Consistent High-Frequency Calibration

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Abstract

Economic models are meant to provide a framework to describe real-world economic activities. In principle, how well a model performs this task can be evaluated by how close the model's simulated activities track the observed ones. A necessary first step in simulating a model is to choose values for the model's parameters in accordance with actual economic data. A fundamental problem in economic modelling, however, is that actual economic data are sampled at time intervals that are typically longer than the decision intervals of actual economic agents.

One popular resolution of this problem is to constrain the length of the decision intervals of theoretical economic agents to be equal to the length of the actual data-sampling intervals. This widely adopted approach makes it feasible to directly calibrate theoretical models to the observed data, but it can introduce substantial biases in the models' empirical performance, as demonstrated by recent research that has allowed the decision intervals to be shorter than the data-sampling intervals. This alternative, high-frequency modelling approach, however, has brought with itself a fundamental issue that direct calibration of the models' parameters is no longer feasible. In response, researchers have employed a simple, yet ad hoc, rule to transform commonly chosen lower-frequency parameter values (which can be calibrated directly from the available data) to their high-frequency counterparts.

We show in this paper that this simple transformation rule has three major drawbacks. First, it produces internal inconsistencies in steady-state equilibrium conditions. Second, it is sometimes at odds with microeconomic evidence. And third, it can result in inaccurate log-linear approximations to the models' true equilibrium solutions by worsening the fit of both the transition dynamic coefficients and the point of approximation itself. We present here an alternative, coherent transformation rule for calibrating high-frequency models that directly addresses these three shortcomings. We then use our consistent transformation rule to calibrate high-frequency versions of two well-known economic models and show how it improves these models' empirical performance.

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1 Introduction

An economic model is meant to provide a simplified framework to describe real-world economic activities. A good economic model is one that captures these economic activities in a parsimonious fashion without relying on overly restrictive simplifying assumptions. In this paper, we focus on one commonly overlooked assumption that is, at times, unnecessarily restrictive.

The usual practice in economic modelling is to set the length of the decision interval in theoretical models equal to the real-world data-sampling interval. For example, if a modeler is endowed with quarterly data, then the usual approach is to assume that economic agents in the model make decisions once per quarter. The quarterly model can then be calibrated by matching the model's steady-state values to the historical averages of the actual time series in the quarterly data or by relying on the embodied microeconomic evidence. The model is then evaluated by comparing its simulated economic activities with statistics from the actual data. The advantage of this widely accepted approach of setting the decision interval equal to the constraining data-sampling interval is that it is (1) feasible to calibrate the model directly to the available data and (2) straightforward to assess the model's empirical performance.

A fundamental issue associated with this modelling approach is that, in reality, actual decisions by economic agents are likely to be made at time intervals that are more frequent than the intervals at which economic data are sampled. Furthermore, this real-world feature may play an important role in shaping the observed behaviors of economic variables. Indeed, research has shown that temporal aggregation may alter the time-series patterns of economic data generated by more frequent decisions (e.g., see Working (1960); Geweke (1982); and Weiss (1984)). Therefore, abstracting from this feature may subject theoretical models to substantial biases in their empirical performances.

To assess the importance of this issue, several recent studies have chosen to examine high-frequency models where the decision intervals are shorter than the data-sampling intervals. The general procedure is to first solve the models, simulate high-frequency artificial data, and then aggregate the data up to the longer data-sampling intervals following the sampling and temporal aggregation procedures used in creating the actual data. With the low-frequency artificial data in hand, one can then compare its statistical properties with the actual dataset (of the same low-frequency) to evaluate the models' empirical performance. The starting point of this exercise is to choose values for the parameters in the high-frequency models. Unfortunately, this can no longer be done by appealing directly to the observed lower-frequency data. Instead, one has to transform the commonly chosen lower-frequency parameter values, which can be calibrated directly from the available data, to obtain their high-frequency counterparts.

The "standard" method for calibrating high-frequency models appears to have begun with Christiano (1989), and has later been followed by Cogley and Nason (1995), Chari, Kehoe and McGratten (2000), and Aadland (2001), among others. The standard high-frequency calibration technique uses simple transformation rules to adjust parameter values across frequencies so that a high-frequency model produces equilibrium dynamics similar to those generated by its lower-frequency counterpart.

We show in this paper that the standard, simple transformation rule has three major drawbacks. First, it produces internal inconsistencies in steady-state equilibrium relations across frequencies. Second, it can be at odds with microeconomic evidence. Third, due to the complexity of modern economic models, true equilibrium solutions are often non-linear and hard to obtain, and it has become a useful practice to log-linearize the models' equilibrium conditions around their steady states. (This common practice usually provides satisfactory approximations to the true equilibrium dynamics if the underlying disturbances are small.) The simple, ad hoc transformation rule, however, can result in inaccurate log-linear approximations to the models' true equilibrium solutions by worsening the fit of both the transition dynamic coefficients and the point of approximations themselves.

To address these issues, we propose a *consistent transformation rule* for calibrating high-frequency models that adheres to the principle that economic variables must be consistently aggregated across time in steady-state equilibria. Consistency means that steady-state stock variables (e.g., capital stock, money supply) must be equal across frequencies, and low-frequency steady-state flow variables (e.g., output, consumption) must equal the temporal sum of their higher-frequency counterparts.

Our consistent transformation rule, by design, ensures consistencies in steady-state equilibrium relations across frequencies. Further, and perhaps more importantly, it possesses other advantages.

