Price Indeterminacy Reinvented:

Pegging Interest Rates While Targeting Prices, Inflation, or Nominal Income

By David Eagle

Abstract

Contrary to Sargent and Wallace (1975), a central bank's use of an interest-rate instrument does determine prices when the central bank pursues either a short-term or long-term price target. However, in order for a central bank's pursuit of a long-term price target to be credible, the public still needs something like a Taylor or McCallum-Woodford rule. The use of an interest-rate instrument also determines prices when the central bank targets nominal income in either the short-term or long-term. However, if the central bank targets interest rates in the short term with a long-term inflation target, then prices are indeterminate.

Keywords: monetary economics, central banking, price targeting, nominal income targeting, inflation targeting, price indeterminacy

David M. Eagle*
Associate Professor of Finance
Department of Management, MS#3
College of Business Administration
Eastern Washington University
668 N. Riverpoint Blvd., Suite A
Spokane, Washington 99202-1660
Phone: (509) 358-2245
Fax: (509) 358-2267
email: deagle@ewu.edu

April 2004

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I. Introduction

This paper revisits the issue of price indeterminacy of pegging the interest rate as discussed by Sargent and Wallace (1975). Sargent and Wallace applied rational expectations to what McCallum (1981) called a "rather orthodox IS-LM-NRPC model".[1] Under short-term price targeting, they concluded that if a central bank uses the interest rate as its instrument, prices are indeterminate. Based on Eagle and Murff's (2004) revision of the procedures to solve expectational difference equations, this paper finds otherwise. Under the same model Sargent and Wallace analyzed and under their same assumptions, this paper finds that a central bank using the interest-rate as its instrument does determine prices when the central bank targets prices each period as assumed by Sargent and Wallace.

Crucial to this paper's different conclusion is in the logic of infinity. Eagle and Murff (2004) argue against the rational expectations precedent of assuming the solution is bounded. In particular, they review the history of that precedent and find the foundations of that precedent to be less than rigorous and they show several finance and infinitely-repeated-game examples where that precedent leads to erroneous conclusions. Instead, they recommend that one solve expectational difference equations by determining the terminal condition in a model with a finite horizon and then taking the limit of that terminal condition as the horizon goes to infinity. Doing that in the model analyzed by Sargent and Wallace forces the central bank in the last period to

[1] The NRPC stands for "Natural-Rate Phillips Curve."

use money as its instrument as no interest rate exists in that last period. This does lead to price determination even though the interest rate is used as the instrument for all the preceding periods.

However, to some extent, this paper's different conclusion is just a theoretical technicality. While the expected future prices are determined in the model, the public's confidence in those expected future prices may diminish to zero as the horizon goes to infinity unless the pegging of the interest rate is combined with some policy such as a McCallum-Woodford rule or a Taylor rule. Thus, the basic policy implications are essentially the same as in the previous literature that was based on the flawed rational expectations precedent of solving expectational difference equations.

This paper does, nevertheless, bring new realizations relevant to policy. In particular, this paper finds that short-term interest-rate targeting combined with long-term targeting of prices or nominal income does determine prices. However, short-term interest-rate targeting combined with long-term inflation targeting leads to price indeterminacy even under Eagle and Murff's revised procedures for solving expectational difference equations. This result provides support for central banks targeting prices or nominal income in the long term rather than inflation.

In an attempt to avoid confusion, this paper uses the word "target" to represent either a short-term or long-term measurable objective that the central bank pursues. A target in this paper may differ from the term "instrument", which refers to what the central bank uses to achieve its target. While Sargent and Wallace only looked at short-term price targeting, this paper analyzes price determinacy under price targeting, nominal-income targeting, and inflation targeting. In addition to considering these as short-term targets, this paper also considers scenarios where these goals are the longer-term targets with the short-term target being the interest rate itself. Therefore, this paper uses the term "interest-rate targeting" to refer to when the short-term target is the interest rate; the long-term objective may then be to target prices, nominal income, or inflation. However, if the central bank's short-term objective is to target prices, nominal income, or inflation, this paper uses the term "pegging the interest rate" not the term "targeting the interest rate."

The next section, section II, presents the model analyzed by Sargent and Wallace and shows that prices are determined when the central bank targets prices in the short term using the interest rate as its instrument. Section III discusses how the public's confidence in the expected future prices may diminish to zero unless the interest-rate pegging is combined with a sufficiently strong McCallum-Woodford or Taylor rule. Section IV shows that pegging the interest rate also determines prices when the central bank's short-term target is nominal income or inflation. Section V looks at price determinacy when a central bank targets interest rates in the short run and in the long-run targets prices, nominal income, or inflation. Section VI summarizes and reflects on the implications of this paper's results.

II. The IS-LM-NRPC Model and Price Determinacy of Pegging the Interest Rate

Sargent and Wallace present five equations for the IS-LM-NRPC model they analyze. However, we need only concern ourselves with the following three:

$$y_t = a_1 k_{t-1} + a_2 (p_t - E_{t-1}[p_t]) + u_{1t} \tag{1}$$

$$y_t = b_1 k_{t-1} - b_2 \left(i_t - (E_{t-1}[p_{t+1}] - E_{t-1}[p_t]\right) + \vec{b}_3 \vec{Z}_t + u_{2t} \tag{2}$$

$$m_t = p_t + c_1 y_t - c_2 i_t + u_{3t} \tag{3}$$

I have rewritten these equations so all coefficients are positive, except $\vec{b}_3$, which is a vector of unspecified coefficients.[2] "Here y_t, p_t, and m_t are the natural logarithms of output, the price level, and the money supply, respectively" (Sargent and Wallace, 2003, p. 243). I use i_t to represent the nominal interest rate (not its logarithm). The variables u_{1t}, u_{2t}, and u_{3t} are stochastic exogenous terms that need not have zero means. The variable k_t represents productivity, which is determined by Sargent and Wallace's equation (4), which I do not reproduce here. All other exogenous variables are represented in the vector variable $\vec{Z}_t$.

To simplify these equations, define $\bar{y}_t \equiv a_1 k_{t-1} + E_{t-1}[u_{1t}]$, $\varepsilon_{1t} \equiv u_{1t} - E[u_{1t}]$,

$\bar{r}_t \equiv \dfrac{b_1 k_{t-1} - \bar{y}_t + \vec{b}_3 E_{t-1}[\vec{Z}_t] + E_{t-1}[u_{2t}]}{b_2}$, and $\varepsilon_{2t} \equiv \vec{b}_3\left(\vec{Z}_t - E_{t-1}[\vec{Z}_t]\right) + u_{2t} - E_{t-1}[u_{2t}]$. We can then rewrite equations (2) and (1) respectively as:

$$y_t = \bar{y}_t - b_2\left(i_t - \bar{r} - \left(E_{t-1}[p_{t+1}] - E_{t-1}[p_t]\right)_t\right) + \varepsilon_{2t} \qquad (4)$$

$$y_t = \bar{y}_t + a_2(p_t - E_{t-1}[p_t]) + \varepsilon_{1t} \qquad (5)$$

where the error terms ε_{1t} and ε_{2t} have zero expected values.

