Labor-Supply Shifts and Economic Fluctuations*

Yongsung Chang
Department of Economics
University of Pennsylvania†

Frank Schorfheide
Department of Economics
University of Pennsylvania‡

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†McNeil Building, 3718 Locust Walk, Philadelphia, PA 19104-6297. Tel.: (215) 898-6691. Email: *yohg@ssc.upenn.edu*.

‡Tel.: (215) 898-8486. Email: *schorf@ssc.upenn.edu*.

Abstract

We investigate the role of labor-supply shifts in economic fluctuations. A new VAR identification scheme for labor supply shocks is proposed. Our method provides an alternative identification scheme, which does not rely on "zero-restrictions". According to our VAR analysis of post-war U.S. data, labor-supply shifts account for about half the variation in hours and less than one-fifth of variation in output. To assess the role of labor-supply shifts in a more structural framework, estimates from a dynamic stochastic general equilibrium model with stochastic variation in home production technology are compared to those from the VAR.

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1 Introduction

A leading question in macroeconomics is the identification of forces that cause the cyclical allocation of time. Modern dynamic stochastic general equilibrium analysis emphasizes random shifts in labor demand due to technological progress. Empirical studies on the decomposition of working hours, e.g., Shapiro and Watson (1988) and Hall (1997), have called for an attention to labor-supply movements. For example, Hall (1997) finds a predominant role of labor-supply shifts for fluctuations in hours worked. He suggests non-market activities such as job-search or home production as possible causes for labor-supply shifts.

This paper examines the importance of labor-supply shifts as a source of economic fluctuations. First, we develop and apply a new identification procedure for vector autoregressions (VAR) to decompose the fluctuation of aggregate hours and output into movements along the short-run labor demand schedule and shifts of the demand curve itself. The former is interpreted broadly as response to a labor supply shock. Our identifying restrictions are based on the notion that in reaction to a temporary labor supply shock hours will rise and labor productivity will fall, as the production capacity is fixed in the short-run and the economy operates along the decreasing marginal-product-of-labor schedule. Second, we impose additional restrictions by estimating a fully-specified dynamic stochastic general equilibrium (DSGE) model. The DSGE model potentially yields a more precise estimate of the relative importance of labor supply shifts. We consider a model in which labor-supply shifts are caused by changes in home production activities. This model was developed by Benhabib, Rogerson, and Wright (1991) and Greenwood and Hercowitz (1991).[1]

The main empirical findings can be summarized as follows. According to the

[1] The Beckerian home production models are motivated by the fact that, in any economy, agents spend a significant amount of time on non-market activities. For example, according to the Michigan Time-Use Survey, a typical married couple in the U.S. allocates about 25 percent of its discretionary time to home production activities, while the couple spends about 33 percent of its time for paid compensation (see Hill (1984), or Juster and Stafford (1991)).

VAR variance decomposition, labor-supply shocks play an important role as a source of fluctuations of hours. Temporary shifts in labor supply account for about half of the cyclical variation of working hours. This finding is consistent with the results reported in Shapiro and Watson (1988) and Hall (1997). Labor-supply shocks are less important for output fluctuations as they explain not more than 15 percent of the variation in output. The DSGE model analysis yields similar results. While more than 50 percent of the variation of hours is attributed to temporary labor supply shifts, only 13 percent of the output fluctuations are due to labor supply movements.

The DSGE model also provides estimates of the evolution of market and home technology over time. The latter measures the attractiveness of non-market activities. According to the home production model, recessions may occur because agents find it optimal to spend more time in non-market activities. While there are alternative explanations for recessions that are not captured by the simple DSGE model, we find it interesting to compare the estimates of the latent technologies to the NBER business cycle dates. Taken at face value, two out of six business cycle troughs during the period from 1960:I to 1997:IV, namely March 1975 and November 1982, coincide with unusually high productivity of non-market activity.

The proposed VAR identification procedure differs from previous approaches. Shapiro and Watson (1988) assume that both hours and aggregate output are non-stationary. Their identification is based on a long-run restriction: labor-supply shocks have a permanent effect on both hours and output, whereas technology shocks only affect output in the long-run. However, the evidence on the non-stationarity of hours is inconclusive. A researcher who believes that hours follow a stationary process will find the data consistent with his belief. Vice versa, there is not much evidence in hours data that would contradict that hours are (locally) nonstationary. Our investigation treats hours as a stationary process. This assumption is consistent with a large class of theoretical DSGE models, including the one presented in this paper, in which stochastic growth is induced by a nonstationary labor augmenting technology process and the economy evolves along a balanced growth path.

Hall (1997) identifies the labor-supply or preference shocks by deriving short-run labor supply and demand functions based on assumptions on consumer preferences and the firms' production technology. He expresses the equilibrium hours as a function of the labor-supply shock and several observable variables contained in the first-order condition of utility maximization of households. Based on the labor-market equilibrium the labor supply shocks are calculated as residuals from the first-order conditions of household labor supply decision.[2] Similar to Hall's analysis, we also exploit the short-run labor market equilibrium to identify the supply shocks. However, our VAR identification scheme does not rely on a specific form of households' preferences.

Unlike many VAR identification schemes that have been used in the literature, our scheme cannot be implemented solely based on zero-restrictions on the long-run multiplier matrix and the contemporaneous impact matrix of the structural shocks. The structural disturbances are recovered conditional on the slope of the short-run labor demand schedule. We place a prior distribution on this slope and on the reduced form VAR parameters and conduct Bayesian inference. Since the distribution of the reduced form parameters is updated through the sample information, the implied distributions of variance decompositions and impulse response functions are updated with every observation. The VAR identification scheme is consistent with the DSGE model. Based on a long sample of observations generated from the log-linearized DSGE model the structural VAR can approximately recover the exogenous shocks that were used to generate the artificial data.

The paper is organized as follows. Section 2 develops the VAR identification scheme. The home production model is presented in Section 3. Section 4 discusses the econometric estimation and inference. Our VAR approach is compared to recently proposed identification schemes that are based on inequality restrictions for the impulse response functions. The empirical findings are summarized in Section 5, while the last section concludes. Data definitions and computational details are col-

[2]The same strategy to identify preference shocks is used in Hall (1986), Parkin (1988) and Baxter and King (1991).

lected in the Appendix.

2 A VAR Model of Labor Market Fluctuations

In this paper labor market fluctuations are viewed as a series of equilibria generated by competitive households and firms whose tastes and technologies are perturbed by stochastic disturbances. To identify sources of labor market fluctuations we will fit a VAR and a DSGE model to three macroeconomic time series: Hours worked in the market $L_{m,t}$, labor productivity P_t, and expenditures on consumer durables $I_{h,t}$. As we discuss in detail below, we also identify permanent shocks that shift both labor demand and supply. Expenditures on consumer durables serve as a proxy for the households' permanent income. In the context of the DSGE model these expenditures are interpreted as investments in the home capital stock.[3] The remainder of this section describes our identification scheme for the VAR. The VAR innovations are decomposed into three orthogonal shocks, denoted by $\epsilon_{a,t}$, $\epsilon_{b,t}$, and $\epsilon_{z,t}$.

2.1 Identifying Assumptions

During the past four decades, labor productivity, spending on consumer durables, and aggregate output exhibited a pronounced trend, whereas aggregate hours and the consumption share did not show an apparent trend. Based on this observation, many dynamic macroeconomic models have been designed to evolve along a balanced growth path. A common stochastic trend in output, consumption, investment, capital, and wages is typically induced by a labor augmenting technology process. Hours worked, however, are stationary on this path as both labor demand and supply – via wealth effect in a conventional utility and via accumulation of consumer durable goods in home production models – shift in the long-run.

[3]Hours worked are denoted by $L_{m,t}$ because the DSGE model introduced in Section 3 distinguishes between market (subscript m) and home (subscript h) production. Expenditures on consumer durables will be interpreted as home investment.

Assumption 1 *The shocks $\epsilon_{a,t}$ and $\epsilon_{b,t}$ have transitory effects on hours, labor productivity, and consumer durables. The shock $\epsilon_{z,t}$ has a permanent effect on labor productivity and consumer durables. It has no effect on hours in the long run.* □

The shock $\epsilon_{z,t}$ induces a common stochastic trend in productivity and consumer durables. It will subsequently be interpreted as permanent technology shock.

We characterize the labor market equilibrium in terms of demand and supply curves. At time t the inverse labor demand of a competitive profit maximizing firm can be written in terms of its capital stock $K_{m,t}$ and the state of market technology S_t:

$$W_t = MPL_t = \varphi^D(L_{m,t}|K_{m,t}, S_t), \tag{1}$$

where W_t represents the real wage rate, MPL_t the marginal product of labor, and $L_{m,t}$ hours employed at time t. Similarly, the inverse labor supply of the representative household can be written in generic form as

$$W_t = \varphi^S(L_{m,t}|\Omega(S_t, T_t)). \tag{2}$$

Ω_t represents endogenous variables that influence the labor supply of the household, such as the real interest rate, consumption, wealth and the preference of households. T_t represents state variables that reflect the taste of households or the productivity of non-market activities.

Assumption 2 *The shock $\epsilon_{b,t}$ has only a contemporaneous impact on T_t, but not on S_t and $K_{m,t}$. Thus, upon impact the shock shifts the labor supply curve, but not the labor demand curve (marginal-product-of-labor schedule).* □

We will interpret the shock $\epsilon_{b,t}$ broadly as a labor supply shock, such as an unanticipated shift of the preference for leisure or the productivity of non-market activities. The capital stock $K_{m,t}$ is inherited from the previous period and therefore not affected by current period shocks. Although the production capacity is fixed in the short-run, labor demand may shift due to varying utilization of capital.

However, we show in Appendix A that Assumption 2 is still valid provided the cost of utilization is reflected in the depreciation rate of capital.

The responses of the marginal product of labor and hours worked (both in logs) to a labor supply shock $\epsilon_{b,t}$ have to satisfy the following relationship

$$\frac{\partial \ln MPL_t}{\partial \epsilon_{b,t}} = \left(\frac{1}{\varphi^D} \cdot \frac{\partial \varphi^D}{\partial \ln L_{m,t}} \right) \cdot \frac{\partial \ln L_{m,t}}{\partial \epsilon_{b,t}} \tag{3}$$

where the factor in parentheses is the slope of the inverse labor demand function. Under a Cobb-Douglas production technology with labor share parameter α one obtains

$$\frac{\partial \ln P_t}{\partial \epsilon_{b,t}} = (\alpha - 1) \frac{\partial \ln L_{m,t}}{\partial \epsilon_{b,t}}. \tag{4}$$

Roughly speaking, conditional on the slope of the labor demand function, it is possible to identify $\epsilon_{b,t}$ through its joint effect on hours and productivity.[4]

2.2 VAR Specification

Define the vector of stationary variables $\Delta y_t = [\Delta \ln P_t, \Delta \ln I_{h,t}, \ln L_{m,t}]'$. Moreover, let $\epsilon_t = [\epsilon_{z,t}, \epsilon_{a,t}, \epsilon_{b,t}]'$. The VAR can be expressed in vector error correction form as

$$\Delta y_t = \Phi_0 + \Phi_{vec} y_{t-1} + \sum_{i=1}^{p-1} \Phi_i \Delta y_{t-i} + u_t, \quad u_t \sim iid\ \mathcal{N}(0, \Sigma_u). \tag{5}$$

The reduced form disturbances u_t are related to the structural disturbances ϵ_t by $u_t = \Phi_* \tilde{\epsilon}_t$, where $\tilde{\epsilon}_t$ is a standardized version of ϵ_t with unit variance.

