$ is strictly concave, hence $\frac{\partial a^{\circ}(\phi)}{\partial \phi} < \frac{a^{\circ}(\phi)}{\phi}$ and $\alpha^{\circ}(\phi) < 1$. For all $\phi > 1$, we know from Proposition 4 that $\frac{\partial a^{\circ}(\phi)}{\partial \phi} < 0$. Hence, $\alpha^{\circ}(\phi) < 1$ and $\bar{\alpha}^{\circ}(\phi) < 0$ for all ϕ. Therefore, $A^{\circ}(\phi)$ is strictly decreasing in ϕ. ■

The efficient industry advertising rate is not unimodal, but strictly decreasing in ϕ, despite the fact that $a^{\circ}(\phi)$ is found to be unimodal in ϕ. Furthermore, $A^{\circ}(\phi)$ reaches its highest value as ϕ converges to 0. For values of $\phi > 1$, an increase in competition, increasing M, decreases ϕ, and increases $a^{\circ}(\phi)$, both contributing to increase $A^{\circ}(\phi)$. However, for values of $\phi < 1$, any increases in M reduces the efficient individual advertising rate. But, the increase in M always dominate the decrease in efficient individual advertising rate, generating a higher efficient industry advertising rate. In such cases, maximizing total surplus for a given market concentration implies taking into account the positive externality associated with advertising.

Under limited entry, the efficient individual advertising rate $a^{\circ}(\phi)$ relative to the individual profit maximizing rate $a^{*}(\phi)$ for all values of ϕ is summarized in the following proposition.

Proposition 6 *For all finite market concentration value $0 < \phi < \infty$, there is underprovision of individual and industry advertising rates, that is*

$$a^{\circ}(\phi) > a^{*}(\phi) \text{ and } A^{\circ}(\phi) > A^{*}(\phi).$$

Furthermore, the extent of inefficiency diminishes as ϕ increases.

Proof. The proof follows from equations, (21) and (41),

$$a^{\circ}(\phi) - a^{*}(\phi) = \left(\ln\left(\phi e^{-\phi} + \beta\right)\right) - \left(\ln\left(p^{*}(\phi)\,\phi e^{-\phi} + \beta\right)\right) > 0 \tag{42}$$

$\forall p^{*}(\phi) < 1$. This condition holds for all $0 < \phi < \infty$ since $p^{*}(\phi) = 1$ only in the limit as ϕ goes to infinity and negative prices are ruled out. It follows directly that

$$A^{\circ}(\phi) - A^{*}(\phi) = \frac{N}{\phi}\left(a^{\circ}(\phi) - a^{*}(\phi)\right) > 0.$$

As ϕ increases, the equilibrium market price monotonically converges to 1, which implies that the inefficiency of advertising rates becomes less important, that is $\left(a^{\circ}(\phi) - a^{*}(\phi)\right)$ diminishes. ■

For finite market concentration values, underadvertising occurs because of the sharing rule upon a match, the equilibrium market price, is lower than the value of a match, which is one. Since the social and private marginal cost of advertising are the same, this is observed using equations (20) and (40) to express the difference social and private marginal benefits: $MSB_a - MPB_a = \frac{\phi e^{-\phi} e^{-a}}{(1-e^{-a})}(1 - p^{*}(\phi)) > 0$, for all $p^{*}(\phi) < 1$, which holds for all $\phi < \infty$. In this model, when ϕ goes to infinity, the market shuts down. The equilirbrium market price is such that sellers bear the full cost of searching (advertising) but get rewarded by a fraction of the entire surplus from a match, which is one.