While the points made in this paper could probably be made in terms of (3), (4), and (5) without any modification and by referring to equation (3) as the money demand function, I as the author cannot do so without continuing some logical errors other economists have made previously. Appendix B discusses these errors, which include theoretical identification errors and a fallacy of confusing velocity and the money demand function.

Note that (4) is supposed to be the aggregate-demand function. However, by using y_t to represent both real aggregate demand and real aggregate supply, equations (3), and (4) are

[2] Also, here I used rationally expected values for all unknown values. I have also taken the liberty to represent the nominal interest rate as i_t instead of r_t.

already a mixture of equilibrium conditions with structural equations and are therefore already reduced forms. As the appendix discusses, economists can make theoretical identification errors when treating reduced forms as though they are structural equations. Therefore, I consider it important to separate out equilibrium conditions from these equations. Also, separating out the equilibrium conditions will make it easier later in this paper to analyze nominal-income targeting.

Let y_t represent the natural logarithm of real aggregate supply, and let n_t represent the natural logarithm of nominal aggregate demand. The equilibrium of the goods market is determined by the interaction of nominal aggregate demand and aggregate supply. In particular, the equilibrium price level should equal nominal aggregate demand divided by aggregate supply. In logarithmic terms, this means that $p_t = n_t - y_t$. Equation (4) is supposed to be the aggregate demand curve. Therefore, to separate the goods market equilibrium condition from (4), we need to replace y_t with $n_t - p_t$. Doing so gives us:

$$n_t = p_t + \bar{y}_t - b_2\left(i_t - \bar{r}_t - \left(E_{t-1}[p_{t+1}] - E_{t-1}[p_t]\right)\right) + \varepsilon_{2t} \qquad (6)$$

Next we need to separate out the goods market equilibrium condition from (3). How we do that depends whether we consider (3) to represent the money demand function or the structural velocity function. If we look at (3) as the money demand function consistent with the microeconomic definition of a demand function, then we would leave (3) as it stands as being a function of aggregate supply. However, that would imply a very complex structural velocity function that would be hard to justify. Instead, even though many economists refer to (3) as the money demand function, I interpret it to be the structural velocity function, which is the relationship between money and nominal aggregate demand. The structural velocity function differs from the inverse of the money demand function as explained in Appendix B. Using that interpretation, I separate (3) from the goods market equilibrium condition by rewriting it as:

$$n_t = m_t + (1 - c_1) y_t + c_2 i_t - u_{3t} \quad (7)$$

where $(1 - c_1) y_t + c_2 i_t - u_{3t}$ is the natural logarithm of the structural velocity of money.[3]

Given that we have extracted the goods market equilibrium condition from equations (3) and (4), we now need to include in our model the goods market equilibrium condition:

$$p_t = n_t - y_t \quad (8)$$

Equation (8) means that the goods market instantaneously moves to equilibrium. While we maintain that assumption, some interesting extensions of the model would include replacing (8) with $p_{t+1} = n_t - y_t$ or with $p_t = (1-\alpha) p_{t-1} + \alpha(n_t - y_t)$ for some α between 0 and 1. For these extensions, the separating out the equilibrating condition from the aggregate demand and structural velocity functions is clearly important.

One of the advantages of having separated out the goods market equilibrium condition from (3) and (4) is that we can see more clearly how prices are determined in the model. The price level is determined by equation (8), the equilibrium condition between nominal aggregate demand and real aggregate supply. Some literature, e.g., Woodford (2003), has lost sight of the importance of nominal aggregate demand in determining the price level.

The model with which we are working consists of equations (5), (6), (7), and (8). Also, included in the model are the definitional equations for $\bar{r}_t$, $\bar{y}_t$, ε_{1t}, and ε_{2t} and Sargent and Wallace's equations (4) and (5) which explain the evolution of variables affecting $\bar{r}_t$, and $\bar{y}_t$.

[3] If we defined v_t to be the natural logarithm of the structural velocity, then $n_t=m_t+v_t$. If we do interpret (3) as the inverse of the structural velocity function, then an issue exists as to whether the term $(1-c_1) y_t$ before the separating out of the equilibrium condition refers to aggregate supply or aggregate demand or some combination of

This is the same model as used by Sargent and Wallace, except I am being more precise by separating the goods-market equilibrium condition from the aggregate demand equation and the structural velocity equation.

Please note that, while equations (5), (6), (7), and (8) are structural equations of the model, they still imply the reduced-form equations (3) and (4), which we may therefore reference when convenient to do so.

By taking expectations of (4) and (5) given the information set at time t-1, we obtain $E_{t-1}[y_t] = \bar{y}_t - b_2\left((E_{t-1}[i_t] - \bar{r}_t - (E_{t-1}[p_{t+1}] - E_{t-1}[p_t])\right)$ and $E_{t-1}[y_t] = \bar{y}_t$, which together imply:

$$E_{t-1}[i_t] = \bar{r}_t + E_{t-1}[p_{t+1}] - E_{t-1}[p_t], \tag{9}$$

Equation (9) is basically a Fisher equation and is equivalent to Sargent and Wallace's equation (26).[4] As did Sargent and Wallace, we assume homogenous expectations which means that the bank is transparent in how it sets the interest rate and that the public and the central bank have the same expectations. Furthermore, we assume the central bank is transparent in how it sets the interest rate. Since the central bank pegs the interest rate based on its (and the public's) expectations at time t-1, the public knows at time t-1 what interest rate the central bank will peg at time t. Let $\tilde{i}_t$ be the interest rate the central bank sets at time t. Then $E_{t-1}[i_t] = \tilde{i}_t$. Therefore, substituting (9) for i_t in (4) and simplifying gives:

$$y_t = \bar{y}_t + \varepsilon_{2t} \tag{10}$$

Substituting this into (5) and solving for p_t gives:

$$p_t = E_{t-1}[p_t] + \frac{\varepsilon_{2t} - \varepsilon_{1t}}{a_2} \tag{11}$$

both. To be honest, I chose this term to refer to aggregate supply because I thought it would be the simplest assumption with which to work.

This shows that if we can determine $E_{t-1}[p_t]$, then p_t will also be determined.

Sargent and Wallace assumed that the central bank tried to minimize a loss function of the weighted squared deviations of output and prices from their targets. Since Sargent and Wallace show that monetary policy under rational expectations cannot affect expected output, the central bank's only remaining goal is to minimize the squared deviations of prices from the price targets. While they assumed one constant price target regardless of the time period, this paper allows the price targets to vary over time. Let p_t^* be the central bank's targeted price level for time t, which is known by the public. Therefore, under Sargent and Wallace's assumptions, the central bank would try to minimize $E_0\left[\sum_{t=0}^{\infty}\delta^{t-1}\left(p_t - p_t^*\right)^2\right]$ where $0 < \delta < 1$. This loss function would create some complex issues for us to deal that are mostly just distractions from what we are trying to do. We can simplify our task considerably by replacing this loss function with the following similar loss function:

$$\sum_{t=0}^{\infty}\delta^{t-1}\left(E_{t-1}[p_t] - p_t^*\right)^2 \qquad (12)$$

By changing the form of this loss function, some may say that I have "cheated" by changing an assumption made by Sargent and Wallace. However, all I am doing is changing an assumption slightly to be consistent with Sargent and Wallace's statement on page 249, in reference to their equations (24) and (25), where they stated that the value of $E_{t-1}[p_t]$ that minimizes the central bank's loss function equals the targeted price level. While that is true for the loss function (11), it is not necessarily true for the loss function Sargent and Wallace actually assumed. This is a technicality that the Appendix A discusses further.

