Controllability and non-neutrality of economic policy: The Tinbergen's approach in a strategic context[†]

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Abstract

In the last 20 years issues of policy effectiveness and neutrality (notably with reference to monetary policy) have been increasingly raised in the context of static LQ (linear-quadratic) policy games. The general conditions ensuring policy non-neutrality in a strategic environment remains however to be inquired. We state these conditions by generalizing the classical theory of economic policy developed by Tinbergen and others to such a context. We also state necessary and sufficient conditions for the existence of Nash and Stackelberg equilibria. We finally show that the conditions for monetary policy effectiveness asserted in the literature respect our general conditions.

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1. Introduction

In the last decade or so effectiveness of specific policy instruments has been analyzed in a context of policy games, mainly with reference to monetary policy, starting from the pioneering articles of Stokey (1990) and Gylfason and Lindbeck (1994). Explicitly or implicitly formal conditions leading to monetary policy ineffectiveness – or neutrality – have been investigated in specific setups within the class of static LQ-games. Such conditions are apparently very different from those stated in the classical theory of economic policy initiated by Tinbergen (1952, 1954) and more formally asserted by Preston and Pagan (1982) and Holly and Hughes Hallett (1989). In the classical theory they are in fact expressed in terms of matrix ranks, whereas in the literature that uses policy games they are usually referred to the nature of the private sector's preference function.[1]

Our aim is twofold: 1) to extend conditions for static controllability of the classical theory of economic policy from a single decision-maker (a parametric context) to a strategic multi-player context; our extension will make us define the game equilibrium properties in terms of existence and policy ineffectiveness; 2) to compare our general statements (expressed in the terms of the traditional counting rule) to the specific conditions for the effectiveness of monetary policy found in policy game settings.

For the sake of simplicity, we restrict ourselves to the common LQ-games in a perfect information static context. Our simple logic can be extended to more complex frameworks. Some intuitions in that direction are provided in the last section.

The rest of the paper is organized as follows. The next section, presents our generalization of the Tinbergen's approach to a strategic context, in which players have strictly quadratic preferences. Conditions for policy neutrality are then asserted. Section 3 provides some further generalizations to the case of linear-quadratic preferences. Section 4 compares the conditions for the effectiveness of monetary policy found in the literature to those deriving from our approach. Section 5 concludes and provides some intuitions for further generalizations. All proofs of the theorems as well as the generalization to the *n*-player case are contained in Appendix B.

2. The target/instruments approach in a strategic context

2.1 Preliminary definitions

Before introduc~~ing~~ the policy game setting, we need redefin~~ing~~ policy neutrality in a context where policy is endogenous. In addition, in order to extend the traditional theory of economic policy to a strategic context, we also need to revisit the traditional definition of controllability.

<u>Definition 1 (endogenous policy neutrality):</u> *Economic policy is neutral with respect to a target variable, if the equilibrium value of such a variable is not affected by any change in policymaker's preferences.*[2]

<u>Definition 2 (static controllability):</u> *A system is controllable if the number of decision-maker's independent targets equals the number of its independent target variables.*

This definition of controllability is *global*, in the sense that, given a linear representation of the targets, it states the general conditions to influence the whole system, i.e. to achieve the vector of target values. However, for our aim, we are also interested in a *partial* form of controllability, which holds when a decision-maker can control only a sub-vector of its targets. We refer to this concept as sub-controllability.

In more formal terms, let us consider the equation system $Ay = Bu + K$, where $y \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ are the decision-maker's targets and instruments; $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $K \in \mathbb{R}^n$ are (full-rank) matrices and vector of parameters. If $m > n$, $C = A^{-1}B$ is rectangular and the decision-maker cannot control its targets. However, if the system is the *sum* of independent systems it might control part of them (sub-controllability), exactly achieving some of the

[1] See e.g. Gylfason and Lindbeck (1994), Acocella and Ciccarone (1997), Guzzo and Velasco (1999), Soskice and Iversen (2000), Jerger (2002), Coricelli *et al.* (2000, 2001), Lawler (2000 and 2001), Cukierman and Lippi (2001), Lippi (2003), Acocella and Di Bartolomeo (2004).
[2] This definition is implicit in Gylfason and Lindbeck (1994).

target values.[3] Thus sub-controllability defines the controllable (target) set, which groups all target variables that are controllable by the decision-maker. We can now state the following formal definition.[4]

Definition 5 (controllable target set). *If a) a decision-maker faces the equation system* $Ay = Bu + K$ *; b)* $col(C)$ *is a basis of C, where C =* $A^{-1}B$ *; c)* $e(i) \in \mathbb{R}^n$ *is an eye vector with the i-th entry equal to one otherwise zero; then* $\Theta = \left\{ \forall y_i \ i \in \{1,2,...,n\} \middle| e(i) \in \text{span}\left[col(C) \right] \right\}$ *is the decision-maker's controllable (target) set.*

It is easy to verify that definition 2 generalizes definition 1. In fact, if the number of independent instruments is equal to (or greater than) the number of independent targets, $col(C) = I$. Therefore, all possible target values are in the controllable set and the system $Ay = Bu + K$ is controllable in the Tinbergen terms.

