The Time Consistency of Optimal Monetary Policy with Heterogeneous Agents

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Abstract

This paper studies the structure and time consistency of optimal monetary policy from a public finance perspective in an economy where agents differ in preference for liquidity and holdings of nominal assets.

I find that the presence of redistributional effects breaks the link between time consistency and high inflation which characterizes representative agent models of optimal fiscal and monetary policy. For a large class of economies, optimal monetary policy is time consistent. I relate these findings to key historical episodes of inflation and deflation.

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1. Introduction

The purpose of this paper is to study the structure and the time consistency of optimal monetary policy from a public finance perspective in an economy where agents are heterogeneous in holdings of currency and other nominal assets. The seminal work of Calvo (1978) and Lucas and Stokey (1983)[1] illustrates that in a monetary economy a benevolent policymaker has the incentive to tax outstanding nominal assets via unanticipated inflation when lump-sum taxation is not available. On this basis, lack of commitment has been advocated as a potential explanation of persistent high inflation and high public deficits. In the presence of nominal assets and distortionary taxation, rational agents anticipate the policymaker's incentive to revise policy in the direction of higher money growth. This leads to high inflation in equilibrium. Moreover, the equilibrium inflation rate is positively correlated with the level of outstanding nominal government debt.

I find that the presence of redistributional motives breaks the link between time consistency and high inflation which characterizes representative agent models of optimal fiscal and monetary policy. The incentive to generate unaticipated inflation depends crucially on the distribution of currency and other nominal assets, as well as on the distribution of political power. Optimal monetary policy is time consistent for a large class of economies.

Lucas and Stokey (1983) and Chamley (1985) argue that time consistency of optimal monetary policy can be achieved if the monetary authority can commit to a path for nominal prices. Persson, Persson, and Svensson (1987) exhibit a particular debt management strategy, involving both nominal and indexed government bonds of various maturities, that can remove the problem of time inconsistency. The findings in this paper suggest that optimal monetary policy could be made time consistent by influencing the distribution of government debt. This argument is not new. Hamilton (1795) argued in favor of the Federal assumption of the states' war debt as a way to reduce the risk of monetization. Debt assumption would provide powerful government creditors with a strong incentive to support Federal tax legislation, making the use of inflation to raise revenues less likely.

I describe a cash-credit good economy in which households have different preferences over cash and credit goods and differ in holdings of nominal bonds. Households chose consumption and labor supply and are subject to proportional labor income taxation. The government issues money and nominal debt and collects labor income taxes to finance an exogenous stream of government spending. Monetary and fiscal policy redistribute resources across households. Inflation weighs more heavily on households who consume a greater fraction of cash-goods and unanticipated inflation hits holders of nominal assets. The share of labor income tax revenues collected from each type of household is proportional to supplied labor, which is inversely related to outstanding nominal wealth.

First, I study optimal monetary and fiscal policy for a benevolent government with the ability to commit to future policy and I trace out the Pareto frontier for

[1] Kydland and Prescott (1977), and Barro and Gordon (1983, a and b) emplore the consequences of the time inconsistency of monetary policy in an expectational Phillips curve environment.

this economy. I concentrate on a class of utility specifications with unitary income elasticity of money demand. As discussed in Chari, Christiano and Kehoe (1996), the Friedman rule is optimal in these economies with no heterogeneity. I find that the Friedman rule is optimal if and only if the government wishes to redistribute to agents with a larger weight on cash goods, otherwise high rates of inflation and government deficits will result. I then proceed to characterize the sufficient conditions for time consistency of the optimal policy and characterize the optimal fiscal and monetary policy when the government choses policy sequentially and has no ability to commit. I assume that in each period fiscal and monetary policy are chosen before households can adjust their holdings of currency, following Svensson (1985).

Distributional goals have ambivalent effects on government incentives. On one hand, the lack of a full menu of redistributional policy instruments may increase the incentive to revise pre-announced policy[2]. On the other, the redistributional effects of inflation may remove the incentive to deviate from the ex ante optimal policy. Therefore, the time consistency of optimal fiscal and monetary policy depends on the balance between redistribution and efficiency.

I find that, for a large class of economies, lack of commitment does not imply a higher equilibrium rate of inflation. With no government debt outstanding, the optimal policy under commitment is time consistent if the Pareto weight on households with higher weight on cash goods is sufficiently high. If the level of government debt outstanding is positive, the government has an incentive to reduc money growth and increase the tax rate on labor, relative to expectations, if households with holdings a larger fraction of government debt are assigned a higher Pareto weight. This induces a rise in the real interest rate, redistributing resources to households who are net creditors to the government. Therefore, if households with a high weight on cash goods have a sufficiently high Pareto weight and hold a sufficiently high fraction of government debt, the optimal policy is time consistent. If households with low weight on cash goods hold a sufficiently high fraction of government debt and have a sufficiently high Pareto weight, the optimal policy under commitment is not time consistent but the optimal deviation involves a fall in the inflation rate relative to expectations. If these conditions do not hold, the time consistent policy involves higher inflation than under commitment. Moreover, the time consistent inflation rate is higher than in an economy with no heterogeneity, given that both efficiency and redistributional incentives push the government towards higher tha exoected money growth. Under the time consistent policy, households with a larger weight on cash goods suffer large welfare losses, reaching 7-10% of per period consumption in a plausibly parametrized version of the economy.

To evaluate the relevance of these findings, I analyze a number of key historical episodes of large inflations and deflation. Descriptive accounts of these episodes provide clear evidence of the importanc of redistributional consequences of unanticipated changes in inflation in shaping government incentives, conditional on the political influence of different groups of agents on monetary and fiscal policy decisions.

[2] Persson and Tabellini (1997) suggest that, with heterogeneity, time inconsistency may arise even with lump-sum taxes, if they are not agent specific. Pearce and Stacchetti (1997) study an economy where this is indeed the case.

The plan of the paper is as follows. The next subsection reviews the literature on the time consistency of optimal fiscal and monetary policy with heterogeneous agents. The model economy is presented in Section 2. Section 3 studies optimal fiscal and monetary policy under commitment and characterizes the sufficient conditions for time consistency. Section 4 reviews a number of historical episodes of inflation and disinflation. The optimal policy without commitment is analyzed in Section 5. Section 6 concludes.

1.1. Previous Literature

This paper is closely related to the work of Nicolini (1998) who evaluates the conditions under which optimal monetary policy is time consistent when agents are heterogeneous in their ability to adjust currency holdings in response to unanticipated changes in inflation. He also analyzes a representative agent cash in advance economy in which households cannot adjust their holdings of currency in response to unanticipated changes in inflation. He finds that optimal monetary policy is not time consistent in general but that for certain conditions the optimal deviation involves a fall in the rate of money growth. The conditions involve the distribution of the Pareto weights and the price elasticity of cash good consumption. Nicolini does not consider labor income taxation and stops short of analyzing the case with nominal government debt.

The findings on the sufficient conditions for time consistency are also related to Caselli (1997). This paper studies public debt runs under alternative assumptions on the distribution of taxes among tax bases, the distribution of debt among classes of taxpayers and the distributive preferences of the government. The results suggest that the probability of a run on government debt, which depends on the perceived credibility of the government, is decreasing in the degree of identification of the government with a specific constituency and in the fraction of debt held by this constituency. Caselli also provides empirical evidence from a sample of high-debt OECD countries broadly consistent with these results.

Rogers (1986) is the first to formally analyze whether a government's distributive goals can inhibit its incentive to renege on announced policies. She analyzes the issue in the context of a two-period, multiple consumer model of optimal wage and interest taxation. She finds that inconsistent interest tax increases may be moderated if they create an unacceptable utility distribution in the economy.

2. A Cash-Credit Good Economy with Heterogenous Households

In this section, I describe a version of Lucas and Stokey's cash-credit good economy with two key modifications. First, there are two types of households having different preferences over cash and credit goods. Second, in each period trade in goods and labor precedes trade in assets. This timing, introduced by Svensson (1985), implies that households cannot adjust the amount of currency available for purchases in the current period to changes in the inflation rate. The economy is populated by households, firms and a government. Households consume cash and credit goods and supply labor. Firms have access to a linear production technology that requires labor for the production of consumption goods. They are perfectly competitive. The government finances an exogenous stream of spending by issuing nominal debt, printing money and taxing labor income at a uniform proportional rate. There is no uncertainty.

I now illustrate the problems faced by the agents in our economy in detail.

2.1. Firms

Firms live for one period. They hire labor to produce consumption goods with a linear technology, given by:

$$\sum_{j=1}^{2} y_{jt} \leq n_t.$$

Here y_{1t} is total production of cash goods and y_{2t} total production of credit goods at time t and n_t is aggregate labor. Perfect competition implies:

$$P_{1t} = P_{2t} = P_t = W_t,$$

where P_t is the price charged for consumption goods and W_t the nominal wage at time t.

2.2. Households

There is a continuum of unit measure of households, divided into two types, where $0 < \nu_i < 1$ is the fraction of type i agents, with $i = 1, 2$ and $\sum_i \nu_i = 1$. Households of the same type are identical. Households have preferences defined over consumption of cash goods c_{i1}, consumption of credit goods c_{i2} and over hours worked n_i. Preferences are given by:

$$\sum_{t=0}^{\infty} \beta^t U^i \left(c_{it}, n_{it}\right),$$

$$c_i = h^i \left(c_{i1}, c_{i2}\right),$$

where $h^i(\cdot)$ is homothetic, U is continuous, concave and twice differentiable and $U_1^i \geq 0$ and $U_2^i \leq 0$. Leisure and c_i are gross substitutes.