3.2 Efficient Entry

With free entry, the social planner now takes the socially chosen individual advertising rate, $a^{\circ}(\phi)$, as given, and chooses ϕ to solves the following problem:

$$\max_{\phi}\left\{M\left(1-\left(1-h\left(a^{\circ}\right)\pi^{*}\right)^{N}\right)-Mc\left(a^{\circ}\right)-MF\right\} \tag{43}$$

In large markets, the social planner's problem becomes:

$$\max_{\phi}\left\{M\left(1-e^{-\phi}\right)-Mc\left(a^{\circ}\right)-MF\right\}=\max_{\phi}\left\{\frac{N}{\phi}\left(1-e^{-\phi}\right)-\frac{N}{\phi}c\left(a^{\circ}\right)-\frac{N}{\phi}F\right\} \tag{44}$$

with the first-order condition:

$$\left(\frac{e^{\phi}-1-\phi}{e^{\phi}}\right)=\beta a^{\circ}(\phi)-\beta\frac{\phi\partial a^{\circ}(\phi)}{\partial\phi}+F. \tag{45}$$

Using the socially chosen individual advertising rate from (41) and its derivative,

$$\left(\frac{e^{\phi^{\circ}}-1-\phi^{\circ}}{e^{\phi^{\circ}}}\right)=\beta a^{\circ}\left(\phi^{\circ}\right)-\beta\phi^{\circ}\frac{e^{-\phi^{\circ}}\left(1-\phi^{\circ}\right)}{\left(\phi^{\circ}e^{-\phi^{\circ}}+\beta\right)}+F \tag{46}$$

Equation (46) fully characterizes the efficient level of entry ϕ°, and hence M°, where the left hand side is the marginal social benefit of entry (MSB_{ϕ}) and the right hand side the marginal social cost of entry (MSC_{ϕ}).

The inefficiency of entry summarized in the following proposition.

Proposition 7 *For a given number of consumers N, the free entry equilibrium number of sellers is higher than the efficient one, that is, $M^{\circ}<M^{*}$(or $\phi^{\circ}>\phi^{*}$).*

Proof. We must show that $\phi^{\circ}>\phi^{*}$. Note first that from the left hand side of (45), MSB_{ϕ} has the same functional form as $R(\phi)$ which is the marginal private benefit of entry (or MPB_{ϕ}). It must be then that that entry is dictated by marginal cost of entry. From the right hand side of (45), the marginal social cost of entry is $MSC_{\phi}=\beta a^{\circ}(\phi)-\beta\phi\frac{e^{-\phi}(1-\phi)}{(\phi e^{-\phi}+\beta)}+F$ while the marginal private cost is $MPC_{\phi}=\beta a^{*}(\phi)+F$. For any values of $\phi\in(0,\infty)$ it follows that:

$$MSC_{\phi}-MPC_{\phi}=\beta\left[a^{\circ}(\phi)-\phi\frac{e^{-\phi}(1-\phi)}{(\phi e^{-\phi}+\beta)}-a^{*}(\phi)\right]>0 \tag{47}$$

From Proposition 6 we know that $a^{\circ}(\phi)-a^{*}(\phi)>0$. For $\phi\geq1$, clearly $MSC_{\phi}>MPC_{\phi}$. For $\phi<1$, it can be shown that the inequality also holds. A numerical example is provided in a further section. This implies that at ϕ° it must be that $MSC_{\phi^{\circ}}>MPC_{\phi^{\circ}}$. But at ϕ° we have $MSB_{\phi^{\circ}}=MSC_{\phi^{\circ}}=R(\phi^{\circ})>MPC_{\phi^{\circ}}$. Hence, at the efficient level of entry, firms are making strictly positive profits. Under free entry, entry occurs, reducing ϕ, until ϕ^{*} where $R(\phi^{*})>MPC_{\phi^{*}}$. Therefore, $\phi^{*}<\phi^{\circ}$. ■

In addition to a higher advertising rate for all concentration levels as shown above, the efficient entry takes into account of how entry affects each seller's advertising cost as shown by the term $\beta\frac{\phi\partial a^{\circ}(\phi)}{\partial\phi}$. These two effects lead to a marginal social cost of entry to be superior to the marginal private cost of entry for all concentration levels. Since social and private marginal benefits are the same for each concentration value, there is excessive entry.

This result shares a similarity with the inefficiency of entry in Mankiw and Whinston (1986). Their model is a standard oligopoly industry and differs from this one in that they do not have capacity constraint, no matching environment, no advertising, consumers have a downward sloping demand, and each firm has more then one unit for sale. They show that excessive entry occurs when the following three conditions are satisfied: (1) postentry equilibrium aggregate output rises with the number of firms entering and approaches some finite bound as entry converges to infinity, (2) output per firm fall as the number of firms increases, what they refer to as business stealing effect, and (3) for any number of entrants, equilibrium price is weakly above marginal cost of production. They argue that the business stealing effect drives a wedge between social and private incentives to enter and yields excessive entry.

