Operating policies in a model with terminal scrapping

By
George C. Bitros[1]
Professor of Economics
And
Elias Flytzanis
Professor of Mathematics

Abstract

We draw on three strands of literature dealing with *utilization*, *maintenance*, and *scrapping* in order to analyze the properties of the respective policies and their interactions. We do so by focusing on the last period of the received multi-period service life model and extending it in three directions: first, by associating the physical deterioration of equipment to the intensity of its *utilization* and *maintenance*; second, by expanding on the range of explainable *operating policies* to allow for *idling*, *mothballing*, *capacity depleting*, *capacity preserving, full capacity, upgrading*, and *downgrading*; and, third, by linking the *operating policies* to the *capital policy* of *scrapping*. Owing to these enhancements, the analysis leads to several important findings. One among them is that *optimal operating policies* behave usually in opposite directions, proceeding in time from *harder* to *softer* or vice versa, depending on the net revenue earning capability of the equipment under consideration. Another is that profit (loss) making equipment is *scrappable* iff on the average the operating capital deteriorates faster (slower), or equivalently improves slower (faster), than the scrapping capital. And still another result is that *operating policies* are determined jointly with scrapping policy *capital policies*, thus suggesting that empirical investigations of their determinants should allow for this simultaneity.

JEL Classification: E22, E32, O16
Keywords: Utilization, maintenance, idling, mothballing, capacity depleting, capacity preserving, upgrading, downgrading, scrapping.

Correspondence:
Professor George C. Bitros
Athens University of Economics and Business
76 Patission Street, Athens 104 34, Greece
Tel: (01) 8223545 Fax: (01) 8203301,
E-mail: bitros@aueb.gr

1. Introduction

Owners' decisions with respect to their durables may be classified into two categories.[2] The first concerns the decisions that are primarily directed at changing the condition of durables themselves and includes *replacement, scrapping, expansionary investment, upgrading, downgrading, overhauling* and *stripping.* Below we shall refer to them as *capital policies*. The second category comprises the decisions that are associated with *utilization* and *maintenance* and we shall refer to them as *operating policies*.

In Bitros and Flytzanis (2002a) we extended the multi-period service life model and laid down the groundwork to derive all these policies from a unified analytical framework based on rational economic behavior. But partly because of the overwhelming attention they have received in the literature and partly because the presentation had to be kept within reasonable limits, in that paper we placed the emphasis on *replacement* and *scrapping* and kept all other policies in the background. As a result this left for us the tasks, on the one hand, to expand the model so as to incorporate the remaining *real capital policies,* and on the other, to investigate the properties of *operating policies* and their interactions with *capital policies.* Thus, having accomplished recently in Bitros and Flytzanis (2002b) the former of these two tasks, our goal in this paper is to pursue the latter.

The long and arduous endeavor to integrate operating with capital policies has evolved along three paths. Following the trail of thoughts by Keynes (1935), the objective in the first path was to allow for the depreciation of durables due to the intensity of their utilization. This started with the contribution by Taubman and Wilkinson (1970); Nadiri and Rosen (1974), Shapiro (1986), Bischoff and Kokkelenberg (1987), and Johnson (1994) developed it further; and progress peaked with the papers by Prucha and Nadiri (1996) and Jin and Kite-Powell (1999).[3] In the second path the goal was to introduce maintenance. This began with Masse (1962); it continued with Naslund (1966), Jorgenson, McCall and Radner (1967), and Thompson (1968); and while it culminated with Kamien and Schwartz (1971), the interest in it has not subsided because of the wide implications and significant relative size of maintenance expenditures.[4] Finally, working in the third path, Bitros (1972, 1976a, 1976b) and Parks (1977,1979) in the 1970's, Epstein and Denny (1980), Everson (1982) and Kim (1988) in the 1980's, and Licandro and Puch (2000), Collard and Kollintzas (2000), and

Boucekkine and Tamarit (2003), more recently, have pushed for a model of capital services with endogenous utilization, maintenance and gross investment.