Households purchase consumption goods, supply labor, accumulate currency and trade one-period nominal discount bonds in each period. They enter a period with M_{it} units of currency and B_{it} holdings of one-period risk-free nominal discount bonds. They are subject to a cash in advance constraint for purchases of cash goods, given by:

$$P_t c_{i1t} - M_{it} \leq 0. \tag{2.1}$$

The asset market session follows trading in the goods and labor market. During the asset market session households receive labor income net of taxes, clear consumption liabilities and trade bonds issued by other households or by the government at price Q_t. Bonds purchased at time t entitle holders to one unit of currency in the asset market section at $t + 1$. I assume that the government cannot default on its debt, which implies that agents are indifferent between holding privately or government issued bonds, which trade at the same price. If the government does not issue debt, the bonds will be in zero net supply. Total holdings of debt by agent i at the end of time t are denoted with B_{it+1} for $i = 1, 2$.

Households face the following constraint on the asset market:

$$M_{t+1} + Q_t B_{it+1} \leq M_{it} + B_{it} - P_t c_{i1t} - P_t c_{i2t} + W_t (1 - \tau_t) n_{it}, \tag{2.2}$$

where W_t is the nominal wage and τ_t is the tax rate on labor income.

2.3. Government

The government finances an exogenous stream of consumption $\bar{g}$ and is subject to the following dynamic budget constraint:

$$P_t \bar{g}_t + M_t + B_t = Q_t B_{t+1} + M_{t+1} + W_t \tau_t n_t, \tag{2.3}$$

where M_t, B_t are the supply of currency and nominal bonds respectively.

2.4. Private Sector Equilibrium

The timing of events in each period is as follows:

1. Households come into the period with holdings of currency and debt given by M_{it} and B_{it}.

2. The government sets policy subject to (2.3).

3. Households, firms and the government trade on the goods and labor markets. The households' purchases of cash goods are subject to (2.1). Equilibrium on the goods market requires:

$$\sum_{i=1,2} \nu_i (c_{i1t} + c_{i2t} - n_{it}) + \bar{g}_t = 0. \tag{2.4}$$

4. Asset markets open. Households purchase bonds and acquire currency to take into the following period subject to the constraint (2.2). Equilibrium in the asset market requires:

$$\sum_{i=1,2} \nu_i B_{it+1} = B_{t+1},$$
$$\sum_{i=1,2} \nu_i M_{it+1} = M_{t+1}.$$

Definition 2.1. *A private sector equilibrium is given by a government policy* $\{\bar{g}_t, \tau_t, M_{t+1}, B_{t+1}\}_{t\geq 0}$ *, a price system* $\{P_t, W_t, Q_t\}_{t\geq 0, j\in[0,1]}$ *and an allocation* $\{c_{i1t}, c_{i2t}, n_{it}, y_{1t}, y_{2t}\}_{i=1,2,t\geq 0}$ *such that:*

1. given the policy and the price system households and firm optimize;

2. government policy satisfies (2.3);

3. markets clear.

The following proposition characterizes the competitive equilibrium.

Proposition 2.2. *An allocation* $\{c_{i1t}, c_{i2t}, n_{it}, y_{1t}, y_{2t}\}_{i=1,2,t\geq 0}$ *and a price system* $\{P_t, W_t, Q_t\}_{t\geq 0, j\in[0,1]}$ *constitute a private sector equilibrium if and only if, for a given government policy* $\{\bar{g}_t, \tau_t, M_{t+1}, B_{t+1}\}_{t\geq 0}$ *, (2.4), (2.3) and the following conditions are verified:*

$$0 < Q_t \leq 1,$$

$$W_t = P_t,$$

$$Q_t = \beta \frac{P_t}{P_{t+1}} \frac{(1-\tau_t)}{(1-\tau_{t+1})} \frac{u_{in,t+1}}{u_{in,t}}, \tag{2.5}$$

$$\frac{-u_{i2t}}{u_{int}} = \frac{1}{(1-\tau_t)} \text{ for } t \geq 0, \tag{2.6}$$

$$\frac{u_{i1t+1}}{u_{i2t+1}} = Q_t^{-1},$$

$$\left(Q_t^{-1} - 1\right)\left(P_{t+1} c_{i1t+1} - M_{it+1}\right) = 0,$$

$$P_{t+1} c_{i1t+1} \leq M_{it+1},$$

for $t \geq 0$*, and:*

$$P_0 c_{i10} \leq M_{i0}, \tag{2.7}$$

$$\sum_{t=0}^{\infty} \beta^t \left[u_{i1t} c_{i1t} + u_{i2t} c_{i2t} + u_{int} n_{it}\right] = \frac{u_{i10}}{P_0} M_{i0} + \frac{u_{i20}}{P_0} B_{i0}. \tag{2.8}$$

for $i = 1, 2$.

Equation (2.8) is the households' intertemporal budget constraint and it incorporates the transversality condition. The proof of this proposition is standard.

3. Optimal Policy with Commitment

I define the Ramsey equilibrium as the private sector equilibrium which maximizes the government's objective function, given by the weighted sum of the households' lifetime utility. The Pareto weight on type i agents is η_i, with $\eta_1 + \eta_2 = 1$. I assume that Pareto weights are time-invariant. The case $\eta_i = \nu_i$ corresponds to a utilitarian government.

The Ramsey equilibrium outcome can be characterized by solving the "primal government problem", where the government chooses an allocation at time 0 subject to the constraint that it constitutes a private sector equilibrium. This problem's choice variables are $\{c_{i1t}, c_{i2t}, n_{it}\}_{i=1,2,t\geq 0}$ and P_0. The level of P_0 determines the real value of nominal assets at time 0 and defines the boundary of the agents' intertemporal budget set. High values of P_0 amount to a tax on outstanding nominal wealth and on consumption of goods purchased with cash at time 0. The government is constrained to tax all nominal assets at the same rate. The extent to which each household is hit by this tax depends on the exogenous distribution of currency and bonds at time 0 and on liquidity preference.

For convenience, I introduce the following notation:

$$p_t = \frac{P_t}{M_t},$$

$$b_t = \frac{B_t}{M_t},\ b_{it} = \frac{B_{it}}{M_{it}},$$

$$m_{it} = \frac{M_{it}}{P_t},\ \phi_m = \frac{m_{20}}{m_{10}},$$

for $t \geq 0$ and $i = 1, 2$. In the Ramsey allocation problem I let the government select m_{10}, which is equivalent to chosing P_0.

Proposition 3.1. *An allocation $\{c_{i1t}, c_{i2t}, n_{it}\}_{i=1,2,t\geq 0}$ and initial level of real balance holdings m_{i0} for $i = 1, 2$ constitute a Ramsey equilibrium if and only if they solve the problem:*

$$\max_{m_{10}, \{c_{i1t}, c_{i2t}, n_{it}\}_{i=1,2,t\geq 0}} \sum_{t=0}^{\infty} \beta^t \sum_{i=1,2} \eta_i U^i \left(c_{it}, n_{it}\right)$$

subject to:

$$\frac{u_{11t}}{u_{12t}} = \frac{u_{21t}}{u_{22t}}, \tag{3.1}$$

$$\frac{u_{i1t}}{u_{i2t}} \leq 1, \text{ for } i = 1, 2, \tag{3.2}$$

$$\frac{u_{12t}}{u_{1nt}} = \frac{u_{22t}}{u_{2nt}}, \tag{3.3}$$

$$\sum_{t=0}^{\infty} \beta^t \left[u_{i1t}c_{i1t} + u_{i2t}c_{i2t} + u_{int}n_{it}\right] = \left[u_{i10} + u_{i20}b_{i0}\right] \frac{m_{i0}}{p_0}, \tag{3.4}$$

$$m_{20} = \phi_m m_{10}, \tag{3.5}$$

and (2.7), (2.4).

A detailed proof of this characterization can be found in Chari and Kehoe (1998).

3.1. Properties of Optimal Policy for $t > 0$

In this section, I illustrate the key properties of Ramsey equilibrium policy for $t > 0$. To sharpen the analysis, I specialize to the following utility specification[3]:

$$h^1(c_{11}, c_{12}) = c_{11}. \tag{3.6}$$

Since $c_{12t} \equiv 0$, constraint (3.1) drops out of the problem, while constraint (3.2) holds for $i = 2$ only.

Type 1 agents are only affected by the wedge $(1 - \tau_t) Q_{t-1}$, while type 2 agents are separately affected by the wedge introduced by the tax rate on labor and positive nominal interest rates. An increase in nominal interest rates - a lower value of Q_t - increases the price of consumption for type 1 agents. For type 2 agents, it corresponds to an increase in the price of cash goods relative to credit goods at time t. If cash and credit goods are gross substitutes, the level of credit good consumption will increase with nominal interest rates. If the weight of credit goods in type 2's utility function and the weight of type 2 in government preferences are high enough, it will be optimal to set the discount rate positive, since this makes the price of consumption lower for type 2 relative to type 1.

I first show that the incentive to use inflation to redistribute resources across households of different types arises when the government does not have access to a full set of redistributional instruments.

Proposition 3.2. *If the government has access to individual specific proportional labor income taxation then in any Ramsey equilibrium $Q_t = 1$ for $t \geq 0$.*

The proof is in Appendix A and is analogous to the proof of the optimality of the Friedman rule in the environment with distorting taxes analyzed by Christiano, Chari and Kehoe (1996). It relies on the homotheticity of the consumption aggregator, which implies a unitary income elasticity of money demand. Intuitively, if the government can set different labor tax rates for different agents, real net wages need not be equalized and constraint (3.3) drops out of the problem. In this case, optimality requires equalizing the relative price of cash and credit goods.