[4] This is the equivalent of Sargent and Wallace's (26), except their (26) had a minor error. The last term in their (26)

The central bank minimizes (11) by choosing its policy instrument so that $E_{t-1}[p_t] = p_t^*$. Having rational expectations, the public knows that $E_{t-1}[p_t] = p_t^*$. Since $E_t[p_{t+1}] = p_{t+1}^*$, $E_{t-1}[p_{t+1}] = E_{t-1}[E_t[p_{t+1}]] = E_{t-1}[p_{t+1}^*] = p_{t+1}^*$. Therefore, we do not need to solve any difference equations to obtain values for $E_{t-1}[p_t]$ and $E_{t-1}[p_{t+1}]$. All we have to do is figure out what interest rates the central bank needs to peg to get $E_{t-1}[p_t] = p_t^*$ for all t. This is the logic behind Proposition 1 below:

Proposition 1: If the central bank follows interest-rate targeting, then the only rational expectations equilibrium where the central bank achieves its target $E_{t-1}[p_t] = p_t^*$ for all time t is where the following conditions hold:

(a) the central bank sets the interest rate at each time t to equal:

$$i_t = \bar{r}_t + p_{t+1}^* - p_t^* \tag{13}$$

(b) the price level at each time t equals:

$$p_t = p_t^* + \frac{\varepsilon_{2t} - \varepsilon_{1t}}{a_2} \tag{14}$$

(c) $E_{t-1}[p_t] = p_t^*$ for all t, and

(d) the money supply for each time t equals:

$$m_t = p_t^* + \frac{\varepsilon_{2t} - \varepsilon_{1t}}{a_2} + c_1(\bar{y}_t + \varepsilon_{2t}) - c_2(\bar{r}_t + p_{t+1}^* - p_t^*) + u_{3t} \tag{15}$$

Proof: Replacing p_t^* for $E_{t-1}[p_t]$ and p_{t+1}^* for $E_{t-1}[p_{t+1}]$ in (9) gives (13), which proves part (a). Substituting p_t^* for $E_{t-1}[p_t]$ in (11) gives (14), which proves point (b). Taking expectations of (14) based on the information set at time t-1 gives $E_{t-1}[p_t] = p_t^*$ for all t, which proves point (c). Now, into (3), substitute (14) for p_t, (10) for y_t, and (13) for i_t to get (15) which proves point (d). Q.E.D.

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should have been $E_{t-1}[b_3 Z_t - u_{1t} + u_{2t}]$.

While proposition 1 states that the only rational expectations equilibrium that is consistent with the central bank achieving its objective of $E_{t-1}[p_t] = p_t^*$ for all t, an issue of uniqueness remains to be proven. We need to determine whether other rational expectations equilibria where $E_{t-1}[p_t] \neq p_t^*$ for some t exist that are consistent with interest rates set by (13).

To address this issue of uniqueness, replace i_t in (9) and solve forward to get:

$$E_{t-1}[p_t] = E_{t-1}[p_T] - \sum_{j=0}^{T-t-1} E_{t-1}[\tilde{i}_{t+j} - \bar{r}_{t+j}] \qquad (16)$$

Equation (16) is essentially the same as Sargent and Wallace's equation (27), where they concluded that they could not solve for the expected price levels and therefore concluded that prices were indeterminate. Later, Sargent (1979) established a precedent of assuming the solution is bounded when solving forward expectational difference equations. However, Eagle and Murff (2004) show that the foundation for this precedent is less than rigorous and present several finance and infinitely-repeated-game examples where this precedent leads to incorrect solutions. They suggest that we follow Sargent's (1979, pp. 195-200) example of assuming a version of the model with a finite horizon to determine the appropriate terminal condition and then taking the limit of that condition as the horizon goes to infinity. This paper follows that approach.

Assume the last period of the economy is period T. In period T, no one can borrow or lend funds and there is no interest rate because there is no next period for the loans to be settled. With no interest rate, the central bank has no choice but to use the money supply as its instrument at time T. With no interest rate to affect velocity, the structural velocity function at time T must be:

$$n_T = m_T + (1 - \tilde{c}_1) y_T - u_{3T} \qquad (17)$$

where $\tilde{c}_1$ is the coefficient on y_T which could be a different parameter from c_1. To achieve its objective of obtaining $E_{T-1}[p_T] = p_T^*$, the central bank would set $m_T = p_T^* + \tilde{c}_1 \bar{y}_T + E_{T-1}[u_{3T}]$. Substituting this value for m_T into (17) after taking expectations given the information set at time T-1 gives $E_{T-1}[n_T] = \left(p_T^* + \tilde{c}_1 \bar{y}_T + E_{T-1}[u_{3T}]\right) + (1-\tilde{c}_1)\bar{y}_T - E_{T-1}[u_{3T}]$, which means that $E_{T-1}[n_T] = p_T^* + \bar{y}_T$. Since $E_{T-1}[p_T] = E_{T-1}[n_T] - \bar{y}_T$ by (8), this implies that $E_{T-1}[p_T] = p_T^*$. Also, $E_{t-1}[p_T] = E_{t-1}\left[E_{T-1}[p_T]\right] = E_{t-1}[p_T^*] = p_T^*$.

Substitute p_T^* for $E_{t-1}[p_T]$ and (13) for $\tilde{i}_t$ in (16) to get

$E_{t-1}[p_t] = p_T^* - \sum_{j=0}^{T-t-1} E_{t-1}[\bar{r}_{t+j} + p_{t+j+1}^* - p_{t+j}^* - \bar{r}_{t+j}]$, which implies that $E_{t-1}[p_t] = p_t^*$. Since this is true for all t, we conclude that the expected prices are uniquely determined and hence prices are uniquely determined. When we let the economy's horizon T approach infinity, this uniqueness is maintained. This then establishes that the equilibrium described in Proposition 1 is unique.

III. The Public Confidence Issue

While following Eagle and Murff's (2004) precedent for solving expectational difference equations does lead to the conclusion that expected future prices and hence prices are determined, solving expectational difference equations says nothing about the public's confidence in those expected prices. This section discusses this confidence issue. Normally, one's confidence in an expected value decreases the further into the future that the random variable occurs. (Another way of saying this is that the size of a confidence interval becomes greater the further into the future the variable occurs.)

Applying the assumption of rational expectations to the public with respect to their price expectations means the public knows what the central bank is trying to do. If the public knows with certainty that the central bank will target prices when period T occurs, and that that price target will be p_T^*, then the public's confidence that $E_{t-1}[P_T] = p_T^*$ will remain the same regardless how far in the future period T occurs. However, while the public may be quite confident that the central bank is currently targeting prices, realistically they will be less confident that the central bank will target prices in the far distant future. Even the central bank itself may have some doubts that it will target prices in the far distant future when the central bank will be under a different administration. Furthermore, even if the central bank does target prices in the far distant future, both the public and the central bank's confidence in what that price target will be will likely diminish the further in the future for which that price target applies. Therefore, realistically, we should expect both the public and central bank's confidence that $E_{t-1}[P_T] = p_T^*$ will diminish as T goes to infinity.

