# An Estimate of the Elasticity of Intertemporal Substitution

# in a Production Economy

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## Abstract

The elasticity of intertemporal substitution (EIS) at the macro level has been estimated mostly based on endowment economy models and these estimates are very sensitive to the choice of interest rates that are used for estimation. Estimates based on production economy models do not need information on interest rates but require endogenous growth models that are free from both scale effects and the strong influence of population growth. Such a model is constructed and EIS is estimated without information on interest rates. The result indicates that EIS at the macro level is as low as 0.09.

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# I. INTRODUCTION

The elasticity of intertemporal substitution (EIS) at the macro level has been estimated mostly based on endowment economy models.[1] Because endowment economy models ignore capital, the marginal product of capital plays no role in the models and an exogenously given real interest rate represents all aspects of production. Hence, information on interest rates or returns on assets are indispensable to estimate EIS based on an endowment economy model. As a result, these estimates are very sensitive to the choice of interest rates or returns on assets that are used for estimation. It is a serious problem because there are various interest rates and returns on assets. They are diverse widely because they are determined not only by the marginal product of capital but by other various factors e.g. taxes, regulations, the depreciation of capital, risks and so on. As a result, very different values of EIS are estimated according to interest rates and returns on assets that are used for estimation. This problem is particularly emphasized in Mulligan (2002, 2004) and McGrattan and Prescott (2003).

The purpose of the paper is to explore an estimation method of EIS that does not require information on interest rates or returns on assets, i.e. an estimation method that is based not on an endowment economy but on a production economy. In a production economy, the familiar Euler equation in case of a Harrod neutral production function is $\rho = \dfrac{\dfrac{\dot{c}_t}{c_t}}{(1-\alpha)\dfrac{y_t}{k_t} - n - \delta - \theta}$

where $\rho$ is EIS, $y_t$ is output per capita, $c_t$ is consumption per capita, $k_t$ is capital input per capita, $n$ is the growth rate of population, $\delta$ is the rate of depreciation, $\theta$ is the rate of time preference, and $\alpha$ is a constant. Thereby, if the values of $n$, $\delta$, $\theta$, $\alpha$ as well as the growth rate of consumption

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[1] There are numerous estimates. See e.g. Mehra and Prescott (1985), Hall (1988), Campbell and Mankiw (1989), Kandel and Stambaugh (1991), Epstein and Zin (1991), Cochrane and Hansen (1992), Obstfeld (1994), and Ogaki and Reinhart (1998).

$\dfrac{\dot{c}_t}{c_t}$ and the output/capital ratio $\dfrac{y_t}{k_t}$ are given, the elasticity of intertemporal substitution $\rho$ can be estimated without information on interest rates or returns on assets. The problem is that this Euler equation is obtained in a model without a mechanism of endogenous growth. If no endogenous mechanism of growth exists, a production economy approaches a steady state such that $(1-\alpha)\dfrac{y_t}{k_t} - n - \delta - \theta = 0$ and $\dfrac{\dot{c}_t}{c_t} = 0$. If exogenous positive technology shocks are given continuously, the data on $\dfrac{\dot{c}_t}{c_t}$ have only information on the growth rate that is attributed to exogenous technology shocks. Hence, it is impossible to estimate EIS by this Euler equation. It implies that the estimation of EIS in a production economy requires an endogenous growth model.

However, endogenous growth models also have several serious drawbacks. Early endogenous growth models like the familiar "*AK*" model has a nature that the growth rate of output depends crucially on the number of population that is called scale effects. As Jones (1995a) argues, scale effects are not supported by observed data. As a result, it is not possible to estimate EIS by these endogenous growth models. The problem of scale effects is partially solved by Jones' (1995b) non-scale model. However, the growth of output crucially depends instead on the growth of population and is irrelevant to the Euler equation in this model. Hence it is still not possible to estimate EIS by this kind of endogenous models. Young (1998), Peretto (1998), Aghion and Howitt (1998), and Dinopoulos and Thompson (1998) propose another type of model that can eliminate the strong influence of population growth as well as scale effects. However, as Jones (1999) argues, this type of models crucially depends on a very special assumption and the growth rate of output is irrelevant to the Euler equation. Hence, it is still not possible to estimate EIS by this type of endogenous growth models.

To estimate EIS in a production economy, therefore, an endogenous growth model that firstly is free from both scale effects and the strong influence of population growth and secondly

has the property that the growth rate of output is determined by the Euler equation is needed. The paper explores such an endogenous growth model. Because the method to estimate EIS that is constructed in the paper is independent of those in the previous literature, the result in the paper will contribute to the argument about the true value of EIS by showing new independent evidence.

The paper relates to Mulligan (2002) that seems to be motivated by the same concern about the problem of using interest rates. Mulligan (2002) attempts to solve this problem by estimating the capital rental rate measured in the National Accounts instead of using data on interest rates.[2] However, the method proposed in Mulligan (2002) seems unsatisfactory because the returns on capital asset are estimated only by rental rates and capital gains are ignored because it is assumed that aggregate capital gains net-of-BEA depreciation can be presumed to be unforecastable. More importantly, because the model used in Mulligan (2002) is based on an endowment economy, it is a variation of the conventional method and is not an alternative estimation method. Contrary to Mulligan (2002), the paper explores an alternative estimation method that provides estimates of EIS in a production economy. It makes use of the data in the National Accounts like Mulligan (2002) but does not require estimates of the returns on capital asset.

The paper is organized as follows. In section II, after considering various problems regarding the estimation of EIS in a production economy, it is concluded that an endogenous growth model that firstly is free from both scale effects and the strong influence of population growth and secondly has the property that the growth rate of output is determined by the Euler equation is needed. Such a model is constructed in section III. In section IV, EIS in a production economy is estimated based on the model. Finally some concluding remarks are offered in section V.

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[2] In Mulligan (2002), the capital rental rate is defined as the amount of capital income net-of-depreciation that is earned per dollar of capital.

