# CPI Inflation Targeting and Exchange Rate Pass-through<sup>∗</sup>

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#### Abstract

This paper analyzes how imperfect exchange rate pass-through affects the transmission of the CPI inflation targeting optimal monetary policy. In the short run, delayed pass-through constraints monetary policy more than incomplete pass-through and interest rate smoothing amplifies this effect. In addition, imperfect pass-through does not increase the variability of the real exchange rate for a subset of strict CPI inflation targeting cases and for flexible CPI inflation targeting. Furthermore, there exists an inverse relation between the pass-through and the insulation of CPI inflation from foreign shocks, and when the pass-through falls, the impact on the trade-off between the stabilization of both CPI inflation and output depends on how strictly the central bank is targeting CPI inflation and on the kind of imperfect pass-through.

JEL Classification: E52, E58, F41.

Key Words: Inflation Targeting; Exchange Rate Pass-through; Open-economy; Direct Exchange Rate Channel; Optimal Monetary Policy.

## 1 Introduction

The last few years have seen an increase in the interest of the international macroeconomics literature in the relationship between the imperfect pass-through of the exchange rate and the working of the economy<sup>1</sup>. This attention is supported by many empirical works spanning over two decades and differing for the countries and the industries considered which provide evidence on imperfect pass-through<sup>2</sup>.

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<sup>&</sup>lt;sup>1</sup>The expression "exchange rate pass-through" denotes the transmission of a change in import costs to domestic prices of imported goods.

<sup>2</sup>For example, Krugman (1987) considering US imports data in the period 1980-1983 finds that, in the machinery and transport sector, 35 to 40 percent of the appreciation of the dollar was not

The way in which changes in costs pass through to import price is a complex mechanism, and several factors may play a role in its determination. Increasing marginal costs and sufficiently large shifts of the import demand, which are determined by exchange rate movements, can lead to a change in the imports price that does not completely reflect the movement of the exchange rate. The positive correlation between inflation and inflation persistence, and the positive impact of the expectations of inflation persistence on the pass-through (via the Taylor staggered price-setting behavior), establish an inverse relation from inflation to the degree of pass-through (Taylor, 2000). Also, the firm's strategy of the pricing to market (PTM) based on international market segmentation and the local currency pricing (LCP) lead to incomplete pass-through (Betts and Devereux, 2000). LCP, in turn, has been justified in two ways: by a low market share of the exporter country in the foreign market coupled with a low degree of differentiation of its goods (Bacchetta and Wincoop, 2002) and by a greater monetary policy stability of the importing country compared to that of the exporting country (Devereux and Engel, 2001). Furthermore, the presence of shipping costs and of non-traded distribution services adds to the previous factors (Engel, 2002). Finally, as it has been noted by Obstfeld and Rogoff (2000), studies on PTM have mostly considered exports rather than consumer prices so that the presence of intermediary firms between the exporters and the consumers is likely to reduce the pass-through more.

These factors remark that imperfect pass-through may be generated by foreign exporting firms and/or the intermediary importing sector. In addition, different behaviors of the firms that produce or import foreign goods lead to different kinds of pass-through. For example, if a fraction of the foreign exporting firms practice LCP instead of producer currency pricing (PCP), then the overall pricing behavior of the exporting sector can lead to incomplete pass-through. On the other hand, staggered price-setting in the importing sector can lead to delayed pass-through.

Taking these factors into account, it has been possible to investigate more general scenarios regarding the pass-through and reach interesting results on the relation between the pass-through and the optimal monetary policy<sup>3</sup>.

In a New Keynesian perspective, considering an emerging market economy with nominal rigidities in both the non-traded goods and import sectors, Devereux and

reflected in a decrease of the import prices. Knetter (1989) for the period 1977-1985 finds that US export prices in the destination market currency tend to be either insensitive to exchange rate fluctuations or to amplify their impact, while German export prices tend to stabilize the exchange rate fluctuations. Considering the sample period 1974-1987, Knetter (1993) shows that Japanese export prices adjustments in the destination country currency offset 48 percent of the exchange rate fluctuations while for U.K. and German export prices this fraction reaches 36 percent. More recently, Campa and Goldberg's (2001) estimation for the period 1975-1999 and a sample of OECD countries supported the complete pass-through hypothesis for the long run but not for the short run.

<sup>&</sup>lt;sup>3</sup>See also Lane and Ganelli (2002) for a survey of the implications of different degrees of passthrough when it is considered also the currency denomination of assets contracts.

Lane (2001) show that in the case of complete pass-through targeting nontradable inflation dominates targeting CPI inflation or an exchange rate peg while, in the case of delayed pass-through, CPI inflation targeting performs better. Devereux (2001) considers a small open-economy with sticky prices in the non-traded goods and import sectors and compares the Taylor rule, a rule that stabilizes non-traded goods inflation, strict CPI inflation targeting and a rule which pegs the exchange rate. He finds that in general, with delayed pass-through, the trade-off between the output and inflation variability is less pronounced; the best monetary policy stabilizes non-traded goods price inflation; and strict CPI inflation targeting performs better with partial passthrough. Smets and Wouters (2002) present an open-economy model calibrated to euro area data with nominal rigidities in the domestic and imported goods sectors and exogenous foreign interest rates, price and output. In this framework, they consider that the welfare costs determined by nominal rigidities in the imported goods sector depend positively on the exchange rate variability. Consequently, they make the point that with delayed pass-through, output gap stabilization is constrained by the minimization of these welfare costs because it leads to a larger exchange rate variability.

In a New Classical perspective, Corsetti and Pesenti (2001) show the importance of the degree of pass-through in affecting the trade-off between output gap stabilization and import costs and the implication for the optimal monetary policy. When the passthrough is incomplete because of LCP, exporters' profits oscillate with the exchange rate, and the hedging behavior of the exporters consequently links the import prices positively to the variability of the exchange rate. Then, if the monetary policy wants to stabilize the output gap, it will increase the variability of the exchange rate and the import prices. Hence, optimal monetary policy has to equate at the margin the cost of the output gap with the cost of higher import price. It follows that the lower the pass-through, the less the socially optimal output gap stabilization.

These models are characterized by a welfare optimization approach to determine or evaluate the monetary policy. Specifically, Deverux and Lane (2001) and Deverux (2001) assume explicit instrument rules for alternative policy regimes and rank them using a welfare measure depending on household expected utility. Smets and Wouters (2002) and Corsetti and Pesenti (2001) assume an explicit welfare optimization framework and derive the optimal monetary policy. A further common feature of these models is that they either consider delayed or incomplete pass-through.

This paper differs from the literature discussed above because it uses inflationforecast targeting instead of an explicit welfare optimization framework, relies on a more complex transmission mechanism with realistic lags for various channels and analyzes both incomplete and delayed pass-through<sup>4</sup>. It broadens the analysis by considering the role of the pass-through in the working of the direct exchange rate channel for a small open-economy that follows CPI inflation targeting. This study is motivated by the fact that central banks which pursue inflation targeting place

<sup>&</sup>lt;sup>4</sup>As to the inflation-forecast targeting procedure, see Svensson (1997) and (1998b).

greater importance on a low and stable CPI inflation than they do on low and stable domestic inflation. In this case, according to the conventional wisdom, and as it is shown in Ball (1998) and Svensson (2000), the direct exchange rate channel, that is, the channel that allows the exchange rate to have an impact on current CPI inflation, plays a prominent role in the transmission of monetary policy. Yet, this channel is considered in these papers with the strict assumption that the pass-through of the exchange rate is complete. This paper relaxes such assumption and, as a result, it is able to address some issues that have been neglected in the literature.

First, it analyzes the way in which the pass-through affects the working of the direct exchange rate channel and examines to what extent this latter channel is reliable for the transmission of the monetary impulse in the short run. Such an analysis is important, as this channel is the only one available to stabilize CPI inflation in the short run.

Second, it considers that optimal monetary policy chooses the extent to use the various available channels according to their relative efficiency in the transmission mechanism. Then, the following question addressed is how imperfect pass-through influences this choice under strict and flexible CPI inflation targeting.

Third, it takes into account that the direct exchange rate channel is also one of the avenues through which shocks originating in the foreign sector propagate to CPI inflation. Indeed, a shock in foreign output or inflation affects the foreign interest rate, which, in turn, via the interest rate parity condition, propagates to the exchange rate and, finally, hits domestic CPI inflation. Thus, the third question addressed is how the way in which the pass-through occurs affects the degree of insulation of CPI inflation from foreign shocks.

Finally, the paper investigates the impact of imperfect pass-through on the tradeoff between output gap and CPI inflation stabilization.

The framework used for the analysis is a small open-economy that pursues discretionary CPI inflation targeting in which different kinds and degrees of exchange rate pass-through are possible in the short and medium term.

The following results are obtained:

(i) With zero pass-through, the direct exchange channel does not work at all and the monetary authority cannot stabilize CPI inflation to any extent in the short run. With imperfect pass-through, it is important to distinguish between incomplete and delayed pass-through. In the case of incomplete pass-through, it is still possible to stabilize CPI inflation to a large extent at the cost of instrument instability. On the other hand, in the case of delayed pass-through, a significant part of the stabilization skill of the central bank is lost compared to the complete case.

(ii) The kind and degree of pass-through affect the extent to which optimal monetary policy uses the direct exchange rate channel most significantly with strict CPI inflation targeting. As long as it is feasible to bring CPI inflation on the target in the short run, strict CPI inflation targeting uses the direct exchange rate channel more aggressively with imperfect pass-through. However, when short run CPI inflation stabilization is no longer feasible (due to low efficiency of the direct exchange rate channel) or desirable (due to interest smoothing and/or flexible CPI inflation targeting cases), imperfect pass-through has a minor impact on the optimal use of the direct exchange rate channel relatively to the other channels. An interesting implication is that in contrast with previous findings in the literature discussed above, when output stabilization is relevant, imperfect pass-through does not increase exchange rate variability.

(iii) There exists an inverse relation between the pass-through and the insulation of the economy from foreign shocks, namely, the lower the pass-through the more the economy is shielded from foreign shocks.

