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Debt-deflation: concepts and a stylised model

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Abstract

This paper proposes a model of how agents adjust their asset holdings in response to losses in general equilibrium. By emphasising the relation between deflation and financial distress, we capture some original features of the early debt-deflation literature, such as distress selling, instability, and endogenous monetary contraction.
The agents affected by a shock sell off assets to prevent their debt from crowding out consumption. But their distress-selling causes a decline in equilibrium prices, and the resulting losses elicit reactions by all agents. This activates several channels of debt-deflation. Yet we show that this process remains stable, even in the presence of large shocks, high indebtedness, and wide-spread default. What keeps the asset market stable is the presence of agents without prior debt or losses, who borrow to exploit the expected asset price recovery. By contrast, debt-deflation becomes unstable when agents try to contain their indebtedness, or when a credit crunch interferes with the accommodation necessary for stability.

JEL Classification: E31, E51, G33, G21, G18.

Keywords: Debt-Deflation, Leverage, Refinancing, Losses, Financial Distress, Distress Selling, Asset Prices, Credit, Inside Money.

Research and Policy Analysis, Monetary and Economic Department. Email: Goetz.von.Peter@bis.org. The views expressed in this paper are my own and do not necessarily reflect those of the BIS.
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Introduction

The world-wide decline of inflation over the last decades has renewed concerns about the threat of deflation, a decline in the price level. But why should we fear deflation? A historically informed answer will certainly include financial distress. DeLong reminds us that during the Great Depression "almost every analyst placed general deflation, and the bankruptcies it caused, at or near the heart of the worst macroeconomic disaster the world had ever seen" (DeLong 1999 p. 231).

In this paper we explore the concept of *debt-deflation*. We propose a stylised model to illustrate its key features, including unexpected losses, distress selling, and distributional effects. These features reflect the central place that financial distress occupies in traditional accounts of deflation. The term debt-deflation was coined by Irving Fisher (1933), and refers to the way debt and deflation destabilise each other. The issue of stability arises because the relation runs both ways: deflation causes financial distress, and financial distress in turn exacerbates deflation. The former was known for centuries, but the latter was, in our view, a key insight of the debt-deflation literature. This 'feedback' from financial distress to deflation can occur through several channels:

- Fisher (1933) argued that borrowers attempting to reduce their burden of debt ('indebtedness') engage in distress selling to raise money for repaying debt. But repayment in aggregate causes a contraction in the money supply and price level deflation.
- Minsky (1982) elaborated the concept to incorporate the asset market. He recognised that distress selling reduces asset prices, causing losses to agents with maturing debts. This reinforces distress selling and reduces consumption and investment spending, which deepens deflation.
- Bernanke (1983) observed that debt-deflation involves wide-spread bankruptcy, impairing the process of credit intermediation. The resulting credit contraction depresses aggregate demand.

Note that these channels involve features that are quite uncommon in today's mainstream macroeconomics: among them are losses and distress selling, the idea that debt and deflation destabilise each other, and the notion that the quantity of money endogenously contracts through the repayment of debt. Note also that some standard methods, including the representative agent and log-linearisation, are not well-suited for exploring this territory. This may explain the shortage of formal work on debt-deflation.

One route forward is to augment a monetary dynamic stochastic general equilibrium model with heterogeneity, assets and debt, distress selling, banking and default. Christiano et al (2004) take a step in this direction.[1] A less ambitious route, pursued here, is to focus on a specific effect of importance for debt-deflation. Existing models focus largely on output effects.[2] The earlier literature

[1] Christiano et al present a quantitative account of the Great Depression. Because their model consists of a stable, linearised propagation mechanism, several large shocks are necessary to replicate the behaviour of key variables. The model yields important insights, but it does not highlight the channels and stability considerations associated with debt-deflation.

[2] Several papers examine why a redistribution of wealth from borrowers to lenders is not neutral in terms of output. In Tobin (1980) and King (1994), aggregate demand falls because lenders have a lower propensity to spend than do borrowers. In Bernanke and Gertler (1989) output falls because borrowers' reduced net worth aggravates the agency problem that constrains investment. (The concept of 'financial accelerator' was further developed by Kiyotaki

suggests, however, that output effects occur rather as a consequence of debt-deflation which, at its core, features the link between deflation and financial distress. These models are neither monetary, nor explicitly about financial distress, leaving this link unaddressed.

We propose a model of how agents adjust their asset holdings in response to losses in general equilibrium. Distributional effects are emphasised, and output effects are put aside, by adopting a frictionless, flexible-price specification in which markets clear and output remains at capacity. In spite of its stylised nature, the model contains assets, debt, default, and banking with inside money, in dynamic general equilibrium. The overlapping generations structure makes it easy to deal with these features, especially with the consequences of default.

The mechanism works as follows. Firms are active for three periods; they purchase assets when young, and sell output (and resell assets) when old. Since their sales revenue arrives in their final period, they must borrow to purchase assets.[3] In a perfect foresight equilibrium, firms simply hold a constant quantity of assets across periods. We then let a redistributive shock set off the debt-deflation process. Having expected more favorable conditions, the shock prompts firms to readjust their assets and debt. Conceptually, three policies can be distinguished,

- *Refinancing*: hold fixed the quantity of assets (and borrow the necessary amount).
- *User cost spending*: hold fixed the fraction of income spent on user cost.[4]
- *Containing indebtedness*: hold fixed debt over net worth.

User cost spending is the optimal policy. Following losses, firms indeed engage in distress selling: they sell off assets they had planned to hold, to prevent that final period consumption gets crowded out by the burden of debt. Distress-selling causes second-round effects, as the changing equilibrium prices cause losses and elicit reactions by all agents. This activates several channels of debt-deflation, but we show that debt-deflation remains stable even in the presence of large shocks, high indebtedness, and wide-spread default.[5] The main reason for stability is that those firms without any prior debt or losses are willing to increase their borrowing to exploit the expected asset price recovery.

This outcome lies half-way between the policy of refinancing, which produces no debt-deflation, and the policy of containing indebtedness, which produces *unstable* debt-deflation as Fisher and Minsky predicted. The notion of unstable debt-deflation ultimately relies on reasons why agents try to

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and Moore (1997), and Bernanke et al (1999), to include a role for asset prices in explaining output effects.) A related question is whether increased price flexibility helps stabilise output, as assumed in much of economics. DeLong and Summers (1986) argue, using a Taylor price-staggering model, that this is typically not the case.

[3] We introduce debt and banking with the following device: all assets are initially owned by households, who only want to consume goods; and all goods are produced by firms, who want to use assets. To achieve the desired pattern of exchange, the banking system intermediates payments and credit, whereby firms become borrowers and households become depositors.

[4] User cost is the effective cost of holding an asset. It is today's asset price minus next period's resale price discounted.

[5] The result that default is not destabilising is due, in part, to the absence of side-effects that might arise if frictions and uncertainty were added. It no longer holds when default affects credit intermediation, as shown in section 5.

contain or reduce their indebtedness. Margin requirements can be one such reason. A credit crunch, which interferes with the accommodation necessary for stability, can be another.

Although our model of debt-deflation derives from writings on the Great Depression, we believe it is of broader interest. First, the logic of the model applies as much to unexpected *disinflation* as to deflation. Second, falling *asset* prices are shown to be as harmful as deflation, and the asset market may well be more liable to instability. Another possibility is risk aversion and precautionary behaviour, but including uncertainty is left for future research.

The paper is organised as follows. Section 1 presents a structured view of the debt-deflation literature to identify the main channels. Section 2 develops the perfect foresight benchmark. Section 3 introduces an exogenous shock and studies agents' reactions to losses. Section 4 examines the channels of debt-deflation, their stability, and the role of distress selling. Section 5 considers two instances of unstable debt-deflation. The final section concludes with a discussion of limitations and relevance.

1 Debt-deflation - selected literature

Deflation had been a regular phenomenon under various specie standards. The consequences of deflation were often benign, but sometimes not (Bordo and Filardo 2004). Thornton (1802) already emphasised that deflation in the presence of rigid wages would produce financial distress.[6] Christiernin (1761) and Attwood (1817) further understood that falling prices reduce the ability to repay nominally fixed debt (see Humphrey 2003).[7] This was well before the link between deflation and financial distress was made abundantly clear by depressions before and during the 1930s.[8]

Thus it has been known for a long time that deflation may produce financial distress. It is our view that the main achievement of the debt-deflation literature, initiated by Fisher (1933), was to recognise that *financial distress in turn worsens deflation*. This feedback reveals a distinctly macroeconomic, general equilibrium understanding of deflation.

1.1 Fisher: price level

Fisher's (1933) article "The Debt-Deflation Theory of Great Depressions" sets out a monetary theory of how financial distress exacerbates deflation. Fisher's argument starts with a state of 'over-

[6] Thornton (1802) cautions that "the bank [of England], in the attempt to produce this very low price [level to encourage exports] may [..] so exceedingly distress trade and discourage manufactures as to impair [...] the restoration of our balance of trade." (p. 152).

[7] The role of financial distress also appeared in the literature on the trade cycle, reaching back to Adam Smith ('overtrading, revulsion and discredit'). See also Kindleberger (1996), chapter 2.

[8] Over time this link became obvious to economic commentators and attracted little economic analysis. I thank Robert Mundell for pointing this out. For instance, Veblen (1904) states "A decline of prices which widely touches business interests brings depression" (p. 234-5), but then, "Secondary effects, such as perturbations of the rate of interest, insolvency, forced sales, and the like, need scarcely be taken up here, although it may be well to keep in mind that these secondary effects are commonly very considerable and far-reaching [...]" (p. 111).

indebtedness'. Agents seek to reduce indebtedness by 'liquidating' debt. The first and most important steps in Fisher's 'chain of consequences' are,

> "(1) *Debt liquidation* leads to *distress selling* and to (2) *Contraction of deposit currency*, as bank loans are paid off, and to a slowing down of velocity of circulation. This contraction of deposits and of their velocity, precipitated by distress selling, causes (3) *A fall in the level of prices*, in other words, a swelling of the dollar."
>
> Irving Fisher (1933) p. 342, Fisher's emphasis.

Fisher views price level deflation as "the root of almost all the evils" that he elaborates in six further steps (1932 p. 39). Note that, rather than taking deflation as given, he explains it as the *consequence* of agents' attempt to reduce their indebtedness.[9] They do so by distress selling, to raise the money for repaying bank loans. Repayment in aggregate reduces the quantity of money, or 'deposit currency', which causes deflation. (This last step presumably relies on his quantity equation.) Since deflation is known to increase indebtedness, Fisher's channel closes the loop of debt-deflation,

> "and if the over-indebtedness with which we started was great enough, the liquidation of debts cannot keep up with the fall of prices which it causes. In that case, liquidation defeats itself. While it diminishes the number of dollars owed, it may not do so as fast as it increases the value of each dollar owed. Then, *the very effort of individuals to lessen their burden of debts increases it, because of the mass effect of the stampede to liquidate in swelling each dollar owed*. Then we have the great paradox which, I submit, is the chief secret of most, if not all, great depressions: *The more the debtors pay, the more they owe.* The more the economic boat tips, the more it tends to tip. It is not tending to right itself, but is capsizing. But if the over-indebtedness is not sufficiently great to make liquidation thus defeat itself, the situation is different and simpler. It is then more analogous to a stable equilibrium; the more the boat rocks the more it will tend to right itself."
>
> Irving Fisher (1933) p. 344-45, Fisher's emphasis.

Fisher's theory was largely ignored by contemporaries, until it was revisited by Tobin, Minsky, and Kindleberger.[10] Yet, Fisher's theory was arguably novel in several respects. First, of course, the theory introduced the idea that financial distress feeds back on deflation. It is based on individually optimal behaviour in the form of distress selling.[11] But in contrast to the "liquidationists", Fisher viewed distress selling in aggregate as *harmful*.[12] Second, in contrast to the price-specie flow mechanism, which emphasised deflation in the context of selling goods to stem external drain, Fisher emphasised

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[9] Minsky puts deflation even further down the causal chain, see Minsky (1982) p. 388.

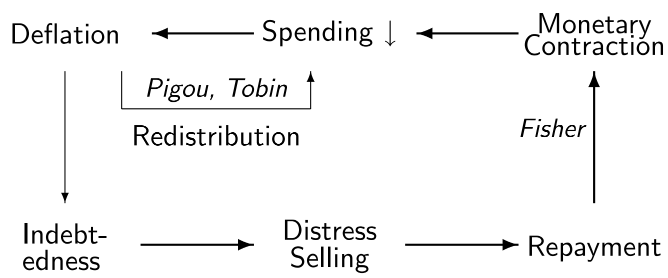
[10] One book review of 1933 argued the theory was not novel (King 1994). This may well be because the effect of deflation on the burden of debt was well understood at the time, while the reverse was not recognised as important.

[11] If Fisher's description of distress selling appears micro-founded in a rather modern way, it may have been informed by his personal misfortune. He had bought stock on margin, and the market crash of 1929 left him bankrupt (Schwartz 1997 p. 104).

[12] Andrew Mellon, Secretary of the Treasury during the Great Depression, was associated with the doctrine "Liquidate labor, liquidate stocks, liquidate the farmers, liquidate real estate.[...] It will purge the rottenness out of the system", quoted in Kindleberger (1996) p. 127.

the selling of *assets*.[13] Finally, in contrast to much of the subsequent literature, Fisher viewed deflation as *destabilizing*.

Fisher's Channel



This last point becomes clear when recasting the problem in terms of aggregate demand. 'Classicals' such as Pigou viewed deflation as stabilizing, as the greater real value of money associated with a lower price level would stimulate aggregate demand.[14] Kalecki (1944) noted that the Pigou effect would in practice be confined to a small base, namely 'outside' money, as every other financial asset is a liability to some private debtor, who is correspondingly hurt by deflation. While Pigou emphasised gains to creditors, Fisher emphasised losses to debtors. Should the opposite forces not simply cancel?[15] Tobin (1975, 1980) argued that Fisher's channel dominates, because debtors generally have a higher propensity to spend than creditors. Debtors borrow *because* they have a higher propensity to spend,

> "Among the stocks fixed in the short run are private debts in the unit of account. These are a heavier burden to debtors the lower the price level, and there are good reasons why transfer of real income and wealth to creditors spells a net deficit of aggregate demand."
> James Tobin (1975) p. 197
> "Aggregation would not matter if we could be sure that the marginal propensities to spend from wealth were the same for creditors and debtors. But if the spending propensity were systematically greater for debtors, even by a small amount, the Pigou effect would be swamped by this Fisher effect.
>
> James Tobin (1980) p. 10

Tobin's variation over Fisher's theme connects spending back to deflation, as shown in the figure. Tobin (1975) also suggested that deflation lowers the value of the capital stock and equity, making both consumption and investment less attractive. The role of asset prices was further explored by Minsky.

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[13] Liquidation involves the selling of the borrower's possessions, "his stocks, his bonds, his farmland, or whatever his available assets may be" (Fisher 1932 p. 13).

[14] Although Pigou (1943) merely considered steady states, it was assumed that deflation would play a stabilizing role in dynamic adjustment, see Pigou (1947).

[15] In the classical model, the Pigou effect on outside money, relabelled 'real balance effect' by Patinkin, has a zero net effect when all money is 'inside' and banks are perfectly competitive (Patinkin 1948, 1971).

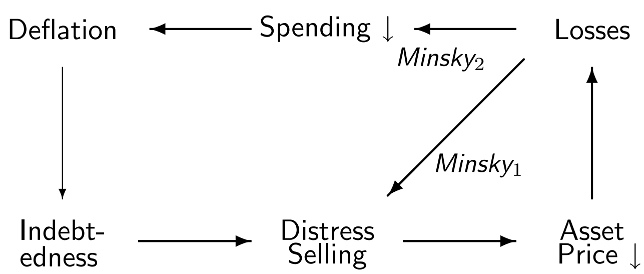
1.2 Minsky: asset prices

Minsky's (1982) elaboration of debt-deflation incorporates the asset market. He recognised that distress selling reduces asset prices, which (1) reinforces distress selling, and (2) worsens deflation. Regarding the first channel, Minsky (1982) wrote,

> "Fisher does not identify the ways a unit can get cash to repay loans that fall due. [...] Once a situation exists where debt payments cannot be made either by cash from operations or refinancing, so that assets have to be sold, then the requirements imposed by the debt structure can lead to a fall in the prices of assets. In a free market, the fall in asset prices can be so large that the sale of assets cannot realize the funds needed to fulfill commitments."
>
> Hyman Minsky (1982) p. 383-84

In other words, when distress selling reduces asset prices, the resulting losses exacerbate indebtedness, and may lead to further distress selling. As in Fisher, distress selling can be self-defeating. The asset market and distress selling feed back on each other (Minsky_1 in the figure). Kindleberger (1996), in his famous "Manias, Panics, and Crashes", takes Minsky's approach as the organising principle for casting the history of financial crises in terms of cycles of speculation on borrowed funds.