2.2 The policy game approach

We consider an economy where two players, the Government and an Agent, interact[5]. We assume that they minimize the following loss functions, respectively:

(1) $$U = (y - \bar{y})' Q (y - \bar{y}) + y'R$$

(2) $$W = (z - \bar{z})' M (z - \bar{z}) + z'H$$

where $y = (y_U, y_S)' \in \mathbb{R}^{n(U)+n(S)}$ and $z = (y_S, y_W)' \in \mathbb{R}^{n(W)+n(S)}$ are each players' targets; $\bar{y} = (\bar{y}_U, \bar{y}_S)$ and $\bar{z} = (\bar{z}_S, \bar{z}_W)$ are target values; *R* and *H* are parameter vectors (in all this

[3] An example can better explain the above concept. Imagine two *distinct* problems, one controllable by the decision-maker and another that is not, e.g. a 2 targets by 2 instruments and a 3 targets by 2 instruments independent systems. Merging the two problems together the decision-maker faces a system of 5 equations (targets) with 4 unknowns (instruments). Although the new system is clearly not controllable in the sense of getting some pre-assigned values for all the 5 targets, the decision-maker can always set the first two equal to its *first best* irrespectively of the other three.

[4] The controllable set is formally derived in Appendix A.

[5] The denomination of the two players is allusive of situations where a public and a private operator interact. However, the two players can be both public (or private, for what matters) subjects.

section we assume they are both zero); Q and M are symmetric positive semi-definite matrices, which for the sake of simplicity we assume to be both diagonal.[6] We assume that players share $n(S)$ targets, i.e. $y_S \in \mathbb{R}^{n(S)}$, while $y_i \in \mathbb{R}^{n(i)}$ for $i \in \{U,W\}$ are each player's peculiar targets.

The following general linear algebraic system describes economic relationships between the relevant variables:

$$(3) \qquad Dx - D_U u - D_W w = \bar{K}$$

where coefficient matrices are $D \in \mathbb{R}^{k \times k}$ with $k = n(U) + n(W) + n(S)$ and $D_i \in \mathbb{R}^{k \times n(i)+n(S)}$ for $i \in \{U,W\}$. The vector of instruments controlled by the Government (Agent) is $u \in \mathbb{R}^{m(U)}$ ($w \in \mathbb{R}^{m(W)}$).

A necessary component in a policy-game approach to controllability is the specification of the kind of interactions between the players. We deal with three well-known solutions in policy games: the *Nash non-cooperative equilibrium,* the *Commitment* solution (with the Government acting as the Stackelberg leader) and the *Discretion* solution (with the Government acting as the Stackelberg follower).

2.3 Decoupling the players' problems

In order to give the policy game a form similar to that of the classical approach, we reduce our decision problem to two separate optimization problems.

We first rewrite the equation system (3) as:

$$(4) \qquad D\begin{pmatrix} y_U \\ y_S \\ y_W \end{pmatrix} - \begin{bmatrix} D_U^U \\ D_U^S \\ D_U^W \end{bmatrix} u - \begin{bmatrix} D_W^U \\ D_W^S \\ D_W^W \end{bmatrix} w = \begin{pmatrix} \bar{K}^U \\ \bar{K}^S \\ \bar{K}^W \end{pmatrix}$$

where $\bar{K}^i \in \mathbb{R}^{n(i)}$; $D_j^i \in \mathbb{R}^{n(i)\times m(j)}$ for $i \in \{U,S,W\}$, $j \in \{U,W\}$.

From (3) we derive two overlapping-equation systems, which define the sub-system relevant for each player:

[6] For the sake of brevity, vector and matrix dimensions are omitted when trivial.

(5) $$y = A^{-1}\begin{bmatrix} D_u^u \\ D_u^s \end{bmatrix} u + A^{-1}\begin{bmatrix} D_w^u \\ D_w^s \end{bmatrix} w + A^{-1}\begin{bmatrix} \bar{K}^u \\ \bar{K}^s \end{bmatrix}$$

(6) $$z = \tilde{A}^{-1}\begin{bmatrix} D_w^s \\ D_w^w \end{bmatrix} w + \tilde{A}^{-1}\begin{bmatrix} D_u^s \\ D_u^w \end{bmatrix} u + \tilde{A}^{-1}\begin{bmatrix} \bar{K}^s \\ \bar{K}^w \end{bmatrix}$$

where squared sub-matrix A ($\tilde{A}$) is obtained by eliminating columns and rows larger (smaller of equal) than $n(U)+n(S)$ from D.

As in single decision-maker's problems, the Government minimizes (1) subject to (5) and the Agent minimizes (2) subject to (6).

