Welfare Costs of Sticky Wages When Effort Can Respond*

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Abstract: We examine the impact of wage stickiness when employment has an effort as well as hours dimension. Despite wages being predetermined, the labor market clears through the effort margin. Consequently, welfare costs of wage stickiness are potentially much, much smaller.

Key Words: Sticky Wage, Endogenous Effort, Welfare Cost

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1. Introduction

There has been renewed interest in exploiting nominal rigidities for explaining business cycle fluctuations and for analyzing optimal monetary policy (e.g., Rotemberg and Woodford, 1997, Goodfriend and King, 1997, Clarida, Gali, and Gertler, 1999, Erceg, Henderson, and Levin, 2000, and Dotsey and King, 2001). For simplicity authors often posit nominal rigidity in output markets. But a number of papers (e.g., Ball and Romer, 1990, Erceg et al., Christiano, Eichenbaum, and Evans, 2001) makes clear that wage rigidities are crucial, in conjunction with price rigidities, for generating large and persistent model responses to monetary shocks and an important short-run policy tradeoff between inflation and output. At the same time, recent papers by McLaughlin (1994, 1999), Kahn (1997), Card and Hyslop (1997), Lebow, Sachs and Wilson (1999), and Hanes (2000) have examined the importance of nominal wage rigidities for explaining changes in individuals' wages. These papers all find some, though varying, role for nominal rigidities.

At an earlier apex of activity on models with nominal rigidities Barro (1977) and Hall (1980), among others, cautioned that the allocative role played by nominal wage (or price) rigidities is not clear. From Hall (pp. 27-28), "The long-standing theory of wage rigidity and determination of employment by demand alone is fully capable of explaining observed movements of the economy. But it amounts to a denial of labor supply in the short run. No good rationalization for this gross departure from standard economic postulates has yet been offered." These concerns have not been echoed during the current resurgence of interest in nominal rigidities.

We examine the impact of sticky nominal wages when labor input varies due to responses in effort as well as hours. A number of papers have exploited effort variations to explain business cycles observations (e.g., Burnside, Eichenbaum and Rebelo, 1993, Bils and Cho, 1994, Basu and Fernald, 2000). We find this fundamentally alters the impact of wages being sticky. If intensity of effort can respond, when the nominal wage does not, then it no longer makes sense to speak of workers as being pushed off their labor supply curves—a central element of the sticky-wage story.

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Figure 1 illustrates. Suppose that a nominal shock, given a sticky nominal wage, drives the real wage above its flexible-wage counterpart. Under the conventional interpretation firms cut back on labor until sufficient to drive the marginal product of labor up to the level of the "too high" real wage. In Figure 1 the market moves from point *E* to $E\boldsymbol{\zeta}$ with labor supply exceeding labor demand. But if labor supply exceeds demand then it is possible, and clearly profitable, for a firm to ask more of its workers. As firms ask more of workers this shifts the demand for hours of labor upward. It also shifts the labor supply curve upward reflecting the greater cost to workers of each hour at the higher effort. This continues until work intensity justifies the "too high" real wage at a point like E^2 . Given the upward shift in labor demand, hours are reduced by less than at $E\boldsymbol{\zeta}^1$

The next section formalizes this story of sticky-wages with endogenous effort within a general equilibrium. Under flexible wages individuals make choices on how hard to labor at work, as well as how many hours to work. Greater exertion at work results in a higher wage, but less energy to devote to consumption and leisure activities.² Under sticky wages this choice is restricted. Workers must produce enough to merit the specified wage in order to maintain employment.

Section 3 quantifies the model for empirical purposes. A key issue is the willingness of workers to trade off exertion and hours in production. We calibrate this willingness based primarily on World War II evidence on how piece-rate workers responded in work efficiency to large swings in their weekly hours of work. We also examine how productivity and wages have responded to other changes in hours, such as mandated hours reductions in Germany in the 1980's and 1990's.

¹ Our results on how effort responses can mitigate the impact of wage rigidities also parallel Carlton's (1989) arguments that variations in delivery lags and other dimensions of servicing customers might alleviate the impact of price rigidities, particularly within wholesale markets.

² Our treatment of effort choice under flexible wages parallels that of Becker (1985). Becker considers differences in intensity of work as an explanation for differences in individuals' wages, particularly between men and women. Relatedly, Oi (1990) depicts the firm-size effect on wage rates as arising partly from a higher effort choice in larger firms. Leamer (1996) examines the role of work intensity in the international distribution of earnings. Hartley (1992) shows how a minimum wage might lead to greater effort, mitigating the legislation's impact on employment. Waller (1989) notes that unexpected inflation will reduce effort in an efficiency-wage model with a predetermined wage. Our results are largely consistent

We then examine the welfare cost of sticky wages in the face of monetary shocks. Wage rigidities do have real effects in our model even if effort responds. Unexpected inflation increases hours and, to a lesser extent, output. But across a number of specifications we find that *effective* leisure and *effective* consumption are much less affected. As a result, the welfare effects of monetary shocks are predicted to be much smaller, by a factor of 100 for our benchmark model, if we allow for the possibility that effort can vary.

We conclude that important costs of wage rigidity require, not only that wages are sticky, but also that variations in exertion are either incredibly disliked or provide no productive gain. It is a long-standing puzzle why wage agreements so seldom condition on inflation and market conditions. Our results suggest one possible explanation. If we recognize that effort can respond, then the benefits of greater wage flexibility are perhaps quite small.

