BARTER ECONOMIES AND CENTRALIZED MERCHANTS

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Abstract

The main goal of this essay is to analyze the emergence of a barter economy, and the rise of centralized merchants and a barter redistribution system out of a primitive barter system. The environment is a spatial general equilibrium model where exchange is costly. Since exchange becomes more complicated as the scope of the economy increases, we prove that, after the economy reaches a critical size, the cost of trade expansion surpasses its benefits. This imposes limitations on the scope of the economy and the production level. To overcome these limitations, rational individuals can develop a more advanced barter system leading to the appearance of centralized merchants. This more sophisticated system is the redistribution system. We also show that under some circumstances, in the presence of transaction costs it may be optimal for individuals to keep using barter instead of adopting a monetary system. This result explains why some primitive economies, like the Incas in Peru and ancient Egypt, did not evolve to a monetary system, and kept barter as their main exchange system.

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I. INTRODUCTION

The understanding of barter economies is a very important matter in economic theory. Although we are sometimes tempted to study them in the context of modern societies, doubtless the natural starting point for studying barter economies are ancient and primitive societies. As anthropologists now realize, see for example Renfrew and Bahn (1996), in the distant past our ancestors lived in a moneyless, primitive communal system in which people produced or found everything they needed to live on, and there were no organized markets. Einzig (1966, pg. 333) summarizes the conditions under which a moneyless primitive communal system existed. They encompass low levels of intelligence, absence of sense of values, low economic development, the absence of private property, no moral or religious taboos, and a scarcity or distrust of a good that could serve well as money. However, once these conditions were overcome, barter economies appeared and in time, converged to some kind of monetary economy.

Hicks (1969) argues that before the appearance of the division of labor and a systematic trading system, societies already had some type of organization. Such an organization requires the existence of cities where people interrelate. An organization like this precedes the emergence of specialized producers. Anas (1992) and Berliant and Konishi (2000) offer two excellent articles on the origins of cities.

After this initial stage of primitive trade, Polanyi (1957) finds economies moving on to economic systems with a more complete division of labor where people exchange their surpluses. He argues that a *redistribution system*, a system in which goods are sent from the production place to a market center and then redistributed, dominates non-monetary economies. In the redistribution system, individuals produce one or a few goods and consume many. They usually go to a market center, often located at the center of the city, and trade the only commodity they have for all other commodities they need. Such trade was made through merchants, which dominate the early trade based on pure barter. Polanyi argues that, although sometimes we find peddler merchants in such a system traveling from city to city, exchanging different kinds of goods and playing an important role for facilitating trade between market centers, their role was less important in the direct exchange with final consumers. Many anthropologists, for example Polanyi (1957), Einzig (1966), Davies (1995) and Renfrew and Bahn (1996), agree that the redistribution system emerged out of a primitive barter system and that, with all likelihood, money appeared under this system. Economists like Ederer (1964), Hicks (1969, 1989), and Clower (1995) also highlight the role of central merchants as a previous stage to a monetary system. However, unlike Kiyotaki and Wright (1989), where the lack of a double coincidence of wants problem is solved by the introduction of a medium of exchange, in this essay individuals overcome the problem by changing their exchange system but still keeping bartering.

Based on these ideas, the main goal of this essay is to analyze the emergence of a barter redistribution system with centralized merchants out of a primitive barter system. The environment is a spatial general equilibrium model where exchange is costly. Since exchange becomes more complicated as the scope of the economy increases, we prove that, after the economy reaches a critical size, the cost of trade expansion surpasses its benefits. This imposes limitations on the scope of the economy and the production level. To overcome these limitations, rational individuals develop a more advanced barter system leading to the appearance of centralized merchants. This more sophisticated system is the redistribution system. This explains why some primitive economies, like the Incas in Peru and ancient Egypt, did not evolve to a monetary system and kept barter as their main exchange system.

Section II shows a spatial model of a pure barter economy where individuals want to consume all goods available. Every individual, who is a specialized producer, meet and barter with every other individual in a central market to acquire the consumption goods they need. To trade successfully, they must pay transaction costs, which include transportation, and meeting costs, all of which are measured in units of time, as stressed in Howitt (1988).

Section III develops a spatial model in which the barter activity is done through centralized merchants allowing producers to lower their transaction cost and increase their production. The appearance of merchants lowers the producer's transaction cost, and induces an increase in aggregate production. Every producer, however, must give up a share of his production to pay for the merchant's services.

Section IV compares the models developed in sections II and III, and determines the conditions under which the barter economy with central merchants dominates the primitive barter economy. It shows that, after the scope of the economy[1] reaches a certain size, and technology has advanced enough, the barter economy with merchants dominates the primitive barter economy developed in section II. It also shows that, within range of the scope of the economy, a barter economy with merchants and centralized markets is optimal and there is no incentive for the emergence of money. Section V offers some conclusions.

II. THE PRIMITIVE BARTER ECONOMY

Assumptions of the Model

Suppose that there is a "long narrow city" on an interval $[-n, n]$, so the city size is $2n$. There is a continuum of N individuals living in that city that specialize in producing only one good at their location that can be totally different or simply differentiated from all others. Individuals can be identified by their location on the interval $[-n, n]$, and hence, by the good that they produce[2]. Notice that in this *Primitive Barter Economy* (*PBE*), the number of individuals is equal to the

[1] The scope of the economy is defined as the number of different or differentiated goods that the economy produces.

[2] The term "long narrow city" refers to a model of urban economics developed by Solow and Vickrey (1972). They developed a model of business land use in a linear business district, in which each business interacts a fixed number of times with every other, incurring transportation cost during the process. We define the number of goods

number of goods and $N = 2n$. Assume that the individual i (located at i) has the following utility function

$$U^i(C^i(j)) = \int_{-n}^{n} \log\left(C^i(j)\right) dj \tag{1}$$

where $C^i(j)$ is the individual i's consumption of good j. This means that every individual wants to consume every good available. Observe that if individual i wants to consume the good produced by another individual j, then i wants to consume only some of j's production but not the entire production of j. Therefore, the coincidence of wants *is not complete* and i must trade the rest of his production with other individuals until he acquires all the goods that he needs.

