Estimation of a Money Demand Function for M2 in the U.S.A. in a Vector Error Correction Model

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The economic debate on the determinants of the demand for money sparked by John Maynard Keynes in “The general theory of employment, interest and money” has since then resulted in voluminous literature that explores the issue on both theoretical and empirical level. A curious feature of this ongoing research effort is that despite the wide variety of often conflicting assumptions, employed in the construction of rivaling theoretical models, the overwhelming majority of these models identify a common set of macroeconomic variables as the main determinants of the desired holdings of real money balances. This set typically includes variables that serve as proxies for the opportunity cost and the own rate of return of money, as well as a scale variable measuring the volume of transactions financed with money. Often, the functional form of the derived relation between the desired real monetary holdings and the indicators described above is assumed to be log-linear:

$$\ln\left(\frac{M_t^*}{P_t}\right) = \alpha + \beta_1 . \ln\left(\frac{Y_t}{P_t}\right) + \beta_2 . \ln I_t^b + \beta_3 . \ln I_t^m \text{ , where} \tag{1}$$

M_t^* - desired nominal money holdings

I_t^b - nominal interest rate on bonds

I_t^m - nominal interest rate on money

Y_t - nominal income or wealth

P_t - aggregate price level

Expression (1) represents a legitimate demand equation only if the following restrictions on its parameters hold: $\beta_1 \geq 0$, $\beta_2 \leq 0$, $\beta_3 \geq 0$ (Hendry and Ericson, 1991, p.23).

The first empirical studies on the demand for money viewed equation (1) as a long-run relation and estimated it with annual data. The implicit assumption was that the presence of transactions costs and other market imperfections prevent the continuous equality between the actual and desired monetary holdings (Goldfeld,1973, p. 581). Subsequent research focused on the short-run performance of the money demand equation. Early attempts to obtain OLS estimates of its coefficients with quarterly or monthly data were not successful, because of the detected strong positive serial correlation between the intertemporal values of real monetary holdings (see Goldfeld (1973), p.581 and Rasche et al (1996), p.8). To overcome this problem, researchers typically augmented equation (1) by adding the lagged value of real monetary balances in the list of explanatory variables. The resulting regression was then evaluated using OLS in combination with the Cochrane-Orcutt technique for dealing with first-order serial correlation in regression residuals:

$$m_t = \alpha + \beta_1 . y_t + \beta_2 . i_t^b + \beta_3 . i_t^m + \beta_4 . m_{t-1} + \nu_t \tag{2}$$

$$\nu_t = \rho . \nu_{t-1} + u_t \text{ , where}$$

$$u_t \sim i.i.d.\, N(0, \sigma^2)$$

x_t - lower case letters denote the natural logarithm of the corresponding variable X_t

Chow (1966) showed that a representation such as (2) can be derived from a "stock adjustment" model, in which agents face quadratic costs of adjusting their actual money holdings to the desired level.[1] Despite the economic appeal and relatively good fit of the data, partial adjustment models based on (2) superfluously impose severe restrictions on the dynamic interactions between the variables entering the money demand function. To see this, following Hamilton (1994, p.324) we rewrite the above system of equations in a single expression:

$$\begin{aligned} m_t = (1-\rho).\beta_0 + \beta_1.y_t - \beta_1.\rho.y_{t-1} + \beta_2.i_t^b - \beta_2.\rho.i_{t-1}^b + \\ + \beta_3.i_t^m - \beta_3.\rho.i_{t-1}^m + (\beta_4 + \rho).m_{t-1} - \beta_4.\rho.m_{t-2} + u_t \end{aligned} \tag{3}$$

It is obvious that equation (3) is a restricted version of a general "autoregressive-distributed lag model of the variables in levels" (Hendry and Ericson, 1991, p.23):

$$m_t = \alpha + \sum_{i=1}^{p} \beta_i.m_{t-i} + \sum_{i=0}^{p} \gamma_i.y_{t-i} + \sum_{i=0}^{p} \delta_i.i_{t-i}^b + \sum_{i=0}^{p} \phi_i.i_{t-i}^m + \varepsilon_t \tag{4}$$

The more recent contributions to the literature on money demand reject such *a priori* imposition of restrictions on the dynamics of the estimated relation in favor of more general specifications in the spirit of (4). There is also a growing consensus among economists that none of the variables entering (4) are covariance-stationary. Hoffman and Rasche (1991) show with monthly data that the 3 months Treasury Bill rate and the real Personal Income are both integrated of order one, while Mehra (1996) obtains the same result for the real M2 and its own rate of return. Therefore, regression (4) needs to be reparameterized to account for non-stationarity and possible cointegration between regressors, before being evaluated with OLS. A standard approach to this problem in the literature is to apply the Granger Representation Theorem (Hamilton, 1994, p. 580). Because all of the variables in (4) are integrated of order one, if there exists a linear combination of them that is stationary (which is implied by the existence of a long-run money demand function[2]), then regression (4) can be represented in an error-correction form. To derive the latter, we first write down the two equations specifying respectively the long-run and the short-run dynamics of the relation between monetary holdings and its determinants:

$$\left| \begin{aligned} & m_t = a_0 + a_1.y_t + a_2.i_t^b + a_3.i_t^m + u_t \\ & m_t = \alpha + \sum_{i=1}^{p} \beta_i.m_{t-i} + \sum_{i=0}^{p} \gamma_i.y_{t-i} + \sum_{i=0}^{p} \delta_i.i_{t-i}^b + \sum_{i=0}^{p} \phi_i.i_{t-i}^m + \varepsilon_t \end{aligned} \right. \tag{5}$$

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[1] See Rasche, 1996, pp. 5-16 for a review of the early empirical literature.
[2] Residual in (1) should be a white noise for it to be interpretable as a long-run money demand function.

The simultaneous existence of the two specifications imply that the long-run elasticities of money demand in respect to real income, treasury bill rate and own rate of return of money, derived from each of these equations should be pairwise equal:

$$a_1 = \frac{\sum_{i=0}^{p}\gamma_i}{1-\sum_{i=1}^{p}\beta_i} \;\; ; \;\; a_2 = \frac{\sum_{i=0}^{p}\delta_i}{1-\sum_{i=1}^{p}\beta_i} \;\; ; \;\; a_3 = \frac{\sum_{i=0}^{p}\phi_i}{1-\sum_{i=1}^{p}\beta_i} \tag{6}$$

The error correction representation of (4) can then be obtained by explicitly embedding the above coefficient restrictions in it. To this end, we first add and subtract the following terms from the RHS of (4):

$$\sum_{i=1}^{p}\beta_i.m_{t-1} \;\; ; \;\; \sum_{i=0}^{p}\gamma_i.y_{t-1} \;\; ; \;\; \sum_{i=0}^{p}\delta_i.i_{t-1}^{b} \;\; ; \;\; \sum_{i=0}^{p}\phi_i.i_{t-1}^{m}$$

Then, we form and group the current and lagged first-differences of all regressors and construct the error correction term ($\lambda.u_{t-1}$):

$$\Delta m_t = \alpha + \sum_{i=1}^{p-1} b_{1i}.\Delta m_{t-i} + \sum_{i=0}^{p-1} b_{2i}.\Delta y_{t-i} + \sum_{i=0}^{p-1} b_{3i}.\Delta i_{t-i}^{b} + \sum_{i=0}^{p-1} b_{4i}.\Delta i_{t-i}^{m} + \lambda.u_{t-1} + \varepsilon_t \tag{7}$$

Where,

$$b_{2i} = \begin{cases} -\gamma_0 & \text{, for } i=0 \\ -\sum_{j=i+1}^{p}\gamma_j & \text{, otherwise} \end{cases} \qquad b_{3i} = \begin{cases} -\delta_0 & \text{, for } i=0 \\ -\sum_{j=i+1}^{p}\delta_j & \text{, otherwise} \end{cases} \qquad b_{4i} = \begin{cases} -\phi_0 & \text{, for } i=0 \\ -\sum_{j=i+1}^{p}\phi_j & \text{, otherwise} \end{cases}$$

$$b_{1i} = -\sum_{j=i+1}^{p}\beta_j \qquad \lambda = -\left(1-\sum_{i=1}^{p}\beta_i\right) \qquad u_{t-1} = m_{t-1} - a_0 - a_1.y_t - a_2.i_t^{b} - a_3.i_t^{m}$$

