The Roles of Money in an Economy and the Optimum Quantity of Money

By E. L. Feige, M. Parkin, R. Avery and C. Stones

In a recent paper in this journal, Perlman [6] examines the implications of two alternative theoretical frameworks which explicitly focus on the role money plays in an economy, and concludes that the different models have different implications concerning the optimal quantity of money that a society should hold.[1] The first framework is a utility maximizing model which incorporates real cash balances as an explicit argument in the utility function. In this theory, which underlies the work of such authors as Friedman [3], Johnson [4, 5], Samuelson [8] and Pesek and Saving [7], money is viewed as yielding non-observable non-pecuniary services (referred to in the literature as liquidity, convenience, security, insurance services, etc.). It is argued that since fiat money can be produced at zero cost, cash balances should be held to the point of satiety, i.e. where the marginal utility of money equals the marginal social cost of producing money which is claimed to be zero. When viewed from the perspective of asking what is the optimal quantity of money in a society, the utility maximizing framework asserts that since the marginal utility of money must be brought to zero, the optimal quantity of money will be achieved by paying interest on money equal to the return on other assets.

The alternate theoretical framework for analysing the role played by money in an economy is an inventory theory model which focuses explicit attention on the costs of transacting in different markets and on the storage costs associated with inventories of different assets. Feige and Parkin [2] have extended the usual inventory theory framework to include holdings of money, bonds, commodity inventories and real productive capital, and have claimed that *if* the real costs of creating *real* cash balances are zero:

> Pareto efficiency requires that interest be paid on cash balances which is equal to the rate of return on bonds and the net rate of return on capital. This conclusion, which has also been derived from the traditional utility maximizing model . . . emerges unscathed when the problem of determining the optimal quantity of money is subjected to an extended inventory theoretic analysis. ([2], p. 348).

Perlman [6], on the other hand, claims on the basis of a similar inventory theory model:

> If the quantity of money is not at its social optimum because for individuals the cost of making transactions into assets are less than the yield,

[1] References in square brackets are listed on p. 431.

> payment of interest on money cannot achieve the social optimum, because it will distort the optimum transaction pattern between money and goods. ([6], p. 246).

Thus, two inventory theory models, which at first sight appear to be almost identical, reach strongly conflicting conclusions. It follows, then, that the Feige–Parkin model and the Perlman model are in fact different in the sense that either their assumptions differ and/or one of them contains a logical error.

There are in fact several differences in the approach and assumptions of the two models. The Feige–Parkin specification views a representative family as maximizing a utility function which depends solely upon its consumption. Perlman on the other hand frames his model in terms of a cost minimization problem. In Section I we outline the model and demonstrate that the Feige–Parkin utility maximum model can indeed be framed as an analogous cost minimization problem which in no way changes the overall optimality conditions for the economy.

A second difference in approach is that the Feige–Parkin framework allows the representative family unit to choose between a portfolio menu of money, bonds, commodity inventories and repoducible capital, whereas the Perlman model of the household provides a portfolio choice between money, commodity inventories and "assets". Perlman is unclear as to whether "assets" refers to financial assets (bonds) or real assets (reproducible capital). In Section II we demonstrate that the omission of either bonds or capital from the portfolio choice problem of the household does not affect the Feige–Parkin result. Instead it is shown that Perlman's assumption that consumption is predetermined is the major specification difference between his model and the model developed by Feige and Parkin. In the Feige–Parkin framework, consumption is endogenously determined along with optimal inventories of all assets. Perlman's assumption that consumption is exogenously determined leads him to misspecify the relevant social constraint in his model of the household, and thus is led to his inconsistent result.

In Section III we analyse Perlman's development of the producing units of the economy. We show that Perlman's development is marred by two logical errors which lead to his erroneous conclusions. In spite of his assertion that income is "already determined" ([6], p. 243), Perlman does in fact allow income to vary by permitting producers to shift commodity inventories into reproductive capital. His errors arise from his incorrect specification of the quantity of inventories that can be converted into capital, and from his failure to recognize that from the producer's viewpoint, money balances have an opportunity cost in terms of foregone income. When these errors are put right, the Feige–Parkin optimality conditions can be derived from Perlman's corrected specification of the producer sector. Thus, Perlman's dichotomization of the economy into households and producers does not affect the conclusion that the optimal transaction pattern and inventory holding pattern for the economy can only be achieved when interest is paid on cash balances.

The final section demonstrates that the foregoing conclusions are crucially dependent upon the assumption that the social costs of producing cash balances are zero. When there exist real social costs of inducing individuals to hold larger real cash balances, the optimality conditions for the economy require that interest be paid on cash balances equal to the net rate of return on capital minus the marginal social costs of instituting a payment mechanism for money. This conclusion is consistent with both the traditional utility maximizing approach and the extended inventory theoretic approach.

I. A Cost Minimization Analogue of the Feige–Parkin Model

The Feige–Parkin inventory model represents a stationary economy which consists of "representative" family units. The family units simultaneously display the behavioural attributes of consuming households and productive units. The family units hold a menu of assets including money, government bonds, commodity inventories and real repoducible capital. Money serves as a medium of exchange and along with bonds can function as a store of value. The economy has a single real capital asset which can either be used as a productive input in the production process or as a non-productive commodity inventory. Since the representative family consumes goods at a steady rate throughout the period, the running down of commodity inventories facilitates the steady consumption pattern.

