Inflation And Its Variation: An Alternative Explanation*

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Abstract

This paper introduces a general objective function for monetary policy that abandons certainty equivalence and features 'prudence'. It provides an alternative explanation for the positive relation between the level and variability of inflation, both across countries and over time. In particular, the model predicts that high (low) inflation tends to be more variable (stable) over time.

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1 Introduction

There is substantial variation in inflation, both over time and across countries. As is well known, inflation and its variation obey a remarkably robust empirical regularity: There is a strong positive relation between the average level and variance of inflation across countries. This paper shows that this relationship can be explained by a surprisingly simple model. Moreover, the model provides an explanation for the variation in inflation over time. The main theoretical innovation in this paper is the introduction of an objective function for monetary policy makers that exhibits 'prudence'. The results indicate that Blinder (1997, p. 6) is right; the conventional assumption of a quadratic loss function is not innocuous.

This paper adopts an elementary version of the Barro and Gordon (1983) framework. The central bank engages in discretionary monetary policy after the public has formed its inflation expectations. The economy is described by an expectations-augmented Phillips curve and is subject to random supply shocks. The central bank maximizes an objective function that depends on the level of inflation and output. As usual, it is quadratic in inflation. But, for output the commonly made assumption of certainty equivalence is abandoned. Instead, I adopt 'prudence' and motivate my decision by presenting microfoundations for the central bank's objective function. Prudence gives rise to the crucial property of the model that inflation is decreasing and convex in the supply shock to output. So, when the economy faces an episode of unfavorable shocks, the central bank operates on the steep end of the inflation response, giving rise to high and variable inflation. When disturbances tend to be favorable, inflation will be low and stable.

Convexity of the inflation response brings about an additional mechanism that induces a positive relationship between the average level and variability of inflation. An increase in the variance of real shocks produces a rise in both the mean and variance of inflation. The positive association between inflation and its variation appears to be extremely robust. It does not depend on differences in (the variance of) supply shocks; it also arises for variation in central bank preferences. In addition, it even holds in the presence of perfect information about the supply shock and the central bank's type. And, in case of an information asymmetry for the private sector, it is independent of the way expectations are formed.

When there is asymmetric information about the level of the supply shock, an increase in the variance of the shocks not only affects average inflation, but it also raises the level of inflation for any realization of the supply shock. This level effect hinges on rational expectations. When people perceive greater variation in supply shocks, they will increase their inflation expectations, thereby boosting the rate of inflation. This may provide an explanation for the fact that the oil shocks in the 1970s brought about such persistently high inflation. It also suggests that exchange rate stabilization could lower the average level of inflation through a reduction of the variance of imported supply shocks.

There is an additional interesting finding in the case of asymmetric information about supply shocks. It allows the central bank to stabilize output, but it also leads to lower average inflation than with perfect information. This means that an information advantage for the central bank has a positive effect on its reputation by reducing inflation expectations.

The approach adopted in this paper distinguishes itself from the existing literature on monetary policy in two important respects. First, it proposes an alternative, more general objective function that produces previously obtained results as limiting cases. Moreover, this paper emphasizes the relevance of *economic* variables for monetary policy. The latter is in sharp contrast to most of the literature which focuses on (uncertainties about) the institutional setting with issues like central bank independence and credibility. Although this paper does not deny their importance, I hope to illustrate that these institutional characteristics are by no means the sole determinants of the outcome of monetary policy.

Earlier explanations of the positive correlation between inflation and its variation, which was already documented by Okun (1971), include Cukierman and Meltzer (1986), Devereux (1989) and Demetriades (1988). The key ingredients in Cukierman and Meltzer (1986) are imperfect monetary control and asymmetric information about the central bank's shifting objectives. Relative instability of objectives induces a reduction in the optimal degree of monetary control, because control errors obscure the shifts in preferences and thereby allow the central bank to reap the benefits from monetary surprises. A higher variance of control errors in turn, increases the variability of inflation but also the mean. The latter arises because people rationally anticipate the central bank's greater incentive to inflate. Hence, variation in political stability produces a positive relationship between the mean and variance of inflation. The driving forces in Devereux (1989) are wage indexation and real disturbances. A higher variance of real shocks raises inflation variability. In addition, it reduces the optimal degree of wage indexation for the private sector. This increases the central bank's ability to benefit from surprise inflation, which raises average inflation. So, differences in the variability of real shocks generate the positive correlation between inflation and its variance. Finally, Demetriades (1988) simply postulates a feedback rule for monetary policy that implies asymmetric stabilization policy and formulates conditions under which a positive relation holds.

There are a few other papers that examine alternative objective functions in monetary policy, including Chadha and Schellekens (1998). In their model, the central bank minimizes a loss function that only depends on inflation, subject to the restriction that inflation follows an autoregressive process with endogenous policy and stochastic supply shocks. Their findings suggest that deviations in the loss function from symmetry and certainty equivalence produce results that resemble those of a quadratic with an adjusted inflation target. The major disadvantage of their approach, however, is that it does not explicitly incorporate the central bank's preferences for output or the role of inflation expectations. Like Chadha and Schellekens (1998), this paper abandons symmetry and certainty equivalence. But it adopts a slightly richer model and shows that the alternative objective function does give rise to qualitatively new results.

The remainder of the paper is organized as follows. Section 2 presents the model and motivates my objective function. Subsequently, section 3 analyzes the case of perfect information, which provides a useful and simple benchmark. Next, asymmetric information about supply shocks is introduced in section 4. Section 5 discusses some interesting extensions and other implications of the model, and section 6 concludes.