The error-correction representation (7) "…generalizes the conventional partial-adjustment model, allowing separate reaction speeds to the different determinants of money demand …, yet via the error correction mechanism ensures that long-run targets are achieved …"(Hendry and Ericson, 1991, p.23). Regressions based on (7) can not be estimated by OLS, because of simultaneous equations bias. There are two alternative solutions to this problem: the use of instrumental variables (Mehra, 1996, p.30) or the estimation of (7) as part of a bigger and more comprehensive vector error-correction model (Johansen and Juselius, 1990). The vector error-correction model (VECM) is a logical extension of the line of reasoning behind the error-correction representation of the money demand function. In VECM, all variables appearing in (4) are treated as endogenous and their short-run dynamic adjustments are explicitly modeled as linear functions of their own lags and the lagged values of the remaining variables, subject to the coefficient restrictions imposed by the long-run cointegrating relation (6). Thus, instead of having only one error-correction representation as in (6), the VECM associated with (4) consists of a system of four error-correction representations:

$$\left| \begin{array}{l} \mathbf{z_t} = \mathbf{A'y_t} \\ \Delta\mathbf{y_t} = \mathbf{a_0} + \mathbf{a}_1\Delta\mathbf{y_{t-1}} + \mathbf{a}_2\Delta\mathbf{y_{t-2}} + \cdots + \mathbf{a_{p-1}}\Delta\mathbf{y_{t-p+1}} - \mathbf{Bz_{t-1}} + \mathbf{u_t} \end{array} \right. \text{, where} \tag{8}$$

$\mathbf{z_t}$ – stationary $(h \times 1)$ vector, where h is the number of cointegrating relations between the members of $\mathbf{y_t}$

$\mathbf{y'_{t-j}} = \begin{bmatrix} m_{t-j} & y_{t-j} & i^b_{t-j} & i^m_{t-j} \end{bmatrix}$, $j = 1,\ldots,p$-1

The main purpose of this paper is to apply the vector error-correction model presented above in the estimation of a money demand function for M2 in the U.S.A with monthly data for the period 1959:01 – 1996:12. To get a feel of the data used in this paper, in Figures 1a to 1d we plot respectively the log levels of real M2, real Personal Income, 3 months Treasury Bill rate and the own rate of return of M2.

Figure 1a

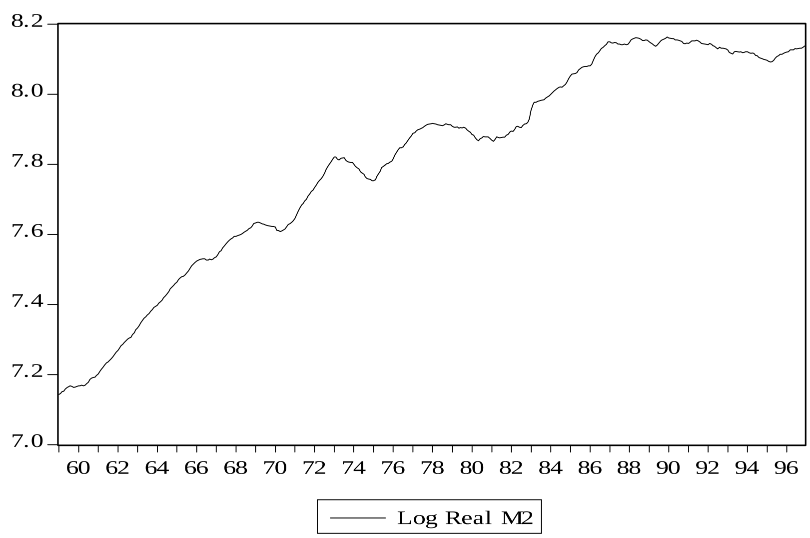


Figure 1b

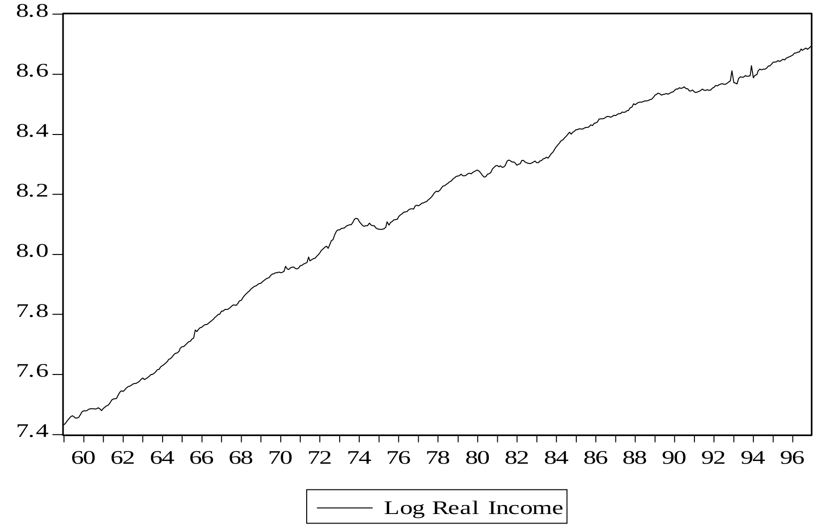


Figure 1c

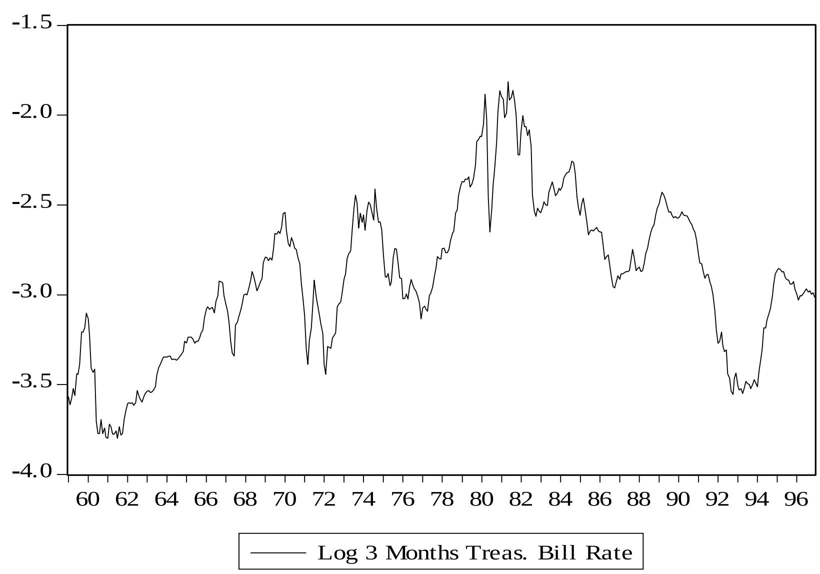
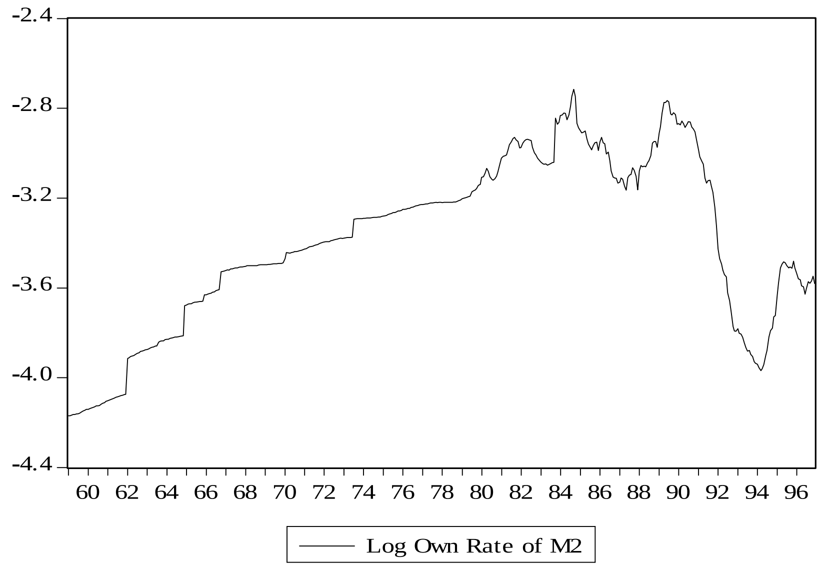


Figure 1d



The visual inspection of Figures 1a to 1d convincingly shows that all of the series are non-stationary. Consequently, any further statistical use of the data should be preceded by a test for the existence of unit-roots. Figures 2a to 2d present the results from the conducted Augmented Dickey-Fuller tests for unit roots in the log levels of real M2, real Personal Income, 3 months Treasury Bill rate and the own rate of return of M2. The null hypothesis under the Augmented Dickey-Fuller (ADF) test is that the time series are generated by unit root processes. "The null hypothesis of a unit root is rejected against the one-sided alternative if the t-statistic is less than (lies to the left of) the critical value." (Eviews 3 User's Guide, 1997, p.333). Figures 2a to 2d show the 5% critical value of the asymptotic ADF t-statistic under the null hypothesis and plots of its estimated values, obtained from an iterative procedure using a rolling sample[3] of 240 monthly observations for each of the four series. All ADF tests were conducted with the least restrictive specification, which allows for the presence of a constant and a trend in the estimated equation, as well as serial correlation in its residuals up to a lag length of twelve.

