Sales and Promotions: A More General Model

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Abstract

We embed the Varian (1980) model in a broader setting that considers how switcher/loyal customer segments are determined. Generally, customer acquisition is deterministic while pricing is randomized. The equilibrium outcome depends on the timing of customer acquisition relative to pricing. If sellers acquire customers before setting prices, the unique equilibrium is asymmetric. If sellers acquire customers and set prices simultaneously, the unique equilibrium is symmetric. Our results provide a fundamental justification for previous analyses that variously assumed the outcome to be asymmetric or symmetric. The comparative statics for the asymmetric and symmetric equilibria are identical.

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1. Introduction

When two or more sellers of an identical product compete on price for consumers, who are variously captives of particular sellers ("captives") or comparison shoppers ("switchers"), the equilibrium outcome is randomized pricing (Butters 1977; Salop and Stiglitz 1977; Rosenthal 1980; Varian 1980; Png and Hirshleifer 1987; Narasimhan 1988; Raju et al. 1990; Baye et al. 1992). Randomized pricing has been interpreted as sales or price promotions.

With few exceptions, previous research has left open the question of how the captive/switcher segments are determined, and the pricing outcomes when the segments are endogenous. McAfee (1994) and Chioveanu (2003) consider settings where sellers first acquire customers and then set prices; they show that the equilibria are asymmetric. Others have focused on more specific scenarios -- two sellers (McGahan and Ghemawat 1994; Roy 2000; Chen and Iyer 2002), more than two sellers but assuming that the equilibrium is symmetric (Baye and Morgan 2001, 2004), and a sufficiently large number of small sellers (Butters 1977).

However, several key issues remain unresolved. First, the research to date has assumed (implicitly) that customer acquisition is deterministic. Since pricing is randomized, it is natural to ask whether customer acquisition would be randomized as well. Second, most research to date has assumed that sellers first acquire customers and then set prices (exceptions include Butters (1977) and Robert and Stahl (1993)). But a great deal of advertising and direct marketing includes prices, and so, violates the assumption of sequential timing. What would be the outcome if customer acquisition and pricing are simultaneous? Third, the previous research has provided few comparative statics propositions. To extract useful business and policy implications, it is important to derive the comparative statics.

In this paper, we consider a setting where, initially, all potential consumers and all sellers are identical. Specifically, consumers have identical ex-ante information about the sellers and prices, and they have no preferences for particular sellers. Within this setting, we analyze the strategies of a finite number of competing sellers on two dimensions – customer acquisition and pricing.

Each seller must incur a cost to acquire potential consumers, which cost we assume to be convex in the number of unique consumers acquired. Given the competing sellers' expenditures on customer acquisition, consumers endogenously divide into captive and switcher segments, depending on whether they are acquired by just one or multiple sellers.

In setting price, each seller must balance two concerns – set a high price to extract the maximum surplus from captive consumers or set a low price to compete for switchers. The result of these two conflicting concerns is randomized pricing.

Our contributions are as follows: First, we show that when the marginal cost of customer acquisition does not increase too fast, sellers will not randomize customer acquisitions (although pricing is randomized).

Second, we show how the equilibrium outcome depends on the timing of customer acquisition relative to pricing. If sellers acquire customers before setting prices, customer acquisition is deterministic and the unique equilibrium is asymmetric. One seller *dominates* while all other sellers *acquiesce* – the dominant seller acquires twice as many customers as each of the acquiescing sellers and sets relatively higher prices. However, if sellers acquire customers and set prices simultaneously, then, customer acquisition is deterministic and the unique equilibrium is symmetric. All sellers acquire the same number of customers and set the same pricing strategy.

Finally, we show how the equilibrium outcome and welfare vary with demand, cost, and competitive conditions. In particular, we show that the comparative statics are similar for both asymmetric and symmetric equilibria.

Our research is of broad significance as the basic scenarios of Butters (1977) and Varian (1980), further developed by Narasimhan (1988), have been applied to a broad range of strategic issues, including:[1]

- Mergers and collusion (McAfee 1994; Manduchi 2004);

[1] At the time of writing, the Social Sciences Citation Index reported the following numbers of citations: Butters (1977) – 191; Varian (1980) – 219; and Narasimhan (1988) – 76.