First, it is consistent with the sampling and temporal aggregation procedure used in practice for creating the actual data. Second, it is sufficiently flexible to incorporate relevant empirical evidence on high-frequency parameter values. Third, through improving the fit of both the transition dynamic coefficients and the point approximations themselves, it leads to better approximations to the true equilibrium solutions. And finally, besides the consequences merely caused by sampling and temporal aggregation themselves (see the literature cited above), an improved fit of high-frequency parameter values and high-frequency point approximations can interact with the temporal aggregation procedure to improve the performance of the time-aggregated lower-frequency dynamics. In this sense, our consistent transformation rule for calibrating high-frequency models complements the existing literature on sampling and temporal aggregation.

We provide two examples to illustrate the advantages of our consistent transformation rule over the simple, ad hoc rule. We do so by calibrating and computing the log-linearized, high-frequency versions of two well-known economic models, and showing how the models' empirical performances are improved under the consistent rule than under the ad hoc rule.

The rest of the paper is organized as follows. Section 2 describes, in general terms, the transformation procedures used in calibrating high-frequency models. It draws special attention to the principles behind the standard transformation rule and our proposed consistent transformation method. In Section 3, we calibrate and compute a weekly version of a standard real business cycle (RBC) model. We show that our consistent transformation method improves the fit of the RBC model by generating additional volatility in hours worked. This is accomplished without altering the typically chosen quarterly values of the structural parameters. Instead, we take these common low-frequency parameter values as given and rely on the principle that variables are consistently aggregated across frequencies, while incorporating the microeconomic evidence that workers are more willing to intertemporally substitute their labors at high frequencies than at low frequencies. In Section 4, we consider a standard consumer-based asset pricing model that is augmented to capture both the durability of consumption and the existence of habit persistence. Starting with an annual model that is calibrated so that habit persistence sets in quickly, we then use our consistent transformation procedure to calibrate and compute a weekly version of the model. We show that our consistent transformation technique naturally introduces local substitutability in consumption, while continuing to maintain long-run habit persistence. As shown by Heaton (1995), the dual existence of local substitution and long-run habit persistence improves the fit of consumer-based asset pricing models. Section 5 concludes the paper. The appendix provides some technical details on the derivation of the consistent, high-frequency relations for our RBC example.

2 Methodologies

In this section, we outline in general terminology the concept of transformation rules for calibrating high-frequency models and, more specifically, the principles behind the standard transformation rule and our consistent transformation method. The task of high-frequency calibration is to establish an isomorphic mapping from low-frequency values of parameters (e.g., quarterly or annual) to their higher-frequency counterparts (e.g., weekly, daily, hourly, etc.). The issue here is not about how to choose the low-frequency values per se — they are taken as given for the purpose of this analysis and are obtained from the routine calibration of low-frequency models based on actual low-frequency data. Rather, it is about how to derive the high-frequency parameter values from their commonly accepted low-frequency counterparts. Naturally, one would wish such a mapping to be consistent with the rules by which actual data are aggregated across time, and with any available microeconomic evidence.

The standard high-frequency calibration approach, however, is not based on this natural transformation principle. Instead, it relies on ad hoc transformation rules designed to produce dynamic behavior within the model that is invariant to the choice of the decision intervals. In contrast, our consistent high-frequency calibration method builds on the natural transformation principle. It transforms the low-frequency parameter values to a higher-frequency level subject to the constraints that (i) the high-frequency steady-state values of economic variables are aggregated across time up to the low-frequency level in the same manner as the actual data are aggregated, and (ii) the high-frequency parameter values do not fall outside the admissible ranges suggested by available microeconomic evidence. In principle, starting with the usual low-frequency parameter values, the higher-frequency values obtained via the consistent method can differ from the higher-frequency values derived under the standard approach.

We now describe the general differences between the two calibration approaches more formally. To keep track of the decision intervals, let τ index the lower frequency (longer) decision interval. In many macroeconomic studies, τ indexes quarterly or annual decision intervals. The higher frequency (shorter) decision interval is indexed by t. We assume that there are n decision intervals in t-time within each decision interval in τ-time.

Under the standard high-frequency calibration approach, one converts the given lower-frequency parameter values into the parameter values for the higher-frequency models by simply raising all parameters explicitly associated with time to the $1/n$ power (e.g., discount factors, depreciation rates). Other parameters are either divided by n (e.g., time endowments, means of flow variables) or are treated as invariant to the changes in the decision intervals (e.g., preference parameters, production share parameters).

Our consistent high-frequency calibration approach starts with the same lower-frequency parameter values as in the standard approach, but by contrast to the standard approach, it insists on the principle that economic variables be temporally aggregated in the steady states in a manner that is consistent with how the actual data are constructed in practice. Denote by F_*, S_*, γ_* the t-time steady-state values of a flow vector, stock vector, and parameter vector, respectively, and F, S, γ their τ-time counterparts. Consider a system of steady-state equations in t-time, $g(F_*, S_*, \gamma_*) = 0$, and a corresponding system of steady-state equations in τ-time, $g(F, S, \gamma) = 0$. To transform γ into γ_*, we rely on the time-aggregation principle that $F_\tau = \sum_{i=0}^{n-1} F_{t-i}$ for flows and the beginning-of-period sampling principle that $S_\tau = S_t$ for stocks, which together imply the steady-state constraints that $F = nF_*$ and $S = S_*$, with which the transformation must be consistent. Solving the system $g(F/n, S, \gamma_*) = g(F, S, \gamma)$ then produces an isomorphic correspondence that maps consistently the low-frequency parameter values one-to-one and onto their high-frequency counterparts according to $\gamma_* = \gamma_*(F, S, \gamma, n)$, provided that the system is exactly identified. If the system is overidentified, then additional parameterizations may be needed and the corresponding parameters may need to be allowed to vary across frequencies. If the system is underidentified, then additional parameter restrictions may be necessary in order to achieve identification. In this case, the additional restrictions should be chosen to respect available microeconomic evidence.

