Some novel implications of replacement and scrapping

By
George C. Bitros

Abstract

The emphasis of capital theory in recent decades has moved away from the implications of useful life as an important economic variable and has turned on the microeconomic and macroeconomic consequences of investment irreversibilities. Thus the voluminous literature that has developed ignores the marked difference between replacement and scrapping and glosses over their significant implications for microeconomic and aggregate dynamics. This paper highlights the gains in explanatory power that result when useful life, replacement and scrapping are placed in the center of the analysis. It does so by considering an economy with two representative firms that differ only in that the one applies replacement and the other scrapping. Among other interesting findings, at the microeconomic level it turns out that the demand for replacement investment is not invariant with respect to the type of capital policy being applied, whereas at the macroeconomic level it is shown that we cannot obtain consistent aggregates of capital stock and replacement investment.

JEL Classification: E22
Keywords: Capital, investment, service life, replacement, scrapping

Correspondence: Professor George C. Bitros
Athens University of Economics and Business
76 Patission Street, Athens 104 34, Greece
Tel: (01) 8223545 Fax: (01) 8203301,
E-mail: bitros@aueb.gr

1. Introduction

Capital goods last for many years. So building a new piece of structure or equipment has fundamental intertemporal implications, the study of which has been as old as modern economics. In the early phase, Böhm-bawerk (1888) and Wicksell (1893) shed some light by focusing on elementary point-input point-output cases in which all capital took the form of goods in process (circulating capital). In the second phase, initially Wicksell (1923) and Åkerman (1923) demonstrated how it was possible to optimize the *useful life* of fixed capital in the absence of technological change and later Hayek (1939) drew on their results to propose a theory of business cycles. Then, in the third phase, Blitz (1958) and Westfield (1958) showed how optimal *capital longevity* or *durability* or *service life* could be computed and what was its importance for economic development. However, in the current phase, which begun with the seminal paper by Arrow (1968), with a few exceptions the emphasis has moved away from the implications of useful life as an important economic variable and has turned on the microeconomic and macroeconomic consequences of investment irreversibilities. As a result, the voluminous literature that has developed ignores the marked difference between replacement and scrapping, and hence it glosses over their significant implications for both microeconomic and aggregate dynamics.

My objective in this paper is to demonstrate the gains in explanatory power when useful life, replacement and scrapping are placed in the center of economic analysis. For this purpose, I consider an economy with two sectors. In each sector there operates a single firm facing a downward sloping demand curve. Technological progress is embodied and proceeds at a constant exogenous rate, so that the productivity of more recent vintages of capital increases at the same rate. In view of the latter condition, the two firms price their products so as to fend off competition from other firms that might attempt to enter by taking advantage of the higher productivity of newer equipment. Moreover, and most importantly, the two firms differ in their capital policies in that the one applies replacement and the other scrapping.

The results that emerge are quite novel. At the microeconomic level it is found that the useful life of capital under scrapping is always higher than under replacement. From this it follows that in the steady state the capital-scrapping firm, if in operation, renews a smaller proportion of its durables relative to the capital-replacing firm, and thus establishes that the demand for replacement investment is not invariant with respect to the type of capital policy being applied. Another result is that under scrapping the useful life of capital depends on the price elasticity of demand for output. So scrapping acts as a channel through which shifts in market demand induce capital adjustments. Still another result is that, as the price elasticity of demand for output tends to infinity, the higher useful life under scrapping converges to the lower useful life under replacement, thus highlighting the existence of a direct link between market structure, capital policy and useful life.

Turning to the macroeconomic level of analysis, it emerges that, as the policies of replacement and scrapping give rise to different useful lives, it becomes impossible to obtain consistent aggregates of capital stock and replacement investment.[1] Under the established approach this problem is ignored and the respective variables are computed by adding up estimates of the undepreciated values of the various classes of durables. These approximations are inconsistent and leave much to be desired. On the contrary, by adopting the approximation first suggested by Haavelmo (1960) this study allows for the time dimension of capital, and hence it leads to surrogates of these variables with less measurement errors. Another novel finding is that what the scrapping firm does regarding its place in the business depends on the value of the parameters, and particularly those of the interest rate and the price elasticity of demand for output. If the parameters are such that operating profits are larger than capital losses, the scrapping firm enters and stays in the industry. But if the parameters change so that operating profits fall short of capital losses, the scrapping firm exits and loses the undepreciated part of its capital. Even though it is derived from a different approach, it is worth noting that this finding corroborates the evidence discovered by Veracierto (2002), according to which investment irreversibilities arise from an increase in the effective depreciation of capital. Last, but not least, the results show that, given the interest rates that were observed in the United states during the 1949-1968 period, if the price elasticity of demand for output is allowed to adjust appropriately, the model tracks very well the replacement data used by Feldstein and Foot (1971) and Eisner (1972).

The rest of the paper is organized as follows. Section 2 presents the model and highlights its microeconomic and macroeconomic properties. Some comments regarding

[1] This impossibility is long known and goes back to the great debates in the 1950s and 1960 regarding the role of capital as a factor of production. Simply put, it arises because the various classes of durables cannot be transformed into durables of some standard durability.

the definitions adopted for the construction of certain key macroeconomic variables are also found in the same place. In Section 3 the analysis focuses on the microeconomic and aggregate dynamics of the model. More specifically, initially it traces the responses of the main variables to a once and for all change in the interest rate and the price elasticity of demand for output by scrapping firms, and then it attempts to replicate the replacement data in the manufacturing sector of the United States from 1949 to 1968. Finally, Section 4 summarizes the conclusions and offers a few suggestions for further research.