According to Assumption 1, the shock $\epsilon_{z,t}$ generates a stochastic trend in productivity and expenditures on consumer durables. The two series are cointegrated with cointegration vector $\lambda = [1, -\lambda_{21}, 0]'$. Instead of restricting λ_{21} to one we

[4]Our analysis does not consider other disturbances such as monetary and fiscal policy shocks. For post-war U.S. data, government policy shocks are often considered to be of secondary importance in business-cycle analysis. For example, according to King, Plosser, Stock and Watson (1991), permanent nominal shocks identified by imposing long-run neutrality explain little of the variability in real variables. The cyclical components of government spending is not highly correlated with output measures – it is less than 0.2 for Hodrick-Prescott filtered data. Also, expanding the list of shocks often invites arbitrary identifying restrictions in the VAR analysis.

estimate the parameter in the VAR analysis to allow for a possibly steeper Engle curve for expenditures on durable goods. We do not impose a cointegration relationship between the cumulative hours-worked process $\sum_{\tau=0}^{t} L_{m,t}$ and productivity or consumer durables. Hence, the rank of $\Phi_{vec} = \mu\lambda'$ is chosen to be one. This rank order was confirmed by a formal selection based on Bayesian posterior odds. The stochastic trend of y_t has the form $C_{LR}\Phi_* \sum_{\tau=0}^{t} \tilde{\epsilon}_t$. Since productivity and consumption expenditures have a common trend, the first two rows of the 3×3 long-run multiplier matrix C_{LR} are proportional.

The structural shocks $\tilde{\epsilon}_t$ are identifiable, if the elements of the 3×3 matrix Φ_* can be uniquely determined based on $\Phi_0, \ldots, \Phi_p$, Φ_{vec}, and Σ_u. Let Ψ_* denote the unique lower triangular Cholesky factor of Σ_u. Any matrix Φ_* such that $\Phi_*\Phi_*' = \Sigma_u$ is an orthonormal transformation of Ψ_*, that is, $\Phi_* = \Psi_* B$ for some orthonormal matrix B. Let $[A]_{ij}$ denote the i'th row and j'th column of a matrix A. According to Assumption 1, the shocks $\epsilon_{a,t}$ and $\epsilon_{b,t}$ only have transitory effects on productivity and consumer expenditures. Thus, the elements $[(C_{LR}\Psi_*)B]_{12}$ and $[(C_{LR}\Psi_*)B]_{13}$ have to be zero. The contemporary effects of the labor supply shock $\epsilon_{b,t}$ on productivity and hours worked are given by $\partial P_t/\partial\epsilon_{b,t} = [\Phi_*]_{13}$ and $\partial L_{m,t}/\partial\epsilon_{b,t} = [\Phi_*]_{33}$. Define $C_* = [1, 0, -(\alpha - 1)]$. According to Assumption 2 and Equation (4) the value of $[(C_*\Psi_*)B]_{13}$ has to be zero. These three orthogonality conditions uniquely determine the orthonormal transformation B.

Conditional on the slope of the labor demand schedule, it is possible to uniquely determine the structural shocks $\tilde{\epsilon}_t$.[5] The slope of the labor demand schedule itself, however, is not identifiable. Our approach to econometric inference in the presence of a nonidentifiable parameter is discussed in Section 4.

3 A Fully Specified Model Economy

The DSGE model presented subsequently provides a more specific interpretation of the three structural shocks and their propagation. It also assists the understand-

[5] The parameter α can be interpreted as an index for a set of identification schemes.

ing of the economic intuition behind our VAR identification scheme in Section 2. The DSGE model imposes further restrictions, in addition to the ones used in the VAR analysis, to identify the vector of structural shocks $\tilde{\epsilon}_t$. If these restrictions are well specified, the DSGE model will yield precise estimates of variance decompositions and impulse response functions. On the other hand, if the restrictions are inadequate, they could lead to misspecification bias.

The model economy consists of identical infinitely lived households who maximize the expected discounted lifetime utility U defined over consumption C_t and pure leisure $1 - L_{m,t} - L_{h,t}$ where $L_{m,t}$ is the fraction of time supplied to the labor market and $L_{h,t}$ is the fraction of hours spent on home production activities (e.g., lawn-mowing, dish-washing, or cooking).

$$U = E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} (\log C_s + \kappa \log(1 - L_{m,s} - L_{h,s})) \right] \tag{6}$$

E_t is the expectation operator conditional on information available at time t and β is the discount factor. Consumption is an aggregate of market consumption $C_{m,t}$ and the consumption of home produced goods $C_{h,t}$:

$$C(C_{m,t}, C_{h,t}) = [\chi C_{m,t}^{\frac{\upsilon-1}{\upsilon}} + (1-\chi) C_{h,t}^{\frac{\upsilon-1}{\upsilon}}]^{\frac{\upsilon}{\upsilon-1}}, \tag{7}$$

where υ is the substitution elasticity, reflecting the household's willingness to substitute market and home-produced goods. Output from home production depends on the state of technology and capital stock at home. It is produced according to a constant-returns-to-scale technology with inputs home capital K_{ht} and labor L_{ht}

$$C_{h,t} = [\psi (X_{h,t} L_{h,t})^{\frac{\tau-1}{\tau}} + (1-\psi) K_{h,t}^{\frac{\tau-1}{\tau}}]^{\frac{\tau}{\tau-1}}, \tag{8}$$

where τ is the substitution elasticity between labor and capital in home production. $X_{h,t}$ is a labor augmenting productivity process that will be specified below. It is important to note that this specification of home production is much more general than the conventional utility with leisure only. In fact, the commonly used separable-in-log utility can be obtained by simply setting $\upsilon = \tau = 1$.

The household owns the market capital stock and rents it to the representative firm. The budget constraint is of the form

$$C_{m,t} + I_{m,t} + I_{h,t} = W_t L_{m,t} + R_t K_{m,t}, \tag{9}$$

where $I_{m,t}$ and $I_{h,t}$ are investments in the capital stock in the market $K_{m,t}$, and at home $K_{h,t}$. In each period t, the household chooses $C_{m,t}$, $C_{h,t}$, $I_{m,t}$, $I_{h,t}$, $L_{m,t}$, and $L_{h,t}$. Market capital and home capital accumulate according to:

$$\begin{aligned} K_{m,t+1} &= \phi(I_{m,t}/K_{m,t})K_{m,t} + (1-\delta)K_{m,t} \\ K_{h,t+1} &= \phi(I_{h,t}/K_{h,t})K_{h,t} + (1-\delta)K_{h,t}, \end{aligned} \tag{10}$$

where δ is the depreciation rate of capital. The capital accumulation is subject to convex adjustment cost: $\phi' > 0, \phi'' \leq 0$.[6]

Output Y_t is produced by a representative firm that operates a Cobb-Douglas technology with the inputs capital $K_{m,t}$ and labor $L_{m,t}$

$$Y_t = K_{m,t}^{1-\alpha}(X_{m,t}L_{m,t})^{\alpha}. \tag{11}$$

$X_{m,t}$ represents a labor augmenting technology process. The firm solves the one-period problem

$$\max_{L_{m,t},K_{m,t}} \; K_{m,t}^{1-\alpha}(X_{m,t}L_{m,t})^{\alpha} - W_t L_{m,t} - R_t K_{m,t}, \tag{12}$$

which leads to an inverse demand function of the form (1). In equilibrium the output produced by the representative firm is equal to the consumption of market goods and the investment in home and market capital:

$$Y_t = C_{m,t} + I_{m,t} + I_{h,t}. \tag{13}$$

[6]Unlike one-sector models, in a multi-sector model, the investment in one sector can increase enormously at the price of the investment in the other sector, without affecting consumption significantly, resulting in unreasonably volatile investments over time. Adjustment costs of capital accumulation generate a more reasonable behavior of sectoral investment (e.g., Baxter (1996) and Fisher (1997)).

The labor augmenting productivity of the market and home technology are of the form $X_{m,t} = \exp[z_t + a_t]$ and $X_{h,t} = \exp[z_t + b_t]$, respectively. Here z_t represents a common technology process that follows a random walk with drift:

$$z_t = \gamma + z_{t-1} + \epsilon_{z,t}. \tag{14}$$

The processes a_t and b_t capture temporary productivity movements that are modeled as stationary first-order autoregressions:

$$a_t = \rho_a a_{t-1} + \epsilon_{a,t} \tag{15}$$

$$b_t = \rho_b b_{t-1} + \epsilon_{b,t}. \tag{16}$$

Define $\epsilon_t = [\epsilon_{z,t}, \epsilon_{a,t}, \epsilon_{b,t}]'$. We assume that ϵ_t is serially uncorrelated with diagonal covariance matrix Σ_ϵ. Its elements will be denoted by σ_z^2, σ_a^2, and σ_b^2, respectively.

Due to the labor augmenting random walk technology process z_t, the model economy evolves along a balanced stochastic growth path. Except for R_t, $L_{m,t}$, and $L_{h,t}$ all endogenous variables grow at the rate z_t. The stochastic trend shifts both the labor supply and demand curves, such that in the long-run a unit shock $\epsilon_{z,t}$ raises the equilibrium wage rate by one percent but does not affect hours worked.

A state-space model for hours worked $\ln L_{m,t}$ and the growth rates of labor productivity $\Delta \ln P_t = \Delta \ln(Y_t/L_{m,t})$ and home investment $\Delta \ln I_{h,t}$ is derived from the log-linearized DSGE model. We regard the expenditures on consumer durable goods as a measure of home investment. Hence, as the VAR in Section 2, the DSGE model provides a probabilistic representation for $\Delta y_t = [\Delta \ln P_t, \Delta \ln I_{h,t}, L_{m,t}]'$. Since the market production function in the DSGE model is Cobb-Douglas, responses of the model economy to a labor supply shock $\epsilon_{b,t}$, i.e. a temporary home productivity shock, satisfy the relationship in Equation (4). The VAR identification scheme proposed in Section 2 is consistent with the DSGE model in the following sense. Let $\tilde{Y}_T$ be a sample of (artificial) observations generated from the log-linearized DSGE model. Based on $\tilde{Y}_T$ and the correct labor share parameter α it is possible to consistently estimate the standardized shocks $\tilde{\epsilon}_t$ with the structural VAR, provided that the lag-length is increased appropriately as the size of the sample grows.

4 Econometric Approach

The goal of the econometric analysis is to assess the relative importance of labor supply shocks for the cyclical variation of output and hours worked based on estimates of the VAR and the DSGE model. To describe our estimation and inference procedure the following additional notation is introduced. The VAR is denoted by $\mathcal{M}_0$ and the overidentified log-linearized DSGE model by $\mathcal{M}_1$. To be consistent with the Cobb-Douglas production technology used in the DSGE model, we will assume that under the VAR specification the slope of the inverse labor demand function is also $\alpha - 1$. Hence, the parameter α is shared by $\mathcal{M}_0$ and $\mathcal{M}_1$. The parameters of model $\mathcal{M}_i$ except for α are stacked in the vector $\theta_{(i)}$, $i = 0, 1$. $\theta_{(0)}$ contains the cointegration parameter λ_{12} and the non-redundant elements of the reduced-form matrices $\Phi_0, \ldots, \Phi_p, \Sigma_u$ in Equation (5).