(iv) Imperfect pass-through influences the trade-off between the stabilization of CPI inflation and output. This effect depends on how strictly the central bank is targeting CPI inflation and the kind and degree of pass-through. Specifically, with incomplete pass-through, strict and flexible inflation targeting face a more and less pronounced trade-off, respectively. With delayed pass-though, the main result is that with strict CPI inflation targeting, the trade-off becomes more pronounced when most of the change in import cost is delayed until the next period. In contrast, when most of the change is not delayed, the trade-off becomes less pronounced.

The paper is structured as follows: section 2 analyzes the relation between the exchange rate pass-through and the transmission mechanism. Section 3 presents CPI inflation targeting under different degrees of exchange rate pass-through; it discusses the optimal reaction functions obtained and the working of the economy via the impulse response functions. Section 4 concludes and presents some directions for further research. Appendix A and B contain, respectively, some technical details and the state-space form of the model.

# 2 The relation between the pass-through and the transmission mechanism

The framework used to study the impact of the exchange rate pass-through on both the transmission of the monetary impulse and foreign shocks to the economy is the model set out in Svensson (2000). Yet, such a model considers only complete passthrough and, therefore, it is extended to take into account incomplete and delayed exogenous pass-through justified with the practice of LCP and nominal rigidities in the importing sector.

The methodology consists of two stages: First, determining the optimal reaction function for the monetary authority that, coupled with the behavior of the private sector, comprises all the information to depict the equilibrium; second, simulating the response of the economy to different shocks under various assumptions on the exchange rate pass-through.

It is assumed that monetary authorities are concerned, first and foremost, with

stabilizing inflation and, to some extent, output around some target values. In pursuing this task, they minimize in each period a loss function taking the expectations of the private sector as given and knowing that they will reoptimize in all subsequent periods.

### 2.1 The model

The behavior of the private sector is described by an aggregate supply and demand equation, a real interest parity condition and some equations describing the foreign variables. A definitory equation is introduced for CPI inflation

The aggregate supply is given by the following Phillips curve

$$
\pi_{t+2} = \alpha_{\pi} \pi_{t+1} + (1 - \alpha_{\pi}) \pi_{t+3|t} + \alpha_y \left[ y_{t+2|t} + \beta_y \left( y_{t+1} - y_{t+1|t} \right) \right] + \alpha_q q_{t+2|t} + \varepsilon_{t+2} \tag{1}
$$

where  $x_{t+\tau|t}$  denotes the rational expectation of  $x_{t+\tau}$  in period  $t + \tau$ , conditional on the information available in period t,  $\pi_t$  stands for domestic inflation in period t and is measured as the deviation of domestic inflation from the constant inflation target,  $y_t$  is the output gap defined as  $y_t \equiv y_t^d - y_t^n$ , where  $y_t^d$  is aggregate demand and  $y_t^n$  is the natural output assumed to be exogenous and stochastic

$$
y_{t+1}^n = \gamma_y^n y_t^n + \eta_{t+1}^n, \qquad 0 \le \gamma_y^n < 1, \qquad \eta_{t+1}^n \sim \text{iid } (0; \sigma^2) \tag{2}
$$

 $q_t$  is the log real exchange rate measured as a deviation from its steady state value and, finally,  $\varepsilon_{t+2}$  is a white-noise term to capture inflation shocks ('cost-push' shock).

The aggregate demand (in terms of output gap) is described by

$$
y_{t+1} = \beta_y y_t - \beta_\rho \rho_{t+1|t} + \beta_y^* y_{t+1|t}^* + \beta_q q_{t+1|t} - (\gamma_y^n - \beta_y) y_t^n + \eta_{t+1}^d - \eta_{t+1}^n \tag{3}
$$

where  $y_t^*$  is foreign output,  $\eta_{t+1}^d$  is a white-noise to capture demand shocks,  $\rho$  is the sum of the current and expected future real interest rate,

$$
\rho_t \equiv \sum_{\tau=0}^{\infty} r_{t+\tau|t} \tag{4}
$$

and  $r_t$  is the short real interest rate defined as

$$
r_t \equiv i_t - \pi_{t+1|t} \tag{5}
$$

where, finally,  $i_t$  is the short nominal interest rate used as an instrument by the central bank and is measured as the deviation of the short run nominal interest rate from the sum of the inflation target and the natural real interest rate. Both aggregate demand and supply are derived with some microfundations described in Svensson (1998a).

The equation for the consumer price index inflation,  $\pi_t^c$ , is

$$
\pi_t^c = (1 - \omega) \pi_t + \omega \pi_t^f \tag{6}
$$

where  $\pi_t^c$  denotes CPI inflation,  $\pi_t^f$  denotes domestic-currency inflation of imported foreign goods and  $\omega$  is the share of imported goods in the CPI.

On the basis of the interest parity condition amended with the risk premium, the following real interest parity condition is derived

$$
q_{t+1|t} = q_t + i_t - \pi_{t+1|t} - i_t^* + \pi_{t+1|t}^* - \psi_t \tag{7}
$$

where  $i_t^*$  is the foreign nominal interest rate assumed to follow a Taylor-type rule,  $\pi_{t+1|t}^*$  refers to foreign inflation and  $\psi_t$  is the foreign exchange risk premium.

The exogenous variables foreign inflation, foreign income and risk premium are respectively described by

$$
\pi_{t+1}^{*} = \gamma_{\pi}^{*} \pi_{t}^{*} + \varepsilon_{t+1}^{*}
$$
\n
$$
y_{t+1}^{*} = \gamma_{y}^{*} y_{t}^{*} + \eta_{t+1}^{*}
$$
\n
$$
\psi_{t+1} = \gamma_{\psi} \psi_{t} + \xi_{\psi, t+1}
$$
\n(8)

where the coefficients are non-negative and less than unity and the shocks are white noises.

Finally, the foreign sector sets the monetary policy according to the following Taylor rule

$$
i_t^* = f_\pi^* \pi_t^* + f_y^* y_t^* + \xi_{it}^* \tag{9}
$$

where the coefficients are positive and  $\xi_{it}^*$  is a white noise<sup>5</sup>.

The behavior of the central bank consists of minimizing the following loss function

$$
E_{t} \sum_{s=0}^{\infty} \beta^{\tau} \left[ \mu \pi_{t+\tau}^{c2} + \lambda y_{t+\tau}^{2} + \nu \left( i_{t+\tau} - i_{t+\tau-1} \right)^{2} \right]
$$
 (10)

where  $\mu$ ,  $\lambda$  and  $\nu$  are weights that express the preferences of the central bank for the CPI inflation target, the output stabilization target and the instrument smoothing target respectively.

In this general equilibrium model, the (column) vectors of predetermined and forward-looking variables are, respectively,

$$
X_t = (\pi_t, \pi_{t-1}, y_t, \pi_t^*, y_t^*, i_t^*, \varphi_t, y_t^n, q_{t-1}, q_{t-2}, i_{t-1}, \pi_{t+1|t})'
$$
  

$$
x_t = (q_t, \rho_t, \pi_{t+2|t})'.
$$

It is worthwhile pointing out that  $\pi_{t+1|t}$  is a predetermined variable. This is apparent if we consider the Phillips curve and take the expectation at time  $t + 1$ because  $\varepsilon_{t+2|t+1} = 0$  and  $\pi_{t+2|t+1}$  is a function of predetermined variables.

<sup>&</sup>lt;sup>5</sup>As to the parameters of the model, I have used the same parameters chosen in Svensson (2000).

Concerning the solution of the optimization problem, the tool used is the dynamic programming technique of the linear stochastic regulator with rational expectations and forward-looking variables<sup>6</sup>.

### 2.2 The transmission mechanism

Transmission of monetary policy to inflation occurs via various channels: the usual aggregate demand and expectations channels that work in a closed economy, on the one hand, and the channels that are introduced by the exchange rate, on the other. The latter channels capture the impact of the exchange rate on (i) the relative price of domestic and foreign goods, which affects both domestic and foreign demand; thus this channel can be thought of as a component of the aggregate demand channel, (ii) the domestic-currency price of imported intermediate goods, which affects the production costs, and (iii) the domestic-currency price of imported final goods, which affects directly the CPI (direct exchange rate channel).

Each channel, other then the direct exchange rate channel, needs some time to convey the monetary policy to CPI inflation. With the aggregate demand channel there is a first lag to transmit the stimulus to the output-gap through the lagged expectations of both future real interest rates and the real exchange rate (that are present in the aggregate demand relation). Then, a second lag is required by the output-gap to affect inflation through the Phillips curve<sup>7</sup>. With the expectations channel there is a lag even though monetary policy can affect inflation expectations immediately. The reason is that the aggregate supply relation requires a lag to affect inflation via wage and price setting behavior. Also, with the channel that transmits the impact of the exchange rate to the domestic-currency price of imported intermediate goods there is a lag that is mirrored by the expected real exchange rate. The only channel for which there is no lag is the direct exchange rate channel. In this case, the real interest parity condition conveys the monetary impulse to the real exchange rate, which, in turn, directly hits CPI inflation.

We can have a better understanding of the impact of the different channels on CPI inflation by considering the case of complete exchange rate pass-through. Here, the domestic currency price of imported foreign goods is given by

$$
p_t^f = p_t^* + s_t, \quad p_t^* = \log \text{ of foreign price}, \quad s_t = \log \text{ of nominal exchange rate.} \quad (11)
$$

When this is the case, domestic-currency inflation of imported foreign goods, defined as  $\pi_t^f \equiv p_t^f - p_{t-1}^f$ , turns out to be

$$
\pi_t^f = \pi_t + q_t - q_{t-1}
$$

Finally, CPI inflation,  $\pi_t^c$ , fulfills

<sup>&</sup>lt;sup>6</sup>The optimization problem is reported in appendix B.

<sup>7</sup>There is substantial agreement in the literature on the view that the aggregate demand channel affects inflation with a two periods lag.

$$
\pi_t^c = \pi_t + \omega \left( q_t - q_{t-1} \right) \tag{12}
$$

In (12) domestic inflation and the real exchange rate determine current CPI inflation. However, the monetary authorities cannot control domestic inflation at time t as the aggregate demand and expectations channels affect it with some lags. On the contrary, they can control the real exchange rate because the direct exchange rate channel affects it directly.

