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Sounding the Alarm on Inflation Indexing and Strict Inflation Targeting

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Abstract

Unanticipated inflation or deflation causes one party of a nominal contract to gain at the expense of the other party, an effect absent in macroeconomic models with one representative consumer or with consumers having identical consumption. In this paper's general dynamic and stochastic equilibrium model, diverse consumers maximize risk-averse utility and rent labor and land to profit-maximizing firms. Both inflation indexing and strict inflation targeting are Pareto inefficient. When Pareto sharing of changes of aggregate supply is proportional, nominal contracts under perfect nominal income targeting are Pareto efficient, while quasi-real contracts are Pareto efficient regardless.

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Sounding the Alarm on Inflation Indexing and Strict Inflation Targeting

1. Introduction

This paper reports the existence of "bombs" that have the potential to explode under extreme economic stress, wreaking havoc across our already weakened economies. Almost all countries have inflation-indexed contracts, the more common type of bomb. A less common bomb is strict inflation targeting, which exists in some economic systems and is recommended by some economists for use in others. While the likelihood that these will explode at a particular time is small, we need to plan for such contingencies to make sure our economic systems with their contracts survive even under extreme situations.

Many economists have encouraged inflation-indexed contracts and strict inflation targeting because of their attitudes towards inflation and their simplistic views of nominal contracts. Their attitudes towards inflation today are about the same as the attitudes medical experts had towards cholesterol twenty years ago when they considered all cholesterol as bad. As medical experts then tried to reduce their patients' cholesterol down to what they considered a positive safe level, many economists today recommend that we should strive for a positive inflation rate of around one or two percent, and they consider any inflation greater than that targeted inflation rate as bad regardless of the source of that inflation.

This paper is aimed at changing economists' views of inflation and nominal contracts, their goals and objectives concerning inflation, their recommendations concerning the monetary and fiscal policies to deal with inflation, and the financial instruments they design to hedge against inflation. Economists need to recognize that there can be both good and bad inflation, just as the medical community now recognizes the existence of both good and bad cholesterol.

Similar to how medical experts today try to reduce their patients' bad cholesterol while maintaining or even increasing their patients' good cholesterol, economists need to strive to reduce or filter out the bad inflation while letting the good inflation fulfill its important allocating roles.

The next section, section 2, discusses scenarios where inflation indexing and strict inflation targeting could explode. Section 3 explains why aggregate-supply-caused inflation or deflation is good, while section 4 discusses why aggregate-demand-caused inflation or deflation is bad. We further discuss inflation indexing and strict-inflation targeting in sections 5 and 6 respectively. We reflect on our conclusions in section 7 and discuss alternatives to pure inflation indexing and strict-inflation targeting.

2. Explosive Scenarios

To show that our use of the word "bomb" is appropriate to inflation indexing, consider the possibility that aggregate supply drops by 50% because of some event such as war, a terrorist attack, or a natural event such as pollution from a series of volcanic eruptions or a large meteorite hitting the earth. Assuming no change in nominal aggregate demand, prices would double. Assuming half of the government's budget is indexed to inflation, the doubling of prices would double the nominal payments on its inflation-indexed obligations, completely crowding out everything else in its budget. Raising taxes at such a time of low output is likely to be politically unfeasible. If the drop in aggregate supply is permanent, borrowing would not be a viable long-term solution.

Havoc would exist outside the government as well. Firms with obligations from inflation-indexed bonds they had issued and from inflation-indexed wages would find themselves squeezed. On average their nominal revenues would be unchanged (because we assumed nominal aggregate demand is unchanged). However, their nominal inflation-indexed obligations on these bonds and wages would have doubled.

To some extent, contract renegotiations will help mitigate the harmful effects of such an extreme situation. However, those contract renegotiations will be costly and slow because those due to receive the inflation-indexed payments will be reluctant to give up those higher payments. Also, some inflation-indexed obligations such as those on government inflation-indexed bonds cannot be renegotiated.

We should strive to write our contracts so they can be upheld even under extreme circumstances. Under the above extreme circumstances, nominal contracts would have worked quite well because aggregate-supply-caused inflation would have decreased the real value of the obligations on these nominal contracts, making those obligations more manageable.

However, even nominal contracts would have been dysfunctional had the monetary authority pursued a strict inflation targeting policy. Instead of keeping nominal aggregate demand constant, a strict-inflation-targeting monetary policy would have responded to a 50% drop in aggregate supply with a 50% drop in nominal aggregate demand, leaving the price level unchanged. Therefore, the government's nominal payments on both its inflation-indexed and non-inflation-indexed obligations would not have changed. However, with a 50% drop in nominal aggregate demand, the government's tax revenue would have dropped by 50% forcing a 50% cut in its budget if no new taxes or borrowing are forthcoming.

Similarly, firms without inflation-indexed obligations would also be squeezed. Their nominal contractual obligations would not have changed. However, if nominal aggregate demand had fallen 50%, then on average the firm's revenues would have also fallen 50% once again squeezing the corporations' cash flow as well as profitability.

In some sense, the "bomb" of strict-inflation-targeting is more dangerous than the "bomb" of inflation indexing because strict inflation targeting affects all contracts regardless whether the contracts are inflation indexed or not. Thus, someone using conventional contracts would have been unable to avoid the havoc caused by strict inflation targeting. (In the final section of this paper, we do discuss a type of indexed contract that can prevent such a squeeze.)

On the other hand, inflation-indexed contracts are contracts that are not easily broken. If such an extreme situation did arise, there would be time for the monetary authorities to realize strict inflation targeting is a mistake and back away from that commitment. In fact, many central banks follow flexible inflation-targeting policies instead of strict ones where they do have escape clauses in case of real shocks.[1] However, some central banks such as the Bank of Canada have statements supporting inflation targeting with no mention of any such escape clauses (Bank of Canada, 2003).[2] Also, some respected economists do recommend strict inflation targeting (e.g., Goodfriend, 2003).

Aggregate-demand and aggregate-supply shifts are the two causes of inflation or deflation. The view that inflation is primarily a monetary phenomenon has blinded many economists to the good that inflation can do when aggregate supply changes.[3] Contributing to

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[1] Bangko Sentral ng Pilipinas, 2001, states "The BSP may use escape clauses or arguments to explain deviations of actual inflation performance from the target level. This is so because there are other factors affecting inflation that are beyond the control of monetary policy, such as changes in tax policy, prices of oil in the world market, and natural disturbances that affect food supply."

[2] The Bank of Canada strictly targets the long-run inflation rate not necessarily the short-run inflation rate. Gavin (2003) argues that real shocks in the short run tend to be reversed in the long run. However, if a substantial real shock occurred which was not reversed in the long run, the Bank of Canada's firm commitment to a long-run inflation target would produce the problems we discuss.

[3] One area of economic literature that has recognized the difference between aggregate-demand-caused inflation and aggregate-supply-caused inflation is the wage indexation literature begun by Gray (1976) and Fischer (1977). While they did recognize that distinction, that literature has proposed indexing that partially filters out both the good and

this blindness is that many of our macroeconomic or monetary economic models assume one representative consumer or consumers who have identical consumption.[4] Such models cannot capture the effect that one party of a nominal contract gains at the expense of the other as a result of inflation or deflation. This effect can only be captured in a model of diverse consumers. The next section uses a model of diverse consumers to shine the light onto aggregate-supply-caused inflation.

3. Aggregate-Supply-Caused Inflation and Deflation

We write this section to show that if aggregate demand does not change, then aggregate-supply-caused inflation or deflation in conjunction with nominal contracts is “good” in the Pareto sense. As the Appendix shows, in order for a monetary equilibrium to be truly Pareto efficient,[5] the model must either assume no money is held from one period to the next, or it must assume other extremely unreasonable assumptions. We choose the first route. We use a monetary-modeling methodology that treats nominal aggregate demand as exogenous or stochastically exogenous, but where no money is held from period to period. So to not distract from the major theme of this paper, we just apply that methodology in this section leaving the defense of the methodology to the Appendix.

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the bad inflation. Also, the goals and objectives of monetary and macroeconomic literature have not changed in light of the discoveries in the wage indexation literature.

[4] For example, Aoki (2001) develops a model in which he argues that strict inflation targeting is Pareto efficient. However, consumers in his model always have the same level of consumption. Therefore, there are no winners or losers as a result of inflation.

[5] Some authors have loosely applied the term "Pareto efficiency". For example, in an economic model which includes a market constraint such as a cash-in-advance constraint, the authors may argue that the model is "Pareto efficient" since there is no way to make one person better off without making someone worse off without violating that market constraint. The conclusions of the first and second fundamental theorems of welfare economics are much stronger since they assume no such market constraints and yet a competitive market is Pareto efficient. We define true Pareto efficiency as being when the consumption allocation matches the consumption allocation that would come from an Arrow-Debreu economy which we know is "truly Pareto efficient." These other concepts of Pareto efficiency are valid concepts, but we would label them as "Pseudo Pareto efficient" so to not distract from the more rigorous "truly Pareto efficient." When this paper uses "Pareto efficient," we mean "truly Pareto efficient".