## II. EIS IN PRODUCTION ECONOMIES

### 1. Non endogenous growth models

In a production economy, the familiar Euler equation in case of a Harrod neutral production function such that $y_t = \dfrac{Y_t}{L_t} = A_t^{\alpha} k_t^{1-\alpha} = A_t^{\alpha} \left( \dfrac{K_t}{L_t} \right)^{1-\alpha}$ is

$$(1) \quad \frac{\dot{c}_t}{c_t} = \rho \left( \frac{\partial y_t}{\partial k_t} - n - \delta - \theta \right) = \rho \left[ (1-\alpha) \left( \frac{A_t}{k_t} \right)^{\alpha} - n - \delta - \theta \right] = \rho \left[ (1-\alpha) \frac{y_t}{k_t} - n - \delta - \theta \right]$$

where $K_t$ is capital input and $L_t$ is labor input, $Y_t$ is output and $A_t$ is knowledge/technology/idea. Hence,

$$(2) \quad \rho = \frac{\dfrac{\dot{c}_t}{c_t}}{(1-\alpha)\dfrac{y_t}{k_t} - n - \delta - \theta},$$

and if the values of $\alpha$, $n$, $\delta$, $\theta$ as well as the growth rate of consumption $\dfrac{\dot{c}_t}{c_t}$ and the

output/capital ratio $\dfrac{y_t}{k_t}$ are given, the elasticity of intertemporal substitution $\rho$ can be estimated

by equation (2) without information on interest rates or returns on assets.

However, this Euler equation is obtained in a model of an economy without technological

progress. As a result, $\lim\limits_{t \to \infty} \dfrac{\dot{c}_t}{c_t} = 0$ and $(1-\alpha) \lim\limits_{t \to \infty} \dfrac{y_t}{k_t} - n - \delta - \theta = 0$. Because of this nature such

that $\dfrac{\dot{c}_t}{c_t} = 0$ and $(1-\alpha)\dfrac{y_t}{k_t} - n - \delta - \theta = 0$ at steady states, i.e. both numerator and denominator

on the right side of equation (2) are zero, it is not possible to estimate EIS by equation (2).

Furthermore, even if exogenous technological progress is assumed, it is still difficult to estimate

EIS by equation (2) because two different processes confound the data on $\dfrac{\dot{c}_t}{c_t}$: one is the shift of

steady state by technological progress and the other is the transition process to steady state after deviation. Just after a positive technology shock, consumption jumps to a transition path to the new steady state and moves on this transition path. The observed data on $\frac{\dot{c}_t}{c_t}$ consists of both these jumps and the following transition processes.[3] Although these jumps by technological progress are irrelevant to equations (2), they are included in the observed data on $\frac{\dot{c}_t}{c_t}$. Hence, without distinguishing these jumps from transition processes, it is impossible to estimate EIS by the observed data on $\frac{\dot{c}_t}{c_t}$ and equation (2). If positive technology shocks occur continuously, the growth rate of consumption is constant such that $\frac{\dot{c}_t}{c_t} = \zeta$ where $\xi$ is a constant and indicates only the growth rate attributed to technological progress. In this case, the observed data on the growth of consumption growth reflect only the growth of consumption caused by technological progress and there is no information on the movement of consumption relating to transition processes. As a result, even if exogenous technological progress is presumed, it is difficult to estimate EIS by equation (2).

If transition processes can be distinguished from jumps initiated by technological progress, it may be possible to estimate EIS by the observed data on $\frac{\dot{c}_t}{c_t}$. One possible way to distinguish transition processes from jumps is to exclude a trend in consumption from the observed data on consumption. However, there are various detrending methods and estimated trends are very different according to these detrending methods. The difference among them appears much wider than that among interest rates. In addition, estimated trends may not reflect only

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[3] The growth rate $\frac{\dot{c}_t}{c_t}$ attributed to the transition process decreases gradually to zero as the accumulation of capital proceeds and an economy approaches the new steady state.

technology shocks but other various shocks. Hence, it seems hard to distinguish transition processes precisely from jumps caused by technological progress. In sum, it is very difficult to estimate EIS in a production economy by equation (2). Therefore EIS has not been estimated based on models of production economy.

## 2. Endogenous growth models

If an endogenous mechanism of growth is expressed by $\alpha$, $n$, $\delta$, $\theta$ as well as $\dfrac{\dot{c}_t}{c_t}$ and $\dfrac{y_t}{k_t}$, and if this mechanism is reflected in the Euler equation, it is not necessary to distinguish the technology progress from the transition process and EIS in a production economy can be estimated without information on interest rates or returns on assets. It implies that an endogenous growth model is needed to estimate EIS in a production economy. However, endogenous growth models have other serious drawbacks and it is still difficult to estimate EIS in a production economy.

In any endogenous growth model with a constant growth rate, the growth rate of consumption is $\dfrac{\dot{c}_t}{c_t} = \rho \left[ \psi \chi \left( \dfrac{A_t}{k_t} \right) - n - \theta \right]$ where $\psi$ is a constant and $\chi(\cdot)$ is a function, and the ratio $\dfrac{A_t}{k_t}$ does not decrease as the stock of capital increases but is constant at any time because of a mechanism of endogenous growth. Since the ratio $\dfrac{A_t}{k_t}$ is constant, the equation

$$\frac{\dot{K}_t}{K_t} - \frac{\dot{L}_t}{L_t} = \varphi_1 \frac{\dot{A}_t}{A_t}$$ holds at any time where $\varphi_1$ is a constant. Early endogenous growth models like the familiar "$AK$" model explicitly or implicitly assume a linear relation between $A_t$ and $K_t (= k_t L_t)$ such that $\dfrac{A_t}{k_t} = \dfrac{\varphi_2 K_t}{k_t} = \varphi_2 L_t$ where $\varphi_2$ is a constant. [4] Hence,

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[4] Early human capital-based endogenous growth models are also categorized to this class of models.

