Dynamical Elements of the Monetary Theory

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ABSTRACT

Here I depict the dynamics behind the monetary theory and define the differential equations describing the driving forces.

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1. Introduction

Dynamics of economic behavior of commodity's supply, demand, and price were described by author in the number of articles [1, 2, 3, 4]. Author also tried in [5] to connect the dynamics of commodity's growth and decline with such monetary concept as an interest rate.

Here author considers the phenomena of money as a peculiar commodity. He describes the dynamics of the money supply, demand, and price by the same differential equations as the ones used to describe the dynamical behavior of a regular commodity.

Then author shows how the dynamical behavior of the monetary volume affects the dynamical behavior of regular commodities. He gives a representative example, where the dynamics of commodity's supply is described as the superposition of two sinusoids.

Later the author presents a joined model of the monetary and a single commodity's markets, and examines its solutions. Next he expands these results onto markets with multiple commodities produced by multiple producers.

2. Dynamics of the Commodity's Market

When commodity's supply (production) and demand coincide, the commodity's market lies in the equilibrium position, and behaves according to the first Newton's law. It means the commodity's supply and demand are both developing with a constant rate and the commodity's price is fixed.

When this balance between the commodity's supply and demand is broken the commodity's market is experienced the economic forces, which act to bring the market to a new equilibrium position.

These market forces are described by the following differential equations regarding to the commodity's supply $V_S(t)$, demand $V_D(t)$, and price $P_C(t)$.

$$\frac{d^2 V_S(t)}{dt^2} = \lambda_1 \cdot \frac{dP_C(t)}{dt} \tag{1}$$

$$\frac{dP_C(t)}{dt} = \lambda_2 \cdot (V_D(t) - V_S(t)) \tag{2}$$

$$\frac{d^2 V_D(t)}{dt^2} = -\lambda_3 \cdot \frac{d^2 P_C(t)}{dt^2} \tag{3}$$

In the equations above the values $\lambda_1,\ \lambda_2,\ \lambda_3 \geq 0$ are constants.

The meaning of equations (1) – (3) is as follows. When commodity's demand and supply are not equal, the price of commodity changes. If commodity's demand exceeds the one's supply, the price goes up, and it goes down in another circumstances. When the commodity's price changes, the commodity's supply changes. When the price increases, the commodity's supply goes up, and it goes down in another circumstances. Also when the commodity's price changes, the commodity's demand experiences the oppressive effect. When the price increases, the commodity's demand goes down, and it goes up in another circumstances.

3. Dynamics of the Monetary Market

Similarly for the monetary market, when money supply and demand for money coincide, the monetary market lies in an equilibrium position, and behaves according to the first Newton's law. The money supply and demand are both developing with a constant rate and the price of money is fixed. The price of money is commonly represented by an interest rate.

When such balance between the money supply and demand is broken, the money market is experienced the economic forces, which act to bring the market to a new equilibrium position.

These market forces are described by the following differential equations regarding to the money supply $M_S(t)$, demand $M_D(t)$, and price $P_M(t)$.

$$\frac{d^2 M_S(t)}{dt^2} = m_1 \cdot \frac{dP_M(t)}{dt} \tag{4}$$

$$\frac{dP_M(t)}{dt} = m_2 \cdot (M_D(t) - M_S(t)) \tag{5}$$

$$\frac{d^2 M_D(t)}{dt^2} = -m_3 \cdot \frac{d^2 P_M(t)}{dt^2} \tag{6}$$

In the equations (4) – (6) above the values $m_1, m_2, m_3 \geq 0$ are constants.

The meaning of equations (4) – (6) is as follows. When the money demand and supply are not equal, the price of money changes. If demand for money exceeds the one's supply, the price of money goes up, and it goes down in another circumstances. When the price of money changes, the money supply changes. When the price of money increases, the money supply goes up, and it goes down in another circumstances. Also when the price of money changes, the demand for money experiences the oppressive effect. When the price of money increases, the demand for money goes down, and it goes up in another circumstances.