By solving the system of focs for this minimization problem gives the Nash equilibrium. Stackelberg equilibria are obtained in a similar manner considering a backward procedure: the follower problem is the same as above, while the leader's is the same augmented with the additional constraint derived from the follower's optimization problem.

2.4 Policy neutrality and controllability

In the above decoupled representation of the policy game, an intuitive condition for neutrality can be defined as follows. Let us refer to the Nash solution. *Provided that equilibrium exists*, the Government's policy is neutral with respect to the targets shared with the Agent, if the system (6) is controllable by the Agent or, more precisely, if the first $n(S)$ targets are in the Agent's controllable set.

Although intuitive, the above condition nests an apparent contradiction, since the Agent's controllability does not exclude that also the Government can control its sub-system. As we will show, the contradiction is only apparent. In fact, were this the case, the equilibrium would not exist. The issue of equilibrium existence indeed is crucially related to that of controllability and neutrality, as the following theorem more formally states.

Theorem 1 (Government's policy neutrality). *(i) The equilibrium of the game exists if and only if the intersection of the players' controllable sets is empty or the players share the same target values for the variables therein contained. (ii) The Government's policy is neutral for all the Government's target variables contained in the Agent's controllable target set.*

It is worth noticing that the Theorem holds irrespectively of the particular concept of solution considered.

With reference to part (i) of Theorem 1, it is worth underlining that, in an unusual way, we have derived the necessary and sufficient condition for the existence of the Nash equilibrium in terms of the classical counting rule of the number of targets and instruments. It is thus finally useful to compare our results to a well-known theorem of existence of Nash equilibrium. In an LQ-context, a sufficient condition for the Nash equilibrium existence[7] is that the space of strategies of each player is convex and compact. If players' controls are unbounded, the Nash equilibrium may not exist. The introduction of quadratic instrument costs would make them bounded, thus assuring the equilibrium existence. In our terms, this would imply that the dimensions of matrices Q and M become $n(S)+n(i)+m(i)$ for $i\in\{U,W\}$. Thus, the number of instruments would always be smaller than that of targets, the system would not be controllable by any player and equilibrium would exist. It is worth noticing, however that our theorem is more general than the mentioned theorem of existence, since that of instrument costs is a particular case.

Let us now briefly refer to part (ii) of Theorem 1. The analysis of policy neutrality has hitherto been conducted in the literature on the basis of a detailed inspection of policymakers' preferences and model constraints and usually requires solving the complete policy game. By contrast, Theorem 1 makes it possible to detect policy neutrality ex ante on the basis of the Agent's controllable target set only, by a simple analysis of the structure of a matrix, in a Tinbergen fashion.

3. The case of LQ-preferences

In the previous section we have assumed strictly quadratic losses; here we generalize our results to LQ-preferences (i.e. R and H different from zero), which are often used in policy

[7] See, e.g. Dasgupta and Maskin (1986).

games. In order to avoid trivial cases, we assume that at least one target variable enters each player's function in a quadratic form.

The extension of conditions for policy existence and neutrality to LQ-losses is simple and can be done along the lines of our previous analysis; however, it nests some further technical complexities. In particular, stronger conditions for the equilibrium existence and a weaker form of neutrality are involved.

We will deal with existence first. Before doing so, we must clarify the implications of LQ-preferences for optimal policies. A generic entry of the LQ-loss for, e.g., the Government is $Q_{i,i}\left(y_i - \overline{y}_i\right)^2 + R_i y_i$, which collapses to a quadratic term for $R_i = 0$. Hence, the *optimum optimorum* for the target variable y_i is $\overline{y}_i - \frac{1}{2}\frac{R_i}{Q_{i,i}}$, instead of $\overline{y}_i$ (as in the quadratic case), and it does not exist as a finite value if $Q_{i,i} = 0$ as in such a case $y_i = \pm\infty$ is optimal for the player according to the sign of R_i. Thus, if a decision-maker is able to control the system, it will optimally set its instrument vector at the value associated with its *optimum optimorum* instead of having zero deviations from the target vector.

Now we can deal with the issue of existence. Because of the linear terms in the loss functions, a specific problem arises, leading to more stringent conditions for the equilibrium existence.[8] If a player can control a system, it sets its instruments in order to achieve values of its target variables equal to its *optima optimorum*, for example $\overline{z}_i - \frac{1}{2}\frac{H_i}{M_{i,i}}$ in the case of the Agent. But, if $M_{i,i} = 0$ for some *i*, the *optima optimorum* for those target variables no longer exist and the Agent's problem cannot be solved (for finite values of instrumental variables). Hence, if a target variable, that enters *U* (*W*) only linearly, is in the Government's (Agent's) controllable target set, the equilibrium does not exist. Generalizing the first part of Theorem 4

[8] This is because in LQ preferences target variables that enter the loss function only linearly imply unbounded payoff functions.

to the case of LQ-preferences, the existence problem can be summarized as in the theorem below, which implies stronger conditions for the equilibrium existence.