2. Model

Consumers:

The distinguishing feature of our model is that the effective amount of labor supplied to the market and to home consumption depends not only on how hours are split between market and home, but also on levels of exertion or effort. Suppose the consumer spends an amount of time n_t in the market and 1- n_t at home for consuming. Let f_t and \tilde{f}_t represent the effort levels he exerts at work and in home activities. We impose a constraint on the available energy that a consumer can exert in market and non-market activity. This "energy constraint" is

$$\boldsymbol{f}_t \boldsymbol{n}_t + (1 - \boldsymbol{n}_t) \boldsymbol{\tilde{f}}_t = 1.$$
⁽¹⁾

with his, despite the fact we consider a competitive labor market and do not impose (as he does) that hours and effort are perfect substitutes.

This presents consumers with a tradeoff. If the consumer works more intensely in the market, he is left with less energy for non-market activities. (Becker, 1985, introduces such a constraint.)

We treat effective labor in the market l_t and effective time at home \tilde{l}_t as

$$l_t = n_t \mathbf{f}_t^{\mathbf{g}}$$
$$\tilde{l}_t = (1 - n_t) \tilde{\mathbf{f}}_t^{\ 1}.$$

The elasticities of effective work and home time with respect to exertion are, respectively, g and I. We assume both g and I are ≥ 0 and < 1.³ We can envision ways, other than literally increasing exertion, by which consumers can show themselves to be more desirable employees. For instance workers could reduce absenteeism at the cost of less flexibility to stay at home on days they deem that as attractive. Or workers could cut back on consumption activities that make them less productive and attractive as employees, such as carousing to late hours during the workweek. In each case, however, this requires sacrificing the ability to enjoy non-work time.

The remainder of the consumer's problem is as follows. At time 0, the representative consumer maximizes expected discounted utility defined over a stream of effective consumption, x_t .

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$$U = E_0 \sum_{t=0}^{\infty} \boldsymbol{b}^t u(x_t), \ 0 < \boldsymbol{b} < 1$$

Following Becker (1965), home activities require an input of time as well as goods. More exactly, effective consumption, x_t , reflects commodities c_t , real balances m_t/P_t , and

³ Requiring both g and l < 1 guarantees consumers will choose an interior solution for market hours n. By contrast, as g goes to 1 (and l < g, as the calibrations below assume) market hours go towards zero with exertion per hour at work becoming arbitrarily high. (See equations (9) and (10) below.) Alternatively, we could define exertion in terms of productivity units, setting both g and l equal to 1, but impose a rising energy requirement per unit of exertion. This would not affect the analysis.

effective labor available for at home \tilde{l}_{t} . We assume the utility function reflects the relations

$$u(x_t) = \log x_t$$

$$x_t = z_t^q \tilde{l}_t^{1-q}$$

$$z_t = [cc_t^{1-\frac{1}{u}} + (1-c)(\frac{m_t}{P_t})^{1-\frac{1}{u}}]^{\frac{u}{u-1}}.$$

Higher values for q imply greater importance of purchased inputs relative to time in producing utility. For the balance of the text we set q = 1/2 to ease exposition. But in calibration we choose a value for q to match a desired steady-state value for hours worked. Goods and real money balances combine through a CES function with substitution elasticity v. This yields a demand for real balances that exhibits unit elasticity with respect to consumption and -v with respect to the nominal interest rate.

In addition to consumer goods, the consumer spends on physical capital investments, i_t , and money holdings, m_t . Income includes labor income, rental income and a cash transfer from the government. Given the nominal wage rate W_t , rental rate R_t , price of goods P_t , and money transfer from government T_t , the budget constraints are⁴

$$c_{t} + \frac{m_{t}}{P_{t}} + i_{t} \le \frac{W_{t}}{P_{t}} n_{t} + \frac{R_{t}}{P_{t}} k_{t} + \frac{m_{t-1}}{P_{t}} + \frac{T_{t}}{P_{t}}$$
(2)

$$k_{t+1} = i_t + (1 - d)k_t.$$
(3)

The money transfer from the government reflects the income from money creation. $T_t = M_t - M_{t-1} = (\mathbf{m}_t - 1)M_{t-1}$, where $\mathbf{m}_t = \frac{M_t}{M_{t-1}}$. Capital depreciates at rate \mathbf{d} .

The effective labor of a worker is $n_t f_t^g$. The wage is related to hourly effort f_t by

⁴ Following the convention in the literature, we use lower case letter for individual variable and capital letters for aggregate variables.

$$\frac{W_t}{P_t} = \boldsymbol{W}_t \boldsymbol{f}_t^{\boldsymbol{g}} \,. \tag{4}$$

 W_t is the market's valuation of labor, determined in general equilibrium as described below. Equation (4) has two separate interpretations depending on whether wages are flexible or sticky. Under flexible wages it represents the ability of a worker to earn a higher wage by exerting greater effort. A worker treats the marginal value of labor, W_t , as given with respect to his actions. Therefore, he views each 1 percent expansion in exertion to result in a g percent real wage increase. Under sticky wages the worker takes not only W_t but also the real wage as given. Therefore, to be employed, the worker has to provide effort dictated by

$$\boldsymbol{f}_t = \left(\frac{W_t}{P_t} / \boldsymbol{W}_t\right)^{1/\boldsymbol{g}}.$$
(4')

Under flexible wages, consumer maximization yields two static first-order conditions for choosing effort and hours worked and two dynamic first-order conditions for choosing investment in capital and investment in real money balances.