## 2.3 The role of the pass-through in the stabilization of CPI inflation

When the assumption of complete pass-through is relaxed, two important effects occur. The first is that the smaller the pass-through, the larger the degree of insulation of CPI inflation from shocks hitting either foreign inflation or the variables that affect the nominal exchange rate (such as the foreign interest rate and the risk premium). The second is a positive relation between the degree of pass-through and the capacity of monetary policy to stabilize current CPI inflation.

The former effect occurs because a foreign shock arrives to CPI inflation mainly via domestic currency inflation of imported goods and the degree of pass-through affects the transmission of the shock. Indeed, this transmission can occur both indirectly, via the impact that the shock in the foreign variable exerts upon the nominal exchange rate (for instance in the case in which the foreign variable affects the foreign interest rate, which, in turn, through the interest parity condition, affects the nominal exchange rate) and directly, in the case of a shock in foreign price. In both cases, the degree of pass-through determines how much of the shock arrives to domestic currency inflation of imported goods.

The latter effect, the positive relation between the degree of pass-through and the capacity of monetary policy to stabilize current CPI inflation, occurs because the possibility to affect current CPI inflation is based exclusively on the availability of the direct exchange rate channel. This channel works best if the pass-through is complete and works not at all when there is no pass-through. In the limiting case in which there is no pass-through, domestic-currency inflation of imported goods is not affected by a change of import costs. As a result, the monetary authorities completely lose the capacity to stabilize current CPI inflation by affecting domestic-currency inflation of imported goods. In addition, as will be explained below, also assuming imperfect pass-through, the capacity of monetary policy to stabilize current CPI inflation is still corrupted.

It is worthwhile noting that the degree of pass-through exerts two opposite effects on the stability of CPI inflation. On the one hand, it negatively affects the insulation of CPI inflation from foreign shocks; on the other hand, it positively affects the capacity of monetary policy to stabilize CPI inflation. This could lead to the conclusion that the degree of pass-through is of second order relevance because these two opposite effects, roughly, offset each other. However, this would be misleading, as CPI inflation, described by (6), is a weighted average of domestic inflation and domestic currency inflation of imported goods. Then, with imperfect pass-through, CPI inflation tends to be shielded by the aforementioned shocks, but it remains exposed to all the shocks that can affect domestic inflation, for example a cost-push shock, with a limited possibility of using monetary policy to absorb them in the short run.

#### 2.3.1 The relaxation of the complete pass-through hypothesis

The crucial role played by the pass-through in the stabilization of current CPI inflation can be shown by introducing a new hypothesis on the way in which the passthrough occurs. Here, (11) is replaced by

$$
p_t^f = \delta_1 \left( p_t^* + s_t \right) + \delta_2 \left( p_{t-1}^* + s_{t-1} \right), \qquad \delta_1, \ \delta_2 \in [0, 1], \quad \delta_1 + \delta_2 \in (0, 1] \tag{13}
$$

This equation allows the consideration of different hypotheses on incomplete and delayed pass-through and, although it does not stem from an explicit optimization, it can be justified by LCP in the foreign exporting sector and staggered price setting in the retails sector. If there is a shock in t to import costs,  $\delta_1$  of the shock passesthrough to  $p_t^f$  and  $\delta_2$  to  $p_{t+1}^f$ . Thus, with  $\delta_1 \in [0,1)$  and  $\delta_2 = 0$ , the monetary authorities face the cases of incomplete pass-through, which is more likely to occur when some exporting firms practice LCP. Instead, with  $\delta_1 \in [0,1]$  and  $\delta_2 \in (0,1]$ , they face the cases of delayed pass-through, more likely with staggered price setting.

In addition, the degree of pass-through in the subsequent periods depends on the persistence of the shock. For example, if the shocks occur both at time t and  $t + 1$ , the response of  $p_{t+1}^f$  will be stronger than the response of  $p_t^f$  and equal to  $\delta_2$  times the first shock plus  $\delta_1$  times the second shock, consistent with the idea that in the medium term the pass-through tends to be complete<sup>8</sup>.

When  $\delta_1 = 1$  and  $\delta_2 = 0$ , (13) boils down to (11) and we have the benchmark case of complete pass—through with CPI inflation determined by (12), which is the case considered in Svensson (2000). Otherwise, domestic-currency inflation of imported foreign goods turns out to be

$$
\pi_t^f = p_t^f - p_{t-1}^f = \delta_1 \pi_t + \delta_2 \pi_{t-1} + \delta_2 (q_{t-1} - q_{t-2}) + \delta_1 (q_t - q_{t-1})
$$

and CPI inflation is given by

$$
\pi_t^c = [1 + \omega(\delta_1 - 1)]\pi_t + \omega\delta_2\pi_{t-1} + \omega\delta_2(q_{t-1} - q_{t-2}) + \omega\delta_1(q_t - q_{t-1}) \tag{14}
$$

In this general equilibrium model, shocks may occur at the beginning of period t and, when this is the case, the vector of current predetermined variables is updated to take them into account. It is assumed that the information set for the monetary authorities comprises current predetermined variable (and so current shocks) and

<sup>8</sup>See Campa and Goldberg (2001).

previous predetermined variables. If a shock occurs, the optimal monetary policy responds to it and this, finally, results in the vector of forward-looking variables at period t.

Now, in order to affect  $\pi_t^c$  at time t the only variable that the monetary policy can control in equation (14) is  $q_t$ . Indeed, this variable is the only forward-looking variable, all the others being predetermined variables. Hence, when there is no passthrough in the first period,  $\delta_1 = 0$ , current CPI inflation is no longer a function of  $q_t$ and becomes a linear function of predetermined variables, acquiring itself the nature of a predetermined variable. It follows that it is impossible for the monetary policy to affect CPI inflation at time t.

# 3 CPI inflation targeting under different degree of pass-through

## 3.1 Optimal monetary policy

In what follows two kinds of CPI inflation targeting are analyzed: 1. strict CPI inflation targeting, which occurs when the only objective of the central bank is to stabilize CPI inflation in the shortest time horizon, and 2. flexible CPI inflation targeting, where the overriding objective is to stabilize CPI inflation but there is also some concern for output-gap stabilization, which results in a more gradual stabilization of CPI inflation. These two cases occur, respectively, when in the loss function of the monetary authorities, eq. (10), it is set ( $\mu = 1$ ,  $\lambda = 0$  and  $\nu = 0$ ) and ( $\mu = 1$ ,  $\lambda = 0.5$ and  $\nu = 0$ ).

For this kind of optimization a general closed form solution does not exist. Yet, via numerical analysis it has been possible to find the coefficient of the optimal reaction function

$$
i_t^{opt} = c_1 \pi_t + c_2 \pi_{t-1} + c_3 y_t + c_4 \pi_t^* + c_5 y_t^* + c_6 i_t^* + c_7 \psi_t + c_8 y_t^n + c_9 q_{t-1} + c_{10} q_{t-2} + c_{11} i_{t-1} + c_{12} \pi_{t+1|t}
$$

reported in Table 1 for the considered cases. The first and the second part of the table report the reaction function for the case of strict and flexible CPI inflation targeting respectively. The parameters  $\delta_1$  and  $\delta_2$  determine the kind and degree of pass-through. The case of complete pass-through,  $\delta_1 = 1$ ,  $\delta_2 = 0$ , is the benchmark case.

Table 1. Reaction-function coefficients



Overall, the qualitative response of the monetary policy instrument to the predetermined variables is rather stable when the pass-through varies. However, the quantitative side exhibits some considerable changes. To obtain some insight, we can exploit the fact that for the cases of strict CPI inflation targeting in which is possible to stabilize CPI inflation in the first period, there is an analytical characterization of the solution and the inflation targeting operating procedure, described by Svensson (1997), allows to find it.

#### 3.1.1 Characterization of the solution for the cases of short run CPI stabilization

Consider the strict CPI inflation targeting case and suppose that it is possible to stabilize CPI inflation in any periods  $t + \tau$ ,  $\tau \geq 0$ . Then, in any period t the optimal monetary policy consists of finding the sequence  $\{i_{t+\tau|t}\}_{\tau=0}^{\infty}$  so as to obtain

$$
\pi_{t+\tau|t}^c = 0, \ \ \tau \ge 0
$$

Focusing on  $\pi_t^c = 0$  and  $\pi_{t+1|t}^c = 0$  we have

$$
\pi_t^c = [1 - \omega(1 - \delta_1)] \pi_t + \omega \delta_2 \pi_{t-1} + \omega \delta_2 (q_{t-1} - q_{t-2}) + \omega \delta_1 (q_t - q_{t-1}) = 0 \quad (15)
$$

and

$$
\pi_{t+1|t}^{c} = \left[1 - \omega(1 - \delta_{1})\right] \pi_{t+1|t} + \omega \delta_{2} \pi_{t} + \omega \delta_{2} \left(q_{t} - q_{t-1}\right) + \omega \delta_{1} \left(q_{t+1|t} - q_{t}\right) = 0. \tag{16}
$$

Combining these equations with the real interest parity condition (7) yields the following reaction function

$$
i_t = -\frac{\delta_2}{\delta_1} \left(\frac{\omega - 1}{\omega \delta_1}\right) \pi_t + \left(\frac{\delta_2}{\delta_1}\right)^2 \pi_{t-1} - \pi_{t+1|t}^* + i_t^* + \psi_t + \left(\frac{\delta_2}{\delta_1}\right)^2 (q_{t-1} - q_{t-2}) + \left(\frac{\omega - 1}{\omega \delta_1}\right) \pi_{t+1|t}
$$
\n(17)

Equation (17) shows how the interest rate reacts to the predetermined variables. To complete the characterization of the optimal monetary policy for the cases in which is feasible to stabilize CPI inflation in the short run, we can refer, for sake of simplicity, to the benchmark case of complete pass-through, i. e.  $\delta_1 = 1, \delta_2 = 0$ . In the appendix A is reported the extension to the general case.