2. The model

Consider an economy with two firms and any number of workers. Each firm consists of two lines of production, one constructing an intermediate good called capital solely by means of labor and another producing a final good by combining each unit of capital with a fixed number of workers. Let firm X produce electricity, i.e. a necessity, and firm Y produce tennis rackets, i.e. a luxury. In year v, firm X uses $K_X(v)$ units of capital, measured in megawatts, whereas firm Y employs a lathe capable of cutting $K_Y(v)$ thousand rackets per year. Usage does not wear capital because its effects are exactly offset by maintenance. But $K_X(v)$ and $K_Y(v)$ lose constantly value because, to protect their markets from new entrants, firms price the goods produced with them so as to transfer all benefits from technological progress to consumers. Finally, assume that while firm X conducts business as if its monopoly will last forever, firm Y plans to exit at the end of the useful life of the lathe and re-enter if the prevailing market conditions at that time warrant it. The question that I want to investigate is whether the difference in the behavior of the two firms regarding the outlook of re-investment opportunities has important implications for the economy. To this end we proceed as follows.

2.1 Microeconomics

Since in the economy under consideration there are two firms, which behave differently at least with respect to their capital policies, we will analyze each one separately.

Firm *X*

Assume that firm X faces a demand curve of the constant elasticity type:

$$X(v) = N_X(v)[P_X(v)]^{\eta_X} \ . \qquad \textbf{(1)}$$

In this equation $X(\upsilon)$ stands for output in year υ, $N_X(\upsilon)$ denotes a multiplicative constant, $P_X(\upsilon)$ is the price of output, and $\eta_X < -1,\ X(\upsilon) > 0,\ N_X(\upsilon) > 0,\ P_X(\upsilon) > 0$.

During year υ the firm uses $K_X(\upsilon)$ units of capital, all of which are equally productive because they embody the same technology. Hence,

$$b_X(\upsilon) = \frac{K_X(\upsilon)}{X(\upsilon)}, \tag{2}$$

defines the productivity of equipment, $b(\upsilon)$ being the capital-output coefficient.

Capital built after year υ is expected to be more productive because of technological progress. So to allow for this consideration we set:

$$b_X(t) = b_X(\upsilon) e^{\mu(t-\upsilon)}, \tag{3}$$

where $\upsilon < t,\ \mu < 0,$ and μ is the economy wide rate of technological progress.

Next, regarding the minimum amount of labor required to built a unit of electricity generating capacity, we assume that:

$$\beta = M_X(0)[b_X(0)]^{\gamma}, \tag{4}$$

where $M_X(0) > 0$ and $\gamma <$ -1. This implies that the minimum labor required to build a unit of capital embodying the new technology exceeds that required to build a unit of capital from older vintages.

Finally, let the service life of electric generators be T_X, so that $K_X(\upsilon)$ is kept in operation for the time interval $\upsilon < t < \upsilon + T_X$. But during these years other firms may enter the market by purchasing newer, and hence more productive, generators. So to fend off potential competition firm X reduces prices at the rate of technological progress by setting:

$$P_X(t) = P_X(\upsilon) e^{\mu_X(t-\upsilon)}. \tag{5}$$

Drawing on the above it can be shown that, if the salvage value of equipment on retirement is zero, the net worth of a unit of new capital at $\upsilon = 0$ is given by:

$$n_X(0)=\int_0^{T_X}[\frac{P_X(0)}{b_X(0)}e^{\mu t}-w]e^{-\sigma t}dt-\beta w=\frac{P_X(0)}{b_X(0)}\frac{1-e^{(\mu-\sigma)T_X}}{\sigma-\mu}-w\frac{1-e^{-\sigma T_X}+\beta\sigma}{\sigma}, \quad \textbf{(6)}$$

where w and σ denote respectively the economy wide rates of wages and interest.

Firm X behaves as if its monopoly will last forever. This implies that at any period it must have no more and no less than the necessary generating capacity to meet the demand for electricity. For if it has less it will be losing sales and if it has more it will be wasting resources. As a result, it is led to maximize the present value of profits from an infinite series of equidistant replacements given by:

$$\Pi(T_X,P_X(0))=\frac{b_X(0)n_X(0)X(0)}{1-e^{-(\mu-\sigma)T_X}}=b_X(0)N_X(0)[P_X(0)]^{\eta_X}\left[\frac{P_X(0)}{b_X(0)}\frac{1}{\sigma-\mu}-\frac{w}{\sigma}\frac{1-e^{-\sigma T_X}+\beta\sigma}{1-e^{(\mu-\sigma)T_X}}\right]. \quad \textbf{(7)}$$

From the first order conditions for $P_X(0)$ and T_X we obtain:

$$P_X(0)=\frac{\eta_X}{1+\eta_X}\frac{\sigma-\mu}{\sigma}\frac{1-e^{-\mu T_X}+\beta\sigma}{1-e^{(\mu-\sigma)T_X}}b_X(0)w\,, \quad \textbf{(8)}$$

$$g(T_X)=\sigma e^{-\mu T_X}-\mu e^{-\sigma T_X}=(1+\beta\sigma)(\sigma-\mu). \quad \textbf{(9)}$$

Equation **(9)** does not permit an explicit solution for T_X. However, it can be established that one and only one positive solution for T_X exists.