To facilitate our discussion, imagine a closed economy with one consumption good where individuals consume all that good; the good cannot be stored from one period to the next. Also, imagine firms producing that consumption good with a constant-returns-to-scale production function of labor and land of the form, $e^{\alpha_t} f(\tilde{L}_t, \tilde{H}_t)$, where $f_L > 0$, $f_{LL} < 0$, $f_H > 0$, $f_{HH} < 0$, and where $\tilde{L}_t$ and $\tilde{H}_t$ are the aggregate amounts of labor and land demanded, and α_t is a stochastic term having zero mean and a positive variance. We model the firm sector with one representative price-taking firm that maximizes its profits:

$$P_t e^{\alpha_t} f(\tilde{L}_t, \tilde{H}_t) - W_t \tilde{L}_t - R_t \tilde{H}_t \qquad \textbf{(1)}$$

where P_t is the price level, W_t is the nominal wage rate, and R_t is the nominal land rent. The necessary and sufficient conditions for this profit to be maximized are that the real wage rate and real land rental rates equal the following:

$$\frac{W_t}{P_t} = e^{\alpha_t} f_L(\tilde{L}_t, \tilde{H}_t) \qquad \textbf{(2)}$$

$$\frac{R_t}{P_t} = e^{\alpha_t} f_H(\tilde{L}_t, \tilde{H}_t) \qquad \textbf{(3)}$$

Assume that the supply of labor and land is fixed each period. Then, (2) and (3) show that the real wage rate and the real land discount rate vary with the realization of α_t. When productivity is higher than expected (i.e., α_t is greater than 0), the real wage rates and real rental rates should be higher than in situations when productivity is lower than expected. If wages and rental rates are set prior to the realization of productivity, ideal contracts (contracts that would help lead to a Pareto-efficient consumption allocation) should be dynamic rather than static, changing the real wage rates and the real land rental rates according to productivity.

Writing such ideal contracts may sound to be too complicated to be very practicable. However, writing these contracts to respond to aggregate productivity is quite easy. By the quantity equation (MV=N=PY), $N_t = P_t Y_t$, where N_t is nominal aggregate demand at time t. If we multiply both (2) and (3) by P_t/N_t and remember that $N_t = P_t Y_t$ and that $Y_t = e^{\alpha_t} f(\tilde{L}_t, \tilde{H}_t)$, we get

$$\frac{W_t}{N_t} = \frac{P_t e^{\alpha_t} f_L(\tilde{L}_t, \tilde{H}_t)}{P_t Y_t} = \frac{e^{\alpha_t} f_L(\tilde{L}_t, \tilde{H}_t)}{e^{\alpha_t} f(\tilde{L}_t, \tilde{H}_t)} = \frac{f_L(\tilde{L}_t, \tilde{H}_t)}{f(\tilde{L}_t, \tilde{H}_t)}$$

$$\frac{R_t}{N_t} = \frac{P_t e^{\alpha_t} f_H(\tilde{L}_t, \tilde{H}_t)}{P_t Y_t} = \frac{e^{\alpha_t} f_H(\tilde{L}_t, \tilde{H}_t)}{e^{\alpha_t} f(\tilde{L}_t, \tilde{H}_t)} = \frac{f_H(\tilde{L}_t, \tilde{H}_t)}{f(\tilde{L}_t, \tilde{H}_t)}$$

These show that the nominal "ideal" wage and land rent are constant as long as nominal aggregate demand stays the same. In other words, contracts that nominally fix wages and land rents are ideal contracts under these assumptions.

It is good inflation and deflation that makes these nominal contracts so dynamic when nominal aggregate demand remains the same. If productivity rises, aggregate supply increases, causing the price level to decrease. The lower price level causes the real wage and the real land rent to increase just as (2) and (3) require. If productivity falls, aggregate supply decreases, causing the price level to increase. The higher price level causes the real wage rate and the real land rent to decrease so that (2) and (3) continue to hold.

We can also see that aggregate-supply-caused inflation or deflation is good by looking at the consumer side of the economy. Very important for our understanding of how inflation affects consumers is what we call the Consumption-Aggregate-Supply Functionality Theorem:

> **The Consumption-Aggregate-Supply Functionality Theorem:** When consumers are strictly risk averse and have time additively separable utility

functions and all the consumption good is consumed, then any particular Pareto-efficient consumption allocation is a function solely of aggregate supply.

The key word here is "function" meaning that it is impossible to have a Pareto-efficient consumption allocation where someone's consumption in a particular period t differs between two states of nature with the same level of aggregate supply in period t. Assume otherwise, that for some period t, there exist two states of nature where aggregate supply at time t is the same but where during period t at least one individual consumes more in one state than in the other state.[6] We will label these two states as states 1 and 2. Define a new consumption allocation where for states 1 and 2 each individual's consumption equals the average of their original consumption in those two different states. For all other states at time t and for all other time periods, the consumption allocation is the same as the original. Since the goods market clears with the original consumption allocation, it must also clear with the newly defined allocation making the newly defined allocation feasible.[7] Any strictly risk-averse consumer with different consumption in the two states where aggregate supply is the same would prefer their newly defined consumption allocation[8] and no one would be made worse off with the newly defined allocation. Therefore, the newly defined consumption allocation is Pareto superior to the original allocation, contradicting the statement that the original allocation was Pareto efficient. Therefore, by proof by contradiction, the above theorem is true.

[6] Actually, if the consumption allocation differs for one consumer, it must also differ for at least one other consumer because the sum of the individual's consumption must equal aggregate supply.

[7] Let c_{jit} be the j's consumption in state i at time t for the original allocation. Let $\hat{c}_{j1t} = \hat{c}_{j1t} = \dfrac{c_{j1t} + c_{j2t}}{2}$ be the new allocation. Since $Y_{1t} = Y_{2t}$, then $\sum_{j=1}^{m} \hat{c}_{j1t} = \sum_{j=1}^{m} \hat{c}_{j2t} = \sum_{j=1}^{m} \dfrac{c_{j1t} + c_{j2t}}{2} = \dfrac{Y_{1t} + Y_{2t}}{2} = Y_{1t} = Y_{2t}$.

[8] A strictly risk-averse individual prefers consuming the average of the two different consumption levels in both of the states over the uncertainty of the different consumption levels in those two states.

Our using the word "function" can lead to some confusion. We are not saying that there is only one Pareto-efficient consumption allocation; we know there normally should be a continuum of Pareto-efficient consumption allocations. However, in an economy with stochastic variables, our specifying any particular Pareto-efficient consumption allocation requires us to specify that consumption for all consumers, for all time periods, and for all states of nature. Therefore, our specification of this particular Pareto-efficient consumption allocation would be a mapping from consumers, time periods, and states to consumption. Another word for "mapping" is "function". Aggregate supply itself is a function of the time period and the state. What the Consumption-Aggregate-Supply Functionality Theorem says, is that any particular Pareto-efficient consumption allocation can be written as a function of the consumers, the time periods, and aggregate supply leaving the state out of the function even though variables other than income may vary with the state of nature. (See footnote 8 for an example to clarify this subtle distinction.)

Throughout this paper, we assume that consumers are strictly risk averse and all the consumption good is consumed. Therefore:

$$\sum_{j=1}^{m} \tilde{c}_{jt}(Y_t) = Y_t \tag{4}$$

where m is the number of consumers, Y_t is the aggregate supply at time t, and $\tilde{c}_{jt}(Y_t)$ is the function depicting how j's consumption in this particular Pareto-efficient consumption allocation changes as aggregate supply changes. It is important to recognize that $\tilde{c}_{jt}(Y_t)$ is a reduced form. It does **not** represent what we normally think of a consumption function of income. To help keep this distinction straight, we will refer to Y_t as aggregate supply, not income.

Differentiating (4) with respect to aggregate supply gives:

$$\sum_{j=1}^{m} \tilde{c}'_{jt}(Y_t) = 1 \tag{5}$$

Also, dividing (4) by Y_t gives:

$$\sum_{j=1}^{m} \frac{\tilde{c}_{jt}(Y_t)}{Y_t} = 1 \tag{6}$$

Subtracting (6) from (5) gives:

$$\sum_{j=1}^{m} \left(\tilde{c}'_{jt}(Y_t) - \frac{\tilde{c}_{jt}(Y_t)}{Y_t} \right) = 0 \tag{7}$$

Equation (7) tells us that on average, an individual's share of a decrease (or increase) in aggregate supply equals the proportion of that individual's consumption to aggregate supply. When aggregate supply decreases, in order for one individual's consumption to decrease by less than her proportionate share, some other individual's consumption must decrease by more than his proportionate share. If this were to happen in a Pareto-efficient economy, in essence a less risk-averse consumer would be providing the higher risk-averse consumer with insurance that would allow consumption to change less for the higher risk-averse consumer than for the less risk-averse consumer.

Many people think about nominal contracts as simple static contracts. While nominal contracts are static in a nominal sense, they are dynamic in a real sense. The real dynamics of nominal contracts help allocate increases and decreases in aggregate supply across the economy. However, we recognize the limitations of nominal contracts. Unless nominal contracts are written as insurance policies, they will usually not provide insurance. For now, let's assume that there is no need of such insurance by assuming that the consumers' utility functions are such that for any particular Pareto-efficient consumption allocation, each individual j's consumption at time t is proportional to aggregate supply,[9] i.e.:

$$\tilde{c}'_{jt}(Y_t) = \frac{\tilde{c}_{jt}(Y_t)}{Y_t} = \tilde{\mu}_{jt} \tag{8}$$

for some constant $\tilde{\mu}_{jt}$. By (8), we are assuming the constant $\tilde{\mu}_{jt}$ cannot change when the state of nature changes as long as we stay with the same particular Pareto-efficient consumption allocation, the same consumer j, and the same time period t.[10]

We will now show that when nominal aggregate demand does not change, inflation in conjunction with nominal contracts will facilitate consumers in planning their consumption allocations to be proportional to aggregate supply. Imagine each consumer j, with his/her endowments of labor and land, maximizing the sum of his or her expected utility[11] over time subject to the following constraints for each time t=1..T where T is the last period of the consumer's life:

$$P_t c_{jt} + B_{jt} = W_t \overline{L}_{jt} + R_t \overline{H}_{jt} + Z_{jt} + B_{j,t-1}(1 + i_{t-1}) \tag{9}$$

where B_{jt} is the amount of the nominal bond j demands at time t, and i_t is the nominal interest rate on that bond. To simplify our analysis, assume only the consumption good enters the consumers' utility functions. Neither labor nor land affects utility directly so that all labor and land is supplied to the market; $\overline{L}_{jt}$ and $\overline{H}_{jt}$ represent the fixed amounts of labor and land

[9] Iff consumers have the same relative risk aversion will equation (6) hold (See, Eagle and Domian, 2003).