2 The Model

Monetary policy is determined by the interaction between the central bank and the private sector. The latter acts first, by forming its expectations π^e about the level of inflation π. It is assumed that the public has rational expectations so that $\pi^e = \mathrm{E}\left[\pi|\Omega\right]$, where Ω denotes the market's information set. The economy is described by the aggregate supply relation

$$y = \bar{y} + \theta\left(\pi - \pi^e\right) + \varepsilon, \qquad (1)$$

where y denotes the level of real output, $\bar{y}$ equals the natural rate of output, ε is a white noise supply shock with variance σ_ε^2, and θ equals the slope of the aggregate supply curve ($\theta > 0$). The central bank observes the supply shock ε and inflation expectations π^e, and subsequently sets the level of inflation π to maximize its objective

$$W = -\frac{1}{2}\alpha\left(\pi - \pi^*\right)^2 + f\left(y - y^*\right), \qquad (2)$$

where π^* is the inflation target, y^* is the output target and α determines the relative importance of the inflation objective ($\alpha > 0$). The output objective $f\left(.\right)$ reflects risk aversion and prudence. More precisely, I assume that $f''\left(.\right) < 0$ and $f'''\left(.\right) > 0$. Furthermore, let $\lim_{x\to\infty} f'\left(x\right) = \gamma$, $\lim_{x\to\infty} f''\left(x\right) = 0$ and normalize $f\left(0\right) = 0$, where the argument of $f\left(.\right)$ is the deviation of output from its target, $y - y^*$.[1] For notational simplicity, it will

[1] In contrast to the previous assumptions on $f\left(.\right)$, these conditions are not essential; all qualitative conclusions hold without them.

be assumed that the natural rate of output equals the target level: $\bar{y} = y^*$. A flexible parameterization that satisfies the assumptions on the output objective is the linear-exponential specification $f(x) = -e^{-\beta x} + \gamma x + 1$, where $\beta > 0$. It is depicted in figure 1 for $\beta = -\gamma = 1/2$.

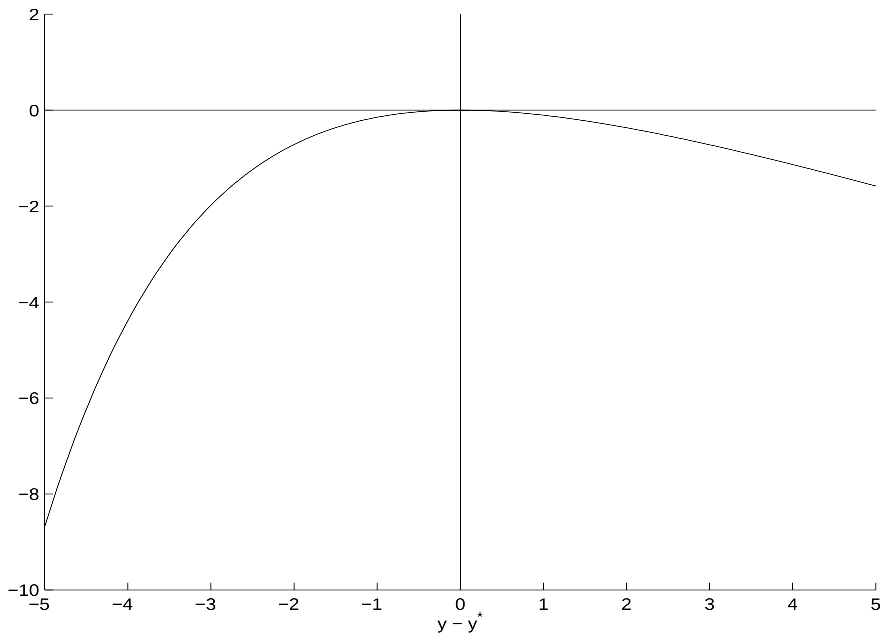

Figure 1: The central bank's objective function $f(y - y^*)$.

Clearly, the model fits into the familiar framework of discretionary monetary policy games first discussed by Kydland and Prescott (1977) and formally analyzed by Barro and Gordon (1983). Previous contributions, however, have adopted central bank objective functions that are either linear increasing or negative quadratic in output. Both specifications have important shortcomings.

The linear case, $f(y - \bar{y}) = \beta(y - \bar{y})$, is characterized by monotonicity and risk neutrality. Discretionary monetary policy always gives rise to an inflationary bias, but there is no role for stabilization policy in response to supply shocks. In addition, inflation expectations π^e, and thereby reputation, do not matter for the level of inflation.

The quadratic case, $f(y - y^*) = -\frac{1}{2}\beta (y - y^*)^2$, displays risk aversion but also certainty equivalence. Supply shocks and reputation are relevant; inflation is decreasing in the supply shock ε and increasing in inflation expectations π^e. It is typically assumed that the target rate exceeds the natural rate of output ($y^* > \bar{y}$). The justification for this assumption usually consists of an appeal to market failures, e.g. due to distortionary taxes or powerful unions. As a result, local nonsatiation obtains at $y = \bar{y}$, and there is an inflationary bias due to discretion.

Compared to the quadratic case, my specification maintains risk aversion, but abandons certainty equivalence. In addition, the assumptions allow for either a bliss point in output ($\gamma < 0$) like the quadratic case, or strict monotonicity ($\gamma \geq 0$) like the linear case.[2] However, the crucial assumptions, $f''(.) < 0$ and $f'''(.) > 0$, give rise to a fundamental asymmetry in the objective function. In particular, the absolute change in the marginal effect for a positive deviation of output is smaller than for a negative deviation, for any level of output. Thus, output losses have a larger impact on the central bank's optimal inflation decision than output gains.

The central bank's objective function (2) could be interpreted as a social welfare function or as a positive description of competing political interests that affect the central bank's behavior (see also Cukierman (1992), p. 43-45). The political approach to the objective function focuses on the influence of advocates of price stability versus those concerned about the level of output. The coefficient α reflects the relative political power of the former, or the degree of central bank independence. For such a political objective function, it seems natural to assume that output losses carry more weight than output gains, justifying $f'''(.) > 0$.