The family unit maximizes a utility function

$$U=U(pq), \tag{1}$$

where U=utility index, pq=value (volume) of commodities consumed, and is subject to both a budget constraint and an asset constraint. The budget constraint is

$$0=Y+\pi-pq-T; \tag{2}$$

and the asset constraint is

$$\bar{A}^*=\bar{M}+\bar{B}+\overline{PQ}+\overline{PK}, \tag{3}$$

where Y=labour income, π=net profit from inventory management, T=taxes, $\bar{A}^*$=average stock of nonhuman capital, $\overline{PK}$=average stock of reproducible capital, $\bar{M}$=average money holdings, $\bar{B}$=average bond holdings and $\overline{PQ}$=average commodity inventories.

The asset constraint takes explicit account of the opportunity cost as seen by the family unit of increasing the holdings of any one asset at the expense of reducing the holdings of another asset dollar for dollar. The net profit from inventory management represents the net returns on all inventory holdings minus the costs of transacting in the goods market and in the bond market. Thus

$$\pi=(r_k-\alpha_k)\overline{PK}+(r_b-\alpha_b)\bar{B}+(r_m-\alpha_m)\bar{M}-\alpha_q\overline{PQ}-\beta_b n-\beta_q m, \tag{4}$$

where r_k = rate of return on physical capital, r_m = rate of return on money, r_b = rate of return on bonds, α_k = the cost per dollar of capital inventories, α_m = the cost per dollar of money inventories, α_b = the cost per dollar of bond inventories, α_q = the cost per dollar of commodity inventories, β_b = cost per bond market transaction and β_q = cost per commodity market transaction.

The individual family receives its income at the beginning of the period and spreads out its consumption pq evenly throughout the period. Since the family consumes evenly throughout the period, it must maintain some capital in the form of commodity inventories in order to avoid infinite trips to the commodity market. The commodity inventories are acquired by making m equally spaced trips to the store and thus the amount spent per trip on commodity inventories is pq/m. Since these inventories are consumed at a constant rate, the average inventory holding for the period is

$$\overline{PQ} = pq/2m. \tag{5}$$

Income is received in the form of money, and the family makes n equally spaced trips to the brokerage market. On the first trip government bonds are purchased, and the subsequent $n-1$ trips bonds are sold. Having received pq/n in cash the individual immediately spends pq/m on commodities. The average cash balance for the period is thus

$$\overline{M} = pq/2n - pq/2m. \tag{6}$$

Average bond holdings are consequently $[(n-1)/2n]pq$, so that

$$\overline{B} = pq/2 - pq/2n, \qquad n \geqq 1. \tag{7}$$

The individual must therefore choose pq, m and n so as to maximize the Lagrangean

$$\begin{aligned} V = U(pq) + \lambda\Big[Y + (r_k - \alpha_k)\left(\overline{A}^* - \frac{pq}{2}\right) + (r_b - \alpha_b)\left(\frac{pq}{2} - \frac{pq}{2n}\right) \\ + (r_m - \alpha_m)\left(\frac{pq}{2n} - \frac{pq}{2m}\right) - \alpha_q \frac{pq}{2m} - \beta_b n - \beta_q m - pq - T\Big]. \end{aligned} \tag{8}$$

When human income and taxes are taken as predetermined, the utility maximizing problem in (8) can be treated analogously as a cost minimizing problem where the family attempts to choose (pq, m and n) so as to minimize the net costs of inventory management. The cost function to be minimized is thus

$$\begin{aligned} C = \beta_b n + \beta_q m + \alpha_q \frac{pq}{2m} + \alpha_k\left(\overline{A}^* - \frac{pq}{2}\right) + \alpha_b\left(\frac{pq}{2} - \frac{pq}{2n}\right) + \\ \alpha_m\left(\frac{pq}{2n} - \frac{pq}{2m}\right) - r_k\left(\overline{A}^* - \frac{pq}{2}\right) - r_b\left(\frac{pq}{2} - \frac{pq}{2n}\right) - r_m\left(\frac{pq}{2n} - \frac{pq}{2m}\right). \end{aligned} \tag{9}$$

This is analogous to the family unit being able to push out its budget constraint by efficient inventory management.