[10] For example, in an economy consisting of just three consumers – A, B, and C, one such Pareto-efficient allocation could be where A, B, and C always respectively consume 90%, 8%, and 2% of aggregate supply. Another Pareto-efficient allocation might be where A, B, and C always respectively consume 10%, 30%, and 60% of aggregate supply. Our assumption of (8) assumes that j's consumption at time t as a proportion of aggregate supply is the same regardless of the state of nature as long as we are talking about one particular Pareto-efficient consumption allocation.

endowed onto j and hence supplied by j at time t. An income-distribution system exists that taxes some consumers and makes payments to other consumers; Z_{jt} represents j's receipt at time t of a receipt from this system, which if negative represents a tax. We call this distribution system "social security", although the theory could also encompass various welfare systems.

Assume that these consumers enter into nominal contracts for wages and land rent prior to the time when the labor or land services are provided. The Appendix shows that the equilibrium, involving nominal contracts between the consumers described above and the firms described earlier, is Pareto efficient when the Pareto-efficient consumption allocation for each consumer is proportional to income. In that equilibrium, the consumer's investment in bonds each period does not depend on the realization of aggregate supply in each period. To help see why nominal contracts facilitate consumers to obtain consumption allocations that are proportional to aggregate supply, subtract both sides of (9) by B_{jt}, replace P_t with N_t/Y_t, , and divide both sides of (9) by N_t to get:

$$\frac{c_{jt}}{Y_t} = \frac{W_t \overline{L}_{jt} + R_t \overline{H}_{jt} + Z_{jt} + B_{j,t-1}(1+i_{t-1})}{N_t} - \frac{B_{jt}}{N_t} \qquad \textbf{(10)}$$

Equation (10) shows us that as long as W_t, R_t, and Z_{jt} have been set in previous periods, and N_t and B_{jt} do not vary across states of nature, individual j's consumption at time t will be proportional to aggregate supply since the values of $B_{j,t-1}$ and i_{t-1} were known the previous period. Therefore, if consumers know N_t with certainty, they then know *a priori* the ratio of their consumption to aggregate supply.

[11] We do assume that each consumer j's utility function at each time t, $U_{jt}(c_{jt})$, is sufficiently well behaved to guarantee the existence and uniqueness of the equilibrium.

The necessary and sufficient condition for an individual's consumption to remain a constant proportion of aggregate supply is that the individual reduces his/her consumption by the same percentage change as the change in aggregate supply. To see this, let μ_{jt} be the constant ratio of consumption to aggregate supply, then $c_{jt} = \mu_{jt} Y_t$, which implies that $\Delta c_{jt} = \mu_{jt} \Delta Y_t$. Dividing both sides by their respective sides of $c_{jt} = \mu_{jt} Y_t$ gives $\frac{\Delta c_{jt}}{c_{jt}} = \frac{\Delta Y_t}{Y_t}$.

To see more simply how aggregate-supply inflation and nominal contracts can cause the percentage change in individuals' consumption to equal the percentage change in aggregate supply, let X_{jt} be consumer j's net cash inflow at time t. Then under the assumptions of this model,

$$c_{jt} = \frac{X_{jt}}{P_t} \tag{11}$$

which implies that $\dot{c}_{jt} = \dot{X}_{jt} - \dot{P}_t$ where the dot above each variable means the infinitesimal percentage change.[12] Therefore, if aggregate supply decreases by 1%, which causes the price level to increase by 1%, then j's consumption will decrease by 1% if the nominal cash inflow j receives at time t does not change. Similarly, if aggregate supply increases by 1%, which causes the price level to decrease by 1%, then j's consumption will increase by 1%. In both cases, the percentage change in consumption matches the percentage change in aggregate supply, resulting with the ratio of j's consumption to aggregate supply staying constant.

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[12] Let η represent any variable that may affect the variables of (11). Taking logarithms of both sides gives $\ln(c_{jt}) = \ln(X_{jt}) - \ln(P_t)$. Totally differentiating with respect to η gives $\frac{dc_{jt}/d\eta}{c_{jt}} = \frac{dx_{jt}/d\eta}{x_{jt}} - \frac{dP_t/d\eta}{P_t}$, which is the same as $\dot{c}_{jt} = \dot{X}_{jt} - \dot{P}_t$. Usually, economists treat η as time but it could actually be any exogenous variable, including an abstract unspecified variable as we are using it here.

If we replace P_t in (11) with N_t / Y_t, we get

$$c_{jt} = \frac{X_{jt}}{N_t} Y_t \qquad \textbf{(12)}$$

Taking the log and differentiating gives:

$$\dot{c}_{jt} = \dot{X}_{jt} - \dot{N}_{jt} + \dot{Y}_{jt} \qquad \textbf{(13)}$$

This shows that as long as the nominal net cash inflow does not change and nominal aggregate demand does not change, the percentage change in consumption will equal the percentage change in aggregate supply. Using differentiation to get the results of (13) only applies to infinitesimal changes. However, if nominal aggregate demand and j's nominal net cash inflow stay the same, then (12) implies $\Delta c_{jt} = \frac{X_{jt}}{N_t} \Delta Y_t$. Dividing both sides by their respective sides of (12) gives $\frac{\Delta c_{jt}}{c_{jt}} = \frac{\Delta Y_t}{Y_t}$. Hence, as long as nominal aggregate demand and j's nominal net cash inflows do not change, then the percentage change in consumption equals the percentage change in aggregate supply for any size change.

In conclusion, nominal contracts are not static contracts; in real terms, they are dynamic, adjusting their real payments to changes in aggregate supply. Aggregate-supply-caused inflation is good, playing an important role in allocating the effects of changes in aggregate supply across the economy. When Pareto-efficiency requires consumers to proportionately share those changes in aggregate supply, nominal contracts are Pareto efficient as long as nominal aggregate demand does not change.

4. Aggregate-Demand-Caused Inflation and Deflation

When nominal aggregate demand stays the same, nominal contracts work very well to allocate changes in aggregate supply across the economy. However, a stochastic component to nominal aggregate demand will cause nominal contracts to work inefficiently. Remember that the Consumption-Aggregate-Supply Functionality Theorem states that as long as consumers are strictly risk averse and all the consumption good is consumed, any particular Pareto-efficient consumption allocation must be a function solely of aggregate supply. This means that as long as aggregate supply remains the same in different states of nature, the Pareto-efficient consumption allocation should also be the same in those states of nature. However, unexpected changes in nominal aggregate demand will cause changes to the consumption allocation even when aggregate-supply remains the same. Therefore, the resulting changes in consumption represent departures from Pareto efficiency.

With most nominal contracts, one party is obligated to make a monetary payment to the other party. If an unexpected increase in nominal aggregate demand causes inflation to be greater than anticipated, the real payment will be less than anticipated. This will benefit the party making the payment but hurt the party receiving the payment. On the other hand, if an unexpected decrease in nominal aggregate demand causes inflation to be less than anticipated, the real payment will be more than anticipated. This will hurt the party making the payment and help the party receiving the payment.

Assuming they are risk averse, both parties will be *a priori* worse off given the possibility of nominal-aggregate-demand causing this unanticipated inflation or deflation. They would prefer not being exposed to this risk.

5. Inflation-Indexed Contracts

The previous section showed that the risk of aggregate-demand-caused unanticipated inflation or deflation makes both parties of nominal contracts worse off *a priori*. Inflation-indexing is a way to filter out this inflation, which would make both parties better off as long as all inflation is caused by changes in nominal aggregate-demand. However, the problem with inflation indexing is that it filters out all inflation whether caused by aggregate demand shifts or aggregate supply shifts.

The obligations on a contract can be inflation indexed by multiplying the agreed-upon base obligations at time t by the ratio of the price level at time t to the base year's price level. For example if $\overline{W}$ is the contractual base wage rate agreed upon at time t-1, the actual nominal wage rate at time t would be $W_t = \overline{W}\frac{P_t}{P_{t-1}}$. This then results with the real wage rate being a constant since $\frac{W_t}{P_t} = \frac{\overline{W}}{P_t}\frac{P_t}{P_{t-1}} = \frac{\overline{W}}{P_{t-1}}$ and since $\overline{W}$ and P_{t-1} are known at time t-1.

When aggregate supply remains the same, Figure 1 contrasts how nominal aggregate demand affects the consumption levels of contractual parties of an inflation-indexed obligation compared to if they had entered into a nominal obligation. Figure 1 assumes the party receiving payment on the obligation relies solely on that payment for her consumption in that period, whereas it assumes the party making the payment has resources equal to twice his expected consumption. To normalize the consumption of both parties, Figure 1 divides their actual consumption levels by the level of consumption they would experience if nominal aggregate demand equals its expected value (N_e).