Before introducing the economic examples, we note that our consistent high-frequency calibration methodology can be easily extended to handle low-frequency parameters that are econometrically estimated. It is becoming increasingly popular to directly estimate the structural parameters in macroeconomic models using system estimation techniques such as generalized method of moments (GMM) and full-information maximum likelihood (FIML), as opposed to relying solely on long-run historical averages or evidence from microeconomic studies to choose parameter values. Notable examples of the former include Christiano and Eichenbaum (1992), Ireland (2001), and Kim (2000). To see how consistent calibration works in this context, begin by letting the low-frequency estimated parameters be represented by $\hat{\gamma}$, with variance-covariance matrix Σ. The consistent high-frequency parameters, similar to the analysis above, are given by the relation $\hat{\gamma}_* = \gamma_*(F, S, \hat{\gamma}, n)$. Furthermore, provided that $\hat{\gamma}$ are consistent and $\gamma_*(F, S, \hat{\gamma}, n)$ is not a function of the sample size, Slutsky's Theorem guarantees that $\hat{\gamma}_*$ will be consistent as well (Greene (2003), p.903). One of the primary attractions of directly estimating the structural parameters with techniques such as GMM or FIML is that the researcher has an explicit measure of the degree of sampling uncertainty in the estimated parameters (i.e., Σ). This in turn allows the researcher to form confidence intervals for standard metrics (e.g., standard deviations, cross correlations, impulse response functions, spectral density functions) and perform Classical hypothesis tests to measure the goodness-of-fit of the model. In order to perform hypothesis tests for a high-frequency model, it is necessary to calculate the variance-covariance matrix for $\hat{\gamma}_*$, which we denote by Σ_*. Since $\gamma_*(\hat{\gamma})$ is typically a non-linear function in $\hat{\gamma}$, we take a first-order linear approximation to γ_* around its true population value γ_*^0. The variance of the linearized version of $\hat{\gamma}_*$ is then given by

$$\Sigma_* = \Gamma_*(\gamma_*^0)\Sigma\Gamma_*(\gamma_*^0)^T, \tag{1}$$

where Γ_* indicates the matrix of first partial derivatives of $\gamma_*(\hat{\gamma})$ with respect to each element in $\hat{\gamma}$ and superscript T denotes the transpose operator. This is often referred to as the "delta method" (Greene (2003), p.913). Lastly, substitution of consistent estimates for γ_*^0 into (1) makes Σ_* operational and allows one to test various hypotheses regarding the fit of the high-frequency model. We turn now to our economic examples.

3 Example #1. Real Business Cycle Theory

If the transformed high-frequency parameter values or the implied behavior of economic variables were not sensitive to which calibration method is used, then the choice of calibration methods would not be a serious issue of concern. Unfortunately, this is not typically the case.

For concreteness, let's now look at a simple RBC example. Consider first the usual version of the model with the longer decision interval indexed by τ. In this model, a representative agent is assumed to maximize an expected stream of discounted utility

$$U(C,L) = E_\tau \sum_{s=0}^{\infty} \beta^{\tau+s} \left[\log(C_{\tau+s}) + \frac{\phi}{\eta} l_{\tau+s}^{\eta} \right] \tag{2}$$

by choosing consumption and leisure paths in τ-time, where E_τ denotes the mathematical expectations operator conditional on all information dated time τ and earlier, β a subjective discount factor, C_τ consumption, ϕ leisure's weight in total utility, $l_\tau = (N - L_\tau)/N$ the proportion of endowed time spent toward leisure, N the endowment of time available for leisure and labor, L_τ labor hours, and $1/(1-\eta)$ the intertemporal elasticity of proportional leisure.[1] Consumption is subject to the resource constraint

$$C_\tau \leq Y_\tau - I_\tau(1 + 0.5\psi q_\tau^2), \tag{3}$$

where Y_τ denotes output, I_τ gross investment into the capital stock K_τ, and $0.5\psi q_\tau^2$ the unit adjustment cost for investment with $q_\tau = I_\tau/K_\tau$. Capital accumulates according to

$$I_\tau = K_{\tau+1} - (1-\delta)K_\tau, \tag{4}$$

and output is given by the production function

$$Y_\tau = (z_\tau L_\tau)^{1-\alpha}(nK_\tau)^\alpha, \tag{5}$$

[1] The steady-state intertemporal elasticity of labor is also $1/(1-\eta)$ under the assumption that equal proportions of time are spent in leisure and labor activities.

where $z_\tau = z_{\tau-1}\exp(\mu + \epsilon_\tau)$ is a stochastic technology process following (in natural logs) a random walk with drift μ and mean-zero white-noise shock ϵ_τ. Capital is scaled by n because a given capital stock (similar to the stock of laborers) provides a stream of services to firms over each smaller decision interval. Without loss of generality, we have normalized the flow of *capital services* to be equal to the capital stock in t-time. Therefore, the τ-time capital stock provides n times the capital services as that at the higher frequency.