4.1 Variance Decompositions and Impulse Response Functions

Variance decompositions and impulse response functions are transformations of the parameters $\theta_{(i)}$ and α. Under both $\mathcal{M}_0$ and $\mathcal{M}_1$ the vector process Δy_t has a moving-average (MA) representation in terms of the standardized structural shocks $\tilde{\epsilon}_t$:

$$\Delta y_t = \mu(\theta_{(i)}, \alpha) + \sum_{j=0}^{\infty} C_j(\theta_{(i)}, \alpha)\tilde{\epsilon}_{t-j}. \tag{17}$$

The population mean μ and the moving average coefficients C_j are model-specific functions of $\theta_{(i)}$ and α. Define the vectors $M_z = [1, 0, 0]'$, $M_a = [0, 1, 0]'$, and $M_b = [0, 0, 1]'$. The impulse responses to the shock $\tilde{\epsilon}_{s,t}$ are given by

$$\frac{\partial \Delta y_{t+j}}{\partial \tilde{\epsilon}_{s,t}} = C_j M_s, \quad j = 0, 1, \ldots, \quad s \in \{z, a, b\}. \tag{18}$$

The h-th order autocovariance matrix of Δy_t can be decomposed into the contributions of the three structural shocks:

$$\Gamma_{\Delta y}(h) = \Gamma_{\Delta y}^{(z)}(h) + \Gamma_{\Delta y}^{(a)}(h) + \Gamma_{\Delta y}^{(b)}(h), \tag{19}$$

where

$$\Gamma^{(s)}_{\Delta y}(h) = \sum_{j=\max\{0,-h\}}^{\infty} C_j M_k M_k' C_{j+h}', \quad s \in \{z, a, b\}.$$

The relative contribution of shock s to the unconditional variance of the j'th element of Δy_t is given by the ratio $[\Gamma^{(s)}_{\Delta y}(0)]_{jj}/[\Gamma_{\Delta y}(0)]_{jj}$. The spectrum of the stationary process Δy_t is

$$S_{\Delta y}(\omega) = \sum_{h=-\infty}^{\infty} \Gamma_{\Delta y}(h) e^{-ih\omega} \tag{20}$$

and represents the contribution of frequency ω to the variance of Δy_t. Just as the autocovariances $\Gamma_{\Delta y}(h)$, for each ω the spectrum can be decomposed into the relative contribution of the three shocks.[7]

In the remainder of Section 4 we will generically represent the variance decompositions and truncated impulse response functions by an $m \times 1$ vector $\varphi = \tilde{\varphi}_i(\theta_{(i)}, \alpha)$.

4.2 Estimation and Inference

The likelihood functions are denoted by $p(Y_T|\theta_{(i)}, \alpha, \mathcal{M}_i)$. We adopt a Bayesian approach and place prior distributions of the form

$$p(\theta_{(i)}, \alpha|\mathcal{M}_i) = p(\theta_{(i)}|\mathcal{M}_i)p(\alpha), \quad i = 0, 1 \tag{22}$$

on the parameters. Equation (22) incorporates the assumption that α is *a priori* independent of $\theta_{(0)}$ and $\theta_{(1)}$. Moreover, the prior distribution of α is the same for both models. Since the population characteristics φ are functions of the parameters $\theta_{(i)}$ and α, Equation (22) implicitly determines its prior distribution $p(\varphi|\mathcal{M}_i)$.

[7] According to $\mathcal{M}_0$ and $\mathcal{M}_1$ the level of output is integrated of order one and its autocovariances do not exist. Let $S^{(s)}_{\Delta \ln Y}(\omega)$ denote the three components of the spectrum of output growth. We define the spectrum of the level of output at frequencies $\omega > 0$ as

$$S^{(s)}_{\ln Y}(\omega) = \lim_{\phi \to 1} \frac{S^{(s)}_{\Delta \ln Y}(\omega)}{1 + \phi^2 - 2\phi cos(\omega)}. \tag{21}$$

The term $1/[1 + \phi^2 - 2\phi cos(\omega)]$ is the power transfer function of the AR(1) filter $[1 - \phi L]^{-1}$, where L denotes the temporal lag operator. Equation (21) implies that the relative importance of the shocks is not affected by the filter that cumulates the growth rates of output.

The data Y_T are used to update the prior distribution by means of the likelihood function. Conditional on model $\mathcal{M}_i$, inference is based on the posterior distribution

$$p(\theta_{(i)}, \alpha | Y_T, \mathcal{M}_i) \propto p(Y_T|\theta_{(i)}, \alpha, \mathcal{M}_i) p(\theta_{(i)}|\mathcal{M}_i) p(\alpha), \tag{23}$$

where $\propto$ signifies proportionality. Draws from this posterior distribution can be generated through Bayesian simulation techniques described in the Appendix C and in Schorfheide (2000). The posterior distribution of population characteristics $p(\varphi|Y_T, \mathcal{M}_i)$ can be simulated by transforming the $[\theta'_{(i)}, \alpha]'$-draws according to $\tilde{\varphi}_i(\theta_{(i)}, \alpha)$. If prior probabilities $\pi_{i,0}$ are placed on the two models, the overall posterior distribution of φ is given by the mixture

$$p(\varphi|Y_T) = \pi_{0,T} p(\varphi|Y_T, \mathcal{M}_0) + \pi_{1,T} p(\varphi|Y_T, \mathcal{M}_1). \tag{24}$$

$\pi_{i,T}$ denotes the posterior probability of model $\mathcal{M}_i$.

While the posterior inference for the DSGE model is conceptually straightforward, it is worthwhile to examine the VAR-based inference more carefully. As pointed out in Section 2, the structural shocks can only be determined if the slope $\alpha - 1$ of the inverse-labor-demand schedule is given. However, the likelihood function is uninformative with respect to α as it only depends on the reduced-form parameters $\theta_{(0)}$, that is,

$$p(Y_T|\theta_{(0)}, \alpha, \mathcal{M}_0) = \tilde{p}(Y_T|\theta_{(0)}, \mathcal{M}_0). \tag{25}$$

The joint posterior density of $\theta_{(0)}$ and α can be expressed as

$$\begin{aligned} p(\theta_{(0)}, \alpha|Y_T, \mathcal{M}_0) &= \frac{\tilde{p}(Y_T|\theta_{(0)}, \mathcal{M}_0) p(\theta_{(0)}|\mathcal{M}_0) p(\alpha)}{\int \left[\tilde{p}(Y_T|\theta_{(0)}, \mathcal{M}_0) p(\theta_{(0)}|\mathcal{M}_0) [\int p(\alpha) d\alpha] \right] d\theta_{(0)}} \\ &= \tilde{p}(\theta_{(0)}|Y_T, \mathcal{M}_0) p(\alpha). \end{aligned} \tag{26}$$

Thus, $p(\theta_{(0)}, \alpha|Y_T, \mathcal{M}_0)$ is the product of the posterior density of the (identifiable) reduced form parameters and the prior density of α. According to the VAR the data Y_T convey no information about α. Hence, the prior density $p(\alpha)$ is not updated after observing Y_T.[8]

[8]Suppose $\mathcal{M}_0$ and $\mathcal{M}_1$ are analyzed jointly by placing prior probabilities $\pi_{i,0}$ on the two models.

4.3 Discussion

Our VAR based inference is a specific example of Bayesian analysis of a nonidentified econometric model. Poirier (1998) provides a comprehensive survey and many additional examples. The basic insight from this literature is the following. If the joint prior distribution for all the model parameters is proper, one obtains a proper posterior distribution. However, the prior is not updated in the directions of the parameter space in which the likelihood function is flat, that is, the directions in which the model is not identified.

Gordon and Boccanfuso (2001) propose to specify a prior distribution on the coefficient matrices of the moving-average representation of a vector time series. This prior is then projected onto the restricted set of MA coefficients that are consistent with a finite-order VAR representation. Although their structural VAR is not identifiable in a classical sense, they obtain a proper posterior distribution for the impulse response functions. However, in general the direct specification of a proper prior distribution for impulse response functions with a reasonable covariance structure is very demanding and their bivariate example is difficult to generalize.

Rather than attempting to specify a prior directly, we use economic intuition developed from assumptions on aggregate preferences, production technologies, and equilibrium relationships to specify a prior for φ indirectly by means of a prior for $\theta_{(0)}$ and α and the mapping $\tilde{\varphi}_i(\theta_{(0)}, \alpha)$. Since the distribution of reduced-form parameters $\theta_{(0)}$ is updated based on the sample information Y_T, the implied distribution of φ is updated with every observation and we learn about the relative importance of structural shocks and the response of the economy. To illustrate the extent of learning, we will report both prior and posterior distributions for the variance decompositions in Section 5.

Despite the presence of the DSGE model $\mathcal{M}_1$ and the informative posterior $p(\alpha|Y_T, \mathcal{M}_1)$ that it generates, it is still true that the VAR impulse responses have to be identified through the prior $p(\alpha)$, not the DSGE model posterior $p(\alpha|Y_T, \mathcal{M}_1)$, or the overall marginal posterior $p(\alpha|Y_T) = \pi_{0,T}p(\alpha) + \pi_{1,T}p(\alpha|Y_T, \mathcal{M}_1)$.

Asymptotically the posterior distribution of φ does not degenerate to a point mass. Even with infinitely many observations there will remain uncertainty about $\varphi = \tilde{\varphi}_0(\theta_{(0)}, \alpha)$ since the uncertainty with respect to α never vanishes. Unlike the approach taken by Gordon and Boccanfuso (2001), our method is explicit about the direction of the parameter space in which learning does not occur. If the dimension of the nonidentifiable component of the parameter vector is low, as in our application, we can assess the robustness of our conclusion by tracing out, for instance, the relative importance of the labor supply shock as a function of α. A similar approach was used by King and Watson (1992) who plotted their statistics of interest against a one-dimensional variable indexing the identification scheme of the VAR. If the dimension of the nonidentifiable component is large, robustness can be examined by the comparison of posteriors obtained from different prior distributions.

The VAR identification proposed in this paper is based on the notion that productivity and hours worked move in opposite directions in response to a labor supply shock. Equation (4) can be qualitatively interpreted as an inequality restriction on the impulse responses:

$$\frac{\partial \ln P_t}{\partial \epsilon_{b,t}} > 0 \quad \text{and} \quad \frac{\partial \ln L_{m,t}}{\partial \epsilon_{b,t}} < 0 \tag{27}$$

Canova and DeNicolo (1998), Faust (1998), and Uhlig (1997) develop identification and inference procedures based on such inequality constraints. For instance, Uhlig (1997) considers a large set of inequalities for initial and subsequent responses to a monetary shock. He uses a loss function to map the reduced-form parameter estimates into structural parameter estimates. The loss function imposes a strong penalty onto the violation of the inequality constraints. Faust (1998) computes bounds for the relative importance of a money supply shock by searching over all possible identification schemes that are consistent with a pre-specified set of constraints. Our approach places a prior distribution on the identification schemes that are consistent with (27) and averages the posterior distribution of population characteristics φ over *a priori* likely values of the unidentifiable parameter α that indexes the identification schemes.

5 Empirical Analysis

Both VAR and DSGE models are fitted to post-war quarterly U.S. data on labor productivity, expenditure on consumer durables and hours worked. The construction of the data set is described in Appendix B. The sample period ranges from 1955:I to 1997:IV. The overall sample size is $T = 172$ and the first $T_* = 20$ observations are used as training sample to initialize lags and parameterize the prior distributions. The data are plotted in Figure 1. Solid vertical lines correspond to the NBER business-cycle peaks, while dashed lines denote troughs. The peaks coincide with periods in which aggregate hours is high, and troughs coincide with periods in which hours and expenditure on consumer durable goods were at a low. The hours series has no apparent trend, yet its movement is quite persistent.