Theorem 2 (equilibrium existence extended). *An equilibrium of the policy game between the Government and the Agent exists if i) the first part of Theorem 1 holds or ii) no player's controllable target set contains any target variable that enters its loss linearly only.*

The existence of target variables that enter the players' loss functions only linearly implies also additional complications from the point of view of policy neutrality. The second part of Theorems 1 becomes:

Theorem 3 (Government's extended policy neutrality). *Provided that either the Nash or the Commitment equilibrium of the policy game between the Government and the Agent exist, the Government's policy is neutral for all the Government's shared target variables, if the number of instruments of the Agent is equal to the number of its quadratic target variables.*

Notice that Theorem 3 generalizes the second part of Theorem 1 only for Nash and Commitment solutions. The reason is easy to explain. Even if Government's neutrality holds, neither the Nash nor the Commitment solutions are first bests for the Agent when some targets enter its preference only linearly. Thus the Agent could raise its utility if it is able to change the equilibrium finite value of linear targets. This is possible under Discretion, where the Agent can use its first-mover advantage internalizing the Government's reaction and taking account of Government's preference parameters, in such a way as to suffer a loss lower than that associated with the Nash or the Commitment solutions. The same can occur in a cooperative equilibrium.

4. A closer look at the literature on monetary policy neutrality

4.1 Barro and Gordon (1983) and inflation aversion[9]

[9] This subsection briefly summarizes a more formal discussion contained in the working paper version of this article (see Acocella and Di Bartolomeo, 2005).

The most celebrated policy game is probably that of Barro and Gordon (1983). The model can be easily represented by using a Lucas supply function and modeling a game between the government, setting inflation to minimize a quadratic loss in inflation and output deviations from desired targets, and the private sector that sets inflation expectations to minimize the output deviation from a natural rate.

In the Tinbergen terminology the controllable set of the private sector contains output since it has one instrument for one target. By contrast, that of the government's controllable set is empty if the government preference is quadratic in output. Neutrality of monetary policy is a straightforward result (see Theorem 1).

Moreover, it is easy to verify that, if inflation does not enter the government's preferences (or enters them linearly), the equilibrium does not exist since the government's controllable set contains output as well (see Theorem 1: part *ii*). By contrast, if the government does not care about output, the private sector is neutral with respect to inflation since this target is in the government's controllable set, as in the well-known case of the conservative central banker (Rogoff, 1985).

Modeling the private sector strategies explicitly in terms of the nominal wage[10], results become more interesting. Assuming that the private sector – or a representative union – cares about real wage and output does not alter the above results, since these variables are not independent. By contrast, including inflation as a quadratic argument of private sector's preferences makes its controllable set empty and neutrality vanishes, as shown by Gylfason and Lindbeck (1994). In such a model, it is also easy to verify that including inflation as a linear argument into the private sector's preferences does not imply monetary non-neutrality under Nash or Commitment solutions, whereas monetary non-neutrality holds under Discretion (as from Theorem 3).[11]

[10] See Stokey (1990) and Gylfason and Lindbeck (1994).

[11] More in general, in a Barro-Gordon context, the direct or indirect inclusion of an additional independent target in the private sector's preferences implies non-neutrality. See e.g. Detken and Gärtner (1994), Acocella and Ciccarone (1997) or Acocella and Di Bartolomeo (2004). The former authors add to private sector's preferences the electoral result and public debt, respectively. The latter break the link between real wage and output, and make them independent targets by considering the difference between product and consumer prices.

4.2 Monopolistic competition and wage setters

More recent contributions stress a new channel of monetary non-neutrality. Among other innovative results directly related to the monetary non-neutrality, Soskice and Iversen (1998, 2000), Coricelli *et al*. (2002) and Cukierman and Lippi (2002) show that a multiplicity of unions and monopolistically competitive markets in a Barro-Gordon framework lead to monetary non-neutrality, even if unions are not directly averse to inflation.

We can describe a model of the above kind by using a simple game between *n* unions and a government (central bank).[12] The government seeks to maximize:

(7) $$G = -\frac{\beta}{2} p^2 - \frac{1}{2} u^2 .$$

where *p* is the price level and *u* is the unemployment rate.

Each union seeks to maximize a LQ-function defined on its members' log real wage ($w_i - p$) and unemployment rate (u_i):

(8) $$U_i = b_1 \left(w_i - p \right) - \frac{1}{2} u_i^2 \qquad i \in \left\{ 1, 2, \ldots n \right\} ,$$

The economy consists of three equations:[13]

(9) $$u_i = \frac{\eta}{\alpha + \eta \left(1-\alpha\right)} \left(w_i - p \right) - \frac{1}{\alpha + \eta \left(1-\alpha\right)} \left(m - p \right)$$

(10) $$p = \alpha w + \left(1-\alpha\right) m$$

(11) $$u = -\frac{1}{1-\alpha} \left(w - p \right)$$

where w_i is the wage set by the *i* union; $\eta > 1$ is the degree of monopolistic competition and $\alpha \in \left(0,1\right)$ is the labor coefficient of the productions function, $w = \sigma w_i + \left(1-\sigma\right) w_j$ is the

[12] We simplify Jerger (2002) and Acocella et al. (2004) to which we refer for more details.