Although this model differs greatly from theirs, their three conditions apply to our model with the following interpretation. Since each seller in the model has only one unit and no production capability, interpret aggregate output as aggregate expected output as determined by the expected number of matches postentry, that is once price and advertising rate are chosen. The expected number of matches is $M(1-(1-\frac{1-(1-h(a))^M}{M}))^N$ in the finite markets. It is easily shown that this term is strictly increasing in M and converges to zero (a finite bound) as M goes to infinity. This is equivalent to condition (1) of Mankiw and Whinston (1986). Second, showing that the probability to make a sale for each seller falls as the number of sellers increases is equivalent to condition (2). This holds for the following reason. When a new seller enters and advertises, it increases the expected number of sellers from which a typical consumer observes advertisements. Hence, each consumer randomizes over more sellers, inducing a lower probability to visit each particular seller, reducing the probability for a seller to make a sale. This is the business stealing effect in action in our model. Finally, there is no production in our model, hence marginal cost of production is zero. Since the equilibrium market price is above zero (marginal cost) in the model then condition (3) is satisfied. Hence, there is excessive entry in the model, also driven by a business stealing effect. However, it is not the entire reason why we find excessive entry. Under free entry, sellers do not consider the social impact of a change in concentration on the optimal advertising rate as evidenced by a higher marginal social cost of entry compared to the marginal private cost. There is therefore a negative externality associated with entry via the business stealing effect that is not internalized by the equilibrium market price and advertising rate.

The implications of Propositions 6 and 7 suggest that too many sellers, each not advertising enough, is a feature of the free entry equilibrium. If an entity was to intervene in the free entry market to implement efficient outcomes, both advertising and entry need to be targeted. Combining a tax on entry raising the set up cost, and an advertising rate subsidy would reduce entry and increase the individual advertising rate. The question of course remains about the feasibility of such a set of policy instruments in term of balanced budget. It can be shown that there exist a combination of an entry tax (τ) and an advertising rate subsidy (σ) which implements efficient outcomes within a balanced budget. Under balanced budget, it must be that $B = M^{\circ}\tau - M^{\circ}\sigma a^*(\phi^{\circ}; \sigma) = 0$, or $\tau = \sigma a^*(\phi^{\circ}; \sigma)$, which is evaluated at the targeted entry ϕ°. However, free entry under such policy, call it ϕ^{**} is determined by $\Pi(\phi^{**}) = 0$ or

$$R(\phi^{**}) = (\beta - \sigma)a^*(\phi^{**}; \sigma) + F + \tau.$$

Substituting in the balanced budget constraint, the free entry condition becomes

$$R(\phi^{**}) = \beta a^*(\phi^{**}; \sigma^{**}) + F.$$

This equation pins down the subsidy level σ^{**} needed to reach $\phi^{**} = \phi^{\circ}$ under balanced budget. Hence the equivalent entry tax is $\tau^{**} = \sigma^{**}a^*(\phi^{**}; \sigma^{**})$. An example of such policy combination is highlighted in the numerical examples section below.

Although the model is highly stylized, there seem to exist markets in which such a combination of policy instruments are used. The assumptions of the model is very conducive for specialized labor markets whereby each firms have one vacancy, workers search for one job, and advertising is informative in nature. Entry tax through required costly accreditations or licensing and subsidized advertising appear to be a feature is many such markets. The US Veterinary Medical Board seem to be one such example, managing entry via licensing practices and subsidize advertising through magazines, web sites, newsletters, etc.

4 Numerical Examples

In this section we provide numerical examples of the model in order to highlight the capabilities and properties of the model, along with an illustration of quantitative aspects of the theoretical implications. We present values for some of the endogenous variables for different values of exogenous variables and parameters. But of course the same can done for all endogenous variables.

First of all, find that equilibrium free entry and advertising rates are invariant in the number of consumers.[16] Therefore we set arbitrarily the number of consumers at $N = 100000$. For simplicity and to make a further property of the outcomes evident, we consider extreme values of advertising costs parameters, that is, we set $\beta = F = .1$ and $\beta = F = .8$.