5.1 Priors

The prior distribution used in the estimation of DSGE model is summarized in columns 3 to 5 of Table 1. The shapes of the densities are chosen to match the domain of the structural parameters. We use informative priors for parameters that can be easily inferred, e.g. labor share, average growth rate of productivity, whereas uninformative priors are used for those that cannot be easily observed, e.g., home production technology.

The prior means for labor share, discount rate, productivity growth, and capital depreciation are $\alpha = 0.666$, $\beta = 0.993$, $\gamma = 0.004$, and $\delta = 0.025$. These values are commonly used in the literature and can be justified based on a training sample that ranges from 1955:I to 1959:IV. Prior means for the steady state hours spent for market work L_m and home work L_h are 0.33 and 0.25, respectively, from the Time Use Survey. A larger standard deviation is allowed for L_h, as hours spent on home work may be measured with a greater uncertainty. The prior mean and standard deviation for the steady-state ratio of home investment to market investment (non-residential fixed investment) I_h/I_m are obtained from the training sample.

We allow for large standard deviations in the prior distributions of home technology parameters as they are not easy to observe. Prior means we use are $\upsilon = 1$ and $\tau = 1$. This case is essentially identical to one-sector model with separable-in-logs in consumption and leisure as the the market consumption and labor supply are not affected by home technology shocks. The prior mean of the labor share ψ in the home production function is also set to 0.666. The weight on leisure χ in the utility is determined by other parameters. For the stochastic process of structural shocks, ρ_a, $\rho_b, \sigma_z, \sigma_a$, and σ_b, we use very diffuse priors. Prior means of persistence parameters for temporary shocks are set to 0.8.

There is no adjustment cost at the steady state: $\phi'(I^*/K^*) = 1$ and $\phi(I^*/K^*) = I^*/K^*$. The elasticity of the investment/capital ratio with respect to Tobin's q, $\eta = (|(I^*/K^*)\phi''/\phi'|^{-1})$ is to be estimated. The prior mean is set to 100 implying small adjustment costs, with a large standard deviation of 100. Finally, we introduce two additional parameters ξ_1 and ξ_2 to adjust the normalization of total hours to one and to capture the average growth rate differential between labor productivity and home investment in the data. The structural parameters are assumed *a priori* independent of each other.

We now describe the prior distribution of the VAR parameters.[9] The DSGE model implies that the cointegration parameter $\lambda_{21} = 1$. This restriction is relaxed and we choose the prior $\lambda_{21} \sim \mathcal{N}(1, 0.025^2)$. Instead of using a model-based prior for the reduced form parameter matrices Φ_0, Φ_1, Φ_2, and Σ_u, that shrinks the estimates toward the DSGE model restrictions (see Ingram and Whiteman (1994)), we construct a data-based prior conditional on λ_{21} from training-sample OLS estimates. Details are provided in Appendix C. The prior for α is the same as in the DSGE model analysis (Table 1). The prior distributions of the VAR and DSGE model parameters induce prior distributions for the variance decompositions, which will be discussed together with the posterior estimates in Section 5.4.

[9]Posterior odds were used to select the lag-length $p = 2$.

5.2 Parameter Estimation

Since the VAR parameter estimates themselves are not of primary interest in our empirical analysis we focus on the DSGE model estimates. The estimates of market labor share, discount factor, productivity growth, and depreciation are similar to those reported in previous studies. Posterior means and standard errors of all parameters are reported in columns 6 and 7 of Table 1.[10] The substitution elasticity between market goods and home goods υ is 2.249. This is slightly higher than the estimates of Rupert, Rogerson and Wright (1995) and McGrattan, Rogerson and Wright (1997). The substitution elasticity between capital and labor in home production τ is 2.381 implying that goods and time are substitutes in home production activity. The labor share in home technology ψ is 0.757 which is slightly higher than that in the market technology. Hours spent on home production activity L_h is 0.170. The temporary home production shock is somewhat more persistent than the market shock: $\hat{\rho}_a = 0.767$ and $\hat{\rho}_b = 0.859$. The nature of stochastic variation of home technology $X_{h,t}$, in particular, its relative magnitude and correlation with market productivity shock $X_{m,t}$, is important for business-cycle analysis. Once we identify the underlying innovation to three structural shocks, conditional on time $t-1$ information, the correlation between the market and home productivity $\ln X_{m,t}$ and $\ln X_{h,t}$ can be obtained:

$$corr_{t-1}[\ln X_{m,t}, \ln X_{h,t}] = \Big([1 + (\sigma_a/\sigma_z)^2][1 + (\sigma_b/\sigma_z)^2] \Big)^{-1/2}. \tag{28}$$

The posterior mean correlation between innovations to market and home productivity $\ln X_{m,t}$ and $\ln X_{h,t}$ is 0.27. The 90-percent posterior confidence interval ranges from 0.18 to 0.37. The estimates are somewhat lower than the values that have been

[10]While McGrattan, Rogerson and Wright (1997) also estimate home production models based on aggregate time series, our analysis distinguishes itself from theirs in several dimensions. First, we focus on variance decompositions and a comparison to results obtained from a structural VAR analysis. Second, microeconomic evidence is incorporated through prior distributions in our Bayesian estimation. Third, we are able to uncover the comovement of innovations to market and home productivity.

used in the literature. Finally, the adjustment cost parameter η is 24.58 implying a small adjustment cost in capital accumulation.

5.3 Time Series Fit

To assess the relative time series fit of the VAR and the DSGE model we compute posterior model probabilities $\pi_{i,T} = \frac{\pi_{i,0} p(Y_T|\mathcal{M}_i)}{\sum_{i=0,1} \pi_{i,0} p(Y_T|\mathcal{M}_i)}$ conditional on the training sample 1955:I to 1959:IV. The marginal data densities are given by $p(Y_T|\mathcal{M}_i) = \int p(Y_T|\theta_{(i)}, \alpha, \mathcal{M}_i) p(\theta_{(i)}, \alpha|\mathcal{M}_i) d(\theta_{(i)}, \alpha)$.[11] The log-marginal data density can be interpreted as a measure of one-step-ahead predictive performance $\ln p(Y_T|\mathcal{M}_i) = \sum_{t=T_*}^{T} p(y_t|Y_t, \mathcal{M}_i)$. The values are $\ln p(Y_T|\mathcal{M}_0) = 1409$ for the VAR and $\ln p(Y_T|\mathcal{M}_1) = 1308$ for the DSGE model, which implies that the posterior probability of the DSGE model is essentially zero. To shed more light on the poor time series fit of the DSGE model, we also computed in-sample root-mean-squared-errors (RMSE) at the posterior mode estimates. While the RMSE's for the growth rates of output and consumer durable expenditures are very similar for the two models, the RMSE of hours is substantially higher for the DSGE model: 0.0076 versus 0.0057 for the DSGE model.

5.4 Variance Decompositions

Our main interest is to unveil the sources of cyclical variation in hours and output. Table 2 presents the variance decomposition of hours, from both VAR and DSGE model, into three structural innovations $\epsilon_{z,t}$, $\epsilon_{a,t}$, and $\epsilon_{b,t}$. It contains prior and posterior means for the decomposition of the unconditional variance and the variance at two business cycle frequencies: 1/32, and 1/12 cycles per quarter.

The common practice of excluding labor supply shocks in business cycle analysis may suggest that many researchers regard them as fairly unimportant. Our prior mean is by and large consistent with this view, as the temporary labor-supply shifts explain only a small fraction of the output variation (about 7 to 12 percent) and

[11] To compute $p(Y_T|\mathcal{M}_i)$ we use a numerical approximation, known as modified harmonic mean estimator, proposed by Geweke (1999).

roughly a quarter of the variation in hours worked. The time series data, however, lead to a revision of this view. According to both VAR and DSGE model based decompositions, the labor-supply shifts play a much more important role for fluctuation of hours. The ϵ_b shock accounts for more than 45 percent of the fluctuation in hours according to the VAR and more than half according to the DSGE model. With respect to output, however, the contribution of the labor-supply shocks is modest as they account for less than 15 percent of the variation.

Most fluctuations of output are due to either permanent (ϵ_z) or temporary (ϵ_a) shifts in labor demand. One interesting observation is that while permanent shifts in technology are important for output variation, their ability to generate movements in hours is limited in the DSGE model. ϵ_z accounts for less than 3 percent of hours variation. This is due to the so-called balanced growth property of this class of models. Common technology shocks tend to shift both labor demand and supply in a similar magnitude leaving hours almost unaffected.[12] Our decompositions are fairly robust across various business-cycle frequencies, beyond the ones reported in Table 2.

A shortcoming of Table 2 is that it does not reflect the uncertainty associated with the decompositions. Figures 2 and 3 visualize the entire distributions for decompositions at frequency 1/20. Since the sum of the shares is always one, the variance decompositions can be depicted in two-dimensional triangular shaped subspaces (simplex) of $I\!R^3$. The three corners z, a, b of the simplex correspond to decompositions that assign 100 percent of the variation to one shock, and 0 percent to the other two shocks. Each dot corresponds to one draw from the prior or posterior distribution. Clusters of dots indicate regions of high density.

The VAR-based decompositions are depicted in Figure 2. The first row of plots visualizes the decomposition of hours. As we move from prior to posterior, a substantial fraction of the probability mass shifts toward the b corner, indicating a

[12]The same is true for the model with conventional utility that supports balanced growth path where income and substitution effect are likely to offset each other in response to a permanent increase in productivity.

more important role for the labor supply shocks. The plots also indicate that there is little evidence in the data that helps distinguishing between permanent (ϵ_z) and temporary (ϵ_a) productivity shocks. As pointed out by Faust and Leeper (1997) decompositions based on long-run restrictions are associated with a high degree of uncertainty. The role of labor-supply shocks in the output decomposition, depicted in the second row of plots in Figure 2 becomes slightly more important as we move from prior to posterior. Fewer draws appear near the bottom edge of the simplex.

The DSGE model-based decomposition in Figure 3 shows a sharp distinction between prior and posterior.[13] The overidentifying restrictions embodied in the DSGE model lead to precise estimates of its structural parameters and hence to a very concentrated posterior distribution of variance shares. Unlike the VAR, the DSGE model is able to separate the effects of permanent and temporary productivity shocks. For instance, the contribution of ϵ_z to the variation in hours is now with high probability less than 5 percent.

The variance decompositions are function $\varphi_i(\theta_{(i)}, \alpha)$. The VAR posterior means reported in Table 2 were computed by integrating out the reduced form parameters $\theta_{(0)}$ with respect to the posterior $\tilde{p}(\theta_{(0)}|Y_T, \mathcal{M}_0)$ and the parameter α with respect to the prior density $p(\alpha)$. Since the data provide no information about α, the inference is potentially sensitive to the choice of the prior $p(\alpha)$. Moreover, our identifying assumption for labor supply shocks exploits the notion that the production capacity is fixed in the short run. Allowing for time-varying utilization could undermine this assumption. In Appendix A we show that if the cost of intensive utilization of capital results in a faster depreciation of capital, our identifying restriction is still valid. The linear relation between labor productivity and hours still exists but one needs to use a higher value for α.