$\dfrac{\dot{c}_t}{c_t} = \rho\left[\psi\chi(\varphi_2 L_t) - n - \delta - \theta\right]$ and the number of population $L_t$ plays a crucial role for economic

growth which is called scale effects. In these models, EIS therefore is expressed by

$$\rho = \dfrac{\dfrac{\dot{c}_t}{c_t}}{\psi\chi(\varphi_2 L_t) - n - \delta - \theta}.$$ Hence, given the values of $\psi$, $\varphi_2$, $L_t$, $n_t$, $\theta$ and $\dfrac{\dot{c}_t}{c_t}$ as well as the

functional form of $\chi(\cdot)$, EIS can be estimated. Among them, the value of $\varphi_2$ is hard to estimate.

The only way to estimate the value of $\varphi_2$ may be to take regression over cross country data on

$\dfrac{A_t}{k_t} = \dfrac{\varphi_2 K_t}{k_t} = \varphi_2 L_t$. However, it is difficult to obtain an appropriate and stable estimate of $\varphi_2$ by

the regression because, as Jones (1995a) argues, scale effects are not supported by observed data

in many countries and many researchers agree that the relation $\dfrac{A_t}{k_t} = \dfrac{\varphi_2 K_t}{k_t} = \varphi_2 L_t$ does not exist

in reality. As a result, it is difficult to estimate EIS by the Euler equation

$$\rho = \dfrac{\dfrac{\dot{c}_t}{c_t}}{\psi\chi(\varphi_2 L_t) - n - \delta - \theta}.$$

The problem of scale effects is partially solved by Jones (1995b). However, his non-scale

model does not solve the problem to estimate EIS in a production economy on the basis of

endogenous growth models because, although non-scale models are free from scale effects, they

are under the strong influence of population growth. Non-scale models focus on the relation

between $L_t$ and $A_t$ instead of the linear relation between $K_t$ and $A_t$ and assume that there is a

linear relation between $\dfrac{\dot{A}_t}{A_t}$ and $\dfrac{\dot{L}_t}{L_t}$ such that $\dfrac{\dot{A}_t}{A_t} = \varphi_3 \dfrac{\dot{L}_t}{L_t}$ where $\varphi_3$ is a constant, and the only

one case such that $\dfrac{\dot{K}_t}{K_t} = (1 + \varphi_1\varphi_3)\dfrac{\dot{L}_t}{L_t} = \dfrac{\dot{Y}_t}{Y_t}$ is selected to be relevant because only this case

satisfies both the equation $\dfrac{\dot{K}_t}{K_t} - \dfrac{\dot{L}_t}{L_t} = \varphi_1 \dfrac{\dot{A}_t}{A_t}$ and a "balanced growth path."[5] A problem of

Jones' (1995b) model is that this model keeps away from investigating the mechanism behind

the linear relation $\dfrac{\dot{k}_t}{k_t} = \varphi_3 \dfrac{\dot{A}_t}{A_t}$ but from the beginning assumes a linearity among $\dfrac{\dot{L}_t}{L_t}$, $\dfrac{\dot{A}_t}{A_t}$ and

$\dfrac{\dot{K}_t}{K_t}$. As a result, the Euler equation is unclear in this type of models. Because of

$\dfrac{\dot{K}_t}{K_t} = \left(1 + \varphi_1 \varphi_3\right) \dfrac{\dot{L}_t}{L_t} = \dfrac{\dot{Y}_t}{Y_t}$, the growth of output principally depends on the growth of population

$\dfrac{\dot{L}_t}{L_t}$ and is irrelevant to the Euler equation. Hence, it is impossible to estimate EIS by this type

of models. To sum up, non-scale models originally developed by Jones (1995b) appear still

inappropriate to estimate EIS in a production economy.

To eliminate the strong influence of population growth, Young (1998), Peretto (1998),

Aghion and Howitt (1998), and Dinopoulos and Thompson (1998) propose the third approach.

They assume a relation between $\dfrac{\dot{A}_t}{A_t}$ and $L_t$ such that $\dfrac{\dot{A}_t}{A_t} = \varphi_4 L_t^{1-\varphi_5}$ where $\varphi_4$ and $\varphi_5$ are

constants. Hence, $\dfrac{\dot{K}_t}{K_t} = \dfrac{\dot{L}_t}{L_t} + \varphi_1 \varphi_4 L_t^{1-\varphi_5} = \dfrac{\dot{Y}_t}{Y_t}$ if the relation $\dfrac{\dot{K}_t}{K_t} - \dfrac{\dot{L}_t}{L_t} = \varphi_1 \dfrac{\dot{A}_t}{A_t}$ holds and an

economy is on a balanced growth path, and thus if $\varphi_5 = 1$ then even though $\dfrac{\dot{L}_t}{L_t} = 0$ an

economy can grow at a constant rate $\varphi_1 \varphi_4$. This type of models can eliminate the strong

influence of population growth as well as scale effects. However, Jones (1999) argues that it

crucially depends on a very special assumption such that $\varphi_5 = 1$, which means that $\dfrac{\dot{A}_t}{A_t} = \varphi_4$, i.e.

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[5] A balanced growth path is defined as a growth path on which all variables are growing at constant (exponential)

rates.

knowledge/technology/idea grows autonomously. As a result, the growth rate of consumption is not determined by the Euler equation but by autonomously growing knowledge/technology/idea. As a result, it is impossible to estimate EIS by the Euler equation.

In sum, considering the drawbacks of the above three types of endogenous growth models, in order to estimate EIS in a production economy, an endogenous growth model that firstly is free from both scale effects and the strong influence of population growth and secondly has the property that the growth rate of output is determined by the Euler equation is needed. Such a model is constructed in the next section.