- Price information (Robert and Stahl 1993; Baye and Morgan 2001; Chen et al. 2002; Iyer and Pazgal 2003; Ghose et al. 2002; Chen and Hitt 2004; Moscarini and Ottaviani 2004);
- Price matching (Png and Hirshleifer 1987; Corts 1996; Moorthy and Winter 2002; Chen et al. 2001);
- Advertising and branding (Meurer and Stahl 1994; Chioveanu 2003; Baye and Morgan 2004; Dukes 2004);
- Various other aspects of marketing strategy (McGahan and Ghemawat 1994; Lal and Villas-Boas 1998; Roy 2000; Chen et al. 2001; Chen and Iyer 2002; Hong et al. 2002; Morgan and Sefton 2003).

2. Previous Research

Baye, Morgan, and Sholten (2006) review the literature in detail. Here, we highlight only the most closely relevant work. Butters (1977) considers a setting where competing sellers simultaneously invest in advertising and set prices. Only consumers who receive an advertisement may purchase the item. When the number of sellers is sufficiently large, there is a unique market equilibrium with specific advertising and sales price distributions. Otherwise, when the number of sellers is finite, the market equilibrium is unique only if every seller prices deterministically. However, Butters did not characterize the individual seller's equilibrium advertising choices. It is not clear if the sellers would behave symmetrically or asymmetrically.

Separately, Robert and Stahl (1993) consider competition where sellers set advertising and prices simultaneously, but, in a subsequent stage, consumers can search among sellers whose advertisements they have not received. The unique equilibrium is symmetric: sellers either set a high price to cater to loyal uninformed consumers, or advertise in a range of prices that is strictly lower than the high price to attract consumers to search. However, Robert and Stahl did not address the equilibrium outcome in the most parsimonious setting, of just customer acquisition and pricing without consumer search.

Chen and Iyer (2002) study competition between two sellers over two stages. In the first stage, the sellers invest in information to address potential consumers. All consumers are addressed: those addressed by only one seller are captive of that seller, while those addressed by both sellers are switchers. In the second stage, the sellers set prices. If cost of information is high or consumers differ greatly, the equilibrium is symmetric in information and pricing, while if the cost is low or consumers are similar in tastes, then the equilibrium is asymmetric. Chen and Iyer did not address the outcome with more than two sellers.[2]

Chioveanu (2003) considers competition between multiple sellers over two stages. In the first stage, sellers invest in advertising to persuade customers. All consumers who are not reached become switchers and buy from the lowest-price seller.[3] In the second stage, sellers set prices. There exist multiple asymmetric pricing equilibria, in which one seller randomizes over a continuous distribution and all other sellers price at the consumers' reservation value with some probability mass. These are supported by asymmetric investments in advertising.

Baye and Morgan (2004) study a setting similar to that of Chioveanu's (2003) except that, in the second stage, sellers decide whether to list at a price-comparison website as well as set prices. Focusing on the symmetric equilibrium, Baye and Morgan show that all sellers invest in advertising, and then randomize between pricing randomly with a listing on the website, and pricing at the consumers' reservation value with no listings.

In an important contribution, McAfee (1994) studies competition between multiple sellers over two stages. In the first stage, sellers invest in "availability" to secure customers (this is similar to "advertising" in Butters (1977) and others, or "addressability" in Chen and Iyer (2002)). Consumers can buy the item only if it is available at the store that they patronize, and then, they buy from the lowest-price seller. In the second stage, sellers set prices. McAfee proves that the unique

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[2] In a similar setting, Roy (2000) focuses on sellers' decisions of whether to address separate or overlapping segments.

[3] Chioveanu's (2003) analysis is, however, silent on the behavior of customers who are reached by more than one seller.

equilibrium is *asymmetric*: a large seller invests in twice the availability of every other seller, and then, the large seller sets relatively higher prices than the other sellers.