Let $\bar{\eta}_1$ denote the Pareto weight such that government policy is neutral from a redistributional standpoint. It is given by the value of η_1 for which the constraint that the net real wage is equal across agents is non-binding, which is defined by the following equation:

$$\frac{u_{12t}/U_2^1}{u_{22t}/U_2^2} = \frac{\bar{\eta}_1}{\nu_1}\left(\frac{\bar{\eta}_2}{\nu_2}\right)^{-1}, \text{ for } t > 0.$$

The following result holds if the labor income tax schedule is linear.

Proposition 3.3. *For $h_{12}^2 \leq h_{22}^2$ and $U_{12}^2 \leq 0$, in the Ramsey equilibrium, $Q_t < 1$ for $t \geq 0$ if and only if $\eta_1 < \bar{\eta}_1$.*

[3] The results under a more general specification are presented in Chapter 3.

The proof is in Appendix A.

In the rest of the analysis, I focus on the utility specification:

$$U^i(c_i, n_i) = \frac{c_i^{1-\sigma} - 1}{1-\sigma} - \gamma n_i, \text{ for } i = 1, 2, \quad (3.7)$$

$$c_i^{1-\sigma} = (1 - z_i) c_{i1}^{1-\sigma} + z_i c_{i2}^{1-\sigma}, \quad (3.8)$$

with $z_1 = 0$, $z_2 \in (0,1)$, $\sigma \in (0,1]$, and $\gamma > 0$. The restriction on σ ensures that labor supply increases with the real wage. The assumption of constant marginal disutility of labor[4] implies that the Ramsey equilibrium government policy at time t depends on $\bar{g}_t$ only. In particular, if government spending is constant, then Ramsey equilibrium policy is constant for $t > 0$. Moreover, the distribution of cash holdings across agents is constant and does not depend on government policy. It is given by[5]:

$$\phi_1 \equiv \frac{c_{21}}{c_{11}} = \left(\frac{1 - z_2}{1 - z_1}\right)^{1/\sigma}. \quad (3.9)$$

The consumption price index for agent i and $t > 0$ is given by:

$$P_t^i \sim \left[(1 - z_{it}) (1/Q_{t-1})^{\frac{\sigma-1}{\sigma}} + z_{it}\right]^{\frac{\sigma}{\sigma-1}}.$$

For $\sigma < 1$, cash and credit goods are substitutes for type 2 households and $P_t^2 < P_t^1$. A lower price of credit goods relative to cash goods redistributes in favor of type 2 agents, since the net real wage, defined as the ratio of the nominal wage in efficiency units to the consumption price index, is higher for type 2:

$$\frac{W_t (1 - \tau_t)}{P_t^2} > \frac{W_t (1 - \tau_t)}{P_t^1}.$$

Therefore, setting the nominal interest rate above the Friedman rule allows the government to redistribute in favor of type 2 households.

To evaluate the impact of redistributional incentives I compute the Ramsey equilibrium as a function of the Pareto weight for a plausibly parametrized version of the economy. I set z_2 to match post-WWII US data on average M2 velocity for the preference specification in (3.7)-(3.8). I set ν_1 to match the percentage of US households having no financial assets except for a checking account, according to the 1995 Survey of Consumer Finances. The value of σ determines both the compensated elasticity of labor supply with respect to the real wage and the interest elasticity of money

[4] A constant marginal disutility of labor implies that the government's intertemporal problem separates into infinitely many indipendent maximization problems, each corresponding to a time period. Therefore, policy across periods will differ in equilibrium only to the extent that the "fundamentals" of the government problem, like tax base elasticities, outstanding nominal liabilities or level of government spending, are different.

[5] Note also that:

$$\frac{c_{12}}{c_{22}} = \left(\frac{z_1}{z_2}\right)^{1/\sigma}.$$

demand. I set σ so that the compensated elasticity of labor supply equals 0.25, which is within the range of values reported by macroeconomic and microeconomic studies. For these values of σ, z_2 and ν_i the interest elasticity of money demand[6] is equal to 3.052 at a nominal interest rate of 8% - slightly lower than estimates reported in the literature (e.g. Mulligan and Sala-i-Martin (2000) and Chari, Christiano and Kehoe (1993)). I set the level of government debt at time 0 to 40% of GDP when $\tau_0 = 0.30$ and the net nominal interest rate is equal to 6%[7].

Table 1: Benchmark Parameter Values

	α	β	γ	ν_1
	0.654	0.97	3	0.56
σ	0.7	0.8	0.9	1
$\overline{g}$	0.0467	0.592	0.0715	0.0833
ψ	0.588	0.564	0.568	0.571
ψ_1	0.851	0.834	0.817	0.802
ψ_2	0.8511	0.834	0.817	0.802

Results are displayed in Figures 3.1-3.2. In each panel, the solid line corresponds to the optimal policy for $t > 0$, the dotted line corresponds to optimal policy at $t = 0$ and a vertical dotted line is placed at $\eta_1 = \bar{\eta}_1$, the Pareto weight for which the government does not pursue redistributional goals. Figure 2.1 displays the Ramsey equilibrium policy as a function of η_1 for $\psi_i = 0$ for $i = 1, 2$. The equilibrium inflation rate reaches 40% for the lowest value of η_1 considered, the tax rate on labor is increasing in η_1, ranging from 10% to 31% for $t > 0$. Figure 2.2 displays the Ramsey equilibrium policy as a function on $\bar{g}$ for two values of η_1. The tax rate on labor increases with $\bar{g}$ for both values of η_1. In the top panels, $\eta_1 < \bar{\eta}_1$. Here, the nominal interest rate is positive and decreases with $\bar{g}$, and at $\bar{g} = 0$ the labor tax rate is negative. For $\eta_1 \geq \bar{\eta}_1$, the nominal interest rate is 0, with the tax rate on labor equal to 0 for $\bar{g} = 0$. These results clearly hinge on the trade-off between redistribution and efficiency. When $\bar{g} = 0$ and there is no debt outstanding, the government need not raise any revenues and should set the inflation and the labor tax rate to 0 to maximize efficiency. However, if $\eta_1 < \bar{\eta}_1$ it is optimal to tax currency holdings and subsidize labor. Since the elasticity of the inflation tax base is higher than that of the labor tax base, this scheme generates a cost which increases with $\bar{g}$ and eventually offsets the incentives to redistribute.

3.2. Sufficient Conditions for Time Consistency

In this section, I illustrate the potential sources of time inconsistency and provide an analytical characterization of the sufficient conditions for time consistency of the Ramsey plan for the preference specification in (3.7).

[6] Computed as:

$$\frac{\partial \log (M/P)}{\partial \log(R)},$$

where M/P are aggregate real money balances.

[7] This is the average value for the post-WWII period in the U.S., as reported by Chari and Kehoe (1998)

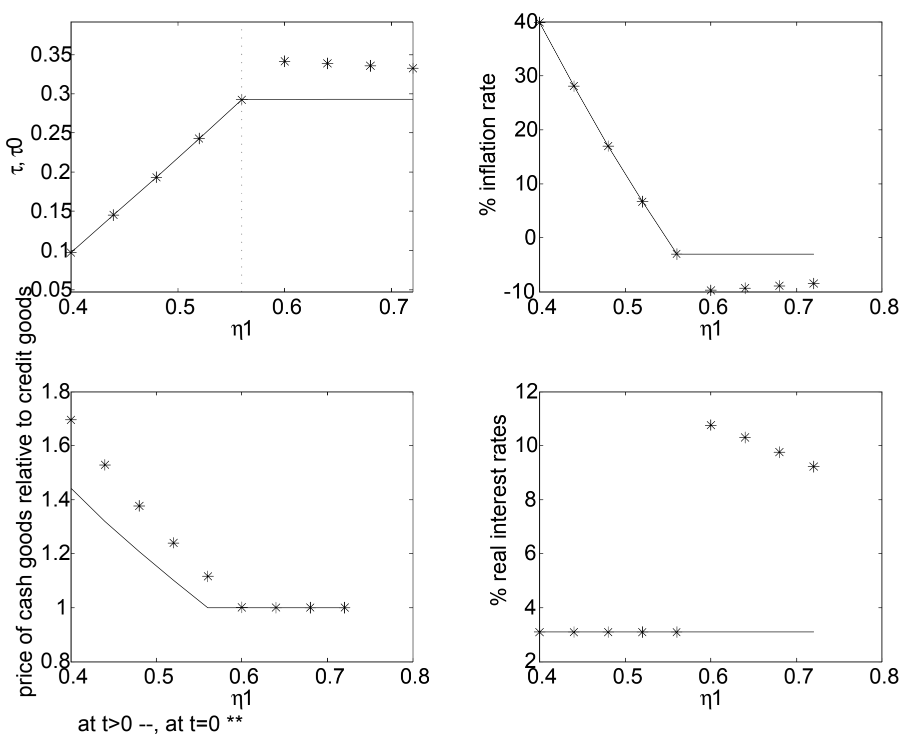

Figure 3.1: Ramsey equilibrium policy as a function of η_1.