Minsky's Channels



Regarding the second channel, Minsky argues that the fall in asset prices reinforces deflation,

> "If payment commitments cannot be met from the normal sources, then a unit is forced either to borrow or to sell assets. Both borrowing on unfavorable terms and the forced sale of assets usually result in a capital loss for the affected unit. However, for any unit, capital losses and gains are not symmetrical: there is a ceiling to the capital losses a unit can take and still fulfill its commitments. Any loss beyond this limit is passed on to its creditors by way of default or refinancing of the contracts. Such induced capital losses result in a further contraction of consumption and investment beyond that due to the initiating decline in income. This can result in a recursive debt-deflation process."
>
> Hyman Minsky (1963) p. 6-7

In other words, losses from the decline of asset values reduce aggregate spending through a wealth effect (labelled Minsky_2).

1.3 Bernanke: credit

Both Fisher and Minsky emphasised the consequence of financial distress for macroeconomic variables: aggregate spending, the price level, and asset prices. Another channel of feedback can arise when financial distress affects the banking system.

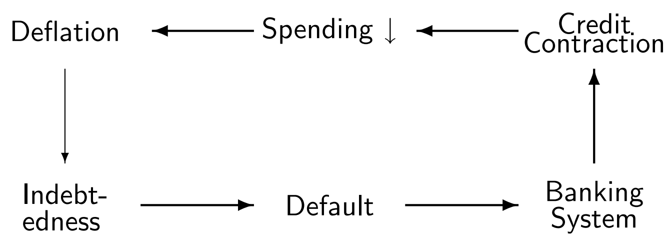
Keynes (1931) argued that the modern world is characterised by financial intermediation, as the banking system advances money on goods, shares, bonds, and real estate. Already by 1931 the practice of financing assets on bank credit was said to have "grown to formidable dimensions" (p. 169). The "margins of safety" (collateral and net worth) built into lending arrangements ordinarily protect the banking system from the consequence falling prices. But sufficiently large price declines affect the banking system: "a decline in money values so severe as that which we are now experiencing threatens the solidity of the whole financial structure." (p. 176).

Keynes only mentioned in passing that new lending would thereby be curtailed (p. 172-73). It was Bernanke (1983) who highlighted the macroeconomic significance of a credit contraction for aggregate spending. In the context of the Great Depression, he emphasized that wide-spread borrower default and bank runs together reduced the effectiveness of financial intermediation, by raising the cost of information-gathering and market-making services,

> "The banking problems of 1930-33 disrupted the credit allocation process by creating large, unplanned changes in the channels of credit flow. [...] plus the actual failures, forced a contraction of the banking system's role in the intermediation of credit." p. 264
>
> "[...] experience does not seem to be inconsistent with the point that even good borrowers may find it more difficult or costly to obtain credit when there is extensive insolvency. The debt crisis should be added to the banking crisis as a potential source of disruption of the credit system." p. 267
>
> "The effects of this credit squeeze on aggregate demand helped convert the severe but not unprecedented downturn of 1929-30 into a protracted depression."
>
> Ben Bernanke (1983) p. 257

In other words, financial distress impairs the process of credit intermediation, and a credit contraction in turn depresses aggregate demand.[16]

Bernanke's Channel



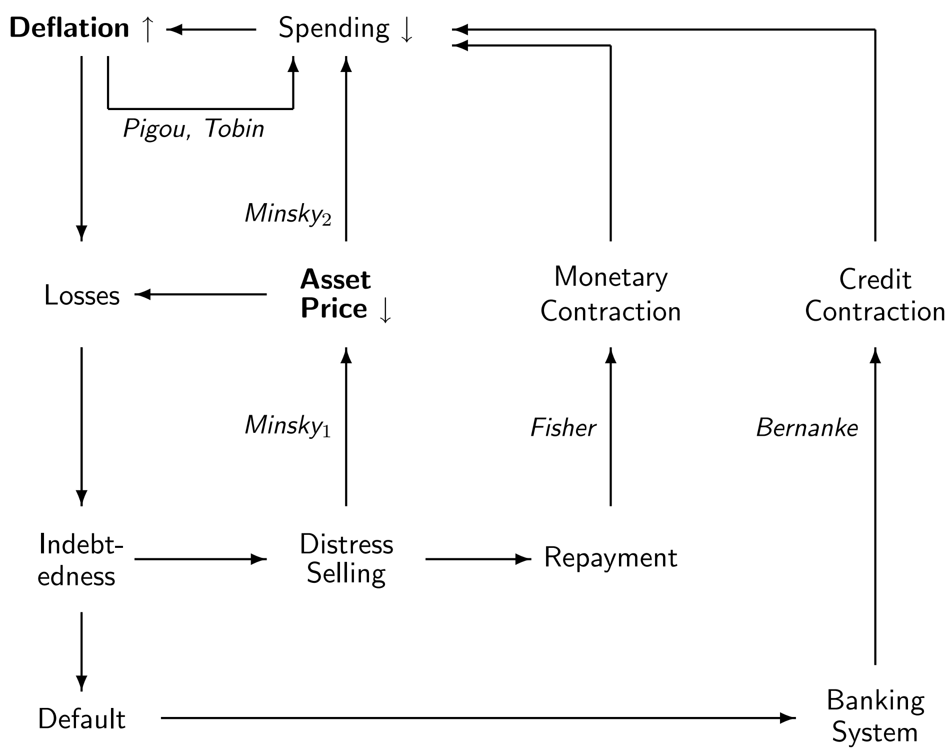
This figure represents Bernanke's channel as a feedback from credit to aggregate spending, subject to two qualifications. First, absent asymmetric information, this channel becomes active only in the

[16] Bernanke provides evidence of the pervasiveness of debtor insolvency, and argues that this contributed to an increase in the cost of credit intermediation by complicating banks' task of distinguishing good borrowers.

presence of wide-spread default. Second, one should expect a credit contraction to depress asset prices as well, based on Keynes's observations.

Figure 1 puts the channels together. The feedback we emphasised, from financial distress to deflation, is represented by the arrows pointing upward.

Figure 1: Channels of Debt-Deflation



1.4 Stability and propagation

Each channel of debt-deflation in figure 1 closes the loop between financial distress and deflation. This naturally gives rise to the questions: what initiates debt-deflation? Under what conditions does the process become unstable? Fisher's second quotation contains an explicit stability condition: if the initial over-indebtedness exceeds a threshold, the process becomes unstable. Debt-deflation would only come to a halt in "almost universal bankruptcy" (p. 346). That the debt structure determines stability is also reflected in Minsky's quotations and in his financial instability hypothesis more generally.[17] A robust debt structure makes the economy "an equilibrium seeking and containing system"; but under a fragile debt structure, it becomes "a deviation amplifying system" (Minsky 1992 p. 7).[18]

[17] The hypothesis is articulated in a series of papers, especially Minsky (1977, 1978, 1992).

[18] Minsky describes robust and fragile debt structures in the following terms:
Hedge Finance: the pattern of cash flow suffices to meet cash commitments *every period*.

Thus both Fisher and Minsky view the debt-deflation process as potentially unstable. Tobin only talks about stability to the extent that deflation weakens the adjustment mechanism following a large shock. First, as discussed above, deflation-induced redistribution can reduce aggregate demand because creditors have a lower propensity to spend. Second, higher real rates discourage consumption and investment.[19] Tobin is less concerned with unstable debt-deflation than with the weakening of the adjustment mechanism that returns the economy to full capacity.

Finally, Bernanke does not specify any stability conditions. He cautions that his theory does not offer a complete explanation of the Great Depression; it neither explains its beginning, nor excludes other theories (Bernanke 1983 p. 258). His aim is to explain the depth and length of the Depression, not what initiated it. This is consistent with his econometric approach, and with the financial accelerator models developed with Gertler which, as their name suggest, amplify and propagate shocks originating elsewhere.

Both Bernanke and Tobin therefore view debt-deflation primarily as a propagation mechanism. For the process to work, deflation, or another form of redistribution, must already be under way. By contrast, Fisher and Minsky view debt-deflation as a process capable of assuming a life of its own. All it takes is that agents engage in distress selling because they feel over-indebted.[20] Conversely, when agents refinance rather than sell assets, debt-deflation does not develop.

> "A reversion from a speculative debt structure is triggered when interest rates rise so that the payments on new debt as well as on refinanced old debt rise relative to expected cash flows. This will affect the willingness and ability of units to go into debt; units will try to *substitute selling out positions for refinancing of positions.*"
>
> Hyman Minsky (1982) p. 386, emphasis added.

Note that Minsky considers debt-deflation a fully endogenous phenomenon, not merely a mechanism that propagates external shocks. More generally, his financial instability hypothesis is a theory of how a capitalist economy endogenously generates a financial structure susceptible to financial crises, and how the normal functioning of financial markets gives rise to financial crises (Minsky 1977 p. 67-8). The two aspects most relevant to our discussion are, first, that over-indebtedness occurs because the system has an endogenous tendency towards a fragile debt structure,

> "As the period over which the economy does well lengthens, two things become evident in board rooms. Existing debts are easily validated and units that were heavily in debt prospered; it paid to lever. [...] the margins of safety built into the debt structures were too great. [...] views about

Speculative Finance: the present value of cash flows exceeds cash commitments, but refinancing is needed.
Ponzi Finance: even interest payments can only be met by new issues of debt.

[19] Tobin (1975) sets up a small Keynesian model whose steady state is locally stable. However, he argues that stability could be lost when a large shock moves the economy far away from the steady state, because falling prices discourage consumption and investment *more* when high real rates depress the present value of capital assets (p. 201).

[20] Fisher (1933) mentions as causes of high indebtedness "new opportunities to invest at a big prospective profit", facilitated by "easy money" (p. 348), but he does not specify when and why agents perceive their indebtedness to be excessive. This realization, he writes, may have many causes but "the chief cause may well be that earnings, current or expected, have begun to disappoint" (Fisher 1932 p. 40).

acceptable debt structure change."

Hyman Minsky (1977) p. 65

The financial instability hypothesis does not rely on exogenous shocks to generate business cycles (Minsky 1992 p. 9). The 'shock' he considers is a rise in interest rates, as evident from Minsky's second last quotation.[21] But the rise is not a policy decision but an endogenous, private response as it becomes apparent that the payment commitments implied by the debt structure approach expected cash flows,

> "the larger the dependence upon speculative and Ponzi finance, the greater the likelihood that a sharp run up in short-term interest rates will occur."
>
> Hyman Minsky (1982) p. 387

In other words, the "shock" is not exogenous, but a result of the endogenous evolution toward a more fragile debt structure.

1.5 Original features

In reviewing these contributions, it becomes clear that some original features of the debt-deflation literature have no counterpart in today's mainstream macroeconomics. First, the notion that the quantity of money endogenously contracts through repayment is no longer encountered, neither in macroeconomics nor in banking theory.[22] The post-War literature discusses monetary contraction mostly in the context of bank runs with no reference to borrower distress.[23] Clearly, Fisher's repayment channel is also operative when banks are not run upon.

Second, the role of financial distress remains relatively unexplored by modern macroeconomics. Compared to the earlier literature, financial distress became less prominent in interwar economics (see Haberler 1960, Laidler 1999), and less prominent still in post-War macroeconomics. Although recent decades witnessed renewed interest in credit issues, this development has not yet reached a stage on which the ideas of the debt-deflation literature would naturally take hold.[24] More generally, a number of standard methods today are not well suited for exploring debt-deflation, for instance the use of representative agent models, and log-linearisation around the steady state. Linearisation distracts from the global stability analysis relevant in the context of debt-deflation.

We now develop a stylised model that attempts to capture some of the key features of debt-deflation.

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[21] He considers as equivalent a fall in asset values, Minsky (1963) p. 91.

[22] Important exceptions include Black (1970) in finance, and McAndrews and Roberds (1995) in banking theory, and much of the Post-Keynesian literature, e.g Moore (1988).

[23] Friedman and Schwartz (1963) argue that bank runs were harmful (only) because they produced a decline in money supply. This becomes clear from their suggestion that the Depression would have been worse had the decline in the money stock been produced some other way, without bank failures, because these also reduced money demand (p. 353). To the extent it is discussed, monetarists put financial distress among non-bank borrowers causally last. Schwartz (1995) argues that financial distress is the result of price level fluctuations (not the reverse), because unexpected deflation distorts loan terms and leads to defaults. Bordo and Wheelock (1998) provide empirical support.

[24] For instance, few models allow for wide-spread default and losses to the banking system. This includes Bernanke et al (1999), and Christiano et al (2004). This issue is taken up in von Peter (2004).

2 The basic model

We build on the model of von Peter (2004) which has several useful features. Firms and households are present; households hold deposits, whereas firms are indebted and hold assets. As in Kiyotaki and Moore (1997), the absence of a rental market generates indebtedness. But instead of productive assets, we use utility-yielding assets, as in the housing literature (e.g. Miles 1995).[25] Credit is intermediated by banks, which makes it possible for money to contract through the repayment of debt. The overlapping generations structure implies active markets and makes dealing with losses and default tractable. The main departure from von Peter (2004) is to incorporate distress selling. The method is to start with a perfect foresight model, and to perturb its steady state with a shock to examine whether a debt-deflation process arises. Note, finally, that the analysis is conducted in nominal terms: relative prices, such as the real asset price, do not convey all the information relevant for the repayment of nominal debt.

2.1 Firms

Firms of each generation are three period-lived (young, mid-age, old) and produce consumption goods. Production takes one period, and so does bringing goods to the market. The typical firm entering in $t-1$ hires labour, and sells its production at price level p_{t+1}. Out of sales revenue $p_{t+1}y(\ )$, the firm pays the wage w_{t+1}, determined on a competitive labour market at the time of hiring $t-1$.[26] The firm chooses labour to maximise profits,

$$\max_{n_{t-1}} \ \Pi_{t+1} = p_{t+1}y(n_{t-1}) - n_{t-1}w_{t+1}.$$

The production function is standard neoclassical. Labour demand is given by equating the marginal product to the real wage,

$$y'(n^d_{t-1}) = w_{t+1}/p_{t+1}. \tag{1}$$

The next question is how profits are disposed. We assume that each firm is owned by an entrepreneur who enjoys final period consumption. Entrepreneurs also derive utility from holding real assets, which can be thought of as housing. Assets provide a service flow proportional to the quantity held from one period to the next. Hence, asset holdings, h, appear directly in entrepreneur's utility function,

$$V = (1-\theta)\ln c_{t+1} + \theta\left[\ln h_{t-1} + \beta \ln h'_t\right]/(1+\beta), \tag{2}$$

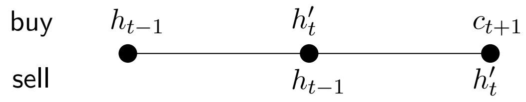
where β is the discount factor, and $\theta \in [0,1]$ measures the preference for assets.[27] An entrepreneur who only values final period consumption ($\theta = 0$) will spend profits entirely on goods. With preferences over both consumption and housing, entrepreneurs will choose $\{h_{t-1}, h'_t, c_{t+1}\}$ all positive,

[25] This assumption is made for simplicity. Results remain qualitatively unchanged if assets enter the production function instead, as in von Peter (2004).

[26] This assumptions makes the wage bill similar to nominal debt. By time t workers have already delivered their labour input, and the firm can no longer modify the nominal wage in response to an unanticipated change in p_{t+1}.

[27] The prime on h'_t denotes next-period demand for assets. We use h_t to refer to the first-period demand of the next generation of firms, entering at t.

buying h_{t-1} when young, adjusting this position to h'_t when mid-age, and reselling this quantity when old,



As production takes time, sales revenue accrues late, and asset holding is accompanied by borrowing B at the gross nominal interest rate $R = \beta^{-1}$,

$$\begin{aligned} q_{t-1}h_{t-1} &= B_{t-1} \\ q_t(h'_t - h_{t-1}) + RB_{t-1} &= B'_t \\ p_{t+1}c_{t+1} + RB'_t &= \Pi_{t+1} + q_{t+1}h'_t. \end{aligned} \tag{3}$$

Combining these period constraints yields the intertemporal budget constraint. It states that spending on consumption and assets (braced) adds up to profits,

$$p_{t+1}c_{t+1} + \underbrace{R^2 u_{t-1}h_{t-1} + Ru_t h'_t} = \Pi_{t+1}. \tag{4}$$

The term $u_t \equiv q_t - q_{t+1}/R$ is the *user cost*, the purchasing price minus the resale price discounted by the cost of borrowing. User cost is the cost of *holding* an asset between periods t and $t+1$, as in Kiyotaki and Moore (1997). Note that user cost is a small fraction of the asset price q_t because assets, unlike goods, can be resold after use. (This will provide a strong incentive to become indebted.)