Prior research failed to address an obvious, rather fundamental question. In these settings, pricing is randomized, but what about customer acquisitions – are they deterministic or randomized? Further, most prior research assumed that sellers acquire customers before setting prices. What if, as assumed by Butters (1977) and often happens in practice, sellers acquire customers and set prices at the same time?[4] Would there be a unique equilibrium, or, as conjectured by Butters (1977, p. 471), are the sellers' strategies indeterminate? Finally, how do the comparative statics depend on the timing of actions and nature of the equilibrium? These are the questions that we address.

3. Setting

Each potential consumer derives benefit v from one unit of some item, which may be a good or service. A potential consumer can purchase the item only if acquired by a seller (Butters 1977; Grossman and Shapiro 1984; McAfee 1994; Stegeman 1991; Roy 2000; Chen and Iyer 2002; Soberman 2005).

The marketing can be interpreted as either advertising in the mass media or direct marketing through mail, telephone, or email. Yet another interpretation is seller's investments to reduce costs that consumers incur to switch products (Klemperer 1987).[5] Let a seller's cost of acquiring A customers be

$$C(A) = c\ln\left(1-\frac{A}{L}\right)\Big/\ln\left(1-\frac{1}{L}\right), \tag{1}$$

where $c > 0$ is a constant and L is the total number of potential consumers.[6] For simplicity, we assume that A and L are continuous variables rather than integers.

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[4] Robert and Stahl (1993) address this issue, but in a context where consumers can spend on search, and so derive results which McAfee (1994) describes as being "qualitatively different".

[5] Models (e.g., Chioveanu 2003; Baye and Morgan 2004) in which sellers need not invest to sell to switchers cannot capture such switching costs.

[6] Specification (1) follows Grossman and Shapiro (1984) and McAfee (1994). In the Appendix, we provide a motivation for this specification and explain why it is more reasonable than the quadratic cost function assumed by Chen et al. (2001a) and Chen and Iyer (2002).

Alternatively, we could interpret A/L as the fraction of consumers acquired, which then would clearly be a continuous variable.

We consider two alternative specifications for the timing of sellers' actions:

- Sellers acquire customers before setting prices, and
- Sellers acquire customers and set prices simultaneously.

A potential consumer who is acquired by a seller will pay up to v for the item. A consumer who is acquired by *more than one seller* will be a switcher and purchase from the seller offering the lowest price, or buy with equal probability from each of several sellers that offer the same price.

We make the following regularity assumptions to ensure that the sellers will earn positive profit from customer acquisition:

$$v > c, \tag{2}$$

and

$$X \equiv \frac{c}{vL} \frac{-1}{\ln(1-1/L)} < 1. \tag{3}$$

The parameter X represents the cost of acquiring a new customer relative to the potential demand. For sufficiently large L, $X \cong c/v < 1$, hence (2) implies (3).

Let $G_i(A)$, with support T_i, where $\underline{A}_i = \inf(T_i)$ and $\hat{A}_i = \sup(T_i)$, represent seller *i*'s acquisition strategy, for $i = 1,...,n$. Without loss of generality, let the sellers be labeled in decreasing order of the suprema of their supports, i.e., such that $\hat{A}_1 \geq \hat{A}_2 \geq ... \geq \hat{A}_n$. Let the seller with the largest *realized* number of customers acquired be labeled m, i.e., $A_m \geq A_i$, all $i = 1,...,n$.

Let $F_i(p)$, with support S_i, where $\underline{p}_i = \inf(S_i)$ and $\hat{p}_i = \sup(S_i)$, represent seller *i*'s pricing strategy. Further, let $\underline{p}$ be the lowest among the infima of the supports of the various pricing strategies, i.e., $\underline{p} = \min(\underline{p}_i)$.