Alternatively, when the central bank is completely isolated from political

[2] The algebraic results reduce to those for the conventional linear and quadratic objective functions for $f'(.) > 0$, $f''(.) = f'''(.) = 0$ and $f'''(.) = 0$, respectively.

influence, the central bank's objective would be to maximize social welfare. For this social welfare approach, the coefficient α captures the welfare cost of inflation deviations relative to output deviations from target. In appendix A, I provide a motivation for $f'''(.) > 0$ by deriving the partial social welfare function for output from an elementary microfounded model. It is based on utility-maximizing consumers who supply labor, the only factor of production. There is a linear production technology and perfect competition prevails. The socially optimal level of output $(y^* = \bar{y})$ is produced when all workers supply the individually optimal amount of labor. Higher levels of output can be achieved by working more hours. For lower output levels, the realistic feature is incorporated that decreases in output are obtained partly through a reduction in working hours, and partly by laying off workers. It is shown that the social welfare function exhibits prudence $(f'''(.) > 0)$, even if certainty equivalence prevails at the individual level.

The optimal policy for the central bank is to maximize (2) subject to (1) and given ε and π^e. Hence, the first order condition gives

$$\alpha(\pi - \pi^*) = \theta f'(\theta(\pi - \pi^e) + \varepsilon). \tag{3}$$

The second order condition for a maximum is satisfied by concavity of $f(.)$. A property that follows directly from (3) is the relevance of inflation expectations. In particular,

$$\frac{\partial \pi}{\partial \pi^e} = \frac{-\theta^2 f''(y - \bar{y})}{\alpha - \theta^2 f''(y - \bar{y})} > 0. \tag{4}$$

This shows that the model captures reputation effects. It also means that the outcomes for static and dynamic games could differ, depending on the information structure. In the presence of uncertainty about the central bank's type, there is an opportunity to engage in reputation-building. However, for the purpose of this paper it suffices to restrict ourselves to the static model.

Regarding the public's information set Ω, two cases will be analyzed: (i) perfect information in section 3; and (ii) asymmetric information about the supply shock in section 4.

3 Perfect Information

First, let us consider the case of perfect information. Although this may not be a realistic assumption, it provides a useful benchmark. More formally, it is assumed that the market's information set equals $\Omega_P \equiv \{\varepsilon, \pi^*, y^*, \bar{y}, \sigma_\varepsilon^2, \theta, \alpha, f(.)\}$. In the presence of perfect information, rational expectations imply $\pi^e = \pi$. Using (3), the optimal level of inflation under perfect information equals

$$\pi_P = \pi^* + \frac{\theta}{\alpha} f'(\varepsilon). \tag{5}$$

An important characteristic is that inflation is decreasing and convex in the supply shock ε. The convexity crucially depends on $f'''(\varepsilon) > 0$. Intuitively, since output losses have a larger impact on central bank objectives than output gains, the temptation to create inflation decreases in the real disturbance ε. The marginal disutility of negative supply shocks increases disproportionately when real shocks become more negative, so the central bank's incentive to inflate rises correspondingly. The outcomes for the linear and quadratic objective functions follow as special cases of (5).[3]

It is straightforward to see that there is a positive relationship between average inflation and the variance of inflation from (5). Suppose the economy is hit by negative shocks with mean $\varepsilon_L < 0$ and variance σ_ε^2. Then the mean of inflation will be high, because π_P is decreasing in ε, and the variance of inflation will be high, due to convexity of π_P. When the economy experiences positive shocks with mean $\varepsilon_H > 0$ and the same variance σ_ε^2, then both the average level and variance of inflation will be low.

However, there is an additional mechanism that follows from convexity of π_P but that does not rely on differences in the mean of supply shocks. Instead, it focuses on variation in the variance of supply shocks. In particular, convexity of π_P implies that the expected value of inflation is increasing in the variance of supply shocks, σ_ε^2. Furthermore, the variance of inflation is

[3] For the linear function $f(y-\bar{y}) = \beta(y-\bar{y})$, $\pi_P = \pi^* + \frac{\theta\beta}{\alpha}$. For the quadratic case in which $f(y-y^*) = -\frac{1}{2}\beta\,(y-y^*)^2$, $\pi_P = \pi^* + \frac{\theta\beta}{\alpha}(y^*-\bar{y}) - \frac{\theta\beta}{\alpha}\varepsilon$.

increasing in σ_ε^2 by monotonicity of π_P. Therefore, there exists a positive relationship between the average level and the variance of inflation. This correspondence is noncausal; it merely reflects the common dependence on the variance of supply shocks.

Alternatively, a positive relation between the mean and variance of inflation follows from the common dependence on the coefficients α and/or θ. For example, an increase in α will reduce both $\mathrm{E}[\pi_P]$ and $\mathrm{Var}[\pi_P]$.

To investigate these relationships quantitatively, approximate expressions for $\mathrm{E}[\pi_P]$ and $\mathrm{Var}[\pi_P]$ will be derived. Taking a second-order Taylor approximation of (5) around $\varepsilon = 0$,

$$\pi_P \approx \pi^* + \frac{\theta}{\alpha} f'(0) + \frac{\theta}{\alpha} f''(0)\,\varepsilon + \frac{1}{2}\frac{\theta}{\alpha} f'''(0)\,\varepsilon^2, \tag{6}$$

so that the expected value of inflation equals

$$\mathrm{E}[\pi_P] \approx \pi^* + \frac{\theta}{\alpha} f'(0) + \frac{1}{2}\frac{\theta}{\alpha} f'''(0)\,\sigma_\varepsilon^2. \tag{7}$$

Notice that there may be an inflationary bias of discretionary monetary policy. Even if there is a bliss point in the output objective at $y = \bar{y}$ so that $f'(0) = 0$, the nonlinear inflation response will induce an average bias ($\mathrm{E}[\pi_P] > \pi^*$) that is decreasing in the degree of central bank independence α, but increasing in the curvature of the inflation response $f'''(0)$ and the variance of supply shocks σ_ε^2. If the output objective is (locally) increasing so that $f'(0) > 0$, then there will be an additional inflationary bias.[4] Equation (7) also shows that an increase in the variance of supply shocks σ_ε^2 increases expected inflation; this follows directly from the convexity of π_P.