Given Π_{t+1}, the firm's problem is to maximise (2) subject to (4). The first order conditions equate the marginal utilities per dollar spent on goods and assets,

$$\frac{1-\theta}{p_{t+1}c_{t+1}} = \frac{\theta\beta/(1+\beta)}{Ru_t h'_t} = \frac{\theta/(1+\beta)}{R^2 u_{t-1}h_{t-1}} = \xi,$$

where $\xi = 1/\Pi_{t+1}$ is the Lagrange multiplier on (4). Thus it is optimal to spend the fixed share $(1-\theta)$ of profits on consumption, and to split the remainder θ equally between user costs,

$$\text{consumption spending} \qquad p_{t+1}c_{t+1} = (1-\theta)\Pi_{t+1} \tag{5}$$

$$\text{user cost spending} \qquad Ru_t h'_t = Ru_{t-1}h_{t-1} = \theta\Pi_{t+1}/(1+R). \tag{6}$$

Remarks. Firms are necessarily indebted while active ($B_{t-1}, B'_t > 0$), because they purchase assets upfront, before earning sales revenue. (3) also shows that debt service tends to reduce consumption spending. Note that firms can reduce their debt in t only by selling assets, i.e. $B'_t < RB_{t-1} \Leftrightarrow h'_t < h_{t-1}$. This will form the basis for distress selling. Yet, do these observations imply that entrepreneurs actually *care* about debt per se? No. Entrepreneurs optimally choose the amount of spending on user cost in (6), regardless of the implied path for debt. Put differently, the notion of user cost is inherently dynamic - it embodies the fact that reselling assets at a higher price makes it easier to repay debt in the future: asset price appreciation effectively reduces the cost of servicing debt.[28] This suggests that the analysis of distress selling requires a context where asset price dynamics are set against debt considerations.

2.2 Households

There is a unit measure of infinitely-lived households who derive utility from consuming goods,[29]

$$U = \sum_{t=2}^{\infty} \beta^t u(c_t^h). \tag{7}$$

Households work every period, providing one unit of labour, and they save in the form of interest-bearing deposits. The flow constraint states that consumption spending ($s_t \equiv p_t c_t^h$) plus deposits are financed by the wage plus accumulated deposits,

$$s_t + D_t = w_t + RD_{t-1}. \tag{8}$$

Households solve a standard intertemporal consumption problem, maximising (7) subject to (8). The slope of optimal consumption is given by the familiar Euler equation,

$$u'(c_t^h) = \beta R \frac{p_t}{p_{t+1}} u'(c_{t+1}^h).$$

In steady state, it must be the case that $R = \beta^{-1}$, the interest rate equals the inverse rate of time-preference, as assumed above.[30] Consumption spending then equals permanent income, $s = w + (R-1)D$. To specify how spending deviates outside steady state, we posit time-separable CRRA utility $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$, and rewrite the Euler equation to relate the evolution of spending directly to price levels,

$$s_t = \left(\frac{p_{t+1}}{p_t}\right)^{\frac{1-\gamma}{\gamma}} s_{t+1}. \tag{9}$$

This expression depends on $1/\gamma$, the intertemporal elasticity of substitution (IES). Note that a low price level today attracts increased spending, provided IES is high (with $\gamma < 1$, $p_t < p_{t+1}$ implies $s_t > s_{t+1}$). We will focus on $\gamma < 1$, so that a higher real interest rate encourages saving, discourages spending. The values of s_t and s_{t+1} will be determined by goods market clearing. Hence lower γ means more responsive household spending, which translates into smaller price level variation. The case $\gamma \to 0$ admits a well-defined fixed-price limit.

Initial wealth and deposits are determined by what happens at 'the beginning of the world'. To bring into existence a large volume of credit, we resort to the device introduced in von Peter (2004).

---

[28] For given Π_{t+1}, entrepreneurs will take on more debt when the user cost falls, $u_t < u_{t-1} \Rightarrow h'_t > h_{t-1}$. At the extreme, a user cost of zero would mean that assets appreciate at the rate of interest, $q_{t+1} = Rq_t$. In that case, demand for assets, and consequently the willingness to incur debt, would become unbounded.

[29] It is at date 2 that goods first become available, produced by firms of generation 0.

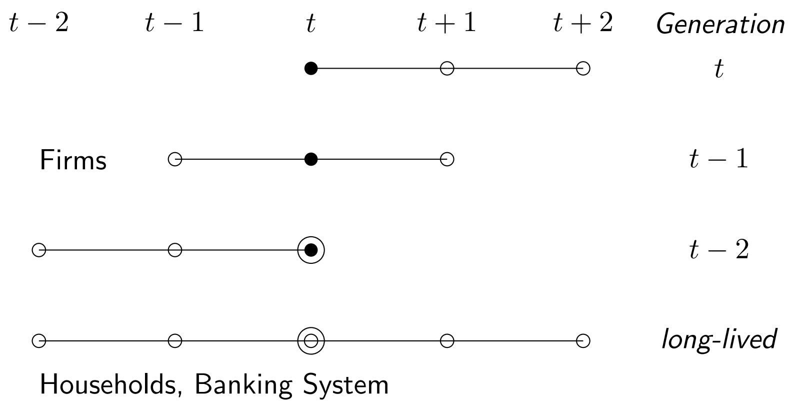
[30] For simplicity, we assume that the nominal rate remains pegged outside the steady state. As interest rate rules are not the subject of this paper, we assume a fixed rate and dispense with nominal price level indeterminacy by imposing $p_2 = 1$. Real interest rate movements are then solely due to price level movements.

Households are initially endowed with all the assets; they receive h assets at dates $t = 0,1$. As households derive utility only from consumption, they will sell h to the first generations of firms.[31] Hence initial household wealth equals $D_1 = (q_1 + q_0 R)\, h$, the counterpart of the borrowing of the first two generations of firms.

2.3 The banking system

We introduce a banking system because the views of Fisher, Keynes, Bernanke and Minsky involve a close relation between deposit money, bank credit and asset prices. Households are the lenders in this economy, while firms are borrowers each period since production takes time. Efficient intertemporal exchange requires that assets be passed down successive generations of firms, in exchange for part of firms' output consumed by households.

Figure 2: Structure of the Economy



In this context we motivate banking as a payment and credit mechanism (as elaborated in von Peter 2004). Deposits serve as the means of payment. They are created through lending, and are held across periods by households. Every period t, the banking system creates deposits worth $q_t h_t$ by extending loans to new firms. New firms then purchase assets by transferring their new deposits to old firms.[32] This allows old firms to repay what they had borrowed when young, spending the remainder on goods consistent with (3). Households spend s_t of their accumulated deposits on firm output, and keep the remainder on deposit consistent with (8).

We consider the banking system as a single competitive entity with no capital and no profits.[33] Thus bank assets consist of credit extended to firms, matched by monetary liabilities in the form of household deposits,

[31] Households save by holding deposits rather than real assets. This is because deposits are interest-bearing, while assets cannot appreciate at the rate of interest in addition to yielding utility to entrepreneurs. (User cost must remain positive and equal to entrepreneurs' marginal utility.)

[32] At dates $t = 0, 1$ these payments go to households selling their asset endowment.

[33] The banking system provides elastic credit at R, consistent with firms' intertemporal budget constraint; there are no separate credit constraints.

Bank Balance Sheet in t

B_t	D_t
B'_t	$(K = 0)$

The bank balance sheet is recorded after the close of markets. Hence the two generations of firms concurrently borrowing are the young, who just started borrowing, and the mid-age, who carry over their borrowing plus interest from last period. (Old firms were removed from the balance sheet upon repayment.) Using (3) and the definition of $u_t \equiv q_t - q_{t+1}/R$, the size of the banking system equals

$$q_t h_t + \left[q_t(h'_t - h_{t-1}) + Rq_{t-1}h_{t-1}\right] = D_t. \tag{10}$$

2.4 Perfect foresight equilibrium

A *perfect foresight equilibrium* is a sequence of prices $\{p_t, q_t, w_t\}$, and quantities $\{c_t, c^h_t, h_t, h'_t, n_t\}$ such that firms maximise (2) subject to (4); households maximise (7) subject to (8) and initial wealth; and the goods, asset and labour markets clear every period. The overlapping generations structure, depicted in figure 2, gives rise to active markets, open at the beginning of every period.

Labour market equilibrium requires $n^d_t = 1$, because a unit measure of firms hire, and an equal measure of *workers* supply one unit each. From (1) the constant equilibrium real wage is $w_t/p_t = y'(1)$, or

$$\begin{aligned} w_t &= \varepsilon p_t y, \text{ and} \\ \Pi_t &= (1-\varepsilon)p_t y \end{aligned} \tag{11}$$

where $y \equiv y(1)$ is output, and $\varepsilon \equiv y'(n)\frac{n}{y}|_{n=1}$ is output elasticity. Consistent with the neoclassical distribution of income, labour earns the competitive wage (the marginal value product), and entrepreneurs earn the remainder.[34]

Goods market clearing requires that produced output y equals the consumption demands of households and old firms, $y = c^h_t + c_t$ (the two larger circles in figure 2). Using (5) and (11), the goods market clears when

$$p_t y = s_t + (1-\theta)\,\Pi_t \qquad \Rightarrow \qquad s_t = \bar{\varepsilon} p_t y \tag{12}$$

where $\bar{\varepsilon} \equiv \varepsilon + (1-\varepsilon)\,\theta$. Pairs of $\{p_t, s_t\}$ are then determined by combining successive clearing conditions with the Euler equation (9).

Asset market equilibrium requires that fixed supply equals the asset demands of young and mid-age firms, $2h = h_t + h'_t$ (the three black circles in figure 2). Multiplying by u_t and using (6) and (11),

$$2u_t h = \frac{\theta\,(1-\varepsilon)\,y}{R(1+R)}\left[p_{t+1} + p_{t+2}\right]. \tag{13}$$

[34] Firm profits can be interpreted as an implicit wage to the entrepreneur's firm-specific labor input, subsumed in the production function.

Aggregate user cost spending equals the spending of young and mid-age firms, which is a fraction $\theta/[R(1+R)]$ of profits, which are in turn a fraction $(1-\varepsilon)$ of sales revenue, hence price levels. Denote the coefficient following the square brackets by uh. The user cost $u_t \equiv q_t - q_{t+1}/R$ being an intertemporal price, (13) can be developed into an asset pricing equation,

$$\begin{aligned} q_t &= \frac{uh}{2h}\left[p_{t+1}+p_{t+2}\right]+\frac{1}{R}q_{t+1} \\ &= \frac{u}{2}\sum_{T=t}^{\infty}\frac{\left[p_{T+1}+p_{T+2}\right]}{R^{T-t}} \end{aligned} \tag{14}$$

after iterating forward and ruling out bubbles. The asset price is the present discounted value of marginal utilities that agents derive from their use, as measured by the shares of income they dedicate to user cost. The expression is proportional to future price levels, since greater sales revenue in the future means agents can afford to spend more on assets today.

Finally, the evolution of debts and deposits is given by (3) and (8). Using equilibrium relations allows to restate the size of the banking system (10) as

$$2q_t h + Ruh\, p_{t+1} = D_t, \tag{15}$$

which reflects primarily the value of the asset market, $2q_t h$, since bank lending finances the purchase of assets. The following provides the benchmark for subsequent results.

Proposition 1 Basic model

(a) The unique perfect foresight equilibrium is the steady state.

(b) Firms hold a constant quantity of assets throughout their lives.

(c) Firms are indebted.

Using the Euler equation (9) to connect successive goods market clearing conditions (12) implies $p_{t+1} = p_t$, hence the price level must be a constant. A constant price level p implies constant wages and profits in (11), $w = \varepsilon py$ and $\Pi = (1-\varepsilon)\, py$. Hence spending on goods and assets, $s = w + (R-1)\, D$, (5) and (6), must also remain constant. Therefore, from (14) and (15),

$$\begin{array}{llll} \text{Market value of assets} & 2qh & \equiv 2uhR/\left(R-1\right) & = \frac{2\theta(1-\varepsilon)}{R^2-1}py \\ \text{Size of banking system} & D & \equiv qh(1+R) & = \frac{\theta(1-\varepsilon)}{R-1}py. \end{array} \tag{16}$$

A constant asset price leads to a constant user cost of $u = q(R-1)/R$ in (6). Consequently, firms hold a constant quantity of assets, $h' = h$, throughout their lives. The evolution of debt, from (3), is therefore $B = qh$, and $B' = Rqh$. Indebtedness is defined as debt over profits (in present value),

$$\text{Indebtedness} \quad L = \frac{B}{\Pi/R^2} = \frac{B'}{\Pi/R} = \frac{\theta R^2}{R^2-1}. \tag{17}$$

Greater preference for assets θ implies greater indebtedness, and $L > 1$ when $\theta > (R^2 - 1)/R^2$. Thus for almost the full parameter range of $\theta \in [0, 1]$, firms choose to be indebted in the sense that debt exceeds profits associated with the firm's underlying productive activity.

Remarks. The incentive to get indebted is now easy to understand. User cost is a small fraction of the asset price q, because assets, unlike goods, are resold after use. Ex post, therefore, the cost of holding an asset is only small fraction of its price, namely $Ru \equiv (R - 1)q$, effectively the *net* interest rate.[35] But to take advantage of this arrangement, young firms must borrow qh upfront to purchase the assets, and carry this volume of debt until they resell assets and repay debt when old. In the interim, they are vulnerable to unexpected asset price declines, the more so the greater their indebtedness.

3 How do agents react to losses?

Agents are never 'over-indebted' in our context of optimal choices under perfect foresight. But the idea that agents engage in distress selling to reduce their indebtedness is central to debt-deflation. We now leave the perfect foresight benchmark by introducing an exogenous shock and studying agents' reactions to the resulting losses (this section). The changes in spending are then aggregated across agents. Once the equilibrium system is in place, we can turn to the question of which channels of debt-deflation are active, and under what conditions they become destabilising (next sections).

3.1 Shock and losses

Following Fisher (1933), the shock should trigger over-indebtedness. Following King (1994), "The key is to examine debt-deflation in a model of purely distributional shocks with no aggregate uncertainty at all." (p. 420). A one-off redistribution from borrowers to lenders satisfies both criteria. We consider a small unexpected payment ρ, due in t from all mid-age firms to households.[36,37] We assume that the failure to pay ρ would preclude firms from selling output in $t+1$. Mid-age firms are the most interesting agents to expose: they are already indebted, yet can still alter their spending on both consumption and assets. Debt and assets carried over from $t-1$ are predetermined, hence the period constraints (3) become

$$\begin{aligned} qh &= B \\ q_t(h'_t - h) + RB + \rho &= B'_t \\ p_{t+1}c_{t+1} + RB'_t + w &= p_{t+1}y + q_{t+1}h'_t. \end{aligned} \tag{18}$$

[35] For example $u \simeq 0.05q$ when the borrowing cost is 5% - dedicating only a fifth of profits to user cost in (6) finances assets worth four times profits.

[36] The reason for the shock is unimportant - examples include additional labour costs, increased corporate pension contributions, or a class action lawsuit.

[37] Similar experiments have been considered in non-monetary or partial-equilibrium models. Bernanke and Gertler (1989) consider a similar redistribution to show that it affects investment. Holmström and Tirole (1998) study firms subject to a liquidity shock, but no counterparty receives any payments. Considering only one side of a payment would be inconsistent in our model (it violates two identities).

This shock has direct implications for liquidity and solvency. It affects solvency because the loss of ρ cannot be recovered, and spending must be modified accordingly. The shock also generates liquidity needs, because firms have no revenue in t to pay with: they must either increase their borrowing by ρ, or sell assets worth ρ, to be able to effect the payment.

The shock merely serves the purpose of setting off the dynamics. It is assumed to be small, as we are not interested in shocks so large as to singlehandedly bankrupt firms. We instead focus on the *system of equilibrium reactions* that follow the initial shock. Intuitively, those directly affected by the shock alter their consumption and asset holdings (the direct effect). But their reactions change equilibrium prices, which produces a new round of reactions by *all* agents and further changes to equilibrium prices (the general equilibrium effect). The new equilibrium is found where all agents' reactions are consistent with each other. Thus, the shock has potentially important general equilibrium implications.

We now explore the reactions to the shock and its consequences simultaneously. (Appendix A1 derives the expressions that follow in detail.) Debt is predetermined, but the ability to repay is not. In $t-1$ firms had borrowed qh, assuming that in $t+1$ they would sell goods and assets at continued steady state prices $\{1, q\}$. Following the shock, a new set of equilibrium prices materialises. A decline in the asset price, $q_t < q$, reduces the value of asset holdings by $qh\delta_t$, where $\delta_t \equiv (q - q_t)/q$ denotes the percentage decline. Likewise, any possible decline in the price level, $p_{t+1} < 1$, involves a loss of sales revenue on goods sold next period, as shown in (18). Thus, mid-age firms face an unexpected total loss of

$$\Omega' \equiv \rho + qh\delta_t + (1 - p_{t+1})\, y/R. \tag{19}$$

Similarly, old firms must now sell goods p_t and assets at q_t. They also face a decline in the value of asset holdings and the loss of sales revenue to deflation. Old firms' loss equals

$$\Omega \equiv qh\delta_t + (1 - p_t)y. \tag{20}$$

Losses imply default only when they exceed planned consumption. Once losses exceed this threshold, limited liability implies that any further losses are transferred as non-performing loans to the banking system (see appendix A1).