[3] We start with a sample of the first 240 observations (1959:01 – 1978:12) and compute the value of the ADF test statistic. Then on the second step of the iterative procedure, we add one additional observation at the front-end of the sample (1979:01) and drop one observation from the back-end of the sample (1959:01). We continue in this fashion until we reach the end of the original data sample (1996:12).

Figure 2a: Log Levels of Real M2

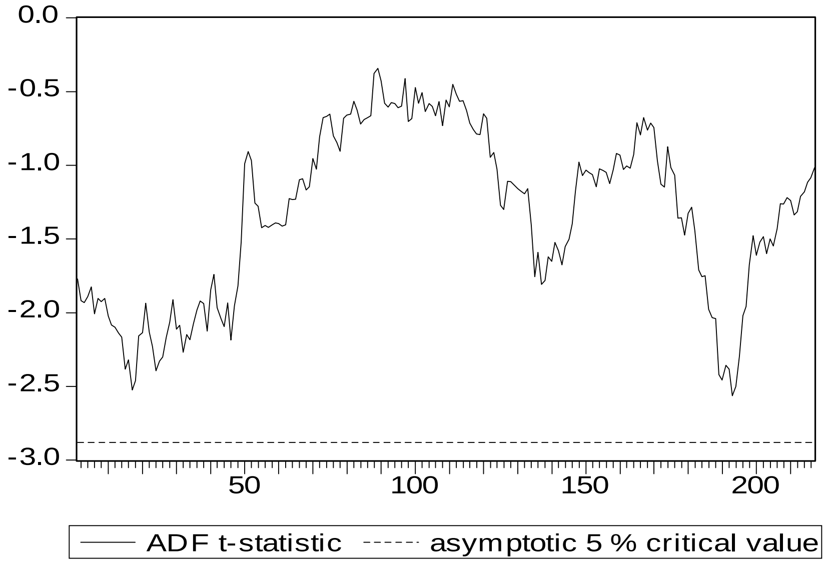


Figure 2b: Log Levels of Real Personal Income

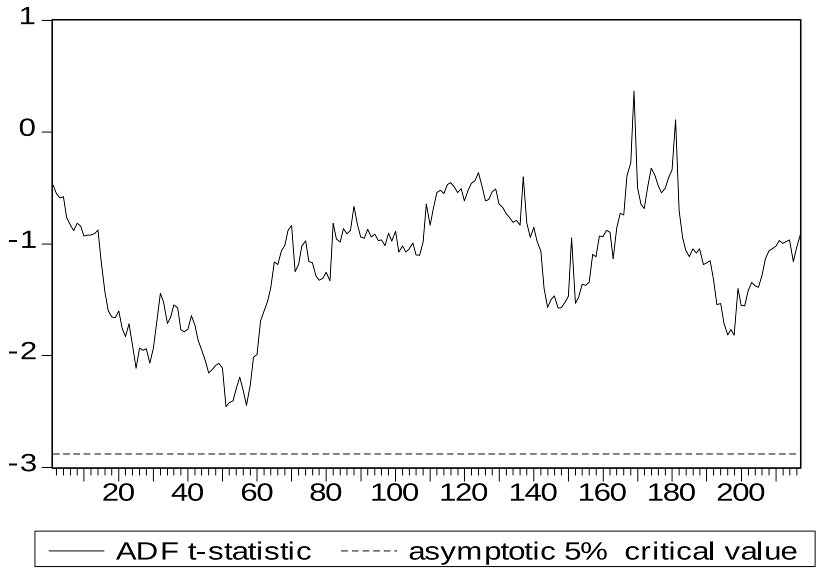


Figure 2c: Log Levels of 3 Month Treasury Bill Rate

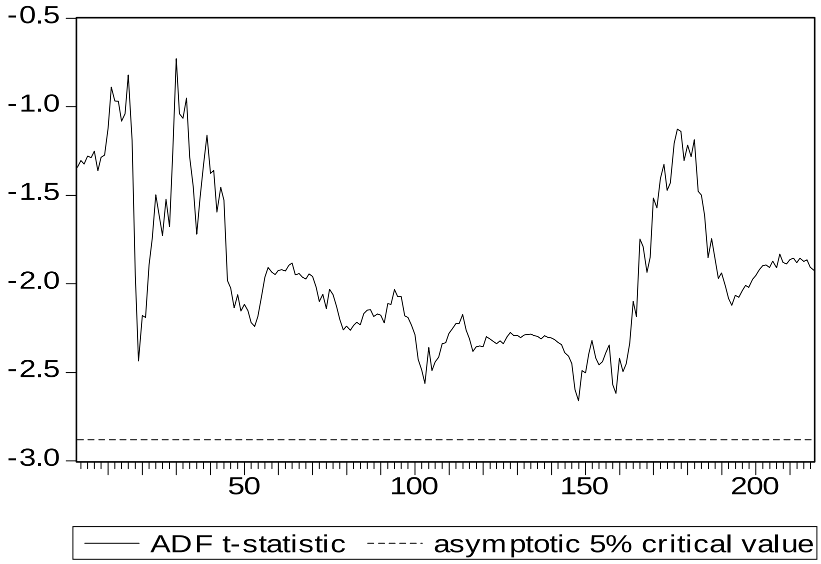
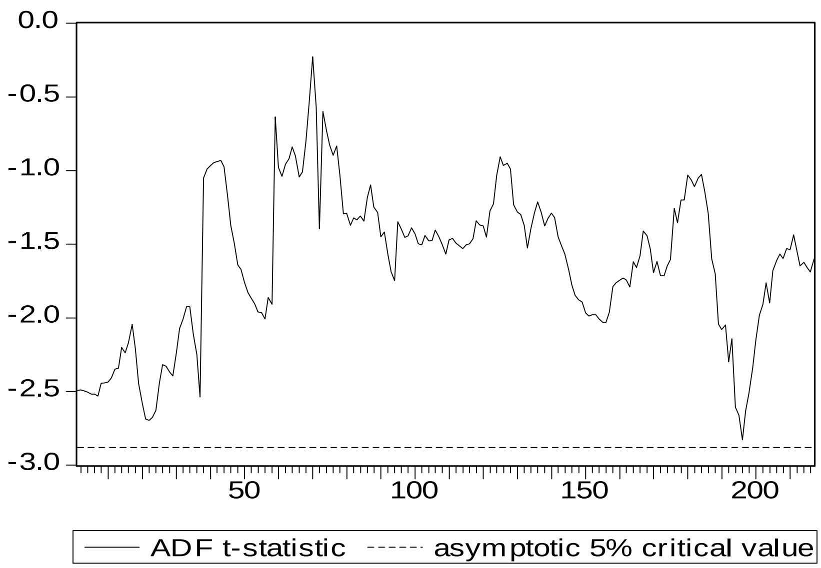


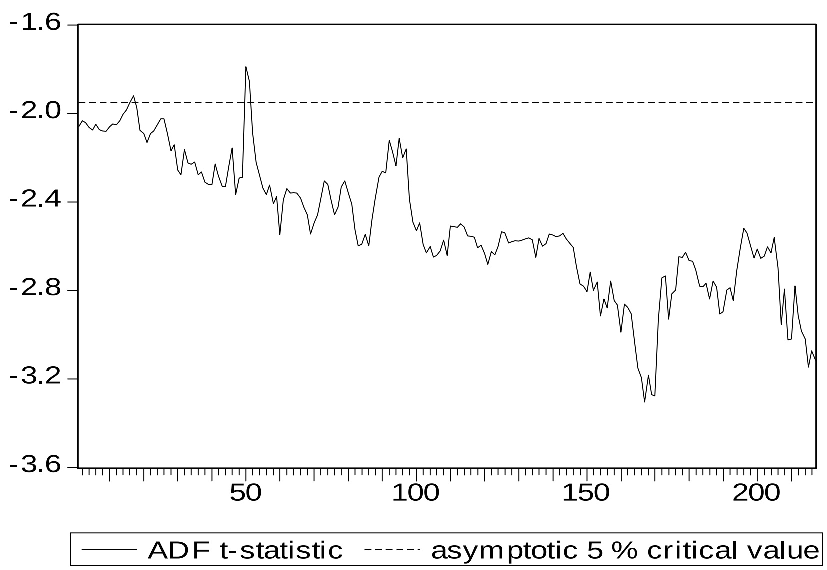
Figure 2d: Log Levels of Own Rate of Return of M2



A major advantage of the iterative research technique used in the construction of the above plots is that it provides a ready test of the structural stability of the tested relation. If the rolling ADF tests consistently accept the null hypothesis of an unit-root in the data over a large number of subsets of the full sample, we can be pretty sure that the obtained results do not hinge on a few outliers or on other peculiarities of the data. Figures 2a to 2d strongly support the empirical observation that all of the tested series are non-stationary and that their log levels contain unit-roots.