As suggested by Hicks (1969), there is a Central Business District (CBD) located at the center of the city, where each individual physically meets every other individual for barter purposes[3]. Suppose also that every good produced belongs to one of the following sets

$\Lambda = \{x \in [-g, g]$ such that it has a perishability rate δ_0 and a transportation cost $\tau\}$ or

$\Phi = \{x \in [-g, g]$ such that it has a perishability rate δ_1 and a transportation cost $\tau_1 = 0\}$

Notice that the set $\{\Lambda, \Phi\}$ is a partition of the interval $[-n, n]$. Assume that the set Φ is of measure zero. We can interpret this in the following way: goods in the set Φ constitute a group small enough so that its size is insignificant in comparison with the whole economy. The aggregate production and the quantity of goods in the set Φ, however, are not zero.

Assume also that the individual located at i produces q_i at the beginning of the period, with a production function $q_i = AL_i$, where L_i is the amount of labor, measured in units of time, used to produce the good i during this period, and A is a constant representing the production

as $2n$ instead of n to create symmetry among locations thus simplifying the mathematics. The assumption has no effect on the results.

[3] Beckmann (1975) and, Borukhov and Hochman (1977) set alternative spatial contact models to analyze urban economies in which every individual meets every other one.

technology. Every individual i is endowed with one unit of time available for production and exchange activities.

Once the individual is in the CBD, he must spend some time moving from one place to another inside the CBD, and meeting every other individual he needs to meet[4]. Let h be a fixed amount of time per meeting that an individual spends during his stay in the CBD. This includes the length of every meeting, and the time spent looking for another individual. Hence, the total amount of time spent in the CBD by each individual is proportional to the scope of the economy and equals to $2hn$. In addition, if the distance from location i to the CBD is its absolute value, $|i|$, it results in the following constraint:

$$L_i + \tau_i \, |i| + 2hn = 1,$$

where τ_i denotes the transportation cost rate faced by the individual located at i. Finally, assume that the producer incurs no transportation cost when he goes back to his production place with his consumption goods.

Equilibrium

Assume that the economy works in the following way. The individual i works L_i hours to produce q_i. After that, he transports the merchandize to the CBD to barter it with other individuals. At the time that he arrives at the CBD, he finds his production perished at the rate δ_i, then he barters the remaining unperished production for consumption goods $C^i(j)$. Let P_i be the price of good i. Therefore, individual i faces the following budget constraint

$$(1 - \delta_i)\,(1 - \tau_i \, |i| - 2hn)\, P_i \, \mathrm{A} \geq \int_{-n}^{n} P_j C^i(j)\,dj \qquad (2)$$

[4] There are some other costs associated with the search technology that we do not consider here since they give no additional insights. That does not mean, however, that they are not important or cannot play an important role in some other context. In particular, we refer to the depreciation cost of the good held for exchange by individuals while they are searching in the CBD. In the present context, however, this cost can be interpreted as an increase in the searching time and hence, equivalently, as a cut in production.

Individual i's problem is to maximize the utility function (1) subject to budget constraint (2). Let us now define the equilibrium.

Definition: Given a long narrow city of size $2n$, and with $2n$ producers, some preference order represented by the utility function (1), a parameter set $\{\delta_0, \delta_1, \tau, h, A\}$, and location specific production functions $q_i = A\, L_i$, we define an equilibrium in this *Primitive Barter Economy* (*PBE*) as the set $\{C^i(j), L_i, P_i\}$ for $i, j \in [-n, n]$, such that each producer maximizes (1) subject to constraints (2); and the supply of good i offered for barter equals the aggregate demand for consumption of this good by all other consumers. That is:

$$\int_{-n}^{n} C^i(k)di = (1-\delta_0)(1-2hn-\tau|k|)A, \qquad \text{if } k \in \Lambda \qquad \text{and} \qquad (3a)$$

$$\int_{-n}^{n} C^i(k)di = (1-\delta_1)(1-2hn)A, \qquad \text{if } k \in \Phi \qquad (3b)$$

From the first order conditions of the maximization problem, we obtain that $P_j\, C^i(j) = P_k\, C^i(k)$ for all goods j, k and every individual i. Substituting this into the budget constraint (2) we obtain the producer's consumption functions in a PBE:

$$C^i(k) = \frac{(1-\delta_0)(1-2hn-\tau|i|)AP_i}{2nP_k} \qquad \text{if } k \in \Lambda \qquad \text{and} \qquad (4a)$$

$$C^i(k) = \frac{(1-\delta_1)(1-2hn)AP_i}{2nP_k} \qquad \text{if } k \in \Phi \qquad (4b)$$

Q.E.D

Theorem 1: In a PBE, the equilibrium prices for Φ goods can be taken as numeraire, that is, P_i is equal to one if $i \in \Phi$ and the equilibrium prices for Λ goods are given by

$$P_j = \frac{(1-\delta_1)(1-2hn)}{(1-\delta_0)(1-2hn-\tau|j|)} \qquad \text{for any } j \in \Lambda. \qquad (5)$$

Proof: From the individual's maximization problem we found that $P_j\, C^i(j) = P_k\, C^i(k)$ for all goods j, k and every individual i. Applying the Lebesgue integral operator to both sides of this identity,

$$P_j \int_\Lambda C^i(j)di = P_k \int_\Lambda C^i(k)di$$

If k and j are in Φ, we see from (3b) that the integrals on both sides are identical. Therefore prices of goods in the set Φ are all identical and we can use the goods in Φ as numeraire. If $j \in \Lambda$ and $k \in \Phi$, the last equation becomes:

$$P_j(1 - \delta_0)(H - \tau|j| - 2gh)\mathrm{A} = P_k\,(1 - \delta_1)(H - 2gh)\mathrm{A}$$

But recognizing that $P_k = 1$, we obtain:

$$P_j = \frac{(1-\delta_1)(H-2gh)}{(1-\delta_0)(H-2gh-\tau|j|)} \qquad \text{for any } j \in \Lambda.$$

Q.E.D

If the perishability rate is the same for both sets, Λ and Φ, or lower for goods in set Φ ($\delta_1 \leq \delta_0$), then the denominator in (5) is smaller than the numerator. In this case, goods in the set Λ with positive transportation costs are more expensive than the goods in the set Φ, which have no transportation cost. However, this conclusion may change if the perishability rate is much higher for goods in Φ than for goods in Λ. Notice also, that goods in Λ become more expensive the farther away the production site. Considering equations (4) and (5), and the fact that Φ goods are numeraire, we can write consumption functions as:

$$C^i(k) = \frac{(1-\delta_0)(1-2hn-\tau|k|)A}{2n}, \qquad \text{for any } k \in \Lambda, \text{ and} \qquad (6a)$$

$$C^i(k) = \frac{(1-\delta_1)(1-2hn)A}{2n}, \qquad \text{for any } k \in \Phi, \qquad (6b)$$

for every individual in the interval [-n, n] wanting to consume good k. Notice that all individuals (regardless of their location) consume the same quantity of a particular good $k \in \Lambda$. This quantity (consumed by all individuals) is lower the farther away from the center this good is produced. Hence, centrally produced goods are consumed in larger quantities because they are cheaper, and peripherally produced goods are consumed in lower quantities because they are more expensive. As for goods $k \in \Phi$, the same quantity is consumed by everyone and for all goods $k \in \Phi$. From the producer's point of view, the increase in price as a function of the location of the production site is completely offset by the higher cost of transporting the output to the CBD. Therefore, after bringing their goods to market and valuing them at market prices, all producers are equally wealthy. Since they also face the same prices, their consumption vectors are perfectly symmetric.