The consumption and labor Euler equations for this problem are

$$C_\tau^{-1} = \beta(1+1.5\psi q_\tau^2)^{-1}E_\tau\left[C_{\tau+1}^{-1}(1+nr_{\tau+1}-\delta+0.5\psi q_{\tau+1}^2(q_{\tau+1}+1.5(1-\delta))\right] \quad (6)$$

$$w_\tau = \phi l_\tau^{\eta-1}C_\tau/N \quad (7)$$

where r_τ is the rental rate for capital services and w_τ the wage rate for labor services:

$$r_\tau = \alpha\frac{Y_\tau}{nK_\tau} \quad (8)$$

$$w_\tau = (1-\alpha)\frac{Y_\tau}{L_\tau}. \quad (9)$$

For the purpose of our analysis, we take as given the parameter values for the low-frequency RBC model $\boldsymbol{\gamma} = (\beta, \delta, \alpha, \eta, \phi, \psi, \mu)$. In the business-cycle literature, these low-frequency parameter values are typically chosen so that the model's steady-state solutions match the historical averages of the corresponding actual time series, or to be consistent with relevant microeconomic evidence. Usually this is performed at either a quarterly or annual decision interval. Once $\boldsymbol{\gamma}$ and the initial conditions K_0 and z_0 are chosen, one can compute the model's equilibrium paths and generate artificial τ-time data for given realizations of the driving technology process.

Our focus in this paper is instead on how one can calibrate the t-time version of the model from a given set of τ-time parameter values. Consider first the standard high-frequency calibration approach which uses the following transformation rule

$$\boldsymbol{\gamma}_*(\boldsymbol{\gamma}) = (\beta^{1/n}, 1-(1-\delta)^{1/n}, \alpha, \eta, \phi, \psi_*, \mu/n). \quad (10)$$

The capital adjustment cost parameters ψ and ψ_*, irrespective of the length of the model's decision interval, are often chosen such that the ratio of investment volatility to output volatility in the model equals the ratio observed in the detrended post-war quarterly U.S. data (e.g., Chari et al. (2000), Huang and Liu (2002)).[2] The high-frequency parameter values $\boldsymbol{\gamma}_*$ obtained from the standard calibration approach can then be substituted into the t-time version (thus $n = 1$) of the model to simulate high-frequency artificial data. This is the approach taken by Christiano (1989), Cogley and Nason (1995), Chari et al. (2000), and Aadland (2001).

In transforming the given τ-time parameter values $\boldsymbol{\gamma}$ (with longer decision interval) into their t-time counterparts $\boldsymbol{\gamma}_*$ (with shorter decision interval), our consistent high-frequency calibration approach insists on the principle that the steady-state values of economic variables in the model are consistently aggregated across time. To implement the consistent method, we impose the following consistency constraints $F = nF_*$ and $S = S_*$ on the steady-state versions of equations (3)-(9) and solves the system jointly with the corresponding steady-state equations in t-time.[3] This generates the following mapping from $\boldsymbol{\gamma}$, n, and l to $\boldsymbol{\gamma}_*$

$$
\begin{aligned}
\beta_* &= \beta n e^{\mu/n}\left(e^{\mu}+\beta(e^{\mu/n}n-e^{\mu})\right)^{-1} && \text{(11a)}\\
\delta_* &= (\delta/n)+(1-e^{\mu/n})+(1/n)(e^{\mu}-1) && \text{(11b)}\\
\alpha_* &= \alpha && \text{(11c)}\\
\eta_* &= \eta+\log(\phi/\phi_*)/\log(l) && \text{(11d)}\\
\psi_* &= \psi n^2 && \text{(11e)}\\
\mu_* &= \mu/n. && \text{(11f)}
\end{aligned}
$$

Since $\boldsymbol{\gamma}_*$ includes seven elements and (11a - 11f) only involves six equations, the system is underidentified (see equation 11d). To identify the system we impose an additional restriction that η_* be consistent with microeconomic evidence on high-frequency intertemporal elasticity of substitution in labor. Although the microeconomic evidence does not pin down an exact value for η_*, the evi-

[2]For Hodrick-Prescott detrended quarterly U.S. data over the period 1948 through 1999, this ratio is approximately 2.25.

[3]Details of this procedure are shown in the appendix.

dence confirms our expectations that individuals are more willing to substitute labor across shorter time periods and provides a general guide to the appropriate magnitude for η_*.

There are many studies that have estimated the willingness of workers to substitute labor across time in response to changes in real wages. Many of these studies are based on life-cycle evidence of hours worked (intensive margin) by men at annual or even lower frequencies. Pencavel (1986) summarizes the findings of these surveys and concludes that the intertemporal labor supply elasticity has a "central tendency of 0.20." Using multi-industry panel or macro-level data sets, several studies find as Browning, Hansen and Heckman (1999) report on p. 616 that "...time [labor supply] is more substitutable over shorter intervals than longer ones." MaCurdy (1983), using monthly data from the Denver Income Maintenance Experiment, finds (intensive) elasticities in the range of 0.3 to 0.7. Abowd and Card (1989) report elasticities that increase as one moves from biennial to annual to semi-annual data. Barsky and Miron (1989) provide indirect evidence of larger intertemporal substitution at higher frequencies by noting that there exists substantial procyclical seasonal (quarterly) variation in total hours worked (i.e., labor hours and employment are higher than average during the boom periods of summer and fall and lower than average in the winter recessions). Hall (1999) states that "the seasonal data suggest reasonable amounts of intertemporal substitution among the quarters of the year." Kimmel and Kniesner (1998) using tri-annual micro panel data of the Survey of Income and Program Participation estimate hours worked elasticities (intensive margin) of approximately 0.5 and employment elasticities (extensive margin) of approximately 1.5. In addition to this multi-industry evidence, there are several industry-specific studies that also suggest larger substitutability at higher frequencies (Treble (1986); Carrington (1996); Oettinger (1999)). Roughly in line with this high-frequency evidence, we choose a conservative baseline value of η_* such that the high-frequency intertemporal elasticity of labor is one and a half times larger than the quarterly elasticity typically applied in business-cycle models. In our opinion, the research cited above supports choosing even greater substitutability at higher frequencies, but applying a more conservative estimate of η_* will keep it from overshadowing the influence of other high-frequency parameters. Once a value for η_* is chosen, ϕ_* is then pinned down by (11d).

