On the Evidence of Non-Linear Structure in Canadian Unemployment.

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Abstract

In this paper, we focus on and examine the empirical evidence of non-linearity in aggregate Canadian unemployment. Contrary to the conclusion reached in Murray et al. (1993), and using a corrected for bias simple non-parametric test (SNT), we reject the null hypothesis of a linear structure for Canadian unemployment.

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1 Introduction

The appropriate modeling assumptions of the equilibrium unemployment rate are an open and unsettled debate. Most of the literature focus on the linear specifications while few address the non-linear aspect of the behavior of the unemployment rate. Of crucial importance to this modeling behavior is the stylised fact of unemployment persistence. We argue that this persistence is often incorrectly articulated and modeled using a linear specification - or at best, as a linear approximation at the equilibrium of an inherent non-linear specification. Measuring unemployment persistence is of concern to macroeconomic policy makers since whenever evidence of persistence is found, there exists the possibility of some room to decrease the unemployment rate without changing any structure in the organization of the labour market. How fast can the unemployment rate be decreased depends on the persistence mechanism. Whenever unemployment non-linearity is present, the persistence mechanism will be mis-specified as a linear process and the short-run adjustment of the economy can take place over a very long period.

From a time series perspective, adopting linear models often and implicitly support the neglect of testing for non-linearities. Prior to investigating the importance of non-linearities as a theoretical possibility, here, we investigate the empirical evidence of non-linearity in aggregate Canadian unemployment.

The literature on non-linear dynamics has proposed a wide range of tests to detect, and models to investigate, non-linearities in macroeconomic time series. Brock, Dechert and Scheinkman (1987) (BDS hereafter) derived and proposed a test for independence and identical distribution based on the correlation integral. Widely adopted as a powerful portmanteau statistic for model specification, the BDS test suffered from a caveat of a negative bias in small samples - namely, under 250 observations. Originally proposed by Grassberger and Procaccia (1984) as a U-statistic estimator, the correlation integral determines the fractal dimension of an attracting set. The class of U-statistics was first introduced by Hoeffdring (1948), wherein he showed that this class could be approximated using a projection representation as the sum of i.i.d. random variables. By changing the kernel used in the BDS statistic, Mizrach (1994) corrected for the failure of the BDS statistic in small samples and derived the simple non-parametric test (SNT hereafter). The SNT was devised to discriminate between linear and non-linear time series models. If non-linearities are ubiquitous, models such as ARCH, GARCH, switching Markov, bilinear time series and others are suggested to capture the non-linearities.[1] This paper uses the SNT test to investigate non-linearity in aggregate Canadian unemployment. If evidence of temporal non-linearities is found, then appropriate non-linear model specification(s) should be undertaken to investigate unemployment persistence.

In economics, Sayers (1986) and Brock and Sayers (1988) argued for the presence in non-linearity in most economic time series. Nott (1996) did not reject a non-linear Phillips curve in Canadian data. Armed with monthly observations, Murray et al. (1993) did not rejected linearity in Canadian unemployment based on the BDS test statistic. They concluded that their result is not an artifact of the aggregative nature of the unemployment series. However, it is acknowledged in their findings (1993, p. 342) that, the rejection of the i.i.d hypothesis could have resulted from the small sample bias of the test. One year after, Mizrach (1994) devised the SNT to correct for the small sample bias. Mizrach (1994, p. 384) reported evidence of non-linear temporal dependence in the French and Italian unemployment series. Here, we apply the SNT to test for non-linear temporal dependence in aggregate Canadian unemployment.