## III. THE MODEL[6]

### 1. The basic nature of the model

The production function is assumed to be $Y_t = F(A_t, K_t, L_t)$, where $Y_t (\geq 0)$ is outputs, $K_t$ $(\geq 0)$ is capital inputs, $L_t (\geq 0)$ is labor inputs, and $A_t (\geq 0)$ is knowledge/technology/idea inputs in period $t$. The model is based on the following assumptions.

**Assumptions:**

**(A1)** The accumulation of capital and knowledge/technology/idea is $\dot{K}_t = Y_t - C_t - v\dot{A}_t - \delta K_t$, where $v(>0)$ is a constant and a unit of $K_t$ and $\frac{1}{v}$ of a unit of $A_t$ are produced using the same amounts of inputs, and $\delta$ is the rate of depreciation.[7]

**(A2)** Every firm is identical and has the same size, and for any period, $m = \dfrac{M_t^{\rho}}{L_t} = \text{constant}$

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[6] The model is based on Harashima (2004). See also Harashima (2005a, b).

[7] Hence, like Jones' (1995b) non-scale model, $A_t$, as well as $K_t$, is produced less as $A_t$ and $L_t$ increase if the usual production function of homogeneous of degree one is assumed.

where $M_t$ is the number of firms and $\rho(>1)$ is a constant.

**(A3)** $\dfrac{\partial Y_t}{\partial K_t} = \dfrac{1}{M_t^\rho} \dfrac{\partial Y_t}{\partial (vA_t)}$ and thus $\dfrac{\partial y_t}{\partial k_t} = \dfrac{1}{mv} \dfrac{\partial y_t}{\partial A_t}$.

Assumption (A1) is standard one in the literature of endogenous growth. Assumption (A2) simply assumes that the number of population and the number of firms in an economy are positively related, which seems intuitively natural. In assumption (A3), the paper assumes that returns to investing in $K_t$ and investing in $A_t$ for a firm are kept equal. However it is also assumed in (A3) that a firm that invents a new technology can not obtain all the returns to investing in $A_t$. This means that investing in $A_t$ increases $Y_t$ but returns of an individual firm that invests in $A_t$ is only a fraction of the increase of $Y_t$ such that $\dfrac{1}{M_t^\rho} \dfrac{\partial Y_t}{\partial (vA_t)} = \dfrac{1}{mL_t} \dfrac{\partial Y_t}{\partial (vA_t)}$. The reason why only a fraction of the increase in $Y_t$ the returns of an individual firm is, is uncompensated knowledge spillovers to other firms.

More specifically, the production function is assumed to have the following functional form: $Y_t = F(A_t, K_t, L_t) = A_t^\alpha f(K_t, L_t)$, where $\alpha(0 < \alpha < 1)$ is a constant. Let $y_t = \dfrac{Y_t}{L_t}$, $k_t = \dfrac{K_t}{L_t}$,

$c_t = \dfrac{C_t}{L_t}$ and $n_t = \dfrac{\dot{L}_t}{L_t}$ and assume that $f(K_t, L_t)$ is homogenous of degree one. Thereby

$y_t = A_t^\alpha f(k_t)$, and $\dot{k}_t = y_t - c_t - \dfrac{v\dot{A}_t}{L_t} - n_t k_t - \delta k_t$. By assumptions (A2) and (A3),

$A_t = \dfrac{\alpha f(k_t)}{mvf'(k_t)}$ because $\dfrac{\partial y_t}{mv \partial A_t} = \dfrac{\partial y_t}{\partial k_t} \Leftrightarrow \dfrac{\alpha}{mv} A_t^{\alpha-1} f(k_t) = A_t^\alpha f'(k_t)$. Since $A_t = \dfrac{\alpha f}{mvf'}$, then

$y_t = A_t^\alpha f = \left(\dfrac{\alpha}{mv}\right)^\alpha \dfrac{f^{1+\alpha}}{f'^\alpha}$ and $\dot{A}_t = \dfrac{\alpha}{mv} \dot{k}_t \left(1 - \dfrac{ff''}{f'^2}\right)$.

For simplicity, the growth rate of population is assumed to be positive and constant, i.e. $n_t = n > 0$ hereafter, and in the paper, only the case of Harrod neutral technological progress

such that $y_t = A_t^\alpha k_t^{1-\alpha}$ and thus $Y_t = K_t^{1-\alpha}(A_t L_t)^\alpha$ is examined.[8] Because the production function is Harrod neutral and because $A_t = \dfrac{\alpha f(k_t)}{mvf'(k_t)}$ and $f = k_t^{1-\alpha}$, then $A_t = \dfrac{\alpha}{mv(1-\alpha)}k_t$

and $\dfrac{f f''}{f'^2} = -\dfrac{\alpha}{1-\alpha}$. The accumulation of capital thereby proceeds by $\dot{k}_t = y_t - c_t - \dfrac{v\dot{A}_t}{L_t} - nk_t - \delta k_t$

$$= \left(\frac{\alpha}{mv}\right)^\alpha \frac{f^{1+\alpha}}{f'^\alpha} - c_t - \frac{\alpha}{mL_t}\dot{k}_t\left(1 - \frac{f f''}{f'^2}\right) - nk_t - \delta k_t.$$ Hence,

$$\dot{k}_t = \frac{\left(\dfrac{\alpha}{mv}\right)^\alpha \dfrac{f^{1+\alpha}}{f'^\alpha} - c_t - nk_t - \delta k_t}{1 + \dfrac{\alpha}{mL_t}\left(1 - \dfrac{f f''}{f'^2}\right)} = \frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\alpha}\left\{\left[\left(\frac{\alpha}{mv}\right)^\alpha(1-\alpha)^{-\alpha} - n - \delta\right]k_t - c_t\right\}.$$

The optimization problem for a representative household therefore is:

Max $E_0 \int_0^\infty u(c_t)\exp(-\theta t)\,dt$,

subject to

$$\dot{k}_t = \frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\alpha}\left\{\left[\left(\frac{\alpha}{mv}\right)^\alpha(1-\alpha)^{-\alpha} - n - \delta\right]k_t - c_t\right\}.$$