From (16), we conclude that regardless of the value of T or j, $\frac{\partial E_{t-1}[p_t]}{\partial E_{t-1}[p_T]} = 1$ and $\frac{\partial E_{t-1}[p_t]}{\partial E_{t-1}[\tilde{i}_{t+j} - \bar{r}_{t+j}]} = -1$. These derivatives imply that the importance of the terms $E_{t-1}[P_T]$ and $E_{t-1}[\tilde{i}_{t+j} - \bar{r}_{t+j}]$ remain the same regardless the value of T or j. However, if the confidence in $E_{t-1}[P_T]$ and $E_{t-1}[\tilde{i}_{t+j} - \bar{r}_{t+j}]$ decrease the greater is T and the greater is j, then the public's confidence in $E_{t-1}[p_t]$ will decrease the greater is T. In the limit the public's confidence could decrease to zero as T goes to infinity. A zero confidence in $E_{t-1}[p_t]$ would correspond to an infinite variance of p_t conditional on the information set at time t-1. While some statistical distributions do result in finite means and infinite variances, the usefulness of the $E_{t-1}[p_t]$ when the variance is infinite may be questionable. If so, then the implications of Sargent and Wallace's original indeterminacy result may still apply even though in some technical sense, prices are determined when pegging the interest rate.

Instead of (13), suppose the central bank sets its interest rate to the following:

$$\tilde{i}_t = \bar{r}_t + p^*_{t+1} - p^*_t + \varphi(E_{t-1}[p_t] - p^*_t) \qquad (18)$$

where φ is a positive constant. Equation (18), which is similar to a McCallum-Woodford rule (See McCallum, 1981, and Woodford, 2003), states that the central bank will set the interest rate at a level higher (lower) than (13) when the public and central bank[5] expect this period's price level to be higher (lower) than the central bank's target.

Equating (9) to (18), subtracting $\bar{r}_t$ from both sides and adding $E_{t-1}[p_t]$ to both sides gives:

$$E_{t-1}[p_{t+1}] = p^*_{t+1} + (1+\varphi)(E_{t-1}[p_t] - p^*_t) \qquad (19)$$

Comment: $\bar{r}_t + E_{t-1}[p_{t+1}] - E_{t-1}$
$E_{t-1}[p_{t+1}] = E_{t-1}[p_t] + p^*_{t+1} -$
$E_{t-1}[p_{t+1}] = p^*_{t+1} + (1+\varphi)(E_{t-}$
$(1+\varphi)(E_{t-1}[p_t] - p^*_t) = E_{t-1}[$

Solving (19) forward gives:

$$E_{t-1}[p_t] = p^*_t + \frac{E_{t-1}[p_T] - p^*_T}{(1+\varphi)^{T-t}} \qquad (20)$$

Comment: $E_{t-1}[p_t] = p^*_t + \frac{E_{t-}}{}$

Following Eagle and Murff's (2004) revised procedures for solving expectational difference equations, we would once again have the central bank setting the money supply in period T to try to get the expected price level equal to its target. Taking the partial derivative of (20) with respect to $E_{t-1}[p_T]$ and then taking the limit as T goes to infinity gives $\lim_{T\to\infty} \frac{\partial E_{t-1}[p_t]}{\partial E_{t-1}[p_T]} = \lim_{T\to\infty} \frac{1}{(1+\varphi)^{T-t}} = 0$, which means that the weight on the terminal expected price decreases to zero as the horizon goes to zero.

Let $\sigma_{t-1}(p_T)$ represent the standard deviation at time t-1 of the price level at time T in the mind of the public. The public's confidence that $E_{t-1}[p_T] = p_T^*$ should be inversely related to $\sigma_{t-1}(p_T)$. It may very well be the case that $\sigma_{t-1}(p_T)$ increases as T increases. However, by (20) if φ is sufficiently large to cause $\lim_{T\to\infty} \frac{\sigma_{t-1}(p_T)}{(1+\varphi)^{T-t}} = 0$, then the McCallum-Woodford rule offsets the decrease in the public's confidence that $E_{t-1}[p_T] = p_T^*$ as T goes to infinity. Furthermore, the act of the central bank following (18) means that the central bank takes steps each period to control inflation rather than waiting until period T to undertake that control. From signaling theory, these steps should even further boost the public's confidence in the central bank.

Now instead of (18), suppose the central bank sets the interest rate according to the following equation:

$$\tilde{i}_t = \bar{r}_t + p_{t+1}^* - p_t^* + \gamma\left(E_{t-1}[\pi_t] - \pi_t^*\right) \tag{21}$$

where $\pi_t \equiv p_t - p_{t-1}$ and $\pi_t^* \equiv p_t^* - p_{t-1}^*$. This is similar to a Taylor rule (See Taylor, 1993). Equating (9) to (21), replacing $E_{t-1}[p_{t+1}] - E_{t-1}[p_t]$ with $E_{t-1}[\pi_{t+1}]$, replacing $p_{t+1}^* - p_t^*$ with π_{t+1}^*, and subtracting $\bar{r}_t$ from both sides gives:

$$E_{t-1}[\pi_{t+1}] = \pi_{t+1}^* + \gamma\left(E_{t-1}[\pi_t] - \pi_t^*\right)$$

Solving this forward gives:

$$E_{t-1}[\pi_t] = \pi_t^* + \frac{E_{t-1}[\pi_T] - \pi_T^*}{\gamma^{T-t}} \tag{22}$$

[5] As did Sargent and Wallace (1975), we assume homogenous expectations, which means that the public and the central bank have the same expectations.

At time T, the central bank would set the money supply so that $E_{T-1}[p_T] = p_T^*$, and it would set the interest rate at time T-1 according to (21), which determines $E_{T-2}[\pi_T]$ and hence $E_{t-1}[\pi_T]$. Also as before, the public's confidence in the value of $E_{t-1}[\pi_T]$ may diminish as T goes to infinity. Let $\sigma_{t-1}(\pi_T)$ be the standard deviation of inflation at time T in the public's mind. Where γ sufficiently exceeds one so that $\lim_{T\to\infty} \frac{\sigma_{t-1}(\pi_T)}{\gamma^{T-t}} = 0$, this Taylor rule offsets the public's diminishing confidence in $E_{t-1}[\pi_T]$ as T goes to infinity.

Pegging the interest rate determines prices in a technical sense when we revise Sargent and Wallace's (1975) analysis to reflect Eagle and Murff's (2004) revised procedures for solving expectational differences. However, when we take into account the public's confidence in the central bank, the policy implications are very similar to what the previous literature has discussed. The use of a McCallum-Woodford rule or a Taylor rule may offset this diminishing public confidence in the central bank in the long run resulting with greater confidence in expected prices in the near term.

IV. Price Determinacy Under Short-Term Nominal-Income and Inflation Targeting

The previous two sections discuss price determinacy when the central bank uses the interest rate as its monetary instrument as it pursues a short-term price target. Worldwide, however, central banks are moving toward inflation targeting. Also, Eagle and Domian (2003 and 2004) use Pareto-efficiency arguments to argue in favor of nominal-income targeting. This section sketches how the price-determinacy issue is affected by the central bank pursuing a short-term inflation or nominal-income target.