With the nominal contract, if nominal aggregate demand exceeds its expected value, unanticipated inflation takes place, reducing the real value of the nominal obligation. The consumption level of the receiver of the nominal obligation will decrease, whereas the consumption level of the payer will increase as a result of this unanticipated inflation.

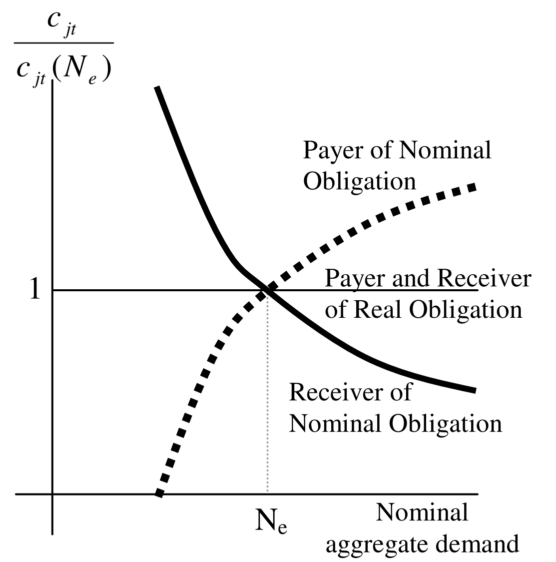

Figure 1: Effects of Unexpected Changes in Nominal Aggregate Demand on the Consumption of Parties to Nominal and Inflation-Indexed Obligations

Conversely, if nominal aggregate demand is less than expected, then the inflation rate will be less than anticipated, increasing the real value of the obligation. This will increase the consumption level of the receiver of the nominal obligation, but will decrease the consumption level of the payer of the obligation. Note that the payer of the obligation could actually become bankrupt if nominal aggregate demand decreased over 50%.

On the other hand, inflation-indexed obligations counteract any changes in nominal aggregate demand making the real payments constant. Hence, the real consumption of the parties to inflation-indexed obligations will be unaffected by the changes in nominal aggregate demand. If both parties knew with certainty that aggregate supply would not change and if they were both risk averse, they would both prefer the certainty of their consumption level provided by inflation-indexed contracts compared to nominal contracts.

However, if aggregate supply does change, inflation-indexed contracts will filter out aggregate-supply-caused inflation as well as aggregate-demand-caused inflation. Figure 2 contrasts how aggregate supply affects the consumption levels of the parties of inflation-indexed contracts compared to nominal contracts when nominal aggregate demand remains the same. To normalize the consumption of both parties, Figure 2 divides their actual consumption by the consumption they would experience if aggregate supply equals its expected value (Y_e).

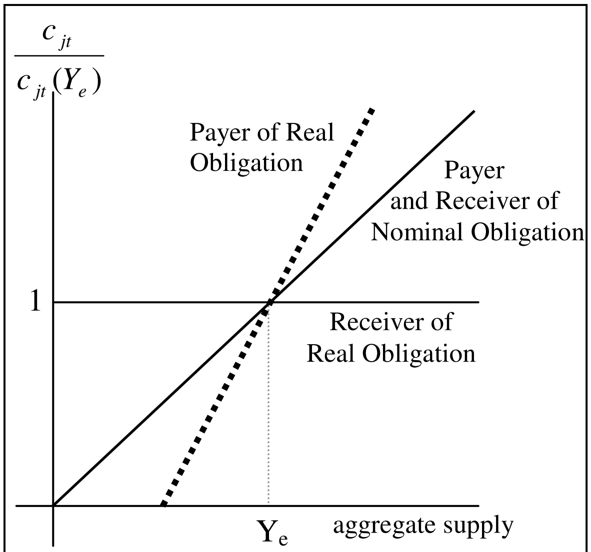

Figure 2: Effects of Changes in Aggregate Supply on the Consumption of Parties to Nominal and Inflation-Indexed Obligations

These are the same parties as discussed in Figure 1. The receiver of the obligation solely relies on that obligation for her consumption. When aggregate supply equals its expected value (Y_e), the payer of the obligation would have resources equal to twice his consumption. Figure 2 assumes the payer's resources increase proportionately with aggregate supply.

As section 2 discussed, the real payment on a nominal obligation will vary proportionately with aggregate supply. Since the receiver of the payment relies solely on that payment for her consumption, her consumption will increase proportionately with aggregate supply as shown by the diagonal line going through the origin in Figure 2. That same diagonal line represents the consumption for the payer of the payment. When aggregate supply decreases, his resources decrease but so does his real obligation from the contract. The result is that both the payer and the receiver of the obligation proportionately share in the reduction of aggregate supply.[13]

[13] Under these assumptions, the payer's consumption, c, equals kY-W/P Since P=N/Y, c=(k-W/N)Y, which implies that c is proportional to Y.

On the other hand, the real payment of an inflation-indexed obligation is constant regardless of the level of aggregate supply. Since the receiver of the inflation-indexed obligation solely relies on that obligation, her consumption will be constant regardless of the level of aggregate supply. The payer of the inflation-indexed obligation is guaranteeing that constant real payment, despite his resources varying with aggregate supply. To meet that guarantee, the payer will need to decrease his consumption more than proportionately when aggregate supply decreases.[14] If the drop in aggregate supply decreased by over 50%, the payer would be bankrupt.

In conclusion, inflation-indexing does eliminate the bad unanticipated inflation and deflation caused by a stochastic nominal aggregate demand. However it also eliminates the good inflation and deflation caused by changes in aggregate supply.

6. Strict Inflation Targeting

Important to understanding inflation targeting is being able to distinguish between various forms of inflation targeting. We define three types of inflation targeting: (a) flexible inflation targeting, (b) strict long-run inflation targeting, and (c) strict short-run inflation targeting. Flexible inflation targeting as defined by Svensson (1999) is when the monetary authority not only tries to minimize the squared deviation of inflation from its target but also to minimize the squared deviations of other variables from their targets such as output and unemployment. Strict short-run inflation targeting is the opposite, where the only thing the central bank concerns itself is minimizing the squared deviation of inflation from its target,

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[14] Since the payers resources equals twice his consumption when aggregate supply equals its expected value, $kY_e=2c(Y_e)$. Since the real payment he makes is constant then the real payment is $kY_e/2$. Therefore his consumption in general will equal $kY-kY_e/2$. The ratio of his actual consumption to his consumption if $Y=Y_e$ is therefore $(Y/Y_e-1/2)$. This means when $Y=Y_e/2$, the payer's consumption will equal 0.

including inflation in the short run. Almost no central bank follows a strict short-run inflation targeting policy. However, many central banks such as the Bank of Canada follow a strict long-run inflation targeting policy, where they make a long-run commitment to a given inflation range no matter what, although they are flexible in the short-run. The problems that we discuss could arise from strict long-run inflation targeting as well from strict short-run inflation targeting as long as the drop in aggregate supply is a permanent drop.

If the central bank is following strict inflation targeting, then even someone using nominal contracts would encounter problems similar to the problems with inflation-indexed contracts. A central bank following strict inflation targeting will try to set $\dot{P} = \hat{\dot{P}}$ where $\hat{\dot{P}}$ is the targeted inflation rate. Since $P=N/Y$, $\dot{P} = \dot{N} - \dot{Y}$. Therefore, the central bank will try to minimize the deviation of $\dot{N}$ from $\hat{\dot{P}} + \dot{Y}$.

If $\dot{Y}$ is known with certainty, then a central bank following strict inflation targeting will try to minimize the deviation of nominal aggregate demand from N_e in Figure 1. This would be good as it would minimize the real effects of aggregate-demand-caused inflation or deflation on nominal contracts. However, if $\dot{Y}$ is greater than expected, then the central bank will try to increase $\dot{N}$ to compensate. On the other hand, if $\dot{Y}$ is less than expected, then the central bank will try to decrease $\dot{N}$ to compensate. In both cases, the central bank will try to offset the effects of aggregate-supply-caused inflation or deflation.

Figure 3 redraws that graph of Figure 2 assuming the central bank is perfectly able to meet its inflation target by compensating for changes in aggregate supply with changes in nominal aggregate demand. Strict inflation targeting eliminates all unanticipated inflation, whether caused by aggregate-demand shifts or aggregate-supply shifts. Strict inflation targeting destroys the dynamics of nominal contracts, making both nominal and real contracts static in both nominal and real terms. In the model presented in section 2, strict inflation targeting is Pareto inefficient.

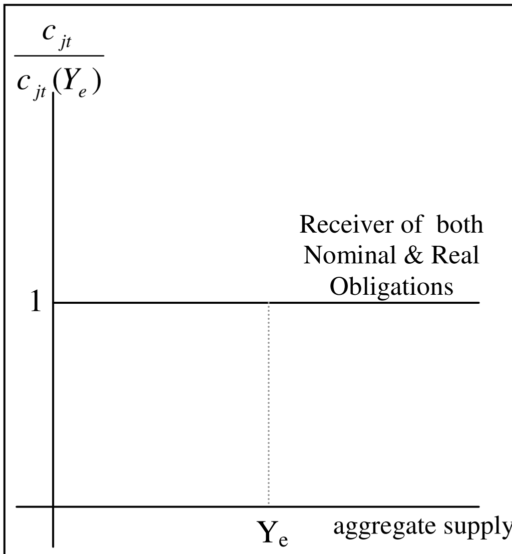

Figure 3: Effects of Changes in Aggregate Supply on Real Consumption of Contractual Parties Under Strict Inflation Targeting

7. Reflections

While nominal contracts are static in a nominal sense, they are “really” dynamic. While inflation-indexed contracts are dynamic in a nominal sense, they are “really” static. While to some, strict inflation targeting appears to be a dynamic policy, it really removes the dynamism out of contracts making both nominal and inflation-indexed contracts static in both the nominal and real senses.