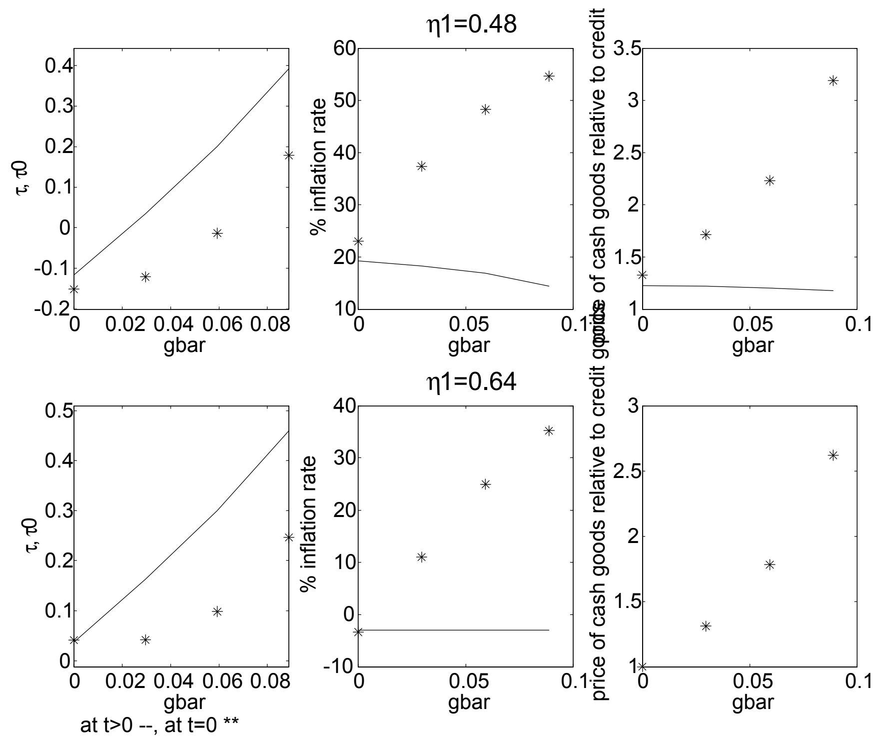

Figure 3.2: Ramsey equilibrium policy as a function of $\overline{g}$.

The fact that government spending and the marginal disutility of labor are constant implies that a government selecting policy for time 0 in the Ramsey equilibrium faces the same incentives as a government setting policy sequentially. At time 0 the elasticity of the inflation tax base is lower than at $t > 0$, since private sector decisions on the accumulation of nominal wealth and on payment arrangements are taken irreversibly before policy is chosen. Time 0 policy can therefore be viewed as the "optimal deviation" from the Ramsey plan, namely, the policy that a government with the same Pareto weights would choose if allowed to re-optimize, when confronted with the same distribution of currency and debt. The government's incentives for departing at time 0 from the policy which is optimal for $t > 0$ depend on the distribution of nominal assets, as well as on the Pareto weight.

I first analyze the case in which outstanding debt holdings are 0 for both types and the distribution of currency at time 0 is the same as the steady state distribution. This corresponds to the case in figure 3.1. For $\eta_1 < \bar{\eta}_1$ nominal interest rates are positive for $t > 0$ and the cash in advance constraint is binding. Since holdings of currency are predetermined at time 0, the price elasticity of cash good consumption is 1, while it is equal to $1/\sigma$ at $t > 0$. For $\sigma < 1$, it is efficient to tax cash good consumption at a higher rate at time 0. This effect is also present in a representative agent economy as illustrated in Nicolini (1998). A higher value of P_0 (or equivalently, a lower value of m_{10}) also corresponds to a lower relative price of credit goods which favors of type 2 households. Since the redistributional effect reinforces the incentive arising from efficiency considerations, the increase of the inflation tax relative to expectations will typically be larger than in the corresponding representative agent economy.

For $\eta_1 > \bar{\eta}_1$, nominal interest rates are 0 for $t > 0$, so that if the government were to follow this policy the cash in advance constraint would be non-binding, making the response of cash good consumption to small changes in the price level the same as for $t > 0$. Larger increases in the price level would cause the cash in advance constraint to be binding and cash good consumption to fall one-to-one with an increase in the price level. Since the government wishes to redistribute to type 1 households, it is optimal to generate a rise in the relative price of credit goods. Since the price of cash goods relative to credit goods is already at its lowest, the only feasible deviation is to increase the relative price of credit goods in the current period relative to future periods. This can be achieved by taxing labor income at a higher rate at time 0, which allows for decreased money growth between time 0 and time 1[8].

If outstanding government debt is positive, the elasticity of the inflation tax base is always lower at time 0 than at $t > 0$, since taxation of nominal debt holdings is non-distortionary. The households' inability to adjust currency holdings in response to inflation imposes an upper bound to the extent to which an increase in inflation is desirable ex post. If debt is evenly distributed across households or $\eta_1 = \bar{\eta}_1$, for which the distribution of wealth is irrelevant, this is the prevailing incentive.

[8] This produce a rise in the real interest rate between time 0 and time 1, given by:

$$r_1 \equiv \frac{u_{220}}{\beta u_{221}} = \frac{P_0}{P_1} Q_0^{-1} = \frac{(1-\tau_1)}{\beta(1-\tau_0)}. \tag{3.10}$$

Here, the second and third equality obtain from the households' first order conditions.

If the distribution of debt is uneven and $\eta_1 \neq \bar{\eta}_1$, the redistributional impact of "unexpected" inflation may offset the gains in efficiency. Whether this occurs depends on the distribution of debt. For $\eta_i > \bar{\eta}_i$ and ψ_i is sufficiently high, it is optimal for the government to reduce money growth between time 0 and time 1, causing P_0 to fall. The resulting rise in the real interest rate between time 0 and time 1 favors type i, who in this case is a net buyer of bonds on the time 0 asset market. Consequently, for $\eta_1 > \bar{\eta}_1$ the Ramsey equilibrium is time consistent for a sufficiently high level of ψ_1, even with government debt. For $\eta_2 > \bar{\eta}_2$, this will not be the case, but for high values of ψ_2 inflation is low and the labor tax high rate relative to expectations in the optimal deviation.

Figure 3.3 shows an example of a distribution of nominal assets for which the Ramsey equilibrium and the Markov equilibrium policy would coincide for η_1 large enough. Here, outstanding government debt equals 40% of the aggregate stock of currency at time 0 and $\psi_1 = 1$ while $\psi_2 = -1.59$. The vertical dashed line corresponds to $\bar{\eta}_1$. Values of η_1 greater than the vertical solid line correspond to economies where the Markov and Ramsey equilibrium would coincide.

I formalize this reasoning in the following proposition[9].

Proposition 3.4. *The Ramsey equilibrium is time consistent if the distribution of currency at time* 0 *is the same as the equilibrium distribution of currency for* $t > 0$ *i.e.* $\phi_m = \phi_1$, *and if:*

1. $\lambda_1 \psi_1 + \lambda_2 \phi_m \psi_2 = 0$ *and* $\sigma = 1$ *for* $\eta_1 < \bar{\eta}_1$,

   or

2. $\lambda_1 (1 + \psi_1) + \lambda_2 \phi_m (1 + \psi_2) = 0$ *and* $\lambda_1 + \phi_m \lambda_2 \geq 0$ *for* $\eta_1 \geq \bar{\eta}_1$.

Here, λ_i is the multiplier on the implementability constraint of type i in the Ramsey allocation problem. The proof is in Appendix B.

This proposition identifies the class of distributions of nominal assets for which the Ramsey equilibrium is time consistent. Given the stationarity properties of the environment studied here, this strategy is equivalent to the more general approach followed by Lucas and Stokey (1983) and Bassetto (1999)[10].

[9] This proposition holds under the assumption that outright default on government debt is not permitted, as in Lucas and Stokey (1983).

[10] The general procedure required to identify the sufficient conditions for time consistency is as follows. First, compute the Ramsey equilibrium at $t = 0$ with the distribution of currency and debt given by ϕ_m and $\{\psi_1, \psi_2\}$, respectively. Derive ϕ'_m and $\{\psi'_1, \psi'_2\}$, the distribution of currency and debt at the beginning of time 1 from the time 0 dynamic budget constraint, evaluated at the prices corresponding to the time 0 Ramsey equilibrium. Then, solve the Ramsey allocation problem at $\{\psi'_1, \psi'_2\}$, with the same Pareto weights, subject to the implementability constraints starting from $t = 1$. Finally, search for the value of $\{\psi_1, \psi_2\}$ such that the Ramsey equilibrium outcome at $\{\psi_1, \psi_2\}$ for $t > 0$ is the same as the Ramsey equilibrium at $\{\psi'_1, \psi'_2\}$ for $t \geq 1$.

Due to the constancy of $\bar{g}$ and of the marginal disutility of labor, which rules out dynamic interdependecies between government policy in subsequent periods, time consistency requires that the Ramsey equilibrium at $t = 1$ be constant and that $\lambda'_i = \lambda_i$, where λ'_i is the multiplier on type i's implementability constraint in the Ramsey equilibrium at $t = 1$. Therefore, the Ramsey equilibrium is time consistent if the allocation is constant for $t \geq 0$.