Notice from (6a, b) that $C^i(k) = C^j(k)$ for every good k and every two individuals i and j. Therefore, we find that *in equilibrium, all individuals*, (*producers/consumers*) *have the same utility level*. This result shows that equilibrium prices in the PBE play the role of land rent functions in spatial economic models, since they adjust to offset differences in the utility levels among individuals. Since all individuals have the same utility regardless of their location, there is no incentive for anyone to relocate in equilibrium.

Welfare Analysis

If we consider any welfare function in which all individuals are treated equally and social welfare is an increasing function of any one individual's utility, since all individuals have the same equilibrium utility level, a welfare optimum is reached by maximizing the equilibrium utility level. The symmetry in goods locations with respect to the CBD location allows us to make one more simplification: we can consider only one side of the city, that is, those goods in the interval [0, n], and can define the social welfare function as:

$$W_{PBE}(n) = \int_{[0,n]} \log\left(C(j)\right) dj$$

Using consumption functions (6a, b) and solving the integral we obtain:

$$W_{PBE}(n)=\frac{[1-(2h+\tau)n]\{1-Log a_0[1-(2h+\tau)n]\}-[1-2hn]\{1-Log a_0[1-2hn]\}}{\tau}$$

$$-\,n\,Log\,(2n) \qquad (7)$$

where $a_0 = (1 - \delta_0)A$, is a technological parameter.

Theorem 2: The scope of the economy ($2n$) is bounded by $2/(2h + \tau)$.

Proof: From (7) we can directly see that the welfare function is defined if and only if the terms inside the logarithm expressions are positive, that is, $1 - 2hn > 0$ and $1 - (2h + \tau)n > 0$. If the second inequality holds, then the first inequality also holds, so this inequality imposes a restriction on the size of the economy.

Q.E.D

From theorem 2, we conclude that the economy cannot be so large (the city can not be so long). Otherwise, it would be impossible to produce some peripheral goods since, at those locations, no time would be left for production after transportation and transaction times are netted out of the total time endowment.

Theorem 3: The welfare function has the following properties:

a) is a concave function of n, has a maximum at n^*, where $0 < n^* < 1/(2h + \tau)$,

b) W_{PBE} tends to zero as n tends to zero,

c) W_{PBE} tends to a positive number as n tends to $1/(2h + \tau)) > 0$, and

d) W_{PBE}' tends to infinity as n tends to zero.

Proof: Taking the derivative of (7) and rearranging terms, we obtain

$$\tau W_{PBE}'(n) = -\tau + Log\,\frac{a_0^{\tau}[1-(2h+\tau)n]^{2h+\tau}}{(1-2hn)^{2h}(2n)^{\tau}}.$$

Substituting n by zero in (7) and this expression shows that $W_{PBE}(0) = 0$ and W_{PBE}' tends to infinity as n tends to zero. Notice also that, as n tends to the population's least upper bound, $1/(2h + \tau)$, W_{PBE} tends to

$$\left(\frac{2h}{2h+\tau}\right)\left[Loga_0\left(\frac{2h}{2h+\tau}\right)-1\right],$$

which is positive is the technological parameter, a_0, is high enough. On the other hand, W_p' = 0 implies that

$$[1-(2h+\tau)n]^{2h+\tau}=\left(\frac{2e}{a_0}\right)^{\tau}(1-2hn)^{2h}(2n)^{\tau}.$$

where e is the Neper number. Computing the derivatives on each side of the this equation directly verifies that the left-hand side is a concave and decreasing function of n, and positive in the interval $[0, 1/(2h + \tau)]$, and the right-hand side is a concave function that passes through the origin, has a maximum in $n = t/[2h(1+\tau)] < 1/2h$, and is positive in the interval $[0, 1/2h]$. Consequently, both function must intersect somewhere between 0 and $1/(2h + \tau)$

Q.E.D

Using this information, the welfare function can be drawn as shown in Figure 1. Note now some interesting properties of the primitive barter economy. First, notice that, in their shopping trip, every individual meets every other individual at the CBD. There is not, however, a "complete" double coincidence of wants, since individual i only exchanges a small share of their production with individual j.

FIGURE 1 COMES HERE

Second, the number of goods produce satisfaction. As in Dixit and Stiglitz (1977), the utility function expresses the fact that the consumer has a taste for variety in consumption goods[5]. As the consumption of a specific good tends to zero, its marginal utility shoots up to infinity. With such a utility function, the consumer will always choose to consume a quantity of any new goods, regardless of their price. Third, as already explained, at equilibrium, every individual consumes the

[5] Dixit and Stiglitz (1977), however, used a C.E.S. utility function instead of the log used here.

same amount of every good. Notice from (5) that prices reflect the unit transportation cost, τ, as well as the time per transaction, h.

Fourth, growth in the PBE has a limitation. Population in the PBE has an upper bound of $1/(\tau + 2h)$ and hence the city size, the scope of the economy and the aggregate production are bounded. There are two reasons for that. The first is that, with the number of goods greater than or equal to $1/(\tau + 2h)$, people dwelling at this distance or farther would spend their whole time traveling to the CBD. The second is that, as population increases, the number of goods also increases, and individuals spend too much time in shopping activities, so the marginal utility of new goods is less than the marginal disutility caused by the shopping cost, causing the individual's utility to decrease.

III. THE BARTER ECONOMY WITH MERCHANTS

The growth limitations in the PBE are given by the time needed for barter and transportation. The incomplete double coincidence of wants causes producers to dispose much time in exchange activities, and hence limit their production capacity causing a growth limitation. We will now set up a model in which a barter economy with centralized merchants overcome the absence of the double coincidence problem, and allow the economy to grow further.