To assess the robustness of our conclusions Figure 4 shows the posterior expected

[13] As we set $\nu = 1$ and $\tau = 1$ for our prior means, market consumption and home consumption become separable in logs, which makes our prior mean of DSGE model essentially identical to standard one-sector model. Thus, Figure 3 also shows the home production model's ability to transmitting the home productivity shocks to market decisions.

contribution of the labor supply shock to the variation in hours and output as a function of α between 0.3 and 0.9. The share of ϵ_b lies between 25 and 55 percent for hours and 5 to 25 percent for output. As we move to higher values of α, which is likely the case as we allow for varying capital utilization, the importance of labor supply shock is reinforced.

Overall, both VAR and DSGE model analysis document that labor supply shocks plays a very important role as a source of economic fluctuations, especially in hours. Our findings are comparable to the work by Shapiro and Watson (1988) where 60 percent of cyclical variation in hours is due to the stochastic trend component in labor supply, and the one by Hall (1997) where almost entire cyclical variation of hours is attributed to preference shocks. However, the fairly concentrated DSGE model based posteriors have to be interpreted with caution, since the weak time series fit of the DSGE model, as documented in Section 5.3, indicates that some of its restrictions are misspecified.

5.5 Impulse Response Functions

We compare the impulse response functions to see if the structural shocks identified from the VAR conform to our economic intuition. Figure 5 depicts one-standard-deviation impulse responses of labor productivity, consumer durable goods (investment in home capital) and market hours to three structural shocks. It shows the DSGE model responses (solid line) and those from the VAR along with the 90 percent confidence interval (dotted lines). Looking at first row, in response to a permanent shock, labor productivity both in the model and data approach the new steady state at a similar pace. Spending on consumer durables also approaches a new steady state. Hours in the market increase immediately in the model, whereas they exhibit somewhat delayed responses, especially for hours, in the VAR. Model responses to a temporary market productivity increase closely trace those from the VAR confirming our interpretation of temporary labor-demand shifts. The response of hours in the VAR is again delayed for about 2 quarters. Finally, in response to a temporary increase in home productivity, while the responses of labor productivity is within the 90 percent confidence interval, it shows a very persistent response in the data, whereas it decays rapidly in the model. Home investment initially decreases and moves above the steady state after 12 quarters in the data, whereas it increases immediately and decays at a much higher pace in the model. Again, hours exhibits somewhat delayed response in the data. Overall, the model, by and large, reproduces the impulse response in the VAR. Yet the response of hours is delayed for about 2-3 quarters in the data suggesting frictions in the labor market.

5.6 Evolution of Latent Technology Processes

According to the home production model, recessions may occur because agents find it optimal to allocate more time in non-market activities. In our DSGE model the attractiveness of non-market activity, or labor-supply shifts in general, is measured by the home technology process. We plot three technology index in Figure 6 together with the NBER business cycle peaks and troughs.[14]

All six recessions during the sample period are associated with low levels of market productivity. Two business cycle troughs, in March 1975 and November 1982, coincide with unusually high productivity of non-market activities. The strong interpretation of this finding is that unusually high productivity or preference shift has contributed to low market employment and output in those recessions. A weaker interpretation is, that in March 1975 and November 1982 the economic downturn cannot solely be explained by an adverse technology shock in the market. The other four recessions are associated with low productivity in both market and home sector.

5.7 Alternative Interpretations of the VAR Identification

In this section, we provide a couple of caveats regarding our identifying restriction. We exploit the competitive labor market equilibrium in identifying temporary shocks

[14] For each draw from the posterior distribution of DSGE model parameters $[\theta'_{(1)}, \alpha]'$ a smoothing algorithm is applied to compute expected values for the technology sequences $\{a_t\}_{t=1}^T$, $\{b_t\}_{t=1}^T$, and $\{z_t\}_{t=1}^T$ conditional on $\mathcal{M}_1$ and the sample of observations Y_T. These sequences of expected values are averaged across the parameter draws and plotted in Figure 6.

to labor demand and supply. While our DSGE model interprets them, respectively, as shifts in market and home technology, the proposed identification scheme is more general and allows an alternative interpretation. In fact, our identification distinguishes between the *shift* of and *movement* along the marginal product of labor, as Equation (4) essentially captures the movement of the economy along the marginal product labor curve.

As an illustrative example, consider a model economy with sticky prices where firms have to produce goods to meet its demand. In this economy, the labor demand is no longer a simple reflection of the marginal product of labor. It is instead jointly determined by the demand for goods and the output-labor elasticity from production technology. Suppose now there is an increase in the demand for goods that is not caused by a productivity shift. This will lead to an increase in the demand for labor at a given level of production capacity. The joint behavior of labor productivity and hours is still dictated by the production function with a downward sloping marginal product of labor. Thus, the same restriction can be used to capture such shocks.[15] Our DSGE model interprets the movement along the marginal product of labor as a shift in labor supply curve caused by a shift in home technology.

Finally, according to Gali (1999) and Basu, Fernald and Kimball (1998), a permanent increase in productivity could lead to a decrease in hours at least during the first few years. This implies that temporary (market) productivity shocks may also generate a negative correlation between labor productivity, which potentially makes our identifying restriction vulnerable. However, according to our VAR estimation result, which exploits a cointegration relationship as well as the stationarity of hours in identifying permanent components, we did not find such a negative response of hours in response to a permanent productivity shock.

[15] In this event, the real wage will increase given the upward sloping labor supply curve. However, labor productivity falls as employed hours increases, and this further justifies our use of labor productivity instead of wage series under the alternative interpretation.

6 Conclusion

We investigate the sources of economic fluctuations in the context of a dynamic general equilibrium. A new VAR identification scheme is proposed that identifies three types of underlying disturbances in the aggregate labor market equilibrium: temporary labor-supply shifts, temporary labor demand shifts, and permanent productivity shocks that eventually move both demand and supply. According to the variance decomposition from the VAR, the labor-supply shifts are an important driving force of the cyclical fluctuation of hours, as they account for about half the variation. However, for output fluctuations, the role of labor-supply shifts is modest. Either permanent or temporary shifts in labor demand, interpreted, respectively, as permanent and temporary productivity shifts, explain more than 80 percent of the variation in output.

To assess the role of labor-supply shifts in an equilibrium model, a home production model with stochastic variation in non-market technology is estimated, and its predictions are compared to those from the VAR. When the equilibrium model is estimated with the same set of structural shocks, again, about half the variation of hours is still attributed to the temporary labor-supply shifts.

In order to make the VAR and DSGE model analysis comparable, it is desirable to use an identification scheme for the VAR that correctly identifies the structural shocks, if the data were in fact generated from the DSGE model. However, for many DSGE models the correct identification cannot be achieved based on simple "zero-restrictions" (Canova and Pina, 2000). To overcome this problem, the DSGE model could be re-specified to make it consistent with the "zero-restrictions", e.g., Rotemberg and Woodford (1998). On the other hand, one could employ an identification scheme that does not solely rely on these "zero-restrictions". We followed the second path. Unlike in recent papers by Canova and DeNicolo (1998) and Uhlig (1997), who achieve identification based on inequality restrictions, we develop a scheme conditional on one non-identifiable parameter. For our analysis, we find it justifiable to specify a tight prior on this non-identifiable parameter. We view this approach as a promising alternative that has potentially a wide application in both macroeconomics and time series analysis.

References

[1] Basu, S., J. Fernald, and M. Kimball (1998) "Are Technology Improvement Contractionary?" International Finance Discussion Paper No. 625, Board of Governors of the Federal Reserve System.

[2] Baxter, M. (1996) "Are Consumer Durables Important for Business Cycles?" Review of Economics and Statistics 78, 147-155.

[3] Baxter, M. and R. King (1991) "Productive Externalities and Business Cycles," Discussion paper 53, Institute for Empirical Macroeconomics, Federal Reserve Bank of Minneapolis.

[4] Becker, G. (1965) "A Theory of the Allocation of Time" Economic Journal 73, 493-508.

[5] Benhabib, J., R. Rogerson, and R. Wright (1991) "Homework in Macroeconomics: Household Production and Aggregate Fluctuations," Journal of Political Economy 99, 1166-1187.

[6] Bils, M., and P. Klenow (1998) "Using Consumer Theory to Test Competing Business Cycle Models," Journal of Political Economy 106, 223-261.

[7] Canova, F. and G. DeNicolo (1998) "Monetary Disturbances for Output for Business Cycle Fluctuations in the G-7," Working Paper 661, Federal Reserve Board, Division of International Finance.

[8] Canova, F. and J.P. Pina (2000) "Monetary Policy Misspecification in VAR Models," Manuscript, Universitat Pompeu Fabra.

[9] Faust, J. (1998) "The Robustness of Identified VAR Conclusions about Money," Carnegie Rochester Conference Series 49, 207-244.

[10] Faust, J. and E. Leeper (1997) "When Do Long-run Identifying Restrictions Give Reliable Results?", Journal of Business & Economic Statistics 15, 345-353.

[11] Fisher, J. (1997) "Relative Prices, Complementarities and Comovement among Companies of Aggregate Expenditures," Journal of Monetary Economics 39, 449-474.

[12] Gali, J. (1999) "Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?" American Economic Review 89, 249-271.

[13] Gelman, A., J.B. Carlin, H.S. Stern, and D.B. Rubin (1995) *"Bayesian Data Analysis,"* Chapman & Hall, New York.

[14] Geweke, J. (1999) "Using Simulation Methods for Bayesian Econometric Models: Inference, Development and Communication," Econometric Reviews, 18, 1-126.

[15] Gordon, S. and D. Boccanfuso (2001) "Learning from Structural Vector Autoregression Models," Manuscript, Universite Laval, Quebec City.

[16] Greenwood, J., and Z. Hercowitz (1991) "The Allocation of Capital and Time over the Business Cycle," Journal of Political Economy 99, 1188-1215.

[17] Hall, R. (1986) "The Role of Consumption in Economic Fluctuations," in R.J. Gordon (ed.) The American Business Cycle: Continuity and Change. Chicago: University of Chicago Press.

[18] Hall, R. (1997) "Macroeconomic Fluctuations and the Allocation of Time," Journal of Labor Economics 15, s223-s250.

[19] Hill, M. S. (1984) "Pattern of Time Use." In F. Thomas Juster and Frank P. Stafford, eds., Time, Goods and Well-Being, Ann Arbor: University of Michigan Press.

[20] Houthakker, H. S., and L. D. Taylor (1970) “Consumer Demand in the United States: Analysis and Projections”, 2nd, enl. ed. Cambridge, Mass., Harvard University Press.

[21] Ingram, B. F. and C. H. Whiteman (1994) “Supplanting the Minnesota Prior – Forecasting Macroeconomic Time Series Using RBC Model Priors,” Journal of Monetary Economics 34, 497-510.

[22] Johansen, S. (1995) “Likelihood-Based Inference in Cointegrated Vector Autoregressive Models,” Oxford University Press, Oxford.

[23] Juster, F. T., and F. P. Stafford (1991) “The Allocation of Time: Empirical Findings, Behavior Models, and Problems of Measurement.” Journal of Economic Literature 29:471-522.

[24] King, G. R., C. I. Plosser, J. H. Stock, and M. Watson (1991) “Stochastic Trends and Economic Fluctuations,” American Economic Review 81 (4), 819-840.

[25] King, R. and M. Watson (1992) “Testing Long-run Neutrality,” Manuscript.

[26] McGrattan, E., R. Rogerson, and R. Wright (1997) “An Equilibrium Model of the Business Cycle with Household Production and Fiscal Policy,” International Economic Review 38, 267-290.