$$\boldsymbol{f}_{t}; \quad \frac{\boldsymbol{I}\boldsymbol{f}_{t}}{1-n_{t}\boldsymbol{f}_{t}} = \boldsymbol{g}\frac{W_{t}}{P_{t}\tilde{c}_{t}}$$
(5)

$$n_t; \quad \frac{1-I}{1-n_t} + \frac{If_t}{1-n_t f_t} = \frac{W_t}{P_t \tilde{c}_t}$$
(6)

$$k_{t+1}; \quad \boldsymbol{b} \widetilde{c}_{t} E_{t} [\frac{R_{t+1}/P_{t+1} + 1 - \boldsymbol{d}}{\widetilde{c}_{t+1}}] = 1$$
(7)

$$m_t; \quad \frac{1-\boldsymbol{c}}{\boldsymbol{c}} \left(\frac{m_t}{P_t \tilde{\boldsymbol{c}}_t}\right)^{-\frac{1}{\boldsymbol{u}}} = 1 - \boldsymbol{b} \boldsymbol{E}_t \left[\frac{P_t \tilde{\boldsymbol{c}}_t}{P_{t+1} \tilde{\boldsymbol{c}}_{t+1}}\right]$$
(8)

where $\tilde{c}_t = c_t \left[1 + \frac{1-c}{c} \left(\frac{m_t}{P_t c_t}\right)^{1-\frac{1}{u}}\right]$ is consumption magnified by a factor equal to one plus the foregone interest cost of real balances held per unit of consumption.

With flexible wages equations (1), (5), and (6) yield the relative time devoted to market work and the relative rates of exertion in market and home activity

$$\frac{n_t}{1-n_t} = \frac{1-\boldsymbol{g}}{1-\boldsymbol{I}} \frac{W_t}{P_t \tilde{c}_t} - 1 \tag{9}$$

$$\frac{\boldsymbol{f}_t}{\boldsymbol{\tilde{f}}_t} = \frac{\boldsymbol{g}(1-\boldsymbol{l})}{\boldsymbol{l}(1-\boldsymbol{g})}.$$
(10)

An economy that exhibits common growth in consumption and real wages exhibits no trend in hours supplied to the market. The ratio of exertion at work to that at home is constant. Effort at work exceeds that at home if $g > \lambda$, with the reverse true for $g < \lambda$.

To examine the importance of wage stickiness, we examine the impact if each worker must post his hourly wage rate one period in advance. If the wage is predetermined then workers are no longer free to choose effort. First-order condition (5) becomes irrelevant, with f_t instead determined by the constraint in (4'). In a typical sticky wage model, if the real wage is too high ex-post, hours are reduced to drive up the marginal product of labor. Here by working harder workers maintain the choice to work longer hours. In fact, even though the wage is sticky the labor market continues to clear. (Recall Figure 1.)⁵

A worker's wage is chosen to trade off the expected benefit in higher earnings against the expected cost in terms of the higher effort that employers will require. The first-order condition for this choice can be written as

⁵ We model hours worked as the only dimension to market hours. If we were to extend the model to allow for employment choices (such as in Cho and Cooley, 1994), then responses in effort would eliminate the involuntary unemployment generated from wage rigidities.

$$W_{t+1}; W_{t+1}E_t[\frac{n_{t+1}}{P_{t+1}\tilde{c}_{t+1}}] = \frac{1}{g}E_t[\frac{n_{t+1}f_{t+1}}{(1-n_{t+1})\tilde{f}_{t+1}}].$$
(11)

Intuitively, a one percent increase in W_{t+1} increases real earnings by $\frac{W_{t+1}n_{t+1}}{P_{t+1}}$, the left-hand side weights this, in expectation, by the marginal utility of more goods, $\frac{1}{\tilde{c}_{t+1}}$. The righthand side of equation (11) is the multiple of the following three effects. Each percent increase in W_{t+1} dictates a 1/g percent increase in f_{t+1} . Each percent increase in f_{t+1} requires, from the energy constraint, a $\frac{n_{t+1}f_{t+1}}{(1-n_{t+1})\tilde{f}_{t+1}}$ percent decrease in \tilde{f}_{t+1} . Finally, each percent decrease in \tilde{f}_{t+1} reduces utility by λ percent.

Firms:

There are a large number of identical firms operating under constant returns to scale in capital and effective labor. Firms hire capital and labor each period to maximize profits $(P_tY_t - W_tN_t - R_tK_t)$ subject to the production function

$$Y_{t} \equiv F(K_{t}, N_{t}, \Phi_{t}) \equiv K_{t}^{1-a} (N_{t} \Phi_{t}^{g})^{a}, \qquad (12)$$

 \boldsymbol{F}_t represents aggregate effort level.

The first order conditions for capital and labor are

$$\frac{R_t}{P_t} = MPK_t = (1-\boldsymbol{a})K_t^{-\boldsymbol{a}}(N_t \Phi_t^{\boldsymbol{g}})^{\boldsymbol{a}}$$
(13)

$$\frac{W_t}{P_t} = MPN_t = \mathbf{a}K_t^{1-\mathbf{a}}N_t^{\mathbf{a}-1}\Phi_t^{\mathbf{ga}}.$$
(14)

The first-order condition for labor yields a wage rate per efficiency unit of labor equal to $\Omega_t = \boldsymbol{a} \left(\frac{K_t}{N_t \Phi_t^g}\right)^{1-a}.$

Rational Expectations Equilibrium:

The markets for goods, labor, and money clear each period. A rational expectation equilibrium is a set of endogenous variables $\{n_t, f_t, c_t, k_{t+1}, m_t, P_t, W_t, R_t\}_{t=0}^{\infty}$ that satisfies (4'), (6)-(8), (11), (13)-(14), market clearing, and aggregate consistency conditions $(n_t = N_t, f_t = F_t, k_t = K_t, m_t = M_t)$ subject to an exogenous monetary process (described below). The model is solved numerically using a log-linear approximation of the system of first order conditions and constraints of the stationary economy around the steady state as in King, Plosser and Rebelo (1988.)⁶

3. Calibration

The labor market clears in our model, despite short-run wage stickiness, thanks to fluctuations in market effort. Nevertheless, monetary shocks do distort allocations by affecting the level of exertion at work versus home. The importance of this distortion depends critically on the parameters g and l. As an example, in the limit as g and l approach one, hours and effort are perfect substitutes. Consider a negative monetary shock that drives up the real wage and market effort by one percent. Workers' hours simply decrease by one percent, with output (and welfare) unaffected. More generally, however, the increase in real wage and effort leads to a lesser decrease in hours, an increase in output, and a fall in effective leisure \tilde{l}_i . For this reason, much of this section focuses on choosing proper values for parameters g and l.