To get a further quantitative feel, we set $n = 13$ so that τ indexes quarterly decision intervals and t weekly decision intervals. The first row of Table 1 depicts the commonly accepted quarterly parameter values, which are taken as the starting point for any higher-frequency calibration. The second and third rows of the table display the implied weekly parameter values under the standard and consistent high-frequency calibration approaches. According to the table, the differences in the standard and the consistent weekly parameter values are small for β_* and δ_*, suggesting that the standard high-frequency calibration method is providing a good approximation to the consistent high-frequency calibration for these two parameters.[4] As can be seen from the table, however, the standard and the consistent weekly parameter values for η_*, ϕ_* and ψ_* are substantially different, reflecting the fundamental differences in the two high-frequency calibration methodologies. Our consistent approach sets high-frequency parameter values according to steady-state consistency criteria across frequencies and microeconomic evidence, while the standard approach uses simple approximating transformations designed to produce dynamics that are invariant across frequencies.

Table 1. Comparison of RBC Parameters Values

	Parameter Values						
	β	$1-\delta$	α	η	ϕ	ψ	μ
Commonly Chosen Quarterly Values	0.9898	0.974	0.34	0	0.883	15.77	0.004
	β_*	$1-\delta_*$	α_*	η_*	ϕ_*	ψ_*	μ_*
Transformed Weekly Values (Standard Calibration)	0.99922	0.99798	0.34	0	0.883	3540	0.0003
Transformed Weekly Values (Consistent Calibration)	0.99921	0.99800	0.34	0.33	1.112	2667	0.0003

The first row of Table 2 depicts common quarterly steady-state values for key economic vari-

[4] In other contexts, such as those described by overlapping generations models with unusually long decision intervals or with less durable capital, the starting low-frequency parameter values for β or $1-\delta$ may be farther away from one. As a result, the differences between the standard and the consistently calibrated high-frequency values for the two parameters will be more stark. For example, if $\beta = 0.5$, $\mu = 0$ and $n = 4$, then the standard high-frequency calibration approach gives $\beta_* = 0.84$ but the consistent high-frequency calibration approach gives $\beta_* = 0.80$. When discounting over even relatively shorter time horizons, differences in β_* of this magnitude can have substantial effects on economic decisions.

ables and their ratios. The second and third rows of the table report the quarterly steady-state values that result from time aggregating the weekly model under the two high-frequency calibration methods. Notice that under our consistent high-frequency calibration approach, weekly parameters are selected in order to equate the steady-state values of the time-aggregated weekly model with those from the model calibrated at the accepted quarterly decision interval. On the other hand, under the standard high-frequency calibration approach, the weekly parameters are selected in an ad hoc fashion and therefore the time-aggregated quarterly steady-state values deviate from the accepted quarterly values. As mentioned above, although the differences between the steady-state values under consistent and standard calibration are not large, they have an impact on the dynamic properties of the weekly model both directly (via differences in the fundamental parameters) and indirectly (via the linear approximation around the steady-state).

Table 2. Comparison of RBC Steady States

	Steady-State Values					
	Y/K	C/Y	L	$0.5\psi q^2$	r	w
Quarterly Values (Quarterly Calibration)	0.11981	0.7478	520	0.00710	0.00313	7.381
Time-Aggregated Quarterly Values (Standard Weekly Calibration)	0.12093	0.7469	520.26	0.00962	0.00316	7.346
Time-Aggregated Quarterly Values (Consistent Weekly Calibration)	0.11981	0.7478	520	0.00710	0.00313	7.381

Finally, we turn to the transition dynamics for the RBC model. Figure 1 depicts the responses of quarterly output, consumption, investment and labor hours to a permanent one-time increase in productivity – the first column depicts the impulse responses from a quarterly RBC model and the second column from a time-aggregated weekly RBC model. (Note that output, consumption and investment are normalized by the technology shock z.) The quarterly dynamics of the RBC model are familiar. The permanent productivity gain increases real wages and rental rates for capital services, inducing workers to increase their hours worked and firms to invest more in capital.

Output increases (recall that although normalized output in Figure 1 is declining, raw output is increasing) due to the productivity gain itself and the increased labor effort and capital stock. Since the standard high-frequency calibration approach treats preference parameters as frequency invariant, it is not surprising that the time-aggregated quarterly dynamics of the system under the standard calibration approach are similar to the dynamics from the model with a quarterly decision interval. Our calibration approach, however, relies on fundamental steady-state relationships for stock and flow variables that should be satisfied across frequencies. Imposing these constraints and relying on micro studies which indicate that workers are more willing to substitute hours worked across shorter time periods, imply substantially different weekly transition dynamics. Most notably, consistent weekly calibration generates a substantially larger response in hours worked.[5] Given that equilibrium business cycle models have historically produced too little volatility in hours worked (Kydland (1995)), the increased volatility of hours worked in the weekly RBC economy may be an important avenue for bridging the gap between economic theory and U.S. aggregate data.[6]

4 Example #2. Local Substitutability and Long-Run Habit Persistence in Consumption Behavior

An ongoing puzzle in macroeconomics and finance is the inability of the standard consumption-based asset pricing model to match various moment conditions in actual data (Campbell (1999)). A well-known example is the equity premium puzzle of Mehra and Prescott (1985). One method that has been proposed for resolving these puzzles is to assume that consumers exhibit habit persistence such that the level of utility derived from current consumption depends upon an average of past consumption to which they have grown accustomed (Constantinides (1990)). When consumers display habit persistence, this tends to increase the volatility of their intertemporal marginal rate

[5] Weekly consumption dynamics are quite similar across the standard and consistent calibration approaches. We suspect this independence from differences in labor-market dynamics and adjustment costs across the two calibration approaches is due to separability between consumption and proportional leisure in the utility function.