From the proceeding it follows that the present paper falls in the last group of studies. But it differs from them in that they fail to characterize the properties of *operating policies* and their interactions with *capital policies*. To substantiate this claim, suppose that we would like to obtain advice on the following questions. When should the representative firm stop operations and proceed to *idle, mothball* or even *scrap* its equipment? Under what conditions is it profitable to *upgrade* or *downgrade* the equipment? Do the analytic forms of the functions relating *utilization* and *maintenance* to cash flow and equipment deterioration matter, and if so, in what way? If one searched for enlightenment in the literature cited above, one would not find much. And the same is true with the literature from such fields as *operations research* and *operations management*. To the best of our knowledge then, this constitutes the first attempt to shed light on these questions.

Owing to the new setting, the results that emerge are quite illuminating. Unlike previous studies that led to indeterminate *utilization*, *maintenance* and *service life* policies, the ones obtained here are determinate and computable to any desired approximation. At his own discretion the owner may run down his equipment through more intensive *utilization* and *downgrading.* Technological improvements permitted under the original design of equipment may be incorporated gradually through *upgrading*. Technological breakthroughs generate uncertainty, which raises the effective rate of discount. If either of the two flow functions relating *utilization* and *maintenance* to cash flow and equipment wear is strictly concave, the optimal path of operating policies is in fact unique and continuous. Otherwise there may be jumps to *operating policies* of lower intensity, i.e. both lower *utilization* and *maintenance*, and vice versa. Last but not least, the owner may stop using his equipment and decide to: a) *scrap* it, b) *idle* it temporarily in order to weather unfavorable market conditions or even *mothball* it for use much later.

Section 2 describes the model, the optimality conditions, and the policies. Since the building blocks of the model have been elaborated extensively in Bitros and Flytzanis (2002a, 2002b), the presentation here is meant to serve only as a vehicle to introduce certain clarifications and to identify the totality of operating and capital policies. In Section 3 we obtain the general solution of the model and analyze the dependence of optimal operating and capital policies on the parameters. In Section 4 we construct an example by adopting separable specifications for the flow rate functions r and w. In Section 5 we highlight the implications of our results for economic theory and policy. In Section 6 we summarize our findings and conclusions, and, finally, in the Appendix we supply some technical material, which supplements the presentation significantly.

2. The model

2.1 Model specification

In Bitros and Flytzanis (2002a), we examined the problem of optimal service life of equipment in the framework of the multi-period replacement model, allowing for any number of consecutive replacements to be followed by terminal scrapping. In particular, we examined the relation between the time durations of the consecutive replacement periods and the terminal scrapping period. Furthermore, we related the above to the case of steady state replacements at equal time intervals. Here we concentrate only in one period of operations, which leads to scrapping. In fact in our previous work we showed that very often the optimal policy is that of scrapping without replacement, and further that even when it is optimal to replace, the last scrapping period is where most of the profit is made.[5] In this scrapping period the objective for the owner of the equipment may be stated as follows:

$$\begin{aligned}&\text{Choose } [T,u(t),m(t)] \text{ so as to maximize:}\\ &A=\tilde{Q}+\tilde{S}=\int_0^T q(u,m,K)\varphi(t)dt+\varphi(T)S(K_T,T)\\ &\text{s.t.}\quad \dot{K}=-s(u,m,K),\ \text{with } K(t_0)=K_0,\ \text{and}\\ &\qquad 0\le u\le 1,\ 0\le m\le 1,\end{aligned} \quad \textbf{(1)}$$

where the various symbols are defined as follows:

$\tilde{Q}=\int_0^T q(u,m,K)\varphi(t)dt$: *Expected net operating revenue for operating horizon T*.

$K=K(t)$: *Used equipment* measured in efficiency units, reflecting its size and age since first put in operation. New or unused equipment will be denoted by $K_0=K(0)$.

$u=u(t)$: *Utilization intensity* relative to some extremal values, with $0\le u\le 1$.

$m=m(t)$: *Maintenance intensity* expressed as expense relative to some extremal values, with $0\le m\le 1$.

(u,m): *Operating policy factors*.

$q(u,m,K)$: *Flow of net operating revenue*.

$s(u,m,K)$: *Flow of net capital wear.* It expresses the effects on equipment of maintenance and usage, including aging.