How do agents react to the losses they face? At the time of the shock, firms are at different points in their life-cycle (see figure 2):

- Young firms (entering in t) can take new prices into account before taking on debt. They face no losses and behave as in the perfect foresight case (section 2.1).
- Old firms (exiting in t) only have final consumption left to do. Their asset holdings are predetermined, hence the loss Ω reduces only spending on consumption.
- Mid-age firms already incurred debt, yet they can still alter both their asset position and final consumption. Hence the loss Ω' affects spending on both consumption and assets.

In view of our discussion of debt-deflation, we can learn most from the behaviour of mid-age firms. In particular, when they incurred debt in $t-1$ they had intended to hold h assets until $t+1$, as shown in proposition 1. How will they modify their asset holdings?

Definition: *Refinancing* occurs if asset positions are held constant as intended, $h'_t = h$.
Distress selling occurs if assets that were intended to be held are sold off, $h'_t < h$.
The value realised by distress selling equals $q_t(h - h'_t)$.

3.2 Firms

We now consider three types of reactions and their implications for spending on consumption and assets.

(1) Optimal user cost spending

A positive loss Ω' means that mid-age firms can no longer spend as intended in steady state, namely pc on goods and uh on assets. Firms *reoptimise* their choices $\{h'_t, c_{t+1}\}$ subject to their reduced budget (appendix A1). It is optimal to revise spending downward in fixed proportions of the loss,[38]

$$u_t h'_t = uh\,\theta'\Omega' \tag{21}$$

$$p_{t+1}c_{t+1} = pc - \left(1-\theta'\right) R\Omega', \tag{22}$$

This is the optimal policy of user cost spending, as in (6). Now, whether this reaction involves distress selling can only be determined once equilibrium prices have been solved for. If user cost were to fall sufficiently, asset holding, debt and indebtedness might all increase.[39] What can be said, however, is that debt and indebtedness *per se* play no role in the optimal reaction to losses, consistent with our earlier remarks (page 12).

(2) Refinancing

An alternative reaction would be for mid-age firms to hold on to all their assets until $t+1$. (This could be the result of fixed adjustment costs, see appendix A2.) Refinancing involves a rise in debt to $B'_t = RB + \rho$, so the firm can effect the payment ρ without selling assets ($h'_t = h$). Ex post, spending on assets simply equals $u_t h$ in (18). Consequently, consumption spending will have to be reduced by the entire loss,

$$p_{t+1}c_{t+1} = pc - R\Omega' + \left(R\delta_t - \delta_{t+1}\right) qh. \tag{23}$$

The last term corrects for the fact that the decline asset values is realised not in t, but in $t+1$. Note that the unit coefficient on Ω' exceeds that in (22), which had *shared* the loss between reduced

[38] This expression holds provided the firm does not default. In case of default, $h'_t = c_{t+1} = 0$, given $u_t > 0$.

[39] From (21), $u_t < u - \theta'\Omega'/h \Rightarrow h'_t > h$, and (18) would imply $B'_t > RB + \rho$, with a commensurate increase in indebtedness. To illustrate this point, suppose the asset price were to appreciate as much as $q_{t+1} > Rq_t$, so user cost u_t would turn negative. Then buying up all assets on credit would be profitable strategy, even for a firm in default.

spending on consumption and assets. While refinancing allows agents to retain assets, the additional debt service can be a burden on consumption.

(3) Containing indebtedness

A third possible reaction is the attempt to reduce or keep constant indebtedness, as suggested by Fisher (1933). (This could be the result of margin requirements or uncertainty, as discussed later.) Recall that indebtedness is measured as debt over profits in (17). Thus, a positive loss Ω' makes firms over-indebted, unless they reduce nominal debt accordingly. Yet firms can raise money for repaying debt only by selling assets in (18), irrespective of prices. Containing indebtedness requires distress selling worth a multiple of the loss,

$$q_t(h - h'_t) = \rho + L\Omega'. \tag{24}$$

This value far exceeds the firm's immediate liquidity needs: ρ must be raised only to keep nominal debt constant - to keep *indebtedness* constant, another $L\Omega'$ must be realised by distress selling. (Note, however, that firms cannot sell more than their holdings: the value of (24) cannot exceed $q_t h$, as $h'_t \geq 0$.) The implication for consumption spending is found to be

$$p_{t+1}c_{t+1} = pc - R\Omega' + Ruh - Ru_t \left[h - \left(\rho + L\Omega'\right)/q_t\right]. \tag{25}$$

The term in square brackets equals h'_t, the remaining stock of assets after distress-selling. Hence the impact of the loss on consumption spending, compared to (23), is moderated by the money saved on carrying fewer assets.[40]

The three types of reactions we have discussed are summarised in

Proposition 2 Reactions to losses

(a) The optimal reaction is a downward revision of both consumption and user cost spending, in proportion to the loss.
(b) Refinancing would instead require a downward revision of consumption spending by the full loss.
(c) Containing indebtedness would require distress selling of assets worth a multiple of the loss.

In each case, there will be different prices, quantities, and losses in general equilibrium. They are determined by aggregating reactions in general equilibrium. Before doing so, we complete this section by describing the behaviour of the remaining agents.

3.3 Households and the banking system

Households and the banking system do not alter their behaviour. The banking system faces non-performing loans λ^+ if firms default. The value of λ^+ simply equals firms' losses (19)-(20), minus

40 Appendix A3 shows how to obtain (24) and (25).

their ability to withstand them (see appendix A1). As there are no bank dividends in this model, these losses do not directly affect aggregate spending; recognising loan losses instead involves reducing the value of deposits, as bank capital is zero. Due to the absence of frictions and capital constraints in our model, the banking system continues to provide elastic credit.[41]

Households are affected in three ways relative to steady state. They are the recipients of the payment ρ by firms. However, their deposits are reduced to $RD - \lambda^+$ should any firms default. Finally, future wages fall to $w_{t+i} < w$ if future price levels $p_{t+i} < 1$. Their intertemporal budget constraint forces them to revise downward future consumption spending s_{t+i} accordingly,

$$\sum_{i=0}^{\infty} \frac{(s - s_{t+i})}{R^i} = \lambda^+ - \rho + \sum_{i=0}^{\infty} \frac{(w - w_{t+i})}{R^i}. \tag{26}$$

Importantly, wealth effects leave the slope of the Euler equation (9) unchanged.

4 Stable debt-deflation

The previous section has determined the changes in spending on consumption and assets in response to arbitrary price deviations. Aggregating these changes across agents will determine the *equilibrium* deviations as those prices and quantities that make all agents' reactions consistent with each other.[42] For optimal user cost spending, this section shows that several channels of debt-deflation are active, but none is destabilising, even when large shocks, high indebtedness, and wide-spread default are considered.

4.1 Equilibrium system

We now develop the following result. In our context of our perfect foresight model with elastic credit, optimal user cost spending leads to

Proposition 3 Stable debt-deflation

(a) *Uniqueness:*
- *for a zero shock ρ, the continued steady state is the unique equilibrium, provided $\gamma < 1$.*
- *Prices are decreasing in the shock, $p_t'(\rho)$, $p_{t+1}'(\rho)$, and $q_t'(\rho) < 0$, until defaults occur.*

Hence debt-deflation is not fully endogenous: an exogenous shock must initiate it.

(b) *Stability:*
- *the system is deviation-amplifying in the channels of Fisher, Minsky, and Tobin, but*
- *debt-deflation remains globally stable, for any indebtedness and any size of shock.*
- *Default per se is not destabilising.*

Hence the channels of Fisher, Tobin and Minsky are not destabilising in this model.

[41] This is modified in section 5.2. More generally, dividends, bank capital, and credit crunches are considered in von Peter (2004).

[42] This method is not specific to the nature, or presence, of the original shock.

Appendix B1 derives the general system, comprising the four possibilities that no-one, old firms, mid-aged firms, or both types default. Appendix B2 then shows that by $t+2$, the system reverts to the previous steady state: the dynamics cease when all firms affected by the shock have left the model. Uniqueness is then established for $\rho = 0$.

This allows us to start our discussion from the equilibrium system without default, (27). It consists of goods markets, the asset market, and Euler equations. Note that the supply side remains unchanged: $2h$ assets turn over every period, and ouput remains at capacity y since the competitive wage clears the labour market. (Appendix B4 considers nominal wage rigidity, unemployment, and output effects.) Goods market equilibrium (GM) relates deflation to the fall in aggregate spending by households and firms, relative to steady state where $p = 1$. The asset market (AM) relates the asset price decline δ_t to reduced user cost spending,

Pigou   Minsky$_2$ Fisher-Tobin

$$\begin{aligned}
\text{GM}_t:&\quad (1-p_t)y = (s-s_t) + [qh\delta_t + (1-p_t)y] \\
\text{GM}_{t+1}:&\quad (1-p_{t+1})\,y = (s-s_{t+1}) + (1-\theta')\left[\rho + qh\delta_t + (1-p_{t+1})\,y/R\right]R \\
\text{AM}_t:&\quad \delta_t = \tfrac{\theta'}{2qh}\left[\rho + qh\delta_t + (1-p_{t+1})\,y/R\right] \\
\text{Euler}:&\quad p_{t+i} = \left(\tfrac{s_{t+i}}{s}\right)^{\frac{\gamma}{\gamma-1}} \qquad \forall i \geq 0,
\end{aligned} \tag{27}$$

Pigou   Minsky$_1$ Fisher-Minsky

and $p_{t+i} = p' = 1$ for $i \geq 2$, $\delta_{t+i} = (1-p') = 0$ for $i \geq 1$, and $w_{t+i} = w$ for $i \geq 0$. Note that the Euler equation admits a well-defined fixed-price limit, $p_{t+i} = 1$ when $\gamma = 0$, for the reasons discussed in (9).

To interpret this system in terms of debt-deflation, consider first goods market equilibrium in t in view of figure 1. The expression in brackets represents the wealth effect from old firms' losses (20). Starting from steady state ($p_t = 1$), one observes that, given $(s - s_t)$, an asset price decline δ_t tends to produce deflation (**Minsky**$_2$ in figure 1). However, $(s - s_t)$ is *not* given: with $\gamma < 1$, a low price level p_t encourages household spending: $p_t < 1 \Leftrightarrow s_t > s$ is implied by the Euler equation. This is an intertemporal **Pigou** effect.[43] Cancelling terms in GM$_t$ shows that households' extra spending in equilibrium exactly offsets the wealth effect,

$$s_t - s = qh\delta_t. \tag{28}$$

[43] More precisely, it is an intertemporal substitution effect reminiscent of Pigou's. In both cases, a lower price level encourages greater real spending. But the Pigou effect, strictly speaking, is a wealth effect on outside money which is absent in this model. (Nor do households enjoy a permanent increase in the real value of deposits, since the price level reverts to $p = 1$.)

The price level would *not* fall if households willingly spent that much more at *unchanged* prices.[44] But the price level must fall to attract households' extra spending. Therefore, an asset price decline produces deflation,

$$p_t = \left(1 + \frac{qh}{s}\delta_t\right)^{\frac{\gamma}{\gamma-1}} < 1 \qquad \text{if} \qquad \delta_t > 0. \tag{29}$$

Deflation then causes a second wealth effect (**Fisher-Tobin**): the sales revenue old firms lose to deflation, $(1-p_t)y$, reduces their profits, and thereby entrepreneurs' spending on other firms' goods. This new gap in aggregate spending is *not* offset. Therefore, although deflation does attract extra spending by lenders, this less than compensates for the reduced spending by borrowers, as Tobin suggested. (Borrowers' marginal propensity to consume effectively equals 1.) As a result, when $\delta_t > 0$, aggregate spending in t falls short of steady state spending in $t-1$, consistent with deflation.

Goods market equilibrium in $t+1$ rests on the same logic. Relative to steady state, the price level falls when households have to 'plug the hole' in aggregate demand. This happens following firms' loss (19), which again consists of asset market losses and sales revenue lost to deflation. However, as a result of reoptimization (22), only a fraction of the loss falls on aggregate demand, the remainder falls on user cost spending, and thereby on the asset market AM$_t$.

It remains to determine whether the asset price in t declines in the first place. To see that it does, suppose first that $\delta_t = 0$. A positive payment ρ implies that firms will spend less on holding assets than they would have in steady state. Similarly, the lower p_{t+1}, the more firms will reduce their spending in (22); the unexpected shortfall in future revenue means that their net worth is lower than expected when they had purchased assets in $t-1$. Hence, deflation tends to reduce the asset price (**Fisher-Minsky** in figure 1).[45]

But once the asset price decline is positive, it feeds back on itself (**Minsky**$_1$): δ_t itself appears on the right-hand side of AM$_t$. The decline causes a loss to firms already holding assets, and is thus reflected in their reduced spending on assets (21). The asset price decline then causes the wealth effect with which we started the argument above.

Note, however, that the system contains no independent effect from money (Fisher) or credit (Bernanke) to the goods and asset markets. The absence of these channels is due to the perfectly elastic provision of credit and inside money, a point taken up in section 5.2.

4.2 Stability and propagation

With the channels of debt-deflation in place, we can examine their stability and propagation for shocks of any size. Considering the limit case of a zero shock means that we only drop the perfect foresight assumption for t. This allows to discriminate whether debt-deflation is fully endogenous, as agents' general equilibrium responses alone produce debt-deflation, or whether it merely propagates shocks, as discussed in section 1.4.

[44] They would only do so with infinite intertemporal elasticity of substitution ($\gamma = 0$, linear utility).

[45] A contemporaneous relation between deflation and asset price decline would be obtained under the slightly different timing assumption that firms sell goods when mid-age, not when old.

The system (27) is "deviation amplifying", as Minsky conjectured. Amplification manifests itself in two ways:

- Feedback within markets: deflation exacerbates deflation, and the asset price decline reacts on itself.
- Feedback across markets: today's asset price decline causes deflation, and future deflation in turn exacerbates today's asset price decline.

Clearly, price deviations amplify each other. It does not follow, however, that debt-deflation becomes unstable. The feedbacks, although positive, are not destabilising, just as the associated multipliers, although greater than one, are finite.

To see this, consider first feedback *within* markets: in AM$_t$, the coefficient on δ_t is $\theta'/2 < 1$, and the asset market multiplier $(1-\theta'/2)^{-1} < 2$ is clearly finite for all $\theta \in [0,1]$.[46] Hence the asset price decline converges,

$$\delta_t = \frac{\theta'}{(2-\theta')\,qh}\left[\rho + (1-p_{t+1})\,y/R\right]. \tag{30}$$

Similarly, solving GM$_{t+1}$ for $(1-p_{t+1})$ results in a deflation multiplier of θ'^{-1}.[47] Similarly, feedback *across* markets is not destabilising either: although the asset price decline and future deflation reinforce each other, combining AM$_t$ and GM$_{t+1}$ shows that

$$\delta_t = \frac{1-\theta'}{2-\theta'}\left[\delta_t + ...\right].$$

As this coefficient lies below $1/2$, feedback remains stable across markets. Finally, note that feedbacks within and across markets fall over time.[48] These results, taken together, show that the system remains stable in the no-default region.

We can now represent the solution by combining goods and asset markets in the $\{\delta_t, p_{t+1}\}$ plane,[49]

$$\begin{cases} \text{AM:} \quad \delta_t = \frac{\theta'}{(2-\theta')qh}\left[\rho + (1-p_{t+1})\,y/R\right] \\ \text{GM:} \quad p_{t+1} = \left[1 - \frac{R}{s}\left(qh\delta_t - \rho\right)\right]^{\frac{\gamma}{\gamma-1}}. \end{cases} \tag{31}$$

Each locus represents combinations of δ_t and p_{t+1} such that asset and goods markets are in equilibrium, as shown in figure 3. The AM-locus has a negative slope: the lower the price level, the greater the asset price decline – deflation depresses the asset price. The GM-locus, provided $\gamma < 1$, is increasing and convex. The thick lines, drawn for $\rho = 0$, intersect at the steady state, $\delta_t = 0$ and

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46 The coefficient θ' equals $\frac{\theta}{1+(1-\theta)R}$ (see appendix A1), and reaches its maximum of 1 as θ reaches 1.

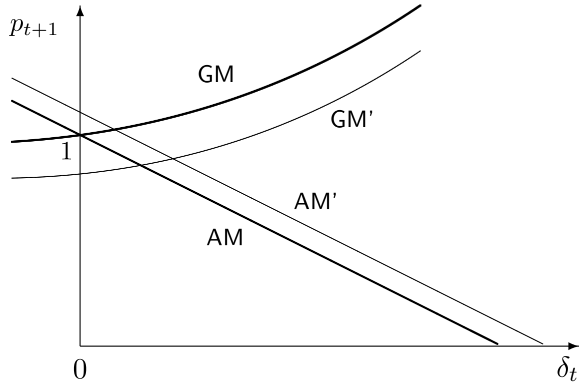
47 The infinite multiplier in GM$_t$ does not induce instability for reasons explained following (28).