The variance of inflation equals

$$\mathrm{Var}[\pi_P] = \mathrm{E}\left[(\pi_P - \mathrm{E}[\pi_P])^2\right] \approx \mathrm{E}\left[\left(\frac{\theta}{\alpha} f''(0)\,\varepsilon + \frac{1}{2}\frac{\theta}{\alpha} f'''(0)\left(\varepsilon^2 - \sigma_\varepsilon^2\right)\right)^2\right]$$

[4]The latter is also the reason for the inflationary bias in the linear case and the quadratic with $y^* > \bar{y}$.

using (6) and (7). Assuming that $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ so that $\mathrm{E}[\varepsilon^3] = 0$ and $\mathrm{E}[\varepsilon^4] = 3\sigma_\varepsilon^4$, this reduces to

$$\mathrm{Var}[\pi_P] \approx \frac{\theta^2}{\alpha^2}[f''(0)]^2 \sigma_\varepsilon^2 + \frac{1}{2}\frac{\theta^2}{\alpha^2}[f'''(0)]^2 \sigma_\varepsilon^4. \tag{8}$$

Suppose the variation in inflation and its variance (over time or across countries) is due to

- variation in σ_ε^2: Solving (7) for σ_ε^2 and substituting into (8) yields

$$\mathrm{Var}[\pi_P] \approx 2\frac{\theta}{\alpha}\frac{[f''(0)]^2}{f'''(0)}\left[\mathrm{E}[\pi_P] - \pi^* - \frac{\theta}{\alpha}f'(0)\right] + 2\left[\mathrm{E}[\pi_P] - \pi^* - \frac{\theta}{\alpha}f'(0)\right]^2.$$

- variation in α or θ: Solving (7) for θ/α and substituting into (8) yields

$$\mathrm{Var}[\pi_P] \approx \left([f''(0)]^2 \sigma_\varepsilon^2 + \frac{1}{2}[f'''(0)]^2 \sigma_\varepsilon^4\right)\left[\frac{\mathrm{E}[\pi_P] - \pi^*}{f'(0) + \frac{1}{2}f'''(0)\sigma_\varepsilon^2}\right]^2.$$

- variation in $f(.)$. In general, it is not possible to derive an analytical expression. However, for the case in which $f(x) = -\frac{1}{\beta}e^{-\beta x} + \gamma x + \frac{1}{\beta}$ one can solve (7) for β and substitute it into (8) to get[5]

$$\mathrm{Var}[\pi_P] \approx 2\frac{\theta}{\alpha}\left[\mathrm{E}[\pi_P] - \pi^* - \frac{\theta}{\alpha}(1+\gamma)\right] + 2\left[\mathrm{E}[\pi_P] - \pi^* - \frac{\theta}{\alpha}(1+\gamma)\right]^2.$$

Hence, in each case, the variance of inflation is positive (and convex) in average inflation. Notice that the relationship is increasing in all cases because $\mathrm{E}[\pi_P] > \pi_P(0)$.

4 Asymmetric Information

Now, let us consider the case of asymmetric information. In particular, assume that the public is not able to observe the supply shock ε when the

[5] Note that variation in γ only affects average inflation, and so it does not yield any association between the mean and variance.

inflation expectations π^e are formed. This could be a consequence of the timing of events, for instance when the market embeds its expectations π^e in nominal contracts well before the central bank acts. Alternatively, the information asymmetry could result from the deliberate withholding of information about economic data by the central bank. Formally, it is assumed that $\Omega = \Omega_P \backslash \{\varepsilon\}$.

In the presence of asymmetric information about the supply shock, perfect foresight no longer prevails and π^e may deviate from π. The first order condition (3) reveals that there is no closed form solution for the level of inflation π_A under asymmetric information. Fortunately, it is still possible to derive major results analytically.

An important consequence of asymmetric information is that inflation expectations π^e are independent of the realization of the supply shock ε. Implicit differentiation of (3) yields

$$\frac{d\pi_A}{d\varepsilon} = \frac{\theta f''(y_A - \bar{y})}{\alpha - \theta^2 f''(y_A - \bar{y})} < 0.$$

Differentiating (1) and substituting for $d\pi_A/d\varepsilon$ gives

$$\frac{dy_A}{d\varepsilon} = \frac{\alpha}{\alpha - \theta^2 f''(y_A - \bar{y})}.$$

Clearly, $0 < dy_A/d\varepsilon < 1$; monetary policy stabilizes output as the supply shock is partially offset by an inflation surprise. In addition, $dy_A/d\varepsilon \to 1$ as $\varepsilon \to \infty$ and

$$\frac{d^2 y_A}{d\varepsilon^2} = \frac{\alpha^2\theta^2 f'''(y_A - \bar{y})}{\left[\alpha - \theta^2 f''(y_A - \bar{y})\right]^3} > 0.$$

So, $y_A - \bar{y}$ is increasing and convex in the supply shock ε, and $y_A - \bar{y} \to \infty$ as $\varepsilon \to \infty$. Furthermore,

$$\frac{d^2 \pi_A}{d\varepsilon^2} = \frac{\alpha^2\theta f'''(y_A - \bar{y})}{\left[\alpha - \theta^2 f''(y_A - \bar{y})\right]^3} > 0.$$

Hence, π_A is decreasing and convex in ε, and it follows from (3) that $\inf \pi_A = \pi^* + \frac{\theta}{\alpha}\gamma$. As a consequence, y_A converges to an asymptote with slope one and intercept $\bar{y} + \theta\left(\pi^* + \frac{\theta}{\alpha}\gamma - \pi^e\right)$ as $\varepsilon \to \infty$. Assuming rational expectations, $\pi^e = \mathrm{E}[\pi_A] > \pi_A(0) > \pi^* + \frac{\theta}{\alpha}\gamma$ by convexity of π_A. Using (1), this also implies that $y_A(0) < \bar{y}$. So, output is below the natural rate when the supply shock equals zero. However, on average output equals the natural rate since rational expectations and (1) imply $\mathrm{E}[y_A] = \bar{y}$.