To sketch the proof, consider Figure 1 below. Setting $T_X=0$, we see that the left-hand side of **(9)** turns into $\sigma-\mu$. Next, letting T_X rise above zero and taking the de-

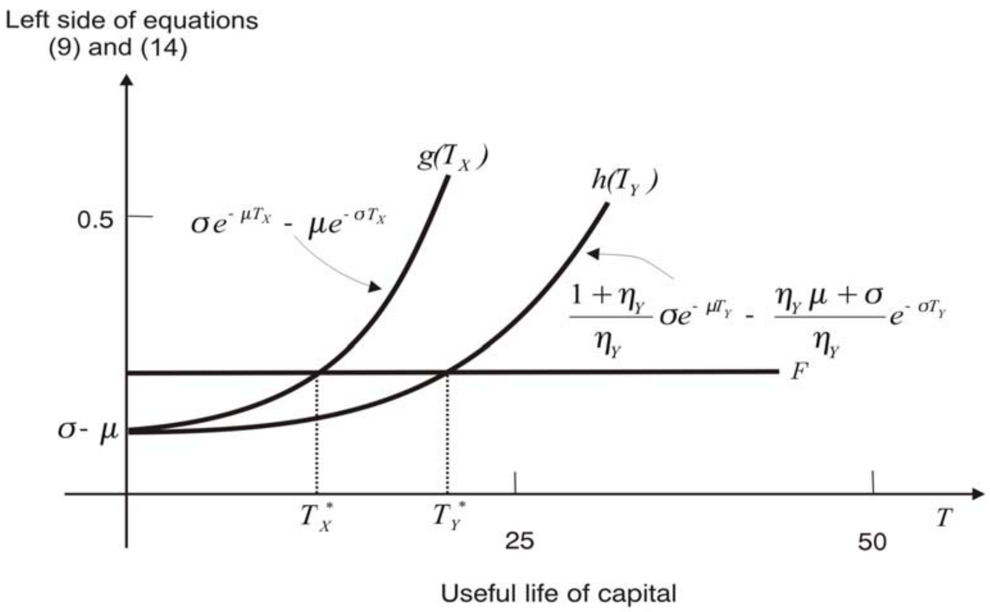


Figure 1

rivative, we can ascertain that the left-hand side of **(9)** rises without bound with rising T_X. These findings are depicted by the upward sloping curve $g(T_X)$. Finally, looking at the right-hand side of **(9),** observe that it defines a horizontal line, which cuts the vertical axis above the value $\sigma - \mu$. Therefore, curve $g(T_X)$ is bound to cut the horizontal line just once, giving the optimal service life T_X^*.

At this point it will prove useful for the analysis later on to find the direction of change of optimal useful life as the parameters in **(9)** change. For this purpose, taking the partial derivatives we ascertain that $\partial T_X^* / \partial \sigma > 0$, $\partial T_X^* / \partial \mu > 0$ and $\partial T_X^* / \partial \beta > 0$. These imply that the optimal useful life of durable producers' goods is longer the higher the interest rate, the slower is technological progress, and the costlier is their construction cost in terms of the minimum required labor to built a single unit. Intuitively, these results make sense since: The costlier the producers' goods and the higher the interest rate, the more urgent it becomes to save capital cost by lengthening their useful life. Also, the slower the technological progress, the less difference between the efficiencies of producers' goods of consecutive vintages, and hence the lower the pressure of retirement.

Introducing T_X^* and **(4)** into **(8)** and using the resulting expression in conjunction with **(1)**, **(2)** and **(6)** we find:

$$n_X^*(0) = \left[\frac{\eta_X}{1+\eta_X} - 1\right] \frac{1 - e^{-\sigma T_X^*} + \beta\sigma}{\sigma} w \quad \textbf{(10)}$$

$$K_X^*(0) = N_X(0)\left[\frac{\eta_X}{1+\eta_X} \frac{\sigma - \mu}{\sigma} \frac{1 - e^{-\sigma T_X^*} + \beta\sigma}{1 - e^{(\mu-\sigma)T_X^*}} w\right]^{\eta_X} \left[\frac{\beta}{M_X(0)}\right]^{\frac{(1+\eta_X)}{\gamma}}. \quad \textbf{(11)}$$

From **(10)** and **(11)** we observe that both the net present unit value $n_X^*(0)$ and the quantity $K_X^*(0)$ of physical capital depend also on T_X^*. But from **(9)** we know that T_X^* depends in turn on the capital policy adopted by the firm. Consequently, under a policy of equidistant replacements the construction cost and the market value of the surviving cost of capital employed by firm X would be respectively $\beta w K_X^*(0)$ and $n_X^*(0) K_X^*(0)$.

Firm Y.

Now let me turn to Firm Y. As I said in the introduction this firm plans to exit at the end of the useful life of the lathe and re-enter if market conditions warrant it. So, assuming again that the value of the lathe on retirement is zero, firm Y maximizes:

$$\Pi_Y(T_Y, P_Y(0)) = b_Y(0)N_Y(0)[P_Y(0)]^{\eta_Y}\left[\frac{P_Y(0)}{b_Y(0)}\frac{1-e^{(\mu-\sigma)T_Y}}{\sigma-\mu} - w\frac{1-e^{-\sigma T_Y}+\beta\sigma}{\sigma}\right], \qquad \textbf{(12)}$$

with respect to T_Y and $P_Y(0)$. In this expression it should be observed that the parameter β of minimum required labor to build a unit of lathe capacity is the same as that in equation **(4)**. This implies that:

$$\beta = M_Y(0)[b_Y(0)]^{\gamma}. \qquad \textbf{(4')}$$

The rational for this assumption is that the minimum required labor to build a unit of productive capacity should be the same across firms, because differences in productivity in their capital building departments would tend to vanish through competitive reallocation of workers among firms.