First consider inflation targeting. If the central bank tries to minimize

$E_0\left[\sum_{t=0}^{\infty}\delta^{t-1}\left(E_{t-1}[\pi_t]-\pi_t^*\right)^2\right]$, it will do so when $E_{t-1}[\pi_t]=\pi_t^*$ for all t. By (9), the central bank sets the interest rate equal to $\tilde{i}_t=\bar{r}_t+\pi_{t+1}^*$. Since $p_t=\pi_t+p_{t-1}$, $E_{t-1}[p_t]=\pi_t^*+p_{t-1}$, and therefore by (11) we get:

$$p_t = p_{t-1}+\pi_t^*+\frac{\varepsilon_{2t}-\varepsilon_{1t}}{a_2} \tag{23}$$

Expected prices and prices will be determined. However, once again something like a Taylor rule may be needed to maintain the public's confidence in the central bank. Such a Taylor rule would be:

$$\tilde{i}_t=\bar{r}_t+\pi_{t+1}^*+\gamma\left(E_{t-1}[\pi_t]-\pi_t^*\right) \tag{24}$$

for a γ sufficiently greater than 1.

Next consider nominal income targeting. Assume the central bank tries to minimize

$E_0\left[\sum_{t=0}^{\infty}\delta^{t-1}\left(E_{t-1}[n_t]-n_t^*\right)^2\right]$ where n_t^* represents the targeted level of nominal aggregate demand. The central bank minimizes its loss function when $E_{t-1}[n_t]=n_t^*$ for all t. Taking expectations of (8) given the information set at time t-1 gives $E_{t-1}[p_t]=n_t^*-\bar{y}_t$. By (9), the central bank sets the interest rate equal to $\tilde{i}_t=\bar{r}_t+\left(n_{t+1}^*-E_{t-1}[\bar{y}_{t+1}]\right)-\left(n_t^*-\bar{y}_t\right)$. Replacing $E_{t-1}[p_t]$ with $n_t^*-\bar{y}_t$ in (11) gives:

$$p_t=n_t^*-\bar{y}_t+\frac{\varepsilon_{2t}-\varepsilon_{1t}}{a_2}$$

A rule like a McCallum-Woodford rule may be needed to firm up the public's confidence in their expected prices. Below I present a McCallum-Woodford-like rule modified to work with nominal income targeting:

$$\tilde{i}_t = \bar{r}_t + \left(n^*_{t+1} - E_{t-1}[\bar{y}_{t+1}]\right) - \left(n^*_t - \bar{y}_t\right) + \varphi(E_{t-1}[n_t] - n^*_t) \qquad (25)$$

This states that the central bank will set the interest rate higher (lower) if the public's expectations of nominal aggregate demand exceeds (falls short of) the nominal-income target.

Setting (9) equal to (25), replacing $E_{t-1}[p_{t+1}]$ with $\left(E_{t-1}[n_{t+1}] - E_{t-1}[\bar{y}_{t+1}]\right)$, replacing $E_{t-1}[p_t]$ with $\left(E_{t-1}[n_t] - \bar{y}_t\right)$, and adding $E_{t-1}[\bar{y}_{t+1}] - \bar{y}_t - \bar{r}_t$ to both sides gives $E_{t+1}[n_{t+1}] - E_{t-1}[n_t] = n^*_{t+1} - n^*_t + \varphi\left(E_{t-1}[n_t] - n^*_t\right)$, which can be rewritten as:

$E_{t+1}[n_{t+1}] - n^*_{t+1} = (1+\varphi)\left(E_{t-1}[n_t] - n^*_t\right)$

Solving this forward gives:

$$E_{t-1}[n_t] - n^*_t = \frac{E_{t-1}[n_T] - n^*_T}{(1+\varphi)^{T-t}}$$

The central bank will set the money supply at time T. Let $\sigma_{t-1}(n_T)$ be the standard deviation of the public's expectation of n_T. If the parameter φ is set sufficiently greater than zero so that $\lim_{T\to\infty} \frac{\sigma_{t-1}(n_T)}{(1+\varphi)^{T-t}} = 0$, then the public should be relatively confident that $E_{t-1}[n_t] = n^*_t$.

Comment: $\bar{r}_t + (E_{t-1}[n_{t+1}] - E_{t-}$

$E_{t-1}[n_{t+1}] - E_{t-1}[n_t] = n^*_{t+1} - ($

$E_{t-1}[n_{t+1}] = n^*_{t+1} - (1+\varphi)n^*_t +$

$E_{t-1}[n_t] - n^*_t = \frac{E_{t-1}[n_{t+1}] - n^*_t}{(1+\varphi)}$

V. Price Determinacy Under Short-Term Interest-Rate Targeting

The central bank in the previous sections had short-term targets of the price level, inflation, or nominal income. However, in the real world, central banks rarely target these in the short term. Instead, in the short term, they target interest rates, reserves, or some other monetary instrument and then in the intermediate-term or long-term make adjustments to meet their long-term goals concerning prices, inflation, or nominal income. This section analyzes price determinacy when the central bank targets the interest rate in the short term and in the long run targets the price level, inflation, or nominal income. In this section, we no longer look at T as being the horizon of the economy. Rather assume T>2 and the central bank targets the interest rate for t=1,2,…,T-1. At time T, the central bank sets its monetary instrument so to meet its long-term goal. The central bank could do that using the money supply. It could also do it with the interest rate at time T set with a McCallum-Woodford rule or a Taylor rule.

First consider long-term price targeting. Then the central bank will choose its monetary instrument at time T so that $E_{t-1}[p_T] = p_T^*$.[6] Substituting this into (16) gives us that:

$$E_{t-1}[p_t] = p_T^* - \sum_{j=0}^{T-t-1} E_{t-1}[\tilde{i}_{t+j} - \bar{r}_{t+j}]$$

Substituting the above into (11) gives:

$$p_t = p_T^* - \sum_{j=0}^{T-t-1} E_{t-1}[\tilde{i}_{t+j} - \bar{r}_{t+j}] + \frac{\varepsilon_{2t} - \varepsilon_{1t}}{a_2} \qquad (26)$$

This means that prices are determined when the central bank targets interest rates for a short-term period followed by targeting prices at time T.

Next consider nominal income targeting in the long run. The central bank would set its monetary instrument at time T so that $E_{T-1}[n_T] = n_T^*$, which implies that $E_{T-1}[p_T] = n_T^* - \bar{y}_T$ by equation (8). Taking expectations of both sides conditional on the information set at time t-1 gives $E_{t-1}[p_T] = n_T^* - E_{t-1}[\bar{y}_T]$. Substituting this into (16) gives:

[6] In the real word, a central bank pursuing a long-term goal would not immediately force that goal to be realized, but would likely just start moving toward that goal. However, in this model, the expected output is unaffected if the central bank moves immediately rather than slowly to meet that goal. Therefore, we assume an immediate meeting of that goal and interpret that as meaning that central bank moves to that goal in the long run.

$$E_{t-1}[p_t] = n_T^* - E_{t-1}[\bar{y}_T] - \sum_{j=0}^{T-t-1} E_{t-1}[\tilde{i}_{t+j} - \bar{r}_{t+j}] \quad (27)$$

Substituting the above into (11) gives:

$$p_t = n_T^* - E_{t-1}[\bar{y}_T] - \sum_{j=0}^{T-t-1} E_{t-1}[\tilde{i}_{t+j} - \bar{r}_{t+j}] + \frac{\varepsilon_{2t} - \varepsilon_{1t}}{a_2} \quad (28)$$

This shows that when the central bank targets nominal income for some finite period of time followed by it targeting nominal income in period T, prices are determined.