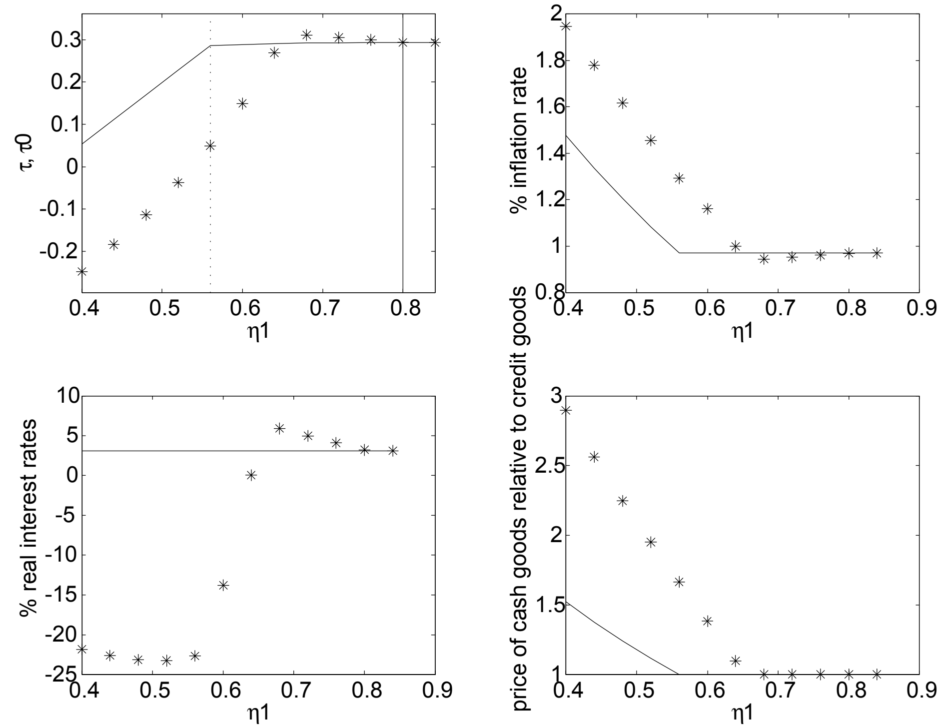

Figure 3.3: Example of a time consistent Ramsey equilibrium.

The first set of conditions are for the case in which type 1 households have a lower Pareto weight. To ensure time consistency there must be no gain in efficiency from taxing cash good consumption at a higher rate at time 0 i.e. labor income taxation is non-distortionary. In addition, the net redistributional gain of taxing nominal debt holdings via changes in the general price level must be 0. To see this note that the multiplier on the households' implementability constraints, λ_i,represents the shadow value to the government of transferring resources to type i. In equilibrium, λ_i is an increasing function of η_i. The second set of conditions is for the case in which the government wishes to redistribute to type 1 agents. In this case, since the cash in advance constraint is non binding in equilibrium the price elasticity of cash good consumption is the same in all periods, so the condition $\sigma = 1$ is not required to ensure time consistency. It is replaced by the second condition, which ensures that the net gain from taxing currency holdings is sufficiently small by imposing a lower bound on the shadow value of economywide liquidity, which guarantees that a unitary relative price of cash and credit goods. The first condition ensures that the net gain of taxing nominal asset holdings via changes in the general price level is 0.

3.3. Does Lack of Commitment Imply High Inflation?

Heterogeneity in holdings of nominal wealth and the demand for cash goods implies that monetary policy has redistributional effects and the time consistency of the Ramsey equilibrium depends on the balance between redistributional and efficiency incentives. This weakens the link between high inflation and time consistency. First, surprisingly high rates of inflation may be optimal even with commitment. Therefore, credibility of government policy does not imply low inflation. On the other hand, due to the redistributional effects of inflation, if the government is allowed to reassess policy, it will not necessarily select a rate of inflation higher than previously announced. If the distribution of nominal wealth and income are sufficiently uneven, potentially high levels of nominal government debt may be consistent with low inflation, even without commitment.

These findings contrast with the results for a representative agent economy, where -as shown in Lucas and Stokey (1983)- it is never possible to guarantee time consistency[11] in a monetary economy where outstanding government debt is positive and denominated in nominal terms. The resulting prediction is that economies in which nominal government debt is mostly held by the economic group having more political power are less subject to high inflation, independently of the level of nominal aggregate debt.

4. Empirical Evidence

The distribution of nominal wealth and the distribution of political power among classes of agents with different exposure to the effects of inflation played a crucial

[11] This is true is the government cannot commit to a pre-announced path of prices, which would make all government debt real, or if debt denominated in real terms is not available, as shown in Persson, Persson and Svensson (1987).

role in shaping monetary policy decisions in a number of historical episodes of large inflations and deflations.

Johnson (1970) provides a detailed description of the behavior of inflation in England in the aftermath of the Glorious Revolution:

> "When the Bank of England recieved its charter [1694] ... its directors cultivated all possible contact with parlamentarians, on whom they relied for periodic renewal of the charter."
>
> "Fierce dispute broke out as to what ... should be the remedy. To return to the good old standard would mean ... bankrupcy of many in trade and enrichment of old creditors. Devaluation would ... protect and stabilize domestic trade though initially hit the foreign trader. ... With landed property predominant in government the issue was never in doubt. The recoinage of 1897-1698 returned to Elisabeth's silver standard."
>
> The same political forces played a role in successive episodes of deflation in England, for example in 1815, as Johnson (1970) reports:
>
> "The unitary in monetary interest in the gold standard... included ...the owners of Consols sold to finance the war with Napoleon at a time of skyhigh prices and interest rates. ...In returning to gold, Lord Liverpool thus handed a large bonus to the landed gentry and to a new monied middle class".

Redistributional concerns can also account for the large monetization which occured in France after the Revolution in 1789. White (1896) reports the following:

> "Mirabeu..showed that he was fully aware of the dangers of inflation, but he yielded to the pressure... partly because he thought it important to sell government lands rapidly to the people, and so develop speedily a large class of landholders, pledged to stand by the government who gave them their titles."
>
> "This outgrowth [in money] was the creation of a great debtor class in the nation, directly interested in the depreciation of the currency in which their debts were to be payed. The nucleus of this debtor class was formed by those who had purchased the Church lands from the Government"

Sargent and Velde (1995) document that the downpayments required to purchase church lands were in the range $12 - 30\%$. The rest of the payment was arranged through promissory notes repayed annually over a period ranging between 10 and 12 years at 5% interest.

Hamilton (1788) also highlights the importance of redistributional concerns for the credibility of government debt policy:

"There are even dissimilar views ... as to the general principle of discharging the public debt. Some of them, either less impressed with the importance of national credit, or because they have little, if any, immediate interest in the question, feel an indifference, if not a repugnance, to the payment of the domestic debt at any rate. ... Others of them, a numerous body of whose citizens are creditors to the public beyond proportion ... in the total amount of the national debt, would be strenuous for some equitable and effective provision."

Based on this view, Hamilton (1795) argued in favor of the Federal assumption of the states' war debt. Debt assumption would provide powerful government creditors with a strong incentive to support the establishment of a Federal tax legislation, thus decreasing the risk of default or monetization.

More recently, Faust (1996) has argued that the Federal Reserve Bank's structure was a response to public conflict over inflation's redistributive effects. The Fed's internal power structure emerged in thirty years of legislation culminating in 1935 with the intent to balance voting interests and give fair representation of the financial, agricultural, industrial and commercial interests in the US.

5. Optimal Policy Without Commitment

In this section, I characterize optimal policy with no commitment for economies in which the Ramsey equilibrium policy is not time consistent. I also quantify the aggregate welfare loss and the redistributional consequences associated with the discretionary outcome.

I provide a general definition of a Markov equilibrium for this economy and illustrate its properties for the preference specification 3.6.

5.1. The Markov Equilibrium

I restrict attention to stationary Markov equilibria. Here, stationarity means that the government follows the same allocation rule in each period. The set up parallels Krusell, Quadrini, Rios-Rull (1997).

Let S be the state of the economy when the government chooses policy. The state can be made up of exogenous variable or endogenous predetermined variables. Let x be the set of choice variables for the government, and let $\hat{X}(S)$ the policy rule that the current government and the private sector expect future governments will follow. Denote by $\Gamma\left(S;\hat{X}\right)$ the set of feasible x's. Feasibility here involves resource constraints, competitive equilibrium restrictions and constraints on policy being satisfied. Let $\Psi(x,S;X)$ be the law of motion for the state, with:

$$S' = \Psi\left(x,S;\hat{X}\right).$$

The government solves the following problem:

$$\begin{aligned} W\left(S;\hat{X}\right) &= \max_{x\in\Gamma\left(S;\hat{X}\right)} \left\{w(x,S) + \beta W\left(S';\hat{X}\right)\right\} \quad (5.1) \\ \text{subject to } S' &= \Psi\left(x,S;\hat{X}\right). \end{aligned}$$

The solution to this problem is a policy rule $X\left(S;\hat{X}\right)$.

A Markov equilibrium is a policy rule $\mathcal{X}(S)$ such that:

$$\mathcal{X}(S) = X(S; \mathcal{X}).$$

Appendix C provides a general algorithm to solve for the Markov equilibrium in economies with a bidimensional state.

5.2. The Cash-Credit Good Economy

In this section, I define a Markov equilibrium for the cash-credit good economy in section 2. This requires identifying the variables S, x and the functions Γ, X, Ψ and $\mathcal{X}$.

The state in this economy is given by the distribution of currency and the distribution of nominal debt. I restrict attention to equilibria where:

$$\frac{M_2}{M_1} \equiv \phi_m = \frac{c_{21}}{c_{11}} \equiv \phi_1.$$

This ensures that the cash in advance constraint will hold with equality for both types and that $u_{11} = u_{21}$ both when the cash in advance constraint is binding and when it is non-binding. Since ϕ_1 does not depend on government policy it does not constitute part of the state. The state of the economy then reduces to:

$$S = \{\psi_1, \psi_2\},$$

where:

$$\psi_i = \frac{B_i}{M_i} \text{ for } i = 1, 2.$$

I restrict attention to $\psi_i \geq 0$ for $i = 1, 2$, since in this case the Ramsey equilibrium is not time consistent.