The cost minimizing conditions for the family are thus[1]

$$C'_m = [(a_m - \alpha_q - r_m)pq]/2m^2 + \beta_q = 0, \tag{10}$$

$$C'_n = [(r_m - r_b + \alpha_b - \alpha_m)pq]/2n^2 + \beta_b = 0. \tag{11}$$

The corresponding problem for society as a whole is simply to minimize the total social cost function, $N(C)$, where N is the number of family units and

$$N(C) = N\left[\beta_b n + \alpha_q m + \alpha_q \frac{pq}{2m} + \alpha_k\left(\overline{W}^* - \frac{pq}{2m}\right) + \alpha_b\left(\frac{pq}{2} - \frac{pq}{2n}\right) + \alpha_m\left(\frac{pq}{2n} - \frac{pq}{2m}\right) - r_k\left(\overline{W}^* - \frac{pq}{2m}\right)\right]. \tag{12}$$

The first two terms represent the real costs to society of transactions in the bond markets and goods markets; the next four terms represent the real social costs of holding inventories of commodities, capital, bonds and money; and the final term represents the social return on productive capital. Since society's average non-human wealth is $N\overline{W}^* = N[\overline{PK} + \overline{PQ}]$, the average stock of reproductive capital is simply

$$N\overline{PK} = N(\overline{W}^* - pq/2m), \tag{13}$$

Eq. (13) represents the opportunity which confronts society of increasing the total stock of reproducible capital by economizing on holdings of commodity inventories. The first-order conditions for a social maximum are thus

$$N(C'_m) = N\left[\frac{(\alpha_k + \alpha_m - \alpha_q - r_k)}{2m^2} pq + \beta_q\right] = 0, \tag{14}$$

$$N(C'_n) = N\left[\frac{(\alpha_b - \alpha_m)}{2n^2} pq + \beta_b\right] = 0. \tag{15}$$

Comparison of the social optimum conditions with the family optimum conditions reveals that families will only be induced to make the socially optimal number of transactions (and consequently to hold the socially optimal quantity of all inventories) when the interest rate on money and bonds equals the net rate of return on capital:

$$r_m = r_b = r_k - \alpha_k, . \tag{16}$$

Thus, the Feige–Parkin framework can be viewed from the perspective of a cost minimization problem with the final optimality conditions being unaffected.

[1] Since pq is considered endogenously determined, there is of course a first order condition for pq as well. The conditions for pq are to be found in Feige–Parkin [2], and are omitted throughout this paper so as to maintain consistency with Perlman's development in which pq is ignored. The final optimality conditions do in fact satisfy the necessary equations for pq. It should be noted that pq depends upon income, wealth and all rates of return. Thus both income and substitution effects are explicitly displayed when the complete system of equations is solved simultaneously. See Feige–Parkin [2], p. 342.

II. The Individual Household Sector

In this section we wish to compare and analyse the differences in the assumptions and the derived conclusions between the Feige–Parkin model and the Perlman specification. In the Feige–Parkin model the family is represented as performing the dual behavioural function of consuming and producing goods. Perlman on the other hand dichotomizes his model to consider the behaviour of individual consumers and producers separately. We thus turn first to Perlman's model of the individual consuming unit. According to Perlman, the individual has a portfolio which includes money, commodity inventories and "assets". The first major ambiguity in the Perlman model is thus an interpretation of the concept of an "asset". Since Perlman never distinguishes between real assets (reproducible capital) and an income yielding financial asset (bonds), one is left unclear as to which "asset" individual households are allowed to hold. The problem is heightened by the fact that the word "bond" never appears in Perlman's paper. Instead Perlman characterizes the behaviour of individuals by the following statements:

> The individual will undertake transactions between the money good and "assets" as long as the resource costs of doing so are less than the yield on the average amount of assets that he will be able to hold over the transaction period by making these transactions. ([6], p. 240).

If individual behaviour is to be separated from producer behaviour and the distinction is to be interesting, one would assume that individuals hold only money, commodity inventories and bonds while the producer sector would be the custodian of reproducible capital. On this interpretation the above quotation would appear to refer to individual's transactions in the bond market. However, Perlman goes on to argue that

> making transactions between the money good and "assets" . . . does not release any of the money good for permanent conversion into assets—it means merely that somewhere else in the economy someone else is holding a larger average quantity of the money good. ([6], p. 240).

This latter quotation suggests that individuals are not indeed transacting between money and government bonds (as in the Feige–Parkin model) but rather are temporarily purchasing real reproducible assets from the producing sector so that they can earn the yield on these assets. But if individuals are not producers, they would not have the technology of production which would enable them to realize the physical yield on real assets. Moreover, if Perlman's intention is to specify a world in which individuals purchase real assets from the producer sector, carry these real assets home, then put them temporarily into production, only to sell them again to realize cash, one wonders what incentive the producer sector has to sell these real productive assets in the first place.

Since the ambiguity of Perlman's interpretation of "assets" is a continual source of confusion, we will develop Perlman's model under both interpretations. Our analysis demonstrates that both interpretations lead to the Feige–Parkin optimality results so long as the treatment is logically consistent. We, however, find the description of households purchasing real reproducible assets troublesome in so far as it vitiates any meaningful distinction between the consuming and producing sector. Let us therefore first assume that Perlman's households hold only money, bonds and commodity inventories.

After this case, the notational equivalences between the Perlman model and the Feige–Parkin model are displayed in Table 1.