Consider seller *i*. Its expected revenue at any price p depends on whether the customers are also acquired by another seller and, if so, the other seller's price(s) relative to seller *i*'s. Seller *i* would sell with certainty to consumers not acquired by another seller *j*, which amounts to

$$A_i \prod_{j \neq i} \left[1 - \frac{A_j}{L} \right].$$

At the other extreme,

$$A_i \prod_{j \neq i} \frac{A_j}{L}$$

consumers are acquired by all *n* sellers. Seller *i* will secure these consumers only if its price, p, is lower than that of all other sellers, which occurs with probability

$$\prod_{j \neq i} [1 - F_j(p)].$$

Accordingly, seller *i*'s expected revenue at any price p,[7]

$$R_i(p) = \left\{ \prod_{j \neq i} \left[1 - \frac{A_j}{L} \right] + \ldots + \prod_{j \neq i} \frac{A_j}{L} \prod_{j \neq i} [1 - F_j(p)] \right\} pA_i = \prod_{j \neq i} \left[1 - \frac{A_j}{L} F_j(p) \right] pA_i. \quad (4)$$

To explain (4), seller *i* would sell to every customer that it acquires, provided that either the customer is not acquired by any other seller, or, the other sellers who acquire her set higher prices.

Following McAfee (1994), the next result generalizes the findings of Narasimhan (1988), Baye et al. (1992), and Chioveanu (2003) to a setting where the captive and switcher segments differ among sellers. It is key to characterizing the pricing outcome.

Lemma 1. Suppose that either customer acquisitions take place before price setting, or customer acquisitions are deterministic. Then, in the pricing equilibrium,

(a) There is no pure-strategy equilibrium;

(b) The supports of the pricing strategies of at least two sellers have supremum at v, i.e., $\hat{p}_i = v$, at least two *i*;

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[7] In Varian (1980), Png and Hirshleifer (1987), and Baye et al. (1992), consumers are acquired by either *one* seller or *all* sellers. Then, the analysis of the 2-seller case can be extended to the $n \geq 3$ sellers by simply redefining the distribution of the competing seller's price as the distribution of the lowest of the $n-1$ competitors' prices. Our setting is more complicated: consumers may be acquired by more than one but fewer than n sellers.

(c) The supports of the pricing strategies of at least two sellers have infimum at $\underline{p}$, i.e., $\underline{p}_i = \underline{p}$, at least two *i*;

(d) Equilibrium pricing strategies do not include any mass points in the interval, $[\underline{p}, v)$; no more than one seller may have a mass point, which must be at v;

(e) The supports of the pricing strategies of all sellers have the same infimum, $\underline{p}$, i.e., $\underline{p}_i = \underline{p}$, all *i*, where

$$\underline{p} = v \prod_{i \neq m} \left[1 - \frac{A_i}{L} \right]; \tag{5}$$

(f) The supports of equilibrium pricing strategies are intervals (and so, do not have any gaps);

(g) For sellers 1 and 2, $\hat{p}_1 = \hat{p}_2 = v$, while among all other sellers, a seller which acquires more consumers will have a support with a higher supremum, i.e., if $A_i > A_j$, then $\hat{p}_i > \hat{p}_j$.

(h) For every seller *i*, expected revenue is

$$R_i = \underline{p} A_i = v A_i \prod_{j \neq m} \left[1 - \frac{A_j}{L} \right]. \tag{6}$$

For brevity, we report the proofs of all results in the online Appendix.[8]

4. Sequential Action

We now analyze the equilibrium outcome when sellers acquire customers before setting prices. In the second stage, when setting prices, every seller knows how many customers the other sellers have acquired.

A preliminary issue is whether sellers will randomize customer acquisitions. Proposition 1 shows that, if the marginal cost of customer acquisition does not increase too fast, the seller will not randomize. This is algebraically equivalent to the condition that the number of potential consumers be sufficiently large.

[8] http://www.comp.nus.edu.sg/~ipng/research/sales_appx.pdf

Proposition 1. If sellers acquire customers before setting prices, for sufficiently large L, customer acquisition is deterministic.

The intuition of Proposition 1 is as follows. Generally, a seller's marginal profit arising from an increase in acquisitions equals its marginal revenue less marginal cost.

The seller's increase in acquisitions affects revenue in two ways. First, it directly raises revenue in proportion to the increase in acquisitions (positive marginal revenue). Second, by Lemma 1, with the probability that the seller's acquisition is not the largest among all sellers, the increase would reduce the price and hence reduce revenue (negative marginal revenue). The negative effect is in proportion to the *square* of the increase in acquisitions because the price effect is multiplied by the acquisitions.