Let Hamiltonian $H$ be

$$H = u(c_t)\exp(-\theta t) + \lambda_t \frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\alpha}\left\{\left[\left(\frac{\alpha}{mv}\right)^\alpha(1-\alpha)^{-\alpha} - n - \delta\right]k_t - c_t\right\}$$

where $\lambda_t$ is a costate variable, thus the optimality conditions are

(3) $\dfrac{\partial u(c_t)}{\partial c_t}\exp(-\theta t) = \dfrac{[mL_t(1-\alpha)+\alpha]}{mL_t(1-\alpha)}\lambda_t$,

(4) $\dot{\lambda}_t = -\lambda_t \dfrac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\alpha}\left[\left(\dfrac{\alpha}{mv}\right)^\alpha(1-\alpha)^{-\alpha} - n - \delta\right]$,

(5)  $\dot{k}_t = \dfrac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\alpha}\left\{\left[\left(\dfrac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-n-\delta\right]k_t - c_t\right\},$

(6)  $\lim\limits_{t\to\infty}\lambda_t k_t = 0.$

The basic nature of the model is as follows.

**Lemma 1:** The growth rate of consumption is $\dfrac{\dot{c}_t}{c_t}=\rho\left\{\dfrac{mL_t(1-\alpha)\left[\left(\dfrac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-n-\delta\right]-\alpha n}{mL_t(1-\alpha)+\alpha}-\theta\right\}$

and thus $\lim\limits_{t\to\infty}\dfrac{\dot{c}_t}{c_t}=\rho\left[\left(\dfrac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-n-\delta-\theta\right].$

**Proof:** By equation (4), $\dot{\lambda}_t = -\lambda_t\dfrac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\alpha}\left[\left(\dfrac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-n-\delta\right]$, and by equation (5),

$\dot{k}_t = \dfrac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\alpha}\left\{\left[\left(\dfrac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-n-\delta\right]k_t - c_t\right\}.$ Hence, by equation (3),

$$\dfrac{\dot{c}_t}{c_t}=\dfrac{\dfrac{mL_t(1-\alpha)\left[\left(\dfrac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-n-\delta\right]-\alpha n}{mL_t(1-\alpha)+\alpha}-\theta}{\left(-\dfrac{c_t u''}{u'}\right)}=\rho\left\{\dfrac{mL_t(1-\alpha)\left[\left(\dfrac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-n-\delta\right]-\alpha n}{mL_t(1-\alpha)+\alpha}-\theta\right\},$$

and thus $\lim\limits_{t\to\infty}\dfrac{\dot{c}_t}{c_t}=\rho\left[\left(\dfrac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-n-\delta-\theta\right]$ because $\dfrac{\dot{L}_t}{L_t}=n>0$  by assumption.

Q.E.D.

**Lemma 2:** If and only if $\lim\limits_{t\to\infty}\dfrac{\dot{c}_t}{c_t}=\lim\limits_{t\to\infty}\dfrac{\dot{k}_t}{k_t}=$ constant, all the optimality conditions are satisfied.

**Proof:**

**(Step 1)** By equation (5), $\dot{k}_t = \frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\alpha}\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-n-\delta\right]k_t - c_t\right\}$ and thus

$\frac{\dot{k}_t}{k_t} = \frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\alpha}\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-n-\delta\right]-\frac{c_t}{k_t}\right\}$. On the other hand, by equation (4),

$\frac{\dot{\lambda}_t}{\lambda_t} = -\frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\alpha}\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-n-\delta\right]$.

Here, $\lim_{t\to\infty}\left(\frac{\dot{\lambda}_t}{\lambda_t}+\frac{\dot{k}_t}{k_t}\right) = -\lim_{t\to\infty}\frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\alpha}\left\{\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-n-\delta-\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-n-\delta\right]+\frac{c_t}{k_t}\right\}$

$=-\lim_{t\to\infty}\frac{c_t}{k_t}$. Thereby if $\lim_{t\to\infty}\frac{c_t}{k_t}>0$, then $\lim_{t\to\infty}\left(\frac{\dot{\lambda}_t}{\lambda_t}+\frac{\dot{k}_t}{k_t}\right)<0$. Hence, the transversality condition

(6) $\lim_{t\to\infty}\lambda_t k_t = 0$ is not satisfied if and only if $\lim_{t\to\infty}\frac{c_t}{k_t}=0$ (Because $c_t \geq 0$ and $k_t \geq 0$).

**(Step 2)** $\lim_{t\leftarrow\infty}\frac{\dot{c}_t}{c_t} = \rho\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-n-\delta-\theta\right]$ = constant by lemma 1, and

$\lim_{t\to\infty}\frac{\dot{k}_t}{k_t} = \lim_{t\to\infty}\frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\alpha}\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-n-\delta\right]-\frac{c_t}{k_t}\right\} = \left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-n-\delta-\lim_{t\to\infty}\frac{c_t}{k_t}$

by equation (5). If $\lim_{t\to\infty}\frac{\dot{k}_t}{k_t}>\lim_{t\to\infty}\frac{\dot{c}_t}{c_t}$, then $\frac{c_t}{k_t}$ diminishes as time passes, then $\frac{\dot{k}_t}{k_t}$ increases.

Hence, eventually $\frac{c_t}{k_t}$ diminishes to zero. Therefore, by (step 1), the transversality condition

(6) is not satisfied. If $\lim_{t\to\infty}\frac{\dot{k}_t}{k_t}<\lim_{t\to\infty}\frac{\dot{c}_t}{c_t}$, then $\frac{c_t}{k_t}$ increases as time passes, then $\frac{\dot{k}_t}{k_t}$ diminishes

and eventually becomes negative. Hence, $k_t$ decreases and eventually becomes negative which

violate the condition $k_t \geq 0$. However, if $\lim_{t\to\infty}\frac{\dot{c}_t}{c_t}=\lim_{t\to\infty}\frac{\dot{k}_t}{k_t}$, then $\lim_{t\to\infty}\frac{c_t}{k_t}$ is constant and thus

$\lim\limits_{t\to\infty}\dfrac{\dot{k}_t}{k_t}$ and $\lim\limits_{t\to\infty}\dfrac{\dot{c}_t}{c_t}$ are constant and identical.