The current government selects $x = \left\{c_{11}, R, \psi_1', \psi_2'\right\}$, where

$$\begin{aligned} R &\equiv \frac{u_{i1}}{u_{i2}} \text{ for } i = 1, 2, \\ \psi_i' &= \frac{B_i'}{M_i'} \text{ for } i = 1, 2. \end{aligned}$$

A prime denotes the following period's variables. The resource constraint is still given by (2.4). Future governments are expected to follow the policy rule:

$$\hat{X}(\psi_1, \psi_2) \equiv \left\{\hat{c}_{11}, \hat{R}, \hat{\psi}_1', \hat{\psi}_2'\right\}(\psi_1, \psi_2).$$

The government problem is (5.1), where:

$$\begin{aligned} w(x) &= \eta_1 U^1\left(h^1(c_{11}, c_{12}), n_1\right) + \eta_2 U^2\left(h^2(c_{21}, c_{22}), n_2\right), \\ \Gamma\left(S; \hat{X}\right) &= \{x \text{ such that } c_{i1}, c_{i2} \geq 0, 1 - n_i, n_i \geq 0 \text{ for } i = 1, 2,\ R \geq 1,\ (2.4) \text{ holds}\}. \end{aligned}$$

and the law of motion for the state $\Psi\left(x, S; \hat{X}\right)$ is obtained from the household's dynamic budget constraints. Further details on the computation are reported in Appendix D.

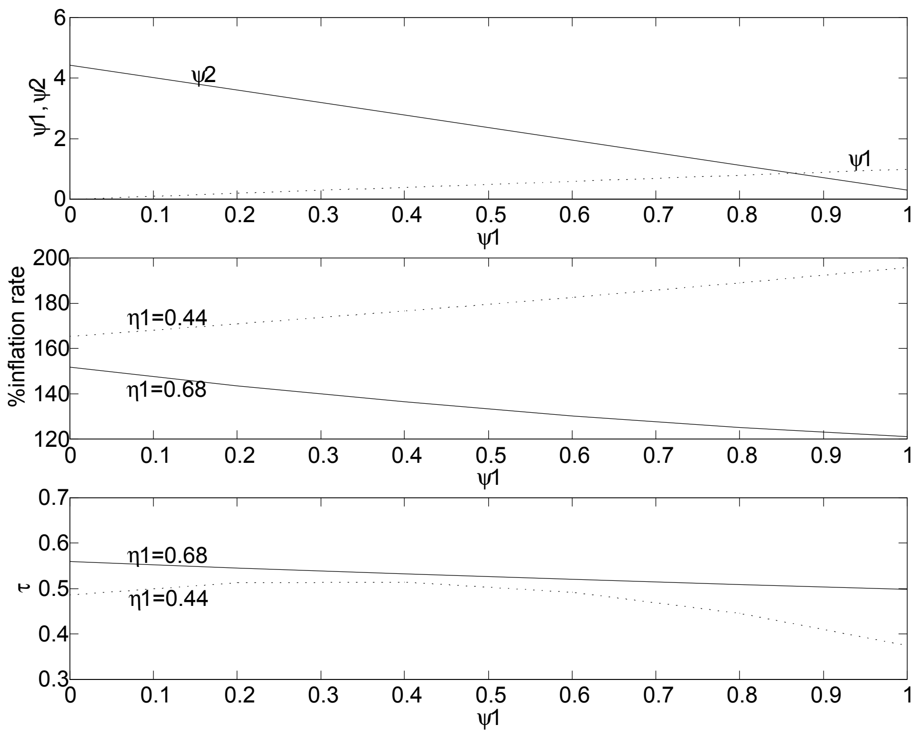

Figure 5.1: Markov equilibrium policy.

5.3. Findings

The equilibrium outcome depends on the distribution of nominal assets, which determines the redistributional impact of unexpected inflation. I compare the Markov and Ramsey equilibrium outcome for different distributions of nominal assets and for different values of the Pareto weight[12]. I find that the Markov equilibrium inflation rate is higher than in the Ramsey equilibrium. The difference decreases with η_1 and increases with government spending and the aggregate level of outstanding debt. As in the Ramsey equilibrium, the tax rate on labor increases with η_1 and $\bar{g}$.

The results for $\sigma = 0.9$, at $\bar{g} = 0.0715$ are displayed in figure 5.1. For $\eta_1 > \bar{\eta}_1$, the Markov equilibrium inflation rate increases with the fraction of nominal debt held by type 1 agents, while the converse is true for $\eta_1 < \bar{\eta}_1$. The tax rate on labor is lower for lower values of η_1 and decreases with ψ_1 for $\eta_1 > \bar{\eta}_1$, while for $\eta_1 < \bar{\eta}_1$ it increases with ψ_1 for low values of ψ_1 and decreases for higher values of ψ_1.

[12] In the Ramsey equilibrium, the initial distribution of debt affects the outcome for $t \geq 0$, while for the Markov equilibrium it only affects the outcome for two consecutive periods. This implies that any comparison of the Markov and Ramsey outcomes is conditional on the initial portfolio allocation in the Ramsey equilibrium.

I also evaluate the welfare costs of lack of commitment by measuring the percentage amount of leisure that an agent would give up per period to switch from the Markov to the Ramsey equilibrium. I compare welfare at the benchmark distribution of debt displayed in Table 1 for different values of σ and of η_1. Lack of commitment hurts type 1 agents more. Welfare costs for type 1 agents range from 7 to 10%, while for type 2 agents are between -5.5 and -1.2%. Aggregate welfare costs, measured as the percentage fall in the planners' utility in the Markov equilibrium relative to the Ramsey, range from 2.4% to 4.8%.

6. Conclusions

I describe a monetary economy in which households are heterogeneous in holdings of currency and other nominal assets. I study optimal monetary and fiscal policy and the sufficient conditions for time consistency. I find that heterogeneity substantially weakens the link between high inflation and government deficits and time consistency. The time consistency of the Ramsey equilibrium depends on the Pareto weights and the distribution of nominal assets, which determines the costs of unexpected inflation. If the distribution of nominal assets is uneven a policymaker acting under discretion will not necessarily select a rate of inflation higher than previously announced. If the Ramsey equilibrium is not time consistent, then the "bias" towards high inflation and large deficits due to lack of commitment is larger than in a representative agent economy, since in this case both efficiency and redistributional objectives reinforce the government's incentive to deviate.

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7. Appendix

7.1. A: Solving the Ramsey problem

The Lagrangian for the Ramsey problem is given by:

$$\max_{\{c_{i1t},n_{it},c_{i2t},m_{10}\}_{t\geq 0}} \sum_{t=0}^{\infty} \beta^t W_t \tag{7.1}$$
$$-\sum_{t=1}^{\infty} \beta^t \left[\sum_{i=1}^{2} \mu_{it} \left(u_{i2t} r_t - u_{i1t}\right) + \delta_t \left(1 - r_t\right)\right]$$
$$-\sum_{i=1,2} \left[\lambda_i \left(u_{i10} + \psi_i u_{i20}\right) m_{i0} + \mu_{i0} \left(u_{i20} - u_{i10}\right) + \chi_i \left(c_{i10} - m_{i0}\right)\right],$$

where

$$W_t = \sum_{i=1,2} \left[\eta_i U_1 \left(h^i \left(c_{i1t}, c_{i2t}\right), n_{it}\right) + \lambda_i \left(u_{i1t} c_{i1t} + u_{i2t} c_{i2t} + U^i_{2t} n_{it}\right)\right] \tag{7.2}$$
$$-\omega_t \left(\sum_{i=1,2} \nu_i \left(c_{i1t} + c_{i2t} - n_{1t}\right) + \bar{g}_t\right) - \zeta_t \left(\frac{u_{12t}}{U^1_{2t}} - \frac{u_{22t}}{U^2_{2t}}\right),$$

$m_{20} = \phi_m m_{10}$, and λ_i and ω_t are the multipliers on the implementability constraints and on the resource constraint for $i = 1, 2$ and $t \geq 0$, respectively. The variable μ_{it} denotes the multiplier for the equality of the relative price of cash and credit goods for the two types of households, while δ_t is the multiplier on the constraint that nominal interest rate is non-negative. The sign of δ_t is positive in equilibrium, while μ_{it} can be negative, though the sign will be independent of i for $t > 0$. The variable $\chi_i \geq 0$ is the multiplier on the cash in advance constraint at time 0, and ζ_t (which can be either positive or negative) is the multiplier on the constraint that the net real wage is the same across agents.