Next, we test for the order of integration[4] of the four series by conducting a rolling ADF tests on sub-samples of 240 observations of their first differences. If the series are integrated of order one, their first differences should be stationary and the estimated values of the ADF t-statistic in Figures 3a to 3d should be smaller (bigger in absolute value) than the 5% critical value under the null hypothesis.

Figure 3a: Log Difference of Real M2



[4] "A series is said to be cointegrated of order *d*, or I(d) if it requires to be differenced *d* times to yield a stationary, invertible, non-deterministic ARMA representation." (Muscatelli and Hurn, 1992, p.2).

Figure 3b: Log Difference of Real Personal Income

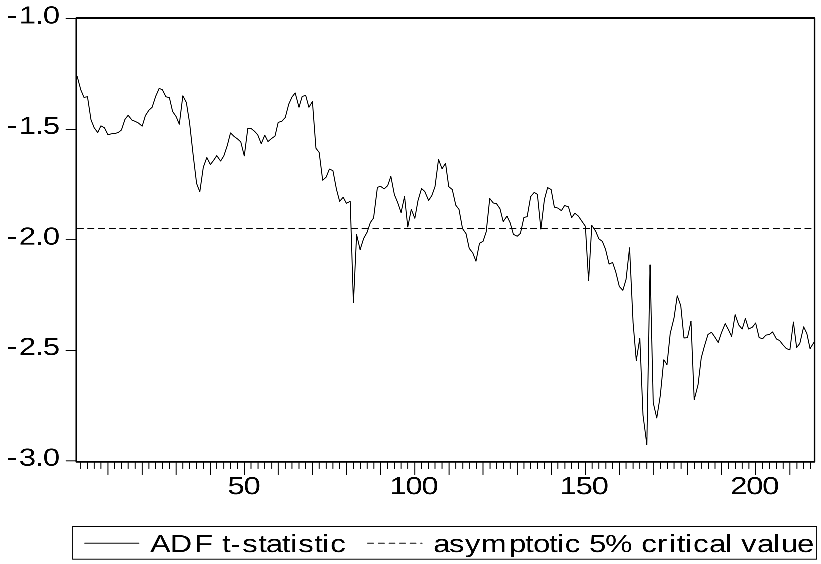


Figure 3c: Log Difference of 3 Month Treasury Bill Rate

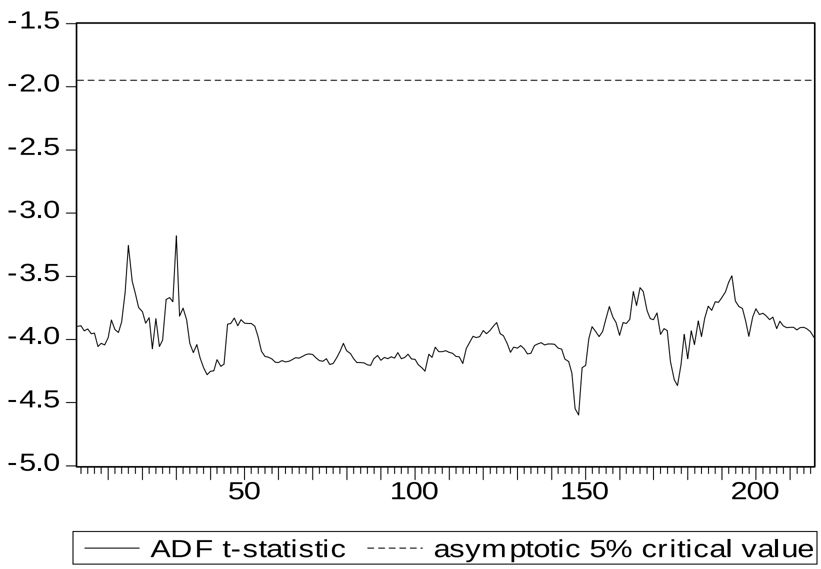
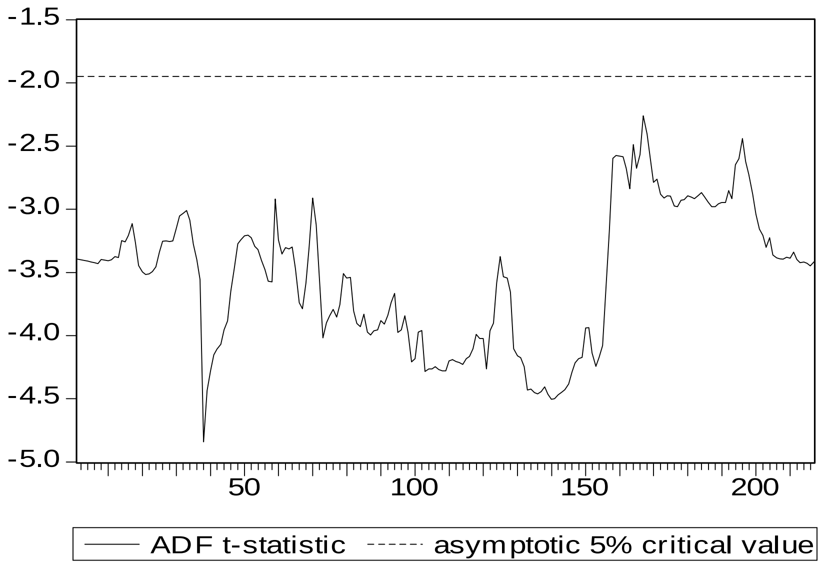


Figure 3d: Log Difference of Own Rate of Return on M2



The above plots clearly show that except for the real Personal Income, all scrutinized variables are consistently integrated of order one. In the case of the real Personal Income, the rolling ADF test indicates that the series was integrated of higher order in samples with a starting date prior to 1971:10 and has become consistently integrated of order one since then. Because cointegration theory does not provide a clear-cut solution to the problem of how should one proceed if the variables tested for cointegration are integrated of different orders (see Muscatelli and Hurn, 1992, p.12), we proceed with the derivation of a VECM using data only from the subsample: 1972:01 – 1996:12.

The second step in the development of a VECM is to test for the existence and the number of cointegrating equations between the log levels of real M2, real Personal Income, 3 month Treasury Bill rate and the own rate of return of M2. Before running any formal tests of cointegration, it is useful to obtain a visual perspective of the relation between the variables under consideration. Figure 3a presents a three-dimensional scatterplot of the log levels of real M2 on the left horizontal axis, real Personal Income on the right horizontal axis and the 3 month Treasury Bill rate on the vertical axis. Figure 3b presents a three-dimensional scatterplot of the log levels of real M2 on the left horizontal axis, real Personal Income on the right horizontal axis and the own rate of return of M2 on the vertical axis. If three integrated of order one series share one unit root[5], their joint realizations should lie on a plane in a three dimensional graph. In general, if we have *n* variables and *h* cointegrating relationships, then they have to lie on an (*n*-*h*) dimensional object in an *n* dimensional graph.

[5] There is only one cointegration relation between them.