[13] Equation (9) refers to the (micro) disaggregate equilibrium conditions whereas Equation (9) and (11) to the (macro) aggregate ones. More in detail, equation (9) is the union's employment function stemming from a traditional labor demand derived by real profit maximization assuming a Blanchard and Kiyotaki's (1987) firm's demand. Equation (9) and (11) are the price level and unemployment rate.

average wage, the general level of prices is defined according to the Dixit-Stigliz tradition as $p = \int_0^1 p_{ij} dj$. After some manipulations, the model reduced form turns out to be:

$$(12) \qquad \begin{pmatrix} p \\ u \\ u_i \end{pmatrix} = \begin{pmatrix} 1-\alpha & \sigma\alpha & (1-\sigma)\alpha \\ -1 & \sigma & (1-\sigma) \\ -1 & \frac{\eta-\alpha\sigma(\eta-1)}{\alpha+\eta(1-\alpha)} & -\frac{\alpha(1-\sigma)(\eta-1)}{\alpha+\eta(1-\alpha)} \end{pmatrix} \begin{pmatrix} m \\ w_i \\ w_{-i} \end{pmatrix}$$

where w_{-i} is the average wage of the unions different from the *i-th*. Notice that the three target variables are independent.

By solving the model the Nash equilibrium is:

$$(13) \qquad p = \frac{(1-\alpha\sigma)(\eta-\alpha(\eta-1))(\alpha-\phi+\alpha\phi)}{(\eta-\alpha\sigma(\eta-1))(1+\phi)} b_1 \geq 0$$

$$(14) \qquad u = \frac{(1-\alpha\sigma)(\eta-\alpha(\eta-1))}{\eta-\alpha\sigma(\eta-1)} b_1 > 0 .$$

where $\phi = \frac{\alpha(1-\alpha)\beta-1}{(1-\alpha)^2\beta+1}$. Commitment yields the same value for unemployment with zero inflation. By contrast, under Discretion (unions' leadership), equilibrium implies:

$$(15) \qquad p = \frac{(1-(\alpha-\phi+\alpha\phi)\sigma)(\alpha-\phi+\alpha\phi)(\alpha+(1-\alpha)\eta)}{\{\eta(1-\sigma(\alpha-\phi+\alpha\phi))+\alpha\sigma(1+\phi)\}(1+\phi)} b_1 \geq 0$$

$$(16) \qquad u = \frac{(1-(\alpha-\phi+\alpha\phi)\sigma)(\alpha+(1-\alpha)\eta)}{\eta(1-(\alpha-\phi+\alpha\phi)\sigma)+\alpha\sigma(1+\phi)} b_1 > 0 .$$

In order to evaluate possible monetary policy ineffectiveness, notice that ϕ is the only parameter containing central bank's preference.

Let us analyze the above results in Tinbergen's terms. First of all, we investigate the government's problem. The sub-system formed by the first two rows of equation (12) is controllable by the central bank only in two cases: if $\beta = 0$ (i.e. $\Theta_G = \{n\}$) or $\beta = +\infty$ (i.e.

$\Theta_G = \{p\}$); otherwise $\Theta_G = \varnothing$. Regarding a representative union, for convenience, after manipulations, we can rewrite its preference function as

(17) $$U_i = -\frac{1}{2}u_i^2 + \frac{\gamma\left[\alpha + \eta(1-\alpha)\right]}{\eta}u_i - \frac{\gamma}{\eta}u$$

Each union has one instrument and two target variables. Thus if γ is finite and different from zero, $\Theta_P = \varnothing$. However, not surprisingly, even if system (12) is not controllable in Tinbergen's terms by the representative union,[14] we can claim that the model implies neutrality in the Nash equilibrium, because the LQ nature of equation (17) as from Theorem 3, and non-neutrality under Discretion as equations (14) and (16) confirm. As a result, if β and b_1 are different from zero, neutrality does not emerge unless $\frac{\alpha+\eta(1-\alpha)}{\eta} = 0$. In fact, for $\eta \to +\infty$ (perfect competition), equation (17) becomes $U_i = -\frac{1}{2}u_i^2$ and standard results discussed above arise. In this case $u_i = u$ and $\Theta_P = \{u_i\}$.

4.3 Fiscal and monetary interactions

It is finally useful to discuss a different application by considering a class of policy games between the government and the central bank recently introduced by Dixit and Lambertini (2001, 2003a, 2003b).

A game of this kind can be briefly described as follows. The policymakers aim to stabilize output and inflation around their desired targets after a shock. Formally, they minimize quadratic preferences constrained by an aggregate demand equation and by an inflation adjustment equation. Their instruments affect the economy in the reduced form linearly.