The individual profit maximizing and efficient advertising rates for different market concentration values are shown in Figure 1 for the different advertising costs.

[16] The number of consumers however affect proportionally the equilibrium number of advertisements sent and the endogenous variables without free entry.

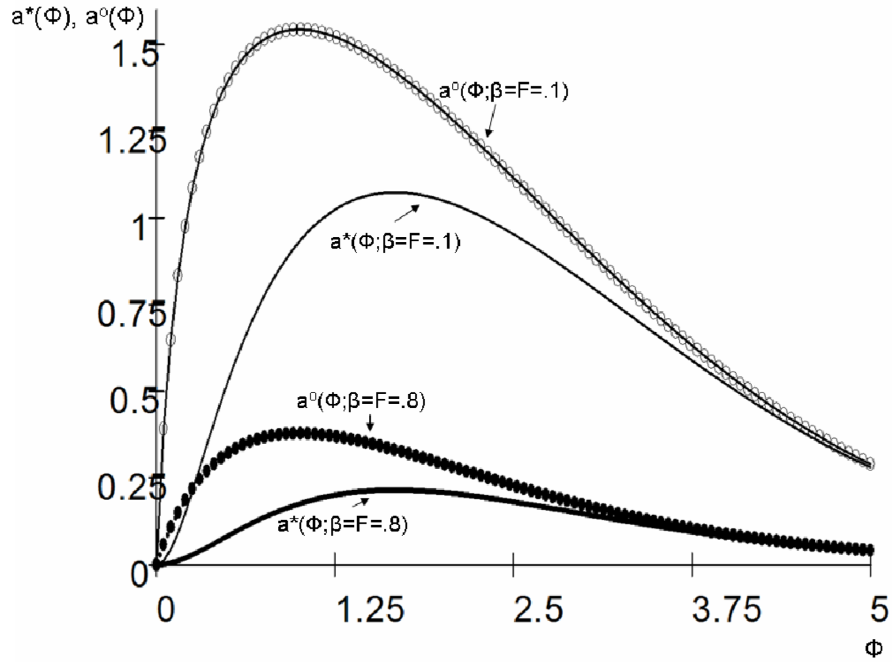

Figure 1: Individual profit maximizing and efficient advertising rates where $a^*(\phi; \beta = F = .1)$ refers to profit maximizing advertising rate under set up and marginal advertising costs being 0.1, etc.

Figure 1 highlights the extent of underadvertising for values of ϕ, and illustrates the implications that advertising rates declines with costs. As the market gets to extremes, that is relatively more concentrated and relatively low concentration, equilibrium and efficient rates converge, but the extent of inefficient advertising rate is maximized when $\phi = 1$.

Figure 2 shows the extent of inefficiency of industry advertising rates.