TABLE 1

NOTATIONAL EQUIVALENCE BETWEEEN THE FEIGE–PARKIN MODEL AND THE PERLMAN MODEL, ASSUMING HOUSEHOLDS HOLD NO PHYSICAL CAPITAL

	Feige–Parkin	Perlman
Number of commodity market transactions	m	α_H
Number of bond market transactions	n	β_H
Consumption	pq	T
Rate of return on bonds	r_b	r
Rate of return on money	r_m	r_m
Cost per transaction between money and goods	β_q	t
Cost per transaction between money and bonds	β_b	b
Storage cost of commodity inventories	$\alpha_q\ \overline{PQ}$	g
Storage cost of money	$\alpha_m\overline{M}$	m
Storage cost of bonds	$\alpha_b\overline{B}$	a
Total cost of transactions and inventory holdings	$\beta_q m+\beta_b n+\alpha_q\overline{PQ}+\alpha_b\overline{B}+\alpha_m\overline{M}$	$C_H(\alpha_H, \beta_H)$
Average money	$\overline{M}=pq/2n-pq/2m$	$\overline{M}=[(\alpha_H-\beta_H)/2\alpha_H\beta_H]T$
Average bond holdings	$\overline{B}=pq/2-pq/2n$	$\overline{A}=[(\beta_H-1)/2\beta_H]T$
Average commodity holdings	$\overline{PQ}=pq/2m$	$T/2\alpha_H$

The individual then minimizes his net cost function which is

$$C=\beta_q m+\beta_b n+\alpha_q\frac{pq}{2m}+\alpha_b\left(\frac{pq}{2}-\frac{pq}{2n}\right)+\alpha_m\left(\frac{pq}{2n}-\frac{pq}{2m}\right)-r_b\left(\frac{pq}{2}-\frac{pq}{2n}\right)-r_m\left(\frac{pq}{2n}-\frac{pq}{2m}\right), \tag{16a}$$

in the Feige–Parkin notation, or

(16b) $$C = C_H(\alpha_H, \beta_H) - rT[(\beta_H - 1)/2\beta_H] - r_m T[(\alpha_H - \beta_H)/2\alpha_H\beta_H],$$

in the Perlman notation ([6], Eq. (4)).

The corresponding first-order conditions for a maximum are then, respectively ([6], Eqs. (5) and (6)):

(17a) (1) $$C'_m = [(\alpha_m - \alpha_q - r_m)pq]/2m^2 + \beta_q = 0,$$
(2) $$C'_n = [(r_m - r_b + \alpha_b - \alpha_m)pq]/2n^2 + \beta_b = 0,$$

(17b) (1) $$C'_{\alpha_H} = C'_{H(1)}(\alpha_H, \beta_H) - (r_m T/2\alpha_H{}^2) = 0,$$
(2) $$C'_{\beta_H} = C'_{H(2)}(\alpha_H, \beta_H) - [(r - r_m)T]/2\beta_H{}^2 = 0.$$

It is clear that there is no difference between the first-order conditions of the original Feige–Parkin model [(Eqs. (10) and (11)] and the Perlman model (Eq. 17) when households are assumed to hold only money, bonds and commodity inventories. We thus turn to the alternative interpretation of Perlman's model in which "assets" represent physical reproducible capital. In this case the notational equivalences between the Feige–Parkin model and the Perlman model change as displayed in Table 2.

TABLE 2

NOTATIONAL EQUIVALENCES BETWEEN THE FEIGE–PARKIN MODEL AND THE PERLMAN MODEL ASSUMING HOUSEHOLDS HOLD NO BONDS

	Feige-Parkin	Perlman
Number of physical capital transactions	n	β_H
Rate of return on physical capital	r_k	r
Cost per transaction between money and capital	β_k	b
Storage costs of physical capital	$\alpha_k \overline{PK}$	a
Total costs of transactions and inventory holdings	$\beta_q m + \beta_k n + \alpha_q \overline{PQ} + \alpha_m \overline{M} + \alpha_k \overline{PK}$	$C_H(\alpha_H, \beta_H)$
Average physical capital	$\overline{PK} = pq/2 - pq/n$	$\bar{A} = [(\beta_H - 1)/2\beta_H]T$

As in the previous case the individual is assumed to choose (m, n) or (α_H, β_H) so as to minimize

(18a) $$C = \beta_q m + \beta_k n + \alpha_q \frac{pq}{2m} + \alpha_m \left(\frac{pq}{2n} - \frac{pq}{2m}\right) + \alpha_k \left(\frac{pq}{2} - \frac{pq}{2n}\right) - r_k \left(\frac{pq}{2} - \frac{pq}{2n}\right) - r_m \left(\frac{pq}{2n} - \frac{pq}{2m}\right)$$

in the Feige–Parkin notation, or

(18b) $C = C(\alpha_H, \beta_H) - r\{[(\beta_H - 1)/2\beta_H]T\} - r_m\{[(\alpha_H - \beta_H)/2\alpha_H\beta_H]T\}$

in the Perlman notation.