[6] In an alternative example, we have examined a weekly monetary business cycle model with nominal wage rigidities, and found again that our consistent high-frequency calibration method helps bring the model's simulations closer to the data along a similar dimension. This example is not presented here due to the space limit, but is available from the authors upon request.

of substitution, and as a result, standard asset pricing models can be made consistent with observed equity premiums without having to rely on unrealistically large levels of momentary risk aversion. One problem with pure models of habit persistence, however, is that they are inconsistent with the notion that consumption tends to be substitutable over relatively short time horizons (Hindy and Huang (1992)). Heaton (1995) addresses this inconsistency by building a model that allows consumers to display both local substitutability in consumption and long-run habit persistence and shows that, with careful treatment of the temporal aggregation problem, the model satisfies the Hansen and Jagannathan (1991) volatility bounds and generally improves the fit over either models of pure substitutability or pure habit persistence.

In this example, we begin with an annual version of the model in Heaton (1995) and use our consistent calibration technique to evaluate a weekly ($n = 52$) version of the model. Starting from an annual model where habit persistence sets in quickly (as one might expect), we show how our calibration technique naturally captures the notion of increasing local substitutability as one moves to higher frequencies.

Following Heaton (1995), assume that a representative consumer maximizes the following expected, discounted stream of utility from consumption services

$$E_\tau \sum_{\tau=0}^{\infty} \beta^\tau \frac{s_\tau^{(1-\nu)} - 1}{(1-\nu)} \tag{12}$$

where $\nu > 0$ is the curvature parameter, and τ indexes annual time.[7] The representative consumer maximizes (12) subject to constraints (3) through (5) introduced in example #1. To focus on consumption behavior, we assume a fixed labor supply and set $\varphi = \mu = 0$. *Final consumption services*

$$s_\tau = c_\tau^{nd} + \pi(s_\tau^d - \kappa x_\tau) \tag{13}$$

are the sum of two parts, with $\pi > 0$ and $0 \leq \kappa \leq 1$. The first, c_τ^{nd}, measures the flow of services from time τ non-durable goods. The second, $(s_\tau^d - \kappa x_\tau)$, weighted by π, measures the

[7] Unlike Heaton (1995), we allow for consumption of non-durable goods and services. Not only does this make the model more realistic but it also provides an additional margin for us to pin down high-frequency calibrated values of the discount factor β.

flow of services from durable goods in a manner that can capture both the local substitutability and long-run complementarity of consumption implied by habit persistence. The term s^d_τ denotes the *intermediate consumption stock* and measures the accumulation of consumption services, c^d_τ, generated by durable goods. The intermediate consumption stock is assumed to follow

$$s^d_\tau = \sum_{j=0}^{\infty} \lambda^j \, c^d_{\tau-j} \tag{14}$$

where $0 \leq \lambda < 1$ governs the durability of consumer goods, or alternatively the substitutability of consumption. The term x_τ is referred to as the *habit stock* and allows for complementarity in consumption over time for positive κ. The habit stock is a weighted average of past intermediate consumption stocks and obeys

$$x_\tau = (1-\theta) \sum_{j=0}^{\infty} \theta^j s^d_{\tau-1-j} \tag{15}$$

where $0 \leq \theta < 1$. Since we treat both s^d_τ and x_τ as stocks, the moving average in (15) is scaled by $(1-\theta)$. The parameter θ governs the rate of decay of the intermediate consumption stock in creating the habit stock. The term κx_τ can also be interpreted as a subsistence level of services from durable goods.

To see the implications for consumption behavior over time, begin by substituting (14) and (15) into (13). Rewriting the resulting expression in lag-operator form produces

$$s_\tau = c^{nd}_\tau + \pi \sum_{j=0}^{\infty} a_j \, c^d_{\tau-j} = c^{nd}_\tau + \frac{\pi(1-\chi L)}{(1-\lambda L)(1-\theta L)} \, c^d_\tau \tag{16}$$

where $\chi \equiv \theta + \kappa(1-\theta)$ and L is the lag operator. At this point, it is instructive to consider a couple of special cases. First, if $\lambda = 0$, then the model is one of pure habit persistence and $a_j < 0$ for all $j > 0$. Second, if $\kappa = 0$, then the model exclusively captures the durability or substitutability of consumption and $a_j > 0$ for all $j > 0$. A third possibility, as noted by Heaton (1995), is that certain combinations of λ, θ and κ will capture both the notion of long-run habit persistence and local substitutability so that $a_j > 0$ for small j and $a_j < 0$ for all larger j. In this sense, the model will capture the notion of local (or adjacent) substitutability and long-run (or distant) complementarity (see Ryder and Heal (1973)).

Now turning to calibration, we begin with annual parameter values $\gamma = (\beta, \delta, \theta, \kappa, \lambda, \pi)$, which are chosen so that the annual model displays habit persistence that sets in quickly, as would be expected in a low-frequency model. These values are depicted in the first row of table 3.