<div align="right">Q.E.D.</div>

Unquestionably rational households will select the initial consumption that leads to a growth path that satisfies all the conditions, i.e. a growth path such that $\lim\limits_{t\to\infty}\dfrac{\dot{c}_t}{c_t}=\lim\limits_{t\to\infty}\dfrac{\dot{k}_t}{k_t}=$ constant.

Hence, it is assumed that given the initial $A_0$ and $k_0$, a representative household sets the initial consumption so as to achieve a growth path that satisfies all the conditions, i.e. a growth path of

$\lim\limits_{t\to\infty}\dfrac{\dot{c}_t}{c_t}=\lim\limits_{t\to\infty}\dfrac{\dot{k}_t}{k_t}=$ constant, while firms adjust $k_t$ so as to achieve $\dfrac{\partial Y_t}{\partial K_t}=\dfrac{1}{M_t^{p}}\dfrac{\partial Y_t}{\partial(vA_t)}$. As a result

of rational behavior of households and firms, the following steady state growth path is achieved.

**Lemma 3:** $\lim\limits_{t\to\infty}\dfrac{\dot{y}_t}{y_t}=\lim\limits_{t\to\infty}\dfrac{\dot{A}_t}{A_t}=\lim\limits_{t\to\infty}\dfrac{\dot{c}_t}{c_t}=\lim\limits_{t\to\infty}\dfrac{\dot{k}_t}{k_t}=$ constant

**Proof:**

(Step 1) Because $\dot{y}_t=\left(\dfrac{A_t}{k_t}\right)^{\alpha}\left[(1-\alpha)\dot{k}_t+\alpha\dfrac{k_t}{A_t}\dot{A}_t\right]$ and $\dot{A}_t=\dfrac{\alpha}{mv}\dot{k}_t\left(1-\dfrac{f f''}{f'^2}\right)=\dfrac{\alpha}{mv(1-\alpha)}\dot{k}_t$,

$\dot{y}_t=\dot{k}_t\left(\dfrac{A_t}{k_t}\right)^{\alpha}\left[(1-\alpha)+\dfrac{\alpha^2}{mv(1-\alpha)}\dfrac{k_t}{A_t}\right]$, and thus $\dfrac{\dot{y}_t}{y_t}=\dfrac{\dot{k}_t}{k_t}\left[(1-\alpha)+\dfrac{\alpha^2}{mv(1-\alpha)}\dfrac{k_t}{A_t}\right]$. Because

$A_t=\dfrac{\alpha}{mv(1-\alpha)}k_t$, $\dfrac{\dot{y}_t}{y_t}=\dfrac{\dot{k}_t}{k_t}\left[(1-\alpha)+\alpha\right]=\dfrac{\dot{k}_t}{k_t}$. Hence $\lim\limits_{t\to\infty}\dfrac{\dot{y}_t}{y_t}=\lim\limits_{t\to\infty}\dfrac{\dot{c}_t}{c_t}=\lim\limits_{t\to\infty}\dfrac{\dot{k}_t}{k_t}=$ constant.

(Step 2) Because $\dot{y}_t=\left(\dfrac{A_t}{k_t}\right)^{\alpha}\left[(1-\alpha)\dot{k}_t+\alpha\dfrac{k_t}{A_t}\dot{A}_t\right]$ and $\dot{A}_t=\dfrac{\alpha}{mv(1-\alpha)}\dot{k}_t$, $\dot{y}_t=\dot{A}_t\left(\dfrac{A_t}{k_t}\right)^{\alpha}\left[\dfrac{mv(1-\alpha)^2}{\alpha}+\alpha\dfrac{k_t}{A_t}\right]$,

and thus $\dfrac{\dot{y}_t}{y_t}=\dfrac{\dot{A}_t}{k_t}\dfrac{mv(1-\alpha)^2}{\alpha}+\alpha\dfrac{\dot{A}_t}{A_t}$. Because $\dot{A}_t=\dfrac{\alpha}{mv(1-\alpha)}\dot{k}_t$, $\dfrac{\dot{y}_t}{y_t}=(1-\alpha)\dfrac{\dot{k}_t}{k_t}+\alpha\dfrac{\dot{A}_t}{A_t}$. Hence,

$\dfrac{\dot{y}_t}{y_t} = \dfrac{\dot{k}_t}{k_t} = (1-\alpha)\dfrac{\dot{k}_t}{k_t} + \alpha\dfrac{\dot{A}_t}{A_t}$  and thus $\dfrac{\dot{k}_t}{k_t} = \dfrac{\dot{A}_t}{A_t}$. Therefore $\displaystyle\lim_{t\to\infty}\dfrac{\dot{y}_t}{y_t} = \lim_{t\to\infty}\dfrac{\dot{A}_t}{A_t} = \lim_{t\to\infty}\dfrac{\dot{c}_t}{c_t} = \lim_{t\to\infty}\dfrac{\dot{k}_t}{k_t} = $constant.

Q.E.D.

These three lemmas indicate that the steady state growth rate

$\displaystyle\lim_{t\to\infty}\dfrac{\dot{c}_t}{c_t} = \rho\left[\left(\dfrac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - n - \delta - \theta\right]$ is independent of the number of population and is not under

strong influence of population growth, which clearly shows that the model is free from both

scale effects and the strong influence of population growth and has the property that the growth

rate of output is determined by the Euler equation such that

$\displaystyle\lim_{t\to\infty}\dfrac{\dot{c}_t}{c_t} = \rho\left[\left(\dfrac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - n - \delta - \theta\right].$ [9] This model thereby can satisfy the criteria for an

endogenous growth model that is used for estimation of EIS in a production economy. Because

it is a model of a production economy, no interest rate is included in the Euler equation. We

therefore can estimate EIS in a production economy by the model without information on

interest rates or returns on assets.