The first-order conditions for an individual maximum are, respectively ([6], Eqs. (5) and (6)):

(19a) (1) $C'_m = [(\alpha_m - \alpha_q - r_m)pq]/2m^2 + \beta_q = 0,$
(2) $C'_n = [(\alpha_k - \alpha_m - r_k + r_m)pq]/2n^2 + \beta_k = 0,$

(19b) (1) $C'_{\alpha_H} = C'_{H(1)}(\alpha_H, \beta_H) - r_m T/2\alpha_H^2 = 0,$
(2) $C'_{\beta_H} = C'_{H(2)}(\alpha_H, \beta_H) - [(r - r_m)T]/2\beta_H^2 = 0.$

Comparison of (19) with (10) and (11) reveals that under the assumption that households hold no bonds but only reproducible physical assets (i.e. $r_k = r_b$, $\alpha_k = \alpha_b$). the individual maximum conditions are identical for both the Feige–Parkin model and the Perlman framework. The differences between the two models must therefore lie in the specification of the social constraint.

For the case of bonds we would specify the social cost minimization problem as choosing m and n such as to minimize

(20a) $$N(C) = N\left[\beta_b n + \beta_q m + \alpha_q \frac{pq}{2m} + \alpha_m\left(\frac{qp}{2n} - \frac{pq}{2m}\right) + \alpha_b\left(\frac{pq}{2} - \frac{pq}{2n}\right) - (r_k - \alpha_k)\left(\overline{W}^* - \frac{pq}{2m}\right)\right],$$

or in Perlman's notation

(20b) $N(C) = N[C_H(\alpha_H, \beta_H) - (r_k - \alpha_k)(\overline{W}^* - T/2\alpha_H)].$

The first term in 20(b) refers to the social costs of transactions in both the commodity markets and bond markets as well as the storage costs of inventories of money, bonds and commodities. The second term reflects the net rate of return on the stock of real capital held by the society, where $N\overline{W}^*$ is society's total stock of physical capital plus commodity inventories. Differentiating 20(a) and (b) with respect to the optimal number of transactions yields

(21a) $C'_m = N\{[(\alpha_m - \alpha_q - r_k + \alpha_k)/2m^2]pq + \beta_q\} = 0,$
$C'_n = N\{[(\alpha_b - \alpha_m)/2n^2]pq + \beta_b\} = 0$

(21b) $C'_{\alpha_H} = N\{C'_{H(1)}(\alpha_H, \beta_H) - [(r_k - \alpha_k)/2\alpha^2{}_H]T\} = 0,$
$C'_{\beta_H} = N[C'_{H(2)}(\alpha_H, \beta_H)] = 0.$

Hence, comparing the social optimum conditions, Eq. (21), with the individual optimum conditions (17) reveals that a socially optimal transaction pattern will be induced if

(22) $r_m = r_b = r_k - \alpha_k,$

which is precisely the result obtained by Feige–Parkin, Eq. (16).

We now turn to the alternate case in which individuals hold real capital. For this case we would specify the social problem as minimizing

$$\text{(23a)} \qquad N(C)=N\left[\beta_k n+\beta_q m+\alpha_q\left(\frac{pq}{2m}\right)+\alpha_m\left(\frac{pq}{2n}-\frac{pq}{2m}\right)-(r_k-\alpha_k)\left(\overline{W}^*-\frac{pq}{2m}\right)\right],$$

or in Perlman's notation

$$\text{(23b)} \qquad N(C)=N\left[C_H(\alpha_H,\beta_H)-a-(r_k-\alpha_k)(W^*-T/2\alpha_H^2)\right].$$

The difference between Eq. (23) and Eq. (20) arises because (i) society is assumed to hold no bonds, and (ii) the *total* cost of holding the existing stock of capital is included in the last term of both (23a and b). Thus, we must subtract the individuals cost of holding capital from the preceding terms in order to avoid double counting of these costs.

Minimizing the social cost function yields the following first-order social conditions:

$$\text{(24a)} \qquad \begin{aligned} C'_m&=N\left[\left(\frac{\alpha_m-\alpha_q-r_k+\alpha_k}{2m^2}\right)pq+\beta_q\right]=0,\\ C'_n&=N[(-\alpha_m/2n^2)pq+\beta_k]=0; \end{aligned}$$

$$\text{(24b)} \qquad \begin{aligned} C'_{\alpha_H}&=N[C'_{H(1)}(\alpha_H,\beta_H)-a'_{H(1)}-(r_k-\alpha_k)\,(T/2\alpha_H^2)]=0,\\ C'_{\beta_H}&=N[C'_{H(2)}(\alpha_H,\beta_H)-a'_{H(2)}]=0. \end{aligned}$$

The social conditions for Perlman's notation (24b) contain the partials of total asset holding costs for the individuals with respect to α_H and β_H. Assuming a linear cost function (i.e., $a=\alpha_k\overline{A}$) implies $a'_{H(1)}=0$ and $a'_{H(2)}=\alpha_k(T/2\beta_H^2)$. Thus (24b) can be simplified to read,

$$\text{(24b}'\text{)} \qquad \begin{aligned} C'_{\alpha_H}&=N[C'_{H(1)}(\alpha_H,\beta_H)-(r_k-\alpha_k)\,(T/2\alpha_H^2)]=0\\ C'_{\beta_H}&=N[C'_{H(2)}(\alpha_H,\beta_H)-\alpha_k(T/2\beta_H^2)]=0, \end{aligned}$$