The first order necessary conditions for c_{i1}, c_{i2} and r in (7.1) for $t > 0$ are (I drop time subscripts to simplify notation):

$$0 = \left(\eta_i + \lambda_i\right) u_{i1} + \lambda_i \left[\sum_{j=1}^{2} \left(U^i_1 h^i_{1j} + U^i_{11} h^i_1 h^i_j\right) c_{ij} + U^i_{21} h^i_1 n_i\right] \tag{7.3}$$
$$-\mu_i \left[r h^i_{21} - h^i_{11}\right] - \tilde{\zeta} \frac{U^i_1 h^i_2}{U^i_2} \left[\frac{h^i_{21}}{h^i_2} + \frac{U^i_{11} h^i_1}{U^i_1} - \frac{U^i_{21} h^i_1}{U^i_2}\right] - \omega \nu_i,$$

$$0 = \left(\eta_i + \lambda_i\right) u_{i2} + \lambda_i \left[\sum_{j=1}^{2} \left(U^i_1 h^i_{2j} + U^i_{11} h^i_2 h^i_j\right) c_{ij} + U^i_{21} h^i_2 n_i\right] \tag{7.4}$$
$$-\mu_i \left[r h^i_{22} - h^i_{12}\right] - \tilde{\zeta}_i \frac{U^i_1 h^i_2}{U^i_2} \left[\frac{h^i_{22}}{h^i_2} + \frac{U^i_{11} h^i_2}{U^i_1} - \frac{U^i_{21} h^i_2}{U^i_2}\right] - \omega \nu_i,$$

$$\sum_{i=1}^{2} \mu_i h_2^i - \delta = 0, \tag{7.5}$$

$$\begin{aligned} \mu_i \left(r h_2^i - h_1^i \right) &= 0,\ h_1^i = r h_2^i, \\ \delta (1 - r) &= 0,\ \delta \geq 0,\ r \geq 1, \\ \zeta \left(\frac{u_{12}}{U_2^1} - \frac{u_{22}}{U_2^2} \right) &= 0,\ \frac{u_{12}}{U_2^1} = \frac{u_{22}}{U_2^2}, \end{aligned} \tag{7.6}$$

where i indexes agents and j indexes goods. In addition, for $i = 1, 2$:

$$\tilde{\zeta}_i = (-1)^{i-1} \zeta.$$

It's easy to show that (7.3) and (7.4) imply that $\zeta_t < 0$ (or $\tilde{\zeta}_2 > 0$ and $\tilde{\zeta}_1 < 0$) for $\eta_2 > \bar{\eta}_2$ and $\zeta_t > 0$ (or $\tilde{\zeta}_1 > 0$ and $\tilde{\zeta}_2 < 0$) for $\eta_1 > \bar{\eta}_1$[13].

In addition:

$$\frac{h_1^i}{h_2^i} = \max \{ r, 1 \},$$

where

$$r = \frac{\frac{\eta_i + \lambda_i}{\lambda_i} + \left[\sum_{j=1}^{2} \left(U_1^i h_{2j}^i + U_{11}^i h_2^i h_j^i \right) \frac{c_{ij}}{U_1^i h_2^i} + \frac{U_{21}^i}{U_1^i} n_i \right] - \frac{\mu_i \left[r h_{22}^i - h_{12}^i \right]}{\lambda_i U_1^i h_2^i} - \frac{\tilde{\zeta}_i}{\lambda_i} \frac{U_1^i h_2^i}{U_2^i} \left[\frac{h_{22}^i}{h_2^i} + \frac{U_{11}^i h_2^i}{U_1^i} - \frac{U_{21}^i h_2^i}{U_2^i} \right]}{\frac{\eta_i + \lambda_i}{\lambda_i} + \left[\sum_{j=1}^{2} \left(U_1^i h_{1j}^i + U_{11}^i h_1^i h_j^i \right) \frac{c_{ij}}{U_1^i h_1^i} + \frac{U_{21}^i}{U_1^i} n_i \right] - \frac{\mu_i \left[r h_{21}^i - h_{11}^i \right]}{\lambda_i U_1^i h_1^i} - \frac{\tilde{\zeta}_i}{\lambda_i} \frac{U_1^i h_2^i}{U_2^i} \left[\frac{h_{21}^i}{h_2^i} + \frac{U_{11}^i h_1^i}{U_1^i} - \frac{U_{21}^i h_1^i}{U_2^i} \right]}$$

Under assumption (3.6), $\mu_1 = 0$ and $\delta = \mu_2 h_2^2$, which implies $\mu_2 \geq 0$.

Proof of Proposition 3.2 If taxes are agent specific, the net real wage need not be equalized across agents in a competitive equilibrium. The first order conditions for the Ramsey problem are the same as for (7.2) with $\zeta_t \equiv 0$ for $t \geq 0$, since the constraint drops out of the problem. By homotheticity:

$$\sum_{j=1}^{2} \left(U_1^i h_{2j}^i + U_{11}^i h_2^i h_j^i \right) \frac{c_{ij}}{U_1^i h_2^i} = \sum_{j=1}^{2} \left(U_1^i h_{1j}^i + U_{11}^i h_1^i h_j^i \right) \frac{c_{ij}}{U_1^i h_1^i} \text{ for } i = 1, 2. \tag{7.7}$$

[13] This can be seen by making three observations. First:

$$\frac{u_{12}}{U_2^i} = \frac{-1}{w_i},$$

where w_i is the net of tax real wage of agent i. Therefore, the expression $\frac{u_{12}}{U_2^i}$ is increasing in the real wage for $i = 1, 2$. Second, u_{i2}/U_2^i is increasing in η_i in an equilibrium in which (3.3) is not imposed and $\bar{\eta}_1$ is defined as the value of the Pareto weight for which (3.3) is non-binding. Lastly, constraint (3.3) can be thought of as a couple of inequality constraints:

$$\begin{aligned} \frac{u_{12}}{U_2^1} &\leq \frac{u_{22}}{U_2^2}, \\ \frac{u_{22}}{U_2^2} &\leq \frac{u_{12}}{U_2^1}, \end{aligned}$$

where $\tilde{\zeta}_1$ and $\tilde{\zeta}_2$ are the respective multipliers. Intuitively, $\tilde{\zeta}_i > 0$ when the government wishes to redistribute to type i.

Then, $r = 1$ if and only if:

$$-\frac{\mu_i}{U_1^i h_2^i}\left[rh_{22}^i - h_{12}^i\right] \leq -\frac{\mu_i}{U_1^i h_1^i}\left[rh_{21}^i - h_{11}^i\right], \tag{7.8}$$

for $i = 2$. By (7.6), $rh_{22}^i = h_{12}^i$ and $rh_{21}^i = h_{11}^i$ in any equilibrium, which implies $r = 1$.■

For the case $h_{12}^2 \geq h_{11}^2$ [14] and $U_{12}^2 \leq 0$, it is also possible to show that $\eta_1 < \bar{\eta}_1$ is required for nominal interest rates to be positive in a Ramsey equilibrium.

Proof of Proposition 3.3 To prove necessity, suppose to the contrary that $Q_t < 1$ and $\eta_1 \geq \bar{\eta}_1$. Then, by (7.3) and (7.4), so that $r > 1$ implies:

$$-\tilde{\zeta}_2\frac{U_1^i h_2^i}{U_2^i}\left[\frac{h_{22}^i}{h_2^i} + \frac{U_{11}^i h_2^i}{U_1^i} - \frac{U_{21}^i h_2^i}{U_2^i}\right] \geq -\tilde{\zeta}_2\frac{U_1^i h_2^i}{U_2^i}\left[\frac{h_{21}^i}{h_2^i} + \frac{U_{11}^i h_1^i}{U_1^i} - \frac{U_{21}^i h_1^i}{U_2^i}\right],$$

for $i = 2$.

Since $\eta_1 \geq \bar{\eta}_1$ implies $\zeta > 0$ and $-\frac{U_1^i h_2^i}{U_2^i} > 0$, this expression simplifies to:

$$\zeta\left[\frac{h_{22}^i}{h_2^i} + \frac{U_{11}^i h_2^i}{U_1^i} - \frac{U_{21}^i h_2^i}{U_2^i}\right] \leq \zeta\left[\frac{h_{21}^i}{h_2^i} + \frac{U_{11}^i h_1^i}{U_1^i} - \frac{U_{21}^i h_1^i}{U_2^i}\right],$$

and further to:

$$\frac{h_{22}^i}{h_2^i} - \frac{h_{21}^i}{h_2^i} \leq (h_1^i - h_2^i)\left(\frac{U_{11}^i}{U_1^i} - \frac{U_{21}^i}{U_2^i}\right).$$

By assumption $h_{12}^2 \leq h_{22}^2$, the LHS of this expression in positive. However, from the concavity of U^2 and $U_{12}^2 \leq 0$, the RHS of this expression is negative, which generates a contradiction.

To prove sufficiency, assume $\eta_2 > \bar{\eta}_2$. By (7.3) and (7.4) $\zeta < 0$ and:

$$\left[\frac{h_{22}^i}{h_2^i} + \frac{U_{11}^i h_2^i}{U_1^i} - \frac{U_{21}^i h_2^i}{U_2^i}\right] \geq \left[\frac{h_{21}^i}{h_2^i} + \frac{U_{11}^i h_1^i}{U_1^i} - \frac{U_{21}^i h_1^i}{U_2^i}\right].$$

This simplifies to:

$$\frac{h_{22}^i}{h_2^i} - \frac{h_{21}^i}{h_2^i} \geq (h_1^i - h_2^i)\left(\frac{U_{11}^i}{U_1^i} - \frac{U_{21}^i}{U_2^i}\right),$$

which is consistent with $r > 1$ since the LHS is positive and the second expression in brackets on the RHS is negative.■

[14] This holds if h is separable in c_{21}, c_{22} or, if it is a CES aggregator, when the elasticity of substitution is less than 1. This implies that c_{21} and c_{22} are NOT gross substitutes.