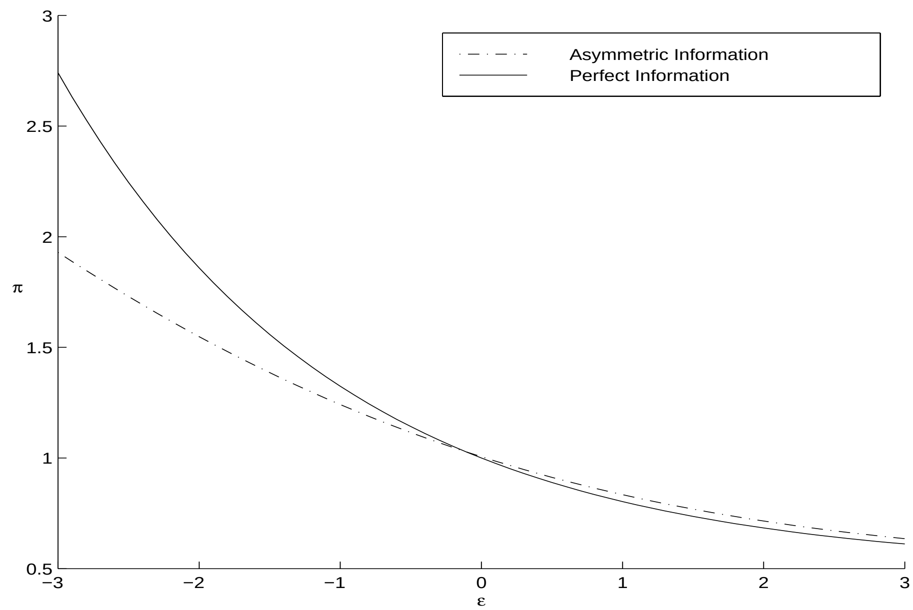

Figure 2: Inflation under perfect and asymmetric information.

The shapes of the inflation and output responses are illustrated in figures 2 and 3.[6] The corresponding outcomes under perfect information are included as well. Clearly, there is a negative, convex relation between inflation π and real disturbances ε. The relation is negative because the central bank adjusts inflation in an attempt to stabilize output. Since output losses

[6] The specification is $f(x) = -e^{-\beta x} + \gamma x + 1$ and it is assumed that ε has a normal distribution. The parameters are $\alpha = \theta = 1$, $\beta = -\gamma = 1/2$, $\sigma_\varepsilon^2 = 1$, $\pi^* = 1$ and $\bar{y} = 100$. Different values give similar shapes. Stochastic simulations gave the corresponding level of $\pi^e = 1.029$ (quite close to 1.032, the analytical approximation in (10)).

carry more weight than output gains, the central bank's response to shocks is asymmetric. It increases the level of inflation more in response to unfavorable shocks, giving rise to the convex shape. Furthermore, figure 2 reveals that the inflation outcome in the case of perfect information shows stronger curvature. This is due to the anticipation of supply shocks under rational expectations. Intuitively, positive shocks decrease inflation expectations under perfect information, which gives rise to lower inflation than in the case of asymmetric information. For negative shocks, however, perfect information raises inflation expectations and thereby produces higher inflation than asymmetric information.[7]

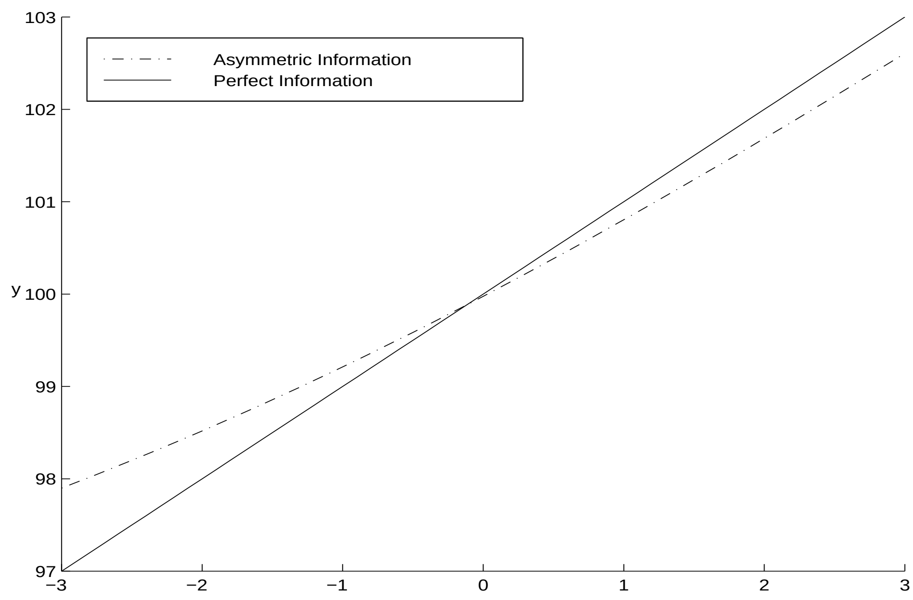

Figure 3: Output under perfect and asymmetric information.

Figure 3 shows how output y is affected by supply shocks. In the case of

[7]Strictly speaking, this is only true for $\varepsilon < \tilde{\varepsilon}$, where $\tilde{\varepsilon}$ is defined by $\pi_P(\tilde{\varepsilon}) \equiv \pi_A(\tilde{\varepsilon})$. Note that (3) and rational expectations imply that $\pi_A(\tilde{\varepsilon}) = \pi_A^e = \mathrm{E}[\pi_A] > \pi_A(0)$. Hence, $\tilde{\varepsilon}$ is unique and satisfies $\tilde{\varepsilon} < 0$.

perfect information, output stabilization is ineffective because people anticipate the central bank's policy. With asymmetric information, lower levels of ε induce greater stabilization efforts, thereby giving rise to the convex shape. The reduction in output for positive real disturbances is the sacrifice the central bank needs to make to enable output stimulation for negative shocks, because rational expectations force output to equal its natural rate on average.