4. Mutual Impacts of Commodity's and Monetary Markets

Let me consider the economical model consisting of the single commodity's market and the monetary market.

At the beginning the commodity's supply and demand remain equal, and the commodity's market is in an equilibrium position. Also the money supply and demand are equal, and the money market is in an equilibrium position as well.

a) Consider a scenario when the demand for money is changed.

The increase of demand for money creates a growth in the price of money (an interest rate) and causes inflow of the money supply. The inflow of money supply increases the demand for commodity, which causes a growth in the commodity's price, and hence an increase of the commodity's supply.

The decrease of demand for money creates a reduction in the price of money (an interest rate) and causes outflow of the money supply. The outflow of money supply decreases the demand for commodity, which causes a reduction in the commodity's price, and hence a decrease of the commodity's supply.

b) Consider a scenario when the price of money is changed.

A growth in the price of money (an interest rate) causes inflow of the money supply. It increases the demand for commodity, which causes a growth in the commodity's price, and hence an increase of the commodity's supply.

A reduction in the price of money (an interest rate) causes outflow of the money supply. It decreases the demand for commodity, which causes a reduction in the commodity's price, and hence a decrease of the commodity's supply.

c) Consider a scenario when the supply of money is changed.

The increase of money supply increases the demand for commodity, which causes a growth in the commodity's price, and therefore an increase of the commodity's supply.

The decrease of money supply decreases the demand for commodity, which causes a reduction in the commodity's price, and therefore a decrease of the commodity's supply.

d) Consider a scenario when the commodity's price is changed.

A growth in the commodity's price increases the commodity's supply.

A reduction in the commodity's price decreases the commodity's supply.

e) Consider a scenario when the commodity's supply is changed.

That scenario is trivial.

5. A Model of Joined Commodity's and Monetary Markets

Let me investigate the scenario (a) how changes on the monetary market impose the changes on the commodity's market. For this purpose I shall operate with the economical model consisting of the single commodity's market and the monetary market.

By using the equations (1) – (6) I am going to formalize the forenamed scenario through ordinary differential equations (ODE).

Consumers' demand for commodity V_D and commodity's supply V_S are equal to V_0 initially. Therefore it holds $\frac{dV_S(t)}{dt} = \frac{dV_D(t)}{dt} = v_0$ until $t \le t_0$ where $v_0 \ge 0$ is a constant. The commodity's price is fixed $P_C(t) = P_0$ for $t \le t_0$.

Money demand M_D and supply M_S are equal initially as well. Therefore it holds $\frac{dM_S(t)}{dt} = \frac{dM_D(t)}{dt} = h_0$ until $t \le t_0$ where $h_0 \ge 0$ is a constant. The price of money (interest rate) is fixed $P_M(t) = I_0$ for $t \le t_0$.

Since both commodity's and money markets are stationary relative to each other, it takes place $n_0 = h_0$ for $t \le t_0$.

According to our scenario at time $t = t_0$ we have the following conditions for the monetary market $M_D(t_0) = M_0 + \Delta$, $M_S(t_0) = M_0$, $P_M(t_0) = I_0$ where $\Delta \ne 0$.

Therefore [1, 2] the deviation between the money supply $M_S(t)$ and the money demand $M_D(t)$ for $t > t_0$ is described by the second-order linear homogeneous ODE with constant coefficients. Solution depends on the roots of characteristic equation. If $m_3/2 \ge \sqrt{m_1/m_2}$ the money supply $M_S(t)$ asymptotically approaches the money demand

$M_D(t)$ in exponential-like manner. If $0 < m_3/2 < \sqrt{m_1/m_2}$ the money supply $M_S(t)$ asymptotically approaches the money demand $M_D(t)$ in damped-oscillations-like manner.

The change in the money supply causes the changes in the consumers' demand for commodity. Let me look at the correlation between the money supply $M_S(t)$ and the consumers' demand for commodity $V_D(t)$ in our model. It is somewhat "natural" to assume that consumers' demand for commodity is inversely proportional to the commodity's price. However in our model the whole money supply is directed to the single commodity's market.