Table 3. Comparison of Annual and Weekly Parameters Values

	Parameter Values					
	β	$1-\delta$	θ	κ	λ	π
Annual Values	0.96	0.9	0.8	0.9	0.1	1.74
	β_*	$1-\delta_*$	θ_*	κ_*	λ_*	π_*
Transformed Weekly Values (Standard Calibration)	0.99922	0.99798	0.9957	0.9	0.9567	1.74
Transformed Weekly Values (Consistent Calibration)	0.99921	0.99800	0.8	0.998	0.9827	1.97

The second row of table 3 depicts the transformed weekly parameter values under the standard approach, which raise parameters associated with time to the $(1/n)$ power while leaving other parameters (e.g., those associated with preferences) unchanged. The weekly parameter values under the standard approach are given by $\gamma_* = (\beta^{1/n}, 1-(1-\delta)^{1/n}, \theta^{1/n}, \kappa, \lambda^{1/n}, \pi)$.

The transformed weekly parameter values, under our consistent approach, are obtained in a manner that guarantees that the steady-state relations are consistently aggregated over time and across frequencies. After substituting out the steady-state habit stock x, there are five equations which can be used under our consistent calibration approach to identify the high-frequency parameters $\gamma_* = (\beta_*, \delta_*, \theta_*, \kappa_*, \lambda_*, \pi_*)$:

$$\begin{aligned} I/K &= \delta && (17a) \\ 1 &= \beta(r+1-\delta) && (17b) \\ (1-\pi) &= -\kappa(1-\beta\lambda)(1-\theta)\sum_{j=1}^{\infty}\beta^j\sum_{k=0}^{j-1}\theta^{j-1-k}\lambda^k && (17c) \\ (1-\lambda) &= c_d/s^d && (17d) \\ s &= c_{nd}+\pi(1-\kappa)s^d. && (17e) \end{aligned}$$

The steady-state equations are the (i) capital accumulation equation; (ii) Euler equation for nondurable goods; (iii) Euler equation for durable goods; (iv) law of motion for the intermediate consumption stock and (v) law of motion for final consumption services. Clearly, application of our consistent calibration principle to the steady-state version of (15) does not call for any change in θ across frequencies nor are we aware of any empirical evidence suggesting that θ should vary across frequencies. Treating θ as invariant across frequencies, it then follows that equations (17a) through (17e) produces a system of five equations and five unknowns. Solving these jointly with an analog system of steady-state equations in t-time produces

$$\gamma_* = (\frac{\beta n}{1+\beta(n-1)}, \frac{\delta}{n}, \theta, \kappa_*, 1-\frac{(1-\lambda)}{n}, \frac{\pi(1-\kappa)}{n(1-\kappa_*)}) \quad (18)$$

where κ_* is found by substituting the elements of γ_* into a high-frequency version of (17c) and numerically solving the resulting expression. The results of this exercise are shown in the third row of table 3. Notice that the differences between standard and consistent high-frequency calibration of κ_* and π_* are nontrivial.

To examine the annual and weekly predictions for the substitutability and complementarity of consumption, we plot in figure 2 the distributed-lag coefficients of final consumption services (i.e., the a_j's from equation 16). Recall that positive values for the a_j's are indicative of the durability of consumption services, while negative a_j's indicate a dominance of habit persistence and the complementarity of consumption services. The key result in figure 2 is that, starting from an annual model that exhibits global habit persistence, our consistent high-frequency calibration technique generates a weekly model that displays local substitutability with long-run habit persistence (while guaranteeing that all steady-state variables are consistently aggregated over time and across frequencies). The dominance of habit persistence effects at quarterly and annual frequencies has been documented by (Ferson and Constantinides (1991)), while the existence of local substitutability at higher frequencies has been advocated theoretically by Hindy and Huang (1992) and documented empirically by (Gallant and Tauchen (1989); Dunn and Singleton (1986)). Furthermore, Heaton (1995) has shown that the existence of local substitutability with long-run habit persistence can improve the fit of models of that rely on either pure substitutability or pure habit persistence.

Lastly, note that the standard high-frequency calibration approach also generates local substitutability, but it is implausibly strong with substitution effects dominating habit effects even as far out as 52 weeks. This appears to be inconsistent with the higher-frequency evidence on consumption substitutability and habit formation. For example, using simulated method of moments, Heaton (1995) estimates that substitutability dominates until somewhere between 12 and 19 weeks when complementarity takes over, which is in agreement with our consistent calibration results.

5 Conclusions

We have outlined a new consistent approach for calibrating high-frequency economic models where agents are allowed to make decisions on a more frequent basis than the data are sampled. This approach has the advantage that it adheres to consistent aggregation of steady-state stock and flow variables across time and is sufficiently flexible to incorporate any relevant empirical evidence on high-frequency parameters. We then show how our high-frequency calibration approach can improve the fit of a standard RBC model by generating additional volatility in hours worked and improve the fit of a standard consumption-based asset pricing model by allowing for local substitutability and long-run habit persistence in consumption.

6 Appendix

In this appendix, we provide additional details on the derivation of the consistent, high-frequency relations for our RBC example. Under our consistent high-frequency calibration approach, we begin by imposing the steady-state constraints $F = nF_*$ and $S = S_*$ on the steady-state relations for the RBC economy. This produces the following steady-state equations in τ-time

$$\begin{aligned} \beta &= e^{\mu}(1+1.5\psi n^2 q_*^2)(1+n\alpha Y_*/K_* - \delta + 0.5\psi n^2 q_*^2(nq_* + 1.5(1-\delta)))^{-1} \\ w &= \phi l_*^{\eta-1} C_*/N_* \\ Y_* &= C_* + K_*(\delta - 1 + e^{\mu})(1+0.5\psi n^2 q_*^2)/n \\ nq_* &= e^{\mu} - (1-\delta) \\ Y_* &= L_*^{1-\alpha} K_*^{\alpha} \\ w &= (1-\alpha)Y_*/L_*. \end{aligned} \tag{19}$$