Next, consider inflation targeting. At time T, the central bank will set its monetary instrument so that $E_{T-1}[\pi_T] = \pi_T^*$, which implies that $E_{t-1}[\pi_T] = \pi_T^*$. Since $\pi_T \equiv p_T - p_{T-1}$, this implies that $E_{t-1}[p_T] = \pi_T^* + E_{t-1}[p_{T-1}]$. Substituting this into (16) gives:

$$E_{t-1}[p_t] = \pi_T^* + E_{t-1}[p_{T-1}] - \sum_{j=0}^{T-t-1} E_{t-1}[\tilde{i}_{t+j} - \bar{r}_{t+j}] \quad (29)$$

If we apply (29) to t=T-1, we get:

$$E_{T-1}[p_{T-1}] = \pi_T^* + E_{T-1}[p_{T-1}] - E_{T-1}[\tilde{i}_{T-1}] - \bar{r}_{T-1} \quad (30)$$

This is the equation that would determine $E_{T-1}[p_{T-1}]$ if it were determined. However, since $E_{T-1}[p_{T-1}]$ is on both sides of the equation, they just cancel; resulting with $E_{T-1}[p_{T-1}]$ being undetermined. If $E_{T-1}[p_{T-1}]$ is undetermined then all $E_{t-1}[p_{T-1}]$ are undetermined, which implies by (29) that $E_{t-1}[p_t]$ is undetermined and hence prices are undetermined.

In conclusion short-term interest-rate targeting followed by inflation targeting at time T leads to price indeterminacy even according to Eagle and Murff's revised procedures for solving expectational difference equations. This results because if all the central bank cares about at time T is the inflation rate at time T, there are an uncountably infinite number of price sequences from time 1,2,…,T-1 that would be consistent with that targeted inflation rate at time T.

Another way to see why prices are indeterminate when the central bank targets interest rates in the short run and the inflation rate at time T is to count equations. If we apply (16) to time t+k and then take expectations conditional on the information set at time t-1, we get:

$$E_{t-1}[p_{t+k}] = E_{t-1}[p_T] - \sum_{j=0}^{T-t-k-1} E_{t-1}[\tilde{i}_{t+j+k} - \bar{r}_{t+j+k}] \qquad (31)$$

Equation (31) applies to k=0,1,2,…,T-t-1 , meaning there are T-t of these equations. With either price targeting or nominal-income targeting, $E_{t-1}[p_T]$ is determined so that the unknowns are $E_{t-1}[p_{t+k}]$ for k=0,1,…,T-t-1, meaning there are T-t unknowns. Since the number of equations equals the number of unknowns and each equation (31) expresses the unknown in terms of the known values, the prices are determined.

However, with inflation targeting $E_{t-1}[p_T] = \pi_T + E_{t-1}[p_{T-1}]$. No longer is $E_{t-1}[p_T]$ a function of known values. We can rewrite this equation as:

$$E_{t-1}[p_{T-1}] = E_{t-1}[p_T] - \pi_T \qquad (32)$$

Thus, we have T-t+1 unknowns, which are $E_{t-1}[p_{t+k}]$ for k=0,1,…,T-t. We also have T-t+1 equations, which consist of (32) and the equations (31) for k=0,1,…,T-t. Applying (31) to k=T-t-1 gives:

$$E_{t-1}[p_{T-1}] = E_{t-1}[p_T] - E_{t-1}[\tilde{i}_T] - \bar{r}_{t+j} \qquad (33)$$

For both (32) and (33), $\frac{\partial E_{t-1}[p_{T-1}]}{\partial E_{t-1}[p_T]} = 1$. Therefore, the Jacobian matrix of the functions that represent the equations is singular causing the prices to be indeterminate.

The Taylor rule (24) does not overcome this price indeterminacy because its correction factor is in terms of the inflation rate differing from the targeted rate of inflation. On the other hand, this is not an issue with the McCallum-Woodford rule since it is a rule consistent with price targeting not inflation targeting.

VI. Summary and Reflections

This paper finds that pegging the interest rate does determine prices when the central bank targets the price level, nominal income, or inflation in the short term as well as the long term. This conclusion differs from Sargent and Wallace (1975) because this paper follows Eagle and Murff's (2004) revised procedures for solving expectational difference equations. These procedures require us to consider the model with a finite horizon to determine the terminal condition and taking the limit of that terminal condition as the horizon goes to infinity. Since no interest-rate exists in the last period, the central bank must use the money supply in the last period to pursue its price target objective. That then determines $E_{t-1}[p_T]$ where T is the last period, which then determines $E_{t-1}[p_t]$ and p_t in general.

Because the price levels are so determined by this terminal price level, the public's confidence in the terminal price level is likely to diminish as T goes to infinity, possibly to the extent to make the value $E_{t-1}[p_T]$ meaningless. However, the central bank may be able to offset this diminishing public confidence by following a McCallum-Woodford rule or a Taylor rule. From a policy standpoint, this leads us to about the same conclusion as the previous literature on price indeterminacy concerning interest-rate targeting, except that the logic behind the conclusion differs.

When the central bank targets interest rates for a finite period followed by it targeting the price level or nominal-income targeting thereafter, prices are determined. On the other hand, when the central bank targets interest rates for a finite period followed by it targeting inflation thereafter, prices are indeterminate. We infer from these results that short-term interest-rate targeting combined with intermediate-term or long-term price targeting or nominal-income targeting does determine prices, whereas short-term interest-rate targeting combined with intermediate-term or long-term inflation targeting does not determine prices. The Taylor rule does not affect this price indeterminacy. The McCallum-Woodford rule, however, does determine prices since it is a price-targeting rule.

Most central banks do pursue long-term objectives as they target interest rates in the short run. This paper's results indicate that if price stability is all that matters, then central banks should pursue price targeting rather than inflation targeting. Since the Taylor rule is in terms of inflation, the central bank should not follow a Taylor rule, but follow something like a McCallum-Woodford rule instead.

On the other hand, Eagle and Domian (2003, and 2004) argue that the Pareto-efficiency of contracts implies that price stability is not all that matters. In their models, Pareto-efficient consumption allocations are proportional to aggregate supply, which is appropriate when all consumers have the same risk aversion. While they recognize aggregate-demand-caused inflation as bad, Eagle and Domian argue that aggregate-supply-caused inflation is good in that it makes the real payments on nominal contracts proportional to aggregate supply. They therefore advocate nominal-income targeting over price targeting or inflation targeting. This paper shows that prices are determined when a central bank targets interest rates in the short run and nominal income in the intermediate and/or long run. A McCallum-Woodford rule modified for nominal-income targeting, equation (25), may be needed to firm up the public's confidence in the central bank targeting nominal income.