Assumptions of the Model

Assume now that there are F *busy and centralized merchants* who produce nothing but devote their time facilitating exchanges. Thus, it remains only $N - F$ individuals producing consumption goods. Let $2g$ denote the number of producers and hence the number of goods. Hence,

$$N = 2n = 2g + F. \qquad (8)$$

A centralized merchant means that the merchant receives the producer's production and exchanges it for all other consumption goods, that is, the merchant receives the only good that the producer offers and gives him a "basket" of the $2g$ goods existing in the economy.

A busy merchant means that the merchant spends his whole time in exchange activities. He devotes his time to two kinds of exchange activities. The first is the exchange with producers, which is basically the time spent in every meeting. Let η be the amount of time that a merchant spends meeting each producer. The second kind of exchange activity is the time that every merchant spends dealing with other merchants. Since there are F centralized merchants that exchange all consumption goods for only one good with the producer, each merchant obtains directly from the producer, at most, $2g/F$ goods. To acquire all other goods that he needs to exchange with the producer, the merchant must deal with other merchants to purvey their storage. Assume that in doing this, he must meet every other merchant. Let φ denote the average amount of time that a merchant spends searching and meeting all other merchants. Hence,

$$F = 2g\,\eta + F\,\varphi, \tag{9}$$

Merchants are located inside the CBD and producer i at a distance $|i|$ away from the CBD, so the city size is now $[-g, g]$.

The economy works as follows. The producer located at i works L_i hours to produce q_i. At the end of the period, he transports his output to the CBD. When he arrives at the CBD he finds that his merchandize has diminished by a factor δ_i. Then, he barters the remaining merchandize with a merchant in exchange for consumption goods. In this way, the producer only needs one meeting for exchange purposes and has more time to devote to production.

Notice that the producer reduces substantially the time spent meeting other individuals. Assume, for simplicity, that it becomes insignificant, so it is set to zero. Note also that, while producers are still located outside the CBD, merchants are locate inside the CBD, so the farthest producer needs to locate at a distance not greater than $2g$, so the city length is also $2g$.

Producer i sells his whole production to the merchant for a price P_i, and buys the consumption goods he needs for a price ξP_i, where ξ is the merchant's markup[6]. As before, there are two sets of goods Λ and Φ with the characteristics as already defined. Hence, the producer's problem is to maximize the utility function (1) subject to the following budget constraints:

$$(1 - \delta_i)\, P_i\, \mathrm{A}_i(1 - L_i - \tau_i\, |i|) = \int_{[-g,g]} \xi P_j C^i(j)dj\,, \tag{10}$$

On the other hand, assume that the F merchants are identical and buy an amount $(1 - \delta_i)\, q_i$ from producer i for a price P_i, and sells it for a markup ξP_i. Assume also that every merchant sets his own markup and knows the markup set by all other merchants. Assuming that merchants act competitively, in equilibrium, all merchants will set the same markup and will have the same profits. Therefore, we can obtain every individual merchant's profit by finding the average merchant profits. Hence, merchant f 's problem is to maximize his utility (1) subject to the budget constraint[7]

$$\frac{(\xi - 1)}{F} \int_{[-g,g]} (1 - \delta_j) P_j q_j dj = \int_{I_f} \xi P_j C^f(j)dj \tag{11}$$

As before, assume that the producer does not incur any cost to transport himself or the consumption goods that he brings back to his place of production.

Equilibrium

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[6] The reader may think that the producer is foolish because he can do better by keeping a portion of his own production and save the merchant's markup on that. The reason for this assumption is mathematical simplicity. Since the producer, however, specializes in producing only one good, its production is insignificant and measures zero in respect to the aggregate production. This means that this assumption has no influence on the results.

[7] For mathematical simplicity, we assume that merchants acquire their own consumption from producers and sell it to themselves charging the markup. However, since the merchant's own consumption is a set of measure zero, this assumption does not affect any result.

Definition: Consider a long narrow city of size $2g$, with N $(= 2n)$ individuals, the preference order implied by (1), a parameter set $\{\delta_0, \delta_1, \tau, \eta, \varphi, A\}$, and a production function $q_i = A_i L_i$. We define an equilibrium in this *Barter Economy with Merchants* (*BEM*) as the set $\{C^i(j), C^f(j), L_i, P_i, \xi, 2g, F\}$ for $i, j \in [-g, g]$, $f \in [0, F]$ such that the following conditions hold

i) Every producer maximizes (1) subject to constraints (10),

ii) Every merchant maximizes (1) subject to constraint (11),

iii) Good i's production offered for barter in every period t equals the aggregate consumption for this good during this period. That is,

$$\int_{-g}^{g} C^i(k)di + \int_0^F C^f(k)df = (1-\delta_0)(1-\tau|k|)A \qquad \text{for } \forall\, k \in \Lambda \tag{12a}$$

and

$$\int_{-g}^{g} C^i(k)di + \int_0^F C^f(k)df = (1-\delta_1)A \qquad \text{for } \forall\, k \in \Phi. \tag{12b}$$

iv) Every individual chooses the occupation that is most convenient for him, that is, individuals choose between being merchants or producers.

v) Producers and merchants are busy, that is, they spend the whole of their available time producing and exchanging goods.

Observe that equilibrium condition (iii) allows us to find the producers' equilibrium prices. On the other hand, from equilibrium condition (iv) we find the equilibrium markups. This condition, in some way, is equivalent to the zero profit equilibrium condition in conventional microeconomic models in which producers maximize profits. Since individuals are free to choose their jobs, if merchants' markup is too high, merchants may achieve a utility level higher than producers', then more producers will become merchants and the markup will decrease. Therefore, merchants' markup will adjust to equalize producers and merchants' utility level. Finally, the equilibrium condition (v) will allow us to determine the number of merchants and producers, so these two variables become endogenous.

From the first order conditions and the budget constraint of the maximization problems we find the following consumption functions:

$$C^i(k) = \frac{(1-\delta_0)(1-\tau|i|)AP_i}{2g\xi P_k} \quad \text{for} \quad i, k \in [-g, g] \quad \text{and} \quad i \in \Lambda \tag{13a}$$

$$C^i(k) = \frac{(1-\delta_1)AP_i}{2g\xi P_k} \quad \text{for} \quad i, k \in [-g, g] \quad \text{and} \quad i \in \Phi. \tag{13b}$$

$$C^f(k) = \frac{(\xi-1)(1-\delta_0)A}{2g\xi F P_k}\int_{\Lambda}(1-\tau|i|)P_i dj \quad \forall\, k \in [-g, g],\ f \in [0, F]. \tag{13c}$$

Theorem 4: In a BEM, the equilibrium prices of Φ goods can be taken as numeraire and the equilibrium prices for Λ goods are given by

$$P_k = \frac{1-\delta_1}{1-\delta_0}\frac{1}{1-\tau|k|} \quad \text{for every } k \in \Lambda \tag{14}$$

Proof: Follows from plugging the consumption functions (9) and (11) into the equilibrium conditions (12) and doing some algebra. See Appendix for details.