[27] Mincer, J. (1963) “Market Prices, Opportunity Costs, and Income Effects,” in Christ, C. et al (eds.) Measurement in Economics: Studies in Mathematical Economics and Econometrics in Memory of Yhuda Frunfeld, Stanford University Press.

[28] Parkin, M. (1988) “A Method for Determining whether Parameters in Aggregative Models are Structural,” Carnegie-Rochester Conference on Public Policy, 29, 215-252.

[29] Poirier, D. (1998) “Revising Beliefs in Nonidentified Models”, Econometric Theory, 14, 183-509.

[30] Rotemberg, J. and M. Woodford (1998) "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy," NBER Macroeconomics Annual.

[31] Rupert, P., R. Rogerson, and R. Wright (1995) "Estimating Substitution Elasticities in Household Production Models," Economic Theory 6, 179-193.

[32] Shapiro, M. and M. Watson (1988) "Sources of Business Cycle Fluctuations," NBER Macroeconomics Annual 1988, 111-48.

[33] Schorfheide, F. (2000) "Loss Function Based Evaluation of DSGE Models," Journal of Applied Econometrics, 15, 645-670.

A Labor Demand with Variable Capital Utilization

Consider a Cobb-Douglas production function with inputs in capital services and hours:

$$Y_t = (u_t K_{m,t})^{1-\alpha}(X_{m,t}L_{m,t})^{\alpha}, \tag{29}$$

where u_t represents the utilization of the capital stock. Suppose the intensive use of capital results in a fast depreciation. At the cost of a more complicated notation, we could work with an alternative decentralization scheme in which firms makes decisions on accumulation. However, since both decentralizations are essentially identical, as in the main text, suppose the firm rents the capital from households. Yet the firm has to compensate households for faster depreciation when the capital is utilized more intensively:

$$\max_{L_{m,t},K_{m,t},u_t} (u_t K_{m,t})^{1-\alpha}(X_{m,t}L_{m,t})^{\alpha} - W_t L_{m,t} - (R_t + \delta(u_t))K_{m,t}. \tag{30}$$

For illustrative purposes, assume that the elasticity of depreciation is constant: $\delta(u_t) = \delta_0 \frac{u_t^{\lambda+1}}{\lambda+1}$, where $\lambda > 0$. As $\lambda \to \infty$, the utilization is held constant and the depreciation rate is fixed. The first order conditions of the profit maximization problem with respect to $L_{m,t}$ and u_t imply that the inverse labor demand schedule still depends on the predetermined capital stock and the market productivity shocks only. However, its slope changes:

$$\frac{\partial \ln W_t}{\partial \epsilon_{b,t}} = \mu(\alpha - 1)\frac{\partial \ln L_{m,t}}{\partial \epsilon_{b,t}}, \quad \mu = \frac{\lambda}{\lambda+\alpha} \leq 1. \tag{31}$$

Therefore, the proposed identification scheme is still valid but the slope of the labor demand schedule is smaller than in the constant utilization case, reflecting an extra margin for the firm to exploit.

B Data Set

The following time series are extracted from DRI: real gross domestic product (GDPQ), consumption of consumer durables (GCDQ), employed civilian labor force (LHEM), civilian noninstitutional population 20 years and older (PM20 and PF20).

Population is defined as $POPQ = 1E6 * (PF20 + PM20)$ and used to convert GDPQ and GCDQ into real dollar per capita terms. Thus, $Y_t = GDPQ/POPQ$ and $I_{h,t} = GCDQ/POPQ$.

¿From the BLS we obtained the series: average weekly hours, private non-agricultural establishments (EEU00500005). Prior to 1963 the BLS series is annual. We used these annual averages as monthly observations without further modification. Our measure of annual hours worked at monthly frequency is $L_{m,t} = 52 * EEU00500005 * LHEM \; / \; POPQ$. Hours are converted to quarterly frequency by simple averaging. Our measure of labor productivity is $P_t = Y_t/L_{m,t}$.

C Vector Autoregression

C.1 Prior

Let ΔY_T be the $(T-p) \times n$ matrix with rows $\Delta y_t'$, $t = p+1, \ldots, T$ (the first p observations are used to initialize lags). Let $k = 3+np$, $X_T(\lambda_{21})$ be the $(T-p) \times k$ matrix with rows $x_t' = [1, t, (1, -\lambda_{21}, 0)y_{t-1}, \Delta y_{t-1}', \ldots, \Delta y_{t-p}']$, U_T be the matrix with rows u_t', and $B = [\Phi_0, \Phi_{tr}, \alpha, \Phi_1, \ldots, \Phi_p]'$. We include a deterministic trend with coefficient vector Φ_{tr} in the specification of $\mathcal{M}_0$ to capture potential long-run shifts in market hours due to structural changes in labor market participation behavior. The reference model can be expressed in matrix form as $\Delta Y_T = X_T(\lambda_{21})B + U_T$. Conditional on λ_{21} the prior for B and Σ is constructed from a training sample $t = p+1, \ldots, T_*$. Let ΔY_* and $X_*(\lambda_{21})$ be matrices with rows $\Delta y_t'$ and x_t' as defined above, $t = p+1, \ldots, T_*$. Define

$$\hat{B}_* = (X_*' X_*)^{-1} X_*' \Delta Y_*, \quad \hat{\Sigma}_{u,*} = (T_* - p)^{-1}(Y_* - X_* \hat{B}_*)'(Y_* - X_* \hat{B}_*). \tag{32}$$

Then we obtain

$$\begin{aligned} \lambda_{21} &\sim \mathcal{N}(1, 0.025^2) \\ \Sigma_u | \lambda_{21}, Y_* &\sim IW\Big((T_* - p)\hat{\Sigma}_{u,*}, T_* - k - p\Big) \\ vec(B) | \Sigma_u, \lambda_{21}, Y_* &\sim \mathcal{N}\Big(vec(\hat{C}_*), \Sigma_u \otimes (X_*' X_*)^{-1}\Big), \end{aligned} \tag{33}$$

where IW denotes the Inverted Wishart distribution. In our empirical analysis the size of the training sample is $T_* = 20$ and the lag-length is $p = 2$.

C.2 Posterior Simulation

A Gibbs sampler is used to generate draws from the posterior distribution of the VAR parameters $(B, \Sigma_u, \lambda_{21})$. We draw successively from the conditional posteriors $p(B, \Sigma_u|\lambda_{21}, Y_T, \mathcal{M}_0)$ and $p(\lambda_{21}|B, \Sigma_u, Y_T, \mathcal{M}_0)$. The distribution of $\Sigma_u|\lambda_{21}, Y_T$ is Inverted Wishart and $B|\Sigma_u, \lambda_{21}, Y_T$ is multivariate normal. The parametrization is given by replacing ΔY_* and $X_*(\lambda_{21})$ with ΔY_T and X_T in Equations (32) and (33). To characterize the posterior distribution of λ_{21}, define $\Delta\tilde{Y}_T$ with rows $\Delta\tilde{y}_t' = [\Delta y_t - \Phi_0 - \Phi_{tr} t - \mu(1,0,0)y_{t-1} - \sum_{i=1}^{p} \Phi_i \Delta y_{t-i}]'$ and $\tilde{X}_t$ with rows $\tilde{x}_t'$, where $\tilde{x}_t = \mu(0,-1,0)y_{t-1}$. Then one obtains

$$\lambda_{21}|B, \Sigma_u, Y_T \sim \mathcal{N}(m_\lambda, v_\lambda), \tag{34}$$

where $v_\lambda^{-1} = 1/0.01 + tr[\Sigma^{-1}\tilde{X}_T'\tilde{X}_T]$, $m_\lambda = v_\lambda \left(\frac{1}{0.01} + \frac{tr[\Sigma^{-1}\Delta\tilde{Y}_T'\tilde{X}_T]}{tr[\Sigma^{-1}\Delta\tilde{X}_T'\tilde{X}_T]} \right)$, and $tr[.]$ denotes the trace operator.

Table 1: PRIOR AND POSTERIOR DISTRIBUTION FOR DSGE MODEL PARAMETERS

Parameters		Prior			Posterior	
Name	Range	Density	Mean	S.E.	Mean	S.E.
α	[0,1]	Beta	0.666	0.020	0.741	0.018
β	[0,1]	Beta	0.993	0.002	0.978	0.003
γ	$I\!R$	Normal	0.004	0.0005	0.004	0.0004
δ	[0,1]	Beta	0.025	0.002	0.016	0.002
L_m	[0,1]	Beta	0.330	0.020	0.340	0.020
ρ_a	[0,1]	Beta	0.800	0.100	0.767	0.030
ρ_b	[0,1]	Beta	0.800	0.100	0.859	0.033
L_h	[0,1]	Beta	0.250	0.050	0.170	0.045
η	$I\!R^+$	Gamma	100.0	100.0	24.58	3.863
ψ	[0,1]	Beta	0.666	0.100	0.757	0.081
ν	$I\!R^+$	Gamma	1.000	2.000	2.249	0.318
I_h/I_m	$I\!R^+$	Gamma	0.700	0.020	0.685	0.020
τ	$I\!R^+$	Gamma	1.000	2.000	2.381	0.405
ξ_1	$I\!R$	Normal	2.960	1.000	3.150	0.005
ξ_2	$I\!R$	Normal	0.000	0.020	0.005	0.0003
σ_z	$I\!R^+$	InvGamma	0.01*	2.000*	0.009	0.0008
σ_a	$I\!R^+$	InvGamma	0.01*	2.000*	0.009	0.0009
σ_b	$I\!R^+$	InvGamma	0.015*	2.000*	0.023	0.0058

Notes: For the Inverse Gamma (u, s) priors we report the parameters u and s. For $u = 2$ the standard error is infinite. The posterior moments are calculated from the output of the Metropolis algorithm.

Table 2: Variance Decomposition by Frequency

Variable	Cycles Per Quarter	Shock	Prior VAR	Prior DSGE	Posterior VAR	Posterior DSGE
$\ln L_{m,t}$	1/32	ϵ_z	0.336	0.415	0.223	0.029
		ϵ_a	0.429	0.356	0.318	0.355
		ϵ_b	0.235	0.229	0.459	0.616
$\ln L_{m,t}$	1/12	ϵ_z	0.325	0.331	0.226	0.015
		ϵ_a	0.455	0.381	0.277	0.457
		ϵ_b	0.220	0.287	0.497	0.528
$\ln L_{m,t}$	Uncond. Variance	ϵ_z	0.335	0.518	0.205	0.099
		ϵ_a	0.440	0.282	0.370	0.314
		ϵ_b	0.225	0.200	0.425	0.587
$\ln Y_t$	1/32	ϵ_z	0.681	0.540	0.467	0.431
		ϵ_a	0.244	0.388	0.388	0.429
		ϵ_b	0.075	0.072	0.145	0.140
$\ln Y_t$	1/12	ϵ_z	0.436	0.444	0.505	0.263
		ϵ_a	0.439	0.456	0.355	0.603
		ϵ_b	0.125	0.100	0.140	0.134
$\Delta \ln Y_t$	Uncond. Variance	ϵ_z	0.367	0.433	0.565	0.248
		ϵ_a	0.512	0.454	0.298	0.624
		ϵ_b	0.121	0.113	0.137	0.128

Notes: Decomposition of unconditional variance and spectral density and for aggregate output $\ln Y_t$ and market hours $\ln L_{m,t}$ at 32, and 12 quarters per cycle. The table reports posterior means.