48 This happens for two reasons. First, reoptimization allows agents to spread their losses across markets. For example, deflation feedback fall from 1 in GM$_t$ to $(1-\theta')$ in GM$_{t+1}$. Second, from $t+1$ onward no unexpected losses arise anymore. The complete system is shown in appendix B1.

49 We have used the fact that the equilibrium values (27) simplify the budget constraint (26) to $(s - s_t) + (s - s_{t+1})/R = -\rho$. This relation allows to replace GM$_{t+1}$ by (28).

$p_{t+1} = 1$. This illustrates proposition 3(a) that a zero shock leaves the economy in steady state.[50]

Figure 3: Stable Debt-Deflation



A positive shock $\rho > 0$ shifts the loci to the positions indicated by thin lines. AM shifts out to AM', because mid-age firms paying ρ spend less on assets, for any given p_{t+1}. GM shifts down, because the price level falls to attract households' extra spending of ρ.[51] The intersection of thin curves shows the new equilibrium, with a small asset price decline and a lower future price level. Substituting AM into GM delivers the implicit solution $p_{t+1}(\rho)$,

$$p_{t+1} = \Big[1 + (1-m)\, R\rho/s - m\,(1 - p_{t+1})\, y/s\Big]^{\frac{\gamma}{\gamma-1}} \tag{32}$$

where $m \equiv \theta' / (2 - \theta')$ simplifies notation. A zero shock leaves the price level unchanged, $p_{t+1}(0) = 1$. The greater the shock, the lower the price level, $p'_{t+1}(\rho) < 0$, for any $\gamma > 0$ (appendix B2 shows the derivation and explicit solutions). From (31) we obtain

$$\delta'_t(\rho) = \frac{m}{qh}\Big[1 + \underbrace{\left|p'_{t+1}(\rho)\right| y/R}_{\text{General Equil.}}\Big] > 0.$$

The direct effect of the shock ρ is to produce an asset price decline of $m\rho/(qh)$. This decline is reinforced by a general equilibrium effect: future deflation hurts mid-age firms who spend less on assets now (Fisher-Minsky). The asset price decline in turn causes a wealth effect in (29), producing deflation in t,

$$p'_t(\rho) = \frac{\gamma p_t^{1/\gamma}}{\gamma - 1}\frac{qh}{s}\delta'(\rho) < 0.$$

Therefore, starting from the steady state ($\rho = 0$), prices fall as the shock increases, $p'_t(\rho)$, $p'_{t+1}(\rho)$, and $q'_t(\rho) < 0$. As a result, firms' losses Ω and Ω' increase monotonically in ρ, and their balance sheets progressively deteriorate. This is the result of several channels of debt-deflation, none of which is destabilising. We now establish that the presence of default leaves this conclusion unaffected.

50 If $\gamma > 1$ instead, then the GM-locus would slope downward. This admits the possibility of self-fulfilling declines in asset and goods prices, where $\delta_t > 0$ and $p_{t+1} < 1$ are consistent with each other even as $\rho = 0$. See appendix B2 for detail.

51 Extra household spending is consistent with aggregate demand below steady state, because a low price level implies reduced profits for, and less spending by, firms in $t + 1$.

Role of default. Perhaps surprisingly, default per se does not destabilise the system: defaulting borrowers no longer spend, hence they cannot *further* reduce their spending when prices continue to fall. In other words, the default of a class of agents removes their reactions from the markets in which they participate, reducing feedback within and across markets. For instance, if old firms default, they spend nothing on goods. Hence GM_t of (27) only consists of household spending,

$$\mathsf{GM}_t \qquad p_t y = s_t$$

Default eliminates both the feedbacks of Minsky$_2$ and Fisher-Tobin in t.[52] Combining GM_t with the Euler equation implies that p_t reaches the floor

$$\underline{p} = \bar{\varepsilon}^{\gamma} = [\varepsilon + (1-\varepsilon)\,\theta]^{\gamma} < 1 \ \text{ for } \gamma > 0.$$

Note that there is no deflation, in spite of default, when IES is infinite ($\gamma = 0$). The less responsive household spending (the greater γ), the more the equilibrium price level must fall to attract spending. The other two market conditions, AM_t and GM_{t+1}, remain unchanged, and jointly determine $\{\delta_t, p_{t+1}\}$ as in (31).

Similar conclusions hold for the default of mid-age firms, which removes all the feedbacks emanating from them. They no longer spend on consumption in GM_{t+1}, nor on assets in AM_t. Correspondingly, $p_{t+1} = \underline{p}$, and AM_t in (27) becomes

$$\bar{\delta} = \frac{R-1}{2R}. \tag{33}$$

Finally, should both old *and* mid-age firms default, then p_t, p_{t+1} and δ_t are *all* constants. Although at a low level, prices stabilize upon default. More generally, in a model containing more types of agents, firms' default can be shown to *slow down* the price decline, which will come to a halt only when all borrowers on the demand side default. The more agents default, the fewer and weaker the feedbacks in the system.[53] In conclusion, the system also remains stable even when wide-spread default is taken into account – default *per se* is not destabilising. This qualifier distinguishes default from its side-effects which *can* be destabilising, as illustrated by Bernanke's credit channel (section 5.2).

Role of indebtedness. Higher indebtedness means that agents default more readily, following smaller shocks. But stability is preserved since default per se is not destabilising. More precisely, firms purchase assets on credit, and the resulting indebtedness can be measured by asset preference θ in (17). The system remains stable even if firms dedicated all revenue to holding assets with maximum leverage $R^2/\left(R^2-1\right)$. This is confirmed by using $\theta = 1$, hence $\theta' = 1$, throughout this section. For example, greater indebtedness implies a greater impact on the asset market: the multiplier in (30)

[52] Formally, $qh\delta_t + (1-p_t)\,y$ in (27) is replaced by a *constant*, $pc = (1-\theta)\,\Pi$. Firms' reduction in spending equals their entire steady state spending.

[53] An earlier version contained a general approach for assessing stability based on the analogy with *input-output tables*: the 'inputs' are the price deviations on the right of (27), including default terms; the outputs are the resulting price changes on the left. Hence the coefficient matrix collects the feedbacks within markets (diagonal) and across markets (off-diagonal), and the *Leontief inverse* yields the corresponding multipliers. While positive feedbacks imply that multipliers exceeding unity, they do not imply system instability. The zeros placed by defaults make the feedback matrix become sparse, and the Leontief inverse approaches the identity matrix, indicating greater stability.

increases, but it remains bounded by 2. Greater indebtedness also reduces firms' ability to withstand losses. Old firms with $\theta = 1$, for instance, would default following any positive loss, since the buffer for absorbing, planned consumption $(1 - \theta)\,\Pi$, would be zero. But again, a greater propensity to default does not destabilise the system when default itself does not.

Role of the shock. Having compared the systems with and without default, it remains to determine the default point. The size of the shock at which mid-age firms default, ρ', is given by[54]

$$\rho' + (1 - \bar{\varepsilon}^{\gamma})\, y/R = uh\left(2 - \theta'\right)/\theta'.$$

It takes a greater ρ' to bankrupt firms when deflation is less pronounced, and the second term disappears completely at the fixed-price limit ($\gamma = 0$).[55] Once mid-age firms default, prices remain at their minima $\underline{p}$ and $\underline{q} \equiv \left(1 - \bar{\delta}\right) q$ for any $\rho \geq \rho'$, because ρ' is the largest feasible payment from firms to households net of loan losses.[56] Therefore, the conclusion that the system remains stable holds for shocks of any size.

To conclude we return to proposition 3. We have shown that the steady state continues unless there is some positive shock. This favours the interpretation of debt-deflation as a *propagation* mechanism, rather than as a fully endogenous phenomenon.[57] Goods and asset prices are decreasing in the size of this shock. This was found to be the result of several active channels of debt-deflation, yet the deviations they produce within and across markets do not amplify each other to the point of producing instability. Indeed, under the optimal policy of user cost spending the system remains *globally stable*. Greater shocks and greater indebtedness produce greater distress selling and wide-spread default. But default per se is not destabilizing in our model without frictions and uncertainty. Also, there are no independent effects from money or credit to the goods and asset markets. Hence the channels of Fisher, Tobin and Minsky are not destabilising, and that of Bernanke does not exist in our model. The economy remains surprisingly resilient to shocks, even with wide-spread default. The predictions of unstable debt-deflation are not borne out under the present assumptions. A useful perspective on these results can be obtained by examining directly a centrepiece of debt-deflation: distress selling.

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54 This threshold is found by equating the loss Ω' with the ability to withstand losses, $pc/R + uh$, or, equivalently, by equating the two expressions for the asset price decline: (33), and (30) evaluated at $\underline{p}$.

55 Conversely, even as $\rho' = 0$, it is still possible that firms default following strong deflation for unfavourable parameters (including $\gamma >> 1$).

56 Any payment exceeding ρ' leads mid-age firms to default; although they will hold no assets, their goods in transit will be sold next period for $\underline{p}y$, unless the payment ρ is not effected. It is in the interest of the bank to finance any payment $\rho < \rho' + \underline{p}y$, as doing so maximizes loss-given-default, $\lambda' = (\rho - \rho')$. Thus when households receive a payment $\rho > \rho'$, the bank also passes through loan losses λ', limiting the effective payment to households to $\rho - \lambda' = \rho'$.

57 See the discussion in section 1.4. Of course we concede that the model does not capture important qualitative features of Minsky's financial instability hypothesis, namely financial innovation, fundamental uncertainty, and the endogenous evolution of the debt structure. It remains unclear how (or even whether) these features can be modelled, see Foley (2001).

4.3 Distress selling versus refinancing

We now quantify distress selling by mid-age firms. The purpose of distress selling is to prevent consumption from bearing all the burden of adjustment. This is why the loss is shared, between reduced spending on consumption *and* assets, under the optimal policy (21). Mapping user cost spending $u_t h_t'$ into distress selling yields[58]

$$q_t\left(h-h_t'\right)=\frac{1-\delta_t}{\frac{R-1}{R}-\delta_t}\left[\theta'\Omega'-qh\delta_t\right].$$

This is the distress selling function for an arbitrary δ_t. But distress selling in turn determines the equilibrium asset price. Using AM$_t$ in (27) to replace $\theta'\Omega'=2qh\delta_t$, the value of distress selling equals

$$q_t\left(h-h_t'\right)=\frac{qh\delta_t\left(1-\delta_t\right)}{\frac{R-1}{R}-\delta_t}\qquad\begin{cases}=0 & \text{if}\quad \rho=0\\ >\rho & \text{if}\quad \rho>0.\end{cases}\tag{34}$$

When $\delta_t=0$, the expression equals 0. (No distress selling occurs in steady state as $\rho=0$.) When $\delta_t=\overline{\delta}$, it equals $\left(1-\overline{\delta}\right)qh$, the remaining value of total asset holdings at the default price. In between those extremes, the relation is positive: the greater distress selling, the greater the asset price decline, and vice versa. The exact dependence on ρ can be obtained by replacing δ_t with (31). In equilibrium, distress selling is unambiguously positive whenever the shock is, and it generally exceeds ρ.[59] This shows that firms sell assets not merely to cover their liquidity needs ρ, but also to reduce their nominal debt. Nonetheless, our earlier finding that debt-deflation remains stable is reflected here in the fact that distress selling does not destabilise the asset market.

The refinancing equilibrium. So far, we have studied the equilibrium resulting from user cost spending. We now compare this to the equilibrium resulting from refinancing (described on page 19). The difference can then be attributed to distress selling.

As mid-age firms refinance their holdings, the asset market comprises old firms selling h assets to young firms. Since $p_{t+2}=1$ as in (27), assets turn over at the steady state price $q_t=q$, consistent with $\delta_t=0$ in (34). Goods market clearing in $t+1$ now requires $s_{t+1}=s+R\rho$.[60] Therefore, refinancing results in the following equilibrium prices,

$$\begin{aligned}\delta_t&=0\\ p_t&=1\\ p_{t+1}^{ref}&=\left(1+\frac{R}{s}\rho\right)^{\frac{\gamma}{\gamma-1}}.\end{aligned}\tag{35}$$

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58 We use the definition $\delta_t\equiv\left(q-q_t\right)/q$ and $u_t=q_t-q_{t+1}/R$, where $q_{t+1}=q$ is known from previous results.

59 To show $q_t\left(h-h_t'\right)-\rho>0$, use (34) and multiply across $\left(R-1\right)/R-\delta_t$. The expression is increasing in δ_t. From (30) we know that $\delta_t\geq\frac{\theta'\rho}{(2-\theta')qh}$ for any $\gamma<1$. Cancelling terms, one finds the sufficient (not necessary) condition that $\theta'/\left(2-\theta'\right)\geq\left(R-1\right)/R$, which translates into $\theta>2(R^2-1)/\left(2R^2-1\right)$. This is a mild restriction on θ (e.g. when $R=1.05$, $\theta=0.17$.)

60 Adapting GM$_{t+1}$ in (27) for refinancing involves setting $\theta'=0$ and $\delta_t=0$.

Refinancing effectively shields the asset market from the shock ρ. There is no asset price decline, no wealth effect, and thus no deflation in t. In this sense, refinancing prevents debt-deflation in t. This provides some support to Minsky's contention that debt-deflation does not develop when agents refinance (see p. 9). But refinancing means to continue financing assets. What is not saved on assets now must be saved on goods next period. Thus refinancing delays the impact of the shock, concentrating it on the goods market in $t+1$. The interest cost of increased borrowing $R\rho$ reduces final period consumption, producing more future deflation than would otherwise be the case, $p_{t+1}^{ref} < p_{t+1}(\rho)$ in (31).[61]

Comparing the general equilibrium consequences of refinancing and distress selling suggests several observations. Distress selling spreads the impact of a redistributive shock across markets, causing prices to fall already in t. Doing so imposes a *negative externality* on other asset holders: old firms would face no losses in the refinancing scenario. Their equilibrium loss Ω in (19) can therefore be attributed to mid-age firms' distress selling and the resulting deflation in t. An equivalent way to see this point is to express old firms' financial condition in real terms. The last line of (3) reads

$$\frac{R^2qh + w - \lambda}{p_t} + c_t = y + \frac{q_t h}{p_t}. \tag{36}$$

This expression shows clearly how the fall in p_t raises the burden of debt and wage liabilities (left), compared to the ability to pay (right).[62] If old firms fail, it is ultimately because they are forced to sell at depressed prices, p_t and q_t, only because *other* firms decided to engage in distress selling. Incidentally, note that knowing the real asset price in (36) is insufficient when the issue is the repayment of nominal debt. This is why we conducted the analysis in nominal terms.

As a final observation, since distress selling affects asset prices, it must also have implications for money and credit.

4.4 Money, credit, and asset prices

Recall from (18) that the debt of mid-age firms evolves as follows,

$$B_t' = RB + \rho - q_t(h - h_t').$$

Under refinancing, firms borrow ρ to effect the necessary payment without selling assets ($h_t' = h$). Their debt thus increases to $B_t' = RB + \rho$. Meanwhile, young firms borrow $B = qh$, as in steady state, since $\delta_t = 0$ in (35). The banking system therefore expands to $B_t + B_t' = (1+R)qh + \rho$. This expansion by ρ, relative to steady state (16), is of course consistent with households leaving the incoming payment ρ on deposit.[63] Thus the banking system must expand to support the refinancing

[61] Refinancing would maintain *all* prices at their steady state values only in the case of a pure liquidity shock. If ρ in t were reversed by a repayment of $R\rho$ in $t+1$, the present value of the loss is zero.

[62] The term λ represents non-performing loans and prevents negative consumption when debt exceed the ability to repay, see (44) in appendix A1.

[63] What encourages households to save their payment until $t+1$ is a higher real interest rate between t and $t+1$, namely $r_t = R\frac{p_t}{p_{t+1}} > R$, using (35).

of the asset market; and deposits, in equilibrium, exactly match the increase in lending.

Fisher's debt-deflation can be thought as a story of why such refinancing will not happen: borrowers do not wish to take on more debt, but engage in distress selling to repay debt and, in doing so, they cause a deflationary contraction of bank deposits (page 4). The present analysis largely supports this view, up to the conclusion.

It is true that we found distress selling to be an equilibrium phenomenon under optimal user cost spending. It is also true that distress selling is used for repaying debt: by virtue of (34), mid-age firms reduce their debt to $B_t' < RB$. But whereas mid-age firms borrow less, young firms borrow more.[64] Just as the repayment of old loans contracts deposits, the creation of new loans expands deposits. The banking system in fact expands. Using (18) in (10) shows that the value of bank assets equals

$$(1+R)qh + (\rho - qh\delta_t).$$

This is, of course, identical to the value of deposits which, using (28), equals

$$D_t = RD + \rho - s_t = D + (\rho - qh\delta_t).$$

Both sides of the balance sheet indicate that the size of the banking system, relative to steady state (15), increases by $\rho - qh\delta_t$.[65] This conclusion remains unaffected by borrower default: the banking system still expands, even after writing off non-performing loans (see page 34).