Now, using (13) and (14), we obtain a producer and a merchant's demand functions for consumption goods:

$$C^i(k) = (1-\delta_0)\frac{(1-\tau|k|)A}{2g\xi} \quad \text{for} \quad k \in \Lambda \quad \text{and} \quad \forall\, i \in [-g, g], \quad \text{and} \tag{15a}$$

$$C^i(k) = (1-\delta_1)\frac{A}{2g}\frac{1}{\xi} \quad \text{for} \quad k \in \Phi \quad \text{and} \quad \forall\, i \in [-g, g]. \tag{15b}$$

$$C^f(k) = (1-\delta_0)(1-\tau|k|)\frac{A}{F}\frac{(\xi-1)}{\xi} \quad \text{for} \quad k \in \Lambda, \tag{15c}$$

$$C^f(k) = (1-\delta_1)\frac{A}{F}\frac{(\xi-1)}{\xi} \quad \text{for} \quad k \in \Phi, \tag{15d}$$

Notice that the difference between individuals' demand for the Λ and Φ goods comes from the time devoted to production and the perishability rate. This is because members of the set Λ have less time available for production since they need to devote some time to transportation.

Theorem 5: In a BEM, the equilibrium number of producers and merchants are

$$2g = \left(\frac{1-\varphi}{1-\varphi+\eta}\right)N, \tag{16a}$$

$$F = \left(\frac{\eta}{1-\varphi+\eta}\right)N. \tag{16b}$$

and the merchant's markup is

$$\xi = 1+\frac{\eta}{1-\varphi} \quad , \tag{16c}$$

Proof: (16a, b) come directly by solving equations (8) and (9). On the other hand, since individuals are free to choose the most convenient occupation, producers may become merchants if merchants' utility is higher and vice-versa. Therefore, in equilibrium, the utility level for both producers and merchants must be equal. That is, $U^i = U^f$ for every producer i and every merchant f. A sufficient condition for this is that $C^i(k) = C^f(k)$ for every $k \in \Lambda$. Then from (15a) and (15c) follow (16c).

Q.E.D

Taking derivatives on (16a, b) we obtain:

$$\frac{\partial 2g}{\partial \varphi} = \frac{-\eta}{(1-\varphi+\eta)^2}N < 0; \qquad \frac{\partial 2g}{\partial \eta} = -\frac{1-\varphi}{(1-\varphi+\eta)^2}N < 0 \qquad \text{and}$$

$$\frac{\partial F}{\partial \varphi} = \frac{\eta}{(1-\varphi+\eta)^2}N > 0; \qquad \frac{\partial F}{\partial \eta} = \frac{1-\varphi}{(1-\varphi+\eta)^2}N > 0$$

This means that the equilibrium number of merchants decreases if merchants spend less time either bartering with producers or among them, that is, if the exchange technology employed by merchants improve, fewer merchants are needed and the number of producers and the scope of the economy increases. This improvement in merchants' technology comes if merchants improve either the barter technology itself or the meeting cost.

On the other hand, (16c) shows that the equilibrium markup is one plus the ratio between the time spent by the merchant for meeting one producer and his total time availability to meet with producers. This ratio is the profit obtained by the merchant for facilitating the exchange of one unit of production. Notice that the equilibrium markup decreases as the merchant's technology for transactions improves, that is, φ or η decreases. Substituting (16a, b, c) into the consumption functions (15a-d) we obtain individual consumption functions:

$$C^i(k) = (1-\delta_0)(1-\tau|k|)\frac{A}{N} \qquad \text{for } k \in \Lambda \text{ and } \forall\, i \in [-g, g], \qquad (17a)$$

and

$$C^i(k) = (1-\delta_1)\frac{A}{N} \qquad \text{for } k \in \Phi \text{ and } \forall\, i \in [-g, g]. \qquad (17b)$$

Observe that all individuals consume the same quantity of a particular good $k \in \Lambda$ and the same quantity of all goods $k \in \Phi$. This means that the total production of every good available for consumption is distributed equally among the whole population, and does not depend on the merchant's cost.

Welfare Analysis

Notice that, unlike the case of the PBE, in the BEM there are $2g$ instead of $2n$ consumption goods. Hence, I define the welfare function for the BEM as

$$W_{BEM}(g) = \int_0^g \log\left(C(j)\right) dj\,.$$

Let's denote $\alpha = (1 - \varphi)/(1 - \varphi + \eta)$, then $g = \alpha\, n$. Hence, using this, (16a) and some algebra, the welfare function can be written as

$$W_{BEM}(n)=\frac{[1-\tau\alpha n]\{1-Log[\alpha a_0(1-\alpha\tau\, n)]\}-1+Log\alpha a_0}{\tau}-\alpha nLog(2\alpha n)\,. \qquad (18)$$

Notice that an individual's welfare increases as the exchange technology that the merchant uses improves, so φ and η decrease. It is also clear that it increases as the transportation cost decreases. This means that there is a strong relationship between transaction costs and individuals' welfare.

Theorem 6: In a BEM, the population is bounded by $2/\alpha\tau$, and the scope of the economy by $2/\tau$.

Proof: From (18) is directly to check that the welfare function is defined only for n less than $1/\alpha\tau$, and considering that $g = \alpha\, n$, the scope of the economy is bounded by $2/\tau$.

Q.E.D

The inequality that follows from theorem 6 imposes a limitation on the BEM. The number of goods cannot grow to more than $2/\tau$, and the population to more than $2/\alpha\tau$. The reason is that, if the number of goods becomes that high, transportation cost would not allow enough time for the production of those goods at their distant locations.

Theorem 7: In the BEM, W_f tends to zero as n tends to zero, it tends to a positive number as n tends to $1/\tau$, and the welfare function is a concave function of n, that has a maximum at

$$0<\ n^{**}=\frac{a_0}{2e+\alpha a_0}<\frac{1}{\tau}\,. \qquad (19)$$

Proof: Taking the limits as n tends to zero, and tends to $1/\tau$, we can easily check that W_f tends to zero, and to $(Log(\alpha\, a_0) - 1)$, which is positive if the technology parameter a_0 is high enough. Taking the derivative of (18) and rearranging terms, we obtain

$$\tau W_{BEM}\,'(n)=-\tau+\tau Log\frac{a_0(1-\alpha\tau\, n)}{2n}\,.$$

n^{**} follows from solving the equation $W_{BEM}(n) = 0$. Finally, note that $n^{**} < 1 < 1/\tau$.