Appendixes

A. The Central Bank Loss Function Switch:

Let x_t be the instrument the central bank uses to conduct its monetary policy. Let $p_t(x_t)$ be the function that represents the price level at time t as a function of x_t. Consider Sargent and Wallace's original loss function, $E_0\left[\sum_{t=0}^{\infty}\delta^{t-1}\left(p_t(x_t)-p_t^*\right)^2\right]$. Taking the derivative of the above with respect to x_t gives:

$$2\sum_{t=0}^{\infty}\delta^{t-1}E_0\left[\left(p_t(x_t)-p_t^*\right)p_t'(x_t)\right]=0 \qquad \text{(A1)}$$

If $E_{t-1}\left[\left(p_t(x_t)-p_t^*\right)p_t'(x_t)\right]=0$ for all t, then

$E_0\left[\left(p_t(x_t)-p_t^*\right)p_t'(x_t)\right]=E_0\left[E_{t-1}\left[\left(p_t(x_t)-p_t^*\right)p_t'(x_t)\right]\right]=0$ for all t, implying (A1) equals zero.

This shows that, technically, the central bank must take into account the derivative of the price as a function of the instrument in order to minimize Sargent and Wallace's original loss function. However, if $p_t'(x_t)$ is a constant, then Sargent and Wallace's original loss function implies the same central bank behavior as does the loss function in (11). For sufficiently, small variations in the stochastic variables of the system, $p_t'(x_t)$ should be arbitrarily close to being constant. If $p_t'(x_t)$ is not constant, then the central bank would set $E_{t-1}\left[\left(p_t(x_t)-p_t^*\right)p_t'(x_t)\right]=0$. However, this would imply an optimal value for x_t, which the public having rational expectations would be able to figure out. They would then be able to figure out $E_{t-1}[p_t(x_t)]$, but they would need to know the joint probability distribution of all stochastic exogenous variables in the system.

B. The Theoretical Identification Error and the Velocity-Money-Demand Fallacy

Monetary economists tend to focus on the money demand function rather than velocity because of a belief that the money demand function is the reciprocal or inverse of velocity. This section shows by a Robinson-Crusoe counterexample that this belief is false. It also discusses that this false belief may have come about from a theoretical identification error or an inconsistency between some economists' definition of the money demand function and the microeconomic definition of a demand function.

Assume one infinitely-lived consumer who receives an exogenous endowment of the consumption good each period and who has perfect foresight. The consumer maximizes his time-additively-separable logarithmic utility:

$$\sum_{t=1}^{\infty} \beta^t \ln\left(\min\left[c_t, \frac{M_{t-1}^d}{P_t} \right] \right) \tag{A2}$$

subject to

$$P_t c_t + M_t^d + B_t \leq P_t y_t - T_t + M_{t-1}^d + B_{t-1}(1+i_{t-1}) \tag{A3}$$

where β is the time preference discount factor, c_t is the consumer's consumption at time t, P_t is the price at time t of the consumption good, M_t^d is the consumer's demand for money at time t, B_t is the consumer's demand for one-period nominal bonds at time t, y_t is the consumer's endowment at time t, T_t is the consumer's tax assessment at time t, and i_t is the nominal interest rate at time t. The variables M_0^d and B_0 are given as well as y_t and T_t for t=1,2,… .

Section C derives the following demands based on microeconomic principles:

$$C_t = \frac{\beta^{t-2}(1-\beta)\left(\tilde{W} + B_0\right)\prod_{s=0}^{t-2}(1+i_s)}{P_t} \tag{A4}$$

$$M_t^d = \beta^{t-1}(1-\beta)(\tilde{W} + B_0)\prod_{s=0}^{t-1}(1+i_s) \qquad \text{(A5)}$$

where $\tilde{W} \equiv \sum_{t=1}^{\infty} \frac{P_t Y_t - T_t}{\prod_{s=0}^{t-1}(1+i_s)}$. (Equation (A4) is for for t=2,3,…,∞, and equation (A5) is for t=1,2,…∞.) The $\tilde{W}$ is a permanent-income type of variable reflecting the present value of future after-tax income. Note that $\frac{\partial \tilde{W}}{\partial i_t} = -\frac{1}{1+i_t} \sum_{s=t+1}^{\infty} \frac{P_t Y_t - T_t}{\prod_{s=0}^{t-1}(1+i_s)} < 0$, which implies that $\frac{\partial M_t^d}{\partial i_t} = \beta^{t-1}(1-\beta)\left(\frac{\partial \tilde{W}}{\partial i_t}\right)\prod_{s=0}^{t-1}(1+i_s) < 0$. Therefore, this money demand function (A5) is somewhat interest elastic.

Microeconomic principles imply that (A5) is the money demand function. To argue that some other function is the money demand function is inconsistant with microeconomic principles. Having agreed (hopefully) that (A5) is the money demand function, we now need to determine the structural velocity function. We define the structural velocity function as the structural relationship between money demand and aggregate demand. In this model, the structural relationship between money demand and nominal aggregate demand is $M_{t-1}^d = P_t c_t$. Since nominal aggregate demand in this Robinson-Crusoe economy at time t equals $P_t c_t$, this says that the money demand of the previous period equals nominal aggregate demand of this period. Note that this relationship is consistent with a cash-in-advance constraint. The structural velocity in this model equals $\frac{P_t c_t}{M_{t-1}^d}$, which is a constant one. That structural velocity is constant in a cash-in-advance constraint is well recognized.

Thus, the money demand function (A5), which is interest elastic, differs from the constant velocity function. My reference in the body of the paper to (3) being the structural velocity function means that I am interpreting that function to be the structural relationship between money demand and nominal aggregate demand.

We should not associate this paper's argument as only being pertinent to an economy with a constant velocity. Monetary economists have thought that the money demand function is the reciprocal of velocity. We have shown by counterexample where the money demand function is not the reciprocal of the structural velocity function. In models where velocity is not constant, the structural velocity function continues to differ from the money demand function.

Below I present the Cambridge argument, which is the most widely known argument as why the money demand function is the inverse of the money demand function. While the Cambridge tradition is usually associated with this argument being applied for a constant velocity; many economists consider that it applies for a variable velocity as well and so I present the argument with a variable velocity. This argument starts out with the equation of exchange:

$$M \cdot V(\vec{u}, \varepsilon) = PY \qquad \text{(A6)}$$

where M is the money supply, P is the price level, Y is aggregate supply, and $V(\vec{u}, \varepsilon)$ is the income velocity function of money. This velocity can be a function of various endogenous and exogenous variables, represented by the vector $\vec{u}$, and a stochastic error term ε. The Cambridge argument divides both sides of (A6) by $V(\vec{u}, \varepsilon)$. It then defines the function, $k(\vec{u}, \varepsilon) \equiv \frac{1}{V(\vec{u}, \varepsilon)}$, which can be referred to as the Cambridge k. The result is:

$$M \cdot = k(\vec{u}, \varepsilon) \cdot PY \qquad \text{(A7)}$$

The Cambridge argument calls (A7) the money demand function.

Often this Cambridge argument is associated with a constant velocity and hence according to the argument, the money demand function is interest inelastic. Nevertheless, the argument depicted above is one that many textbooks give to explain why the money demand function is really the inverse of velocity,

Where did the Cambridge argument make its logical flaw? I have found two possible reasons: First, it may have defined the money demand function in a manner that was inconsistent with microeconomic principles. Second, it may have confused a reduced-form with a structural form.