2 U-Statistics and BDS statistic

This section briefly introduces the U- and the BDS statistics.[2] U-statistics are generalizations of sample averages. The components of a U-statistic include a kernel, a symmetric measurable function $h : R^m \to R$, and a permutation operator, $\sum_{n,m}$ that sums over the $\binom{n}{m}$ distinct combinations of m-elements in a sample space of size n. Let $\{x_i\}$ be a strictly stationary stochastic process with a distribution function F, and let $\{X_1, ..., X_n\}$ be a sample of size n. Define the canonical mapping,

$$U_n = U(X_1, ..., X_n) \equiv \binom{n}{m}^{-1} \sum_{n,m} h(X_1, ..., X_n) \tag{1}$$

Two examples follow to show how this U statistic relates to the sample moments. If $m \equiv 1$, $h(x_i) \equiv x_i$, $\binom{n}{1}^{-1} = \frac{1}{n}$, then $U(X_1, ..., X_n) = \overline{X}$, i.e., the sample mean. If $m \equiv 2$, $h(x_i - x_j) \equiv \frac{(x_i - x_j)^2}{2}$, $\binom{n}{2}^{-1} = \frac{2}{n(n-1)}$, then $U(X_1, ..., X_n)$ equals the sample variance. Now, consider the vector valued version of (1). Let $x_t^m \in R^m$ be a random vector in R^m, and let $F(x_t^m)$ be its joint distribution. Define the kernel as, $h : R^m \times R^m \to R$,

$$h(x_t^m, x_s^m) = I[\| x_t^m - x_s^m \| < \varepsilon] \equiv I(x_t^m, x_s^m, \varepsilon) \tag{2}$$

where I is the indicator function, and $\| \,.\, \|$ denotes the max norm,

$$I(x_t^m, x_s^m, \varepsilon) = I[\max_i \prod_{i=0}^{m-1} |x_{t+j} - x_{s+j}| < \varepsilon] \tag{3}$$

The correlation integral is given by,

$$C(m, \varepsilon) = \int_X \int_X I(x_t^m, x_s^m, \varepsilon) dF(x_t^m) dF(x_s^m) \tag{4}$$

A consistent estimator of the correlation integral is given by,

$$C(m, N, \varepsilon) \equiv \frac{2}{N(N-1)} \sum_{t=1}^{N-1} \sum_{s=t+1}^{N} I(X_t^m, X_s^m, \varepsilon) \tag{5}$$

where $N = n - m + 1$. Note that $C(m, N, \varepsilon)$ is the expected number of m-vectors less than ε away from any given m-vector. In other words, $C(m, \varepsilon)$ measures the probability that any particular pair in the time series are 'ε-close'. The

BDS test statistic is computed as,

$$\sqrt{N}\frac{C(m,N,\varepsilon)-C(1,N,\varepsilon)^m}{\sqrt{var(C(m,N,\varepsilon)-C(1,N,\varepsilon)^m)}} \rightarrow^d N(0,1) \tag{6}$$

If the series is linear but exhibits autocorrelation, then the BDS will reject the null. Therefore in practice, the BDS is usually applied to the residual of a linear model.

3 The Simple Nonparametric Test (SNT)

The SNT has three advantages over the BDS. It involves simpler computation at the order of N rather than N^2. The variance of SNT is similar to that of a binomial random variable. And most importantly, the SNT is properly sized in small samples. At a sample size of 50 observations, the BDS rejects five times more frequently than it should under a conventional 5 percent test. This high rate of error of Type I, led Mizrach (1991) to propose the SNT test. By replacing the kernel in the BDS test statistic by $h : R \rightarrow R$, where $h(.)$ is defined as follows,

$$h(x_t) = I[x_t < \varepsilon] = \left\{ \begin{array}{l} 1, \text{ if } x_t < \varepsilon \\ \\ 0, \text{ otherwise} \end{array} \right\} \equiv I(x_t, \varepsilon) \tag{7}$$

Due to the choice of this kernel, the correlation integral (CI) at dimension m sums m-independent[3] events under the assumption of $i.i.d.$

$$\theta(m,\varepsilon) = \int_{x\in X} \prod_{i=1}^{m} I(x_{t+i},\varepsilon)dF(x_{t+i}) \tag{8}$$