Following the same argument as under perfect information, monotonicity and convexity of π_A give rise to a positive relationship between the average level and variance of inflation when there is variation in the mean or variance of supply shocks. Observe that the two key properties of π_A do not depend on the level of π^e. Hence, the positive relation between inflation and its variation holds regardless of the way inflation expectations are formed, whether there are rational expectations, adaptive expectations or some rule-of-thumb.

Again, approximate expressions can be derived for the expected value and the variance of inflation assuming that $\varepsilon \sim N\left(0, \sigma_\varepsilon^2\right)$. In the case of asymmetric information, (3) yields

$$\mathrm{E}\left[\pi_A\right] = \pi^* + \frac{\theta}{\alpha} \mathrm{E}\left[f'\left(y_A - \bar{y}\right)\right].$$

Taking a second-order Taylor approximation of $\pi_A(\varepsilon)$ around $\varepsilon = 0$ using $y_A(0) \approx \bar{y}$,[8]

$$\pi_A \approx \pi^* + \frac{\theta}{\alpha} f'(0) + \frac{\theta f''(0)}{\alpha - \theta^2 f''(0)} \varepsilon + \frac{1}{2} \frac{\theta}{\alpha} \frac{\alpha^3 f'''(0)}{\left[\alpha - \theta^2 f''(0)\right]^3} \varepsilon^2. \tag{9}$$

So, the expected value of inflation equals

$$\mathrm{E}\left[\pi_A\right] \approx \pi^* + \frac{\theta}{\alpha} f'(0) + \frac{1}{2} \frac{\theta}{\alpha} \frac{\alpha^3}{\left[\alpha - \theta^2 f''(0)\right]^3} f'''(0)\, \sigma_\varepsilon^2. \tag{10}$$

Comparing this to (7), it appears that $\mathrm{E}\left[\pi_A\right] < \mathrm{E}\left[\pi_P\right]$; in the case of asymmetric information, the expected level of inflation is lower than with

[8] Simulations for $f(x) = -e^{-\beta x} + \gamma x + 1$ and a normal distribution indicate that this approximation is quite accurate.

perfect information. Figure 2 indicates that this is driven by the relatively low levels of inflation under asymmetric information when the economy is hit by negative supply shocks. Equation (10) also reveals that the average inflationary bias, $\mathrm{E}[\pi_A] - \pi^*$, is decreasing in the degree of central bank independence, α, and increasing in the slope of the objective function, $f'(0)$, the curvature of the inflation response, $f'''(0)$, and the variance of supply shocks σ_ε^2.

As before, expected inflation increases with the variance of supply shocks. Again, it hinges on $f'''(0) > 0$. In contrast to the case of perfect information, however, there are now two effects at work: For given inflation expectations, expected inflation increases with the variance of supply shocks due to convexity of $\pi_A(\varepsilon)$. In addition, this raises π^e, thereby increasing the level of inflation for any supply shock ε. The magnitude of this level effect can be found by implicit differentiation of (3) and it equals

$$\frac{\partial \pi_A}{\partial \sigma_\varepsilon^2} = \frac{-\theta^2 f''(y-\bar{y})}{\alpha - \theta^2 f''(y-\bar{y})} \frac{\partial \pi^e}{\partial \sigma_\varepsilon^2} \approx \frac{1}{2} \frac{\theta}{\alpha} \frac{-\theta^2 f''(y-\bar{y})}{\alpha - \theta^2 f''(y-\bar{y})} \frac{\alpha^3}{\left[\alpha - \theta^2 f''(0)\right]^3} f'''(0).$$

This level effect is decreasing in ε and it goes to 0 as $\varepsilon \to \infty$. Thus, the variance of supply shocks σ_ε^2 matters not only for average inflation, but also for the level of inflation given any supply shock ε. As a result, a reduction in the variance of supply shocks, e.g. through exchange rate stabilization, will lead to a decrease in the level of inflation.

Using (9) and (10), the variance of inflation equals

$$\mathrm{Var}[\pi_A] = \mathrm{E}\left[(\pi_A - \mathrm{E}[\pi_A])^2\right] \approx \frac{\theta^2 [f''(0)]^2}{\left[\alpha - \theta^2 f''(0)\right]^2} \sigma_\varepsilon^2 + \frac{1}{2} \frac{\alpha^4 \theta^2 [f'''(0)]^2}{\left[\alpha - \theta^2 f''(0)\right]^6} \sigma_\varepsilon^4. \tag{11}$$

Suppose the variation in inflation and its variance (over time or across countries) is due to

- variation in σ_ε^2: Solving (10) for σ_ε^2 and substituting into (11) yields

$$\mathrm{Var}[\pi_A] \approx 2\left[\alpha - \theta^2 f''(0)\right] \frac{\theta [f''(0)]^2}{\alpha^2 f'''(0)} \left[\mathrm{E}[\pi_A] - \pi^* - \frac{\theta}{\alpha} f'(0)\right]$$

$$+2\left[\mathrm{E}\left[\pi_A\right] - \pi^* - \frac{\theta}{\alpha} f'(0)\right]^2.$$

- variation in α, θ, or $f(.)$: It is not possible to derive closed-form expressions. But, simulations suggest that a positive association tends to hold.

Hence, under asymmetric information there will also be a positive relationship between the variance and the mean of inflation.