These findings cast some doubt on Fisher's conclusion, whereby the contraction of money, brought about by distress selling, causes a fall in the price level. First, no monetary contraction takes place when new borrowing exceeds the repayment by distress-selling agents. Second, and more importantly, even if a monetary contraction took place, it would *not* be the cause of deflation here. In our frictionless environment, credit is elastic and money causally last. The quantity of money (deposits) depends on s_t, which is determined by the equilibrium asset price decline. It is the price of assets, rather than the quantity of money, that drives debt-deflation. In this respect the model lends less support to Fisher's than to Minsky's channel.

We collect the above findings in

Proposition 4 Distress selling and monetary implications

(a) Distress selling versus refinancing:
- *Distress selling exceeds liquidity needs and reduces firms' nominal debt.*
- *Distress selling imposes losses on other asset holders, by spreading the impact of the shock.*
- *Refinancing, by contrast, would prevent debt-deflation in t.*

(b) Money, credit, and asset prices:
- *Young firms increase their borrowing more than mid-age firms reduce theirs.*
- *The banking system expands, and the asset price decline remains small.*
- *The price of assets, rather than the quantity of money, drives debt-deflation.*

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64 Young firms spend $u_t h_t = uh$, hence $q_t h_t = \frac{q_t}{u_t} uh$, which exceeds qh since $\frac{q_t}{u_t} = \frac{R}{R - q/q_t} > (R-1)/R \equiv q/u$.

65 This difference is positive since we found that $p_{t+1} < 1$ in (31).

Finally, note that the expansion of the banking system also means that the asset price decline remains very modest. This is *not* due to our assumption of a perfect asset market.[66] Instead, it is the policy of user cost spending that sustains both the level of asset prices and the size of the banking system. User cost, as a dynamic concept, incorporates the fact that every discount comes with as much expected appreciation. It is agents' willingness to get more indebted and buy assets at a discount that keeps this discount so small. Indeed, for user cost to remain positive, the asset price decline cannot exceed $(R-1)/R$, and it does not exceed $\overline{\delta} = (R-1)/(2R)$ in the present context; and since the asset price decline is modest, so is the wealth effect and deflation.

5 Unstable debt-deflation

The previous section concluded that debt-deflation remains stable under the optimal policy of user cost spending, as agents are able and willing to borrow and exploit the expected asset price recovery. In this section we demonstrate that debt-deflation can become unstable when agents are unable or unwilling to follow this course. First, in the spirit of Fisher, we consider what happens when agents actively contain their indebtedness (see page 4). Second, in the spirit of Bernanke, we consider what happens when banks, following borrower default, actively reduce bank lending (page 7).

5.1 Containing indebtedness

We defined indebtedness as debt relative to net worth in (17), and assume that firms observe this level, for instance as a result of margin requirements (see page 37). Following the shock, firms become over-indebted at the existing level of nominal debt. Net worth falls by the loss Ω', and real indebtedness increases accordingly, unless nominal debt is reduced. As was shown in (24), keeping real indebtedness constant calls for distress selling worth a multiple of the loss. We now consider the general equilibrium implications of this policy. This is best done in two steps, to illustrate how far distress selling can drive the asset price.

(a) Suppose first that *only* mid-age firms, those directly affected by the shock, seek to contain their indebtedness.[67] Distress selling, $(h - h'_t) = (\rho + L\Omega')/q_t$, must be absorbed by young firms' demand, $h_t = uh\, p_{t+2}/u_t$. Since equilibrium requires $h_t + h'_t = 2h$, one obtains

$$\text{AM:} \quad \frac{u}{u_t} p_{t+2} = 1 + \frac{\rho + L\Omega'}{q_t h.}$$

While the value of distress selling can be substantial, it cannot exceed total holdings $q_t h$. The last term reaches 1 when mid-age firms sell out. Even so, user cost falls at most to half its steady

[66] A previous version illustrated this claim with two examples. The first, in the spirit of Shleifer and Vishny (1992), assumed that assets are not fully redeployable (e.g. residential versus commercial property). The second example mimicked the absence of unconstrained investors by removing the young generation. In neither case was the asset price decline substantially greater.

[67] Meanwhile, young firms follow the optimal policy of user cost, derived in appendix A1.

state value,[68]

$$u_t \geq u/2 \qquad \Rightarrow \qquad \delta_t \leq \frac{R-1}{2R}. \tag{37}$$

To absorb all assets of defaulting firms, it suffices to halve the user cost to motivate young firms to buy twice as many assets as in steady state. This can be achieved with a very small asset price decline, such that the subsequent appreciation covers half the interest cost of debt. Again, even wide-spread distress selling hardly reduces the asset price when young firms follow their policy of user cost spending (as on page 26). In fact, the presence of a young generation, without any prior debt or losses, is like assuming that there are investors with unlimited access to funds, ready to arbitrage any opportunities. As before, young firms are able and willing to borrow and exploit the expected asset price recovery, which was shown to stabilise the market.

(b) Suppose now that *all* firms seek to contain their indebtedness. This replaces user cost spending by the sort of reaction Fisher had in mind. Young firms' keeping their indebtedness equal to (17) means that their debt cannot exceed a certain value, irrespective of the saving afforded by future appreciation. They now spend a fixed amount on assets, rather than on user cost (appendix A3),

$$q_t h_t = qh\, p_{t+2}.$$

The asset market clears when the price decline produced by distress selling attracts an equal measure of extra demand from young firms,

$$\begin{aligned} \text{AM:} \qquad qh\left[\delta_t - (1-p_{t+2})\right] &= \rho + L\Omega' \\ &= \rho + L\big[qh\delta_t + \rho + (1-p_{t+1})\, y/R\big]. \end{aligned} \tag{38}$$

The coefficient on δ_t equals $qh\,(1-L)$. According to its sign, two cases arise.

Low indebtedness $(L < 1)$**.** Low indebtedness in (17) would follow from a very low preference for assets in (2), namely $\theta < \left(R^2-1\right)/R^2$. In this case, all previous results remain qualitatively unchanged. A zero shock ($\rho = 0$) requires no distress selling, and the continued steady state remains the only equilibrium, as in proposition 3.[69] A small positive shock leads to an asset price decline of

$$\delta_t = \frac{L}{(1-L)\,qh}\big[\rho + (1-p_{t+1})\, y/R\big] + z,$$

where z is a residual.[70] Comparing with the earlier expression (30) indicates stronger feedback, as the multiplier is greater here than under user cost spending $(L > \theta > \theta')$. Nonetheless, with low indebtedness, the response of the asset price and, consequently that of price levels, remains stable as in section 4. The difference is mainly quantitative: for a given ρ, asset and goods prices deviate more from the steady state than was the case under user cost spending.

[68] As appendix B2 remains valid, the price level still reverts to $p_{t+2} = 1$, thus the asset price reverts to $q_{t+1} = q$. Using $q_t - q/R = (q - q/R)/2$ then yields the last expression.

[69] This can be shown as follows: assuming $p_{t+1} < 1$ implies $\delta_t > 0$. This leads to $p_t < 1$ and, through the intertemporal budget constraint $p_{t+1} > 1$, contradicting the initial assumption. The only solution is $\delta_t = 0$ and $p_t = p_{t+1} = 1$.

[70] The residual equals $z \equiv \left[\rho/(qh) + (1-p_{t+2})\right]/(1-L)$.

High indebtedness $(L > 1)$**.** Major qualitative differences arise when indebtedness exceeds unity, as is normally the case (θ exceeds $\left(R^2 - 1\right)/R^2$ in most of its range). The negative coefficient $qh\,(1 - L)$ means that a falling asset price itself is generating more distress selling than attracting new spending. Not even the losses arising from δ_t can be covered, much less the exogenous loss ρ. Distress selling now *destabilises* the market, and any positive shock ρ triggers the unstable debt-deflation process Fisher and Minsky envisioned.[71]

One can imagine the dynamics as follows. In steady state, with a zero shock, no distress selling is necessary and there is no asset price decline. Now mid-age firms get hit by the shock ρ, which reduces their net worth by as much at current prices. To keep their *debt* constant, they must raise ρ through distress selling. But they are still over-indebted, since their net worth has fallen. To keep their *indebtedness* constant, they must raise an additional $L\rho$. If this were of no consequence to equilibrium prices, the process would stop right there, since the balance between debt and net worth would be restored.

At this point the general equilibrium effects set in. As an aggregate shock, ρ affects all mid-age firms. Their distress selling, however small, reduces the asset price, as was the case in (34). All asset holders now realise that $\delta_t > 0$ reduces their net worth and leaves them all over-indebted. To contain their indebtedness, they aggressively sell assets to repay debt. But in aggregate this attempt is self-defeating, as raising L times the asset market loss that resulted from the initial sales will further depress prices. At the same time, falling asset prices cause wealth effects on the goods markets, and deflation adds further losses and further distress selling.[72]

In sum, firms sell assets so aggressively that, in aggregate, prices and net worth fall faster than the repayment that distress selling was meant to achieve. The unstable process comes to a halt only when the whole stock of assets has been sold in distress. This happens when the right-hand side of (38) reaches $q_t h$, hence

$$q_t = \frac{q}{2} p_{t+2} \qquad \Rightarrow \qquad \delta_t = \left(1 - p_{t+2}/2\right). \tag{39}$$

It remains to determine the future price level (see appendix B3). It equals

$$p_{t+2} = \left[1 + \left(1 - \frac{q_{t+2}}{q_t}\right)\frac{qh}{s}\right]^{-\gamma}. \tag{40}$$

The asset price decline (39) is substantial, close to 50% when $p_{t+2} \to 1$ (when $\gamma \to 0$). Under user cost spending, it was sufficient to halve the *user cost* (37) to motivate young firms to buy up all the assets. When containing indebtedness, however, it becomes necessary to halve the *asset price* (39) to allow them to do so. This large a decline invariably leads to wide-spread default among firms, for most $\theta \in [0, 1]$.

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[71] With reference to figure 1, the destabilising channel is the asset market feedback of Minsky$_1$, and deflation is produced through the channel of Minsky$_2$ (see figure 1). This is reinforced by deflation, through the channels of Fisher-Tobin. The channels are the same as before, and so are the sources of losses. The only change we have made is to replace the optimal user cost spending by a policy of constant indebtedness in the spirit of Fisher.

[72] As the asset price keeps falling, the wealth effects on aggregate demand grow larger, and produce more and more deflation until p_t and p_{t+1} hit their lower bound $\underline{p} = \bar{\varepsilon}^{\gamma}$.

Fisher's and Minsky's main predictions of section 1 now materialise: the more the economic boat tips, the more it tends to tip – the system *does* amplify deviations to the point of instability. Distress selling is self-defeating, and a recursive debt-deflation process ensues, ending in almost universal bankruptcy. This stands in sharp contrast to previous results where debt-deflation remained stable even in the presence of large shocks, high indebtedness, and wide-spread default.

5.2 Impaired intermediation

Bernanke's credit channel has remained inactive so far, because our elastic credit specification has the banking system accommodate credit demand. (With neither agency problems nor uncertainty, the banking system is willing to lend in line with firms' intertemporal budget constraints. It is also able to do so since neither reserve, nor settlement, nor capital constraints are imposed here.) So far, even wide-spread default did not produce any feedback from the banking system to the economy. We now consider what happens when the banking system actively reduces bank lending, following borrower default.

Credit expansion. We first observe that the size of the banking system would normally increase following the shock. This can be shown using the bank balance sheet identity. Bank assets, given by (10) and (18), are diminished by loan losses of mid-age firms λ' (see page 21).[73] Equivalently, bank deposits are diminished by loan losses and household spending,

$$(1+R)\,qh + \rho - qh\delta - \lambda' \equiv RD + \rho - \lambda^{+} - s_t. \tag{41}$$

where $D = (1+R)\,qh$ is the size of the banking system in steady state (16). Both sides show that the banking system expands by

$$\rho - qh\delta_t - \lambda' > 0.$$

This expression was shown to be positive, absent default (page 30). Writing off loan losses merely moderates the expansion to $(\bar{\varepsilon}^{\gamma} - \bar{\varepsilon})\,y/R > 0$.[74] The expansion remains positive because the maximum asset price decline, $\bar{\delta}$, was found to be small under user cost spending. Thus elastic credit leads to a natural, unconstrained size of the banking system greater than in steady state.

Credit contraction. We now allow for the possibility that bank credit becomes less accommodative following borrower defaults. As emphasised by Bernanke (1983), borrower insolvencies impair the channels of credit intermediation (page 7). While there is no role for screening and monitoring in our model, we can replicate Bernanke's channel by assuming that loan losses lead to a credit contraction or an equivalent increase in the loan rate. As this mechanism was developed in detail in von Peter (2004), we only sketch the aspects most relevant for debt-deflation.

Suppose the banking system reduces the supply of credit, while positive, by a coefficient κ times the

[73] The balance sheet at time t records lending to young and mid-age firms. Old firms have left the model and are no longer recorded (see page 15). Their loan losses, if positive, are already accounted for.

[74] The expression is valid whether or not old firms default. It is found by using the definition of λ' in (46), steady state parameter relations, and the equilibrium default values $\bar{\delta}$ and $\underline{p}$ (page 26).

loan losses it suffers,

$$\text{Credit Supply:} \qquad (1+R)\,qh - \kappa\lambda^{+}.$$

Passively writing off loan losses corresponds to $\kappa = 1$, while actively reducing bank credit involves $\kappa > 1$. Note that this new constraint is relevant only if firms default.[75] Once this happens, the constraint starts binding for any $\kappa > 0$, since credit demand at R was shown to exceed $(1+R)qh$ in (41). The credit contraction will therefore raise the loan rate, $R_t > R$, bringing credit demand down to reduced credit supply.[76] Moreover, if credit is constrained, so are asset prices. The contraction will therefore drive the asset price decline beyond $\overline{\delta}$.

It becomes apparent that this form of impaired bank intermediation can produce unstable debt-deflation. Equating credit demand (41) with constrained credit supply, one obtains an expression for asset prices,

$$qh\delta - \rho = \kappa\lambda + (\kappa - 1)\,\lambda'.$$

Loan losses λ or λ' are positive only if old or mid-age firms default. If so, then λ and λ' contain the term $qh\delta$, since firms' losses on assets contribute to loan losses. The asset price is no longer determined by a forward-looking asset pricing equation, but constrained by current credit availability,

$$\text{If only mid-age firms default:} \qquad \delta = \ (\kappa - 1)\,\delta + ...$$

$$\text{If both types of firms default:} \qquad \delta = \ (2\kappa - 1)\,\delta + ...$$

This equation can be compared to the earlier expressions in the stable case (27), and the unstable case (38). Here, the asset price decline appears on the right-hand side because of Bernanke's, not Minsky's, channel: δ causes losses to firms, hence loan losses to banks - this disrupts credit supply, which in turn accelerates the asset price decline.

This feedback from the banking system destabilises the asset market when the loss coefficient κ exceeds a threshold. If only mid-age firms default, writing off λ' already contributes 1 to κ, and $\kappa > 2$ is sufficient for instability. When old firms also default, instability already ensues with $\kappa > 1$. As a rule of thumb, instability ensues when κ exceeds a passive write-off by one (see footnote 73). Credit supply then systematically falls short of credit demand until both collapse to zero ($\delta = 1$). The unstable debt-deflation process again involves falling asset prices, increasing losses, and deflationary wealth effects on the goods market, as described in section 5.1.[77] The main difference is that credit contraction, rather than distress selling, now destabilises the asset market.

While proposition 3(b) showed that default *per se* is not destabilising, this no longer holds when default affects credit intermediation. The problems of credit intermediation that Bernanke has in mind, or other constraints, may well place κ above the instability threshold. In von Peter (2004)

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[75] The channel becomes active only when default takes place, after the "margins of safety" have been exhausted. This may be contrasted with Fisher and Minsky's channel (section 5.1), or with models where financial frictions drive up the borrowing rate as soon as net worth falls (e.g. Bernanke et al 1999).

[76] For a given spending on user cost in (6) and (21), a higher loan rate translates into a smaller loan.

[77] Although Bernanke considered the effect on aggregate demand, the credit channel here affects asset prices, because assets rather than goods are purchased on bank credit, as emphasised by Keynes (section 1.3).

we show that this is indeed the case when banks choose, or capital adequacy forces them to keep their leverage constant; the coefficient κ then equals the inverse of bank leverage, which exceeds the stability threshold.