Figure 3a: A 3-D Scatterplot of real M2, real Personal Income and 3 Month Treasury Bill Rate

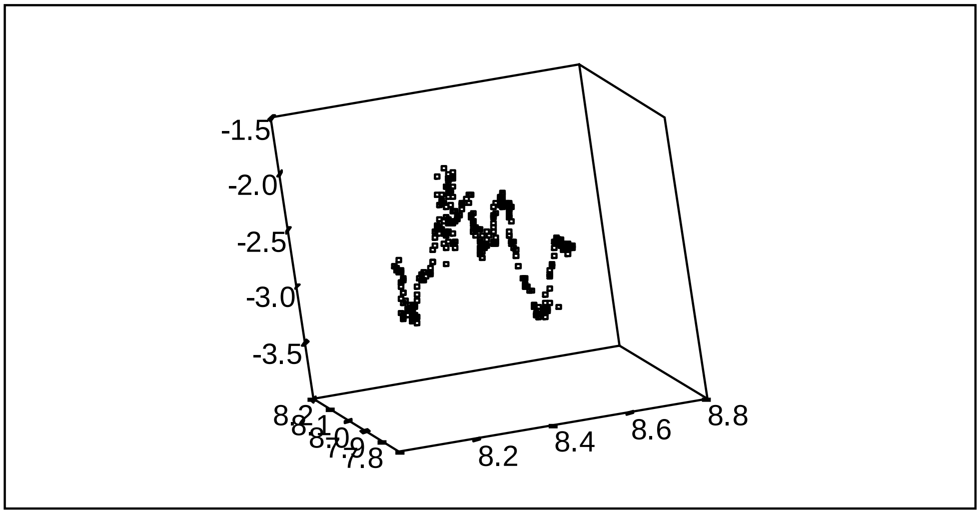
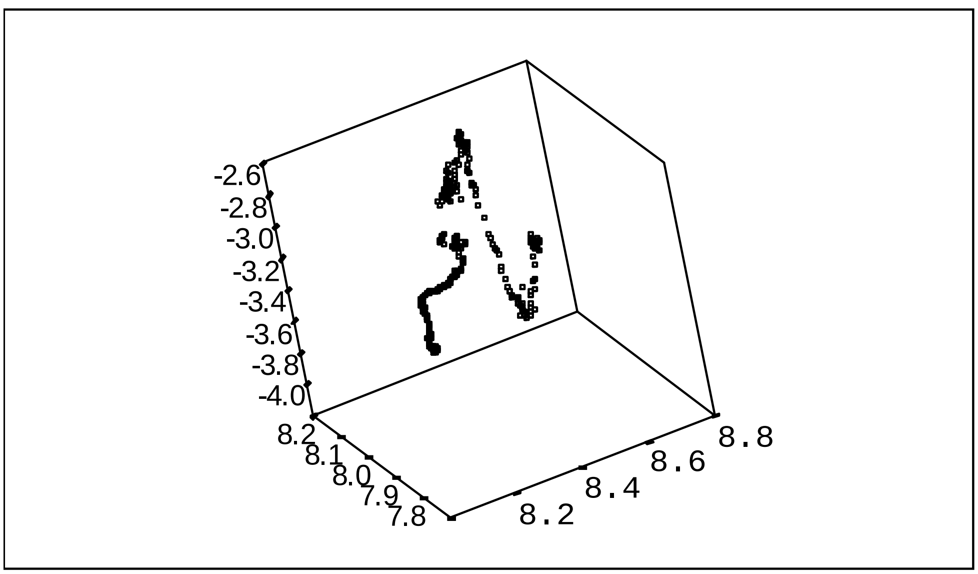
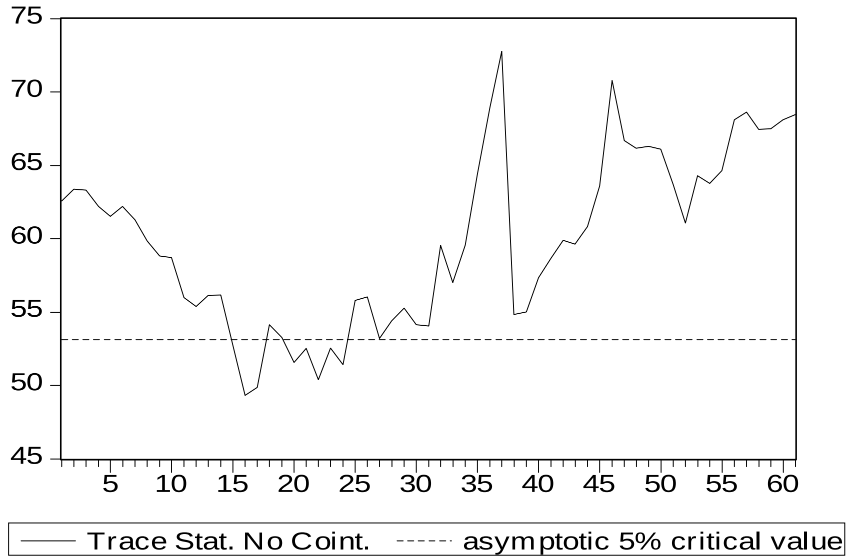


Figure 3b: A 3-D Scatterplot of real M2, real Personal Income and Own Rate of Return of M2



Figures 3a and 3b convincingly show that there indeed exist one or two cointegrating equations between the log levels of real M2, real Personal Income, 3 month Treasury Bill rate and the own rate of return of M2. Next, we test formally for the presence of cointegrating relationship between these variables using the Johansen trace and maximum eigenvalue tests of cointegration. The Johansen trace test compares the hypothesis of *r* cointegrating equations against the hypothesis that the series are stationary. If the estimated trace statistic exceeds the 5% critical value, we can reject the restriction *r* on the number of cointegrating equations. “To determine the number of cointegrating relations r, subject to the assumptions made about the trends in the series, we can proceed sequentially from r = 0 to r = k-1 until we fail to reject.” (Eviews 3 User’s Guide, 1997, p.511). Figure 4a show the 5% critical value of the asymptotic Johansen trace statistic and a plot of its estimated values for the hypothesis of zero cointegrating equations between the four series. The results are obtained from an iterative procedure using a rolling sample[6] of 240 monthly observations under the specification of no deterministic trend in the data, and an intercept but no trend in the cointegrating equation.

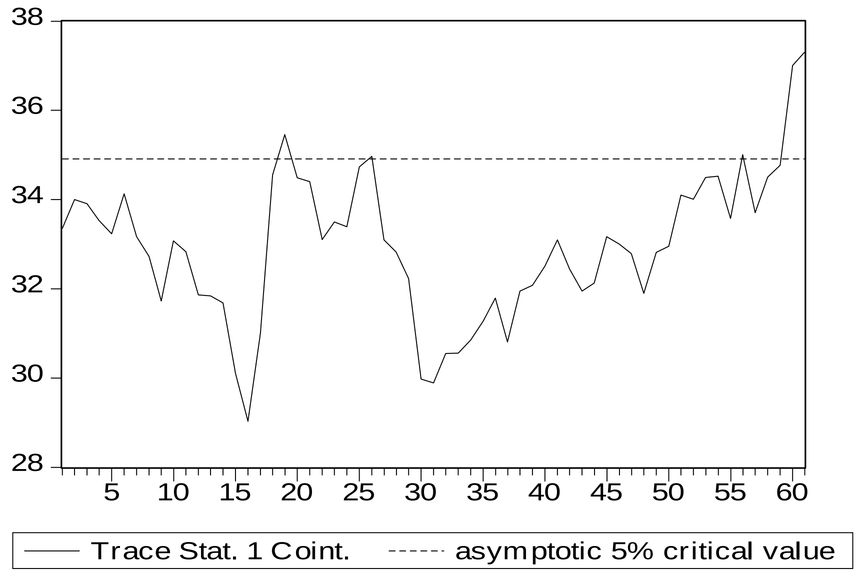
Figure 4a: Johansen Trace Test of Zero Cointegrating Equations



The plot of the values obtained from the rolling Johansen trace test of zero cointegrating equations shows that with the exception of a small number of subsets, we are able to reject the tested hypothesis over the whole range of the data.). Consequently, in Figure 4b we plot the estimated values from a rolling Johansen trace test of the hypothesis of one cointegrating equations against the 5% critical value.

[6] We start with a sample of the first 240 observations (1972:01 – 1991:12) and compute the value of the Johansen trace test. Then on the second step of the iterative procedure, we add one additional observation at the front-end of the sample (1992:01) and drop one observation from the back-end of the sample (1972:01). We continue in this fashion until we reach the end of the original data sample (1996:12).