5 Extensions

The model presented above can easily be extended to more realistic settings. For example, the central bank is probably not able to forecast shocks perfectly. Introducing central bank forecast errors to the asymmetric information case gives rise to some interesting additional results. In particular, better forecasting by the central bank gives rise to lower inflation for a given level of inflation expectations; again, this is due to the convexity of inflation in response to supply shocks. Under rational expectations, the reduction in inflation will be reinforced by a positive reputation effect. This could explain why central banks tend to invest in a large staff of economists to produce superior forecasts. Furthermore, the model suggests that this benefit would be reduced if the central bank forecasts are shared with the private sector and incorporated into inflation expectations. Thus, it would be better to keep those forecasts secret. This could provide a reason why many central banks do not publish their economic forecasts.[9]

The beneficial effect of more accurate central bank forecasts also could provide an explanation for the gradualism that is characteristic of monetary policy. Moving small steps, one at a time, allows the central bank to obtain more accurate forecasts and fine-tune its policy further.

[9] For instance, the Federal Reserve forecasts used by the FOMC are published in the "Green Book", but it is only released after five years.

Another implication of the model is that for a symmetric, unimodal distribution of the supply shocks ε, inflation will have a distribution that is positively skewed. This has an important implication for the design of monetary policy targets. Central banks that adopt a symmetric inflation target centered around the median or mode of inflation are more likely to overshoot than to undershoot their target. This could have a negative impact on the credibility of the target and thereby the reputation of the central bank. Instead, an asymmetric interval with more room on the upside would overcome this problem. Using the median instead of the mode as a focal point has the advantage that inflation will be right on target in the absence of supply shocks and unanticipated demand shocks, requiring no further explanations by the central bank. If a symmetric inflation target is centered around the expected value of inflation, then the central bank is more likely to undershoot its target. This may create the impression that monetary policy is excessively restrictive. An asymmetric target would not help in this case, because inflation will be below the focal point, $\mathrm{E}[\pi]$, most of the time. Therefore, the optimal inflation target would be similar to a confidence interval for inflation, with the median as focal point and an asymmetric interval biased towards higher levels of inflation.

Other interesting theoretical extensions would be uncertainty about the central bank's goals and the effect of reputation in a dynamic context. These will be left for future research.

6 Concluding Remarks

This paper introduces an alternative objective function for monetary policy that exhibits prudence. It provides an explanation for the positive relation between inflation and its variation across countries. However, it should be emphasized that this is by no means the only theoretical explanation. This relationship is also predicted by other models, including Cukierman (1992) who emphasizes the importance of the degree of central bank independence.

In fact, it also holds for the model in this paper but with a quadratic objective function, $W = -\frac{1}{2}\alpha(\pi - \pi^*)^2 - \frac{1}{2}\beta(y - y^*)^2$, provided the target level y^* exceeds the natural rate $\bar{y}$, because both the mean and variance of inflation depend negatively on α. But, the explanation in this paper has the advantage that it applies more generally. In contrast to the other ones, it depends neither on market failures that cause $y^* > \bar{y}$, nor on asymmetries in information, nor on rational expectations.

In light of the extensive empirical evidence on the positive relationship between the average level and standard deviation of inflation across countries (e.g. Okun (1971), Logue and Willett (1976), Gale (1981), Logue and Sweeney (1981), Hafer and Heyne-Hafer (1981), Ram (1985), and Chowdhury (1991)), it would be interesting to investigate whether this can be attributed to central bank independence, wage indexation, political instability, or the prudent monetary objective function presented in this paper. This will be the topic of my future research.

Moreover, the model in this paper predicts that over time, high (low) inflation tends to be more variable (stable). This could capture the difference in the behavior of inflation in the 1970s and the 1990s. In that sense, there is no 'new economy', contrary to claims in the popular press. The 'new era' of low and stable inflation merely reflects the optimal response to shocks by prudent central bankers.

A Appendix: Microfoundations for the Central Bank's Objective Function

Let us consider the case in which the central bank is free from political influence and maximizes social welfare, and provide a justification for the use of an objective function for monetary policy that exhibits prudence in output ($f'''(.) > 0$). The microfoundations for the partial social welfare function for output can be obtained from the following rudimentary model.

For analytical convenience, suppose that there is a continuum of individuals $i \in [0, N]$. Individual i consumes c_i, supplies labor l_i, and enjoys utility $u(c_i, l_i)$, where $\partial u(c,l)/\partial c > 0$, $\partial^2 u(c,l)/\partial c^2 < 0$, $\partial^3 u(c,l)/\partial c^3 \geq 0$, and $\partial u(c,l)/\partial l < 0$, $\partial^2 u(c,l)/\partial l^2 < 0$; for simplicity it is assumed that $u(c,l)$ is additively separable. Each individual i is subject to the budget constraint $c_i = wl_i$, the time constraint $l_i \leq l_{max}$, and the nonnegativity constraints $c_i \geq 0$ and $l_i \geq 0$, where w is the real wage. Normalize $N = 1$, $l_{max} = 1$ and $u(0,0) = 0$. Assume there is a constant-returns-to-scale production technology for aggregate output: $y = A\int_0^1 l_i di$ (so, $0 \leq y \leq A$). Perfect competition and profit maximization imply $w = A$. Hence, the equilibrium condition, $y = \int_0^1 c_i di$, is always satisfied. Substituting the budget constraint, the utility function reduces to $v(l_i) \equiv u(Al_i, l_i)$, where $v(0) = 0$, $v''(l) < 0$ and the optimal labor supply l^* satisfies $v'(l^*) = 0$.[10] Thus, optimal output equals $y^* = Al^*$.