The results of this section are summarised in

Proposition 5 Unstable debt-deflation

(a) *When borrowers contain their indebtedness:*
- *Debt-deflation becomes unstable if indebtedness exceeds the threshold $L = 1$.*
- *Distress selling destabilises the asset market.*
- *The process stops when asset prices reach about half their steady state value.*

(b) *When default impairs credit intermediation:*
- *Debt-deflation becomes unstable if the loss coefficient exceeds the threshold $\kappa = 2$.*
- *The credit contraction destabilises the asset market.*
- *The process stops when asset prices and credit collapse to zero.*

The two instances of unstable debt-deflation share some features. Both can be thought of as interfering with optimal user cost spending, since agents do not or cannot take on more debt. In both instances the asset market is liable to instability, and warrants a central place in the debt-deflation process. Money, credit and asset prices contract in line with each other. (Figure 1 purposely aligns them at the same height.) In both cases it is the price of assets, rather than the quantity of money, that drives debt-deflation; the price of assets was driven by distress selling in the first case, and by a credit contraction in the second.

6 Discussion

The stylised nature of our model of debt-deflation raises some issues worth mentioning. The discussion addresses stability, modelling assumptions, and the relevance of debt-deflation for today.

6.1 Stability and indebtedness

The main results of the paper can be restated in terms of three possible reactions to losses. Under refinancing, debt-deflation does not develop as agents refrain from distress selling. Under user cost spending, debt-deflation remains stable, because agents are willing to borrow and exploit the expected recovery, which stabilises asset prices. Under constant indebtedness, however, debt-deflation can become unstable. A credit crunch similarly constrains the indebtedness agents can incur.

But the optimal policy was shown to be user cost spending, with the property that agents are not concerned with debt *per se* (remarks on page 12). Rather than an explicit reaction to debt, distress selling in (34) was the result of allocating losses between spending on goods and assets. A much greater amount of distress selling arose under constant indebtedness. This raises the question: why would agents choose to contain their indebtedness?

One reason could be uncertainty, as discussed below. Another reason could be regulation, for instance margin requirements. Margin is the fraction a customer must contribute in own funds toward an asset position. The amount purchased on credit is thereby limited to a fraction of the value of assets.[78] Margin requirements therefore enforce constant indebtedness (leverage). Indeed, Fisher's original exposition already emphasised margin accounts, and he observed that falling stock prices made (callable) brokers' loans almost disappear (Fisher 1932, p. 87ff). Accordingly, our results on distress selling can be interpreted in terms of margin calls, and our notion of unstable debt-deflation can be compared with what has, or could have, happened in actual market crashes as 1929 and 1987.[79]

This interpretation suggests a policy lesson. Indebtedness is as much a blessing as a curse: greater indebtedness produces more distress selling, but it also enlarges the ability of other agents to absorb the extra supply, which was shown to be stabilising. Margin requirements can jeopardise this mechanism in a falling market. The theory suggests three cases. First, in a world of optimal user cost spending, introducing margin requirements could do more harm than good, because they force agents to contain their indebtedness which was shown to be destabilising. By contrast, a world where agents curb their indebtedness for some other reason is already potentially unstable. In that case, introducing margin requirements ex ante may mitigate the inherent instability of the system, provided they are strict enough to maintain initial indebtedness below the critical threshold. Finally, imposing or tightening requirements at the time of a crisis fails on both counts: it does not prevent high indebtedness ex ante, but constrains the increase in borrowing necessary for absorbing distress sales ex post.

6.2 Welfare, real effects, and extensions

The analysis was conducted in the context of an overlapping generations model. This structure implies active markets and makes dealing with losses and default tractable. But it also reduces persistence artificially, since affected agents eventually exit the model. The corresponding arrival of young agents is in fact one of the main reasons for the stability of the model: the presence of a young generation, without any prior debt or losses, is like assuming investors with unlimited access to funds.[80] While considered good practice in asset pricing theory, the presence of such investors tilts our results in favour of stability.[81]

Perhaps more contentious is our choice to illustrate debt-deflation in a frictionless model with flexible prices. We opted for price flexibility because falling prices are at the very heart of debt-deflation. But

[78] In the housing market, the corresponding notions are downpayment and loan-to-value.

[79] These analogies are worked out in von Peter (2004b). Aiyagari and Gertler (1999) show that margin requirements can explain overshooting of asset prices, but they do not consider instability.

[80] Recall that credit is provided elastically, except in section 5.2. Margin requirements were found to be destabilising because they restricted young firms' access to credit.

[81] We did, however, assume infinitely-lived households. (They can equivalently be thought of as overlapping generations with an operative bequest motive). Households would otherwise spend all deposits upfront; a long horizon helps maintain the large volume of credit, deposits and intermediation. That households do not buy assets is a simplification of little consequence, as long as the model contains other unconstrained agents, able and willing to purchase assets (young firms in this case).

by the same token we obtain market clearing, full employment, and output at capacity. This means that the real effects are mostly distributional. During any period, output is split between firms and households, and assets are split between young and mid-age firms. When deflation occurs, firms lose in real terms, whereas households gain as their nominal spending buys a greater fraction of goods supply. Similarly, when the asset price falls, mid-age firms lose assets to young firms who enter the market free of losses and inherited debt. Compared with the steady state, such redistributions are welfare-reducing in the aggregate. This is most obvious in the case of default, since defaulting entrepreneurs consume nothing ($V(0) = -\infty$). Distributional effects have, in reality, often caused considerable socio-economic problems.

But while our specification highlights distributional effects, it neglects important aggregate effects. As such, the model is clearly inadequate as a description of actual episodes, such as the Great Depression (Christiano et al 2004 provide a recent attempt). Modelling output effects would certainly be a useful extension. A possibility that suggests itself is wage rigidity, an important factor in the Depression. Appendix B4 shows that nominal wage rigidity indeed depresses employment and output to the extent that the price level remains low, due to high real wages. Another possibility is to consider assets (capital) in the production function. Distress selling will then have output effects, whether in the form of interrupted production or inefficient distribution of assets across firms. The models of Shleifer and Vishny (1992), Kiyotaki and Moore (1997), or Bernanke, Gertler and Gilchrist (1999) all possess features suitable for pursuing this direction.[82] Results on deflation will be less clear-cut when aggregate supply falls alongside aggregate spending. We have not pursued this line here, to keep in sharp relief the key features of the model.

Perhaps the most promising extension would be to incorporate uncertainty.[83] Doing so would bring the model closer to Fisher's and especially Minsky's views, and could change the results considerably. We found that debt-deflation remains stable when agents follow their optimal policy of user cost spending. The absence of uncertainty makes agents willing to take on more debt and purchase undervalued assets, which stabilises the asset market. But agents' attempt to contain their indebtedness was shown to produce unstable debt-deflation. We suspect that the presence of uncertainty would also work in a destabilising direction, since agents will be concerned with the prospect of bankruptcy that comes with higher indebtedness; they would no longer exploit expected future appreciation as aggressively as under perfect foresight. Whether uncertainty makes agents become as defensive as described by Fisher and Minsky remains a question for future research.

6.3 Relevance for today

Can a theory inspired by the Great Depression be relevant for today? The theory does not require an environment of absolute price level deflation. The analysis was conducted relative to a steady state with constant prices. Alternatively, one could take a steady state with a constant inflation rate π as the benchmark. When all nominal prices $\{p, w, q, u\}$ and values $\{B, \Pi, D, s\}$ grow at the rate π,

[82] Necessary modifications would include nominal contracting and avoiding linearisation.

[83] Note also that the combination of frictions and uncertainty might afford further insights. Default may no longer be so inconsequential; and the money supply might play a more causal role.

and the nominal interest rate equals $R = (1+\pi)\,\beta^{-1}$, one arrives at the same real allocation as in proposition 1. In particular, life-time asset holding again remains constant, $h' = h$. Following the shock, the same qualitative results hold, including those on distress selling and stability. Prices may not fall in absolute terms, but they fall relative to debt, because debt grows at nominal interest reflecting steady state inflation. Hence we believe that the mechanism studied in this model applies as much to unexpected disinflation as to deflation. A difference would arise if the model were extended to incorporate cash, bonds, and interest rate policy.[84]

Apart from the price level, the model highlighted the role of asset prices. Falling asset prices are as harmful as deflation, and the asset market may well be more liable to instability. Asset price swings can be a major concern, even in an environment of price stability (Borio and Lowe 2002). The notion of debt-deflation must include asset prices. This also makes it applicable to recent episodes of financial distress. Wolfson (1996) made this point in reference to the 1987 stock market crash. Broadening the scope by adding the exchange rate as a special asset price, could further extend the domain of applicability to emerging market crises.

A related observation is that distress selling remains a relevant concept. Financial history is rich in episodes of distress selling, as recounted by Kindleberger (1996). The concept starts to receive attention from corporate finance (e.g. Shleifer and Vishny 1992, Brown et al 1994, Pulvino 1998). It is also of interest from a macroprudential perspective: distress selling on a significant scale may be rare, but it raises policy concerns when it does occur. For example, the collapse of Long Term Capital Management prompted the Federal Reserve to organise a private sector rescue and ease monetary policy. This course of action was justified with the words

> "The size and nature of the positions of this fund [LTCM] were such that their sudden liquidation in already unsettled financial markets could well have induced further financial dislocations..."
>
> FOMC Minutes, September 1998 p. 183

This statement refers to the threat that distress selling might destabilise the market. Similar concerns followed the 1987 stock market crash, as margin calls produced considerable selling pressure (Brimmer 1989). Thus, while the role of distress selling in debt-deflation was inspired by the Great Depression, the notion arguably remains relevant when extended appropriately. From that perspective it is useful to consider distress selling in general equilibrium, as was attempted in the present paper.

6.4 Conclusion

The Great Depression inspired the theory of debt-deflation, describing how debt and deflation destabilise each other. In this paper we examined the channels of Fisher, Minsky, Tobin and Bernanke, in the context of a model with assets, banking, distress selling, and default. In spite of its stylised

[84] For instance, the inflationary economy would have more room for reducing interest rates without hitting the zero lower bound. This paper has left aside the role of policy and monetary regimes. On these aspects during Great Depression, see e.g. Eichengreen (1992), and Friedman and Schwartz (1963).

nature, it is a dynamic monetary model, specifying how agents adjust their asset holdings in response to losses, and how this determines deflation and asset prices in general equilibrium.

The main finding was that several channels of debt-deflation are active, but none is destabilising, even when large shocks, high indebtedness, and wide-spread default are considered. Stability is largely due to the optimal policy of user cost spending, and the presence of agents without any prior debt or losses. Their ability and willingness to take on more debt and purchase assets stabilises the asset market which we found to be central to the debt-deflation process. By contrast, when margin constraints or a credit crunch prevents agents from following this course, debt-deflation can become unstable.

Debt-deflation does not develop when agents forego distress selling and refinance their assets instead, as Minsky conjectured. But refinancing requires accommodation, an expansion of the banking system. This is certainly feasible in our frictionless inside money model, but may no longer be so when reserve or capital requirements bind, or when the economy is subject asymmetric information and frictions. In the absence of such complications, the idea that monetary contraction directly produces deflation receives little support. It is the price of assets, rather than the quantity of money, that drives deflation.

These results are subject to the limitations of our stylised model. While our frictionless, flexible price specification serves to highlight distributional effects, it neglects output effects that accompany actual episodes of debt-deflation. Similarly, while the absence of uncertainty puts into sharp relief key features of the mechanism, it neglects precautionary behaviour that could work in a destabilising direction. Extensions along these lines would bring the theory closer to Fisher and Minsky's original intent and make it more relevant for today. In developing such extensions it is worth retaining the model's basic premise that stability hinges on agents' reactions to losses.

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Appendix A: Changes in spending

A1. Optimal user cost spending

Each generation of firms is affected differently by the shock in t. In computing agents' changes in spending, we take their period constraints (3) as the starting point, where all variables, including debt, are at their steady state values until t. Denote by $\delta_t = (q - q_t)/q$ the percentage decline in the asset price, and by $(1 - p_t)$ deflation relative to the steady state price level $p = 1$ prevailing until t.

Young firms can factor in new goods and asset prices before taking on debt. Hence they behave as in perfect foresight. Optimal spending on goods and assets is given by (5) and (6), forwarded once. As there will be a competitive labor market at date t, we use (11), forwarded once, to write profits Π_{t+2} in terms of steady state profits as $\Pi_{t+2} = \Pi\, p_{t+2}$. Thus

$$p_{t+2}c_{t+2} = (1-\theta)\Pi\, p_{t+2} \tag{42}$$

$$Ru_{t+1}h'_{t+1} = Ru_t h_t = \frac{\theta\Pi}{1+R}p_{t+2}. \tag{43}$$

This is simply p_{t+2} times steady state spending, pc and Ruh. Subsequent generations' spending is of the same form, with subscripts forwarded accordingly.

Old firms only have their debt repayment and final consumption left to do. Hence, any unexpected losses show up as a wealth effect on consumption spending. Coming out of steady state, their budget constraint (3) reads

$$p_t c_t + R^2 qh + w = p_t y + q_t h.$$

Debt is predetermined, whereas the ability to repay, on the right, is not. They had borrowed qh in $t-2$, expecting to sell assets and goods at continued steady state prices q and $p = 1$, but prices are now q_t and p_t, respectively. The combined loss relative to steady state equals

$$\Omega \equiv qh\delta_t + (1 - p_t)y,$$

the percentage decline in the value of asset holdings, plus the loss of sales revenue to deflation. (Deviations can be negative.) Limited liability requires that final consumption be non-negative; should the loss Ω exceed planned consumption spending $pc = (1-\theta)\Pi$, firms would pass on the difference in the form of non-performing loans,[85]

$$\lambda \equiv \max\{0, \Omega - (1-\theta)\,\Pi\}. \tag{44}$$

The firms' effective loss is thus limited to $\Omega - \lambda$. With only consumption left to do, consumption spending is reduced by this amount (relative to steady state),

$$p_t c_t = pc - (\Omega - \lambda) \geq 0. \tag{45}$$

Mid-age firms already incurred debt, yet they can still alter both their asset position and final consumption. The loss (19) has been discussed in the text. The largest loss mid-age firms can absorb is $\Pi' \equiv (1-\theta)\,\Pi/R + uh$, the amount they would free up by spending nothing on consumption and on assets. Limited liability again implies that losses exceeding this buffer would become non-performing loans,

$$\lambda' \equiv \max\left\{0, \Omega' - \Pi'\right\}. \tag{46}$$

[85] Non-performing loans can equivalently be written as total debt minus repayment ability, $R^2qh - (p_t y + q_t h)$. Expanding by $Rqh - Rqh + qh - qh + py - py$ to bring up user costs, expressing t dated terms as deviations from these steady state values, and replacing $\Pi - R(1+R)uh = (1-\theta)\,\Pi$ yields (44).

Mid-age firms therefore see their budget set tighten by $[\Omega' - \lambda']$. This is consistent with combining the equations in (18) to obtain

$$p_{t+1}c_{t+1} + R^2uh + Ru_th'_t = p_{t+1}y - w - R\left(\rho + qh\delta_t\right),$$

and subtracting the same expression in steady state (drop subscripts, and set $\rho = \delta_t = 0$),

$$p_{t+1}c_{t+1} + Ru_th'_t = \begin{cases} [\Pi' - \Omega']\, R & \text{if} \quad \lambda' = 0 \\ 0 & \text{if} \quad \lambda' > 0. \end{cases} \tag{47}$$

This expression equates remaining spending to the remaining budget which, relative to steady state, is reduced by the loss Ω'. The three reactions (section 3.2) differ in the way they distribute the loss Ω' between reduced spending on consumption and assets. Reoptimisation determines how best to respond to the loss by adjusting $\{h'_t, c_{t+1}\}$ over the remaining horizon,

$$\max \quad V' = (1-\theta')\ln c_{t+1} + \theta' \ln h'_t \quad \text{s.t. (47).} \tag{48}$$

The modified share $\theta' < \theta$ reflects the fact that the remaining horizon for holding assets is now shorter.[86] Proceeding as before (page 12), one finds *revised* spending $p_{t+1}c_{t+1}$ and $Ru_th'_t$ equal to the fixed proportions $(1-\theta')$ and θ' of the *remaining* budget. In steady state, the same proportions would have resulted in steady state spending, pc and Ruh. Therefore, spending is revised downward in proportion to the loss,

$$u_th'_t = uh - \theta' \left[\Omega' - \lambda'\right] \tag{49}$$

$$p_{t+1}c_{t+1} = pc - \left(1-\theta'\right)\left[\Omega' - \lambda'\right] R. \tag{50}$$

Note that the revision is nil if the loss Ω' is zero. At the other extreme, the demands h'_t and c_{t+1} smoothly merge to zero as the firm approaches insolvency: the revision reaches the full value of steady state spending, $c_{t+1} = h'_t = 0$ when $\Omega' \geq \Pi'$.[87] Reoptimisation implies the continuation of user cost spending (21).

The Banking system neither holds real assets nor sells goods. It is nonetheless exposed to market prices through the possible default of its borrowers. If any generation of firms defaults, non-performing loans become positive,

$$\lambda^+ \equiv \lambda + \lambda' \geq 0.$$

As bank profits and capital are zero, recognizing loan losses amounts to reducing the value of deposits in the same measure.