Accordingly, for levels of acquisitions below the highest possible, a seller's revenue is concave in acquisitions. By (1), the cost of acquisitions is convex. Hence, the seller's profit is concave in acquisitions, and hence no sellers would randomize so long as some "big" seller chooses a high level of acquisition and deterministically. A similar argument shows that the "big" seller would also not randomize acquisitions. This implies that, if sellers randomize acquisitions, at least two sellers, say 1 and 2, must have the same supremum, say $\hat{A}$, in the supports of their acquisition strategies.

Now, consider seller 1's expected profit from raising acquisitions to slightly above $\hat{A}$. As explained above, this will directly raise revenue. However, since $\hat{A}$ is the highest level of acquisition among the other sellers, the increase would not reduce price. Accordingly, the marginal revenue for $A_1 > \hat{A}$ is greater than the marginal revenue for $A_1 < \hat{A}$. If the marginal cost does not increase too fast, the variation would raise profit, which means that any set of strategies with more than one seller randomizing at $\hat{A}$ would not constitute an equilibrium.

Given that sellers do not randomize customer acquisition, we can now state Proposition 2, which characterizes the equilibrium outcome. In part, this result applies McAfee's (1994) Theorem 3.

Proposition 2. If sellers acquire customers before setting prices, the unique equilibrium comprises, in the first stage, sellers $i = 2,...,n$ acquiring A^* customers, where

$$\left[1-\frac{2A^*}{L}\right]\left[1-\frac{A^*}{L}\right]^{n-1} = X \ , \tag{7}$$

and seller 1 acquiring $A_1 = 2A^*$ customers, and, in the second stage, sellers $i = 2,...,n$ setting prices according to the atomless distribution,

$$F(p) = \frac{L}{A^*}\left\{1-\left[1-\frac{A^*}{L}\right]\left[\frac{v}{p}\right]^{\frac{1}{n-1}}\right\}, \tag{8}$$

on the support $[\underline{p}, v)$, where

$$\underline{p} = \left[1-\frac{A^*}{L}\right]^{n-1} v \ , \tag{9}$$

and seller 1 setting prices according to the distribution $F_1(p) = 1/2 \ F(p)$ on the support $[\underline{p}, v)$, with a mass point of weight ½ at the price v.

The intuition of Proposition 2 is as follows. Generally, a seller considering whether to increase acquisitions must balance three consequential effects on expected profit: the increase as such would raise sales (raising profit), the increase would through (5) reduce price (reducing profit), and the increase would raise cost (reducing profit). However, if one seller were to increase acquisition beyond every other seller's acquisition, then, by (5), any further increase would not affect the price, and hence, it would have a greater incentive to increase acquisitions. Since the other sellers are *ex-ante* identical, they choose the same level of acquisitions. Accordingly, the equilibrium is asymmetric, with one seller acquiring twice as many customers as the others.

By (7), sellers' expenditures on customer acquisition are strategic substitutes. To interpret condition (7), substitute for X from (3),

$$\left[1-\frac{2A^*}{L}\right]\left[1-\frac{A^*}{L}\right]^{n-1} vL = \frac{-c}{\ln(1-1/L)} .$$

The left-hand side is the probability that a consumer has not been acquired by any seller, multiplied by the consumer's reservation value and the number of potential consumers. Hence, it is related to the expected marginal revenue. The right-hand side is related to the marginal cost of customer acquisition.

It is difficult to characterize the asymmetric equilibrium explicitly because (7) is a polynomial of degree n. However, we can derive bounds on A^*. Since, by (7),

$$\left[1-\frac{2A^*}{L}\right]^n < \left[1-\frac{2A^*}{L}\right]\left[1-\frac{A^*}{L}\right]^{n-1} < \left[1-\frac{A^*}{L}\right]^n,$$

we infer that

$$\frac{1}{2}\left[1-\sqrt[n]{X}\right]L < A^* < \left[1-\sqrt[n]{X}\right]L. \qquad (10)$$

5. Simultaneous Action

In this case, we suppose that sellers acquire customers and set prices at the same time. Then, obviously, sellers cannot condition prices on customer acquisition. As in the case of sequential action, we first prove that sellers will not randomize customer acquisitions.