In the case  $\delta_1 = 1, \delta_2 = 0$ , equations (15) and (16) boil down, respectively, to

$$
q_t = -\frac{\pi_t}{\omega} + q_{t-1} \tag{18}
$$

$$
q_t = \frac{\pi_{t+1|t}}{\omega} + q_{t+1|t} \tag{19}
$$

Now, equation (19) can be solved forward for  $q_t$  and yields

$$
q_t = \lim_{\tau \to \infty} q_{t+\tau|t} + \sum_{\tau=1}^{\infty} \frac{\pi_{t+\tau|t}}{\omega}
$$

Recalling that the real exchange rate is measured as a deviation from its steady state value, it follows that  $\lim_{\tau \to \infty} q_{t+\tau|t} = 0$ . Then we can rewrite this equation as

$$
q_t = \frac{\pi_{t+1|t}}{\omega} + \sum_{\tau=2}^{\infty} \frac{\pi_{t+\tau|t}}{\omega} \tag{20}
$$

where the only predetermined variable is  $\pi_{t+1|t}$ . Since  $q_{t-1}$  is a predetermined variable, without loss of generality we can assume that in period  $t-1$  the economy was in steady state so that  $q_{t-1} = 0$ . Now combining (20) with (18) yields

$$
-(\pi_t + \pi_{t+1|t}) = \sum_{\tau=2}^{\infty} \pi_{t+\tau|t}
$$
\n(21)

This equation says that a necessary condition to have  $\pi_t^c = 0$  and  $\pi_{t+1|t}^c = 0$  is that the central bank determine expectations on domestic inflation from period  $t + 2$ on in a way such that their sum is equal to  $-(\pi_t + \pi_{t+1|t})$  which is predetermined. The central bank can determine these expectations because they are forward looking variables. What does this mean in terms of the interest rates? From (21) and the reaction function (17) we obtain (see appendix A for details)

$$
\sum_{\tau=0}^{\infty} i_{t+\tau|t} = \frac{\omega - 1}{\omega} (-\pi_t) + \sum_{\tau=0}^{\infty} i_{t+\tau|t}^* - \sum_{\tau=0}^{\infty} \pi_{t+\tau+1|t}^* + \sum_{\tau=0}^{\infty} \psi_{t+\tau|t}
$$
(22)

Recalling that the foreign variables and the risk premium are exogenous in the model, it turns out that the right hand side of (22) is predetermined. Thus this equation establishes a condition that the sum of current and expected future interest rates have to satisfy to guarantee  $\pi_t^c = 0$ . Specifically, since  $(\omega < 1)$ , if there is a positive shock to inflation, then the central bank has to rise the sum of all the current and expected interest rates in a proportional way. This condition together with the reaction function (17) completely characterizes the optimal monetary policy when the central bank can stabilize CPI inflation in the short run and  $\delta_1 = 1, \delta_2 = 0$ .

Equation (17) and the generalization of equation (22) reported in appendix A hold for all the strict CPI inflation targeting cases in which it is *feasible* to stabilize CPI inflation in the short run. These cases turn out to be complete and incomplete pass-through for  $\delta_1 \geq 0.1$  and delayed pass-through with  $\delta_1 \geq 0.52$  and  $\delta_2 = 1 - \delta_1$ . Hence, for these cases, the coefficients in Table 1 and in equation (17) are the same.

#### 3.1.2 Monetary policy analysis

Now let us define, in general terms, the efficiency of a channel as its capacity to convey a certain impulse. Then, once the pass-through is no longer complete, the efficiency of the direct exchange rate channel decreases. Indeed, according to (14), the direct exchange rate channel conveys only  $\frac{\partial \pi_i^c}{\partial q_t} = \omega \delta_1$  of an impulse instead of  $\omega$ in the case of complete pass-through.

With complete pass-through, full stabilization of CPI inflation is carried out by the exchange rate channel, and this explains the large absolute value of the coefficient of  $\pi_{t+1|t}$  and the zero values for all the other coefficients of the predetermined variables that do not enter in the real interest parity condition. In the incomplete case, when the pass-through falls, for example, to  $\delta_1 = 0.5$ , the direct exchange rate channel becomes less efficient, but it can still stabilize  $\pi^c$  in the short run with a monetary policy that becomes more aggressive according to  $\frac{\partial (\omega - 1)}{\partial \delta_1}$ ´  $\frac{\omega b_1}{\partial \delta_1} > 0$  in (17). However, in the cases of delayed pass-through with  $\delta_1 < 0.52$ , equation (17) fails. Here, no matter how aggressive the monetary policy is, the direct exchange rate channel can no longer stabilize  $\pi^c$  in the short run, and the other channels, which take longer lags in the transmission mechanism, enter into the stabilization of  $\pi^c$  in the medium and long term. This change in the transmission mechanism is reflected in the monetary policy reported in Table 1 by the presence of the coefficients for  $y$  and  $y_t^n$  and by the lower size, in absolute value, of the coefficients of  $\pi_t$  and  $\pi_{t+1|t}$ , contrary to what equation (17) indicates. For this case of delayed pass-through, since an analytical version of the reaction function is not available, we use the impulse response functions reported in subsection 3.2 to describe the relation between the pass-through and the optimal monetary policy.

In the flexible CPI inflation targeting cases, the stabilization of the output gap becomes important and, consequently, it is no longer possible to have current and future expected CPI inflation equal to zero. Then, as in the previous cases, the impulse response functions will be useful to describe the relation between the passthrough and the optimal monetary policy. Yet some preliminary remarks are possible. First, by comparing flexible with strict CPI inflation targeting, the absolute value of the coefficient of  $\pi_{t+1|t}$  is much lower indicating a smaller role played by the direct exchange rate channel in the transmission mechanism. Second, by comparing delayed with complete pass-through, some qualitative changes stand out, which suggest a change in the optimal monetary policy.

It is interesting to note that for all the cases reported in Table 1, the coefficient

of  $\pi_{t+1|t}$  is negative. This result seems counter-intuitive because states that when a positive shock to  $\pi_t$  occurs and, consequently  $\pi_{t+1|t}$  rises through the Phillips curve, the central bank should cut the interest rate. The simplest way to explain it is to consider the case  $\delta_1 = 1, \delta_2 = 0$  and recall that the reaction function (17) has been determined combining three conditions:  $\pi_t^c = 0$ ,  $\pi_{t+1|t}^c = 0$ , and the real interest parity. Since  $\pi_{t+1|t}^c = 0$  is equivalent to equation (19), which can be rewritten as

$$
q_{t+1|t} - q_t = -\frac{\pi_{t+1|t}}{\omega},
$$
\n(23)

a positive shock to  $\pi_t$ , and therefore to  $\pi_{t+1|t}$ , requires a proportional expected appreciation of the real exchange rate. Then, combining (23) with the real interest parity (7), and assuming that the foreign variables and the risk premium do not change, this appreciation can occur if and only if the central bank decreases the interest rate in a way proportional to  $\pi_{t+1|t}$ .

Finally, it is worth noting that to have clearer results, so far the analysis has been carried out assuming that the central bank does not care about smoothing the interest rate, i.e. setting  $\nu = 0$  in (10). Introducing interest smoothing, i.e.  $\nu > 0$ , constrains the manoeuvre of the interest rate and as a result the capacity of the central bank to stabilize CPI inflation in the short run decreases.

#### 3.1.3 Optimal use of the direct exchange rate channel and pass-through

It is important to note that a decrease of the pass-through affects the efficiency of the direct exchange rate channel only. When short run CPI inflation stabilization is no longer feasible (delayed pass-through case with  $\delta_1$  < 0.52) or desirable (interest smoothing and/or flexible inflation targeting cases) the decrease of the pass-through results in the optimal monetary policy resorting less on the direct exchange channel and more on the other remaining channels. The simplest way to illustrate this point is to consider the strict CPI inflation targeting case with incomplete pass-through and a small weight given to the interest smoothing objective,  $\nu = 0.0001$ . Figure 1 a, b, and c, show, respectively, the unconditional standard deviations of  $\pi$  and i and the short run value of CPI inflation,  $\pi_1^c$ , after a cost-push shock for an increasing level of  $pass\text{-}through<sup>9</sup>$ .

<sup>&</sup>lt;sup>9</sup>The cost-push shock is determined by setting  $\varepsilon_{t+2} = 1$  in the Phillips curve.



Figure 1. Std(i), Std( $\pi$ ) and  $\pi_1^c$  for an increasing  $\delta_1$ .

When  $\delta_1 > 0.07$ , optimal monetary policy stabilizes  $\pi^c$  mainly via the direct exchange rate channel and, secondarily, via the other channels, for simplicity's sake, the aggregate demand channel. The fast rise in the variability of  $i$  (Figure 1, a) reflects the large change in i required by the low and declining efficiency of the direct exchange rate channel to stabilize  $\pi^c$ . The slightly declining variability of  $\pi$  (Figure 1, b) reflects a minor use of the aggregate demand channel that allows  $\pi^c$  to be stabilized in the next periods. The low values of  $\pi_1^c$  (Figure 1, c) show that the direct exchange rate channel, even if low efficient, plays the main role in the stabilization of  $\pi^c$ .

When  $\delta_1$  < 0.07, on the other hand, the low efficiency of the direct exchange rate channel causes a too high cost in terms of interest smoothing, making no longer optimal for monetary policy to stabilize  $\pi^c$  mainly with this channel. Consequently, the aggregate demand channel plays a larger role in the stabilization of  $\pi^c$ . Indeed, the variability of the monetary instrument falls sharply, denoting the minor use of the direct exchange rate channel, and the variability of  $\pi$  decreases more quickly, reflecting the larger role played by the aggregate demand channel.

In synthesis, a lower pass-through implies that the optimal CPI inflation targeting policy tend to exploit the direct exchange channel less and the other channels more.

### 3.2 Impulse responses

In Figures (2-5) the impulse responses for the two targeting cases under different assumptions on the pass-through are reported. They are generated assuming that the economy is in steady state and then is hit by a certain shock. The various shocks are present in equations (1-3) and (8-9) and their value is set equal to 1.