From the first order conditions for maximization of **(12)** we obtain:

$$P_Y(0) = \frac{\eta_Y}{1+\eta_Y}\frac{(\sigma-\mu)}{\sigma}\frac{1-e^{-\sigma T_Y}+\beta\sigma}{1-e^{(\mu-\sigma)T_Y}}b_Y(0)w \qquad \textbf{(13)}$$

$$h(T_Y) = \frac{1+\eta_Y}{\eta_Y}\sigma e^{-\mu T_Y} - \frac{\eta_Y\mu+\sigma}{\eta_Y}e^{-\sigma T_Y} = (1+\beta\sigma)(\sigma-\mu)\ . \qquad \textbf{(14)}$$

Looking at equation **(14)** we observe that, as T_Y goes to zero, the left-hand side turns into $\sigma-\mu$. Hence, both $g(T_X)$ and $h(T_Y)$ start from the same point on the vertical axis in Figure 1. Next let T_Y rise above zero and take the derivative of $h(T_Y)$. As T_Y rises without bound, this derivative remains positive, which means that $h(T_Y)$ always rises. Then the question is whether $h(T_Y)$ rises to the left or to the right of $g(T_X)$. Comparing the derivatives of the left-hand sides of equations **(9)** and **(14)** we can establish that $h(T_Y)$ rises always to the right of $g(T_X)$. This implies in turn that $h(T_Y)$ will cut the horizontal line F to the right of T_X^*, say at T_Y^*, so the optimal service life of the lathe will be longer. Therefore, since in the absence of parameter changes firm Y will find it profitable to exit from and re-enter into its market every T_Y^*, the stationary values for $n_Y^*(0)$ and $K_Y^*(0)$ can be found as before. In particular, using:

$$n_Y(0) = \int_0^{T_Y} [\frac{P_Y(0)}{b_Y(0)} e^{\mu t} - w] e^{-\sigma t} dt - \beta w = \frac{P_Y(0)}{b_Y(0)} \frac{1 - e^{(\mu-\sigma)T_Y}}{1-\mu} - w \frac{1 - e^{-\sigma T_Y} + \beta\sigma}{\sigma}, \quad \textbf{(6′)}$$

in conjunction with **(1)**, **(2)** and **(13)**, we obtain:

$$n_Y^*(0) = \left[\frac{\eta_Y}{1+\eta_Y} - 1\right] \frac{1 - e^{-\sigma T_Y^*} + \beta\sigma}{\sigma} w, \quad \textbf{(15)}$$

$$K_Y^*(0) = N_Y(0) [\frac{\eta_Y}{1+\eta_Y} \frac{\sigma - \mu}{\sigma} \frac{1 - e^{-\sigma T_Y^*} + \beta\sigma}{1 - e^{(\mu-\sigma)T_Y^*}} w]^{\eta_Y} [\frac{\beta}{M_Y(0)}]^{\frac{(1+\eta_Y)}{\gamma}} . \quad \textbf{(16)}$$

From these we surmise that, once we find the optimal useful life of the lathe T_Y^*, the other two key variables, i.e. the net present unit value $n_Y^*(0)$ and the stock of physical capital $K_Y^*(0)$, are fully determined.

Finally, notice that in the absence of parameter changes the expenditures for replacement investment by each firm, I_X^r, I_Y^r, would amount to:

$$I_X^r = \beta w \frac{K_X^*(0)}{T_X^*}, \quad \textbf{(17)}$$

$$I_Y^r = \beta w \frac{K_Y^*(0)}{T_Y^*} . \quad \textbf{(18)}$$

2.2 Macroeconomics

Let us turn now from microeconomics to macroeconomics. Since the two firms produce their goods by means of different equipment, the question that arises is how to define and measure the capital employed in the economy. If electricity generators and lathes were perishable goods like lemons and oranges, the answer would be very easy.

Simply, we would multiply for each price times quantity and we would sum the results to compute their aggregate value in the economy. But this approach is untenable under the present circumstances because the amount of durables employed in the economy is determined by two variables, i.e. quantity and useful live. Hence, we must devise a different approach.

According to Haavelmo (1960, pp. 95-102), in the absence of technological change, a reasonable approximation would be the following:

$$K^*(0) = w\beta[K_X^*(0) + K_Y^*(0)\frac{1-e^{-\sigma T_X}}{1-e^{-\sigma T_Y}}]. \quad \textbf{(19)}$$

where the new symbol $K^*(0)$ is the undepreciated purchase cost of aggregate capital in the economy. The rationale of this formulation is that the multiplication of $K_Y^*(0)$ by the indicated ratio adjusts it to the same useful life as $K_X^*(0)$, and hence $K^*(0)$ becomes a measure of the aggregate capital stock of standard durability. However, observe that since the difference in capital policies leads to $T_X^* < T_Y^*$, the proposed adjustment results always in underestimation of the capital stock at the individual level of firm Y. For this reason, this procedure should be considered a rough approximation, but certainly an improvement over the customary, yet unfounded method of measuring $K^*(0)$ simply as $K_X^*(0) + K_Y^*(0)$.

Finally, in the light of the preceding adjustment, the expenditure for aggregate replacement investment, I^r, is given by:

$$I^r(0) = w\beta[I_X^*(0) + I_Y^*(0)\frac{1-e^{-\sigma T_X}}{1-e^{-\sigma T_Y}}] . \quad \textbf{(20)}$$

3. Responses of equilibrium solutions to changes in key parameters

Assume that the parameters in the problem take the following values:

$$\sigma = 0.05,\ \mu = -0.02,\ \beta = 8,\ \eta_X = -4,\ \eta_Y = -20$$
$$\gamma = -4,\ N_X(0) = 100000,\ N_Y(0) = 10000,\ M_X(0) = 0.1$$
$$M_Y(0) = 0.01,\ w = 7.$$

Solving first equations **(9)** and **(14)**, we obtain the equilibrium values for T_X^* and T_Y^*. Then introducing these values into the equations of the model, there emerges the following solution:

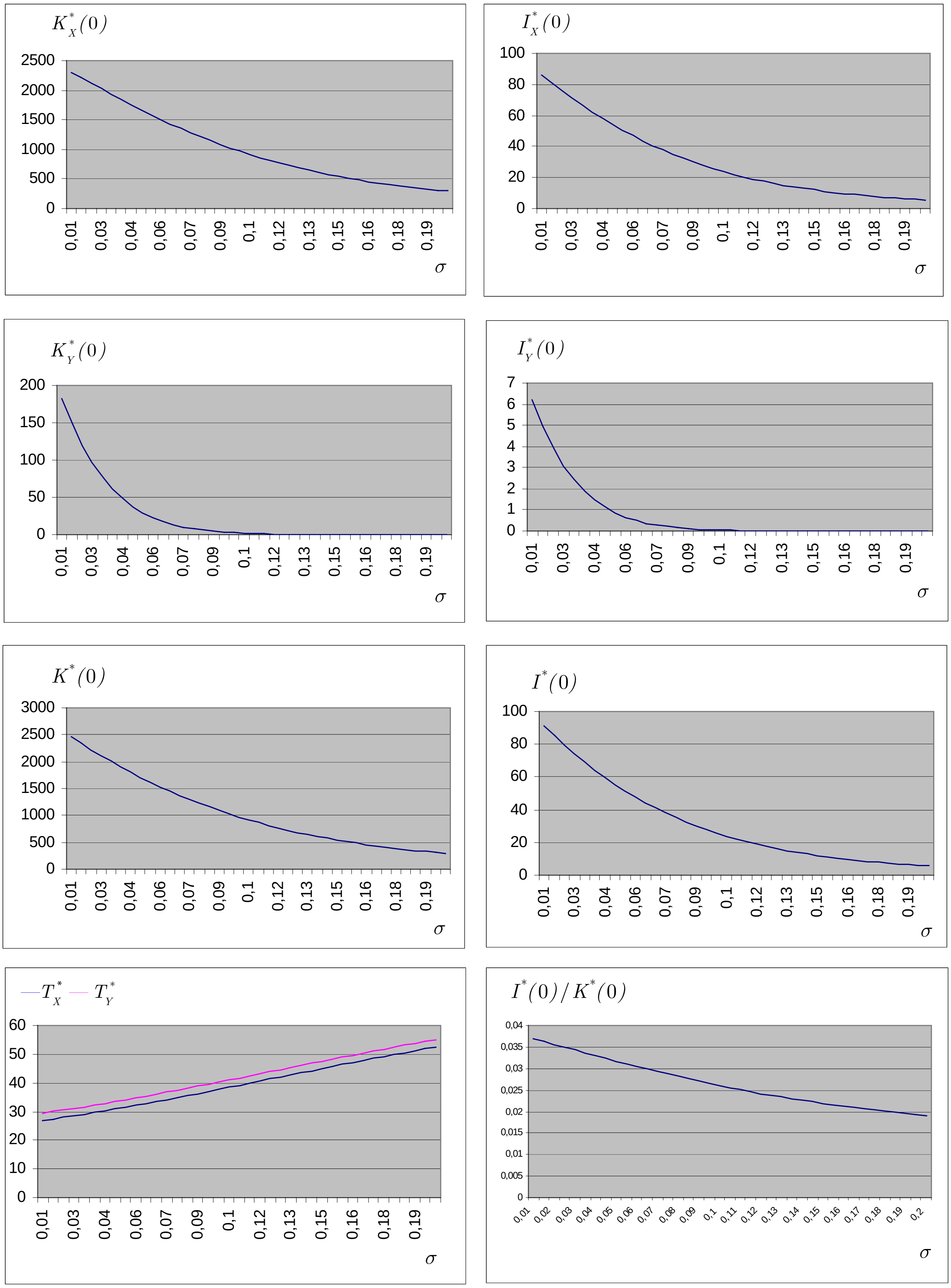
$$T_X^*(0) = 31.48 \quad T_X^*(0) = 34.07 \quad n_X^*(0) = 55.66 \quad n_Y^*(0) = 8.97$$
$$K_X^*(0) = 1589.89 \quad K_Y^*(0) = 29.24 \quad K^*(0) = 1616{,}14$$
$$I_X^*(0) = 50.49 \quad I_Y^*(0) = 0.86 \quad I^*(0) = 51.32$$

where all capital and replacement investment related variables are expressed in construction cost prices. From this solution it can be observed that the Haavelmo approximation to aggregate capital and replacement investment is very successful since $K^*(0)$ and $I^*(0)$ come extremely close to the sum of the underlying microeconomic variables and the useful life of aggregate capital is $T_X^*(0) = 31.48$ periods. Of course, as long as the parameters remain fixed, the two firms and the economy will continue to be in stationary equilibrium.

Now let us investigate what happens to the equilibrium solution, if we allow the interest rate to take on different values. The results from this experiment are shown in Spreadsheet 1 and the associated graphs in Figure 1. Looking at the values of $K_Y^*(0)$ and $I_Y^*(0)$, we observe that, when the interest rate rises above 12.5%, both variables become nearly zero. This implies that the scraping firm leaves the industry of tennis rackets and takes a charge for the cost of the undepreciated value of capital stock. However, observe that at this particular level of the interest rate the net present unit value of capital continues to be positive, meaning that if the firm remained in business it would realize some operating profits. So the question is why does it decide to close down. The answer is that, since the firm must balance constantly the operating results with the results on capital account, it cannot afford to stay in business because the rise in the interest rate renders capital loses higher than operating profits. This is exactly the Veracierto (2002) effect of irreversibility that was mentioned above.