Proposition 3. If sellers acquire customers and set prices simultaneously, customer acquisition is deterministic.

Intuitively, when acquisition and pricing occur simultaneously, each seller's profit is concave in its own acquisitions. Hence, randomizing over any two levels of acquisitions would be less profitable than choosing some intermediate level of acquisition with certainty.

The next proposition shows that the equilibrium is unique and symmetric.

Proposition 4. If sellers acquire customers and set prices simultaneously, the unique equilibrium comprises acquisition

$$A_s = \left[1-\sqrt[n]{X}\right]L \ , \qquad (11)$$

and the pricing strategy

$$F_s(p) = \frac{1 - \sqrt[n]{X}\left[\frac{v}{p}\right]^{\frac{1}{n-1}}}{1 - \sqrt[n]{X}}, \tag{12}$$

with support

$$\left[vX^{1-\frac{1}{n}}, v\right]. \tag{13}$$

For the case of simultaneous action, Butters (1977) derived the market advertising and sales price distributions, but did not characterize the behavior of individual sellers. Robert and Stahl (1993) proved that the equilibrium is unique and symmetric, but in a context including consumer search. Much previous research, for instance, Varian (1980) and Grossman and Shapiro (1984), simply assumed that the equilibrium was symmetric. By contrast, Proposition 4 shows that the equilibrium is unique and *symmetric* when sellers acquire customers and set prices simultaneously. Accordingly, it provides a fundamental justification for a focus on symmetric outcomes.

6. Duopoly

The two-seller case of Varian's (1980) and related models has been widely applied to issues of business and marketing strategy (e.g., McGahan and Ghemawat 1994; Chen and Iyer 2002; Chen et al. 2001a and 2001b; Ghose et al. 2002). An important question then is whether and how such models scale up to the case of more than two sellers. We focus on the case of sequential action. [9]

By Proposition 2, with $n = 2$, solving (7) and substituting in (8), the unique equilibrium comprises in the first stage, seller 2 acquiring A^* customers where

$$A^* = \left[\frac{3}{4} - \frac{1}{4}\sqrt{1+8X}\right]L,$$

and seller 1 acquiring $A_1 = 2A^*$ customers, and, in the second stage, seller 2 setting prices according to the atomless distribution,

[9] It is easy to see the results with simultaneous action from Proposition 4 – each seller would acquire fewer customers as the number of sellers increases.

$$F(p) = \frac{4 - [1 + \sqrt{1+8X}\,]\frac{v}{p}}{3 - \sqrt{1+8X}},$$

and seller 1 setting prices according to the distribution $F_1(p) = 1/2\ F(p)$, with a mass point of weight ½ at the price v.

Whether there are two or more sellers, the unique equilibrium is asymmetric, and a dominant seller acquires twice as many customers as the other acquiescing sellers. Hence, the model scales up in a particular way – as the number of sellers increases, the dominant seller becomes smaller relative to the market. Further, in the next section, we show that, as the model scales up, the dominant seller actually becomes smaller on an absolute basis as well. More importantly, for managerial implications, we show that the impact of changes in cost and demand parameters on the profit-maximizing acquisition and pricing strategies remains qualitatively the same as the number of seller increases beyond two.

7. Comparative Statics

Given the number of sellers, n, and the demand and cost parameters, v, L and c (which together determine X), we can solve (7) numerically for A^*. Figure 1 shows the equilibrium acquisitions as a function of X at various levels of n.