The First and Second Fundamental Theorems of Welfare Economics state that under fairly general assumptions, an Arrow-Debreu equilibrium is Pareto efficient and any Pareto-efficient consumption allocation can be represented by an Arrow-Debreu equilibrium. What makes an Arrow-Debreu equilibrium Pareto efficient are its state-contingent securities. Each state-contingent security is dynamic around the state for which it pays and static everywhere else. Iff that state occurs, will that security pay. The problem with state-contingent securities is that we need so many of them to complete markets and it is difficult to identify different states of nature. Even with sequential markets involving a numeraire good, we still need n_t securities where n_t is the number of states at time t. Instead of state-contingent securities, imagine securities that are everywhere dynamic, adjusting their real payments to different states of nature just as needed by Pareto efficiency. If such continuously dynamic securities existed, then maybe, an equilibrium could be Pareto-efficient with fewer securities than in an Arrow-Debreu equilibrium.

In the model of section 2, we found that if any Pareto-efficient consumption allocation for each consumer is everywhere proportional to aggregate supply, then the dynamic changes in real obligations resulting from nominal contracts do change with aggregate supply exactly as needed for Pareto efficiency. However, the problem with nominal contracts is that they are too dynamic. The real obligations on nominal contracts change when nominal aggregate demand changes as well as when aggregate supply changes. What is needed is a contract where the real obligations are affected by aggregate-supply changes but unaffected by nominal-aggregate-demand changes.

Eagle and Domian (1995) propose quasi-real bonds that adjust for aggregate-demand-caused inflation but not for aggregate-supply-caused inflation. These quasi-real bonds achieve this by multiplying the base principal by the ratio $\frac{N_t / N_0}{(1+g)^t}$ where g is the expected long-run growth rate in real GDP. Eagle and Domian argue that under rational expectations, the market would adjust the quasi-real interest rate to compensate for imperfections in the estimate of the contractual parameter g.

With other types of indexation such as wage indexation, nothing like an interest rate exists to absorb such imperfections. Furthermore, the arguments we presented in section 2 that consumption should increase proportionately with changes in aggregate supply, apply regardless whether that increase is anticipated or not. Therefore, we propose making the quasi-real adjustment by multiplying only by the ratio of the current nominal aggregate demand to the nominal aggregate demand in the base year.

For example, under a quasi-real wage contract entered into one period ahead of time t, the nominal wage at time t equals $\overline{W}\frac{N_t}{N_{t-1}}$ where $\overline{W}$ is the base wage rate. The real wage rate at time t therefore equals $\frac{\overline{W}}{P_t}\frac{N_t}{N_{t-1}} = \frac{\overline{W}}{P_t}\frac{P_t Y_t}{N_{t-1}} = \frac{\overline{W}}{N_{t-1}} Y_t$. Since $\overline{W}$ and N_{t-1} are known at time t-1, this means that the real wage rate is proportional to aggregate supply regardless of nominal aggregate demand at time t.

In a multi-period wage contract negotiated at time 0, the quasi-real contract would adjust the nominal wage at time t to equal $\overline{W}\frac{N_t}{N_0}$, resulting with the real wage rate equaling $\frac{\overline{W}}{N_0} Y_t$. Hence, the real wage rate is still proportional to aggregate supply. If we assume the perfectly competitive firms in section 2 and that the aggregate supply of labor and land are fixed at any point in time, the contract negotiations should result with the base wage being set to $P_0 e^{\alpha_0} f_L(\tilde{L},\tilde{H})$, which is what W_0 should equal according to (2). Given that base wage, the nominal wage at time t under quasi-real contract will equal:

$$\overline{W}\frac{N_t}{N_0} = P_0 e^{\alpha_0} f_L(\tilde{L},\tilde{H})\frac{N_t}{N_0} = P_0 e^{\alpha_0} f_L(\tilde{L},\tilde{H})\frac{P_t Y_t}{P_0 Y_0} = P_t e^{\alpha_0} f_L(\tilde{L},\tilde{H})\frac{e^{\alpha_t} f(\tilde{L},\tilde{H})}{e^{\alpha_0} f(\tilde{L},\tilde{H})} = P_t e^{\alpha_t} f_L(\tilde{L},\tilde{H})$$

which is just exactly what (2) says it should equal. Quasi-inflation-indexed contracts are therefore ideal contracts in this model. The appendix does show that quasi-inflation-indexed contracts for wages, rent, social security and bonds result in a Pareto-efficient equilibrium in the model discussed in section 2.

While there will be practical issues in the application of quasi-real contracts such as the reliability and lagged availability of nominal GDP information, such issues are similar to the practical issues of inflation-indexed contracts.[15] We will leave those practical issues to be dealt with in future literature.

Since the nominal wage rate under the quasi-real contract would be $\overline{W}\frac{N_t}{N_0}$, the real wage rate will equal $\omega = \frac{\overline{W}}{P_t}\frac{N_t}{N_0} = \frac{\overline{W}}{P_t}\frac{P_t Y_t}{N_0} = \frac{\overline{W}}{N_0}Y_t$. Taking the logarithm of both sides and differentiating with respect to time gives $\dot{\omega} = \dot{Y}$ since $\overline{W}$ and N_0 are predetermined. This means that quasi-real wage rates will insure that the real wage rate changes proportionately with aggregate supply.

Some examples can help show how the real obligations on quasi-real contracts will remain proportional to aggregate supply. Let us continue to discuss the wage rate for these examples, although the discussion would apply to all quasi-real obligations. Suppose nominal aggregate demand does not change, but that aggregate supply increases by 1%. Then the nominal wage rate under the quasi-real contract would be $\overline{W}\frac{N_t}{N_0}$ and would not change.

[15] Given technology, a better measure of nominal aggregate demand itself might be developed which would be more timely than nominal GDP, which is a supply measure. If such a measure was developed, quasi-real contracts may then have less timing issues that do pure inflation-indexed contracts.

However, the price level will decrease by 1%, causing the real wage to increase by 1%, which matches the 1% increase in aggregate supply.

Next, suppose nominal aggregate demand increases by 1% while aggregate supply remains the same. Since it equals $\overline{W}\frac{N_t}{N_0}$, the nominal wage rate would increase by 1%. However, the price level would also increase by 1%, offsetting the increase in the nominal wage rate. Therefore, the real wage rate remains unchanged.

Finally, suppose that aggregate supply decreases by 1% and that the central bank follows strict inflation targeting and causes nominal aggregate demand to also decrease by 1%. Therefore, the price level would not change. However, since the nominal wage rate equals $\overline{W}\frac{N_t}{N_0}$, the 1% decrease in nominal aggregate demand will cause the nominal wage rate to fall by 1%. Since the price level does not change, that means the real wage rate also falls by 1%, which matches the 1% drop in aggregate supply. Thus, quasi-real contracts retain the dynamic proportional relationship between real obligations and aggregate supply even when the central bank follows strict inflation targeting.

Given that not all contracts will be rewritten as quasi-real contracts, that some contracts will continue to be nominal, what should a central bank try to do if not strict inflation targeting? Our answer is nominal income targeting.

Technically, the model of section 2 indicates that the central bank should keep nominal aggregate demand from changing even if aggregate supply is expected to increase,[16] which would mean we would expect deflation. However, as the Appendix discusses, the modeling methodology of section 2 does not include some realities of the problems of deflation; in particular, in reality nominal interest rates cannot be negative because in reality one can hold money from one period to another. As normally recommended by proponents of nominal income targeting, we recommend targeting nominal aggregate demand to grow at a rate so that the expected inflation rate is somewhere around 1 to 2 percent. Let g_e be the expected growth rate in real GDP and let $\hat{\pi}$ be the desired expected inflation rate. We recommend that the central bank announce its intentions to increase nominal aggregate demand at the rate $g_e+\hat{\pi}$, which will result in an inflation rate of $\hat{\pi}$ if real GDP does grow as expected. However, the central bank should not deviate the growth rate of nominal aggregate demand even if GDP grows at a different rate.

To some extent, strict nominal income targeting is not that different from flexible inflation targeting with escape clauses for real shocks. However, one of the justifications for inflation targeting is the transparency of monetary policy. Strict nominal income targeting is much more transparent than is "flexible inflation targeting with exceptions". In particular, "flexible inflation targeting with exceptions" does not state what should be done when aggregate supply changes unexpected and/or permanently. As Federal Reserve Board Governor Gramlich said on January 13, 2000, in the case of aggregate-supply-caused inflation, the "most flexible and competent central bank in the world would be faced with a difficult dilemma in such circumstances--forestall the recession by making inflation worse or limit the inflation by making the recession worse." With strict nominal income targeting, the central bank would be clear as to what it should try to do and the central bank can be monitored and judged by the performance measure of how closely the central bank meets its published nominal-income target. The lack of

[16] We surmise that a more elaborate model with population growth may imply that nominal aggregate demand should increase with population growth.

transparency with flexible inflation targeting makes it difficult for economic agents to predict the economic effects of the actions of the central bank since those actions are so nebulous.