## *2. The estimation method*

The equation that is used for the estimation is shown in the following proposition. EIS is

expressed without interest rates or returns on assets in this model when an economy is on a

steady state growth path described in the above three lemma.

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[9] The growth rate of consumption is affected by the growth rate of population $n$, but, unlike Jones' (1995b) model,

the growth rate of population is clearly not an essential factor for the growth of consumption.

**Proposition:** $\rho = \dfrac{\lim\limits_{t \leftarrow \infty} \dfrac{\dot{c}_t}{c_t}}{\dfrac{y_t}{k_t} - n - \delta - \theta}$.

**Proof:** By lemma 1, $\lim\limits_{t \to \infty} \dfrac{\dot{c}_t}{c_t} = \rho \left[ \left( \dfrac{\alpha}{mv} \right)^{\alpha} (1-\alpha)^{-\alpha} - n - \delta - \theta \right]$. On the other hand,

$\left( \dfrac{\alpha}{mv} \right)^{\alpha} (1-\alpha)^{-\alpha} = \dfrac{y_t}{k_t}$ because $y_t = A_t^{\alpha} k_t^{1-\alpha}$ and $A_t = \dfrac{\alpha}{mv(1-\alpha)} k_t$. Hence,

$\rho = \dfrac{\lim\limits_{t \leftarrow \infty} \dfrac{\dot{c}_t}{c_t}}{\dfrac{y_t}{k_t} - n - \delta - \theta}$.

Q.E.D.

The equation includes the limit of the growth rate of consumption $\lim\limits_{t \leftarrow \infty} \dfrac{\dot{c}_t}{c_t}$. However, if

the number of population is sufficiently large and thereby it is possible to assume that

approximately $\dfrac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\alpha} = 1$ and $\dfrac{\alpha n}{mL_t(1-\alpha)+\alpha} = 0$, it is not necessary to know the limit

of the growth rate of consumption. Usually the number of population is sufficiently large in

most industrialized economies and thus it seems natural to assume that $\dfrac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\alpha} = 1$ and

$\dfrac{\alpha n}{mL_t(1-\alpha)+\alpha} = 0$.[10]

**Corollary 1:** If the number of population $L_t$ is sufficiently large and thus approximately

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[10] See Harashima (2004).

$$\frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\alpha}=1 \text{ and } \frac{\alpha n}{mL_t(1-\alpha)+\alpha}=0, \text{ then } \rho=\frac{\dfrac{\dot{c}_t}{c_t}}{\dfrac{y_t}{k_t}-n-\delta-\theta}.$$

**Proof:** By lemma 1, $\displaystyle\lim_{t\to\infty}\frac{\dot{c}_t}{c_t}=\rho\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-n-\delta-\theta\right]$. If $L_t$ is sufficiently large,

$\dfrac{\dot{c}_t}{c_t}=\rho\left[\left(\dfrac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-n-\delta-\theta\right]$, and thus $\displaystyle\lim_{t\to\infty}\frac{\dot{c}_t}{c_t}=\frac{\dot{c}_t}{c_t}$. Hence, by proposition 1,

$$\rho=\frac{\dfrac{\dot{c}_t}{c_t}}{\dfrac{y_t}{k_t}-n-\delta-\theta} \quad \text{if } L_t \text{ is sufficiently large.}$$

<div align="right">Q.E.D.</div>

Corollary 1 indicates that given the proper values of $\alpha$, $n$, $\delta$, $\theta$ as well as the growth rate of consumption $\dfrac{\dot{c}_t}{c_t}$ and the output/capital ratio $\dfrac{y_t}{k_t}$, EIS in a production economy can be estimated without information on interest rates or returns on assets.

### 3. EIS and the rate of time preference

An appropriate value of the rate of time preference (RTP) needs to be given in order to estimate EIS. One way to obtain the value of RTP is to use the equation derived in models with exogenous technological progress such that $\theta=\dfrac{\partial y_t}{\partial k_t}-n-\delta$ at steady states, i.e. RTP equals the marginal product of capital plus some adjustment terms. This is the familiar relation derived

from the Euler equation when models with exogenous technological progress are used.[11] Most previous estimates of RTP at the macro level basically used this relation. However, in the model in the paper, RTP does not necessarily equal the marginal product of capital plus some adjustment terms on steady state growth paths. RTP may generally equal the marginal product of capital plus some adjustment terms but it is not guaranteed in the model.

Nevertheless, because it seems that there is no other appropriate estimate of RTP at the macro level than estimates based on this relation, the second best way to calibrate RTP in the model appears to assume that the relation $\theta = \dfrac{\partial y_t}{\partial k_t} - n - \delta$ holds still in the model and to use an estimate of RTP based on the relation. If an estimate of RTP based on this relation is used, the estimation of EIS in the model can be simplified. The estimation of EIS requires only the values of $\alpha$, $\dfrac{\dot{c}_t}{c_t}$ and $\dfrac{y_t}{k_t}$.