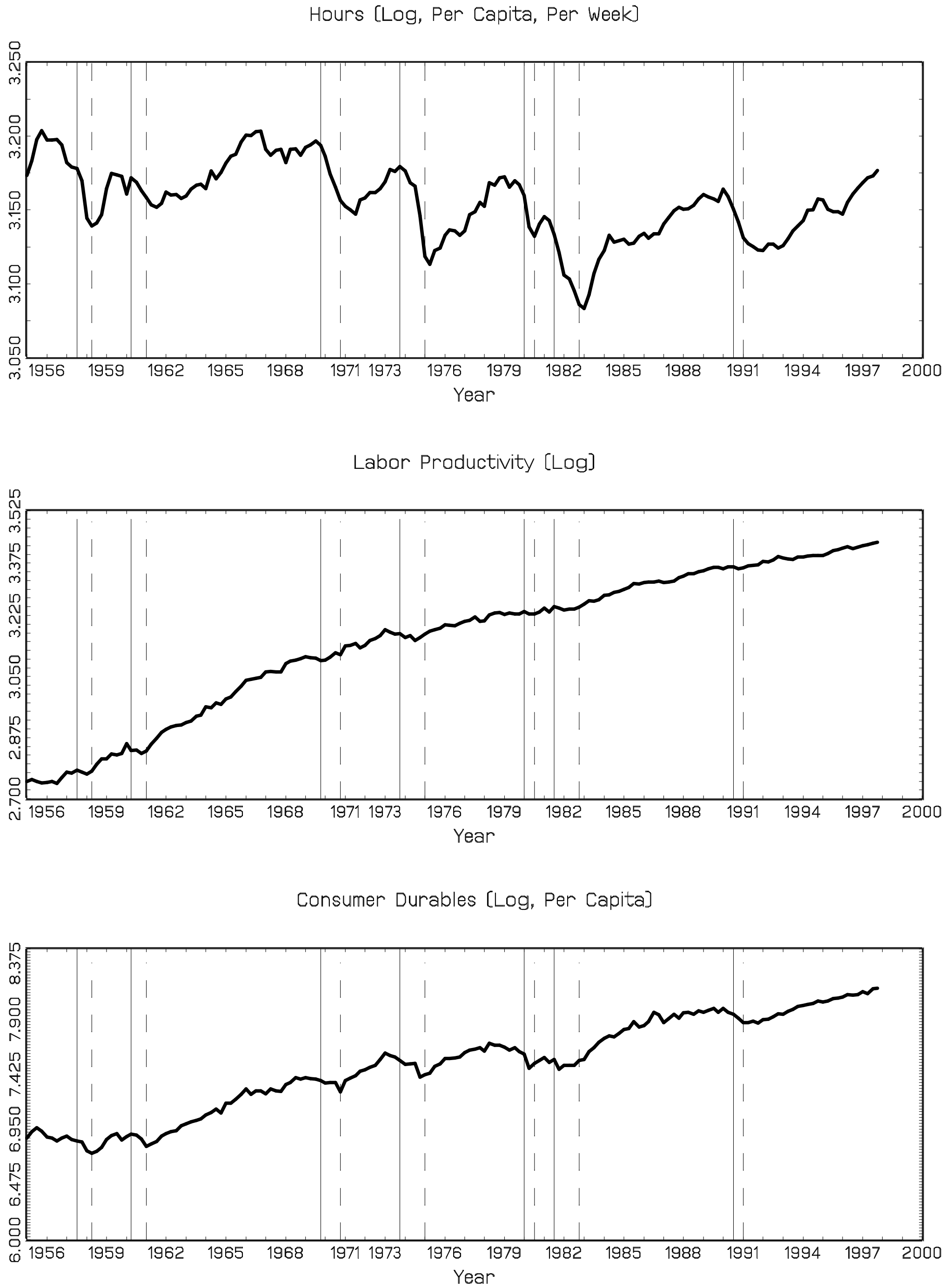


Figure 1: Time series of Hours, Labor Productivity, and Expenditures on Consumer Durables

Notes: Solid vertical lines correspond to business cycle peaks, dashed lines denote business cycle troughs (NBER Business Cycle Dating).

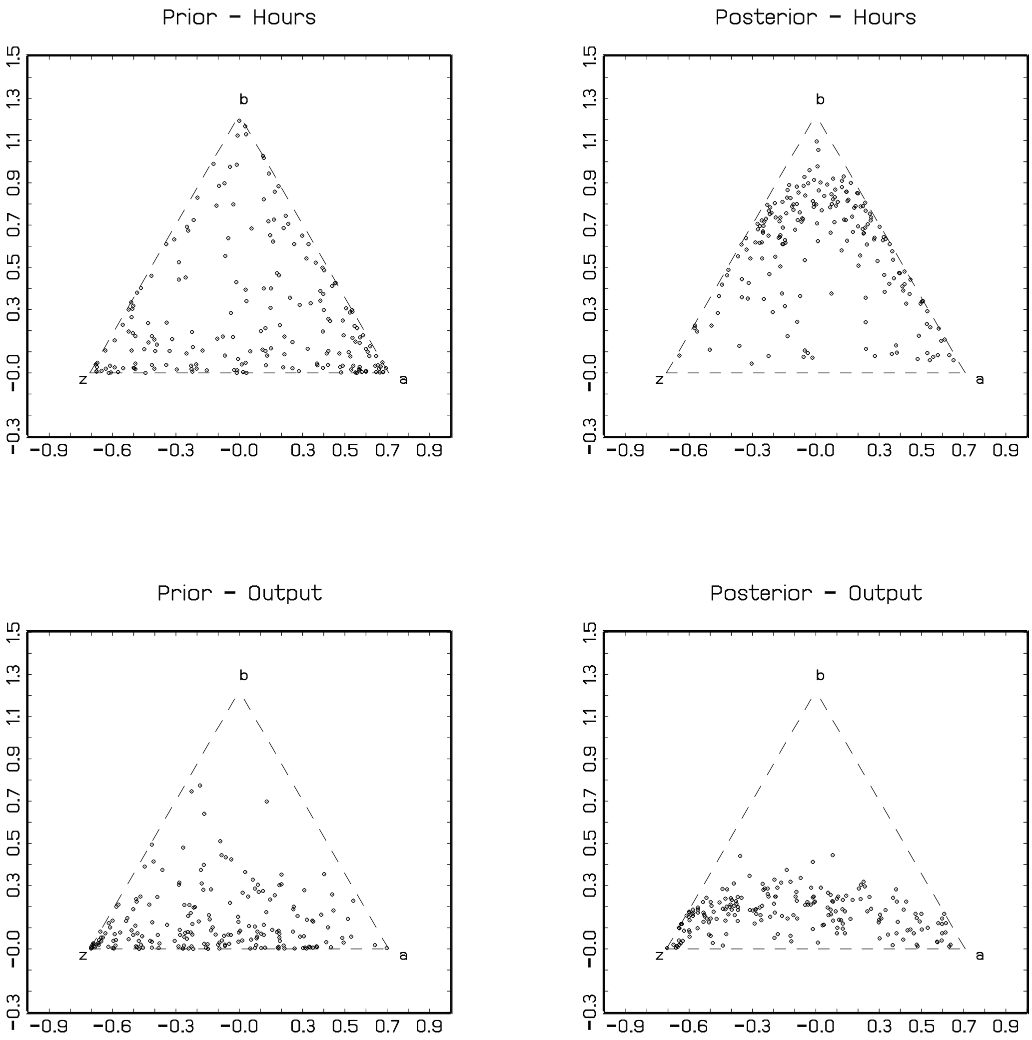


Figure 2: Spectral Density Decomposition, VAR

Notes: The frequency is 1/20 cycles per quarter. Dots represent 200 draws from prior and posterior distributions, respectively.

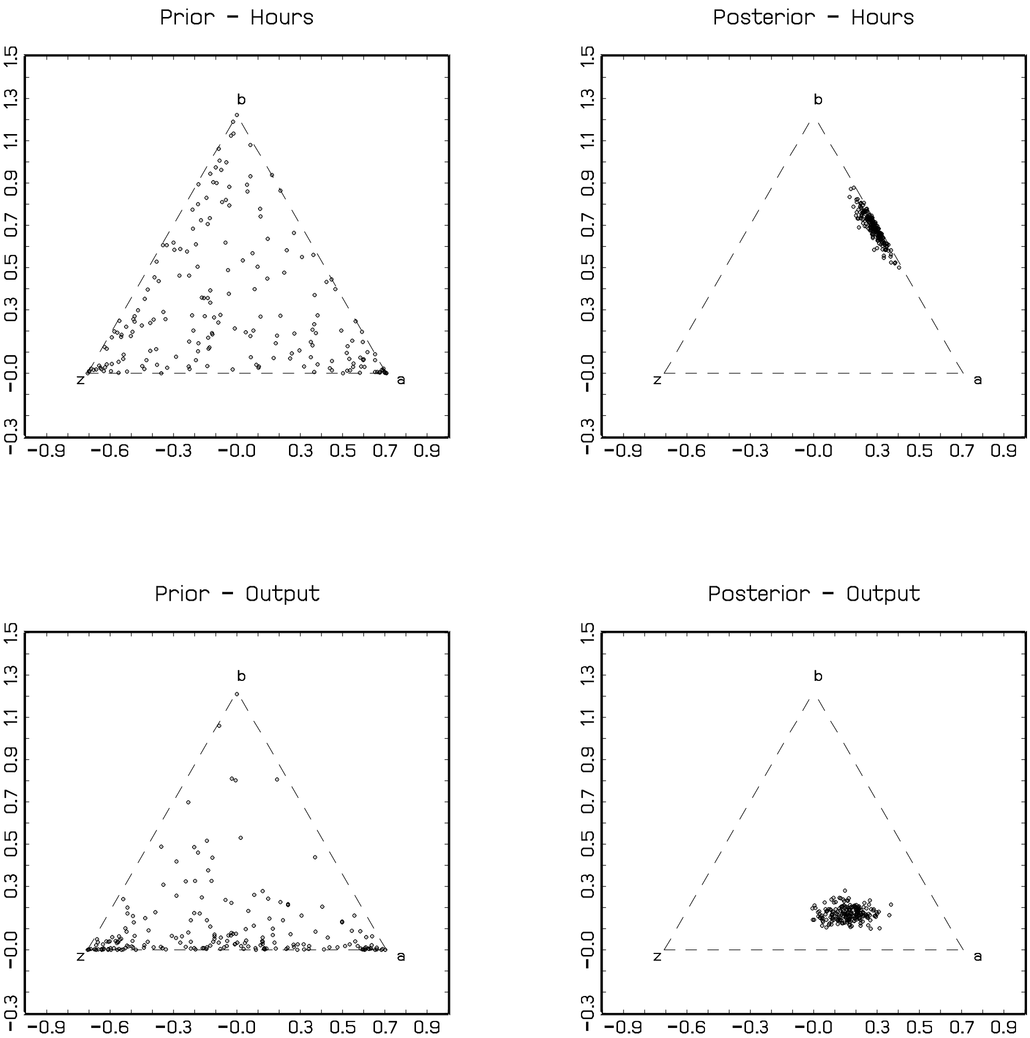
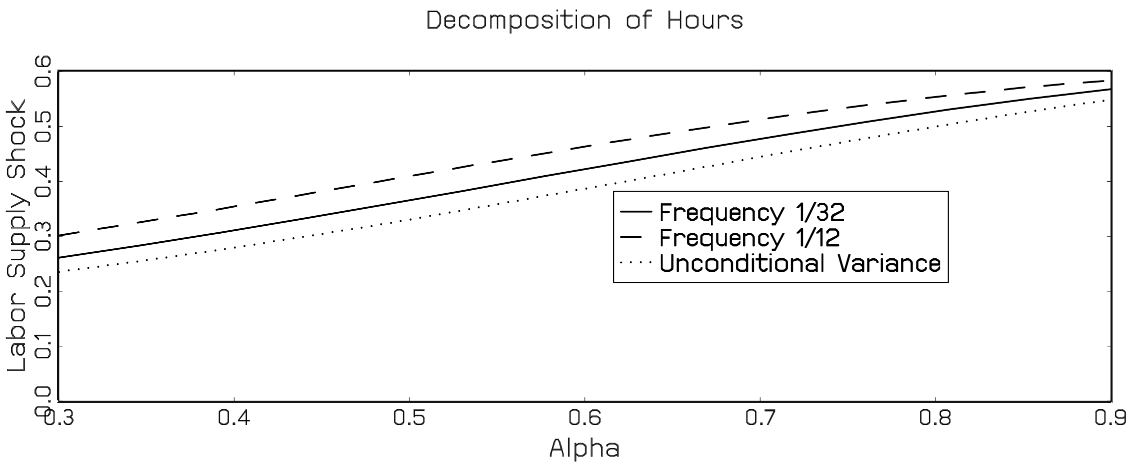


Figure 3: Spectral Density Decomposition, DSGE Model

Notes: The frequency is 1/20 cycles per quarter. Dots represent 200 draws from prior and posterior distributions, respectively.