(q,s): *Operating policy flows.*

$S = S(K_T, T)$: *Scrap value* of used equipment at T. For the scrap value of unused equipment we set $S_0 = S(K_0, 0)$.[6]

$\varphi(t) = e^{-\sigma t}$: *Effective discount factor.* Let $F(t)$ denote the probability of a *technological breakthrough* by time *t,* with $F(0) = 0$ and $F(t) < 1$ for all t. Assuming a constant discount rate ρ, the discount factor would be $e^{-\rho t}$. To account for technological uncertainty this is multiplied by $[1 - F(t)]$. In keeping with the specification of time invariance, we consider only the usual exponential case: $F(t) = 1 - e^{-\theta t}$. Then, since $\varphi(t) = e^{-(\theta+\rho)t}$, the effect of uncertainty is equivalent to introducing a revised *effective discount rate*, expressed by $\sigma = \theta + \rho$.

Expression **(1)** describes the general setting of an optimal control problem. We will proceed with a more specific model by assuming q and s of the following type:

$q = rK^{\varepsilon}$: Where $r = r(u,m)$ is the *operating net revenue rate*. Usually positive, but it can also be negative. Increasing in *u,* decreasing in *m,* concave in *(u, m)*.

$s = wK$: Where $w = w(u,m)$ is the *capital stock wear rate*. Increasing in *u,* decreasing in *m,* convex in (u,m). It expresses the effect on equipment of maintenance and usage, including aging. Usually positive but it can also be negative, if aging causes upgrading or if investment type of maintenance overbalances the wear of equipment, allowing K to even rise above the original K_0.

(w,r): *Operating policy rates*

These rate functions characterize the operating features of the equipment. They have been taken to be time invariant. However, we will allow time variations for the prices, of the constant percentage type, by setting:

$S = pe^{\eta T}K$: *Scrap value* of equipment at time T, where:

η: *Relative rate of price change*. It is the difference between equipment price change and operating revenue price change, because any common part can be subtracted from the discount rate σ. It can have either sign, or be zero.

With the help of these specifications, we will investigate the dependence on the parameters $\{\varepsilon,\sigma,\eta,p,K_0\}$, of: a) the *operating policies* defined by the optimal rates of utilization and maintenance as functions of time: $\{u=u(t),m=m(t)\}$, and b) the *scrapping policy* defined by the optimal duration T^*.

2.2 Policy types

Concerning *scrapping policy*, we will say that the equipment is *nonprofitable,* if $T^*=0$, *scrappable,* if $0<T^*<\infty$, and *durable,* if $T^*=\infty$.

As for the operating policies we refer to Figure 1(a) below. We will say that a policy pair $v:(u,m)$ is of:

Higher intensity, if both utilization and maintenance are higher,
Lower intensity, if both utilization and maintenance are lower.

In this ordering, we distinguish the two extremal policies, of lowest and highest intensity:

$$v_0:(u=0,\ m=0)\ \ \&\ v_1:(u=1,\ m=1).$$

More important is their ordering according to the resultant wear-revenue rates: (w,r). We will say that a policy pair is:

Harder, if it gives higher rates both for wear and revenue,
Softer, if it gives lower rates both for wear and revenue.

In this ordering, we distinguish the two extremal policies: [7]

$\underline{v}:(u=0,m=1)$*: Softest,* with the lowest rates: $(\underline{w},\underline{r})$
$\overline{v}:(u=1,m=0)$*: Hardest* with the highest rates: $(\overline{w},\overline{r})$

Moreover, we will say that a policy pair (w,r) is:

Profit making, if $r>0$, *loss making,* if $r<0$,
Downgrading, if $w>0$, *upgrading,* if $w<0$
Break even of zero revenue, if $r=0$,
Capacity preserving of zero wear, if $w=0$.

For particular equipment any of the above policy types may or may not be available. Classifying equipment according to the totality of the available policies, we say that it is:

Profit making, if $r \geq 0$, *loss making* if $r \leq 0$,
Revenue-flexible, if both profit making and loss making policies are available,
Downgrading, if $w \geq 0$, *upgrading,* if $w \leq 0$,
Wear- flexible, if both upgrading and downgrading policies are available.

Finally, we will say that the equipment is:

Special, if it has policies that are both profit making and upgrading at the same time,
Common, if it is not special.

We will find that the equipment behaves differently depending mainly on its revenue type.