It gives the following condition for the money and commodity's markets in our model $V_D(t) = M_S(t)$. If we place the solution for the consumers' demand for commodity $V_D(t)$ into the second-order ODE describing the commodity's supply $V_S(t)$ [1, 2], we obtain the second-order linear nonhomogeneous ODE with constants coefficients describing the commodity's supply $V_S(t)$ for $t > t_0$. It is known that general solution of such equation represents the sum of the particular solution of the equation in question and the general solution of the corresponding ordinary differential equation [8].

6. A Simple Example of Model's Solution

Consider the following example where at time $t = t_0$ we have the following conditions for the monetary market $\frac{dM_S(t_0)}{dt} = \frac{dM_D(t_0)}{dt} = 0$, $M_D(t_0) = \Delta$, $M_S(t_0) = 0$ where $\Delta \neq 0$.

Let me assume in this example for simplicity that $m_3 = 0$. Then from (4) – (6) the second-order ODE describing the money supply $M_S(t)$ is,

$$\frac{d^2 M_S(t)}{dt^2} + m_1 \cdot m_2 \cdot (M_S(t) - M_D(t)) = 0 \tag{7}$$

or since $\dfrac{d^2 M_D(t)}{dt^2} = 0$,

$$\frac{d^2 (M_S(t) - M_D(t))}{dt^2} + m_1 \cdot m_2 \cdot (M_S(t) - M_D(t)) = 0 \tag{8}$$

where $M_S(t_0) - M_D(t_0) = -\Delta$ and $\dfrac{d(M_S(t_0) - M_D(t_0))}{dt} = 0$.

Therefore

$$M_S(t - t_0) - M_D(t - t_0) = -\Delta \cdot \cos\left(\sqrt{m_1 \cdot m_2}(t - t_0)\right) \tag{9}$$

and

$$M_S(t - t_0) = -\Delta \cdot \cos\left(\sqrt{m_1 \cdot m_2}(t - t_0)\right) + \Delta \tag{10}$$

At the same time $t = t_0$ we have the following conditions for the commodity's market $\dfrac{dV_S(t_0)}{dt} = \dfrac{dV_D(t_0)}{dt} = n_0$, $V_S(t_0) = V_D(t_0) = M_0$, $P_C(t_0) = P_0$ where $n_0 = h_0 \geq 0$ and the following condition connecting both these markets $V_D(t) = M_S(t)$.

Since the consumers' demand for commodity $V_D(t)$ is driven by the money supply $M_S(t)$ without any feedback then coefficient $l_3 = 0$. Hence from (1) – (3) the second-order ODE describing the commodity's supply $V_S(t)$ is,

$$\frac{d^2 V_S(t)}{dt^2} + l_1 \cdot l_2 \cdot (V_S(t) - M_S(t)) = 0 \tag{11}$$

where $V_S(t_0) = 0$ and $\frac{dV_S(t_0)}{dt} = 0$.

Consequently from [8]

$$V_S(t - t_0) = \Delta \cdot \left[\frac{m_1 \cdot m_2}{l_1 \cdot l_2 - m_1 \cdot m_2} \cdot \cos\left(\sqrt{l_1 \cdot l_2}\,(t - t_0)\right) - \frac{l_1 \cdot l_2}{l_1 \cdot l_2 - m_1 \cdot m_2} \cdot \cos\left(\sqrt{m_1 \cdot m_2}\,(t - t_0)\right) + 1 \right] \tag{12}$$

Thus the solution of the studied example represents the sum of two sinusoid-like functions.