However, if real GDP is expected to increase, then if everyone's Pareto-efficient consumption is proportional to real GDP, then the real obligations on contracts should increase with real GDP even if that increase in real GDP is expected. Under the nominal-income-targeting policy we propose above, such would not be the case with nominal contracts, but it would be the case with quasi-real contracts where the adjustment multiplier is N_t/N_0 where 0 is the base year. Therefore, the combination of quasi-real contracts with nominal-income-targeting would move our economies closer to Pareto efficiency.[17]

To make the points of this paper, we needed to make certain assumptions. One assumption was that the Pareto-efficient consumption allocation was proportional to aggregate supply for all consumers. If consumers have different risk aversion, that assumption would no longer hold as some form of insurance contracts would be needed so that the less-risk-averse consumers could sell insurance to the more-risk-averse consumers. Normal quasi-real contracts are not insurance contracts and so they by themselves would not be able lead the economy to Pareto efficiency.

Another assumption was that the consumers' endowments of land and labor were not stochastic. If they were, then other types of insurance contracts would be needed to reallocate that risk.

The Consumption-Aggregate-Supply Functionality Theorem is a very important Theorem having applications that extend well beyond this paper. For example, if the hypotheses of this theorem hold, the Theorem implies that there is only one risk that matters in a Pareto-efficient

[17] The optimal quasi-real adjusting multiplier may differ in a model that includes a growing propulation. We leave such a model to future research.

equilibrium and that risk concerns aggregate supply. Since the risk on aggregate supply cannot be diversified away, a one-factor model of risk would apply and that one factor would be aggregate supply.

However, the Theorem does rest on the assumption that all the consumption good is consumed. Storage of the consumption good would affect the conclusion. More importantly, the existence of capital would mean that part of aggregate supply will be used for purposes other than for consumption. In particular, if the expected marginal productivity of capital depends on past states of nature, an individual's consumption no longer can be written as a function solely of income. Even so, we do not think the conclusions we have reached in this paper will be significantly affected. However, that change substantially affects the pricing of risk resulting possibly with many factors affecting the pricing of risk.

Very fundamental to policy are the goals policymakers try to achieve. Economists' current attitudes towards inflation permeate their goals. Macroeconomists and monetary economists usually encourage monetary and fiscal authorities to pursue the goal of minimizing the squared deviation of inflation from its targeted rate in conjunction with other goals such as minimizing squared deviations of output from its targeted level (e.g., see Grey, 1976; Fischer, 1977; and Svensson, 2000). We argue that those goals are flawed in that they do not distinguish good inflation from bad. Pareto efficiency should be our ultimate goal. In this paper, Pareto efficiency indicates the monetary authority should target aggregate demand (which currently is measured by nominal GDP).

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Appendix

This appendix consists of the following sections:

A. Derivation of the Arrow-Debreu equilibrium for the model of section 2 (This provides the basis to evaluate what is and what is not a Pareto-efficient consumption allocation.)
B. Proof that the model of section 2 with quasi-real discount bonds and other quasi-real contracts is Pareto efficient
C. Proof that the model of section 2 with quasi-real, interest-bearing, one-period bonds and other quasi-real one-period contracts is Pareto efficient
D. Proof that the model of section 2 with nominal bonds and other nominal contracts is Pareto efficient when nominal aggregate demand is constant
E. Argument that an equilibrium with money being held from one period to another cannot be Pareto efficient except under extreme assumptions
F. Defense of the "No-Money-Held" methodology for modeling a monetary economy

For parts A through D, we assume that the Pareto-efficient consumption allocation for each consumer is a constant proportion of aggregate supply so $\tilde{c}_{jit} = \tilde{\mu}_{jt} Y_{it}$ for all j, i, and t.[18]

Arrow-Debreu equilibrium for the model of section 2

Our approach to determine true Pareto efficiency is to compare the consumption allocation of non-Arrow-Debreu equilibria to the allocations under an Arrow-Debreu equilibrium. The model of section 2 as applied to an Arrow-Debreu equilibrium consists of $c_{jit} = \tilde{\mu}_{jt} Y_{it}$ for all j, i, and t, and the following additional conditions:

1. Taking prices, wages, and land rental rates as given, the representative firm chooses its demand for labor and land over time to maximize

[18] While this assumption is a restriction on a endogenous result of the model, we can generate this result assuming consumers have identical CRRA utility functions. Also, Eagle and Domian (2003) show that this result will occur iff all consumers have the same coefficient of relative risk aversion; this coefficient can vary across different time periods and different states, but not across different consumers in order to come up with this result.

$$P_0 e^{\alpha_0} f(\tilde{L}_0, \tilde{H}_0) - W_0 \tilde{L}_t - R_0 \tilde{H}_t + \sum_{t=1}^{T} \pi_{it} \left(\Omega_{it} e^{\alpha_{it}} f(\tilde{L}_t, \tilde{H}_t) - \Psi_{it} \tilde{L}_t - \varphi_{it} \tilde{H}_t \right)$$

The prices paid at time 0 for the state-contingent securities of the consumption good, of labor, and of land respectively are $\pi_{it}\Omega_{it}$, $\pi_{it}\Psi_{it}$, and $\pi_{it}\varphi_{it}$. We can think about Ω_{it}, Ψ_{it}, and φ_{it} as the prices for the state-contingent securities that would be paid if the probability of state i occurring was one. Because the actual probabilities are less than one, the actual prices paid would be the probability multiplied by Ω_{it}, Ψ_{it}, and φ_{it}. We formulate the Arrow-Debreu economy in this manner so as to more readily interpret our results.

2. Each consumer j maximizes $U_{j0}(c_{j0}) + \sum_{t=1}^{T} \beta^t \sum_{i=1}^{n_t} \pi_{it} U_{jt}(c_{jit})$ subject to j's budget constraint, where $U_{jt}(\cdot)$ is j's utility function at time t, c_{j0} is j's consumption at time 0, c_{jit} is j's consumption in state i at time t, β is the time preference factor, and π_{it} is the probability of state i occurring at time t. The budget constraints these consumers would face in an Arrow-Debreu equilibrium are that

$$P_0 c_{j0} + \sum_{t=1}^{T} \sum_{i=1}^{n_t} \pi_{it} \Omega_{it} c_{jit} = W_0 \overline{L}_{j0} + R_0 \overline{H}_{j0} + \Lambda_{j0} + \sum_{t=1}^{T} \sum_{i=1}^{n_t} \pi_{it} \left(\Psi_{it} \overline{L}_{jt} + \varphi_{it} \overline{H}_{jt} \right) \quad \textbf{(A 1)}$$

The symbol Λ_{j0} represents a subsidy to j at time 0 (if negative then a tax) for redistribution purposes such as social security. We assume the government's budget is balanced meaning that $\sum_{j=1}^{n} \Lambda_{j0} = 0$. Since c_{jit} equals j's demand at time 0 for the state-contingent security that pays one consumption good at time t iff state i occurs, we just used c_{jit} instead of inventing a new symbol to represent j's demand for that state-contingent security. Similarly, in place of the state-contingent securities for labor and land for state i at time t, we use $\overline{L}_{jt}$ and $\overline{H}_{jt}$.

3. Equilibrium exists when the state-contingent securities markets clear for the consumption good, labor, and land for all periods and all states.

The first order necessary conditions for the firm's maximization problem are that:

$$\frac{W_0}{P_0} = e^{\alpha_0} f_L(\tilde{L}_0, \tilde{H}_0) \quad \textbf{(A 2)}$$

$$\frac{R_0}{P_0} = e^{\alpha_0} f_H(\tilde{L}_0, \tilde{H}_0) \quad \textbf{(A 3)}$$

$$\frac{\Psi_{it}}{\Omega_{it}} = e^{\alpha_{it}} f_L(\tilde{L}_t, \tilde{H}_t) \quad \textbf{(A 4)}$$

$$\frac{\varphi_{it}}{\Omega_{it}} = e^{\alpha_{it}} f_H(\tilde{L}_t, \tilde{H}_t) \quad \textbf{(A 5)}$$

If we divide (A4) and (A5) by $Y_{it} = e^{\alpha_{it}} f(\tilde{L}_t, \tilde{H}_t)$, we get $\frac{\Psi_{it}}{\Omega_{it} Y_{it}} = \frac{e^{\alpha_{it}} f_L(\tilde{L}_t, \tilde{H}_t)}{e^{\alpha_{it}} f(\tilde{L}_t, \tilde{H}_t)} = \frac{f_L(\tilde{L}_t, \tilde{H}_t)}{f(\tilde{L}_t, \tilde{H}_t)}$ and $\frac{\varphi_{it}}{\Omega_{it} Y_{it}} = \frac{e^{\alpha_{it}} f_H(\tilde{L}_t, \tilde{H}_t)}{e^{\alpha_{it}} f\ (\tilde{L}_t, \tilde{H}_t)} = \frac{f_H(\tilde{L}_t, \tilde{H}_t)}{f(\tilde{L}_t, \tilde{H}_t)}$. Multiply both sides of both equations by $\Omega_{it} Y_{it}$ and then taking expectations, we get:

$$E[\Psi_{it}] = \frac{f_L(\tilde{L}_t, \tilde{H}_t)}{f(\tilde{L}_t, \tilde{H}_t)} E[\Omega_{it} Y_{it}] \quad \textbf{(A 6)}$$

$$E[\varphi_{it}] = \frac{f_H(\tilde{L}_t, \tilde{H}_t)}{f(\tilde{L}_t, \tilde{H}_t)} E[\Omega_{it} Y_{it}] \quad \textbf{(A 7)}$$

since the supply of labor and land is constant at any point of time.