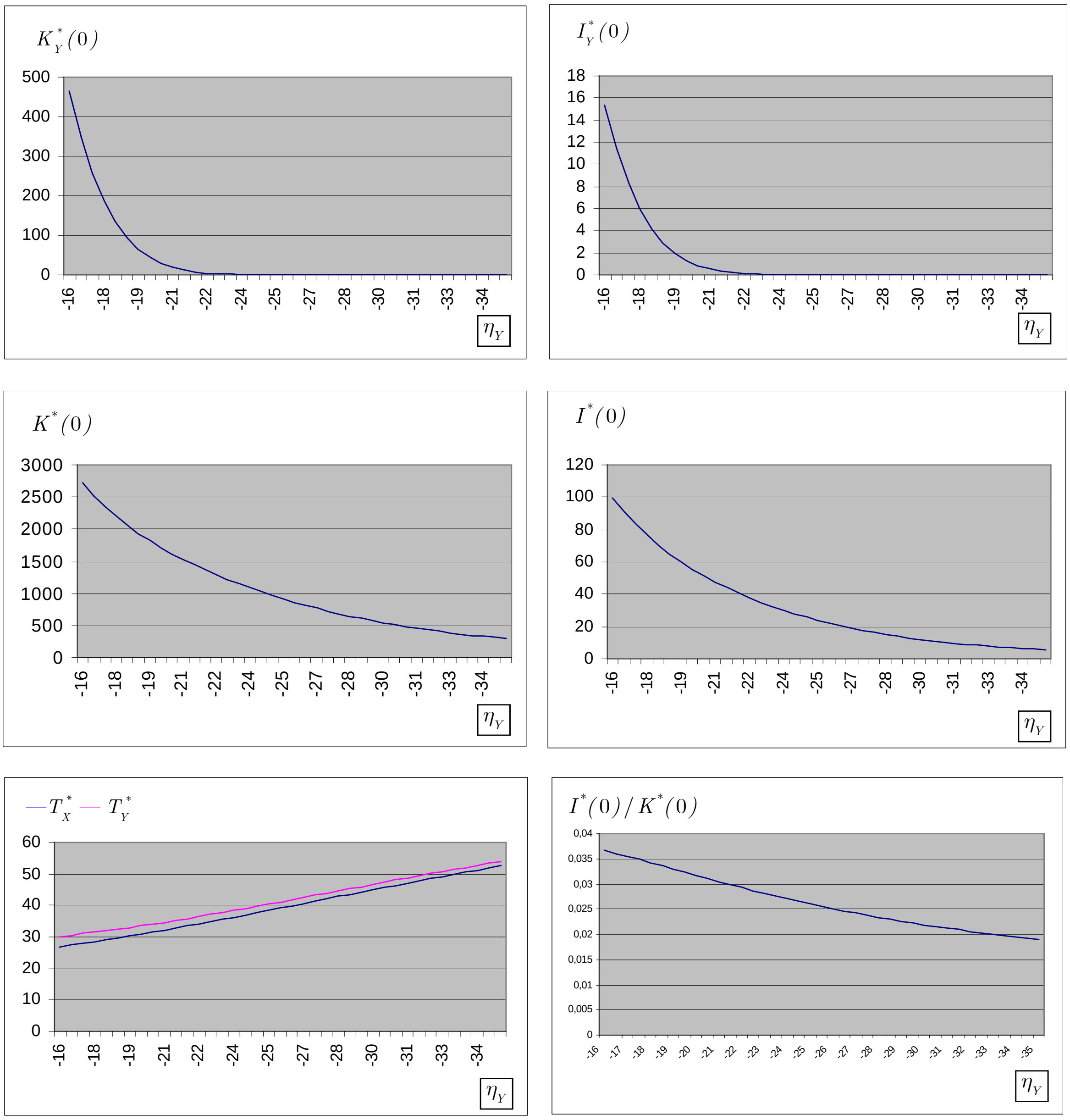
In the next experiment the interest rate and the price elasticity of demand for tennis rackets were allowed to change independently by certain steps. In particular, the said price elasticity was assumed to change in the same direction with the interest rate on the presumption that, as the interest rate increases, the economic conditions worsen, thus

Figure 1: Equilibrium solutions for different values of σ



raising the price elasticity of demand for tennis rackets and slowing down the purchasing of luxuries. The graphs in **Figure 2** report the results for this experiment. From them it turns out that, as the elasticity of demand for tennis rackets increases, the adverse effects of the rising interest rate are strengthened, so Firm Y is forced to quit from the industry earlier. Why may this finding reveal a genuine effect of shifting output demand on the capital decisions of the scrapping firm is not difficult to explain.

Figure 2: Equilibrium solutions for different values of σ and η_Y



This happens because with the increasing price elasticity of demand for tennis rackets the marginal revenue declines faster than if the demand curve remained stable. This hurts the profitability of the scraping firm and precipitates its exit. So, whereas in the absence of shifts in demand for tennis rackets the firm would quit the industry at an interest rate of 12.5%, now with the shifting demand the scrapping firm is forced to quit at an interest rate of 9%.

From the joint graphs of T_X^* and T_Y^* it is also worth observing that as the price elasticity of demand for tennis rackets increases going to minus infinity, the useful life of capital under scrapping converges to the useful life of capital under replacement. In particular, from the following proof:

$$h(T) = \frac{1+\eta}{\eta}\sigma e^{-\mu T} - \frac{1+\mu\eta}{\eta}e^{-\sigma T} \Rightarrow$$