With the information provided by theorems 6 and 7, the welfare function can be drawn as shown in Figure 2.

FIGURE 2 COMES HERE

V. THE PRIMITIVE BARTER ECONOMY AND THE BARTER ECONOMY WITH MERCHANTS COMPARED

Let's now compare the two exchange systems. Consider the difference $\Delta W = W_{BEM} - W_{PBE}$. If this difference is positive, the BEM dominates the PBE and the economy with merchants prevails. Otherwise, the PBE dominates and no merchant is around. From (7) and (18), this difference can be written as

$$\Delta W(a_0, n) = \frac{[1-(2h+\tau)n]\{1 - Log a_0[1-(2h+\tau)n]\} - [1-2hn]\{1 - Log a_0[1-2hn]\}}{\tau}$$

Given parameters h and τ, the sign of ΔW depends of the population size, n, and the technological parameter, a_0. Solving $\Delta W \geq 0$ for $Log(a_0)$, we obtain that

$$Log\ a_0 \leq 1 + Log(2n) + \frac{1}{(1-\alpha)\tau n} Log \frac{[1-(2h+\tau)n]^{1-(2h+\tau)n}}{(1-2hn)^{1-2hn}(1-\alpha\tau\ n)^{1-\alpha\tau\ n}} \tag{20}$$

Taking the first derivative on the equality in (20) we find

$$\frac{d(Log\ a_0)}{dn} = \frac{1}{(1-\alpha)\tau n} Log \frac{1-(2h+\tau)n}{(1-2hn)(1-\alpha\tau\ n)}$$

This expression is positive for $n > 0$. This means that $Log(a_0)$ is an increasing function of n. Above this curve the PBE dominates the BEM.

Let's denote by K_0 the technological level at the moment when the producer specializes and hence starts to exchange. Then we can use Figure 3 to understand the dynamics of the emergence of a BEM system. Below the horizontal line K_0, the producer has not specialized yet. Hence, there is no trade, and the economy (city size) can increase indefinitely. The PBE cannot growth beyond $n = 1/(\tau+2h)$. Consequently, the PBE prevails in the upper area of the curve

$ABCK_0$. On the other hand, the BEM cannot grow beyond $n = 1/\tau$, and it prevails in the area inside the curve *ABCDE*.

There are several facts that are worthy to note in comparing the BEM and the PBE. First, observe that the appearance of merchants allows the economy to grow beyond the PBE limitations, in terms of population size, without the need to evolve into a monetary economy.

Second, in time, both population and technology increase, so the evolution process follows a path like one of the arrows v_i shown in Figure 3. Arrows v_1, v_2, and v_3 show that the economy can evolve from a primitive communal economy to either a PBE or a BEM. If the technology is still too rudimentary and the population is small enough, the division of labor leads to a PBE (arrow v_1). However, if the division of labor occurs in a city that has already a population big enough, it would be very costly for individuals to barter directly with each other and somebody will realize quickly that it is a good business to become a merchant, and the economy would evolve directly to a BEM (arrow v_2). Notice that arrows v_3 and v_4 together show that it is possible for an economy to have several drawbacks in its evolutionary process. It can evolve first to a BEM, and after return back to a PBE. However, the force of the population growth will always push it to the better organized barter system with merchants.

On the other hand, the BEM cannot grow to more than $1/\tau$ goods. Nevertheless, unlike the PBE, in this case the limitation only comes because the transportation cost to the CBD makes it impossible to produce beyond a radius of $1/\tau$. Notice, however, that the producer's limitation due to the time spent in shopping activities disappears, and he only consumes a fixed amount of time (η) in interacting with the merchant. This means that the BEM can grow bigger than the PBE. This is observed from theorems 2 and 6 and illustrated in Figure 3. As the number of goods grows, the BEM will eventually dominate the PBE, as shown by arrows v_4 and v_5. These strong conclusions explain Polanyi's findings that a redistribution system dominates non-monetary economies.

Third, note that the appearance of the centralized merchant allows the incomplete double coincidence problem to be overcome since the producer already knows that the merchant will accept his entire production in exchange for the consumption goods that he needs.

Fourth, in the BEM, producers go to the central CBD only to meet the merchant and not every other individual. This gives them much more time to produce. On the other hand, they must give up a share of their production to the merchant who facilitates exchange activities.

Fifth, in both models, goods in Φ are chosen as numeraire since all of them have the same price. However, unlike the PBE, in the BEM prices in the set Λ reflect only the transportation cost (τ). This means that, in the BEM, prices play the role of the land rent function in urban economic models. On the other hand, merchants' markup (ξ) reflects the exchange technology employed by the merchant, represented by parameters η and φ. Notice then, that the specialization between producers and merchants allows setting prices reflecting the activity and job performed.

Sixth, in both the PBE and the BEM, different individuals consume the same amount of every good. This is a consequence of the individuals' utility function, in which all consumption goods have the same weight, and the assumption that every individual chooses the most convenient occupation and hence has the same income.

Seventh, one of the main features of ancient BEMs was the existence of a strong centralized government that organized the redistribution of goods. In our model, this can be interpreted as follows: the government sets the CBD in a central place and plays the role of the merchants. It needs, however, to hire people to do the merchant's job, so F is the optimal amount of merchants that the government should hire and ξ is the tax revenue that the government receives for its services.

Notice that in this case, if the government decides to increase taxes excessively[8], that is ξ increases excessively, then welfare would decrease excessively also, and the whole system may fall. This was, perhaps, as argued by Einzig (1966, pg. 194), the main reason for the downfall of Egypt's IVth Dynasty.

VI. CONCLUSION

The appearance of centralized merchants represents one step in the specialization process that allows increasing the advantages of trade. This is possible because the appearance of centralized merchants allows producers to save time in exchanging activities and hence increase their production. Consequently, individuals obtain a higher level of consumption and welfare.

Notice that although the assumptions of the model determine the individual who bears a particular cost (transaction, perishability or transportation cost), it is clear that as the economy becomes more complex, the bearing of the costs becomes more specialized. Different costs are shifted among agents as the economy evolves, allowing producers to increase their production and individuals' utility level.