Since $c_{jit} = \tilde{\mu}_{jt} Y_{it}$, we can rewrite (A1) as:

$$P_0 c_{j0} + \sum_{t=1}^{T} \tilde{\mu}_{jt} \sum_{i=1}^{n_t} \pi_{it} \Omega_{it} Y_{it} = W_0 \overline{L}_{j0} + R_0 \overline{H}_{j0} + \Lambda_{j0} + \sum_{t=1}^{T} \left(\overline{L}_{jt} \sum_{i=1}^{n_t} \pi_{it} \Psi_{it} + \overline{H}_{jt} \sum_{i=1}^{n_t} \pi_{it} \varphi_{it} \right)$$

which can further be rewritten as:

$$P_0 c_{j0} + \sum_{t=1}^{T} \tilde{\mu}_{jt} E[\Omega_{*t} Y_{*t}] = W_0 \overline{L}_{j0} + R_0 \overline{H}_{j0} + \Lambda_{j0} + \sum_{t=1}^{T} \left(\overline{L}_{jt} E[\Psi_{*t}] + \overline{H}_{jt} E[\varphi_{*t}] \right) \quad \textbf{(A 8)}$$

where the * in place of the i within the expectations operator means that the expectation was taken over all i.

Equations (A2) through (A8) must hold in an Arrow-Debreu equilibrium. The Arrow-Debreu equilibrium is such that all markets exist at time 0 only; even if markets were open in subsequent periods, no one would trade even if the probabilities of the states change because of new information (assuming homogeneous expectations).

Pareto-efficiency of quasi-real equilibrium with discount bonds

With quasi-real contracts entered into at time 0, the payment at time t will equal the base obligation multiplied by N_t / N_0. Therefore, where $\widehat{W}$ and $\widehat{R}$ are the base wage and land rent obligations, the nominal wage and land-rent payments will be $\widehat{W}\frac{N_t}{N_0}$ and $\widehat{R}\frac{N_t}{N_0}$. The model of section 2 as applied to quasi-real contracts where those contracts are set at time 0 consists of $c_{jt} = \tilde{\mu}_{jt} Y_t$ for all j, i, and t, and the following additional conditions:

1. Taking prices, wages, and the land rental rate as given, the representative firm chooses its demand for labor and land at each time t to maximize

$$P_t e^{\alpha_t} f(\tilde{L}_t, \tilde{H}_t) - \widehat{W}_t \frac{N_t}{N_0} \tilde{L}_t - \widehat{R}_t \frac{N_t}{N_0} \tilde{H}_t$$

2. Each consumer j maximizes the $U_{j0}(c_{j0}) + \sum_{t=1}^{T} \beta^t \sum_{i=1}^{n_t} \pi_{it} U_{jt}(c_{jit})$ subject to the constraints

$$P_0 c_{j0} + \sum_{t=1}^{T} V_t Q_{jt} = W_0 \overline{L}_{j0} + R_0 \overline{H}_{j0} + Z_{j0}$$

$$P_{it} c_{jit} = \left(Q_{jt} + \widehat{W}\overline{L}_{jt} + \widehat{R}\overline{H}_{jt} + \widehat{Z}_{jt}\right)\frac{N_{it}}{N_0}$$

where Q_{jt} is j's demand at time 0 for the quasi-real discount bonds that mature at time t, where the bond pays one monetary unit multiplied by N_{it} / N_0 and V_t is the price of that bond at time 0. The symbol $\widehat{Z}_{jt}$ represents the quasi-real social security obligation of the government to j (or if negative from j to the government) at time t. We assume that the government's budget balances at each time t so that $\sum_{j=1}^{m} \widehat{Z}_{jt} = 0$. The consumer constraints can be collapsed into the following single budget constraint:

$$P_0 c_{j0} + \sum_{t=1}^{T} V_t N_0 \mu_{jt} = W_0 \overline{L}_{j0} + R_0 \overline{H}_{j0} + Z_{j0} + \sum_{t=1}^{T} V_t \left(\widehat{W}\overline{L}_{jt} + \widehat{R}\overline{H}_{jt} + \widehat{Z}_{jt}\right) \qquad \textbf{(A 9)}$$

3. Equilibrium exists when the goods, labor, land rental, markets clear for each time t=0..T and the quasi-real bond market at time 0 clears.

We now show that quasi-real discount bonds in conjunction with quasi-real indexing of wages, rents, and social security results in the same Pareto-efficient consumption allocation as does an Arrow-Debreu economy with complete markets. Let $V_t = \frac{E[\Omega_{*t} Y_{*t}]}{N_0}$,

$\widehat{W}_t = \frac{E[\Psi_{*t}]}{V_t}$, and $\widehat{R}_t = \frac{E[\varphi_{*t}]}{V_t}$ in (A9). Also in (A8), let $\Lambda_{j0} = Z_{j0} + \sum_{t=1}^{T} V_t \widehat{Z}_{jt}$. Then, when

$c_{jt} = \tilde{\mu}_{jt} Y_t$, (A9) holds iff (A8) holds for all j, i, and t. Hence, the consumers' optimization problem is satisfied.

The FONC of the firm's problem is that for t=0 equations (2) and (3) hold, and for t=1..T:

$$\frac{\widehat{W}_t}{P_t}\frac{N_t}{N_0} = e^{\alpha_t} f_L(\tilde{L}_t, \tilde{H}_t) \quad \textbf{(A 10)}$$

$$\frac{\widehat{R}_t}{P_t}\frac{N_t}{N_0} = e^{\alpha_t} f_H(\tilde{L}_t, \tilde{H}_t) \quad \textbf{(A 11)}$$

Since $V_t = \frac{E[\Omega_{*t} Y_{*t}]}{N_0}$ and by (A6) and (A7), $\widehat{W}_t = \frac{E[\Psi_{*t}]}{V_t} = \frac{E[\Psi_{*t}] N_0}{E[\Omega_{*t} Y_{*t}]} = N_0 \frac{f_L(\tilde{L}_t, \tilde{H}_t)}{f(\tilde{L}_t, \tilde{H}_t)}$ and

$\widehat{R}_t = \frac{E[\varphi_{*t}]}{V_t} = \frac{E[\varphi_{*t}] N_0}{E[\Omega_{*t} Y_{*t}]} = N_0 \frac{f_H(\tilde{L}_t, \tilde{H}_t)}{f(\tilde{L}_t, \tilde{H}_t)}$, both of which are constants as they should be since they are set at time 0. Therefore, since $Y_{it} = \frac{N_{it}}{P_{it}}$, the real wage and land rental rates are

$$\frac{\widehat{W} N_{it}}{P_{it} N_0} = N_0 \frac{f_L(\tilde{L}_t, \tilde{H}_t)}{f(\tilde{L}_t, \tilde{H}_t)} \frac{Y_{it}}{N_0} = \frac{f_L(\tilde{L}_t, \tilde{H}_t) e^{\alpha_{it}} f(\tilde{L}_t, \tilde{H}_t)}{f(\tilde{L}_t, \tilde{H}_t)} = e^{\alpha_{it}} f_L(\tilde{L}_t, \tilde{H}_t), \text{ and}$$

$$\frac{\widehat{R}_t N_{it}}{P_{it} N_0} = N_0 \frac{f_H(\tilde{L}_t, \tilde{H}_t)}{f(\tilde{L}_t, \tilde{H}_t)} \frac{Y_{it}}{N_0} = \frac{f_H(\tilde{L}_t, \tilde{H}_t) e^{\alpha_{it}} f(\tilde{L}_t, \tilde{H}_t)}{f(\tilde{L}_t, \tilde{H}_t)} = e^{\alpha_{it}} f_H(\tilde{L}_t, \tilde{H}_t)$$

Therefore, the firm's optimization problem is also satisfied by this equilibrium. Since this is the same consumption allocation as in the Arrow-Debreu model, markets clear. Q.E.D.

Pareto-efficiency of quasi-real equilibrium with one-period contracts

With one-period quasi-real contracts entered into at time t-1, the payment at time t will equal the base obligation multiplied by N_t / N_{t-1}. Therefore, where $\widehat{W}_t$ and $\widehat{R}_t$ are the base wage and land rent obligations, the nominal wage and land-rent payments will be $\widehat{W}_t \frac{N_t}{N_{t-1}}$ and $\widehat{R}_t \frac{N_t}{N_{t-1}}$.

The model of section 2 as applied to these one-period, quasi-real contracts consists of $c_{jit} = \tilde{\mu}_{jt} Y_{it}$ for all j, i, and t, and the following additional conditions:

1. The firm's optimization problem is the same as in the previous section's model.
2. In this quasi-real economy, consumers will face the following constraints:

   $$P_0 c_{j0} + Q_{j0} = W_0 \overline{L}_{j0} + R_0 \overline{H}_{j0} + Z_{j0}$$

   $$P_t c_{jt} + Q_{jt} = \left(\widehat{W}_t \overline{L}_{jt} + \widehat{R}_t \overline{H}_{jt} + \widehat{Z}_{jt} + Q_{j,t-1}(1+q_{t-1})\right)\frac{N_t}{N_{t-1}} \text{ for t=1..T-1, and}$$

   $$P_T c_{jT} = \left(\widehat{W}_T \overline{L}_{jT} + \widehat{R}_T \overline{H}_{jT} + \widehat{Z}_{jT} + Q_{j,T-1}(1+q_{T-1})\right)\frac{N_T}{N_{T-1}}$$

   (The subscripts for the states are suppressed above.)
   Working backwards, these can be collapsed into the following single budget constraint:

   $$\sum_{t=0}^{T} \frac{P_t c_{jt} \, N_0 / N_t}{\prod_{k=0}^{t-1}(1+q_k)} = W_0 \overline{L}_{j0} + R_0 \overline{H}_{j0} + Z_{j0} + \sum_{t=1}^{T} \frac{\left(\widehat{W}_t \overline{L}_{jt} + \widehat{R}_t \overline{H}_{jt} + \widehat{Z}_{jt}\right) N_0 / N_{t-1}}{\prod_{k=0}^{t-1}(1+q_k)} \qquad \textbf{(A 12)}$$

3. Equilibrium is where the goods market, the labor market, the land market, and the quasi-real bond market clear each period.