Hence both players face two non-controllable problems and no policy is ineffective since the intersection of their controllable sets is empty (see Theorem 1: part *i*). However, it is easy to verify that if the central bank cares about inflation only, fiscal policy becomes neutral with respect to inflation stabilization since inflation is in the central bank's controllable set (see

[14] I.e. union *i*'s equilibrium unemployment is not zero.

Theorem 1: part *ii*).[15] If both players only care about the same targets, the equilibrium does not exist unless they share the same target values (see Theorem 1: part *i*). It is also easy to verify the claims of Theorems 2 and 3 about the LQ-preferences by modifying the original model appropriately.

5. Concluding remarks

This paper generalizes the classical theory of economic policy to the more recent strategic approach of policy games. We have shown how a revised version of Tinbergen's traditional theory can deal with policy neutrality problems. Also we have shown how the theory can be profitably used to deal with equilibrium existence conditions in policy games.

In a game theoretical perspective, controllability and neutrality are dual concepts. Controllability for one player implies neutrality for the others. Of course, the static controllability of Tinbergen's approach must be reinterpreted in a strategic context. Once this has been done, neutrality merely becomes an instrument/target accounting problem.

We have shown that controllability of a subset of variables by a player always implies neutrality of all the other's policies for the same subset (if an equilibrium exists), and that the reverse does not hold. In particular, by generalizing our investigation to the case of constant marginal rates of substitution between targets, we have shown that neutrality can emerge if the counting rule is violated. However, this kind of neutrality has a different nature since it does not imply the realization of the player's *optimum optimorum* (which does not exist because of the non-satiation). Hence, it leaves room for different arrangements like cooperation or policy leadership, which could be associated with lower losses and non-neutrality.

The main limiting assumptions implicitly or explicitly used in this paper can be removed, at least in principle.

[15] Similar results hold each time only one player cares no more than one target.

Appendix A – Controllable (target) set

In order to derive the controllable set, we assume[16] $m > n$ and imagine that, by using a permutation matrix $P \in \mathbb{R}^{n \times n}$, the original system $Ay = Bu + K$ can be rewritten as:

(A.1) $Py = PCu + PA^{-1}K$

with $PC = PA^{-1}B = \begin{bmatrix} M_1 & 0 \\ 0 & M_0 \end{bmatrix}$, where M_1 is a full-rank square matrix and M_0 is a rectangular matrix.

If it is possible to find a matrix M_1 and rewrite $Ay = Bu + K$ in the form of (A.1), the original system nests two independent sub-systems, of which one is controllable. Thus the decision-maker can set the values of the first $rank[M_1]$ targets of the system (A.1) independently of the problem of setting the last $n - rank[M_0]$. We can define the set of the controllable targets as the set of the first $rank[M_1]$. Of course, the same target variables can be controlled also in the original system.

A column set for *PC* in system (A.1) can be easily derived as:

(A.2) $col(PC) = \begin{bmatrix} I & 0 \\ 0 & col(M_0) \end{bmatrix} \in \mathbb{R}^{n \times m}$

where $I \in \mathbb{R}^{rank(M_1) \times rank(M_1)}$ is an identity matrix. Clearly an entry of *y* is in the controllable set if and only if it is one of the first $rank[M_1]$ of *Py* or equivalently y_i is in the controllable set if $Pe(i) \in \text{span}[col(PC)]$, i.e. $e(i) \in \text{span}[col(C)]$ as in definition 3.

Appendix B – A compact proof of the theorems

This appendix gives a proof of the theorems and generalizes our results to an *n*-player context. For the sake of brevity, we focus on the Nash case without sub-controllable targets. Then we discuss generalizations.[17] We consider an economy where *n* players interact (*N* is their set). Each player minimizes a LQ-criterion, U_i, defined over $t(i) = q(i) + l(i)$ variables, where $l(i)$ variables enter only linearly:

(B.1) $$U_i = \tfrac{1}{2}\left(y_i - \overline{y}_i\right)' Q_i \left(y_i - \overline{y}_i\right) + y_i{}' R_i + \tilde{y}_i{}' L_i \qquad \forall i \in N$$

where $y_i \in \mathbb{R}^{q(i)}$ ($\tilde{y}_i \in \mathbb{R}^{l(i)}$) is a vector of target second (first) order variables; $\overline{y}_i \in \mathbb{R}^{q(i)}$ is a vector of target values; Q_i is a full-rank diagonal matrix, R_i and L_i are vectors. All the control (target) vectors are sub-vectors of $u \in \mathbb{R}^M$ ($y \in \mathbb{R}^K$). Each player *i* controls a sub-vector of *u*, i.e. $u_i \in \mathbb{R}^{m(i)}$. Of course, $\sum_{i \in N} m(i) = M$. For the sake of simplicity, we also assume $m(i) \le t(i)$. Players can share some target variables: $\sum_{i \in N} t(i) \ge K$.