Remark 1
Referring to various policy types, in practice we often use the following terminology:
1. Among the minimal utilization policies: $u = 0$, we distinguish the following:
   (i) *Closedown,* with no maintenance. It is the policy of lowest intensity:
   $$v_0 : (u = 0,\ m = 0)$$
   (ii) *Idling,* with some maintenance $(u = 0,\ m > 0)$.[8]
   (iii) *Mothballing,* with full maintenance. It is the *softest policy*:
   $$\underline{v} : (u = 0,\ m = 1),$$
   with the lowest rates: $(\underline{w}, \underline{r})$.[9]
2. Among the maximal utilization policies: $u = 1$, we distinguish the following:
   (i) *Capacity depleting,* with no maintenance. It is the *hardest policy*:
   $$\overline{v} : (u = 1,\ m = 0),$$
   with the highest rates: $(\overline{w}, \overline{r})$.[10]
   (ii) *Full capacity,* with maximal maintenance. It is the policy of highest intensity
   $$v_1 : (u = 1,\ m = 1).$$[11]

2.3 Two revenue based measures of capital

We examine first some preliminary notions that will help us interpret the results. We start by distinguishing the two sources of revenue, the operating revenue and the scrap revenue. The capacity of capital to produce these two revenues is affected by operations and also by time discounting. But their effects are exercised in different ways, as follows:

Remark 2
1. Concerning the effect of operations on the two revenues, we have:
   $-\dot{q} / q = \varepsilon w$: *Deterioration rate of operating revenue*, of either sign.
   $-\dot{S} / S = w - \eta$: *Deterioration rate of scrapping revenue*, of either sign.
   We note that if $\varepsilon > 1$, then operations affect the services more than the equipment, after we account for price changes due to η. The opposite is the case if $\varepsilon < 1$.
2. Concerning the effect of time discounting, we note that for the same operating policies, K units of capital at time T are equivalent presently to:
   $rK_{oc}^{\varepsilon} = rK^{\varepsilon} e^{-\sigma T} \Rightarrow K_{oc} = Ke^{-\sigma T / \varepsilon}$ capital units for operating revenue.

$pK_{sc} = pKe^{\eta T} e^{-\sigma T} \Rightarrow K_{sc} = Ke^{-(\sigma-\eta)T}$ capital units for scrapping revenue.

Thus, we have two discounting rates for future capital:

σ / ε: *Discounting rate for operating revenue*, positive

$\sigma - \eta$: *Discounting rate for scrapping revenue*, of either sign.

We note that if $\varepsilon > 1$, then future capital is more heavily discounted for its scrap value than for its services, after accounting for the price changes due to η. The opposite is the case if $\varepsilon < 1$.

3. We can summarize these differences by considering two measures of capital:

   K : *Scrapping capital,* determining the scrap revenue.

   K^{ε} : *Operating capital,* determining the operating revenue,

   where as noted above ε is the *deterioration (improvement) coefficient* for the services rendered by the equipment relative to the downgrading (upgrading) of the equipment itself.

4. At the beginning of the operating period the unit prices of the two capital measures are defined respectively by:

   $\lambda_0 = pK_0 / K_0 = p$: *Owner's unit logistic value for new scrapping capital.*

   $\mu_0 = pK_0 / K_0^{\varepsilon} = pK_0^{1-\varepsilon}$: *Owner's unit logistic value for new operating capital.*

   If K_0 is fixed, i.e. if it is not a parameter, then we can choose capital units and also adjust r, so that the two initial values are equal: $K_0 = 1 \Rightarrow \lambda_0 = \mu_0 = p$

The main results so far can be summarized as follows:

Remark 3

1. Scrapping policy is determined mainly by the deterioration rates: $\{\varepsilon w, w - \eta\}$
2. Operating policies are determined mainly by the discount rates: $\{\sigma / \varepsilon, \sigma - \eta\}$
3. In all cases the policies depend on whether the price p is "low" or "high".

.

2.4 Optimality conditions

Examining the problem in the setting of optimal control theory, we consider the *total profit flow* given by the current value *Hamiltonian:*

$$H = [q(u,m)K^{\varepsilon} - \lambda s(u,m)K] = [q(u,m) - \lambda K^{1-\varepsilon} s(u,m)]K^{\varepsilon},$$

with co-state variable

$$\lambda = \lambda(t): \textit{Owner's unit logistic value for scrapping capital.}$$

In place of $\{H, \lambda\}$, we have also the pair $\{h, \mu\}$, where

$h = H / K^{\varepsilon} = r(u,m) - \mu w(u,m)$: *Total profit flow rate per unit of operating capital*