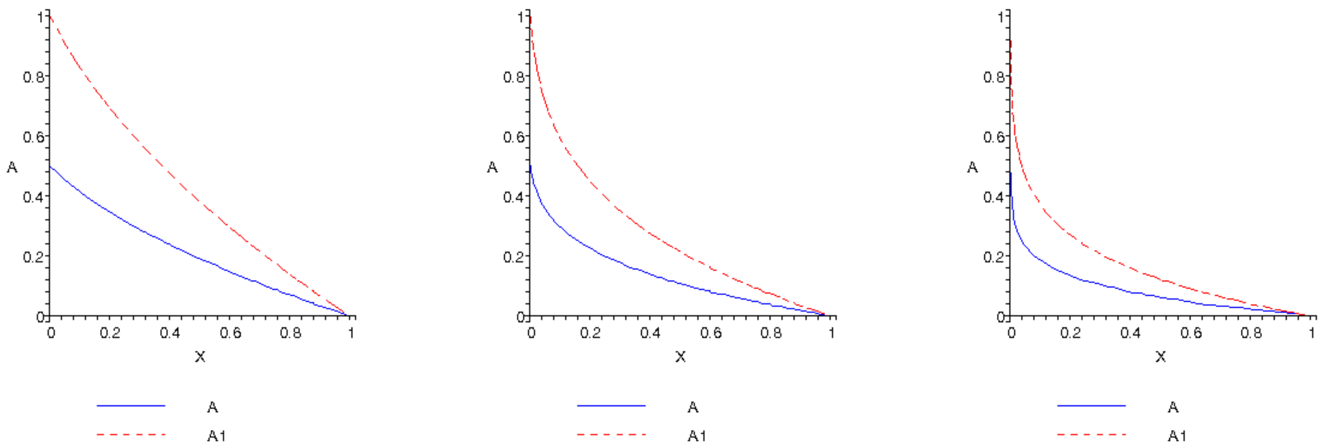

Figure 1. Equilibrium Acquisitions, *n* = 2, 5, 10

Generally, Table 1 below reports comparative statics with respect to changes in the cost of customer acquisition, the consumers' reservation value, the number of potential consumers, and the number of sellers. More importantly, for managerial strategy and public policy implications, the comparative statics for the sequential and simultaneous cases (which correspond to the asymmetric and symmetric outcomes respectively) are identical.

Table 1. Comparative Statics

	A^*	F	F_1 [#]	$\underline{p}$	A_s	F_s
Increase in c	–	–	–	+	–	–
Increase in v	+	+	+	–	+	+
Increase in L [##]	+	–	–	+	+	–
Increase in n	–	? [###]	? [###]	–	–	? [##]
$n \to \infty$	0	Mass point at Xv	Mass point at Xv	Xv	0	Mass point at Xv

[#] In addition, the dominant seller's pricing strategy includes a mass point at v.
[##] The results apply when L is sufficiently large.
[###] The price distribution becomes more extreme -- sellers tend towards either low or high prices and the impact on the average price is ambiguous.

Referring to Figure 1, the equilibrium number of customers acquired, A^*, is a convex function of the cost of acquisitions relative to the market parameter, X. With more sellers, the function becomes relatively more convex, so A^* is more sensitive to changes in X at low levels of X, and less sensitive at higher levels. Accordingly, A^* is more sensitive to changes in c, v, and L at low levels of X (i.e., when the market is lucrative), and less sensitive at higher levels of X (i.e., when the market is not so attractive to sellers).

In the limit of both asymmetric and symmetric equilibria, as $n \to \infty$, the price essentially converges to Xv, which is sufficient for the seller's revenue to slightly exceed its cost of customer acquisition. Hence, as the number of sellers increases without limit, the equilibrium converges to perfect competition.

It is useful to contrast our pricing results with previous models of price competition where consumer segments were exogenous. In the latter, when a new seller enters, it would raise competition for segment of switchers, without affecting any seller's captive segment. So, for each seller, the balance in pricing would shift toward charging a higher price to extract profit from captive consumers (Rosenthal 1980; Png and Hirshleifer 1987; Iyer and Pazgal 2002). Hence, in equilibrium, prices would be higher.

In our more realistic setting, there is a countervailing effect – when a new seller enters, it acquires both consumers who had been acquired by only one seller (captive to that seller), as well as consumers who had been acquired by more than one seller (switchers). The net effect on pricing is ambiguous as sellers are torn between reducing price to compete for the switchers and raising price to capture more revenue from the captive consumers.

8. Welfare

Generally, in our context, social welfare is the benefit to consumers less the sellers' cost of customer acquisitions. In turn, the benefit to consumers is v times the net number of consumers served (net of overlaps among the sellers' acquisitions).