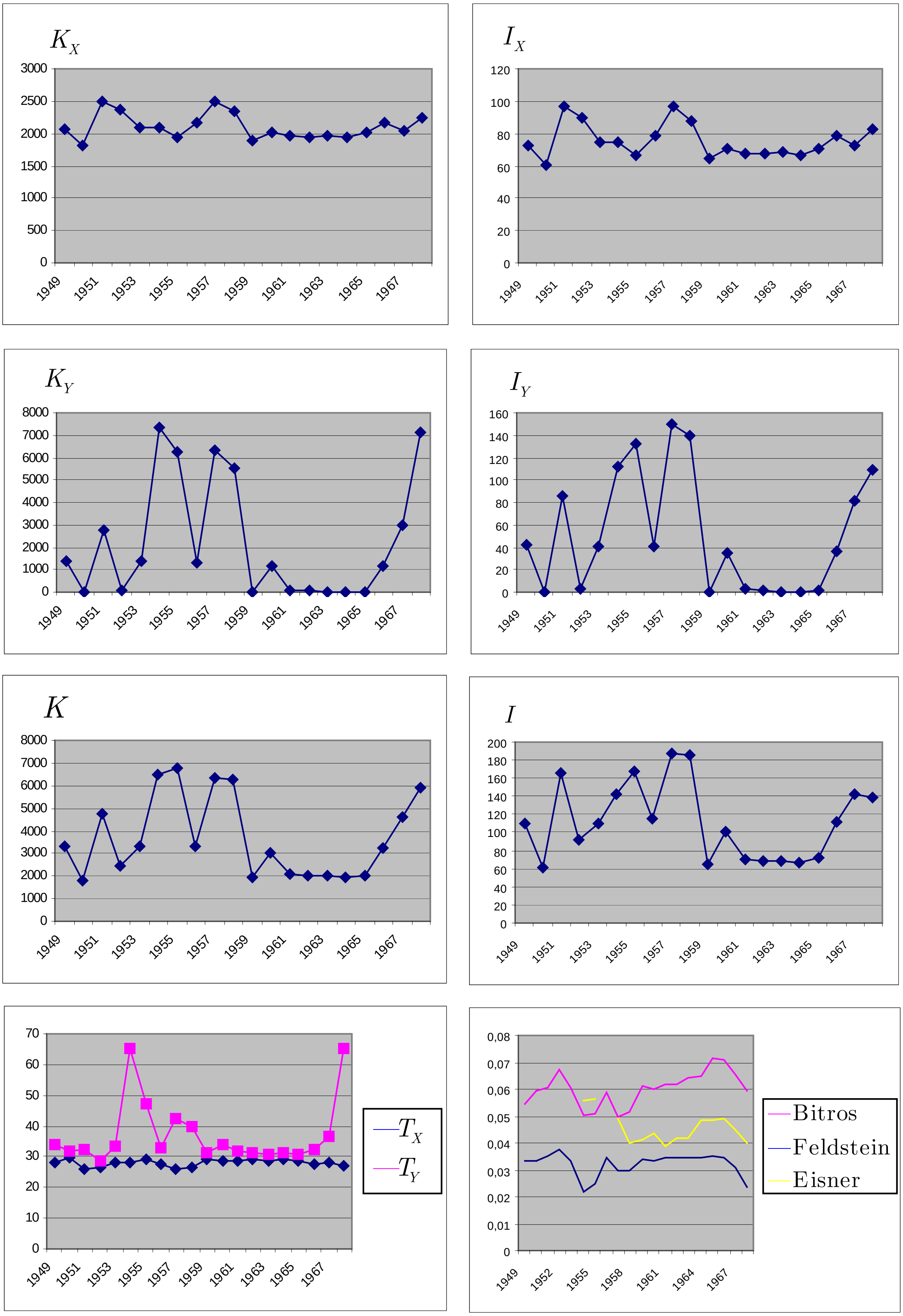
$$= (1+\frac{1}{\eta})\sigma e^{-\mu T} - (\mu + \frac{\sigma}{\eta})e^{-\sigma T}. \qquad \textbf{(21)}$$

$$Lim_{\eta\to-\infty} h(T) = Lim_{\eta\to-\infty}\sigma(1+\frac{1}{\eta})e^{-\mu T} - (\mu + \frac{\sigma}{\eta})e^{-\sigma T} = \sigma e^{-\mu T} - \mu e^{-\sigma T} = g(T),$$

we may surmise that, even if the market for tennis rackets became perfectly competitive, in which case the useful lives of the two types of capital would become identical, the differences in microeconomic and macroeconomic dynamics due to the differences in replacement and scrapping would still continue to exist.

The last experiment was design to shed light on the following question. Assume that a change in the interest rate alters the price elasticity of demand for output produced by scrapping firms in the same direction. According to the preceding analysis, such a change would affect the useful age of capital in the respective activities and filter through to the aggregate replacement investment and capital stock. So, given an actual economy, to what extent changes in η_Y caused by changes in the interest rate might be responsible for the variation observed in the replacement investment capital stock ratio? To highlight it I employed the data reported by Feldstein and Foot (1971) and Eisner (1972) for U. S Manufacturing during the 1949-1968 period. In particular, using the series of real long term interest rates in the USA during this period, in conjunction with the parameter values indicated above, the computer was programmed to calculate the values for η_Y which brought each year the replacement investment capital stock ratio from the model closest to the one observed. As shown in **Figure 3** at the microeconomic level the capital stock and the replacement investment for **Firm *X*** evolve over time by moving smoothly and narrowly around a flat trend. On the contrary, the same variables for **Firm *Y*** exhibit violent and wide variations around a flat trend, mainly because the shifts in η_Y render the optimal useful life of the associated capital stock

Figure 3: Replacement investment capital stock ratio and the explanatory power of η_Y



T_Y^* very volatile. Moreover, when we turn to the macroeconomic level of analysis, we observe that the aggregates of capital stock and replacement investment are influenced predominantly by the decisions of the scrapping firm. From them we observe that the changes in the price elasticity of demand for output by scrapping firms are capable of explaining a large percentage of the variation in the observed replacement investment capital stock ratio. Moreover, in order to quantify the explanatory power of the model, we regressed this ratio on the interest rate and the price elasticities of demand and obtained the following results:

$$\frac{I_t}{K_{t-1}} = \underset{(21.1)}{0.029} - \underset{(-2.24)}{0.147\sigma_t} - \underset{(-5.6)}{0.0004\eta_Y}, \quad \bar{R}^2 = 0.61, \quad \text{NOB}=20.$$