Calibrating $\frac{f}{\tilde{f}}$:

As discussed above, the optimal ratio of effort in market work relative to that in consumption is $\frac{f}{\tilde{f}} = \frac{g(1-1)}{I(1-g)}$. Thus knowing $\frac{f}{\tilde{f}}$ is sufficient to determine a relationship

between the parameters **l** and **g**. Quantifying $\frac{f}{\tilde{f}}$ exactly is not feasible. We start from a presumption that $\frac{f}{\tilde{\epsilon}}$ is greater than one. Passmore, et al. (1974), in the World Health Organization publication Handbook on Human Nutritional Requirements, present energy expenditures for work in various occupations as well as for a range of leisure activities. These calculations are shown in Table 1. Results are given separately for a 65 kg man and a 55 kg woman. All figures are in terms of kilo-calories expended over an 8 hour period. For the 65 kg man leisure activities exhibit a range per 8 hours of 700 to 1500 kilo-calories, for the 55 kg woman the range is 580 to 980 kilo-calories. Occupations are classified as light, moderately active, very active, and exceptionally active. The exceptionally active occupations (lumberjacks, blacksmiths, rickshaw pullers) are relatively uncommon in modern rich economies. If we compare moderately active occupations to leisure activities, we see that work is associated with about 30 percent greater energy expenditure than the midpoint of the range for leisure activities. In addition to physical and caloric considerations, most leisure and consumption activities are presumably less tiresome mentally than typical market work. Based on these considerations, we set the ratio $\frac{f}{\tilde{F}}$ at $\frac{3}{2}$. (We explore the robustness of the results to this choice.) This implies **I** is related to **g** as $I = \frac{2g}{3-g}$.

Calibrating gfrom WWII experiences:

How desired hours respond to exogenous changes in effort reflects parameter *g*. But we do not see a practical way to directly estimate this response. During World War II, even before U.S. entry, many workers in U.S. manufacturing faced dramatic changes in their workweeks. The U.S. Bureau of Labor Statistics (BLS) took advantage of these variations to study the impact of hours of work on efficiency and absenteeism. The results of these studies are reported in *Studies of the Effects of Long Working Hours*, BLS

⁶ For convenience we have omitted technological growth. However, our model fluctuations can be understood as deviations around a growth path driven by deterministic technological growth, with all variables judged relative to their values along that growth path.

(1944) and *Hours of Work and Output*, BLS (1947). By measuring how workers vary effort in response to an enforced change in hours, this provides an alternative way to judge to what extent hours and effort are substitutes.

Worker productivity equals $\mathbf{f}_{t}^{\mathbf{g}}$. From equation (5), $\frac{\mathbf{f}_{t}n_{t}}{1-n_{t}\mathbf{f}_{t}} = \frac{\mathbf{g}}{\mathbf{l}}\frac{W_{t}n_{t}}{P_{t}\tilde{c}_{t}}$. If

consumption expenditures move one-for-one with earnings then $f_t n_t$ is constant. This implies changes in productivity are related to changes in hours according to

$$\boldsymbol{D}\ln(\frac{y_t}{n_t}) = \boldsymbol{g}\boldsymbol{D}\ln(\boldsymbol{f}_t) = -\boldsymbol{g}\boldsymbol{D}\ln(n_t).$$
(15)

Thus the response of productivity to the wartime changes in hours yields a value for g^7

The BLS examined 78 case studies covering 3,105 workers in 34 plants across a variety of industries. Each case included from 1 to 5 changes in hours.⁸ This calibration assumes that workers can choose a level of effort. We are concerned that this may not be true for workers paid at an hourly rate. Therefore we restrict our attention to cases

$$\Delta \ln(\frac{y_t}{n_t}) = \mathbf{g} \Delta \ln(\mathbf{f}_t) \approx -\mathbf{g} [\frac{n\mathbf{f}}{1 - \mathbf{g}(1 - n\mathbf{f})}] \Delta \ln(n_t) = -(\frac{3\mathbf{g}}{5 - 2\mathbf{g}}) \Delta \ln(n_t),$$

where \approx reflects a first-order approximation. The last equality follows from setting f_n equal to 3/5 to be consistent with our other calibrated values. Productivity falls less if consumption does not vary. Based on the WWII data, we estimate directly below that productivity declined by 0.25 percent for each percent increase in hours. From the equation above, this implies a value for g of $\frac{5}{14}$ if consumption failed to respond at all. This value is well within the range of values for g we consider and, in fact, is very close to our preferred value for g of $\frac{1}{2}$.