Figure 4b: Johansen Trace Test of One Cointegrating Equations



The plot of the values of the rolling Johansen trace test of one cointegrating equation in combination with the results from the preceding test suggest that in the majority of subsamples we are not able to reject the restriction that there is only one cointegrating equation between the four series. In Figure 4c, we summarize the results from the complete sequential Johansen trace test of the number of cointegrating equations.

Figure 4c: Number of Cointegrating Equations Derived from the Rolling Johansen Trace Test

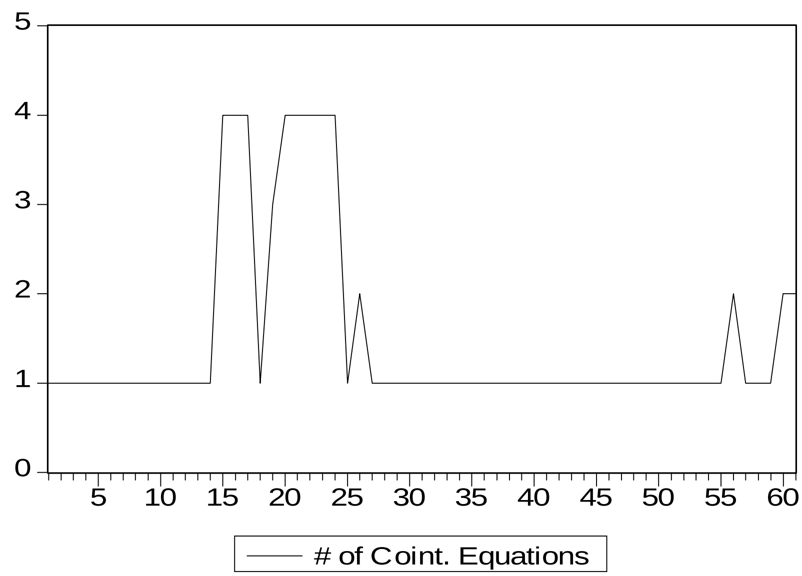


Figure 4c confirms our previous finding that except for a few outliers the real M2, real Personal Income, 3 month Treasury Bill rate and the own rate of return of M2 consistently share one unit root throughout the whole sample 1972:01 – 1996:12. It also shows that this stable long-run relation temporary broke down in some subsamples with starting dates prior to 1974:01. Because of this, we have decided to use the reduced sample of observations 1974:01 – 1996:12 in the last step of the construction of the vector error correction model (VECM).

In summary, the results from the preliminary analysis of the data used in the estimation of a VECM show that the log levels of real M2, real Personal Income, 3 month Treasury Bill rate and the own rate of return of M2 are integrated of order one in the sample 1972:01 – 1996:12 and stably cointegrated after 1974:01. Now, we are in position to obtain the long-run elasticities of money demand, derived from the VECM given in (8). In the vector error correction analysis presented below, we use an iterative technique similar to the ones used earlier in this paper. On each step of the procedure, we estimate a VECM for a rolling sample of 240 observations[7]. The number of cointegrating equations for each subsample is determined from the results of the Johansen trace test presented in Figure 4c. All VECMs are estimated under the specification of no deterministic trend in the data, and an intercept but no trend in the cointegrating equations. We use 12 lags of all participating variables to capture the short-run dynamic interactions between them and to assure that the resulting residuals are not serially correlated. The resulting OLS estimates of the long-run cointegrating coefficients between the four series, as well as the value of the error-correction term in the short-run money demand function are presented in Table 1. The coefficient estimates in columns 3, 4 and 5 are directly interpretable as the long-run elasticities of the demand for real M2 in respect to real Personal Income, 3 month Treasury Bill rate and the own rate of return of M2 if taken with opposite signs. The numbers below the bolded entries in the second and third columns are asymptotic 5% critical values of the Johansen trace test. The corresponding numbers in columns 3 to 6 are the standard errors of the corresponding OLS estimates.

In almost all subsamples, the estimated long-run elasticities of real money demand are in line with the theoretical requirements $\beta_1 \geq 0$, $\beta_2 \leq 0$, $\beta_3 \geq 0$ and are of reasonable magnitude. Consequently, we can conclude that throughout the whole period 1974:01 – 1996:12 there existed a stable long-run money demand function linking real M2 with real Personal Income, 3 month Treasury Bill rate and the own rate of return of M2. The intertemporal dynamics of the corresponding long-run elasticities of real M2 are also quite interesting. In the first 15 subsamples all coefficients are statistically insignificant but have the correct signs and expected magnitudes. In the rest of the cases, the coefficients are in general significant except in subsamples 26, 27, 28 and 29, in which the standard errors of the cointegrating coefficients explode to two and three digit

[7] We start with a sample of the first 240 observations (1974:01 – 1993:12) and estimate the VECM. Then on the second step of the iterative procedure, we add one additional observation at the front-end of the sample (1994:01) and drop one observation from the back-end of the sample (1974:01). We continue in this fashion until we reach the end of the original data sample (1996:12).

numbers. The estimated short-run speed of adjustment of actual monetary holdings to their desired level, which equals one plus the error-correction term (see (7)) is quite high, even though it is also insignificant in the first 15 subsamples. Throughout the whole sample, agents consistently adjust their real monetary holdings to eliminate 99% " of the discrepancy between their current desired real balances and their previous real balance holdings…" (Hoffman and Rasche, 1996, p.11). The long-run elasticity of M2 in respect to real Personal Income is always less than one, pointing at the existence of economies of scale of holding money.

The full estimation output of the VECM (8) obtained from the entire sample 1974:01 – 1996:12 is presented in Appendix 1. The full sample long-run elasticities of real M2 in respect to real Personal Income, 3 months Treasury Bill rate and own rate of return of M2 are respectively: 0.70, -0.22 and 0.30. All are statistically significant at the 95 % level of confidence. The estimated short-run speed of adjustment is 99.99%.

Table 1: Estimates of Error Correction Parameters from Rolling VECMs

Sample	Trace Tests		Real Personal Income	3 Months Treasury Bill Rate	Own Rate of Return of M2	Error Correction Term
	r=0	r=1				
1	**55.81**	**34.73**	**-1.16**	**-0.75**	**0.88**	**0.00166**
	53.12	34.91	0.75	1.01	1.50	0.00179
2	**56.04**	**34.97**	**-1.18**	**-0.75**	**0.90**	**0.00176**
	53.12	34.91	0.77	1.01	1.52	0.00179
3	**53.21**	**33.09**	**-1.16**	**-0.78**	**0.91**	**0.00164**
	53.12	34.91	0.77	1.07	1.58	0.00173
4	**54.41**	**32.82**	**-1.07**	**-0.79**	**0.87**	**0.00126**
	53.12	34.91	0.72	1.15	1.63	0.00165
5	**55.27**	**32.23**	**-0.92**	**-0.57**	**0.54**	**0.00104**
	53.12	34.91	0.44	0.67	0.94	0.00217
6	**54.15**	**29.98**	**-0.97**	**-0.68**	**0.69**	**0.00136**
	53.12	34.91	0.55	0.90	1.24	0.00190
7	**54.06**	**29.89**	**-0.84**	**-0.51**	**0.43**	**0.00070**
	53.12	34.91	0.36	0.58	0.79	0.00232
8	**59.54**	**30.55**	**-0.79**	**-0.51**	**0.41**	**0.00042**
	53.12	34.91	0.36	0.63	0.83	0.00219
9	**57.01**	**30.56**	**-0.72**	**-0.48**	**0.34**	**-0.00010**
	53.12	34.91	0.33	0.62	0.78	0.00214
10	**59.57**	**30.85**	**-0.66**	**-0.46**	**0.28**	**-0.00001**
	53.12	34.91	0.32	0.61	0.75	0.00216
11	**64.41**	**31.28**	**-0.45**	**-0.55**	**0.28**	**-0.00029**
	53.12	34.91	0.54	1.06	1.11	0.00151
12	**68.94**	**31.79**	**-0.42**	**-0.54**	**0.25**	**-0.00049**
	53.12	34.91	0.58	1.06	1.09	0.00146
13	**72.77**	**30.81**	**-0.52**	**0.13**	**-0.45**	**-0.00029**
	53.12	34.91	0.70	1.57	1.61	0.00119
14	**54.84**	**31.95**	**-0.37**	**1.53**	**-2.16**	**-0.00055**
	53.12	34.91	1.19	4.71	5.93	0.00074
15	**55.02**	**32.08**	**-0.55**	**0.50**	**-0.90**	**-0.00109**
	53.12	34.91	0.34	0.52	0.73	0.00209
16	**57.34**	**32.51**	**-0.42**	**0.26**	**-0.69**	**-0.01230**
	53.12	34.91	0.15	0.10	0.20	0.00568
17	**58.67**	**33.10**	**-0.52**	**0.31**	**-0.70**	**-0.00250**
	53.12	34.91	0.22	0.20	0.34	0.00357