A consistent estimator for the CI is,

$$\theta(m,N,\varepsilon) = \sum_{i=1}^{N} \prod_{i=0}^{m-1} I(x_{t+i},\varepsilon)/N \tag{9}$$

The expected number of m-chains with a value of 1 in a sample of size N is,

$$\mu = N\theta(m,\varepsilon) = \sum_{x=0}^{N} x \binom{N}{x} \theta(m,\varepsilon)^x (1-\theta(m,\varepsilon))^{N-x} \tag{10}$$

The variance is given by,

$$\sigma_{B_N}^2 = \mu_2^{'} - \mu^2 = \sum_{x=0}^{N} x^2 \binom{N}{x} \theta(m,\varepsilon)^x (1-\theta(m,\varepsilon))^{N-x} - N^2\theta(m,\varepsilon)^2 \tag{11}$$

$$= N\theta(m,\varepsilon)(1-\theta(m,\varepsilon)) \tag{12}$$

Using both moments, Mizrach (1991) constructed the following statistic and showed that it has an asymptotic normal distribution,

$$SNT \equiv \sqrt{N}\frac{\theta(m,N,\varepsilon) - \theta(m-1,N,\varepsilon)\theta(1,N,\varepsilon)}{\theta(m-1,N,\varepsilon)\theta(1,N,\varepsilon)(1-\theta(m-1,N,\varepsilon))(1-\theta(1,N,\varepsilon))} \to^d N(0,1) \tag{13}$$

Mizrach (1994, pp. 381-382) studied the small sample properties of the SNT statistic.

4 Data and Results

To compute the SNT test, we use the log of the quarterly Canadian unemployment level series. The series is from CANSIM and labeled D980712. It covers the period from 1976:1 to 1998:4. Any linear dependency in the data should be removed and given that the autoregressive component captures it, we fit ARIMA$(p, 1, 0)$ models, where the choice of p is carried to minimize the Akaike criterion. We experimented with different values for p and concluded that $p = 4$ provides the minimum value for the Akaike statistic. Then, we added two dummy variables to offset the effect of the 80s and 90s recessions in the data. Finally, we computed the residuals from the estimated linear ARIMA$(4, 1, 0)$ that includes the dummy variables.[4] The SNT test[5] is carried out on these residuals. Table 1 reports the results.

[Insert Table 1 here]

At the 95 percent confidence level, the SNT rejects the null of i.i.d. for the Canadian unemployment. We also investigated the robustness of these results with respect to the outliers in the series and reached a similar conclusion. This conclusion also holds under alternative values for the embedding dimension m up to and including $m = 5$.

5 Conclusion

In this paper, we focus on and examine the empirical evidence of non-linearity in aggregate Canadian unemployment. Contrary to the conclusion reached in Murray et al. (1993), and using a corrected for bias simple nonparametric test (SNT), we reject the null hypothesis of a linear structure for Canadian unemployment.

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Notes

[1]Note that rejecting the null of i.i.d. on the residuals of fitted linear time series models does not imply that the alternative is only a non-linear time series model. This point is similar in essence to that of Poirier (1997). The rejection of the null simply implies that the linear specification is not accurate; no hints are given regarding its alternative.

[2]For a detailed derivations, see Mizrach (1994) and Cromwell, Labys and Terraza (1994, pp. 32-36).

[3]To examine this 'spatial' correlation, the time series $x(t)$ must be embedded in m-space by constructing a vector. The choice of m for the dimensionality of the vectors is subjective. See Cromwell, Labys and Terraza (1994, p. 33) for details.

[4]For replication purposes, the data and the Fortran program are available at <http://www.bus.ucf.edu/omikhail>

[5]We acknowledge the support of Bruce Mizrach in making the FORTRAN code available. We modified the code to compute the SNT directly. The SNT test was compiled and run on a Linux (kernel 2.4.20.19-7) server.

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6 Table

Table 1: The SNT for Canadian Unemployment

	SNT
$m = 1$	13.53
$m = 2$	9.98
$m = 3$	2.01