An important feature of this model is that the appearance of the centralized merchants allows a lowering of production costs and the creation of new goods. This allows the economy to grow beyond the limitation imposed by the primitive barter system and increases the scope of the economy and individuals' welfare. Another important point of the model is that an improvement in transportation technology allows growth in both the number of goods produced and the individuals' welfare.

[8] In this case this could be proxied by the hiring of too many merchants.

Note also that an improvement in the exchange system allows an increase in the efficiency of the economy and the individual’s welfare without the need to use a medium of exchange. Moreover, this occurs as a consequence of the individual's rational decisions. However, it is necessary that the economy reaches a certain level of production technology and population size before this evolution takes hold.

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APPENDIX A

PROOF OF THEOREM 4:

Let's find first the producers' equilibrium prices. First, assume that $k \in \Phi$, from (12b) we have

$$\int_{\Lambda} C^i(k)di + \int_{\Phi} C^i(k)di + \int_{[0,F]} C^f(k)df = (1-\delta_1)A \qquad \Rightarrow$$

$$\int_{\Lambda} \frac{(1-\delta_0)(1-\tau|j|)AP_j}{2g\xi P_k} dj + \int_{[0,F]} \left(\frac{(\xi-1)(1-\delta_0)A}{2g\xi F P_k} \int_{\Lambda} (1-\tau|j|)P_j dj \right) df$$

$$= (1 - \delta_1) A \qquad \Rightarrow$$

$$\frac{(1-\delta_0)}{2g\xi P_k} \int_{\Lambda} (1-\tau|j|)P_j dj + \frac{(\xi-1)(1-\delta_0)}{2g\xi P_k} \int_{[0,F]} (1-\tau|j|)P_j df = (1-\delta_1) \qquad \Rightarrow$$

$$P_k = \frac{1-\delta_0}{1-\delta_1} \frac{1}{2gH} \int_{\Lambda} (1-\tau|j|)P_j dj \,. \tag{A1}$$

Notice that P_k does not depend on location k if $k \in \Phi$, and that all are identical, thus $P_j = P_k$ if both $j, k \in \Phi$. This allows us to choose goods in Φ to be numeraire so that

$$P_k = 1, \qquad \text{if } k \in \Phi. \tag{A2}$$

Now, since $P_k = 1$ for $k \in \Phi$, we obtain, from (A1),

$$\int_{\Lambda} (1-\tau|j|)P_j dj = \frac{1-\delta_1}{1-\delta_0} 2g \tag{A3}$$

Let's now find prices for goods in Λ. Assume that $k \in \Lambda$ and $j \in \Phi$. Notice that from the first order conditions of both the producer and the merchant problem we find

$$P_j \, C^i(j) = P_k \, C^i(k) \qquad \text{for} \qquad i, j \in [-g, g]$$

Hence, in particular, for $j \in \Phi$ we obtain:

$$P_k \, C^i(k) = C^i(j).$$

And applying the integral operator to both sides:

$$P_k \int_{[-g,g]} C^i(k)di = \int_{[-g,g]} C^i(j)di = \int_{\Lambda} C^i(j)di \qquad \Rightarrow$$

$$P_k \int_{[-g,g]} C^i(k)di = \int_{\Lambda} \frac{(1-\delta_0)(1-\tau|i|)AP_i}{2g\xi} di \qquad \Rightarrow$$

$$\int_{[-g,g]} C^i(k)di = \frac{(1-\delta_0)A}{2g\xi P_k} \int_{\Lambda} (1-\tau|i|)P_i di \ ,$$

and substituting (A3) in the last expression we obtain:

$$\int_{[-g,g]} C^i(k)di = \frac{(1-\delta_0)A}{2g\xi P_k} \frac{1-\delta_1}{1-\delta_0} 2g \qquad \Rightarrow$$

$$\int_{[-g,g]} C^i(k)di = \frac{(1-\delta_1)}{\xi P_k} A \tag{A4}$$

On the merchant side, we know that:

$$P_k \, C^f(k) = P_j \, C^f(j) = C^f(j) \qquad \Rightarrow$$

$$P_k \int_{[0,F]} C^f(k)df = \frac{(\xi-1)(1-\delta_0)A}{2g\xi P_j} \int_{\Lambda} (1-\tau|i|P_i di \qquad \Rightarrow$$

$$\int_{[0,F]} C^f(k)df = \frac{(1-\delta_1)}{P_k} \frac{\xi-1}{\xi} A \tag{A5}$$

Plugging (A4) and (A5) into (12a) and rearranging we obtain

$$P_k = \frac{1-\delta_1}{1-\delta_0} \frac{1}{1-\tau|k|} \qquad \text{for every } k \in \Lambda.$$