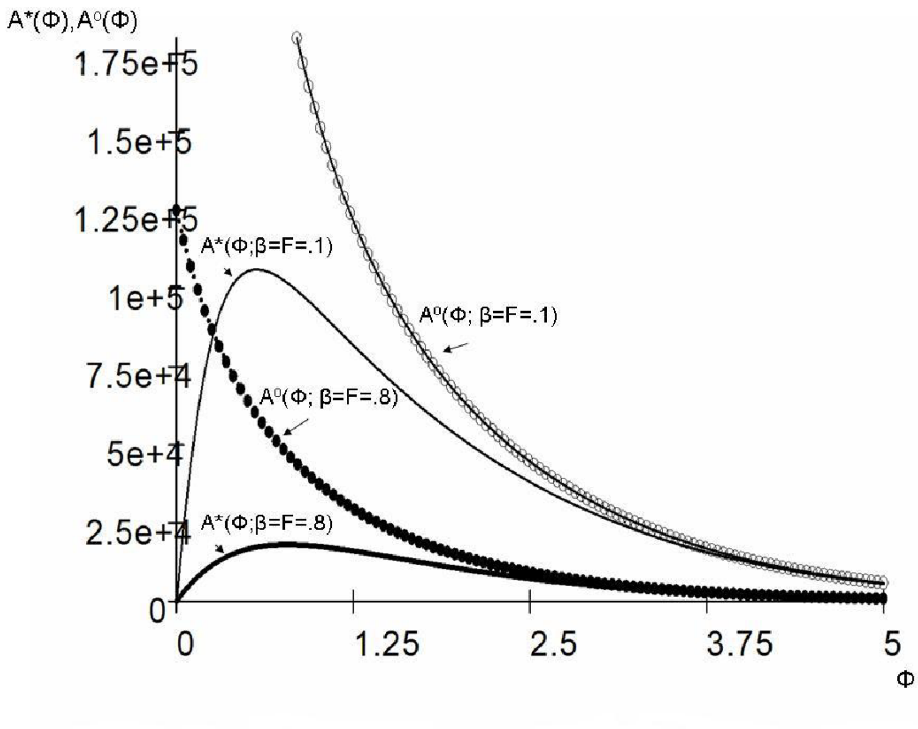

Figure 2: Industry profit mazimizing and efficient advertising rates.

The figure not only highlights the extent of underadvertising at the industry level and how costs affect those rates, but also the result that efficient industry rate is not unimodal in concentration. This implies that the equilibrium industry advertising rate converges to the efficient level only when ϕ gets large, and the extent of inefficient advertising rate for the industry is maximized when $\phi \to 0$. Finally, Figure 3 shows how costs affect equilibrium and efficient entry, ϕ^* and ϕ^o. Figure 3 shows that in this model, higher advertising fixed cost increases ϕ^* and ϕ^o, and hence reduces entry, but also that the difference between ϕ^* and ϕ^o increases with cost and then decrease again, suggesting that there is an advertising fixed cost value such that the extent of excessive entry reaches a maximum.

Parameters\Values	$p^*(\phi^*)$	$a^*(\phi^*)$	$a^{\circ}(\phi^{\circ})$	$A^*(\phi^*)$	$A^{\circ}(\phi^{\circ})$	$\mu^*(\phi^*)$	$\mu^{\circ}(\phi^{\circ})$
$\beta = .1, F = .1$							
$\phi^* = 0.75, \phi^{\circ} = 0.95$	0.3472	0.8101	1.5388	101263.64	170980.90	52971.86	59520.52
$\beta = .8, F = .8$							
$\phi^* = 3.70, \phi^{\circ} = 5.10$	0.9062	0.0986	0.0381	2664.77	747.65	2537.62	733.58

Table 1: Values for the decentralized market

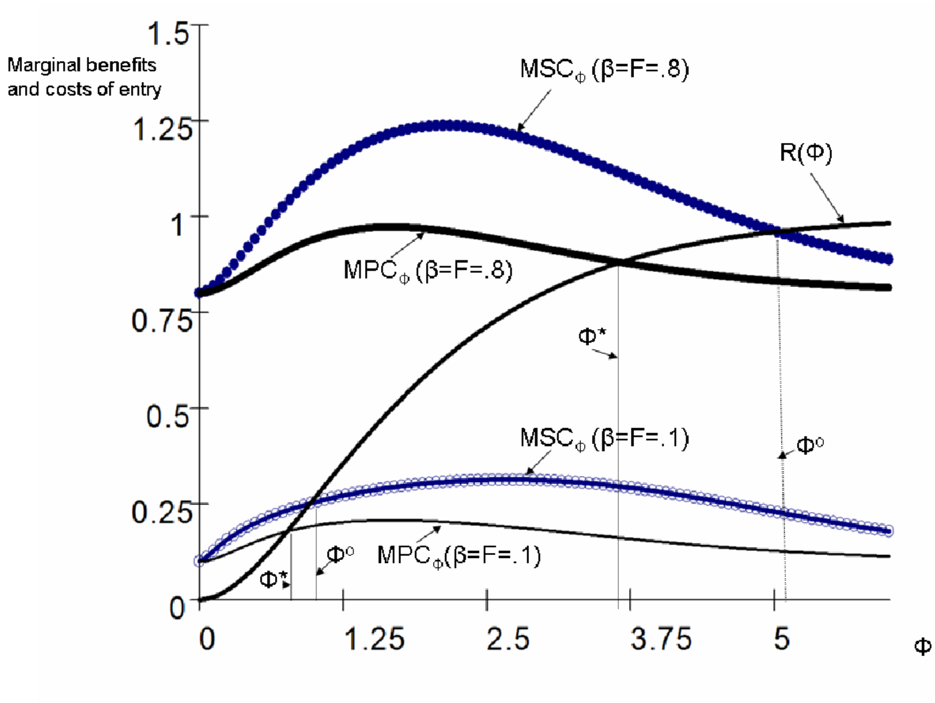

Figure 3: Equilibrium and efficient levels of entry for different advertising costs.