Comparing the social optimum conditions (24a) and (24b′) with the corresponding individual optimum conditions (19a) and (19b) reveals that individuals will be induced to undertake the socially optimal transaction pattern when

$$\text{(25)} \qquad r_m=r_k-\alpha_k.$$

Perlman, on the other hand, arrives at the conclusion that optimality in the money-goods market requires that $(r_m=0)$ and that optimality on the money-asset margin requires $r_m=r$ (see [6], p. 243). These contradictory results (for any rate of return on real assets greater than zero) arise from the fact that Perlman's specification of the net social cost minimization problem differs from our suggested specifications in Eqs. (20) and (23). According to Perlman ([6], p. 243), "from the point of view of the economy, the individual should minimize $C_H(\alpha_H,\beta_H)$". Minimizing $C_H(\alpha_H,\beta_H)$ yields Perlman's conditions for a social optimum ([6], Eqs. (7) and (8)):

$$\text{(26)} \qquad \begin{aligned} &(1) \quad C'_{\alpha_H}=C'_{H(1)}(\alpha_H,\beta_H)=0,\\ &(2) \quad C'_{\beta_H}=C'_{H(2)}(\alpha_H,\beta_B)=0. \end{aligned}$$

Perlman justifies his social conditions by arguing that "the yields both on assets and on money as seen by the individual are illusory from the point of view of the economy, only the costs are real" ([6], p. 243). While we agree that the return on money and government bonds are illusory—being only neutral tax transfer payments—we disagree about the yield on real assets. If Perlman's concept of assets corresponds to bonds, then he has not taken proper account of the fact that someone (i.e. producers) in the society holds real assets which yield a real return. If, alternatively, Perlman's individual hold real assets, then once again the return on these assets are not illusory from society's viewpoint. To see this more clearly, Perlman's Eq. (26) above equates the marginal social resource cost of an additional transaction between money and goods with zero, while we have equated this cost, Eq. (24b), with the expression $(r_k-\alpha_k)(T/2\alpha_H^2)$.

The consequence of an additional transaction in the commodity market is to reduce society's average holdings of commodity inventories. Since average inventories are simply defined as $T/2\alpha_H$, the decrease in inventory holdings resulting from an additional transaction is simply $T/2\alpha_H^2$. Since from the viewpoint of society a reduction of inventories frees resources which can be used as productive capital earning a return of $(r_k-\alpha_k)$, the marginal social revenue of an increased transaction equals $(r_k-\alpha_k)(T/2\alpha_H{}^2)$. This is then the appropriate marginal revenue to be equated with the marginal cost of an extra transaction. The Feige–Parkin model takes explicit account of this source of profit to society as a whole resulting from efficient inventory management.

Perlman's error in this regard arises partly from his spurious comparison between a money-asset economy and an economy with interest-bearing fiat money (see [6], p. 242). Perlman is correct when he argues that "one cannot increase the stock of assets temporarily simply by making transactions between goods and the money asset" (p. 241) if money is indeed a real asset. However, this argument does not apply to an interest-bearing fiat money which does not enter into the social budget constraint. At the social level, individuals can undertake an additional money-commodity transaction, thereby reducing their inventories of real goods and increasing their inventory of cash balances. As long as the social costs of inducing individuals to hold larger money balances is zero, society can produce extra money balances, which in turn, free commodity inventories for additions to the real stock of reproducible capital.

In the original Feige–Parkin framework, where total consumption (or output) is endogenous, the gain in welfare from paying interest on money can be measured precisely from the increased output capacity of the economy resulting from a reduction in the average holdings of commodity inventories. Perlman's treatment of consumption as predetermined has thus led him to ignore the increase consumption opportunities for the society as a whole from shifting commodity inventories into reproducible capital.

We are thus led to the conclusion that Perlman has misspecified the constraint for the social optimization problem and in so doing has been led to the wrong conclusions for his specification of the household sector.

III. The Producer Sector

We now turn our attention to the development of an appropriate specification of the producing sector. For the producer sector, Perlman eliminates his previous ambiguity concerning "assets" by assuming that "the producer does not go into assets temporarily". Thus the producer is assumed never to purchase bonds or real assets. His activities are thus limited to producing output from his given stock of real assets and selling this output to the household sector. Since output is produced at a constant rate during the period, the producer is assumed to accumulate inventories of goods between transaction dates. Since total output for the period is pq, m money-commodity transactions imply that the average inventories of goods are,

$$\overline{PQ}=pq/2m. \tag{27}$$

Since the producer does not go to the bond market, his average money holdings for the period are,

$$\overline{M}=pq/2+pq/2m. \tag{28}$$

The producer's wealth constraint is simply,

$$\overline{A}^*=\overline{PQ}+\overline{M}+\overline{PK}, \qquad \text{or} \qquad \overline{PK}=\overline{A}^*-pq/2-pq/2m, \tag{29}$$

where $\overline{A}^*$ denotes his average fixed stock of assets (Feige and Parkin [2], p. 339). Eq. (29), the wealth constraint, captures the opportunity cost as seen by the producer of adding to cash balances or commodity inventories, since any such transfers can only be accomplished by sacrificing an equivalent dollar volume of capital. The individual producer will thus seek to minimize the following cost function:

$$\begin{aligned} C=\beta_q^p m+\alpha_q^p\frac{pq}{2m}+\alpha_m^p\left(\frac{pq}{2}+\frac{pq}{2m}\right)+\alpha_k\left(\overline{A}^*-\frac{pq}{2}-\frac{pq}{m}\right)\\ -r_m\left(\frac{pq}{2}+\frac{pq}{2m}\right)-r_k\left(\overline{A}^*-\frac{pq}{2}-\frac{pq}{m}\right). \end{aligned} \tag{30}$$

The first term represents the producer's real costs of transacting in the goods market, the next three terms represent the producer's respective costs of holding commodity inventories, money and real capital. The fifth term is simply the realized return on money holdings, and the final term the realized return on physical capital.