7.2. B: Proof of Proposition 3.4

For this particular utility specification, it is more convenient to set up the Ramsey problem in the following form:

$$\max_X \sum_{t=0}^{\infty} \beta^t W_t$$
$$-\sum_{i=1,2} \left[\lambda_i \left(u_{i10} + \psi_i u_{i20}\right) m_{i0} + \mu_{i0} \left(u_{i20} - u_{i10}\right) + \chi_i \left(c_{i10} - m_{i0}\right)\right],$$

with

$$\begin{aligned} W_t &= \sum_{i=1,2} \left[\eta_i U_1 \left(h^i \left(c_{i1t}, c_{i2t}\right), n_{it}\right) + \lambda_i \left(u_{i1t} c_{i1t} + u_{i2t} c_{i2t} + U_{2t}^i n_{it}\right)\right] \\ &\quad -\omega_t \left(\sum_{i=1,2} \nu_i \left(c_{i1t} + c_{i2t} - n_{1t}\right) + \bar{g}_t\right) - \sum_{i=1}^{2} \mu_{it} \left(u_{i2t} - u_{i1t}\right), \end{aligned}$$

and impose the restrictions implied by (3.7) and (3.8), namely[15]:

$$\frac{c_{1j,t}}{c_{2j,t}} \equiv \phi_j, \text{ for } t > 0,$$

directly into the problem. This implies that:

$$X = \left[\left\{c_{11t}, c_{12t}, n_{1t}, n_{2t}\right\}_{t \geq 0}, c_{210}, m_{10}\right].$$

The value of ϕ_1 also corresponds to the ratio of currency held by type 2 to currency held by type 1 for $t > 0$.

The first order necessary conditions for $t > 0$ are given by:

$$\left(\eta_1 + \eta_2 \phi_1\right) u_{11,t} + \left(\lambda_1 + \lambda_2 \phi_1\right)(1-\sigma) u_{11,t} - \sigma \left(\mu_{1t} + \phi_1 \mu_{2t}\right) \frac{u_{11,t}}{c_{11,t}} - \omega_t \left(\nu_1 + \nu_2 \phi_1\right) = 0, \tag{7.9}$$

$$\left(\eta_1 + \eta_2 \phi_2\right) u_{12,t} + \left(\lambda_1 + \lambda_2 \phi_2\right)(1-\sigma) u_{12,t} + \sigma \left(\mu_{1t} + \phi_2 \mu_{2t}\right) \frac{u_{12,t}}{c_{12,t}} - \omega_t \left(\nu_1 + \nu_2 \phi_2\right) = 0, \tag{7.10}$$

$$-\gamma \left(\eta_i + \lambda_i\right) + \omega_t \nu_i = 0, \tag{7.11}$$

$$\mu_{2t} \left(u_{12,t} - u_{11,t}\right) = 0, \;\; \mu_{2t} \geq 0, \tag{7.12}$$

where here μ_t is the multiplier on the constraint that the nominal interest rate be non-negative.

---

[15] Equivalently:

$$u_{12t} = u_{22t} \text{ for } t \geq 0,$$
$$u_{11t} = u_{21t} \text{ for } t > 0.$$

The first order conditions for $t = 0$ are:

$$\left[\eta_i + \lambda_i (1-\sigma) - \frac{\sigma \mu_{i0}}{c_{i10}}\right] u_{i10} + \sigma u_{i10} \frac{m_{i0}}{c_{i10}} \lambda_i - \chi_i - \nu_i \omega = 0, \tag{7.13}$$

for $i = 1, 2$, and

$$\begin{aligned} 0 \quad = \quad & \left[(\eta_1 + \eta_2 \phi_2) + (\lambda_1 + \lambda_2 \phi_2)(1-\sigma) + \frac{\sigma (\mu_{10} + \phi_2 \mu_{20})}{c_{12,0}}\right] u_{12,0} \quad (7.14) \\ & + \frac{\sigma u_{12,0} m_{10}}{c_{12,0}} [\lambda_1 \psi_1 + \lambda_2 \phi_m \psi_2] - \omega (\nu_1 + \nu_2 \phi_2), \end{aligned}$$

$$-\lambda_1 [u_{11,0} + u_{22,0} \psi_1] - \lambda_2 \phi_m [u_{21,0} + u_{22,0} \psi_2] + \chi_1 + \chi_2 \phi_m = 0, \tag{7.15}$$

$$\mu_{i0} (u_{22,0} - u_{i1,0}) = 0, \ \mu_{i0} \geq 0, \ u_{i1,0} - u_{22,0} \geq 0, \tag{7.16}$$

$$\chi_i (c_{i1,0} - m_{i0}) = 0, \ \chi_i \geq 0, \ m_{i0} - c_{i10} \geq 0. \tag{7.17}$$

To ensure that the allocation at time 0 and at time 1 coincide the following two equations have to hold:

$$\left[\frac{\sigma (\mu_1 + \mu_2 \phi_1)}{c_{11}} - \frac{\sigma (\mu_{10} + \mu_{20} \phi_1)}{c_{11}}\right] u_{11} + \frac{\sigma u_{11} m_{10}}{c_{11}} (\lambda_1 + \lambda_2 \phi_1) - \chi_1 - \chi_2 \phi_1 = 0, \tag{7.18}$$

$$-\left[\frac{\sigma (\mu_1 + \mu_2 \phi_1)}{c_{12}} - \frac{\sigma (\mu_{10} + \mu_{20} \phi_1)}{c_{12}}\right] u_{22} + \frac{\sigma u_{12} m_{10}}{c_{12}} (\lambda_1 \psi_1 + \lambda_2 \phi_m \psi_2) = 0, \tag{7.19}$$

and $\phi_m = \phi_1$.

If $u_{21} > u_{22}$, which occurs if $\eta_1 < \bar{\eta}_1$, the cash in advance constraint should be binding at time 0 to ensure time consistency. A binding cash in advance constraint at time 0 implies $\chi_i > 0$ and $\mu, \mu_{i0} = 0$. Therefore, from (7.19):

$$\begin{aligned} -(\lambda_1 \psi_1 + \lambda_2 \phi_m \psi_2) &= 0, \\ \lambda_1 + \lambda_2 \phi_1 &\geq 0. \end{aligned}$$

Combining (7.18) and (7.15) and using $u_{210} = u_{110}$ yields:

$$-u_{11} (\lambda_1 + \lambda_2 \phi_1)(1-\sigma) = 0.$$

This condition can only be satisfied for $\sigma = 1$, since $\lambda_1 + \lambda_2 \phi_1 = 0$ implies $\chi_i = 0$ which contradicts the assumption that the cash in advance constraint is binding.

If $u_{21} = u_{22}$, which occurs when $\eta_1 \geq \bar{\eta}_1$, the cash in advance constraint has to be non-binding at time 0. This implies $\chi_i = 0$ and by (7.15):

$$(\lambda_1 + \lambda_2 \phi_m) + (\lambda_1 \psi_1 + \lambda_2 \phi_m \psi_2) = 0.$$

This restriction, together with (7.18), (7.19), and $\phi_m = \phi_1$, makes (7.19) redundant. The expression in (7.18) reduces to:

$$(\mu_1 + \mu_2 \phi_1) - (\mu_{10} + \mu_{20} \phi_1) + m_{10} (\lambda_1 + \lambda_2 \phi_1) = 0.$$

$m_{10} \geq c_{11,0}$ and $m_1 = c_{11}$ for $t > 0$ imply $(\mu_1 + \mu_2 \phi_1) - (\mu_{10} + \mu_{20} \phi_1) \leq 0$, given that by assumption. It follow that:

$$\lambda_1 + \lambda_2 \phi_1 \geq 0.$$

7.3. C: Computing the Markov equilibrium

Consider an economy with a bidimensional state, given by $S = [s_1, s_2]$. The Markov equilibrium policy can then be obtained as follows:

1. Select by trial and error a suitable bidimensional interval $\{[\underline{s}_1, \bar{s}_1]\,[\underline{s}_2, \bar{s}_2]\}$ and select the grid points as the roots of the $n + 1st$ order Chebyshev polynomial, where n is the order of the polynomial interpolation.

2. Provide an initial guess for the rule $\hat{X}$ at each of the n^2 grid points.

3. Use this initial guess to approximate $W\left(S; \hat{X}\right)$. Approximate the policy functions based on the initial guess and use Chebyshev interpolation outside the grid points. Substitute the policies into the one period utility function, giving rise to the value function. Accelerate the algorithm by assuming that the conjectured policy function is the one corresponding to the Markov equilibrium and will be followed henceforth.

4. Given the expected policy and the corresponding value function, solve the government's maximization problem at each $[s_1, s_2]$ on the bidimensional grid defined at step 1, obtain a new government policy function on the grid points.

5. Iterate on steps 3 to 5 until convergence.

7.4. D: Markov Equilibrium for the Cash-Credit Good Economy

Here, I show that all components of the competitive equilibrium allocation, prices and policy can be identified from the restrictions imposed in section 5.2, based on knowledge of the choice variables and of the conjectured policy rule. The restriction $u_{11} = u_{21}$ implies $c_{21}/c_{11} = \phi_m$ and the cash in advance constraint determines the price level. From R and c_{11} c_{i2} can be computed according to:

$$c_{i2} = \left(\frac{z_i R}{1 - z_i}\right)^{1/\sigma} c_{i1}.$$

Since $u_{12} = u_{22}$, the tax rate on labor is then given by:

$$\tau = 1 - \frac{\gamma}{u_{22}}.$$

Equilibrium labor supply can be determined from the dynamic budget constraint. To do this we first rewrite the wealth evolution equations in terms of the choice variables and the expected policy rule:

$$\left(1 + \frac{\psi_i'}{R'}\right) u_{i1}' c_{i1}' = u_{i2}\psi_i c_{i1} + \gamma n_i - u_{i2} c_{i2}, \tag{7.20}$$

where:

$$
\begin{aligned}
c'_{11} &= \hat{c}_{11}\left(\psi'_1, \psi'_2\right), \\
u'_{i1} &= (1 - z_i)\left(c'_{i1}\right)^{-\sigma}, \\
u'_{11}\xi_1 &= u'_{21}\xi_2.
\end{aligned}
$$