These establish clearly that market conditions, as measured by the price elasticity of demand for output by scrapping firms, constitute a key determinant of replacement investment to capital stock ratio. Also they suggest that by focusing on scrapping rather than replacement we may be able to understand better the way in which capacity adjustments at the plant level influence the macroeconomic dynamics of investment. In this regard it may be useful to recall the business-cycle model proposed by Hayek (1939). His intuition was that during a period of prosperity eventually it becomes profitable for firms to decrease the capital intensity (durability) of their operations by switching their orders from heavy to less durable equipment. In turn, this switching of orders causes financial difficulties to heavy-equipment manufacturers, which by spreading throughout the economy bring about recession. So, if instead of linking this effect to prices *a la* Hayek, we assume that as prosperity advances the interest rate and the elasticity of demand decline, what we would expect to observe according to the model would be an increase in average durability, and hence a decline in the replacement investment capital stock ratio. But then, after the interest rate and the price elasticity of demand start to climb, due to Hayek's effect average durability begins to decline and the replacement investment capital stock ratio to rise, thus pushing the economy into recession. This sequence of events is nicely corroborated by the results of the regression, which show a statistically significant negative relationship between the replacement investment capital stock ratio, on the one hand, and the interest rate and the price elasticity of demand on the other.

4. Conclusions and suggestions for further research

Observe that the price elasticity of output demand is present in **(14)** but absent from **(9)**. This implies that changes in market conditions introduced through changes in η_Y would affect service lives under scrapping but not under replacement. To highlight the importance of this finding for microeconomic and aggregate dynamics of replacement of investment, a general equilibrium model was proposed in which firms are of two types: one operating under replacement and another operating under scrapping. From its solution there emerged three main results. The first of them is that the useful life of capital under scrapping is always higher than that under replacement. The second is that the differences in useful lives under replacement and under scrapping make it extremely difficult, if not impossible, to construct economy-wide aggregates for replacement investment and capital stock. And last, but not least, the third finding is that, whereas scrapping under perfect competition leads to the same useful life as under replacement, the differences of the two capital policies at the macroeconomic level continue to be significant.

Moreover, the model was solved numerically in order to investigate the properties of equilibrium solutions to changes in certain key parameters. The results from one experiment showed that, when the interest rate rises above a certain level, the scrapping firm exits form its industry and absorbs the losses due to the early abandonment of the undepreciated value of its capital stock. This corroborates the finding by Veracierto (2002) regarding the channel through which investment irreversibility affects capital policies. Another experiment showed that, if in addition to interest rate, the price elasticity of demand for output produced by the scrapping firm shifts in the same direction, the exit of the scrapping firm from its industry occurs at an even lower rate of interest. Finally, in a third experiment, I employed the data reported in Feldstein and Foot (1971) and Eisner (1972) to check on the ability of the model to explain the replacement investment capital stock ratio in U. S. Manufacturing from 1949 to 1968. From the results it emerged that, if the price elasticity of demand for output by scrapping firms is computed so as to bring the computed value of this ratio as close as possible to the one observed, the proposed model demonstrates significant explanatory power.

In light of the above analytical and computational results, several extensions appear to be in order. One such extension is to compare the implications of replacement and scrapping in the framework of a real business cycle model. Another important one would be to investigate the consistency of aggregate capital series used in growth accounting. For if the differences in the useful lives of various categories of real assets are not allowed for properly, this series may be beset by measurement errors of unknown magnitudes and directions. Still another extension would be to embed scrapping in empirical models of investment and contrast their properties to conventional type models involving steady state replacement.

Bibliography

1. Åkerman, G., (1923), **Realkapital and Kapitalzins,** Stockholm,.
2. Arrow, K. J., (1968), "Optimal Capital Policy with Irreversible Investment," in J. N. Wolfe, ed., **Value, capital and growth: Papers in Honour of Sir John Hicks** (Chicago: Aldine Publishing Company), pp.1-19.
3. Blitz, R. C., (1958), "Capital Longevity and Economic Developm*ent", American Economic Review*, Vol. 48, pp. 313- 329.
4. Eisner, R., (1972), "Components of Capital Expenditures: Replacement and Modernization Versus Expansion," *The Review of Economics and Statistics,* Vol. LIV, pp. 297-305.
5. Feldstein, M. S., and Foot, D. K., (1971), "The Other Hall of Gross Investment: Replacement and Modernization Expenditures", *The Review of Economics and Statistics*, Vol. LIII, pp. 49-58.
6. Haavelmo, T., (1960), A Study in the Theory of Investment (Chicago: The University of Chicago Press).
7. Hayek, F. A., (1939), ***Profits, Interest and Investment*** (London: Routledge & Kegan Paul, Ltd.).
8. Veracierto, M. L., (2002), "Plant-Level Irreversible Investment and Equilibrium Business Cycles," *American Economic Review*, Vol. 92, pp 181-197.
9. Westfield, F. M., (1958), "A Mathematical Note on Optimum Longevity," *American Economic Review*, Vol. 48, pp. 329-332.
10. Wicksell, K., (1893), **Über Wert, Kapital und Rente,** Jena, especially pp. 90-143.
11. ----------------- (1923), "Realkapital och Kapitalranta," *Ekonomisk Tidskrift*, Vol. 25, pp. 145-180. Translated as the second appendix to **Lectures in Political Economy, 1**, London, 1934.