The *K* target variables are linked together by the following linear equation system:

(B.2) $$y = Au + F .$$

We assume that the column set of *A* is the identity matrix, which means that system (B.2) cannot be reduced to many independent sub-systems.[18]

From equation (B.2), we can extract y_i and $\tilde{y}_i$, obtaining the relevant sub-system for player *i*:

(B.3) $$\begin{bmatrix} y_i \\ \tilde{y}_i \end{bmatrix} = \begin{bmatrix} C_i \\ \tilde{C}_i \end{bmatrix} u_i + \sum_{j \in N/i} \begin{bmatrix} E_{ij} \\ \tilde{E}_{ij} \end{bmatrix} u_j + F_i .$$

where $C_i \in \mathbb{R}^{q(i)\times m(i)}$, $\tilde{C}_i \in \mathbb{R}^{l(i)\times m(i)}$, $E_{ij} \in \mathbb{R}^{q(i)\times m(j)}$, $\tilde{E}_{ij} \in \mathbb{R}^{l(i)\times m(j)}$, $F_i \in \mathbb{R}^{t(i)}$ are parameters.

The equilibrium can be found by solving a set of *n* problems (i.e. minimizing equation (B.1) subject to (B.3) for each of the *n* players), which is the set of the reaction correspondences.

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[16] It trivial that for $m \le n$ all the targets are in the controllable set since the decision maker can control the system in the Tinbergen's terms.

[17] More formal proofs are contained in the working paper version of this article.

[18] The extension of the proof to sub-controllability is trivial.

With reference to this representation of the policy game, we can state the following theorems, which generalize Theorems 1, 2 and 3 to *n* players.

Theorem A (existence): *The equilibrium of the policy game described does not exist if and only if (a) for at least one player:* $L_i \neq 0$ *and* $t(i) = m(i)$*; or (b) the intersection of the players' controllable sets is not empty.*

Theorem B (ineffectiveness): *If an equilibrium exists, player i's policy is ineffective for all the target variables contained in the union of all players' controllable sets a) under the Nash equilibrium or player i's leadership and b) under player j's (j≠ i) leadership if the intersection between player j's controllable set is empty or if* $L_i = 0$.

Proof of Theorem A. The optimization problem of each player implies the following *n* focs:

(B.4) $$\frac{\partial U_i}{\partial u_i} = C_i' Q_i C_i u_i + C_i' Q_i \sum_{j \in N/i} E_{ij} u_j + C_i' \left(R_i + L_i - Q_i \bar{y}_i + Q_i F_i \right) = 0 \qquad \forall i \in N .$$

Grouping them together, the Nash equilibrium results from the solution of the system below:

(B.5) $\Delta u = -\Gamma$ with $\Delta \in \mathbb{R}^{M \times M}$ and $\Gamma \in \mathbb{R}^{M}$.

where $$\Delta = \begin{bmatrix} C_1' Q_1 C_1 & C_1' Q_1 E_{12} & \dots & C_1' Q_1 E_{1n} \\ C_2' Q_2 E_{21} & C_2' Q_2 C_2 & \dots & C_2' Q_2 E_{2n} \\ \dots & \dots & \dots & \dots \\ C_n' Q_n E_{n1} & C_n' Q_n E_{n2} & \dots & C_n' Q_n C_n \end{bmatrix} \text{ and } \Gamma = \begin{bmatrix} C_1' \left[R_1 + L_1 + Q_1 \left(F_1 - \bar{y}_1 \right) \right] \\ C_2' \left[R_2 + L_2 + Q_2 \left(F_2 - \bar{y}_2 \right) \right] \\ \dots \\ C_n' \left[R_n + L_n + Q_n \left(F_n - \bar{y}_n \right) \right] \end{bmatrix}.$$

A necessary and sufficient condition for a solution existence is that the inverse of Δ exists.

Now, let us consider the case of player 1 without loss of generality. Re-partitioning Δ as:

(B.6) $$\Delta = \begin{bmatrix} \left[C_1' Q_1 C_1 \right] & \left[C_1' Q_1 E_{12} \quad \dots \quad C_1' Q_1 E_{1n} \right] \\ \begin{bmatrix} C_2' Q_2 E_{21} \\ \dots \\ C_n' Q_n E_{n1} \end{bmatrix} & \begin{bmatrix} C_2' Q_2 C_2 & \dots & C_2' Q_2 E_{2n} \\ \dots & \dots & \dots \\ C_1' Q_1 E_{12} & \dots & C_n' Q_n C_n \end{bmatrix} \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$

By a well-known formula, we have that $\det(\Delta) = \det(P_{11})\det(P_{22} - P_{21}P_{11}^{-1}P_{12})$. By noticing that if $m(1) = t(1) > q(1)$, $\det(C_1'Q_1C_1) = 0$,[19] $\det(\Delta) = 0$. In other words, if player 1 has a number of instruments equal to the number of its targets and at least one target enters its criterion linearly, the Nash equilibrium does not exist.