⁷ Of course, the ratio of consumption to earnings may have varied with the large changes in hours. As an extreme, suppose that consumption was not affected by the wartime changes in hours. From equations (4) and (5), changes in productivity are, in this case, related to changes in hours by

⁸ The BLS visited over 800 plants; but most did not meet the requirements of the study. From Bulletin 917: "In order to permit the effects of various schedules of hours to emerge clearly, it was necessary to rule out all other variables. A plant, therefore, could not be studied if, during the various periods to be surveyed, hours were not maintained consistently at fairly fixed and definite schedules. During these periods, the operations performed by the workers to be studied had to remain essentially unchanged, and the number of identical workers involved had to be reasonably large."

involving piece rate workers.⁹ We further restrict attention to cases where the pace of work is described as dictated by the worker, rather than machine. We also require that terms of compensation were essentially unaltered across the changes in hours. We are left with 27 groups of workers and 62 hours changes.¹⁰

Implicit in this calculation is that productivity is not affected by an increase in hours worked per week except through effort. If an expansion in hours is associated with a decrease in capital per worker this could lower productivity. We do not believe this was the case for these workers. Adding work on Saturdays or Sundays or lengthening a shift should not reduce capital per worker. Furthermore, the study selected groups of workers where the number and mix of workers in the group did not change too significantly.

Many of the large increases in hours are associated with the United States' entry into the war. Conversely, many of the large decreases reflect the end of the war. Entry into war may have provided a strong psychological stimulus to productivity, particularly for workers producing munitions and other goods directly related to the war effort. For the case of 18 workers involved in machining metal parts BLS (1947) states, "*It is pertinent to an analysis of the findings in this plant that workers became aware of the demand for company's products in the war effort* ..." For this reason, we introduce a control variable, ΔWar , that equals 1 for periods reflecting entry into the war, equals -1 for periods reflecting the end of the war, and equals 0 otherwise.

Results are presented in Table 2. The first column regresses the reported percent change in efficiency solely on the percent change in hours. (The change in hours is net of any change in absenteeism.) The estimated impact on efficiency, though negative, is fairly small and only marginally significant. The regression in Column 2 controls for ΔWar . The war is associated with a very significant positive impact on productivity of 5.6 percent. Productivity now shows an estimated elasticity of -.26 with respect to hours

⁹ The study was heavily weighted toward groups paid by piece-rate as this made it easier to measure workers' output. Bulletin 917 reports that, on average, workers on hourly rates maintained greater efficiency in response to an increase in hours than did those on piece rates.

¹⁰ One case corresponds to 150 workers. All others correspond to a number of workers between 8 and 48. 11 of these changes occurred prior to Pearl Harbor. 16 represent increases in hours from before to after Pearl Harbor. 18 are changes during the war. 16 represent changes from before to after VJ Day. 1 occurred entirely after the war.

(with a standard error of .07). Lastly, Column 3 interacts Δ War with a zero/one dummy variable, that equals one for the 42 changes in hours that are for workers producing goods directly related to the war effort. The impact of the war on productivity is about 8.0 percent for these workers.¹¹ The impact of hours on productivity is estimated fairly precisely at an elasticity of -.25 (standard error of .06), implying a value for g of .25

In the late 1800's and early 1900's a number of manufacturing enterprises exhibited dramatic reductions in the workweek. This often occurred in conjunction with adopting three-shift operation in place of two shifts. Goldmark (1912) reports the impact these changes had on productivity per hour for several companies where particularly good records were maintained. In Table 3 we report these results for 3 cases pertaining to piece-rate workers. In each case we believe it is reasonable to assume that the ratio of labor to effective capital was relatively unaffected by the altered schedule. The two factories that cut the workweek in 1893, the Salford Iron Works in Manchester England and the Engis Chemical Works near Liege, Belgium, both displayed increases in productivity that offset considerably more than half the reduction in hours per week. For the Zeiss Optical Works in Jena, Germany, which reduced daily hours from 9 to 8 in 1900, productivity increased more than proportionately to the reduction in hours, actually resulting in an increase in production. All three cases support values for g of 0.5 or higher, suggesting greater willingness to substitute between hours and effort than implied by the WWII evidence.

More recent evidence also suggests the ability and willingness of workers to substitute between effort and hours worked. Starting in 1985, (West) German unions began to reduce standard hours on an industry-by-industry basis with a stated purpose of expanding employment. Hunt (1999) exploits the cross-industry variation in the timing of these reductions in standard hours to examine the impact on actual hours worked, wages, and employment. From data on individuals from the German Socio-Economic Panel, she shows that hourly wages increased enough to offset much of the decline in actual hours worked. In the manufacturing sector, for example, a one-hour reduction in standard

¹¹ Examples of products not directly related to war are dental equipment, tobacco products, candles, and cough drops. The variable Δ War is not significant for these workers.

weekly hours reduced actual hours worked by between 0.88 and 1 hours. This constitutes a decrease in hours worked of nearly 2.5%, based on a 40-hour workweek. At the same time, the reduction in standard hours increased straight-time wage rates by between 1.3% and 2.4%, depending on the type of worker considered and how the author treats overtime pay. By contrast, an industry's employment was little affected by reducing the workweek. One explanation for the ability of workers to maintain earnings is that effort increased to offset much of the impact of reduced hours on output. This would again suggest a value for g of 0.5 or higher.

Based largely on the WWII evidence, we choose a benchmark value for g equal to $\frac{1}{3}$. We explore robustness to a wider range of values. Note that higher values for γ , as suggested by the recent German experience with hour restrictions or the earlier European case studies, imply greater ability to substitute between hours and effort. Therefore, these higher values would yield even smaller costs of sticky wages when effort can respond.

Calibrating Money Demand and Monetary Shocks:

First-order conditions (7) and (8) imply that money demand is proportional to consumption and has an interest rate (i_t) elasticity of -v. There is a very large empirical literature estimating v. We take v = 0.1 as a reasonable value given the range of estimates (e.g., Laidler, 1985). We set the parameter χ equal to .9999. For a value of 0.1 for v and a nominal interest rate of 8 percent this implies a real balance-consumption ratio, $\frac{m_t}{P_t c_t}$, of 0.51. For the United States for 1954 to 1996 the observed ratio for $\frac{m_t}{P_t c_t}$, measuring money by M2, is 0.24. The small value of $\frac{m_t}{P_t c_t}$ renders the effect of real balances on consumption demand nearly zero.