By letting $V_t = \frac{E[\Omega_{*t} Y_{*t}]}{N_0}$, $\widehat{W}_t = \frac{E[\Psi_{*t}]}{V_t}$, and $\widehat{R}_t = \frac{E[\varphi_{*t}]}{V_t}$ in (A12) and letting $\Lambda_{j0} = Z_{j0} + \sum_{t=1}^{T} V_t \widehat{Z}_{jt}$ in (A8), and then starting at t=1 and letting $q_t = \frac{1}{V_{t+1} \prod_{k=0}^{t-1} (1+q_k)} - 1$ for t=1..T-1; we can see that (A12) will hold iff (A8) holds for $c_{jit} = \tilde{\mu}_{jt} Y_{it}$ for all j, i, and t. Also, the first order necessary conditions for the firm's optimization will be satisfied just as they were for the previous section. Q.E.D.

Pareto-efficiency of equilibrium with one-period nominal contracts when nominal aggregate demand does not change

When nominal aggregate demand does not change, quasi-real contracts and nominal contracts behave the same way. Therefore, since one-period quasi-real contracts in the previous section were Pareto efficient, so must these one-period nominal contracts given that nominal aggregate demand does not change. Q.E.D.

Argument that normally a money equilibrium cannot be truly Pareto Efficient when consumers differ in their consumption.

The two fundamental theorems of welfare economics as applied to Arrow-Debreu equilibria state that under fairly general assumptions (1) an Arrow-Debreu equilibrium is Pareto efficient and (2) any Pareto-efficient consumption allocation can be represented by an Arrow-Debreu equilibrium. Therefore, if we can get a non-Arrow-Debreu equilibrium that has the same consumption allocation as does an Arrow-Debreu equilibrium, then that consumption allocation is Pareto efficient; if we cannot, then the consumption allocation is not Pareto efficient. Holding money from one period to another will generally create a distortion from an Arrow-Debreu equilibrium, which will then be a distortion from Pareto-efficiency.

The logic of the above paragraph creates a “catch 22”. If our model has money being held from period to period, we will not be able to get true Pareto efficiency. Without money being held, then how can we get nominal effects?

The above “catch 22” may have contributed to economists’ blindness to why aggregate-supply-caused inflation is good in the Pareto sense. We answer this “catch 22”, with a methodology where money is not held from one period to another, but where nominal aggregate demand is exogenous or exogenously stochastic.

The above logic does depend on our statement that holding money will create a distortion from an Arrow-Debreu economy. To help see this, let’s simplify our model by assuming perfect foresight. With perfect foresight, the state-contingent securities become the equivalent of pre-paid future contracts. Let F_{jt} be j’s demand for the future contract that pays off at time t. Then (A1) becomes:

$$\tilde{P}_0\tilde{c}_{j0} + \sum_{t=1}^{T}\tilde{\Omega}_t\tilde{F}_{jt} = \tilde{W}_0\overline{L}_{j0} + \tilde{R}_0\overline{H}_{j0} + \tilde{\Lambda}_{j0} + \sum_{t=1}^{T}\left(\tilde{\Psi}_t\overline{L}_{jt} + \tilde{\varphi}_t\overline{H}_{jt}\right) \quad \textbf{(A 13)}$$

and

$$\tilde{c}_{jt} = \tilde{F}_{jt} \quad \textbf{(A 14)}$$

for all t=1..T, where the tilde mark above each variable indicates it is the equilibrium value for the Arrow-Debreu economy.

If we add money being held to this Arrow-Debreu economy, these constraints become:

$$P_0c_{j0} + \sum_{t=1}^{T}\Omega_t F_{jt} + M_{j0} - Z_{j0} = W_0\overline{L}_{j0} + R_0\overline{H}_{j0} + \Lambda_{j0} + \sum_{t=1}^{T}\left(\Psi_t\overline{L}_{jt} + \varphi_t\overline{H}_{jt}\right) \quad \textbf{(A 15)}$$

$$c_{jt} + \frac{M_{jt} - M_{j,t-1}R_{jt} - Z_{jt}}{P_t} = F_{jt} \qquad \textbf{(A 16)}$$

for all t=1..T, where M_{jt} is the money demanded by j at time t to be held to time t+1, R_{jt} is the gross nominal return on money and Z_{jt} is j's monetary endowment at time t. If one sets these monetary endowments all to zero, our argument will still remain valid (and even more obvious). However, monetary endowments may be necessary to get money into the system.

Next we derive the conditions that must exist between the changes in the money demands and the monetary endowments in order for the consumption allocation to be the same as in the Arrow-Debreu economy. To do so, from here on we assume that $c_{jt} = \tilde{c}_{jt}$ for all j and for all t. We also assume the values of the economy do not change, i.e., $\Omega_t = \tilde{\Omega}_t$, $\Psi_t = \tilde{\Psi}_t$, $\varphi_t = \tilde{\varphi}_t$, as changing these values will almost certainly create distortions from the Arrow-Debreu equilibrium.

Special Case: $F_{jt} = \tilde{F}_{jt}$. In this case, the holding of prepaid futures contracts does not change. Subtracting (A13) from (A15) and (A14) from (A16) we conclude that $M_{j0} = Z_{j0}$ and $M_{jt} - M_{j,t-1}R_{jt} = Z_{jt}$. In other words the monetary endowments in each period must equal the change in money demand (net of the return on money) to avoid a distortion away from the Arrow-Debreu consumption allocation. Since we see no reason for these conditions to hold, we conclude that, in the special case where the holdings of prepaid future contracts do not change, holding money will generally cause a distortion from the Arrow-Debreu consumption allocation.

General case: Subtracting (A13) from (A15) and (A14) from (A16), we get:

$\sum_{t=1}^{T} \Omega_t \left(F_{jt} - \tilde{F}_{jt}\right) + M_{j0} - Z_{j0} = 0$ and $\frac{M_{jt} - M_{j,t-1}R_{jt} - Z_{jt}}{P_t} = F_{jt} - \tilde{F}_{jt}$, which can be combined to yield the following condition where $\Omega_0 = 1$:

$$\sum_{t=0}^{T}\Omega_t\left(\frac{M_{jt}-M_{j,t-1}R_t}{P_t}\right)=\sum_{t=0}^{T}\Omega_t\left(\frac{Z_{jt}}{P_t}\right) \quad \textbf{(A 17)}$$

This means that the sum of the real present values of the change in the j's money demand (net of money's return) over j's life must equal the sum of the real present values of j's monetary endowments over j's life. If (A17) does not hold, then holding money will create a distortion from the Arrow-Debreu equilibrium. Since we can see no reason why (A17) would hold, we conclude that holding money will generally create such a distortion.

We are not saying this distortion will be significant, but in welfare economics, we usually do not talk about "almost" Pareto efficiency. If the distortions are insignificant, then that is just another reason to use our "no-money-held" methodology. The final section of the Appendix further defends this methodology.

Defense of no-money-held methodology

In response to the "catch 22" of the previous section, the no-money-held methodology assumes an exogenous or stochastically exogenous nominal aggregate demand with no money being held from one period to the next. While unrealistic, we believe the distortions from money being held are in many situations insignificant and can be ignored. However, we do recognize that we have to be careful. In particular, with our methodology, nominal interest rates can be negative. However, in the real world where money can be held from one period to another, nominal interest rates must be nonnegative. Even so, we should recognize that if the economy does get to the point where nominal interest rates "should" be negative but cannot, then the economy will then not be able to reach Pareto efficiency.

In today's monetary economies, monetary aggregates are only loosely connected to nominal aggregate demand. The no-money-held methodology changes the focus from the monetary aggregates to nominal aggregate demand.

While we do recognize that the assumption of no money being held from one period to another is unrealistic, by the previous section we must make that assumption if we are to have any hope of obtaining true Pareto efficiency. Even if no money being held is unrealistic, we can present a story that will make the model consistent. The following sequence of events within each period does create a nominal aggregate demand without any money being held from period to period:[19]

1. The central bank announces the level of nominal aggregate demand this period.
2. Firms produce; α_{it} is realized meaning aggregate output is realized.
3. Firms enter into agreements with the central bank, where each firm j agrees to sell consumers x_j units of the consumption good at the price P_t where x_j is its output and where $P_t = N_t/Y_t$. In return, the central bank "lends" the firms money equal to $P_t x_j$.
4. Firms pay wages to their laborers and rent to their landlords. Since each firm makes zero economic profit, the amount it pays out will equal $P_t x_j$.
5. The government will collect taxes and will pay social security.
6. The consumers will use their money to meet their bond obligations and buy more bonds.
7. The consumers will pay money to the firms for the consumption good.
8. The firms will return the money to the central bank by the end of the period. The central bank does not charge interest as long as the money is returned within the period. Thus, neither firms nor consumers will hold any money from period to period.

[19] Unfortunately, the story we present is one where velocity is a constant one, whereas we really do not wish this methodology to be associated with a constant velocity. Other stories consistent with the No-Money-Held methodology probably could be told with a varying velocity, but we do not attempt to do so in this paper.