The corresponding steady-state equations in t-time are

$$\begin{aligned} \beta_* &= e^{\mu_*}(1+1.5\psi_* q_*^2)(1+\alpha_* Y_*/K_* - \delta_* + 0.5\psi_* q_*^2(q_* + 1.5(1-\delta_*))^{-1} \\ w_* &= \phi_* l_*^{\eta_*-1} C_*/N_* \\ Y_* &= C_* + K_*(\delta_* - 1 + e^{\mu_*})(1+0.5\psi_* q_*^2) \\ q_* &= e^{\mu_*} - (1-\delta_*) \\ Y_* &= L_*^{1-\alpha_*} K_*^{\alpha_*} \\ w_* &= (1-\alpha_*)Y_*/L_*. \end{aligned} \tag{20}$$

We then solve (19) and (20) jointly for $\boldsymbol{\gamma}_*$ in terms of $\boldsymbol{\gamma}$, n and l, which generates the following equations that are reported in the text:

$$\begin{aligned}
\beta_* &= \beta n e^{\mu/n}\left(e^{\mu} + \beta(e^{\mu/n}n - e^{\mu})\right)^{-1} \\
\delta_* &= (\delta/n) + (1 - e^{\mu/n}) + (1/n)(e^{\mu} - 1) \\
\alpha_* &= \alpha \\
\eta_* &= \eta + \log(\phi/\phi_*)/\log(l) \\
\psi_* &= \psi n^2 \\
\mu_* &= \mu/n.
\end{aligned}$$

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Figure 1. Real Business Cycle Dynamics

Normalized Responses to a One-Time Permanent Productivity Shock

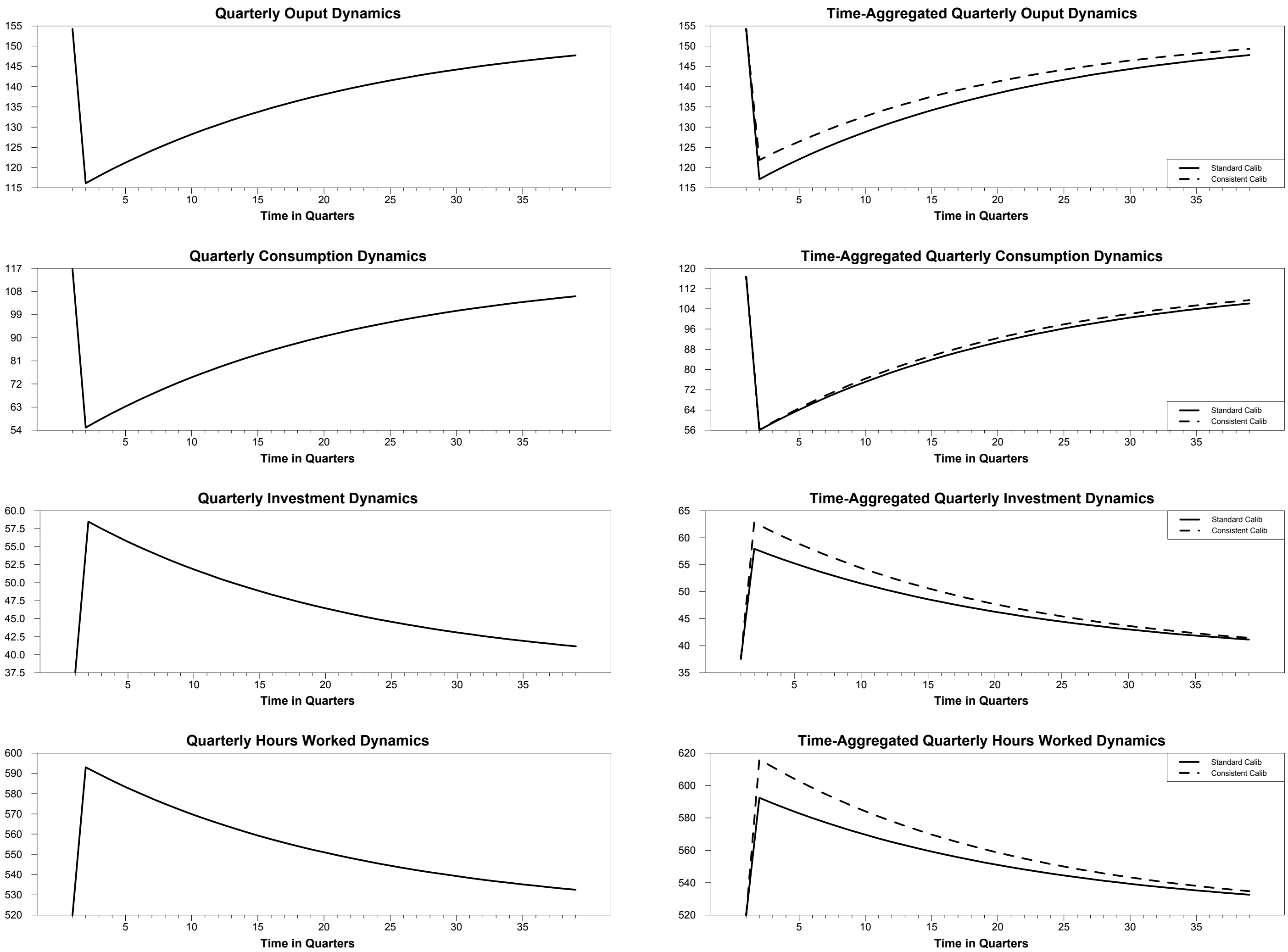

Figure 2. Final Consumption Services Dynamics