Denote deviations in the rate of money growth from its steady-state value by \hat{m} . We assume these monetary shocks follow an AR(1) process

$$\hat{\boldsymbol{m}}_{t} = \boldsymbol{r}\hat{\boldsymbol{m}}_{t-1} + \boldsymbol{e}_{t} , \qquad (16)$$

with parameter $\mathbf{r} < 1$, and innovations \mathbf{e}_t distributed normally with standard deviation \mathbf{s} . We set $\mathbf{r} = 0.78$ based on estimating (16) on the U.S. (per capita) monetary base for 1955-1996. (The standard error of $\hat{\mathbf{r}}$ is 0.088.) Our results on the importance of an effort response are not sensitive to this choice for \mathbf{r} . Conditional on this value for \mathbf{r} , we set \mathbf{s} for welfare calculations so that the model economy exhibits price volatility comparable to that observed in the U.S. GDP deflator for 1954 to 1996.

Table 4 summarizes the parameter values we have discussed in this section. It also lists parameters we have calibrated at values common in the literature, such as labor's share \boldsymbol{a} and the depreciation rate \boldsymbol{d} .

We choose a value for preference parameter \boldsymbol{q} in order to set *n*, the steady-state fraction of time allocated to the market, equal to one half.¹² For our model the elasticity response of hours to the real wage, holding current consumption expenditures constant, is approximately $\frac{1-n}{n}$. Thus we are implicitly setting this elasticity equal to one. This value is lower than typically employed in simulations of real business cycles, but is large relative to most estimates from cross-sectional and panel data (Ghez and Becker, 1975, MaCurdy, 1981, and Altonji, 1986).

4. The Impact of Monetary Shocks

Responses to a monetary shock

We examine responses to a shock of one percent to the monetary growth rate, first for a "standard" sticky-wage model with no effort response, then for the model with effort responding. Results for the sticky wage model without effort response appear in Figure 2. The impact on hours and output is very dramatic. The decline in real wages induces a transitory increase in hours of nearly 4 percent and in output of about 2.5 percent. The expansion is almost entirely through investment spending, which increases by about 7

¹² This is roughly consistent with time-use studies (e.g., Juster and Stafford, 1991), if one compares market time (including commuting time) to time spent in social and leisure activities.

percent. The monetary shock creates a persistent expansion in consumption, but not in output.

With endogenous effort, Figure 3, the impact of the monetary shock is almost neutralized by the decrease in effort brought about by the decrease in the real wage. Hours still expand significantly by about 2 percent, as opposed to 4 percent. But this increase in hours is largely offset by a decline in effort so that the increase of effective labor in the market is very small. The impact on consumption, investment, and output is also small relative to the case of sticky wages with no effort response.

Our focus in this paper is on the ability of an effort margin to offset the welfare consequences of wage stickiness. A related, but separate, question is whether allowing for an effort margin helps sticky-wage models replicate business cycle facts. In Bils and Chang (2001) we address this question in a setting that entertains real as well as monetary shocks. In our model unexpected inflation deteriorates real wages and, consequently, the effort level of workers. For this reason, the impulse responses in Figure 3 show that the monetary shock, by increasing inflation, results in a rise in output together with a drop in TFP. This is counterfactual--it is well known that TFP is procyclical. For an economy subject to real as well as monetary shocks, however, we find that variations in output are largely driven by the real shocks if effort can respond.¹³ As a result, consistent with the data, the model produces prices that are countercyclical and productivity that is very positively correlated with output but little correlated with hours. By contrast, a model with sticky wages and no effort response continues to predict, counterfactually, a strong negative relationship between labor productivity and hours worked.¹⁴

¹³ In Bils and Chang (2001) we allow for productivity shocks (measured by the Solow residual adjusted for effort variations based on the model), disturbances that shift how consumers allocate spending across time, and monetary shocks.

¹⁴ A prediction more unique to our model is that positive monetary shocks, by lowering real wages and effort, reduces TFP. Inflation and TFP are strongly negatively correlated. For instance, for 1962 to 1997, the correlation between the annual rate of inflation and the annual rate of TFP growth is -0.55. This, of course, does not isolate the relation between nominal shocks and effort, particularly if there are important cyclical shocks to productivity that also lead to lower inflation. One would like to isolate the response of TFP to innovations in inflation due solely to exogenous monetary shocks. In practice, identifying such policy shocks is difficult. But, for instance, when we instrument for the inflation rate with the innovations to policy shocks—the innovations to the ratio of non-borrowed to total reserves in the VAR–suggested in Strongin (1995), the instrumented variations in inflation rates remain very negatively correlated with the rate of TFP growth (correlation of -0.50). We discuss the relation between innovations in measures of

Welfare costs of wage stickiness

From Figure 3, consumption expenditure and effective leisure do not move significantly in response to unexpected increase in money growth. This suggests that the welfare consequences of wage stickiness are far lower for our model than in a standard sticky-wage model. We illustrate this in Table 5 for the case of r = 0.78, with the standard deviation of shocks set equal to .5 percent to be roughly consistent with observed price variability.

Under sticky wages and constant effort the welfare cost of the wage rigidity is equivalent to a loss of 0.674 percent of steady-state consumption. (Details on calculating welfare costs are contained in the appendix.) This is consistent with estimates in Cho, Cooley, and Phaneuf (1997), which range from .05 to 1.2 percent of steady-state output.