In each figure, the first and the second column refer, respectively, to the cases of complete and incomplete pass-through and the third and the fourth column to the cases of delayed pass-through. For the strict and flexible case, Figure 2 and 4 report the impulse responses to a cost-push shock and Figure 3 and 5 the impulse responses to a foreign inflation shock.

In the various cases, interest smoothing is sometimes introduced, and the choice of  $\nu$  balances two contrasting goals: To avoid a completely unrealistic monetary policy, which may occur for values of  $\nu$  that are too low and, on the other hand, to avoid an inefficient monetary policy as to the stabilization of CPI inflation, which, instead, occurs when  $\nu$  is not sufficiently small.

#### 3.2.1 Strict CPI inflation targeting

Before carrying out this analysis, it is useful to remember that in this model shocks occur at the beginning of period 1 and the vector of predetermined variables depends on them (and on previous variables). Then, the monetary authorities react to the shock through the optimal reaction function and this, finally, determines the value in period 1 of the forward-looking variables.

Let us start by considering in strict CPI inflation targeting a cost-push shock under the assumption that the pass-through is complete, i.e.  $\delta_1 = 1$  and  $\delta_2 = 0$  in (14), (Figure 2, column 1). To asses the extent to which monetary policy is able to absorb the shock via the direct exchange rate channel, we can consider the initial variation of both the real exchange rate,  $\Delta q_{ini}$ , and CPI inflation,  $\Delta \pi_{ini}^c$ , occurring before the setting of the monetary instrument. Since the real exchange rate and CPI inflation are forward-looking variables<sup>10</sup>, their first period values,  $q_1$  and  $\pi_1^c$ , are determined after the interest rate is set. Thus, the difference between the CPI inflation after and before the setting of the monetary instrument represents a measure of how much the direct exchange rate channel can absorb the shock in the first period. Now, when a cost-push shock occurs, domestic inflation rises to 1 (row 2). By definition, there is a decrease of the real exchange rate,  $\Delta q_{ini} = -1$ , and, consequently, CPI inflation rises by  $\Delta \pi_{ini}^c = \Delta \pi_1 - \omega \Delta q_{ini} = 0.7$  according to  $(14)^{11}$ . Then, the response of monetary policy is (i) to cut the current nominal interest rate  $i_1$ , (row 4), in a way proportional to  $\pi_{2|1}$  in order to determine the expected real appreciation of the exchange rate that guarantees  $\pi_{t+1|t}^c = 0$ , as explained in section 3.12 and (ii) to choose  $\{i_{t+\tau|t}\}_{\tau=0}^{\infty}$ so that condition (22) is satisfied<sup>12</sup>. Since  $\pi_{t+1|t}$  is predetermined when the shock occurs, there is a decrease of the real interest rate (row 5). Finally, there is a second real appreciation of the exchange rate (equal to  $-2.2$ ) which, together with the first appreciation, brings the first period value of the real exchange rate to  $-3.2$ , (row 6).

According to (14), this second appreciation reduces CPI inflation by  $\omega 2.2=0.66$ and, as a result, the first period value of CPI inflation is 0.04 (row 1). Under the assumption that the pass-through is complete, the direct exchange rate channel has been the only channel capable of countering the shock in the same period in which

 $10$ CPI inflation is a linear function of a state variable and a forward-looking variable. Thus, it is a dependent forward-looking variable.

<sup>&</sup>lt;sup>11</sup>Another way to see this intermediate change is to refer directly to equation  $(6)$ . In  $(6)$ , before that the interest rate is set, the intermediate change of CPI inflation is given by the variation of domestic inflation times the share of domestic goods in the CPI.

<sup>&</sup>lt;sup>12</sup>Since in this case  $\nu = 0.05$  and the analytical reaction function holds for  $\nu = 0$  but becomes an approximation of the one found by numerical analysis for low values of  $\nu$ , we have  $i_1$  larger than the value that we would have if  $\nu = 0$ .

it has occurred. Specifically, it has absorbed more than 94% of the shock, thereby stabilizing CPI inflation almost completely. From the second period on, the decrease in the real interest rate and the large appreciation of the exchange rate determine the descent of the output gap (row 3), and the aggregate demand channel starts to work, thereby placing itself side by side to the direct exchange rate channel in the stabilization of CPI inflation in the subsequent periods.

The second appreciation of the real exchange rate follows directly from (18). Yet this result deserves some special attention because it could seem counter-intuitive that a cut of the interest rate determines an appreciation of the real exchange rate. The intuition for how the exchange rate appreciation comes about is that what matters is not only the current interest rate  $i_t$  but the whole monetary policy, i. e. the sequence  $\{i_{t+\tau|t}\}_{\tau=0}^{\infty}$ , specifically, the sum of all the interest rates decided at time t. To see this point, we can solve the interest parity forward for  $q_t$  and obtain

$$
q_{t} = \lim_{\tau \to \infty} q_{t+\tau+1|t} - \sum_{\tau=0}^{\infty} i_{t+\tau|t} + \sum_{\tau=0}^{\infty} \pi_{t+\tau+1|t} + \sum_{\tau=0}^{\infty} i_{t+\tau|t}^{*} - \sum_{\tau=0}^{\infty} \pi_{t+\tau+1|t}^{*} + \sum_{\tau=0}^{\infty} \psi_{t+\tau|t}
$$

Since in the long run we are in steady state,  $\lim_{\tau \to \infty} q_{t+\tau+1|t} = 0$ . Thus

$$
q_{t} = -\sum_{\tau=0}^{\infty} i_{t+\tau|t} + \sum_{\tau=0}^{\infty} \pi_{t+\tau+1|t} + \sum_{\tau=0}^{\infty} i_{t+\tau|t}^{*} - \sum_{\tau=0}^{\infty} \pi_{t+\tau+1|t}^{*} + \sum_{\tau=0}^{\infty} \psi_{t+\tau|t}
$$

Then, given the foreign variables and the risk premium, the current real exchange rate depends on the sum of the current real interest rate and all the expected real interest rates that the central bank decides in period  $t$ . With a shock in domestic inflation the sum of the nominal interest rate is positive, as it is shown by condition (22) and the sum of expected inflation is negative, as it is shown by equation (21). Then, the central bank is, overall, raising the real interest rates leading the real exchange rate to appreciate.

When the pass-through is incomplete, with  $\delta_1 = 0.5$ ,  $\delta_2 = 0$  (second column), the impact of the cost-push shock on CPI inflation in the initial stage does not change. Indeed, according to (14),  $\Delta \pi_{ini}^c = [1 + \omega(\delta_1 - 1)]\Delta \pi_1 - \omega \delta_1 \Delta q_{ini} = 0.7$ . Yet, as will be explained soon, after that the monetary instrument is set, when all the variables will have reacted to the shock, the impact of the shock on CPI inflation in the first period will be *four times* as large as in the former case. Let us see what happens. As we said above, a partial pass-through results in a loss in the efficiency of the direct exchange rate channel. Thus, to affect CPI inflation in the first period, the optimal monetary policy will have to increase the intensity of the impulse. This is apparent in row 4, where the decrease of the nominal interest rate in the first period is larger than in the previous case of complete pass-through. The same happens to the real interest rate (row 5) and, as a result, there is a second appreciation of the real exchange rate (equal to  $-3.5$ ). This appreciation reduces CPI inflation of  $\omega \delta_1 3.5 \simeq 0.52$  and, coupled with the previous one, leads the real exchange rate in the first period to  $-4.5$  (row 6).

This notwithstanding, CPI inflation in the first period is 0.17 (row 1), four times as large as in the complete pass-through case, implying that the direct exchange rate channel is able to absorb only 74% of the shock on CPI inflation (against more than 94% in the previous case). This result shows that even with a partial pass-through, the central bank loses to a significant extent the capacity to stabilize CPI inflation in the short run.

The cases of delayed pass-through reported in the third and forth columns have been selected to show the behavior of the economy when  $\nu = 0.05$  and the passthrough is just enough to stabilize  $\pi_1^c$ ,  $(\delta_1 = 0.7, \delta_2 = 0.3)$ , and when that is no longer feasible  $(\delta_1 = 0.4, \delta_2 = 0.6)$ . What stands out is a different manoeuvre of the interest rate in the short and medium term (row 4), which is motivated by a different working of the transmission mechanism. When the direct exchange rate channel no longer allows  $\pi^c$  to be completely stabilized (row 1 fourth column), monetary policy tends to replace it with the aggregate demand channel. This effect can be seen in the slightly lower variability of domestic inflation and the output gap (second and third rows respectively).

Let us consider now shocks that come from the foreign side. The first general remark is that the shocks in foreign inflation or income or exchange risk premium tend to have a similar qualitative impact on the economy due to the fact that they share, to a large extent, the same transmission mechanism. Indeed, the foreign economy is assumed to set the monetary policy according to the Taylor rule. Thus a change in either foreign inflation or income has the same qualitative impact on the foreign interest rate, and this latter rate and the foreign exchange risk premium, in turn, have the same effect on the domestic interest rate through the uncovered parity condition. In addition, foreign inflation and income have a similar impact on the demand of domestically produced goods via the aggregate demand channel.

If we consider, for example, a foreign inflation shock (Figure 3), we see that CPI inflation either in the case of complete or incomplete and delayed pass-through is completely insulated (row 1). This insulation is due to the fact that the monetary policy uses the direct exchange rate channel in a way such that the shock does not propagate to CPI inflation through the real exchange rate. Indeed, for the first two periods, the real exchange rate is kept at its long run equilibrium value (row 6). It is interesting to note that when the pass-through is incomplete or delayed, there is less variability of the domestic inflation and the output gap (second and third row). The reason is that with imperfect pass-through a significant part of the change in the real exchange rate stemming from the shock does not arrive to CPI inflation.