Table 1 below shows equilibrium and efficient entry with some associated endogenous variables for different parameter values of β and F, while $N = 100\ 000$.

Before discussing the implications of the numerical example of Table 1, we give an interpretation of the numbers. First, set up and marginal costs in the order of 10% of consumers' valuation (which is 1), induces a free entry concentration level of $\phi^* = 0.75$, meaning that $M^* = 133\ 333$ sellers enter to serve 100 000 consumers, while the efficient level calls for $M^o = 105\ 263$ sellers. Each seller under free entry send $s^*(\phi^*) = Na^*(\phi^*) = 81010$ advertisements while efficiency calls for sellers to send $s(\phi^{\circ}) = Na^{\circ}(\phi^{\circ}) = 153880$ advertisements each. At the industry level, each consumers receive 101263.64 advertisement in total from the the 133333 sellers under free entry, while efficiency calls for consumers to receive 170980.9 advertisements from 105263 sellers. The expected number of matches under free entry is found to be 52973. Hence, 52.9% of consumers are satisfied, and since there are 133333 sellers, only 39.73% make a sale. Under efficiency, nearly 60% of consumers are satisfied while 56.5% of sellers make a sale.

For set up and marginal costs in the order of 80% of consumers' valuation, free entry induces a concentration of $\phi^* = 3.7$, or $M^* = 27027$ sellers enter while the efficient entry calls for a concentration $\phi^{\circ} = 5.1$, or $M^{\circ} = 19607$ sellers. Each seller under free entry sends 9860 advertisements, while under efficiency each sends 3810 ads. At the industry level, each consumer receive nearly 2665 ads from all the 27027 sellers under free entry, while under efficiency each consumer receives 748 ads only from 19607 sellers. Finally, from the expected number of matches, 2.53% of consumers are satisfied while 9.38% of sellers make a sale under free entry compared to 0.73% of satisfied consumers and 3.74% of sellers making a sale under efficiency.

Table 1 highlights the implication that the extent of excessive entry is less important for low costs of advertising. However, as shown in figure 3, the extent of excessive entry peaks for intermediate values of advertising costs. The interesting features in Table 1 are that the difference between free entry individual and industry advertising rates and efficient ones are reverse going from low to high advertising costs. The same holds for the expected number of matched. For low advertising costs, the free entry equilibrium allocation calls for lower advertising rates than the efficient ones. Therefore one would observe larger number of sellers, each advertising less relative to sellers under efficient entry. However, for high advertising costs, it is reversed. One would observe larger number of sellers, each advertising more relative to sellers under efficient entry. It can be shown that there exists a set of advertising costs such that sellers choose the same advertising rate under free entry equilibrium as sellers under efficient entry. Once again, this example shows that one cannot simply assess efficiency of advertising without considering market concentration.

Finally, we present an example of how the use of a combination of policy instruments can induce free entry to be efficient and yield efficient advertising rate. This is shown in Figure 4 below.