$\mu = \lambda K / K^{\varepsilon}$: *Owner's unit logistic value for operating capital*

From Leonard & Van Long (1995) or Seierstadt & Sydsaeter (1986), we obtain the following necessary conditions for optimality:[12]

(i). For the operating policies (u,m) the maximality principle:

$$\max_{u,m} H \Rightarrow \max_{u,m}\{h=[q(u,m)-\mu s(u,m)]\| \quad 0\le u\le 1, \quad 0\le m\le 1\}$$

(ii). For the capital stock:

$$\dot{K}=s(u,m)K\,, \ \ with\ K-initial\ condition\ K(0)=K_0 \tag{2}$$

(iii). For the logistic value:

$$\dot{\lambda}=-H_K'+\sigma\lambda \Rightarrow \dot{\mu}=-\varepsilon h+\sigma\mu=\varepsilon\mu(\sigma/\varepsilon-h/\mu)$$
$$\text{with } T-\text{final condition: } \lambda_T=S_K'(K_T,T) \ \ \Rightarrow \mu_T=pe^{\eta T}K_T^{1-\varepsilon}$$

(iv). For the duration, the scrapping $H-$terminal condition:

$$H=\sigma S-S_T' \ \Rightarrow h/\mu=\sigma-\eta$$

The solution will be obtained by the following procedure. First we solve the maximality principle 3(i), to express $\{u,m\}$ as functions of μ. Then for given duration T we solve the autonomous dynamical equation 3(iii) for μ. This gives the optimal solution for given $T: 0<T<\infty$. For $T\to 0$ we find $h_0=r_0-\mu_0 w_0$, and we consider the initial condition:

$$H(0)>\sigma S(K_0,0)-S_T'(K_0,0)\Rightarrow h_0/\mu_0>(\sigma-\eta): Profitability\ condition \tag{3}$$

If it is not satisfied then the optimal duration is zero: $T^*=0$, and the equipment is *non-profitable*.[13] If it is satisfied then it is profitable, and we consider two possibilities. If the terminal condition 1(iv) does not have solution, then the optimal duration is un-bounded: $T^*=\infty$, and the equipment is *durable*. If it has solution, then the equipment is *scrappable,*[14] and we take the first such solution as the scrapping duration $T^*=T_s$.[15] In this case we examine also the operating policies.

As can be seen from the conditions, pivotal role is played by the quantity:

$$i=H/\lambda K=h/\mu : Total\ profit\ index \tag{4}$$

It expresses the *total profit flow per unit logistic value of capital,* expressed either in terms of the operating capital or in terms of the scrapping capital.[16]

3. Equipment characteristics

3.1 Optimal path

The maximality principle 3(i) determines for given μ the optimal $(u,m)-$policies. By convex programming and by the monotonicity properties of the functions involved, the totality of available optimal policies can be obtained also as solutions of either of the following constrained optimization problems, where the Lagrange multiplier of the first problem coincides with μ:

$$\begin{aligned} &\textbf{(i)}\ \max_{u,m}\{r(u,m) \| w(u,m)=w,\ 0\le u\le 1,\ 0\le m\le 1\}, \text{for any } w \\ &\textbf{(ii)}\ \min_{u,m}\{w(u,m) \| r(u,m)=r,\ 0\le u\le 1,\ 0\le m\le 1\},\ \text{for any } r \end{aligned} \qquad \textbf{(5)}$$

They can be characterized as follows:

Remark 4

1. Among the policies that give the same rate of capital wear, optimal are those that maximize the rate of operating revenue, and
2. Among the policies that give the same rate of operating revenue, optimal are those that minimize the rate of capital wear.

Actually the above constrained optimization problems determine pairs of optimal rates: (w,r). As indicated in Figure 1a, for each such pair the contact points of their isorate curves in the (u,m) plane give the corresponding policies, or else they are boundary. These points form a path in the (u,m) plane, which we will call *optimal path.* In general, each contact consists of a single point and then the optimal path is uniquely determined and continuous. In special cases, it may be only upper-semicontinuous, with portions where the policies are not uniquely determined, in the sense that they give the same (w,r) values. In practice, these appear as discontinuity jumps to policies of higher or lower intensity, like part $\widehat{AB}$ in Figure 1a. As μ increases, the optimal path moves from