#### 3.2.2 Flexible CPI inflation targeting

Figures 4-5 show the impulse response functions of the economy to a cost-push shock and a foreign shock when the central bank pursues flexible CPI inflation targeting. All in all, monetary policy stabilizes the output gap more at the cost of higher CPI inflation. When the cost push shock hits the economy (Figure 4), delayed passthrough results in higher CPI inflation in the short run. On the contrary, when the foreign shock hits the economy (Figure 5), moving from the case of complete to the cases of incomplete and delayed pass-through, CPI inflation turns out to be more insulated. The reason is that with imperfect pass-through, CPI inflation is more shielded by foreign shocks due to the lower efficiency of the direct exchange rate channel. At the same time, flexible inflation targeting moves the focus on the medium term stabilization of CPI inflation in favor of a more stable output gap. This implies that the monetary policy uses the direct exchange rate channel less and the aggregate demand channel more.

The minor use of the direct exchange rate channel is reflected in the very similar variabilities of the real exchange rate for different kinds and degrees of pass-through (last row of Figure 4 and 5). Further simulations not reported here show that these variabilities converge to the complete pass-through case variability when some interest rate smoothing is introduced at the cost of a slightly larger CPI inflation variability. This means that flexible CPI inflation targeting stabilizes both the output gap and the real exchange rate (in addition to CPI inflation). This outcome differs from some previous results obtained in the strict CPI inflation targeting cases. There, as long as the optimal monetary policy could bring CPI inflation on target, a fall in the passthrough determined a more aggressive use of the direct exchange rate channel and a larger variability of the real exchange rate.

It is noteworthy that this result is in contrast with previous findings in the literature discussed in the introduction where imperfect pass-through leads to a larger exchange rate variability.

#### 3.3 Short run CPI inflation stabilization

It could be thought that as soon as there is some degree of pass-through in the first period,  $\delta_1 \in (0,1], \delta_2 \in [0,1]$ , and no interest smoothing, i.e.  $\nu = 0$ , current CPI inflation immediately goes back under the full control of monetary policy, as this variable is again a function of the real exchange rate. Yet, this is not generally true. In the case of delayed pass-through, the central bank cannot stabilize  $\pi^c$  in the short run for a wide range of values of  $\delta_1$ .

The reason is that when  $\delta_1$  decreases, the direct exchange rate channel is less efficient and consequently larger changes in  $q_t$  are required to stabilize  $\pi_t^c$ . These changes in  $q_t$  introduce new shocks on  $\pi^c$  in periods  $t + 1$  and  $t + 2$  according to (14), and their absolute values are directly linked to the value of  $q_t$ . In particular, the shock in period  $t + 2$  has the same sign of the exogenous shock in period t and its impact on  $\pi_{t+2}^c$  depends on  $\frac{\partial \pi_t^c}{\partial q_{t-2}} = -\omega \delta_2$ . When  $\delta_2 \ge \delta_1$ , this impact has an absolute value, respectively, larger or equal to the efficiency of the direct exchange channel,  $\omega \delta_1$ , so that if the monetary policy used the direct exchange rate channel to absorb the shock in period t then it would create a shock larger or equal in period  $t + 2$ . Thus, when  $\delta_2 \simeq 0.5$  optimal monetary policy has to reduce the use of the direct exchange rate channel to avoid instability<sup>13</sup>.

This result can be illustrated considering the impulse response functions to a cost-push shock for different degrees of the pass-through. In Figure 6, the first, second, third and fourth columns plot the impulse response functions in the cases of  $(\delta_1 = 0.6, \delta_2 = 0.4), (\delta_1 = 0.56, \delta_2 = 0.44), (\delta_1 = 0.52, \delta_2 = 0.48)$  and  $(\delta_1 = \delta_2 = 0.5).$ 

The first three graphics in the fourth row show how the manoeuvre of the interest rate becomes more sophisticated when the share of the delayed pass-through rises and the monetary policy uses the direct exchange rate channel to completely stabilize CPI inflation. The tendency towards a more apparent 'roller coaster path' for the interest rate is determined by the fall in the efficiency of the direct exchange rate channel and the increasing relevance of the subsequent shocks introduced by the monetary policy. These shocks are too big to be absorbed for  $\delta_1 < 0.52$  and stand out, all of a sudden, in the last graphic of the first row.

The case of delayed pass-through with  $\nu = 0$  is informative in pointing to the best that the monetary authorities can do to stabilize CPI inflation. However, Figure 6, row 4 shows that it is somehow unrealistic. The same problem occurs in the case of incomplete pass-through with  $\nu = 0$ , where strict CPI inflation targeting can stabilize  $\pi_1^c$ , but the interest rate is characterized by huge variations.

When the weight on instrument smoothing is more realistic, the relationship between CPI inflation and the degree of pass-through shifts upwards, and the degree of pass-through required to fully stabilize CPI inflation in the short run increases.

This further result for the case of incomplete pass-through is reported in Figure 7, which shows  $\pi_1^c$  as a function of the degree of exchange rate pass-through considering different values of the weight  $\nu$  placed on instrument smoothing. Also this experiment has been carried out assuming the strict CPI inflation targeting case and a cost-push shock.



<sup>13</sup>The role of the shocks introduced by the exchange rate movements in generating instability is considered also by Ball (1998) in the discussion of the perils of inflation targeting.

Figure 7. First period CPI inflation and degree of pass-through for different interest rate smoothing parameters.

This result is more pronounced in the case of delayed pass-through that is not reported here.

Summing up, the degree and the kind of pass-through and the degree of interest smoothing significantly influence the capacity of monetary policy to stabilize CPI inflation in the short run.

#### 3.4 Medium term CPI inflation stabilization

A crucial factor for a successful monetary policy is the management of the private sector expectations on future monetary policy. To steer effectively the expectations the central bank needs high credibility and, consequently, if credibility is an issue, a quick way to build it is targeting inflation strictly at the cost of less output stability. Once the central bank is credible, strict inflation targeting loses much of its appeal and output gap stabilization gains more attention. For this reason, in the medium term, flexible inflation targeting is very likely to occur. When we consider the medium term it is interesting to focus on how the pass-through affects the variability of CPI inflation and the output gap rather than the short run levels of these variables. Figure 8 and 9 refer to the cases of incomplete and delayed pass-through and show the Taylor curves for different values of  $\delta_1$  and  $\delta_2$ . In order to focus on the role of the pass-through, it is assumed that there is no interest smoothing, i.e.  $\nu = 0$ .

In the case of incomplete pass-through, a decrease of  $\delta_1$  worsens the trade-off between CPI inflation and output stability for low values of the variance of CPI inflation but improves it for larger values. What happens here is based on the decreasing efficiency of the direct exchange rate channel and on the impact of the shock generated by the monetary policy in period t and referred to period  $t + 1$ , i.e.  $\frac{\partial \pi_c^c}{\partial q_{t-1}}$  in eq. (14). When the inefficient direct exchange rate channel is the main way to stabilize CPI inflation, monetary policy has to use it more intensively. As a result, the larger impact of the shock determined by  $q_{t-1}$  causes larger output gap variability, which worsens the trade-off. The second one, the improving of the trade-off, depends on the larger insulation from foreign shocks and the minor use of the direct exchange rate channel that occur with flexible inflation targeting.



Figure 8. Taylor curves for incomplete pass-through cases.

In the case of delayed pass-through the behavior of the trade-off becomes more complex. Figure 9 (a) and (b) show, respectively, the cases in which  $\delta_1 \in [0, 0.52)$ and  $\delta_1 \in [0.52, 1]$  and  $\delta_2 = 1 - \delta_1$ .



Figure 9. Taylor curves for delayed pass-through cases.

In (a), the trade-off behaves exactly as in the case of incomplete pass-through: when the pass-through is more delayed the trade-off worsens under strict CPI inflation targeting and improves under flexible targeting. On the contrary, in (b) the opposite result occurs: the trade-off improves under strict inflation targeting and worsens under flexible.

When  $\delta_1 \in [0, 0.52)$ , the lower  $\delta_1$  is, the harder it is to exert some effect on CPI inflation in the short run. Then, under strict CPI inflation targeting, the trade-off worsens. When  $\delta_1 \in [0.52, 1]$ , it is possible to stabilize CPI inflation in the first period and the impact of the shock generated by the monetary policy in the second period via  $q_{t-1}$  in eq. (14) decreases with  $\delta_1^{14}$ . Thus, it turns out to be easier to stabilize

$$
{}^{14}\text{Indeed, }\frac{\partial\pi_t^c}{\partial q_{t-1}} = \omega\left(\delta_2 - \delta_1\right) \in [-0.3, -0.04] \text{ and } \left|\frac{\partial\pi_t^c}{\partial q_{t-1}}\right| \text{ decreases for }\delta_1 \longrightarrow 0.52.
$$

CPI inflation in the short run and, consequently, the trade-off improves under strict CPI inflation targeting. At the same time, the impact of the shock generated by the monetary policy in the third period is inversely linked to  $\delta_1$ , and this dependence tends to worsen the trade-off under flexible CPI inflation targeting.

These findings are noteworthy because they introduce the consideration of the kind and degree of pass-through in the decision of how gradual the stabilization of CPI inflation should be. Thus, it is important for the central bank to determine where the pass-through is generated and to asses its size.

In addition, low and stable inflation may lead to delayed pass-through via the expectations of inflation persistence (Taylor, 2000) and/or to incomplete pass-through by promoting LCP (Devereux and Engel, 2001). Consequently, the monetary policy could have an important role to determine the kind and degree of pass-through in the medium term.

## 4 Conclusions

In this paper I have analyzed the relation between the exchange rate pass-through and the optimal monetary policy for a small open-economy pursuing inflation-targeting. The framework used is the Svensson (2000) model extended to take into account imperfect pass-through.

The results obtained are, first and foremost, that the kind and degree of passthrough play an important role in the capacity of the central bank to stabilize CPI inflation in the short run. With no pass-through, optimal monetary policy cannot affect CPI inflation at all in the short run, and with imperfect pass-through and some realistic interest smoothing, it loses a significant part of its latitude. In particular, delayed pass-through turns out to reduce monetary policy effectiveness more than incomplete pass-through. These results contrast with the conventional wisdom, according to which the ability to move the exchange rate allows the central bank to affect CPI inflation in the short run.