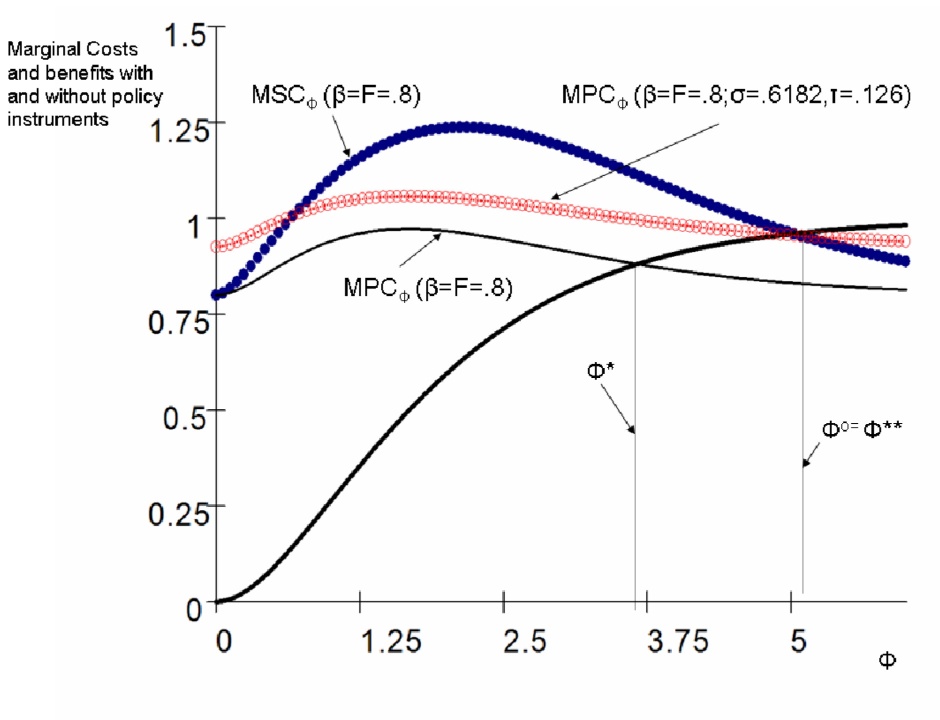

Figure 4: Example of policy instrument values generating efficiency under free entry.

The curve identified $MPC_\phi(\beta = F = .8; \sigma = .6182, \tau = .126)$ is the marginal private cost under the policy instrument values. In this case, for set up and marginal cost of advertising of 0.8, an entry tax of 0.126 raising the set up cost to 0.926 and an advertising rate subsidy of 0.6182 reducing the marginal cost of advertising to 0.1818 would induce the entry equilibrium to be efficient, $\phi^{**} = \phi^o$, as well as the individual advertising rate, $a^*(\phi^{**}; \sigma, \tau) = a^\circ(\phi^\circ)$.

5 Conclusion

This paper constructs a matching model of directly informative advertising and price featuring capacity constrained sellers and hence contributes to the small literature on multi-firm informative advertising models. We find that the relationship between market concentration and both, individual and industry advertising rates are unimodal (inverted U-shape). These findings have significant empirical support as documented in Bagwell (2003) but almost inexistent theoretical underpinnings. The findings in this paper offer one theoretical explanation. The model also allow a more detailed analysis of the relationship between advertising rate and market price. The literature in economics and marketing has focused on explaining a negative association between advertising and market prices, arguing that more ads increase consumers' sensitivity to price changes translating into lower prices. However, by placing this association in perspective with the market concentration, we find that for relatively low concentration industries, increases in market price brought about by an increased in concentration creates a positive association between advertising rate and the market price, yielding more ads per seller, and more ads in the industry. On the other hand, for relatively high concentration industries, an increase in concentration leads to a negative association. Empirical studies in the 70's and early 80's considering the eye glasses, eye exams and prescription medicines have concluded a negative association between advertising and prices.[17] Since the nature of advertising for these items are more on the informative side, and during that period, those industries were relatively concentrated, this may also provide some support for our findings. Contrary to some previous findings, we obtain that individual and industry profit maximizing advertising rates are inefficient and entry is excessive. Using numerical examples, we show how the equilibrium free entry allocation may differ from the efficient one and how it depends on advertising costs. Finally, we conclude by showing how a tax on entry and an advertising rate subsidy can induce the free entry equilibrium to be efficient within a balanced budget.

The model has a direct application to the labor market where sellers can be viewed as firms, each with one vacancy and consumers as unemployed workers, each searching for one job. In such market, advertising is highly informative in nature, so the model shows how search intensity by firms (advertising rate) affects the market outcomes including the endogenous matching rate.