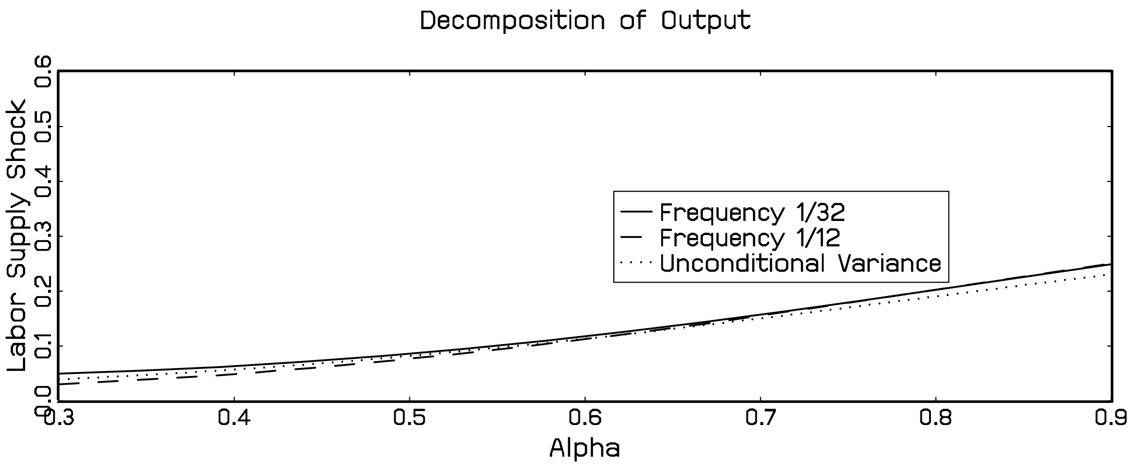


Figure 4: Robustness of Variance Decomposition

Notes: Posterior mean for the percentage of variation due to the labor supply shock. The spectral density is decomposed at 32 and 12 quarters per cycle. For output we report the decomposition of the unconditional variance of output growth.

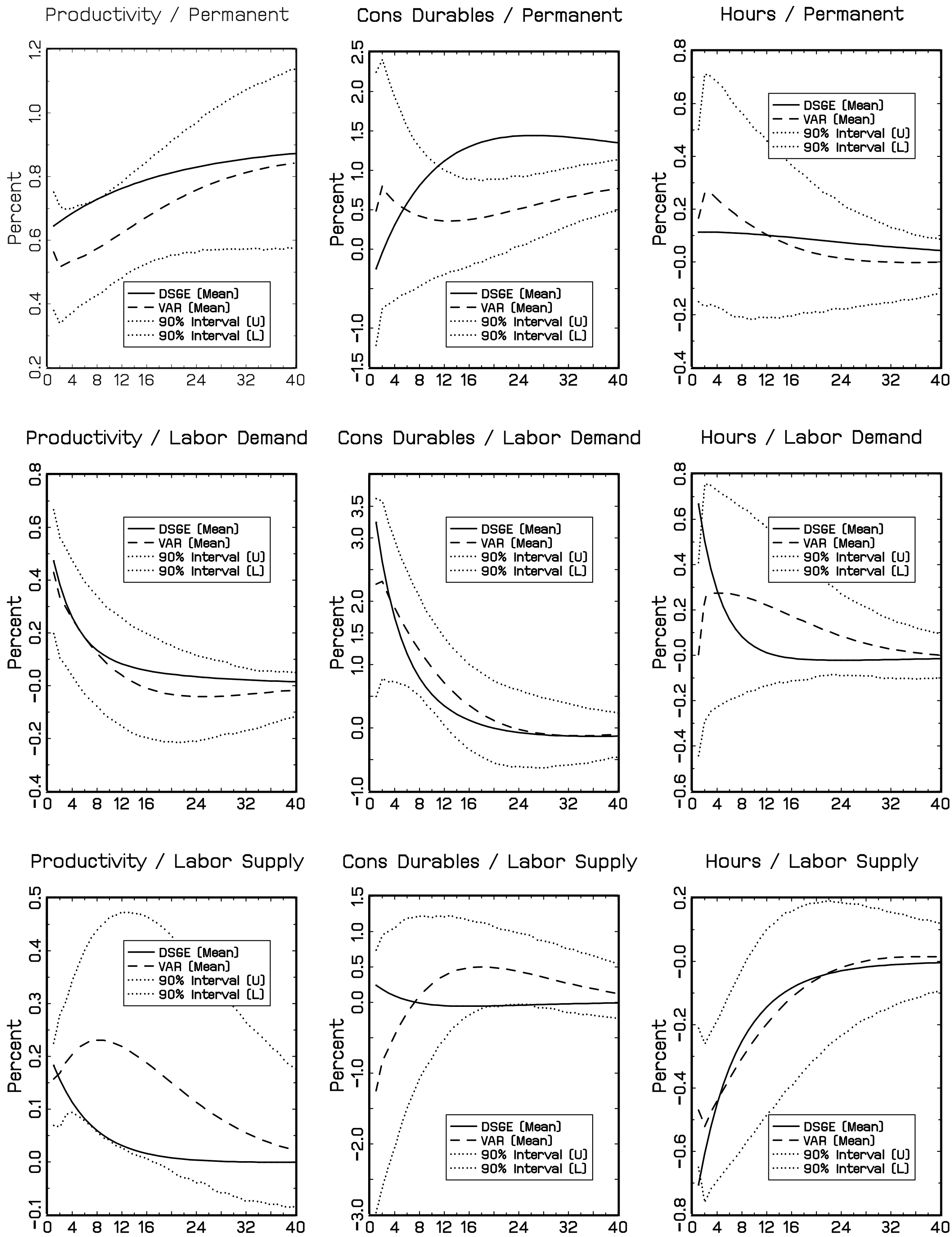


Figure 5: Impulse-response Functions

Notes: Figure depicts VAR posterior mean (solid), 90 percent Bayesian confidence interval (dotted) based on VAR posterior, and posterior mean responses of home production model (dashed).

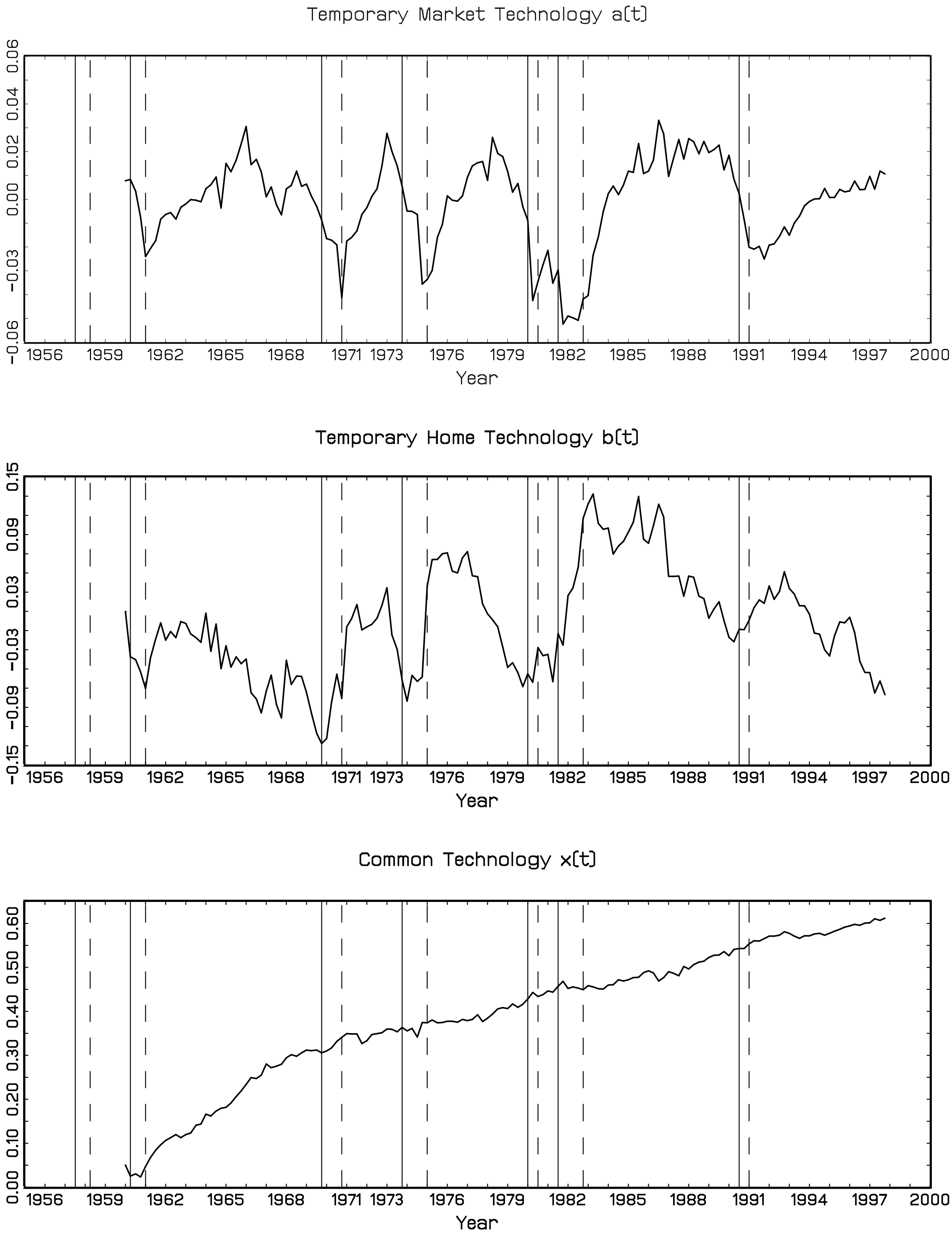


Figure 6: FILTERED TECHNOLOGY PROCESSES a_t, b_t, AND x_t

Notes: The posterior mean estimates of the latent technology processes are based on the DSGE model. Solid vertical lines correspond to business cycle peaks, dashed lines denote business cycle troughs (NBER Business Cycle Dating).