Sample	Johansen Trace Tests		Real Personal Income	3 Months Treasury Bill Rate	Own Rate of Return of M2	Error Correction Term
	r=0	r=1				
18	**59.91**	**32.45**	**0.13**	**1.75**	**-2.84**	**-0.00054**
	53.12	34.91	2.91	6.18	9.24	0.00070
19	**59.63**	**31.95**	**-0.35**	**0.22**	**-0.70**	**-0.02047**
	53.12	34.91	0.14	0.07	0.16	0.00739
20	**60.82**	**32.13**	**-0.37**	**0.25**	**-0.73**	**-0.01368**
	53.12	34.91	0.16	0.09	0.20	0.00646
21	**63.61**	**33.17**	**-0.18**	**0.26**	**-0.89**	**-0.01379**
	53.12	34.91	0.24	0.10	0.28	0.00573
22	**70.80**	**33.00**	**-0.36**	**0.20**	**-0.70**	**-0.01142**
	53.12	34.91	0.15	0.07	0.18	0.00700
23	**66.70**	**32.79**	**-0.01**	**0.27**	**-1.07**	**-0.00621**
	53.12	34.91	0.37	0.13	0.44	0.00456
24	**66.18**	**31.90**	**0.07**	**0.32**	**-1.31**	**0.00116**
	53.12	34.91	0.77	0.28	1.04	0.00249
25	**66.31**	**32.82**	**0.79**	**0.41**	**-2.12**	**0.00132**
	53.12	34.91	2.18	0.53	2.74	0.00155
26	**66.10**	**32.95**	**2.05**	**0.69**	**-3.77**	**0.00094**
	53.12	34.91	7.98	1.82	10.23	0.00079
27	**63.70**	**34.10**	**-8.18**	**-2.15**	**10.43**	**-0.00032**
	53.12	34.91	81.23	24.18	115.56	0.00022
28	**61.07**	**34.01**	**-15.68**	**-4.67**	**20.76**	**-0.00015**
	53.12	34.91	267.46	84.65	374.58	0.00013
29	**64.31**	**34.50**	**-0.71**	**0.13**	**-0.28**	**-0.00028**
	53.12	34.91	71.84	21.65	96.92	0.00026
30	**63.78**	**34.53**	**-0.81**	**0.12**	**-0.20**	**-0.00343**
	53.12	34.91	1.45	0.44	1.98	0.00175
31	**64.66**	**33.58**	**-0.78**	**0.19**	**-0.29**	**-0.00657**
	53.12	34.91	0.48	0.16	0.63	0.00334
32	**68.11**	**35.01**	**-0.83**	**0.26**	**-0.33**	**NA**
	53.12	34.91				
33	**68.64**	**33.71**	**-0.78**	**0.25**	**-0.35**	**-0.01241**
	53.12	34.91	0.23	0.08	0.29	0.00492
34	**67.47**	**34.50**	**-0.78**	**0.23**	**-0.33**	**-0.01097**
	53.12	34.91	0.27	0.09	0.34	0.00451
35	**67.49**	**34.77**	**-0.64**	**0.23**	**-0.46**	**-0.01293**
	53.12	34.91	0.23	0.07	0.27	0.00499
36	**68.11**	**37.01**	**-0.61**	**0.26**	**-0.53**	**NA**
	53.12	34.91				
37	**68.48**	**37.30**	**-0.62**	**0.23**	**-0.50**	**NA**

Finally, we test the forecasting sharpness of our model, by conducting recursive, static, one-period ahead forecasts of real M2. We start with a sample of the first 240 observations and estimate the parameters of the VECM. Then, we use the obtained coefficients of the model to forecast the real M2 in the next period. On the second step of the procedure, we add the actual 241-st observations of the four series and reestimate the parameters of the model, using the new estimates to forecast the real M2 in 242 period from the beginning of the full sample. Figure 5a presents a plot of the actual values and the corresponding one-period ahead forecasts of the log of real M2. Figure 5b plots the difference between the one-period ahead forecasts and the actual values of M2.

Figure 5a: Actual Values and One-Period Ahead Forecasts of Real M2 for the Period 1994:01 – 1996:12

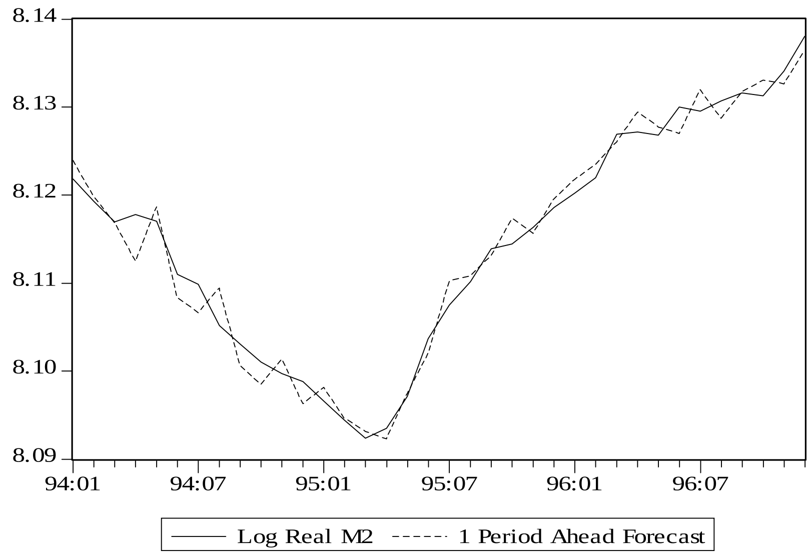
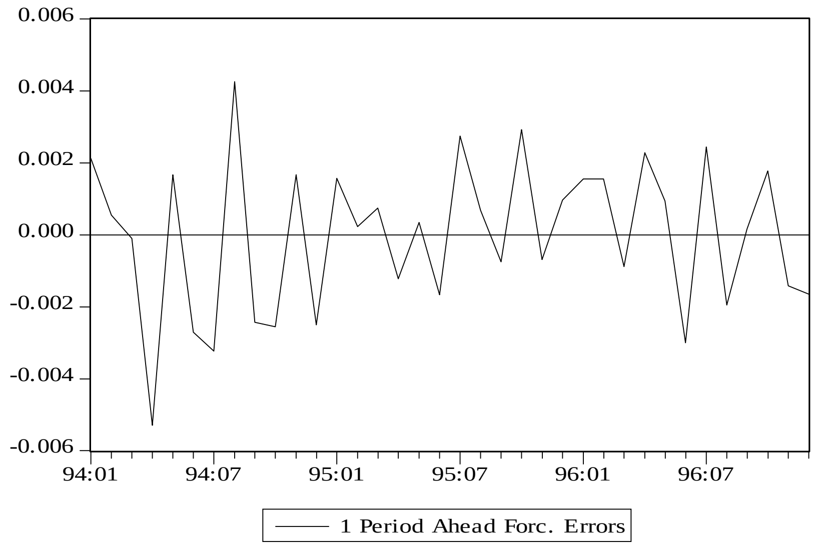


Figure 5b: Difference Between the One-Period Ahead Forecasts of Real M2 and its Actual Values for the Period 1994:01 – 1996:12



The visual inspection of Figures 5a and 5b suggest that our VECM has a very good predictive power in one-period ahead forecasting of real M2. To get a quantitative measure of the forecasting power of the model, we calculate the root mean squared error (RMSE) and the mean absolute percentage error (MAPE) associated with this forecasting exercise. The RMSE of the one-period ahead recursive forecasts of real M2 for the period 1994:01 – 1996:12 is 0.002, whereas the MAPE is equal to 0.0002. Both measures have extremely small values, indicating that the average difference between the one-period ahead forecast and the actual value of real M2 is less than two hundredth of a percent of the actual value.

Appendix 1: Full Estimation Output from the VECM with the entire sample 1974:01 – 1996:12

VECM estimated under the specification of no deterministic trend in the data and an intercept, but no trend in the cointegrating equation. Standard errors are in parenthesis.