Second, considering how inflation targeting chooses optimally the various channels in the transmission mechanism, I find that imperfect pass-through increases the variability of the real exchange rate only in a subset of strict CPI inflation targeting cases. The size of this subset, in turn, depends on the degree of interest smoothing and the kind and degree of pass-through. Taking into account that CPI flexible inflation targeting is more relevant in practice than strict CPI inflation targeting, and that central banks seem to smooth the interest rate to some extent, the model shows that the impact of imperfect pass-through on the variability of the real exchange rate is limited to few cases. This outcome tends to be in contrast with previous findings in the literature where, in general, imperfect pass-through leads to a larger exchange rate variability. Considering the policy implications of this result, further research is important to explain how the transmission mechanism and the lag structure determine this difference.

Third, the insulation of CPI inflation from shocks that come from the foreign side is inversely linked with the degree of pass-through.

Fourth, the kind and degree of pass-through affect the trade-off between the stabilization of both CPI inflation and the output gap. Specifically, with incomplete pass-through, a fall in the degree of pass-through sharpens the trade-off under strict CPI inflation targeting but improves it under flexible CPI inflation targeting. With delayed pass-through, when most of the change in import costs is delayed until the next period, the trade-off becomes significantly more pronounced under strict CPI inflation targeting but slightly improves under flexible CPI inflation targeting. On the other hand, when a small part of the change in import costs is delayed until the next period, the trade-off improves. Hence, kind and degree of pass-through are relevant in the decision of how gradual the stabilization of CPI inflation should be.

Finally, interest smoothing constrains the ability of the central bank to stabilize CPI inflation in the short run and, with delayed pass-through this constrains is stricter than with incomplete pass-through.

These findings are all grounded on the fact that the pass-through affects the efficiency of the direct exchange rate channel in the transmission of the monetary impulse and foreign shocks and, on the other hand, makes more relevant the endogenous shocks generated by the monetary policy itself.

Two natural extensions of this analysis are introducing (i) endogenous passthrough and (ii) the casual relation from the monetary policy to the kind and degree of pass-through in a Taylor (2000) or Devereux and Engel, (2001) style. In this case, considering that the trade-off between the stabilization of CPI inflation and the output gap could be improved by affecting the kind and the degree of pass-through, there could arise a more efficient equilibrium.

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## A Conditions on the sum of the interest rates

### A.1 Benchmark case

To obtain equation (22) in the text, first substitute  $\delta_1 = 1$ ,  $\delta_2 = 0$  in (17). Then, leading the resulting equation by  $\tau$  periods and taking the expectations at period  $t$  yields

$$
i_{t+\tau|t} = \frac{\omega - 1}{\omega} \pi_{t+\tau+1|t} + i_{t+\tau|t}^{*} - \pi_{t+\tau+1}^{*} + \psi_{t+\tau|t}
$$

Next, summing over  $\tau$  we obtain

$$
\sum_{\tau=0}^{\infty} i_{t+\tau|t} = \frac{\omega - 1}{\omega} \sum_{\tau=0}^{\infty} \pi_{t+\tau+1|t} + \sum_{\tau=0}^{\infty} i_{t+\tau|t}^{*} - \sum_{\tau=0}^{\infty} \pi_{t+\tau+1|t}^{*} + \sum_{\tau=0}^{\infty} \psi_{t+\tau|t}
$$

and considering (21) we finally arrive to (22).

### A.2 General case

Solving equation (15) for  $q_t$  yields

 $q_t = \frac{1}{\omega \delta_1} \left\{-\left[1-\omega(1-\delta_1)\right] \pi_t - \omega \delta_2 \pi_{t-1} - \omega \delta_2 \left(q_{t-1} - q_{t-2}\right)\right\} + q_{t-1}$ 

Since the variables  $q_{t-1}$ ,  $q_{t-2}$  are predetermined, assuming that in  $t-1$  and  $t-2$  the economy was in steady state implies  $q_{t-1} = 0$ ,  $q_{t-2} = 0$ . Then the previous equation yields

$$
q_t = -\frac{\left[1 - \omega(1 - \delta_1)\right]}{\omega \delta_1} \pi_t \tag{24}
$$

Solving equation (16) for  $q_t$  yields

$$
q_t = \frac{1}{\omega(\delta_2 - \delta_1)} \left\{-\left[1 - \omega(1 - \delta_1)\right] \pi_{t+1|t} - \omega \delta_2 \pi_t + \omega \delta_2 q_{t-1} - \omega \delta_1 q_{t+1|t}\right\}
$$

Assuming that in  $t - 1$  and  $t - 2$  the economy was in steady state, and letting  $a \equiv \frac{1}{\omega(\delta_2 - \delta_1)}, \ b \equiv [1 - \omega(1 - \delta_1)], \ c \equiv \omega\delta_2 \text{ and } d \equiv -\omega\delta_1 \text{ the previous equation}$ can be rewritten as

$$
q_t = -ab\pi_{t+1|t} - ac\pi_t + adq_{t+1|t}
$$

which can be solved forward to obtain

$$
q_t = -ab \sum_{\tau=0}^{\infty} (ad)^{\tau} \pi_{t+1+\tau|t} - ac \sum_{\tau=0}^{\infty} (ad)^{\tau} \pi_{t+\tau|t} + \lim_{\tau \to \infty} (ad)^{\tau} q_{t+\tau+1|t}
$$

Since  $\lim_{\tau \to \infty} q_{t+\tau+1|t} = 0$  and  $ad = \frac{-\omega \delta_1}{\omega(\delta_2 - \delta_1)}$  if  $\delta_1 \neq \delta_2$  it follows that  $\lim_{\tau \to \infty} (ad)^{\tau} q_{t+\tau+1|t} =$ 0 and we obtain

$$
q_t = -\frac{1}{\omega(\delta_2 - \delta_1)} \sum_{\tau=0}^{\infty} \left( \frac{-\omega \delta_1}{\omega(\delta_2 - \delta_1)} \right)^{\tau} \left\{ \left[ 1 - \omega(1 - \delta_1) \right] \pi_{t+1+\tau|t} - \omega \delta_2 \pi_{t+\tau|t} \right\} \tag{25}
$$

Now, combining (24) with (25) yields the equation that the expected inflation forward looking variables have to fulfil to have  $\pi_t^c = 0$ ,

$$
-\frac{\left[1-\omega(1-\delta_{1})\right]}{\omega\delta_{1}}\pi_{t} = -\frac{1}{\omega\left(\delta_{2}-\delta_{1}\right)}\sum_{\tau=0}^{\infty} \left(\frac{-\omega\delta_{1}}{\omega\left(\delta_{2}-\delta_{1}\right)}\right)^{\tau} \left\{\left[1-\omega(1-\delta_{1})\right]\pi_{t+1+\tau|t} - \omega\delta_{2}\pi_{t+\tau|t}\right\}
$$
\n(26)

Note that for  $\delta_1 = 1$  and  $\delta_2 = 0$  we obtain  $-\pi_t = \sum_{n=0}^{\infty} \frac{1}{n^2}$  $\tau = 0$  $\pi_{t+1+\tau|t}$  which is equivalent

to eq. (21) determined before.

Using the previous notation and expanding the infinite sum, we can simplify eq (26) so as to obtain

$$
\frac{b}{d}\pi_t = -a \{-c\pi_t + (b - dac)\pi_{t+1|t} + da(b - dac)\pi_{t+2|t} + (da)^2 (b - dac)\pi_{t+3|t} + \dots\}
$$
  
\n
$$
\iff (\frac{b}{d} - ac)\pi_t = -a(b - dac) \sum_{\tau=0}^{\infty} (da)^{\tau} \pi_{t+\tau+1|t}
$$

and, finally,

$$
-\frac{1}{da}\pi_t = \sum_{\tau=0}^{\infty} (da)^{\tau} \pi_{t+\tau+1|t}
$$
 (27)

Now consider equation (17) in the paper and, to ease the notation, recall that  $\pi^*_{t+1|t} = \gamma^*_{\pi} \pi^*_{t}$  and let

$$
e \equiv -\frac{\delta_2}{\delta_1} \left(\frac{\omega - 1}{\omega \delta_1}\right), \quad f \equiv \left(\frac{\delta_2}{\delta_1}\right)^2, \quad g \equiv \left(\frac{\omega - 1}{\omega \delta_1}\right) \text{ to obtain}
$$
\n
$$
i_t = e\pi_t + f\pi_{t-1} - \pi_{t+1|t}^* + i_t^* + \psi_t + f\left(q_{t-1} - q_{t-2}\right) + g\pi_{t+1|t}
$$

Then, leading any variable  $\tau$  periods, taking the expectation at period t and multiplying both side for the factor  $(da)^{\tau}$  yields

$$
(da)^{\tau} i_{t+\tau|t} = e (da)^{\tau} \pi_{t+\tau|t} + f (da)^{\tau} \pi_{t-1+\tau t} - (da)^{\tau} \pi_{t+1+\tau|t}^{*} + (da)^{\tau} i_{t+\tau|t}^{*} + (da)^{\tau} \psi_{t+\tau|t} + f (da)^{\tau} (q_{t-1+\tau|t|} - q_{t-2+\tau|t}) + g (da)^{\tau} \pi_{t+1+\tau|t}
$$

and summing over  $\tau$  yields

$$
\sum_{\tau=0}^{\infty} (da)^{\tau} i_{t+\tau|t} = (da)^{\tau} \left( e\pi_{t+\tau|t} + f\pi_{t-1+\tau|t} + g\pi_{t+1+\tau|t} \right) \n+ \sum_{\tau=0}^{\infty} (da)^{\tau} \left( i_{t+\tau|t}^{*} - \pi_{t+1+\tau|t}^{*} + \psi_{t+\tau|t} \right) + f \sum_{\tau=0}^{\infty} (da)^{\tau} \left( q_{t-1+\tau|t|} - q_{t-2+\tau|t} \right) \n\iff
$$

$$
\sum_{\tau=0}^{\infty} (da)^{\tau} i_{t+\tau|t} = [e\pi_t + e(da)\pi_{t+1|t} + e(da)^2 \pi_{t+2|t} + \dots + f\pi_{t-1} + fda\pi_t + f(da)^2 \pi_{t+1|t} \n+ \dots + g\pi_{t+1|t} + g(da) \pi_{t+2|t} + g(da)^2 \pi_{t+3|t} + \dots] \n+ \sum_{\tau=0}^{\infty} (da)^{\tau} (i_{t+\tau|t}^{*} - \pi_{t+1+\tau|t}^{*} + \psi_{t+\tau|t}) \n+ f[q_{t-1} + (da) q_t + (da)^2 q_{t+1|t} + \dots - q_{t-2} - (da) q_{t-1} - (da)^2 q_t - \dots]
$$

$$
\sum_{\tau=0}^{\infty} (da)^{\tau} i_{t+\tau|t} = \left\{ (e + fda) \pi_t + [da (e + fda) + g] \pi_{t+1|t} + da [da (e + fda) + g] \pi_{t+2|t} + ... \right\}
$$

$$
+ \sum_{\tau=0}^{\infty} (da)^{\tau} (i_{t+\tau|t}^{*} - \pi_{t+1+\tau|t}^{*} + \psi_{t+\tau|t}) - fq_{t-2}
$$

$$
\iff
$$

$$
\sum_{\tau=0}^{\infty} (da)^{\tau} i_{t+\tau|t} = (e + fda) \pi_t + [da (e + fda) + g] \sum_{\tau=0}^{\infty} (da)^{\tau} \pi_{t+1+\tau|t} \qquad (28)
$$

$$
+ \sum_{\tau=0}^{\infty} (da)^{\tau} (i_{t+\tau|t}^{*} - \pi_{t+1+\tau|t}^{*} + \psi_{t+\tau|t}) - f q_{t-2}
$$