Suppose the labor market is organized as follows. A person is either employed or unemployed. Individuals $i \in [0, E]$ are employed and have a labor supply of $l_i = l_e > 0$. For the unemployed (if any) $i \in (E, 1]$, labor supply equals $l_i = 0$. For $y = y^*$, everyone is employed ($E = 1$) and supplies the optimal amount of labor ($l_e = l^*$). For $y > y^*$, every person works more than optimal to generate the increase in production ($E = 1$ and $l_e > l^*$). For $y < y^*$, the decline in output is obtained by a reduction in both the number of workers ($E < 1$) and the number of hours worked by those who remain

[10] It is presumed that there is an interior solution with $0 < l^* < 1$ (and $0 < y^* < A$).

employed ($l_e < l^*$). In particular, a fraction p of the relative reduction in the work force $L \equiv El_e$ is realized by laying off workers, and a fraction $1-p$ by cutting the hours per worker, where $0 < p < 1$. More precisely, $\dot{E} = p\dot{L}$ and $\dot{l}_e = (1-p)\dot{L}$, where $\dot{x} \equiv \partial x / x$ denotes the relative change in x. As a consequence, $\partial E/\partial L = p/l_e$ and $\partial l_e/\partial L = (1-p)/E$.

Assume the purely utilitarian social welfare function $W = \int_0^1 u(c_i, l_i)\, di$, which reduces to $W = Ev(l_e)$. Social welfare W depends implicitly on the level of aggregate production $y = AEl_e$. Clearly, W is maximized at the optimal level of output y^*. However, suppose that the level of aggregate output y deviates from y^*, e.g. due to business cycles or macroeconomic policy. Then, the marginal effect on social welfare $\partial W/\partial y$ and other properties of the partial social welfare function can be easily determined.

First, consider the case in which $y > y^*$. Using $\partial E/\partial L = 0$ and $\partial l_e/\partial L = 1$,

$$\left.\frac{\partial W}{\partial y}\right|_{y>y^*} = \frac{1}{A} v'(l_e) < 0$$

since $l_e > l^*$. Furthermore,

$$\left.\frac{\partial^2 W}{\partial y^2}\right|_{y>y^*} = \frac{1}{A^2} v''(l_e) < 0,$$

and

$$\left.\frac{\partial^3 W}{\partial y^3}\right|_{y>y^*} = \frac{1}{A^3} v'''(l_e),$$

which is positive if $v'''(l_e) > 0$.

Next, consider the case in which $y < y^*$. Using $\partial E/\partial L = p/l_e$ and $\partial l_e/\partial L = (1-p)/E$,

$$\left.\frac{\partial W}{\partial y}\right|_{y<y^*} = \frac{v(l_e)}{l_e}\frac{p}{A} + v'(l_e)\frac{1-p}{A} > 0$$

since $v'(l_e) > 0$ for $l_e < l^*$. In addition,

$$\left.\frac{\partial^2 W}{\partial y^2}\right|_{y<y^*} = \left\{ \frac{v'(l_e)\, l_e - v(l_e)}{(l_e)^2}\frac{p}{A} + v''(l_e)\frac{1-p}{A} \right\} \frac{1-p}{AE} < 0$$

because $v(l_e)/l_e > v'(l_e)$ by concavity and $v(0) = 0$. Furthermore,

$$\left.\frac{\partial^3 W}{\partial y^3}\right|_{y<y^*} = \left\{\left[\frac{v(l_e)}{l_e} - v'(l_e)\right]\frac{p}{(l_e)^2}(2-p) + (1-p)^2 v'''(l_e)\right\}\frac{1-p}{A^3E^2}$$

which is positive for $v'''(l_e) \geq 0$.

In the special case in which $p = 0$, i.e. nobody is laid off, the properties of $v(l)$ translate into similar properties for W. So, $v''(l) < 0$ implies $\partial^2 W/\partial y^2 < 0$, and $v'''(l) > 0$ implies $\partial^3 W/\partial y^3 > 0$. However, even if certainty equivalence prevails at the individual level ($v'''(l_e) = 0$ for all l_e), then the social welfare function still features prudence for $0 < p < 1$. For example, for the quadratic utility function $v(l) = -l^2 + 2l^*l$, $\left.\partial^3 W/\partial y^3\right|_{y<y^*} = (2-p)(1-p)p/A^3E^2l_e > 0$. This third derivative is larger for bigger negative deviations from optimal output. But for positive deviations it is zero in this special case.[11]

Also, notice that the model implies an asymmetric marginal effect on welfare for deviations from y^*, assuming $v(l)$ is symmetric about l^*. Formally, symmetry amounts to $-v'(l^*+\varepsilon) = v'(l^*-\varepsilon)$ for $|\varepsilon| \leq \min\{l^*, 1-l^*\}$. Take $\varepsilon = l^* - l_e > 0$, so that $l_e = l^* - \varepsilon$. Then

$$\begin{aligned}\left.\frac{\partial W}{\partial y}\right|_{y<y^*} &= \frac{p}{A}\left(\frac{v(l^*-\varepsilon)}{l^*-\varepsilon} - v'(l^*-\varepsilon)\right) + \frac{1}{A}v'(l^*-\varepsilon) \\ &> \frac{1}{A}v'(l^*-\varepsilon) = -\frac{1}{A}v'(l^*+\varepsilon) = \left|\left.\frac{\partial W}{\partial y}\right|_{y>y^*}\right|.\end{aligned}$$

Hence, a negative deviation from the optimal level y^* produces a marginal effect on welfare that is bigger in magnitude than for a positive deviation. Since $\partial W/\partial y \to 0$ as $y \to y^*_+$ this implies a social welfare function that is asymmetric: $W(y^*) > W(y^*+\varepsilon) > W(y^*-\varepsilon)$ for $\varepsilon > 0$. In words, a positive deviation from the bliss point y^* is not as bad as a negative deviation.[12]

[11] For the linear-exponential parameterization $f(x) = -e^{-\beta x} + \gamma x + 1$, the behavior of $f'''(x) = \beta^3 e^{-\beta x}$ is quite similar, especially when $\beta < 1$.

[12] The specification $f(x) = -e^{-\beta x} + \gamma x + 1$ with $\gamma = -\beta < 0$ (so that there is a bliss point at $x = 0$), also displays these asymmetries. (See also figure 1). The asymmetry vanishes as $\beta \to 0$.

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