7. An Extended Example of Model's Description

Let me combine the equations (1) – (3) and (4) – (6) into one ODE system,

$$\frac{d^2 V_S(t)}{dt^2} = l_1 \cdot \frac{dP_C(t)}{dt} \tag{13}$$

$$\frac{dP_C(t)}{dt} = l_2 \cdot (M_S(t) - V_S(t)) \tag{14}$$

$$\frac{d^2 M_S(t)}{dt^2} = m_1 \cdot \frac{dP_M(t)}{dt} - l_3 \cdot \frac{d^2 P_C(t)}{dt^2} \tag{15}$$

$$\frac{dP_M(t)}{dt} = m_2 \cdot (M_D(t) - M_S(t)) \tag{16}$$

$$\frac{d^2 M_D(t)}{dt^2} = -m_3 \cdot \frac{d^2 P_M(t)}{dt^2} \tag{17}$$

We can rewrite the system (13) – (17) into following ODE system,

$$\frac{d^2(M_S(t) - V_S(t))}{dt^2} + l_2 \cdot l_3 \cdot \frac{d(M_S(t) - V_S(t))}{dt} + l_1 \cdot l_2 \cdot (M_S(t) - V_S(t)) = m_1 \cdot m_2 \cdot (M_D(t) - M_S(t)) \tag{18}$$

$$\frac{d^2\left(M_D(t)-M_S(t)\right)}{dt^2}+m_2\cdot m_3\cdot\frac{d\left(M_D(t)-M_S(t)\right)}{dt}+m_1\cdot m_2\cdot\left(M_D(t)-M_S(t)\right)=l_2\cdot l_3\cdot\frac{d\left(M_S(t)-V_S(t)\right)}{dt} \tag{19}$$

According to [8, 9] let us replace $\left(M_S(t)-V_S(t)\right)=a\cdot e^{k\cdot t}$ and $\left(M_D(t)-M_S(t)\right)=b\cdot e^{k\cdot t}$ in order to reproduce (18) – (19) as follows

$$\left(k^2+l_2\cdot l_3\cdot k+l_1\cdot l_2\right)\cdot a-m_1\cdot m_2\cdot b=0 \tag{20}$$

$$-l_2\cdot l_3\cdot k\cdot a+\left(k^2+m_2\cdot m_3\cdot k+m_1\cdot m_2\right)\cdot b=0 \tag{21}$$

Then one looks for the roots of the characteristic equation

$$\begin{vmatrix} \left(k^2+l_2\cdot l_3\cdot k+l_1\cdot l_2\right) & -m_1\cdot m_2 \\ -l_2\cdot l_3\cdot k & \left(k^2+m_2\cdot m_3\cdot k+m_1\cdot m_2\right) \end{vmatrix}=0, \tag{22}$$

determines a and b from system (20) and (21), and finds the general solutions for $\left(M_S(t)-V_S(t)\right)$ and $\left(M_D(t)-M_S(t)\right)$ as a linear combination of corresponding particular solutions in accordance with [8, 9].

8. A Case of Multiple Producers of Single Commodity

Let me consider following [6] the economical model where the single commodity is produced by multiple producers, and explore the impact of the monetary market.

At the beginning the commodity's total supply and demand remain equal, and the commodity's market is in an equilibrium position. Also the money supply and demand are equal, and the money market is in an equilibrium position as well.

a) A scenario when the commodity's supply by one of the producers is changed.

The increase of the commodity's supply by one of the producers creates a reduction of the commodity's price. The first result of the commodity's price drop is the decrease of commodity's supply by another producers. Another result of the commodity's price drop is an increase of the demand for commodity, which causes respectively the inflow of the money supply, a growth in the price of money (an interest rate), and a decrease of the demand for money.

The decrease of the commodity's supply by one of the producers creates a growth of the commodity's price. The first result of the commodity's price advance is the increase of commodity's supply by another producers. Another result of the commodity's price advance is a decrease of the demand for commodity, which causes respectively the outflow of the money supply, a reduction in the price of money (an interest rate), and an increase of the demand for money.

b) A scenario when the commodity's price is changed.

A growth of the commodity's price increases the commodity's supply by another producers and decreases the demand for commodity. The latter one respectively causes the outflow of the money supply, a reduction in the price of money (an interest rate), and an increase of demand for money.