Minimizing (30) with respect to m yields the following condition for the individual producer:

$$C_m'=[(r_m-2r_k+2\alpha_k-\alpha_m^p-\alpha_q^p)/2m^2]pq+\beta_q^p=0, \tag{31}$$

For society as a whole the social cost function is simply

$$(32)\qquad N(C)=N\left[\beta_q^p+\alpha_q^p\frac{pq}{2m}+\alpha_m^p\left(\frac{pq}{2}+\frac{pq}{2m}\right)\right.$$
$$\left.+\alpha_k\left(\overline{W}^*-\frac{pq}{2m}\right)-r_k\left(\overline{W}^*-\frac{pq}{2m}\right)\right],$$

where N is the total number of producers, and

$$(33)\qquad \overline{W}^*=\overline{PQ}+\overline{PK}.$$

The construction of the social cost function parallels Perlman's approach of dividing the economy into two sectors and treating the social constraints applicable to each sector separately. Thus the terms in the producer's social cost function include only those social costs which are directly affected by the producer's behaviour.

Eq. (33) simply represents the wealth constraint for society, and indicates that reductions of commodity inventories can increase the amount of productive capital. The social cost function differs from the producer cost function in recognizing that the net yield or opportunity cost on money is illusory from the point of view of society. The social optimal condition then becomes

$$(34)\qquad N(C_m')=N\left[\left(\frac{-r_k+\alpha_k-\alpha_m^p-\alpha_q^p}{2m^2}\right)pq+\beta_q^p\right]=0.$$

Comparing the producer optimum with the social optimum yields conditions under which producers can be induced to undertake the socially optimal transaction pattern, namely when

$$(35)\qquad r_m=r_k-\alpha_k,$$

which is identical with the Feige–Parkin result.

Perlman's presentation of the producer model differs from the preceeding approach in two important respects. The first point of departure is Perlman's treatment of commodity inventories held by the producer. Perlman correctly states that "unlike the household, he (the producer) can convert commodity inventories permanently into assets even within a given transaction period, while yet continuing to supply as much as before" ([6], p. 244).

This asset substitution possibility is implicit in our Eq. (29), and we agree that careful account must be taken of the alternatives as seen from the viewpoint of the producer. Indeed, Perlman's failure to recognize this point in his treatment of the individual sector led him to misspecify the social constraint for that sector. Unfortunately, Perlman goes on to specify incorrectly the magnitude of the commodity-asset substitution effect. He argues that "the maximum amount of goods that can be converted permanently into assets is $T/\alpha p$, [or (pq/m)] where T is the amount supplied over the transactions period" (and α_p is the number of goods transactions) ([6], p. 244). Thus Perlman claims that the maximum inventories that can be converted to assets is pq/m whereas in fact the maximum that can be converted is only $pq/2m$. If production proceeds at a constant rate, then immediately after a market transaction the producer holds zero inventories. Between transaction periods, inventories are accumulated at a steady rate until they are again built up to pq/m. Thus for the period as a whole the maximum inventories that can be converted into assets is $pq/2m$.

The second point of departure is Perlman's treatment of the money holdings of the producer sector. Having correctly recognized the first substitution possibly implied by Eq. (29), Perlman has neglected the fact that from the producer's viewpoint there also exists an opportunity cost of holding cash balances. This omission is apparent in his specification of the cost function which the producer wishes to minimize ([6], Eq. (10)):

$$(37) \qquad C=[(r_k-\alpha_k)T]/\alpha_p+C_p(\alpha_p)-r_m\overline{M}_p,$$

where, $\overline{M}_p=T/2+T/2\alpha_p$.

Perlman identifies the first term of the cost function as the opportunity cost of commodity inventories. As pointed out above, this term overstates by a factor of two the amount of inventories which can be converted into capital, and should read $(r_k-\alpha_k)(T/2\alpha_p)$. The second term simply reflects the storage costs of all inventories and all transactions. The final term reflects the *actual* return on money holdings and thus neglects the *opportunity cost* of those money holdings as viewed by the producer, namely $(r_k-\alpha_k-r_m)\overline{M}_p$. The correct specification taking account of both of the above points becomes

$$(38a) \qquad C=[(r_k-\alpha_k)T]/2\alpha_p+C_p(\alpha_p)+(r_k-\alpha_k-r_m)(T/2+T/2\alpha_p),$$

or in the Feige–Parkin notation

$$(38b) \qquad C=(r_k-\alpha_k)\frac{pq}{2m}+\alpha_q^p\left(\frac{pq}{2m}\right)+\alpha_m^p\left(\frac{pq}{2}+\frac{pq}{2m}\right)+\beta_{qm}^p$$
$$+(r_k-\alpha_k-r_m)\left(\frac{pq}{2}+\frac{pq}{2m}\right).$$