**Corollary 2:** If $\theta = \dfrac{\partial y_t}{\partial k_t} - n - \delta$ and if the number of population $L_t$ is sufficiently large, then

$$\rho = \frac{k_t}{\alpha y_t} \frac{\dot{c}_t}{c_t}.$$

**Proof:** Because $\dfrac{\partial y_t}{\partial k_t} = (1-\alpha)\dfrac{y_t}{k_t}$ , $\dfrac{y_t}{k_t} - n - \delta - \theta = \alpha\dfrac{y_t}{k_t} + (1-\alpha)\dfrac{y_t}{k_t} - n - \delta - \theta = \alpha\dfrac{y_t}{k_t}$ if

$\dfrac{\partial y_t}{\partial k_t} = \theta + n + \delta$ . By corollary 1, if the number of population $L_t$ is sufficiently large,

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[11] The Euler equation is $\dfrac{\dot{c}_t}{c_t} = \rho\left[\dfrac{\partial y_t}{\partial k_t} - n - \delta - \theta\right]$, and $\dfrac{\partial y_t}{\partial k_t} - n - \delta - \theta = 0$ at steady state because $\dfrac{\dot{c}_t}{c_t} = 0$.

$$\rho = \frac{\dfrac{\dot{c}_t}{c_t}}{\dfrac{y_t}{k_t} - n - \delta - \theta}$$ . Hence, If the number of population $L_t$ is sufficiently large and if

$\dfrac{\partial y_t}{\partial k_t} = \theta + n + \delta$ , then $\rho = \dfrac{k_t}{\alpha y_t} \dfrac{\dot{c}_t}{c_t}$ .

Q.E.D.


## IV. THE ESTIMATION OF EIS IN A PRODUCTION ECONOMY


### 1. Estimates in the previous literature

EIS or the degree of relative risk aversion (RRA) [12] at the macro level has been estimated mostly based on endowment economies and using information on interest rates or returns on assets in the previous literature.[13] Estimates disperse widely from near zero to over unity. Mehra and Prescott (1985), Hall (1988), Campbell and Mankiw (1989), Kandel and Stambaugh (1991), Cochrane and Hansen (1992), and Obstfeld (1994) argue that EIS is near zero, i.e. RRA is 10 or much larger. On the other hand, Arrow (1971) and Hansen and Singleton (1982, 1984) argue that EIS is unity or much larger, i.e. RRA is unity or much smaller. Epstein and Zin (1991) use a recursive utility function and argue that EIS is spanning the range from 0.05 to 1 and RRA is spanning the range from 0.4 to 1.4. Ogaki and Reinhart (1998) suggest that EIS is around 0.4, and Jorion and Giovannini (1993) argue that RRA is 5.4 - 11.9.


### 2. The estimation of EIS in a production economy

To begin with, the values of the share of labor input $\alpha$, the growth rate of population $n$, the

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[12]  RRA is the inverse of EIS if a constant elasticity utility function is assumed.

[13]  There are also numerous estimates of EIS at the micro level that has been estimated based on various micro data in many field or experimental researches.

rate of depreciation $\delta$, the growth rate of consumption $\frac{\dot{c}_t}{c_t}$, and the output/capital ratio $\frac{y_t}{k_t}$ are calibrated. Those variables and parameters usually take roughly same values across industrialized economies. Here the following typical values that are roughly same as those observed in the U.S are used.[14]

The share of labor input $\alpha$ : 0.7

The output/capital ratio $\frac{y_t}{k_t}$ : 0.33

The annual growth rate of consumption $\frac{\dot{c}_t}{c_t}$ : 0.02

The annual growth rate of population $n$: 0.01

The annual rate of depreciation $\delta$: 0.05

The remaining parameter RTP is calibrated based on the Euler equation in a model with exogenous technology shocks such that $\theta = (1-\alpha)\frac{y_t}{k_t} - n - \delta$ as was argued in the previous section. By the above values of $\alpha$, $\frac{y_t}{k_t}$, $n$ and $\delta$, RTP is estimated to be 0.039. The result that RTP is 4 % annually appears similar to most previous estimates based on the Euler equation in endowment economy models. By using this value of RTP, i.e. $\theta = 0.039$, EIS is estimated by the equations in corollary 1 and 2:

The elasticity of intertemporal substitution: $\rho = \dfrac{\dfrac{\dot{c}_t}{c_t}}{\dfrac{y_t}{k_t} - n - \delta - \theta} = \dfrac{y_t}{\alpha k_t}\dfrac{\dot{c}_t}{c_t} = 0.087$

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[14] The values are roughly same as those used for the calibration of the U.S. economy in Cooley and Prescott (1995).

The degree of relative risk aversion: $\dfrac{1}{\rho} = 11.6$

The result that EIS = 0.087 may be seen as a middle or a little lower estimate compared with estimates in the previous literature. Since the estimate in the paper does not use information on interest rates or returns on assets, it is basically independent of the estimates based on endowment economies for which information on interest rates or returns on assets are essential. Hence, the estimate provides independent evidence that supports the conjecture that EIS at the macro level is not unity but is much lower like 0.1 and RRA at the macro level is as high as 10 as Mehra and Prescott (1985), Hall (1988), Campbell and Mankiw (1989), Kandel and Stambaugh (1991), Cochrane and Hansen (1992), and Obstfeld (1994) argue.

## V. CONCLUDING REMARKS

EIS at the macro level has been estimated mostly based on endowment economy models. Because an exogenously given real interest rate represents all aspects of production in endowment economy models, information on interest rates or returns on assets are indispensable to estimate EIS. A problem of this estimation method is that estimates are very sensitive to the choice of interest rates or returns on assets. To escape this problem, it is necessary to estimate EIS in a production economy. However, it is difficult to estimate EIS in a production economy if a model without a mechanism of endogenous growth is used. Furthermore, endogenous growth models have serious drawbacks: scale effects and the strong influence of population growth. In order to estimate EIS in a production economy, therefore, an endogenous growth model that firstly is free from both scale effects and the strong influence of population growth and secondly has the property that the growth rate of output is determined by the Euler equation is needed. The paper constructs such an endogenous growth model and estimate EIS in a

production economy based on the model.

By using the calibrated value of the rate of time preference $\theta = 0.039$, EIS is estimated to be 0.087. It may be seen as a middle or a little lower estimate compared with estimates in the previous literature. Since the estimate in the paper does not use information on interest rates or returns on assets, it is basically independent of the estimates based on endowment economies for which data on interest rates or returns on assets are essential. Hence, the estimate in the paper provides independent evidence that supports the conjecture that EIS at the macro level is not unity but is much lower like 0.1 and RRA at the macro level is as high as 10.

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