Let us consider the second part of the theorem focusing on the first two players without any loss of generality. In order to compute its determinant, matrix Δ can be re-partitioned as:

(B.7) $$\Delta = \begin{bmatrix} \begin{bmatrix} C_1'Q_1C_1 & C_1'Q_1E_{12} \\ C_2'Q_2E_{21} & C_2'Q_2C_2 \end{bmatrix} & \begin{bmatrix} \dots & C_2'Q_2E_{2n} \\ \dots & \dots \end{bmatrix} \\ \begin{bmatrix} \dots & \dots \\ C_n'Q_nE_{n1} & C_n'Q_nE_{n2} \end{bmatrix} & \begin{bmatrix} \dots & \dots \\ \dots & C_n'Q_nC_n \end{bmatrix} \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$

Moreover, matrix P_{11} can be rewritten as the product of two partitioned square matrices:

(B.8) $$P_{11} = \begin{bmatrix} C_1'Q_1 & \varnothing_1 \\ C_2'Q_2 & \varnothing_2 \end{bmatrix} \begin{bmatrix} C_1 & E_{12} \\ E_{21} & C_2 \end{bmatrix} = \Gamma_1\Delta_1,$$

where $\varnothing_1 \in \mathbb{R}^{m(1)\times q(2)}$ and $\varnothing_2 \in \mathbb{R}^{m(2)\times q(1)}$ are zero (rectangular) matrices. By inspecting the dimensions of matrix Γ_1 and Δ_1 closely, it is easy to verify that these are rectangular matrices if $m(i) > q(i)$. By contrast, if $m(i) = q(i)$, the matrices are square and, therefore, $\det(\Gamma_1) = 0$. Hence Δ is singular and cannot be inverted. Thus, as claimed, if $m(i) = q(i)$ for at least two players sharing at least one target variable, the Nash equilibrium does not exist. ■

Proof of Theorem B. Focs in terms of quasi-reaction functions are:

(B.9) $$\frac{\partial U}{\partial u_i} = C_i'Q_i(y_i - \bar{y}_i) + C_i'R_i + \tilde{C}_i'L_i = 0 \qquad \forall i \in N$$

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[19] In such a case, Q_1 equals T_1T_1' where $T_1 = \left[\sqrt{Q_1} : \varnothing\right] \in \mathbb{R}^{q(1)\times t(1)}$, i.e. T_1 is obtained in two steps: first by considering a $q(1)\times q(1)$ matrix with the square roots of each element Q_1 and then by adding $l(i)$ columns of zeros. Thus $\det(C_1'Q_1C_1) = \det(C_1'T_1)\det(T_1'C_1)$ equals zero since it is easy to verify that all the elements of the last $l(i)$ columns of $C_1'T_1$ are zeros.

Equations (B.9) represent the optimal value of the target variables that assure the minimization of player *i*'s criterion, given the policy of the others. Thus, by definition, all the conditions (B.9) have to be mutually verified to ensure the Nash equilibrium. Formally, equations (B.9) map the vector of target variables into that of the desired target values.[20] If $m(i) = q(i)$, then condition (B.9) becomes:

(B.10) $y_i = \bar{y}_i - Q_i^{-1} R_i - \left(C_i' Q_i \right)^{-1} \tilde{C}_i' L_i$

If the Nash equilibrium exists, it is unique, because of the LQ-structure considered. Hence, if the equilibrium exists and $m(i) = q(i)$ for player *i*, the Nash equilibrium will satisfy equation (B.10) and any other player will not be able to affect the value of any of the variables in y_i. ■

<u>Proof extensions.</u> The above proofs hold for a) the Nash equilibrium; b) the case where there is no sub-controllability. Extensions to the Stackelberg cases are trivial. More in details, in Theorem A if $L_i \neq 0$ and $t(i) > m(i)$ a finite solution of player *i*'s problem does not exist independently of the strategic context and if a player is able to control some variable its strategy is independent of its and other players' ability to commit policy. The extension of Theorem B is also trivial since the proof is based on the reaction function of a player that controls its subsystem and it applies to both Nash and Stackelberg equilibrium (in such a case its reaction has to be verified), but not to the case of the Stackelberg leader that controls its system unless $L_i = 0$. In this case, in fact, the Stackelberg optimal policy is the same as in the Nash case, since the latter implies the achievement of the first best and thus the player has no interest in internalizing the followers' policies. Extensions to the sub-controllability cases are immediate as well since controllability (thus neutrality) and inconsistencies determining non-existence will emerge for a part of the original system in a very similar manner. ■

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[20] It is worth noticing that for each player condition (B.9) represents the dual problem of that described by equation (B.4). If the reaction function system (B.4) is over-determined, the quasi-reaction function system is under-determined; and vice versa.

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