The first-order conditions for a producer cost minimization become

$$(39a) \qquad C'_{\alpha_p}=[(2\alpha_k-2r_k+r_m)T]/2\alpha_p^2+C'_p(\alpha_p)=0,$$

or

$$(39b) \qquad C'_m=[(2\alpha_k-2r_k+r_m-\alpha_q^p-\alpha_m^p)pq]/2m^2+\beta_q^p=0,$$

It should be noted that the first-order conditions derived above [Eq. (39)] are mathematically equivalent to the first-order conditions derived in Perlman's paper (Eq. 11). This similarity, however, is purely coincidental since Perlman's misspecification of the maximum quantity of commodity inventories available for conversion is just counterbalanced by his omission of the opportunity costs of cash balances. Since the opportunity costs of cash balances are illusory from the point of view of society, Perlman's error on the inventory specification carries over into his social constraint. Perlman's social first-order conditions are thus,

$$(40)\qquad N(C'_{\alpha_p})=N\left[\frac{(\alpha_k-r_k)T}{\alpha p^2}+C'_p(\alpha_p)\right]=0,\ \text{[Perlman, Eq. (12)]}$$

whereas the correct social optimum should be

$$(41\text{a})\qquad N(C'_{\alpha_p})=N\left[\frac{(\alpha_k-r_k)T}{2\alpha_p^2}+C'_{\alpha_p}(\alpha_p)\right]=0,$$

or

$$(41\text{b})\qquad N(C'_m)=N\left[\frac{(\alpha_k-r_k-\alpha_q^p-\alpha_m^p)pq}{2m^2}+\beta_q^p\right]=0.$$

Comparing the social conditions, Eq. (41), with the producer conditions, Eq. (39), reveals that the only way in which producers can be induced to undertake the socially optimal transaction pattern is to pay interest on cash balances equal to the net rate of return on capital, that is for

$$(42)\qquad r_m=r_k-\alpha_k.$$

Perlman's conclusion to the contrary that "payment of interest on money distorts the money-goods margin for producers" ([6], p. 245) is erroneous, and results from his oversight of the opportunity costs of cash balances to the producer and his misspecification of the maximum quantity of inventories which can be converted into capital.

The value of Perlman's differentiation between the household and production sectors seems to be that a more realistic representation of receipts and payments practices is permitted. Households receive their income in one lump sum at the end of a transactions period and proceed to spend it evenly through the next period so that producers get even receipts over that period. However, a more important potential pay-off from analysing a model with both households and producers is not taken advantage of. That is the determination by the production sector of the optimal factor payment frequency and the treatment of transactions costs as variables to be determined by the interactions of transactions rather than as parameters in the system.

IV. The Social Costs of Producing Real Cash Balances

In the preceding sections we have demonstrated that regardless of whether the economy is characterized to include representative family units or households and producers, socially optimal transaction patterns and inventory holdings can be induced by paying interest on money and bonds equal to the net rate of return on capital. This conclusion is valid, however, only under the assumption that it is in fact costless to society to institute and operate such an interest payments mechanism.

Perlman has assumed throughout his paper that it is socially costless to increase the supply of cash balances. In the previous sections we have shown why Perlman's conclusion that paying interest on money is non-optimal is erroneous. In this section we show that if the real costs of inducing individuals to hold larger real cash balances are non-zero, these costs must be taken into account when deriving the social optimal conditions.

In the original Feige–Parkin paper the real costs of instituting an interest payments mechanism where specified by

(43) $h(r_m)=k$ if $r_m>0$; $h(r_m)=0$ if $r_m=0$.

A more realistic cost function would take account of the fact that in a modern society with financial intermediaries, the marginal cost of creating real cash balances might not be zero. Thus the cost function could be represented as

(44) $h(r_m)+\alpha_c(\bar{M})$,

where $h(r_m)=k$ if $r_m>0$, $h(r_m)=0$ if $r_m=0$, and $\alpha_c>0$.

When the cost function in Eq. (44) is included as part of the social constraint, the final condition for a social optimum becomes

(45) $r_m=r_b=r_k-\alpha_k-\alpha_c$.

Thus the rate of return on money and bonds must be equated with the net rate of return on capital minus the marginal social cost of inducing individuals to hold optimal quantities of financial assets. If these costs are sufficiently high, they could of course outweigh whatever benefits are to be derived from inducing individuals to hold greater cash balances. This point becomes particularly important when one considers deflation as a means of inducing individuals to hold larger cash balances. In such a case, the real costs of adjusting to a continuous rate of deflation are likely to be considerable. Thus the challenge to the optimal money supply literature is to consider not only the potential gains in welfare from interest payments on money but also to consider the real costs associated with such a policy.

University of Wisconsin.
University of Manchester.

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