Several directions for further research include, but not limited to, assuming different pricing mechanisms such as auctions or bidding instead of price posting; allowing for different capacities as in Burdett, Shi and Wright (2001) and Deneckere and Peck (1995); allowing for consumers to choose only over a subset of firms from which they have observed advertisements; and allowing for intermediaries to form. Finally, a dynamic version would warrant new results, and in a labor market application such as Julien, Kennes and King (2000), it could be discovered how a more compelling equilibrium unemployment rate would be affected by recruitment intensity. Finally, we have focused on directly informative advertising, however, the structure of the model is rich enough to incorporate the indirectly informative and persuasive advertising views, by introducing a variable affecting consumers' values directly.

References

[1] Bagwell, Kyle., (2001): "The Economics of Advertising", *Critical Writing in Economics*, 136.

[2] Bagwell, Kyle., (2003): "The Economic Analysis of Advertising", Manuscript Columbia University.

[17] See Benham (1972), Cady (1976), Comanor and Wilson (1979), Haas-Wilson (1986) and Steiner (1973).

[3] Buxton, A. J., Davies, S. W. and B. R. Lyons (1984): "Concentration and Advertising in Consumer and Producer Markets," Journal of Industrial Economics, 32.4, June, 451-64.

[4] Benham, L., (1972): "The Effect of Advertising on Eyeglasses", *Journal of Law and Economics*, 4, 337-352.

[5] Burdett, Kenneth, Shi, Shouyong and Wright, Randall, (2001): "Pricing and Matching with Frictions", *Journal of Political Economy*, 109, 1060-1085.

[6] Butters, Gerard R., (1977): "Equilibrium Distribution of Sales and Advertising Prices", *Review of Economic Studies*, 44, 465-491.

[7] Cady, J., (1976): "Advertising Restrictions and Retail Prices", *Journal of Advertising Research*, 16, 27-30.

[8] Comanor, W.S. and Wilson, T.A., (1979): "The Effects of Advertising on Competition", *Journal of Economic Literature*, 17, 453-476.

[9] Deneckere, Raymond and Peck, James (1995): "competition Over Price and Service Rate When Demand is Stochastic: A Strategic Analysis", *The Rand Journal of Economics*, 26, 148-162.

[10] Dorfman, R and Steiner, P.O., (1954): "Optimal Advertising and Optimal Quality", *The American Economic Review*, 44, 826-836.

[11] Haas-Wilson, D., (1986): "The Effects of Commercial Practice Restrictions: the Case of Optometry", *Journal of Law and Economics*, 19, 165-186.

[12] Julien, Benoît, Kennes, John and King, Ian, (2000): "Bidding for Labor", *Review of Economic Dynamics,* 3, 619-649.

[13] McAfee, R.Preston, (1993): "Mechanism Design by Competing Sellers", Econometrica, 61, 1281,1312.

[14] McAfee, R. P. (1994): "Endogenous Availability, Cartels, and Merger in an Equilibrium Price Dispersion," Journal of Economic Theory, 62.1, February, 24-47.

[15] Mankiw, N. Gregory and Whinston, Michael D., (1986): "Free Entry and Social Efficiency", *Rand Journal of Economics*, 17, 48-58.

[16] Nelson, P., (1970): "Information and Consumer Behavior", *Journal of Political Economy*, 78, 311-329.

[17] Peters, Michael, (1991): "Ex Ante Price Offers in Matching Games Non-Steady States", Econometrica, 59, 1425-1454.

[18] Robert, J. and D. O. Stahl, II (1993): "Informative Price Advertising in a Sequential Search Model," Econometrica, 61.3, May, 657-86.

[19] Stegemen, Mark, (1991): "Advertising in Competitive Markets", *American Economic Review*, 81, 210-223.

[20] Steiner, R.L., (1973): "Does Advertising Lower Consumer Prices?", *Journal of Marketing*, 37, 19-26.

[21] Stahl, Dale O., (1994): "Oligopolistic pricing and Advertising", *Journal of Economic Theory*, 64, 162-177.

[22] Sutton, C. John, (1974): "Advertising, Concentration and Competition", *Economic Journal*, 84, 56-69.

[23] Uri, N. D. (1987), "A Re-examination of the Advertising and Industrial Concentration Relationship," Applied Economics, 19.4, April, 427-35.