Identification errors can also be made in theoretical work as well as empirical work if we are not careful. In particular, to avoid confusing reduced-form equations with structural ones, we must recognize that when we combine equilibrium conditions with other structural equations, we obtain reduced-form equations, equations that are only true in equilibrium. Structural equations are true both in and out of equilibrium.

Suppose we have a demand function, $Q^D = 20 - 5P$. This is a structural equation; it is defined both in and out of equilibrium. When we draw a demand curve such as in Figure 1, we draw it not only at the equilibrium point, but as a function of the price level; that function is defined for both disequilibrium and equilibrium values even though that demand curve might be a notional demand curve whose behavior assumes no shortages or surpluses. However, in equilibrium, $Q^D = Q^S$. If we substituted this equilibrium condition into the demand curve, we get $Q^S = 20 - 5P$. This latter equation is a reduced-form equation; it is not a structural equation; it is an equation that is true in equilibrium but only in equilibrium.

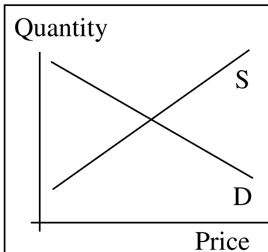

Figure 1. Demand and supply curves defined both in and out of equilibrium

Suppose the velocity function is $V(\vec{u},\varepsilon) \equiv \frac{PY^{AD}}{M^d}$ where Y^{AD} is real aggregate demand.

Then $M^d \cdot V(\vec{u},\varepsilon) \equiv PY^{AD}$. We can define $k(\vec{u},\varepsilon) \equiv \frac{1}{V(\vec{u},\varepsilon)}$ and then write this as:

$$M^d = k(\vec{u},\varepsilon) \cdot PY^{AD} \tag{A8}$$

However, calling the above the money demand function is not consistent with microeconomic foundations. The money demand function should be the relationship between money demand and one's own income. . Equation (A8) is the relationship between money demand and aggregate demand. When people do spend money as reflected by nominal aggregate demand, that spending does in some sense result in other people receiving income. However, the resulting income is someone else's income, not the income of the individual demanding the money. It is true that in equilibrium, aggregate demand equals income or aggregate supply. However, substituting income for aggregate demand converts (A8) into a reduced-form equation, an equation that is true only in equilibrium, not true out of equilibrium.

C. Derivations of Demand Functions for Section B:

We can rewrite the consumer's optimization problem as to minimize $\sum_{t=1}^{\infty} \beta^t \ln(c_t)$ subject to (A3) and $P_t c_t = M_{t-1}^d$, which is the cash-in-advance constraint. Substituting the cash-in-advance constraint into the budget constraint (A3), dividing both sides by $\prod_{s=0}^{t-1}(1+i_s)$, and then summing from t=1 to n gives:

$$\sum_{t=1}^{\infty} \frac{P_{t+1}c_{t+1}}{\prod_{s=0}^{t-1}(1+i_s)} + \sum_{t=1}^{\infty} \frac{B_t}{\prod_{s=0}^{t-1}(1+i_s)} \le \sum_{t=1}^{\infty} \frac{P_t y_t - T_t}{\prod_{s=0}^{t-1}(1+i_s)} + \sum_{t=1}^{\infty} \frac{B_{t-1}(1+i_{t-1})}{\prod_{s=0}^{t-1}(1+i_s)}$$

The bond terms on both sides cancel except for B_0, which means:

$$\sum_{t=1}^{\infty} \frac{P_{t+1} c_{t+1}}{\prod_{s=0}^{t-1} (1+i_s)} \le \sum_{t=1}^{\infty} \frac{P_t y_t - T_t}{\prod_{s=0}^{t-1} (1+i_s)} + B_0 \qquad \text{(A9)}$$

The consumers optimization problem is equivalent to minimizing $\sum_{t=1}^{\infty} \beta^t \ln(c_t)$ subject to the

above. The Largrangian is therefore $\sum_{t=1}^{\infty} \beta^t \ln(c_t) - \lambda \left(\sum_{t=1}^{\infty} \frac{P_{t+1} c_{t+1}}{\prod_{s=0}^{t-1} (1+i_s)} - \sum_{t=1}^{\infty} \frac{P_t y_t - T_t}{\prod_{s=0}^{t-1} (1+i_s)} + B_0 \right)$, and

the first order necessary conditions are $\frac{\beta^{t+1}}{c_{t+1}} - \lambda \frac{P_{t+1}}{\prod_{s=0}^{t-1} (1+i_s)} = 0$ for t=1,2,… which implies that

$$\frac{\beta^{t+1}}{\lambda} = \frac{P_{t+1} c_{t+1}}{\prod_{s=0}^{t-1} (1+i_s)}. \qquad \text{(A10)}$$

Substituting this into (A9) gives $\sum_{t=1}^{\infty} \frac{\beta^{t+1}}{\lambda} = \tilde{W} + B_0$, which implies that $\frac{\beta^2}{\lambda(1-\beta)} = \tilde{W} + B_0$.

Hence, $\frac{1}{\lambda} = \beta^{-2}(1-\beta)(\tilde{W} + B_0)$, which with (A10) implies $\beta^{t-1}(1-\beta)(\tilde{W} + B_0) = \frac{P_{t+1} c_{t+1}}{\prod_{s=0}^{t-1} (1+i_s)}$,

which implies the money demand function (A5). Dividing this by P_{t+1} gives the demand for the consumption good at time t+1, from which comes (A4).

References

Eagle, David M. and Dale Domian, (2003). "Sounding the Alarm on Inflation Indexing and Strict Inflation Targeting." Working Paper

Eagle, David M. and Dale Domain, (2004). "Quasi-Real Indexing – The Pareto-Efficient Solution to Inflation Indexing." Working Paper

Eagle, David M. and Elizabeth Murff, (2004), "Solving Expectational Difference Equations Correctly and A Case of Mathematical Abuse in the New Keynesian Economics," Working Paper

Eagle, David M. (2004a), "True Price Determinacy in a Model of a Cashless Economy," Forthcoming Working Paper

McCallum, Bennett T. (1981). "Price Level Determinancy with an Interst Rate Policy Rule and Rational Expectations," *Journal of Monetary Economics* 8:319-329.

Rudin, Walter (1976). *Principles of Mathematical Analysis* (McGraw-Hill: Columbus, OH), 3rd edition,

Sargent, Thomas J. (1979). *Macroeconomic Theory – Economic Theory, Econometrics, and Mathematical Economics* (Academic Press – New York).

Sargent, Thomas J. (1986). *Dynamic Macroeconomic Theory* (Harvard University Press – Cambridge, Massachusetts).

Sargent, Thomas J. and Neil Wallace (1975). "'Rational' Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule," *Journal of Political Economy* 83:241-254.

Taylor, John B. 1993. "Discretion Versus Policy Rules in Practice," *Carnegie-Rochester Conference Series on Public Policy*, 39, pp. 195-214.

Woodford, Michael (2003). *Interest and Prices – A Foundation of a Theory of Monetary Policy* (Princeton University Press – Princeton, New Jersey).