In the case of sequential action, the consumer benefit is

$$vL\left\{1-\left[1-\frac{2A^*}{L}\right]\left[1-\frac{A^*}{L}\right]^{n-1}\right\}=vL[1-X], \tag{14}$$

after substituting from (7). Further, the sellers' cost of customer acquisitions,

$$\begin{aligned}&\frac{c\ln(1-2A^*/L)}{\ln(1-1/L)}+[n-1]\frac{c\ln(1-A^*/L)}{\ln(1-1/L)}\\&=\frac{c}{\ln(1-1/L)}\left[\ln\left(1-\frac{2A^*}{L}\right)+\ln\left(1-\frac{A^*}{L}\right)^{n-1}\right]=\frac{c\ln X}{\ln(1-1/L)},\end{aligned} \tag{15}$$

after substituting from (7).

Hence, social welfare is

$$W=vL[1-X]-\frac{c\ln X}{\ln(1-1/L)}. \tag{16}$$

It is straightforward to show that in the case of simultaneous action, social welfare is also given by (16). Hence, the number of consumers served and social welfare are the same whether sellers act sequentially or simultaneously.

Further, the welfare in (16) is independent of the number of sellers. This result follows from the nature of the cost function, (1). As discussed in the Appendix, the cost function (1) exhibits what McAfee (1994) terms "constant returns to scale" in the sense that a seller's total cost and net expected yield (after removing duplicates) are the same whether customers are acquired in a single batch or several smaller batches.

We consider this cost function as a useful benchmark case. If the cost of customer acquisition were to exhibit *decreasing* returns to scale, then an increase in the number of sellers would raise welfare, and similarly, if there are *increasing* returns, then increasing the number of sellers would reduce welfare.

9. Concluding Remarks

We have embedded the Varian (1980) model in a broader setting that considers how customer segments are determined. Our first result is that if the marginal cost of customer acquisitions does not increase too fast, then customer acquisitions are deterministic.

Second, we have shown that the unique equilibrium is *asymmetric* if customer acquisition takes place before price setting, but the unique equilibrium is *symmetric* if customer acquisition and price setting are simultaneous. Our findings provide a fundamental rationale for previous analyses that exogenously assumed that the equilibrium outcome would be asymmetric (Narasimhan 1988; McGahan and Ghemawat 1994) or symmetric (e.g., Varian 1980; Grossman and Shapiro 1984; Png and Hirshleifer 1987; Meurer and Stahl 1994; Baye and Morgan 2001 and 2004; Iyer and Pazgal 2003).[10]

Besides the contribution to the pure analytics of price competition among multiple sellers of an identical product for consumers who vary in loyalty to particular sellers, our results also contribute to understanding the dynamics of

[10] The focus on symmetric equilibria can also be justified empirically by the many experiments showing that subjects tend to select symmetric rather than asymmetric outcomes (Van Huyck et al. 1990; Battalio et al. 2003).

customer acquisition, be it interpreted as mass media advertising, availability, direct marketing, consumer addressability, or investments to reduce consumer switching costs.

Following McAfee (1994), our analysis can easily be extended to allow consumers to have elastic demand. Let k be the marginal cost of producing the item and $q(p)$ be the individual consumer's demand at price p. Then, in the key equations like (5) and (6), each seller equilibrates between the contribution margin, $m = [p-k]q(p)$, at the various prices, rather than equilibrating between the prices themselves. With this change, all the major results continue to apply.

The immediate direction for future research is to analyze the equilibrium outcome when potential consumers have different individual demand curves. This would reduce the price elasticity of demand around a competitor's price and dilute the incentive for a seller to just under-cut its competitor's prices. We conjecture this change would reinforce the tendency towards asymmetric equilibria.

Another direction for future research is to take account of consumer search (Butters 1977; Stahl 1989; Robert and Stahl 1993; Banks and Moorthy 1999; Anderson and de Palma 2003; Baye and Morgan 2004). The equilibrium outcome would then depend on both sellers' investments in customer acquisition and consumers' investments in search. Robert and Stahl (1993) show that if consumers search after sellers set advertising and prices simultaneously, the unique equilibrium is symmetric. The outcome when sellers acquire customers before setting prices, and then consumers search, appears to be an open question.

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