Cointegrating Eq:	CointEq1			
LN_RM2(-1)	1.000000			
LN_RINC(-1)	-0.698170			
	(0.06165)			
LN_TB3M(-1)	0.219772			
	(0.05935)			
LN_IM2(-1)	-0.304559			
	(0.06714)			
C	-2.616061			
	(0.49192)			
Error Correction:	D(LN_RM2)	D(LN_RINC)	D(LN_TB3M)	D(LN_IM2)
CointEq1	-9.29E-05	-0.030027	0.148343	0.005008
	(0.00353)	(0.00807)	(0.07871)	(0.03343)
D(LN_RM2(-1))	0.521595	0.247247	2.860538	0.395379
	(0.06784)	(0.15496)	(1.51116)	(0.64173)
D(LN_RM2(-2))	-0.038073	-0.291236	-4.030616	-0.616235
	(0.07689)	(0.17565)	(1.71288)	(0.72739)
D(LN_RM2(-3))	-0.031299	0.318649	2.924974	0.619819
	(0.07591)	(0.17340)	(1.69100)	(0.71810)
D(LN_RM2(-4))	0.035896	0.347613	1.673768	0.454640
	(0.07633)	(0.17437)	(1.70040)	(0.72209)
	(0.47026)	(1.99356)	(0.98434)	(0.62961)
D(LN_RM2(-5))	0.016992	-0.417299	-3.715916	-0.528154
	(0.07579)	(0.17314)	(1.68842)	(0.71701)
D(LN_RM2(-6))	0.009455	0.236702	3.147256	-0.263304
	(0.07587)	(0.17332)	(1.69019)	(0.71776)
D(LN_RM2(-7))	-0.010947	-0.049858	-3.587245	0.244375
	(0.07619)	(0.17405)	(1.69727)	(0.72076)
D(LN_RM2(-8))	-0.051295	-0.113007	2.448257	0.852408
	(0.07575)	(0.17303)	(1.68736)	(0.71656)
D(LN_RM2(-9))	0.180392	0.191439	-0.566912	1.172586
	(0.07557)	(0.17262)	(1.68334)	(0.71485)

D(LN_RM2(-10))	-0.009069	-0.120981	-1.535977	-1.067887
	(0.07557)	(0.17263)	(1.68341)	(0.71488)
D(LN_RM2(-11))	-0.050160	0.198991	0.652114	0.291490
	(0.06220)	(0.14207)	(1.38547)	(0.58835)
D(LN_RINC(-1))	0.021147	-0.252000	1.452138	0.517584
	(0.03016)	(0.06890)	(0.67192)	(0.28534)
D(LN_RINC(-2))	-0.001469	-0.068223	1.877751	-0.113360
	(0.03152)	(0.07201)	(0.70220)	(0.29820)
D(LN_RINC(-3))	0.006734	-0.194249	0.375858	0.038145
	(0.03117)	(0.07120)	(0.69428)	(0.29483)
D(LN_RINC(-4))	0.010985	-0.052284	-0.708143	-0.129041
	(0.03149)	(0.07192)	(0.70136)	(0.29784)
D(LN_RINC(-5))	0.077357	0.025311	0.470064	-0.403774
	(0.03145)	(0.07184)	(0.70057)	(0.29750)
D(LN_RINC(-6))	-0.027376	-0.053925	0.268373	-0.140394
	(0.03160)	(0.07218)	(0.70384)	(0.29890)
D(LN_RINC(-7))	0.017863	0.053578	0.597090	-0.286051
	(0.03137)	(0.07166)	(0.69879)	(0.29675)
D(LN_RINC(-8))	0.081960	0.079209	0.304958	-0.169151
	(0.03093)	(0.07065)	(0.68899)	(0.29259)
	(2.64990)	(1.12110)	(0.44261)	(-0.57812)
D(LN_RINC(-9))	0.044525	-0.103122	-0.216133	-0.456103
	(0.03047)	(0.06959)	(0.67865)	(0.28820)
D(LN_RINC(-10))	-0.018320	0.063370	0.315768	-0.183970
	(0.03073)	(0.07021)	(0.68463)	(0.29074)
D(LN_RINC(-11))	0.041701	-0.058779	0.249312	0.119713
	(0.02938)	(0.06711)	(0.65440)	(0.27790)
D(LN_TB3M(-1))	-0.017226	0.021564	0.420929	0.112462
	(0.00327)	(0.00746)	(0.07275)	(0.03089)
D(LN_TB3M(-2))	0.002104	0.012668	-0.262981	0.037614
	(0.00364)	(0.00832)	(0.08117)	(0.03447)
D(LN_TB3M(-3))	-0.013365	0.005329	-0.029802	0.022422
	(0.00370)	(0.00844)	(0.08235)	(0.03497)
D(LN_TB3M(-4))	-0.007924	0.022169	0.016661	0.091457
	(0.00371)	(0.00848)	(0.08272)	(0.03513)

D(LN_TB3M(-5))	-0.010083	-0.014320	0.070232	0.022091
	(0.00383)	(0.00876)	(0.08538)	(0.03626)
D(LN_TB3M(-6))	-0.008254	0.012115	-0.221265	0.034056
	(0.00365)	(0.00833)	(0.08125)	(0.03450)
D(LN_TB3M(-7))	-0.001106	0.001272	0.043222	0.006990
	(0.00377)	(0.00860)	(0.08387)	(0.03562)
D(LN_TB3M(-8))	-0.004370	-0.003658	-0.032497	0.037905
	(0.00368)	(0.00841)	(0.08206)	(0.03485)
D(LN_TB3M(-9))	-0.009490	0.011450	0.173174	0.038953
	(0.00359)	(0.00819)	(0.07990)	(0.03393)
D(LN_TB3M(-10))	-0.000798	0.007896	-0.076644	0.016500
	(0.00351)	(0.00802)	(0.07825)	(0.03323)
D(LN_TB3M(-11))	-0.005107	-0.007064	0.049345	0.034803
	(0.00334)	(0.00762)	(0.07433)	(0.03156)
D(LN_IM2(-1))	0.006531	-0.004081	0.073415	0.134362
	(0.00714)	(0.01632)	(0.15911)	(0.06757)
D(LN_IM2(-2))	0.009381	-0.011706	0.202310	0.034749
	(0.00716)	(0.01636)	(0.15957)	(0.06776)
	(1.30956)	(-0.71540)	(1.26783)	(0.51279)
D(LN_IM2(-3))	0.016932	-0.002740	-0.084698	0.024503
	(0.00715)	(0.01632)	(0.15917)	(0.06759)
D(LN_IM2(-4))	0.017146	0.010223	0.009849	-0.017429
	(0.00707)	(0.01615)	(0.15747)	(0.06687)
D(LN_IM2(-5))	0.016344	0.003100	-0.078577	-0.031094
	(0.00711)	(0.01625)	(0.15842)	(0.06727)
D(LN_IM2(-6))	0.004136	0.017536	0.194117	0.037252
	(0.00704)	(0.01608)	(0.15676)	(0.06657)
D(LN_IM2(-7))	-0.003682	-0.016512	-0.018360	0.056938
	(0.00704)	(0.01608)	(0.15681)	(0.06659)
D(LN_IM2(-8))	0.003919	-0.010221	0.055613	0.141740
	(0.00694)	(0.01585)	(0.15460)	(0.06565)
D(LN_IM2(-9))	0.008898	-0.010054	-0.205410	0.022629
	(0.00699)	(0.01598)	(0.15580)	(0.06616)
D(LN_IM2(-10))	5.22E-05	-0.005179	-0.022757	-0.061555
	(0.00700)	(0.01598)	(0.15586)	(0.06619)

D(LN_IM2(-11))	0.006176	0.002420	-0.122187	-0.056636
	(0.00669)	(0.01528)	(0.14903)	(0.06329)
R-squared	0.661632	0.239662	0.346663	0.360062
Adj. R-squared	0.597180	0.094836	0.222217	0.238169
Sum sq. resids	0.001527	0.007966	0.757568	0.136617
S.E. equation	0.002571	0.005872	0.057267	0.024319
Log likelihood	1278.873	1050.879	422.3028	658.6870
Akaike AIC	-8.941111	-7.288980	-2.734079	-4.447007
Schwarz SC	-8.350828	-6.698697	-2.143796	-3.856724
Mean dependent	0.001207	0.002082	-0.001511	-0.001046
S.D. dependent	0.004051	0.006172	0.064935	0.027862

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