A reduction of the commodity's price decreases the commodity's supply by another producers and increases the demand for commodity. The latter one respectively causes the inflow of the money supply, a growth in the price of money (an interest rate), and a decrease of demand for money.

c) A scenario when the supply of money is changed.

The inflow of money supply causes a growth in the price of money (an interest rate), and a decrease of demand for money.

The outflow of money supply causes a reduction in the price of money (an interest rate), and an increase of demand for money.

d) A scenario when the price of money (an interest rate) is changed.

A growth in the price of money (an interest rate) causes the decrease of demand for money.

A reduction in the price of money (an interest rate) causes the increase of demand for money.

e) A scenario when the demand for money is changed.

That scenario is trivial.

Let me write an ODE system, which describes the dynamics of the commodity's and monetary markets, where two different producers are supplying a single commodity,

$$\frac{d^2V_{S1}(t)}{dt^2} = \lambda_{11} \cdot \frac{dP_C(t)}{dt} \tag{23}$$

$$\frac{d^2V_{S2}(t)}{dt^2} = \lambda_{12} \cdot \frac{dP_C(t)}{dt} \tag{24}$$

$$\frac{dP_C(t)}{dt} = \lambda_2 \cdot \left(M_S(t) - \left(V_{S1}(t) + V_{S2}(t)\right)\right) \tag{25}$$

$$\frac{d^2 M_S(t)}{dt^2} = m_1 \cdot \frac{dP_M(t)}{dt} - l_3 \cdot \frac{d^2 P_C(t)}{dt^2} \tag{26}$$

$$\frac{dP_M(t)}{dt} = m_2 \cdot \left(M_D(t) - M_S(t)\right) \tag{27}$$

$$\frac{d^2 M_D(t)}{dt^2} = -m_3 \cdot \frac{d^2 P_M(t)}{dt^2} \tag{28}$$

where the values $\boldsymbol{l}_{11} \geq 0$ and $\boldsymbol{l}_{12} \geq 0$ are constants.

We can rewrite ODE system (23) – (28) into the following one using subsequent notation for the total commodity's supply $V_S(t) = V_{S1}(t) + V_{S2}(t)$,

$$\frac{d^2\left(M_S(t) - V_S(t)\right)}{dt^2} + l_2 \cdot l_3 \cdot \frac{d\left(M_S(t) - V_S(t)\right)}{dt} + \left(\sum_{i=1}^{N} l_{1i}\right) \cdot l_2 \cdot \left(M_S(t) - V_S(t)\right) = m_1 \cdot m_2 \cdot \left(M_D(t) - M_S(t)\right) \tag{29}$$

$$\frac{d^2\left(M_D(t) - M_S(t)\right)}{dt^2} + m_2 \cdot m_3 \cdot \frac{d\left(M_D(t) - M_S(t)\right)}{dt} + m_1 \cdot m_2 \cdot \left(M_D(t) - M_S(t)\right) = l_2 \cdot l_3 \cdot \frac{d\left(M_S(t) - V_S(t)\right)}{dt} \tag{30}$$

where all values $l_{1i} \geq 0$ are constants, and $N \geq 1$ is the number of producers.

9. A Case of Multiple Producers and Multiple Commodities

Here I describe markets with multiple commodities supplied by the multiple producers based on some results of [7].

At first let me consider the dynamics of the following commodity's and monetary markets. There are multiple distinct commodities, each one is produced a single producer, and the supply/demand of commodities relate in the Leontief-like manner [10, 11].

Therefore let me introduce a square matrix A, which consists of elements $a_{ij} \geq 0$ expressing an amount of commodity i needed for the production of one unit of a commodity j, and $M \geq 1$ is the number of commodities.