Now, substituting (27) into (28) yields

⇐⇒

$$
\sum_{\tau=0}^{\infty} (da)^{\tau} i_{t+\tau|t} = (e + fda) \pi_t + [da (e + fda) + g] \left( -\frac{1}{da} \pi_t \right) \tag{29}
$$

$$
+ \sum_{\tau=0}^{\infty} (da)^{\tau} \left( i_{t+\tau|t}^* - \pi_{t+1+\tau|t}^* + \psi_{t+\tau|t} \right) - f q_{t-2}
$$

Since the right hand side is predetermined, this equation establishes the condition that the sum of all current and expected interest rate have to satisfy to have  $\pi_t^c = 0$ . As in the case of  $\delta_1 = 1, \delta_2 = 0$ , the central bank use this condition and the reaction function (eq.  $(17)$  in the paper) to determine in any period  $t$  the the optimal monetary policy, i.e. the sequence  $\{i_{t+\tau|t}\}_{\tau=0}^{\infty}$ .

Note that setting  $\delta_1 = 1$ ,  $\delta_2 = 0$  we have  $da = 1$ ,  $e = 0$ ,  $f = 0$ ,  $g = \frac{\omega - 1}{\omega}$  and (29) boils down to (22).

# B State-space form of the model

The model can be set out in the following state-space form

$$
Min_{\{i_{t+\tau|t}\}_{\tau=0}^{\infty}} E_t \sum_{\tau=0}^{\infty} \beta^{\tau} Y'_{t+\tau} K Y_{t+\tau}
$$
\n
$$
subject \ to \ \left[ \begin{array}{c} X_{t+1} \\ x_{t+1|t} \end{array} \right] = A \left[ \begin{array}{c} X_t \\ x_t \end{array} \right] + Bi_t + B^1 i_{t+1|t} + \left[ \begin{array}{c} v_{t+1} \\ 0 \end{array} \right]
$$
\n
$$
where \ Y_t \equiv C_Z \left[ \begin{array}{c} X_t \\ x_t \end{array} \right] + C_i i_t
$$

and where  $X_t$  is the (column) vector of predetermined variables,  $X_t = (\pi_t, \pi_{t-1}, y_t, \pi_t^*, y_t^*, i_t^*, \varphi_t, y_t^n, q_{t-1}, q_{t-2}, i_{t-1}, \pi_{t+1|t})',$  $x_t$  is the (column) vector of forward-looking variables,  $x_t = (q_t, \rho_t, \pi_{t+2|t})'$ ,  $\nu_t$  is the (column) vector of innovations to the predetermined state variables,  $\nu_t = \left(\varepsilon_t, 0, \eta_t^d - \eta_t^n, \varepsilon_t^*, \eta_t^*, f_\pi^* \varepsilon_t^* + f_y^* \eta_t^* + \xi_{it}^*, \xi_{\varphi t}, \eta_t^n, 0, 0, 0, \delta_y \beta_y \left(\eta_t^d - \eta_t^n\right)\right)',$  $Y_t$  is the (column) vector of goal variables,  $Y_t = (\pi_t^c, \pi_t, y_t, i_t, i_t - i_{t-1})'$ , and the matrixes  $A, B, B<sup>1</sup>, C<sub>Z</sub>$  and  $C<sub>i</sub>$  are given, respectively, by

 $\overline{\phantom{a}}$ 

 $\mathbf{I}$  $\mathbf{I}$  $\frac{1}{2}$  $\mathbf{I}$  $\mathbf{I}$  $\mathbf{I}$  $\mathbf{I}$  $\mathbf{I}$  $\mathbf{I}$  $\frac{1}{2}$  $\mathbf{I}$  $\mathbf{I}$  $\mathbf{I}$  $\mathbf{I}$  $\mathbf{I}$  $\mathbf{I}$  $\mathbf{I}$  $\frac{1}{2}$  $\mathbf{I}$  $\mathbf{I}$  $\mathbf{I}$  $\mathbf{I}$  $\mathbf{I}$  $\perp$  $\mathbf{I}$ 

$$
A = \begin{bmatrix} e_{12} & e_{12} \\ e_{1} & e_{1} \\ \frac{e_{13}}{\gamma_{y}^{*}e_{5} + \beta_{q}A_{13} - (\gamma_{y}^{n} - \beta_{y})e_{8}} \\ \frac{e_{12}}{\gamma_{y}^{*}e_{4}} & \frac{e_{13}}{\gamma_{y}^{*}e_{5}} \\ f_{\pi}^{*}\gamma_{\pi}^{*}e_{4} + f_{y}^{*}\gamma_{y}^{*}e_{5} \\ \frac{e_{13}}{\gamma_{y}^{n}e_{8}} & e_{13} \\ e_{9} & e_{0} \\ e_{15} & e_{12} + A_{4} - e_{6} - e_{7} \\ e_{12} + e_{14} \\ A_{n} \end{bmatrix}
$$

where

$$
A_n = \frac{1}{1-\alpha_{\pi}} \left( -\alpha_{\pi}e_{12} + \left[ 1 + \alpha_y \left( \beta_{\rho} + \beta_q \right) + \alpha_q \right] e_{15} - \alpha_y \beta_y A_3 + \alpha_y \beta_{\rho} A_{14} - \alpha_y \beta_y^* \gamma_y^* A_5 - \left( \alpha_y \beta_q + \alpha_q \right) A_{13} + \left( \alpha_y \beta_q + \alpha_q \right) \left( A_6 - \gamma_{\pi}^* A_4 + \gamma_{\varphi} e_7 \right) + \alpha_y \left( \gamma_y^n - \beta_y \right) \gamma_y^n e_8 \right)
$$

$$
B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -1 \\ \frac{-1}{1-\alpha\pi} \left[ \alpha_y \left( 1 + \beta_y \right) \left( \beta_\rho + \beta_q \right) + \alpha_q \right] \\ \mathcal{C}_Z = \begin{bmatrix} \left( 1 - \omega \delta \right) e_1 + \omega \delta e_2 + \omega \delta \left( e_9 - e_{10} \right) + \omega \left( 1 - \delta \right) \left[ e_{13} - e_9 \right] \\ e_1 \\ e_3 \\ - e_{11} \end{bmatrix}, \quad C_i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix},
$$
 
$$
C_i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}
$$

where, using the Svensson's definitions,  $e_j$ ,  $j = 0, ..., 15$  stands for a 1x15 row vector that for  $j = 0$  has all the elements equal to zero and for  $j \neq 0$  has element j equal to unity and all other elements equal to zero;  $A_i$  stands for row j of the matrix A.

Finally, the matrix K is a diagonal matrix with  $(\mu^c_\pi, \mu_\pi, \lambda, \mu_i, \nu_i)$  along the main diagonal.

At this point the algorithm used by Oudiz and Sachs (1985) as well as by Backus and Driffil (1986) for the solution of the optimization problem in the dynamic programming framework is applied and a detailed description is provided by Soderlind (1998) and (1999). For the problem raised by the presence of future controls, see the working paper version of Svensson (2000).

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Figure 2. Strict CPI inflation targeting. IRF to a cost-push shock.  $\nu = 0.05$ . In the first, second, third and fourth column,  $\delta_1$  and  $\delta_2$  are equal, respectively, to  $(1, 0)$ ,  $(0.5, 0), (0.7, 0.3)$  and  $(0.4, 0.6)$ .



Figure 3. Strict CPI inflation targeting. IRF to a foreign inflation shock.  $\nu = 0$ . In the first, second, third and forth columns,  $\delta_1$ ,  $\delta_2$  are equal, respectively, to  $(1, 0)$ ,  $(0.5, 0), (0.7, 0.3)$  and  $(0.4, 0.6)$ .



Figure 4. Flexible infation targeting. IRF to a cost-push shock.  $\nu = 0$ . In the first, second, third and forth columns,  $\delta_1$ ,  $\delta_2$  are equal, respectively, to  $(1, 0)$ ,  $(0.5, 0)$ , (0.7, 0.3) and (0.4, 0.6).



Figure 5. Flexible inflation targeting. IRF to a foreign inflation shock.  $\nu = 0$ . In the first, second, third and forth columns,  $\delta_1$ ,  $\delta_2$  are equal, respectively, to (1, 0), (0.5, 0), (0.7, 0.3) and (0.4, 0.6).



Figure 6. Strict CPI inflation targeting. IRF to a cost-push shock.  $\nu = 0$ . In the first, second, third and fourth column,  $\delta_1$ ,  $\delta_2$  are equal, respectively, to (0.6, 0.4),  $(0.56, 0.44), (0.52, 0.48)$  and  $(0.5, 0.5)$ .