For our benchmark model with endogenous effort, with $\gamma = \frac{1}{3}$ and steady-state $\frac{f}{f}$ equal to $\frac{3}{2}$, the welfare cost of nominal shocks is only 0.0059 percent of steady-state consumption. Thus the cost is reduced by two orders of magnitude. For annual consumption of \$30,000 this translates into a cost of only \$1.77 per year. It is a long-standing puzzle why wage agreements do not build in more indexing and flexibility. Our model, by allowing an effort response, suggests the cost of wage stickiness may be extremely small. Thus it may require only very small costs of writing wage flexibility into labor arrangements to rationalize sticky wages in practice.

In Table 5 we examine a number of other parameter values for the model, all in an attempt to increase the welfare costs under endogenous effort. Looking across the first row, we let marginal variations in effort be less productive by setting $\gamma = \frac{1}{10}$, or higher, while maintaining $\frac{f}{f}$ equal to $\frac{3}{2}$. This increases the welfare cost of wage rigidity to slightly over .01 of one percent of steady-state consumption. Thus it does not materially affect welfare costs. Secondly, looking down the first column, we increase the steady-

monetary policy and TFP in greater detail in Bils and Chang (2001). Given the difficulty of identifying policy shocks, we view the empirical relation between monetary shocks and TFP as an open issue.

state ratio $\frac{f}{\tilde{r}}$, holding g equal to $\frac{1}{3}$. This has somewhat more impact. Increasing $\frac{f}{\tilde{r}}$ to 3 increases the welfare cost to .0366 of one percent of steady-state consumption, but this remains only five percent of the welfare cost under sticky wages without an effort response. Even increasing $\frac{f}{\tilde{r}}$, implausibly, to 100, the welfare cost remains only one-fifth as large as without any effort response. From Table 5 we see that to produce welfare costs comparable to the case without an effort response requires both a very low value for g and a very high value for $\frac{f}{\tilde{r}}$. For instance, setting g = 1/10 and $\frac{f}{\tilde{r}} = 10$, welfare costs are about one third those with no effort response. Finally, for g = 1/100 and $\frac{f}{\tilde{r}} = 100$, welfare costs are 80 percent as large as for the setting with no effort response. We view these cases with significant welfare costs as very unrealistic, however, as they require parameter values that are extremely far removed from those chosen and defended in Section 3.

5. Conclusions

We examine the impact of wage stickiness when employment has an effort as well as hours dimension. Despite wages being predetermined, the labor market clears through the effort margin. Sticky wages do create inefficient fluctuations in exertion at work relative to at home. But the consumption–leisure margin is much less distorted than if no response in effort is allowed.

Although movements in real wages for our model behave very much as in the typical sticky wage setting, fluctuations in hours worked are partially muted. Furthermore, the impact the sticky wage has on hours worked is largely offset by variations in effort at work. As a result, output and consumption behave much as if wages were flexible. Consequently, welfare costs of wage stickiness appear to be much smaller, by perhaps two orders of magnitude, if one allows an effort dimension. In summary, to resurrect important costs of wage rigidity requires assuming, not only that wages are sticky, but also that variations in exertion are either incredibly disliked or provide no material gain.

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It is an open question as to why wage contracts do not provide more contingencies for inflation and employment outcomes. Our model provides a partial rational in that, with an effort margin, the benefits of greater wage flexibility are very small.

Appendix: Welfare costs of wage stickiness

Monetary disturbances are essentially neutral with respect to flexible-wage versions of the model. Therefore, for monetary disturbances, the welfare cost of sticky wages can be measured based on the economy's deviations from its steady state. Let \overline{U} denote the lifetime utility associated with consumption of goods, real balances, and leisure for the economy calculated at its steady-state level. Similarly, let U_s denote lifetime utility for the realizations of

the economy under sticky wages. Then $\hat{U} = U_s - \overline{U} = \sum_{t=0}^{\infty} \boldsymbol{b}^t E_0[\hat{u}_t]$, where \hat{u}_t is the utility cost incurred in *t*. Let D_s represent the welfare cost from sticky wages in consumption units, so

that $\log(x - D_s) - \log x = \hat{U}$. We can express D_s in terms relative to steady-state consumption as $\frac{D_s}{x} = 1 - e^{\hat{U}}$.

Ignoring money balances, which are unimportant, the period t utility cost can be written

$$E[\hat{u}_t] = E[\boldsymbol{q} \log c_t + (1-\boldsymbol{q}) \log \tilde{l}_t] - [\boldsymbol{q} \log c + (1-\boldsymbol{q}) \log \tilde{l}],$$

where c and \tilde{l} are steady state values. Its second-order Taylor approximation is

$$E[\hat{u}_t] \approx \frac{\boldsymbol{q}}{c} E(c_t - c) + \frac{1 - \boldsymbol{q}}{\tilde{l}} E(\tilde{l}_t - \tilde{l}) - \frac{1}{2} \frac{\boldsymbol{q}}{c^2} E(c_t - c)^2 - \frac{1}{2} \frac{1 - \boldsymbol{q}}{\tilde{l}^2} E(\tilde{l}_t - \tilde{l})^2$$

Generally speaking, $E[c_t] \neq c$ under sticky wages. We approximate $E[c_t]$ as follows.

$$E[c_{t}] \approx S_{c}E[y_{t}] = S_{c}E[L_{t}(\frac{K}{L})_{t}^{1-a}]$$

$$\approx S_{c}E[L(\frac{K}{L})^{1-a} + (\frac{K}{L})^{1-a}(L_{t}-L) + L(1-a)(\frac{K}{L})^{-a}((\frac{K}{L})_{t} - \frac{K}{L})$$

$$-\frac{1}{2}a(1-a)L(\frac{K}{L})^{-a-1}((\frac{K}{L})_{t} - \frac{K}{L})^{2} + (1-a)(\frac{K}{L})^{-a}((\frac{K}{L})_{t} - \frac{K}{L})(L_{t}-L)]$$

where $L_t = N_t \Phi_t^g$ and S_c is the ratio of consumption to output in steady-state.