To operate with multiple commodities it is convenient to work with vector notation where the vector $\overline{V}_S(t) = \left(V_S^1(t) \quad \dots \quad V_S^M(t)\right)'$ serves for commodities' supply, the vector $\overline{V}_D(t) = \left(V_D^1(t) \quad \dots \quad V_D^M(t)\right)'$ serves for commodities' demand, and the vector $\overline{P}_C(t) = \left(P_C^1(t) \quad \dots \quad P_C^M(t)\right)'$ serves for commodities' prices.

Also let me introduce the vector $\widetilde{V}_S(t)$ of total values of commodities' supply and the vector $\widetilde{V}_D(t)$ of total values of commodities' demand, both of which include the redistributed portions of commodities intended for internal purposes. Similarly to [10, 11] the vector $\left(\widetilde{V}_D(t) - \widetilde{V}_S(t)\right)$ relates to the vector $\left(\overline{V}_D(t) - \overline{V}_S(t)\right)$ through the following matrix equation,

$$\left(\widetilde{V}_D(t) - \widetilde{V}_S(t)\right) = (I - A)^{-1} \cdot \left(\overline{V}_D(t) - \overline{V}_S(t)\right) \tag{31}$$

where I is a square identity matrix of size M. Note that an inverse matrix $(I - A)^{-1}$ exists, and consists of non-negative elements if and only if the Frobenius eigen-value of matrix A is strictly less than one [12].

Let me start with the ODE system, which describes the dynamics of a standalone commodities' market, where multiple producers are supplying multiple commodities.

The second derivative of the vector of commodities' supply $\overline{V}_S(t)$ is linearly proportional to the change of commodities' prices $\overline{P}_C(t)$,

$$\frac{d^2\overline{V}_S(t)}{dt^2} = \overline{\Lambda}_1 \cdot \frac{d\overline{P}_C(t)}{dt} \tag{32}$$

where the vector $\overline{\Lambda}_1 = \left(\Lambda_1^1 \quad \ldots \quad \Lambda_1^M\right)'$ is defined through the following expression

$\Lambda_1^j = \sum_{i=1}^{N_j} \boldsymbol{l}_{1i}^j$, all values $l_{1i}^j \geq 0$ are constants, and $N_j \geq 1$ is the number of producers i for the commodity j.

The change of commodities' prices $\overline{P}_C(t)$ is linearly proportional to the vector of total commodities' deficit/surplus $\left(\tilde{V}_D(t) - \tilde{V}_S(t)\right)$ on the commodity's market,

$$\frac{d\overline{P}_C(t)}{dt} = \overline{\Lambda}_2 \cdot (I - A)^{-1} \cdot \left(\overline{V}_D(t) - \overline{V}_S(t)\right) \tag{33}$$

where the vector $\overline{\Lambda}_2 = \left(\boldsymbol{l}_2^1 \quad \ldots \quad \boldsymbol{l}_2^M\right)'$ consists of constant values $l_2^j \geq 0$.

The second derivative of the vector of commodities' demand $\overline{V}_D(t)$ is inversely-linearly proportional to the second derivative of commodities' prices $\overline{P}_C(t)$,

$$\frac{d^2\overline{V}_D(t)}{dt^2} = -\overline{\Lambda}_3 \cdot \frac{d^2\overline{P}_C(t)}{dt^2} \tag{34}$$

where the vector $\overline{\Lambda}_3 = \left(\boldsymbol{l}_3^1 \quad \ldots \quad \boldsymbol{l}_3^M\right)'$ consists of constant values $l_3^j \geq 0$.