Q.E.D

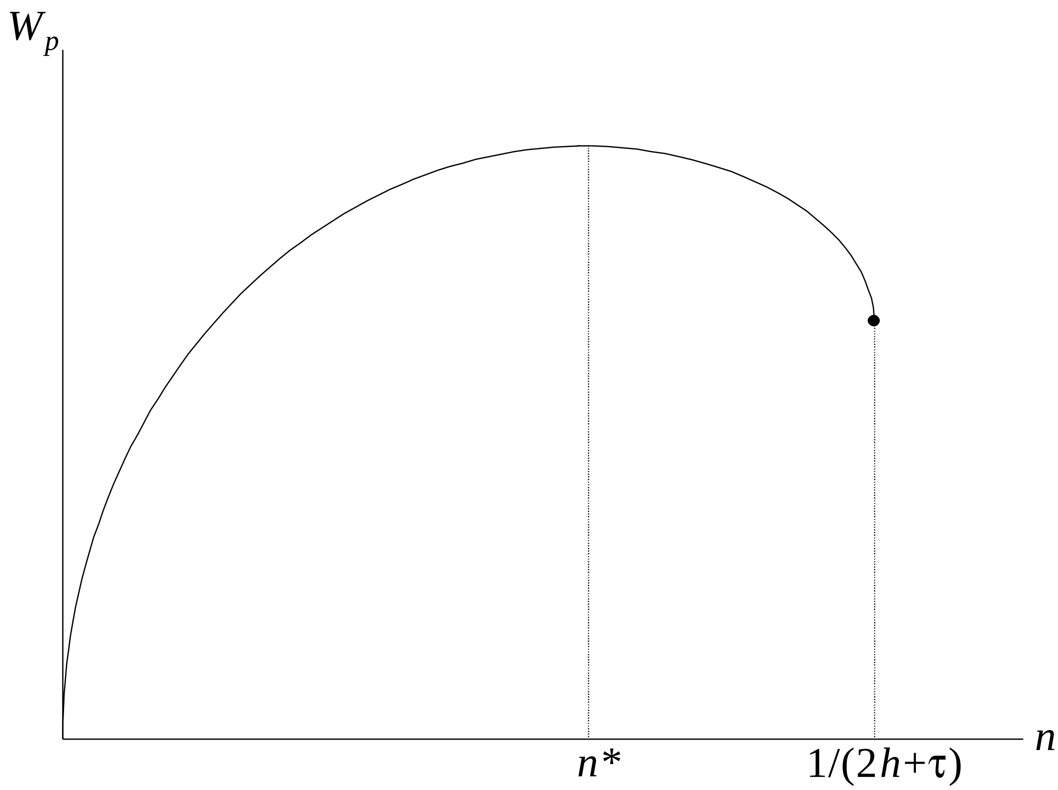

FIGURE 1. The Social Welfare Function in the Primitive Barter Economy

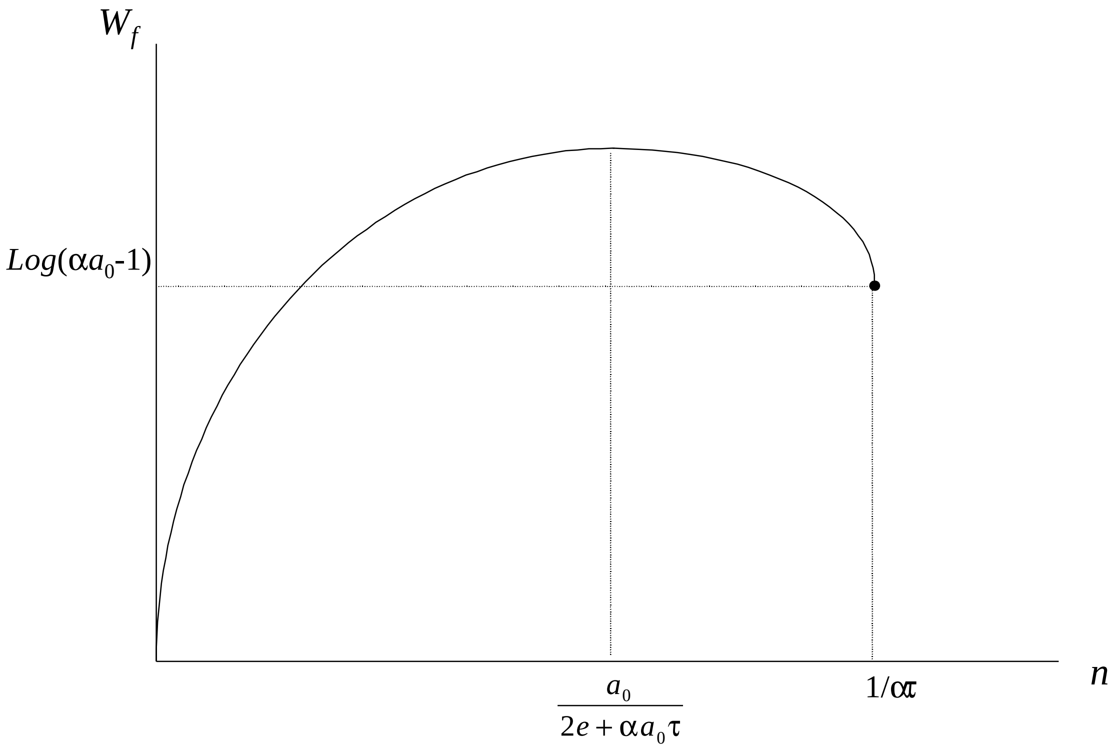

FIGURE 2. The Social Welfare Function in the Merchant Barter Economy

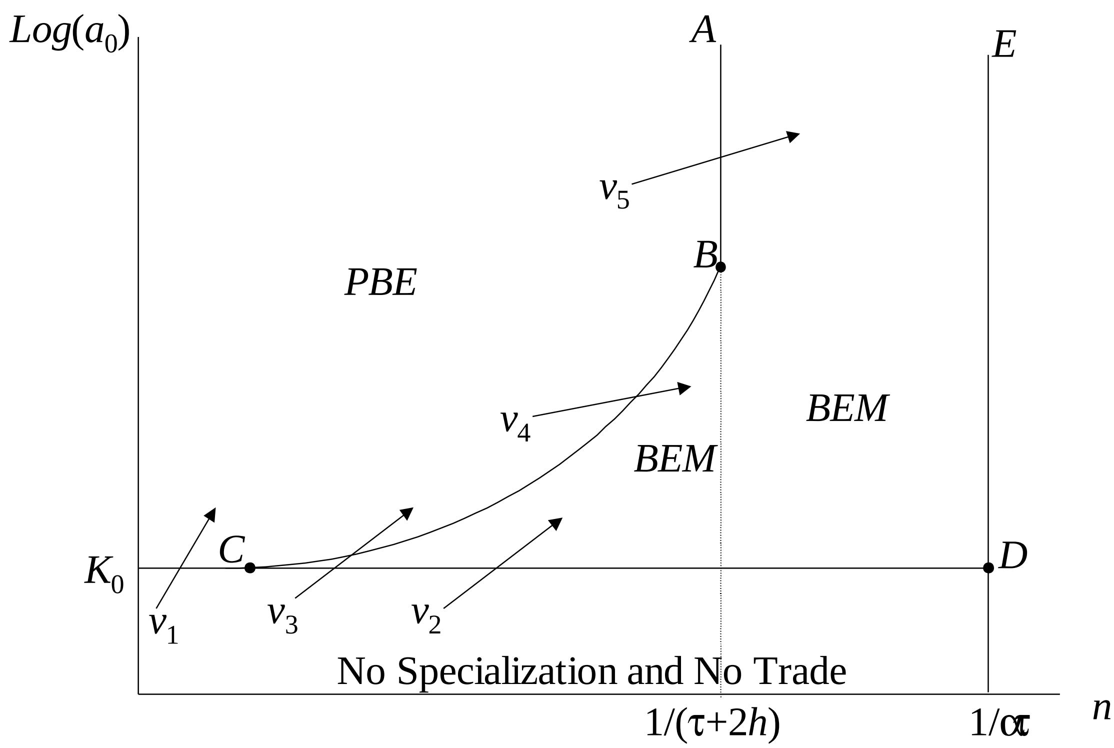

FIGURE 3. The Primitive Barter Economy and the Merchant Barter Economy Compared