Finally, \hat{u}_t can be expressed in terms of percentage deviations of variables from steadystate as

$$E[\hat{u}_{t}] \approx -\frac{1}{2} \boldsymbol{q} E[\hat{c}_{t}]^{2} - \frac{1}{2} (1-\boldsymbol{q}) E[\hat{\tilde{l}}_{t}]^{2} - \frac{1}{2} \boldsymbol{q} S_{c} \boldsymbol{a} (1-\boldsymbol{a}) E[(\frac{\hat{K}}{L})_{t}]^{2} + \boldsymbol{q} S_{c} (1-\boldsymbol{a}) E[(\frac{\hat{K}}{L})_{t} \hat{L}_{t}]$$

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	65 Kg Man	55 Kg Woman
Sleeping	500	420
Non-working Activities	700 – 1500	580 - 980
Working Activities [*]		
Light	1100	800
Moderately active	1400	1000
Very active	1900	1400
Exceptionally active	2400	1800

Table 1 – Energy Expenditures (in kilo-calories per 8 hours) for Sleeping, Non-work Activities, and Working, from Passmore, et. al., *Handbook on Human Nutritional Requirements* (1974)

^{*}Classifications of occupations by intensity:

Light

- Men: Office workers, professionals (lawyers, doctors, accountants, teachers, etc.), shop workers.
- Women: Office workers, housewives with mechanical household appliances, teachers and most other professionals

Moderately active

Men: Men in light industry, students, construction workers (excluding heavy laborers), many farm workers, soldiers on inactive service, fisherman.
 Women in light industry, housewives without mechanical household appliances, students, department-store workers.

Very active

Men: Some agricultural workers, unskilled laborers, forestry workers, army recruits, soldiers on active service, mine workers, steel workers, athletes.Women: Some farm workers (especially in peasant agriculture), dancers, athletes.

Exceptionally active

Men: Lumberjacks, blacksmiths, rickshaw pullers. Women: Construction workers.

	Column 1	Column 2	Column 3
Intercept	.047 (.011)	.045 (.009)	.042 (.008)
DHours Hours	123 (.065)	264 (.069)	250 (.059)
ΔWar		.056 (.014)	
∆War•War related			.080 (.015)
\overline{R}^2	.04	.22	.34
Ν	62	62	62

Table 2 – Response of Productivity to Hours Worked

Case	Description	Results
Salford Iron Works, Manchester England, 1893	March 1, 1893, reduce weekly hours from 53 to 48. Productivity for 3/1/93 to 2/28/94 Compared to previous six years.	Hours reduced by almost 10 percent Productivity (per hour) increased by about 8.8 percent.
Engis Chemical Works, near Liége, Belgium, 1893 [*]	Beginning 1893 change from 2- shift production to 3-shift production, reduce daily hours from 10 to 8. Productivity for 1893 compared to 1889-1892.	Hours reduced by 20 percent. Productivity increased by 12.4 percent.
Zeiss Optical Works, Jena, Germany, 1900 [†]	April 1, 1900 daily hours reduced from 9 to 8. Productivity for 4/1/99 to 3/31/00 compared to 4/1/00 to 3/31/01.	Hours reduced by about 11 percent. Productivity increased by 16 percent. (Results are similar across ages and occupations.)

Table 3 – Results from Studying the Impact of Reduced Workweeks on Productivity of Piece-Rate Workers, described in Goldmark, *Fatigue and Efficiency* (1912)

^{*} L.G. Fromont, the engineer who founded and managed the Engis works reported his findings on the shortened workweek to the Belgian Chemical Society and the Association of Engineers of the Liége School in 1897.

[†] Ernst Abbé, a physicist, university professor, inventor, and owner of the Zeiss Optical Works, reported his results in two lectures before the Society for Political Economy of Jena in 1901.

Parameters	Description
$\alpha = 2/3$	Labor share in output
$\beta = 0.98$	Discount factor
$\delta = 0.1$	Depreciation rate
n = 0.5	Steady-state hours of work
$\boldsymbol{f}/\boldsymbol{\tilde{f}}=3/2$	Ratio of effort in the market and home
$\gamma = 1/3$	Ability to substitute hours and effort in the market
$\lambda = 1/4$	Ability to substitute hours and effort at home
υ = .1	Interest rate elasticity of money demand
$\chi = .9999$	Relative share of consumption in utility
q = 0.6332	Preference parameter
r = 0.78	Autocorrelation of disturbance to monetary growth
s = 0.005	Standard deviation of monetary shock

Table 4 – Parameter Values for the Benchmark Case

Table 5 – Welfare Costs of Monetary Shocks

 $r_m = 0.78 \ s_m = 0.005$

Welfare cost under standard sticky wage model = 0.674

	$\gamma = 1/3$	$\gamma = 1/10$	$\gamma = 1/100$
$\frac{f}{\tilde{c}} = 1.5$	0.0059	0.0106	0.0128
f	$(\lambda = 0.25)$	$(\lambda = 0.069)$	$(\lambda = 0.0067)$
$\frac{f}{2} = 3$	0.0366	0.0742	0.094
f	$(\lambda = 0.1429)$	$(\lambda = 0.0357)$	$(\lambda = 0.0034)$
$\frac{f}{\tilde{z}} = 10$	0.0945	0.229	0.326
f	$(\lambda = 0.0476)$	$(\lambda = 0.011)$	$(\lambda = 0.001)$
$\frac{f}{2} = 100$	0.132	0.372	0.538
f	$(\lambda = 0.005)$	$(\lambda = 0.0011)$	$(\lambda = 0.0001)$