Thus the vector $\left(\overline{V}_D(t) - \overline{V}_S(t)\right)$ for the standalone commodities' market can be defined through the following matrix equation,

$$\frac{d^2\left(\overline{V}_D(t) - \overline{V}_S(t)\right)}{dt^2} + \overline{\Lambda}_2 \cdot \overline{\Lambda}_3 \cdot (I - A)^{-1} \cdot \frac{d\left(\overline{V}_D(t) - \overline{V}_S(t)\right)}{dt} + \overline{\Lambda}_1 \cdot \overline{\Lambda}_2 \cdot (I - A)^{-1} \cdot \left(\overline{V}_D(t) - \overline{V}_S(t)\right) = 0 \tag{35}$$

Now let me combine the standalone commodities' market described by the ODE system (32) – (34) with the monetary market described earlier by the ODE system (4) – (6). Then let me assume that changes in the money supply are reflected in the demand for each individual commodity in proportions defined by the vector of commodities' preferences $\overline{\Psi} = \left(y^1 \quad \dots \quad y^M\right)'$, which consists of constant values $y^j \geq 0$ and $\sum_{j=1}^{M} y^j = 1$. On the other hand, the changes in the total demand for all commodities are reflected in the money supply. Therefore, the following ODE system describes the dynamics of the joined commodities' and monetary markets,

$$\frac{d^2 \overline{V}_S(t)}{dt^2} = \overline{\Lambda}_1 \cdot \frac{d\overline{P}_C(t)}{dt} \tag{36}$$

$$\frac{d\overline{P}_C(t)}{dt} = \overline{\Lambda}_2 \cdot (I - A)^{-1} \cdot \left(\overline{V}_D(t) - \overline{V}_S(t)\right) \tag{37}$$

$$\frac{d^2 \overline{V}_D(t)}{dt^2} = -\overline{\Lambda}_3 \cdot \frac{d^2 \overline{P}_C(t)}{dt^2} + \overline{\Psi} \cdot m_1 \cdot \frac{dP_M(t)}{dt} \tag{38}$$

$$\frac{d^2 M_S(t)}{dt^2} = m_1 \cdot \frac{dP_M(t)}{dt} - \sum_{j=1}^{M} \left(l_3^j \cdot \frac{d^2 P_C^j(t)}{dt^2} \right) \tag{39}$$

$$\frac{dP_M(t)}{dt} = m_2 \cdot \left(M_D(t) - M_S(t)\right) \tag{40}$$

$$\frac{d^2 M_D(t)}{dt^2} = -m_3 \cdot \frac{d^2 P_M(t)}{dt^2} \tag{41}$$

where $M_S(t) = \sum_{j=1}^{M} V_D^j(t)$.

We can rewrite the ODE system (36) – (41) into the following one using the definition of the sum of vector's coordinates, $sum(\overline{x}) = \sum_{j=1}^{M} x^j$, where $\overline{x} = \left(x^1 \quad \ldots \quad x^M\right)'$,

$$\frac{d^2\left(\overline{V}_D - \overline{V}_S\right)}{dt^2} + \overline{\Lambda}_2 \cdot \overline{\Lambda}_3 \cdot (I - A)^{-1} \cdot \frac{d\left(\overline{V}_D - \overline{V}_S\right)}{dt} + \overline{\Lambda}_1 \cdot \overline{\Lambda}_2 \cdot (I - A)^{-1} \cdot \left(\overline{V}_D - \overline{V}_S\right) = \overline{\Psi} \cdot m_1 \cdot m_2 \cdot \left(M_D - sum\left(\overline{V}_D\right)\right) \tag{42}$$

$$\frac{d^2\left(M_D - sum\left(\overline{V}_D\right)\right)}{dt^2} + m_2 \cdot m_3 \cdot \frac{d\left(M_D - sum\left(\overline{V}_D\right)\right)}{dt} + m_1 \cdot m_2 \cdot \left(M_D - sum\left(\overline{V}_D\right)\right) = sum\left(\overline{\Lambda}_2 \cdot \overline{\Lambda}_3 \cdot (I - A)^{-1} \cdot \frac{d\left(\overline{V}_D - \overline{V}_S\right)}{dt}\right) \tag{43}$$

10. Conclusion

Here I attempted to expose the dynamics of the commodity's and monetary markets, and describe them through the ODE systems.

I anticipate that future development of this approach will serve a thorough understanding of driving forces in market economy, and may expand the operating control of a market economy from the current management of an interest rate to much more sophisticated apparatus.

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