Households face losses if deposits are reduced by λ^+, and may earn lower future wages in (8). On the other hand, they receive the payment ρ. Going forward, their spending must respect the new budget constraint

$$\sum_{i=0}^{\infty} \frac{s_{t+i}}{R^i} = \left(RD - \lambda^+\right) + \rho + \sum_{i=0}^{\infty} \frac{w_{t+i}}{R^i}.$$

In steady state, this constraint would be $\sum_{i=0}^{\infty}(s-w)/R^i = RD$. Taking the difference yields (26) in the text and shows by how much households must revise downward future spending when deposits and future wages fall relative to steady state.

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[86] We normalized the Cobb-Douglas exponents relating to c_{t+1} and h'_t in (2) to sum to one, hence

$$\theta' = \frac{\theta\beta/(1+\beta)}{1-\theta+\theta\beta/(1+\beta)} = \frac{\theta}{1+(1-\theta)\,R}.$$

[87] The parameters satisfy $\theta'\Pi' = \theta\Pi/\left[R\,(1+R)\right] = uh$, and $(1-\theta')\,R\Pi' = (1-\theta)\,\Pi = pc$.

A2. Refinancing

The question of refinancing only concerns mid-age firms; generations $t-2$ and t need not be considered. Suppose there is a fixed cost F of adjusting asset holdings. (Such costs are plausible especially in the context of real estate.) Reoptimisation now involves comparing utility between (1) the optimal consumption and asset holding given F; and (2) residual consumption after refinancing assets. Part (1) leads to the same optimal spending as in (49), with the fixed cost F added to the loss Ω'.

Part (2) is found using $h'_t = h$ in (47),

$$\begin{aligned} p_{t+1}c_{t+1} + Ru_t h &= (1-\theta)\Pi + Ruh - R\Omega' \\ \text{i.e.} \qquad p_{t+1}c_{t+1} &= pc + R(u-u_t)h - R\Omega'. \end{aligned}$$

Since $(u-u_t)\,qh \equiv (\delta_t - \delta_{t+1}/R)\,qh$, one obtains (23) in the text,

$$p_{t+1}c_{t+1} = pc - \left[R\rho + qh\delta_{t+1} + (1-p_{t+1})\,y\right].$$

Note that this expression must remain positive, otherwise refinancing is not feasible (it is not affordable, a fortiori, for defaulting firms, assuming $u_t > 0$). The decision to refinance then compares these two allocations in terms of utility (48) at equilibrium prices. Refinancing will be chosen if the fixed cost F is large enough to offset the gain from optimally adjusting both assets and consumption.[88]

A3. Constant indebtedness

Mid-age firms. From (17) we know indebtedness in steady state remains constant and equal to $L = RB'/\Pi$. Not allowing indebtedness to exceed this constant requires $RB'_t \leq L\Pi_{t+1}$. Since profits are now reduced to $\Pi - R\Omega'$, constant indebtedness requires a reduction of debt to $B'_t = B' - L\Omega'$. Using this along with $B' = RB$ in the second line of (18) yields (24). Of course, the value of distress selling no longer increases when $\rho + L\Omega'$ reaches $q_t h$, the value of asset holdings.

Solving (24) for h'_t shows how much user cost is paid on the remaining assets,

$$u_t h'_t = u_t\left[q_t h - \left(\rho + L\Omega'\right)\right]/q_t.$$

Substituting this expression into (47) delivers consumption spending (25).

Young firms. Their debt equals $B_t = q_t h_t$, from (3) forwarded once. Their profits are given by continued labour market clearing (11), $w_{t+2} = w\,p_{t+2}$ and $\Pi_{t+2} = \Pi\,p_{t+2}$. Thus, keeping indebtedness equal to (17) implies

$$\frac{B_t}{\Pi_{t+2}/R^2} = L \qquad \Rightarrow \qquad q_t h_t = qh\,p_{t+2}.$$

Note that firms will not adjust their asset positions over their life-time: young firms' indebtedness remains constant when assets do. When no assets are sold, $h'_{t+1} = h_t$ hence $B'_{t+1} = RB_t$ from (3). Since B_t satisfies the equation above, B'_{t+1} also satisfies $B'_{t+1}/\left(\Pi_{t+2}/R\right) = L$. Using the results $B_t = qh\,p_{t+2}$ and $h'_{t+1} = h_t$ in the budget constraint (3) delivers consumption spending

$$p_{t+2}c_{t+2} = p_{t+2}\left[\Pi - qh\left(R^2 - q_{t+2}/q_t\right)\right].$$

[88] We do not pursue the issue of finding the optimal (S,s) policy for a given F, or finding the critical F for which refinancing is preferred in equilibrium. When discussing refinancing in the text, we assume that F is large enough for agents to opt for refinancing

Appendix B: Aggregation and equilibrium

B1. Aggregation

Agents' market participation can be read off figure 2. Due to flexible prices and the absence of frictions, the supply side is unchanged with output y and assets $2h$. In what follows, the terms λ and λ' only serve to keep spending non-negative in the event of default.

Labour market. Recall that wages are determined on a competitive labour market at the time of hiring (footnote 26). Therefore, w_t and w_{t+1} were determined before the shock in t occured, and equal $w = \varepsilon y$. From $t+2$ onward the perfect foresight equilibrium (11) applies, following the exit of all firms affected by the shock. Thus spending deviates from that in steady state to the extent future prices deviate from 1.

Goods market. As in (12), the value of aggregate demand consists of spending by households and old firms. For date t, we use (45) to obtain $p_t y = s_t + pc - [qh\delta_t + (1-p_t)\,y - \lambda]$, or, since $py = s + pc$,

$$(1-p_t)y = (s - s_t) + [qh\delta_t + (1-p_t)y - \lambda]\,. \tag{51}$$

The goods market of $t+1$ similarly aggregates spending, using (50),

$$(1-p_{t+1})\,y = (s - s_{t+1}) + \left(1-\theta'\right)\left[\rho + qh\delta_t + (1-p_{t+1})\,y/R - \lambda'\right]R. \tag{52}$$

By $t+2$ the goods market reverts to the perfect foresight equilibrium, because all agents entering t or later take into account new equilibrium prices. Using (42) in deviations,

$$(1-p_{t+2})\,y = (s - s_{t+2}) + (1-p_{t+2})\,pc. \tag{53}$$

Successive goods market clearing conditions are connected by the Euler equation (9).

Asset market. Asset market equilibrium in t equates $2u_t h$ to the value of user cost spending by young and mid-age firms in (43) and (49), respectively,

$$2u_t h = uh\,p_{t+2} + uh - \theta'\left[\Omega' - \lambda'\right].$$

To bring out the asset price decline, we write in deviations $(u - u_t) \equiv q\,(\delta_t - \delta_{t+1}/R)$. When unexpected losses reduce spending (the right-hand side is positive), the user cost must fall relative to steady state ($u_t < u$). That requires the percentage decline of q_t to exceed that of q_{t+1} (both relative to q),

$$2qh\left(\delta_t - \frac{\delta_{t+1}}{R}\right) = \theta'\left[\rho + qh\delta_t + (1-p_{t+1})\,y/R - \lambda'\right] + uh\,(1-p_{t+2})\,. \tag{54}$$

The asset market in $t+1$ features firms of generations t and $t+1$, none of whom suffered unexpected losses. Hence, equilibrium in $t+1$ reverts to perfect foresight,

$$2qh\left(\delta_{t+1} - \frac{\delta_{t+2}}{R}\right) = uh\Big[(1-p_{t+2}) + (1-p_{t+3})\Big].$$

Subsequent asset market conditions are of the same form.

B2. Equilibrium

To simplify the equilibrium system, we work backwards.

Reversion to steady state. Due to the overlapping generations structure, the economy reverts to a steady state in $t+2$. Goods market clearing (53) simplifies to $s_{t+i} = s\,p_{t+i}$ from $i = 2$ onward. Substituting into the Euler equation (9) implies that the price level remains constant, $p_{t+i} = p_{t+2}$, for all $i \geq 2$. Call the new steady state price level p'; then wages and profits equal $w_{t+i} = w\,p'$ and $\Pi_{t+i} = \Pi\,p'$, for all $i \geq 2$, and spending in (42) remains constant, hence all other variables, too. We show below that $\rho = 0$ leads to the same real allocation as in the original steady state. Hence we require that p' also be equal to the original $p = 1$; this normalisation remains valid for $\rho > 0$, in that the equilibrium conditions are consistent with households' intertemporal budget constraint (26).

This allows to simplify the Euler equations and households' budget constraint. One can invert the Euler equation (9) to obtain $p_t = p_{t+1}\left[s_{t+1}/s_t\right]^{\gamma/(1-\gamma)}$, and $p_{t+1} = p'\left[s\,p'/s_{t+1}\right]^{\gamma/(1-\gamma)}$. With $p' = 1$ these expressions simplify to[89]

$$p_{t+i} = \left(\frac{s_{t+i}}{s}\right)^{\frac{\gamma}{\gamma-1}}, \text{ for } i = 0, 1. \tag{55}$$

Households' intertemporal budget constraint also simplifies. Since $p_{t+i} = 1$, $s_{t+i} = s$, $w_{t+i} = w$, for all $i \geq 2$, (26) becomes

$$(s - s_t) + (s - s_{t+1})/R = \lambda^+ - \rho. \tag{56}$$

This relation shows that the deposits carried into the new steady state are indeed equal to what they were in the old steady state, $D_{t+1} = D$.[90]

Equilibrium system. Combining (51), (52), and (54) yields

$$\begin{aligned}
\text{GM}_t: \quad & (1-p_t)y = (s - s_t) + \left[qh\delta_t + (1-p_t)y - \lambda\right] \\
\text{GM}_{t+1}: \quad & (1-p_{t+1})\,y = (s - s_{t+1}) + (1-\theta')\left[\rho + qh\delta_t + (1-p_{t+1})\,y/R - \lambda'\right]R. \\
\text{AM}_t: \quad & \delta_t = \tfrac{\theta'}{2qh}\left[qh\delta_t + (1-p_{t+1})\,y/R + \rho - \lambda'\right],
\end{aligned}$$

together with (55), $w_{t+i} = w$ for $i \geq 0$, $\delta_{t+i} = (1-p') = 0$ for $i \geq 1$, and $p_{t+i} = p' = 1$ for $i \geq 2$. This system is consistent with the intertemporal budget constraint: substituting GM$_t$ and GM$_{t+1}$ into (56) makes it hold identically. Hence we use (56) to replace (52), and arrive at

$$\begin{aligned}
\text{GM}_t: \quad & s_t = s + qh\delta_t - \lambda \\
\text{GM}_{t+1}: \quad & s_{t+1} = s - R\left[qh\delta_t - \rho + \lambda'\right] \\
\text{AM}_t: \quad & \delta_t = \tfrac{\theta'}{(2-\theta')qh}\left[\rho + (1-p_{t+1})\,y/R - \lambda'\right].
\end{aligned} \tag{57}$$

Existence and uniqueness. Consider (57) when $\rho = 0$. Clearly, setting all deviations to zero solves this system; the continued steady state is *a* solution. To show uniqueness requires expanding the terms in non-performing loans. The system (57), as the definition of λ^+, comprises the four possibilities that no-one, old firms, mid-aged firms, or both types default. Yet only two cases need to be considered (the possible default of old firms does not change the nature of the system).

Case 1. If mid-age firms do not default ($\lambda' = 0$). Combining AM$_t$ in GM$_{t+1}$, and substituting into

[89] This shows that $p' = 1$ is equivalent to the assumption that the Euler equation (9) holds between $t-1$ and t in spite of the shock: $p_{t-1} = p_t\left[s_t/s\right]^{\gamma/(1-\gamma)}$ also yields (55), since $p_{t-1} = 1$.

[90] Using (8), $D_{t+1} = RD_t + w - s_{t+1}$. Inserting $D_t = RD - \lambda^+ + \rho + w - s_t$, and recalling that $s - w = (R-1)\,D$, one obtains $D_{t+1} = D - R\left[\lambda^+ - \rho - (s_t - s)\right] - (s_{t+1} - s) = D$ by virtue of (56).

(55) yields the implicit equation

$$p_{t+1} = [1 - \Theta(1 - p_{t+1})]^{\frac{\gamma}{\gamma-1}}.$$

where $\Theta \equiv \frac{\theta' y}{(2-\theta')s}$. The right-hand side has the slope $\Theta p_{t+1}^{1/\gamma}\gamma/(\gamma-1)$ and is convex. Therefore, the only solution is $p_{t+1} = 1$.[91] With $p_{t+1} = 1$, the steady state continues: $\delta_t = 0$, $p_t = 1$, hence old firms do not fail either ($\lambda = 0$).

Case 2. If mid-age firms do default ($\lambda' > 0$). In this case, mid-age firms spend nothing, $h'_t = c_{t+1} = 0$. As shown in (33), this leads to $p_{t+1} = \bar{\varepsilon}^{\gamma} \equiv \underline{p}$ and $\delta_t = (R-1)/(2R) \equiv \bar{\delta}$.[92] Hence, assuming $\lambda' > 0$ returns a solution that results in $s_{t+1} = \underline{p}y$ and $s_t = s + qh\bar{\delta}$ (or $s_t = \underline{p}y$ should old firms also default). But if $\gamma < 1$, this solution is not an equilibrium when $\rho = 0$, because deflation $(1-\underline{p})$ is *not* sufficient to make mid-age firms default, which contradicts the assumption $\lambda' > 0$.[93] Equivalently, this solution violates the budget constraint (56).[94] By contrast, if $\gamma > 1$, then the above solution *can* be an equilibrium for those combinations of $\{\varepsilon, \gamma, \theta\}$ that imply mid-age firms default. It is easy to check that, given $\gamma > 1$ and $\lambda' > 0$, the budget constraint (56) holds identically at prices $\underline{p}$ and $\bar{\delta}$.

In conclusion, at $\rho = 0$, the unique equilibrium is the continued steady state, provided $\gamma < 1$; when $\gamma > 1$, a default equilibrium with reduced prices exists for some parameter combinations.

Explicit solutions. Write the solution (32) as $p_{t+1} = [1 - a + ap_{t+1} + b\rho]$, where $a \equiv m/\bar{\varepsilon}$ and $b \equiv (1-m)R/s$, recalling from (12) that $y/s = \bar{\varepsilon}$. The slope of the solution function $p_{t+1}(\rho)$ is given by

$$p'_{t+1}(\rho) = \frac{-b}{a + \frac{1-\gamma}{\gamma}p_{t+1}^{-1/\gamma}} < 0,$$

Explicit solutions can be readily obtained for quadratic cases. For $\gamma = 1/2$ we solve $p_{t+1}^{-1} = [1 - a + ap_{t+1} + b\rho]$, and for $\gamma = 2$, $p_{t+1} = [1 - a + ap_{t+1} + b\rho]^2$. The quadratic solutions can be written as[95]

$$\begin{aligned} \gamma = 1/2: \qquad p_{t+1} &= 1 + \frac{1}{2a}\left[\sqrt{(1+a+b\rho)^2 - 4ab\rho} - (1+a+b\rho)\right] \\ \gamma = 2: \qquad p_{t+1} &= 1 + \frac{1}{2a^2}\left[1 - 2a - 2ab\rho \pm \sqrt{(2a-1)^2 - 4ab\rho}\right]. \end{aligned}$$

In both cases $p_{t+1}(0) = 1$, $p_{t+1}(\rho) < 1$, and $p'_{t+1}(\rho) < 0$.

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[91] If $\gamma < 1$, the slope is negative and cuts the 45^0 line at 1. If $\gamma > 1$, the slope is positive cuts the 45^0 line at 1 and possibly at some smaller value too. But it can cut twice only if the right-hand side, evaluated at $p_{t+1} = 0$, exceeds zero (and only for some $\gamma > 1$). This is contradicted by the fact that $\Theta < 1$. The latter holds because, from the relations $s = \bar{\varepsilon}y$ in (12), and $\theta' \equiv \theta[1 + (1-\theta)R]^{-1}$. ($\Theta'(\varepsilon) < 0$, $\Theta'(\theta) > 0$, thus Θ reaches its maximum at $\varepsilon = 0$ and $\theta' = \theta = 1$.)

[92] The latter comes from using $\Omega' - \lambda' = \Pi'$ in AM$_t$ above. Hence $\delta_t = \frac{\theta'\Pi'}{2qh} = \frac{uh}{2qh} = \frac{R-1}{2R}$.

[93] To see this, losses Ω' exceed the buffer Π' if

$$(1-\underline{p})\,y/R + qh\bar{\delta} > (pc/R + uh).$$

This is equivalent to $s - \underline{p}y > Rqh\bar{\delta}$, ie. $(\bar{\varepsilon} - \bar{\varepsilon}^{\gamma})\,y > \frac{\theta\Pi}{2(1+R)}$. This cannot hold when $\gamma < 1$, since the left-hand side would be negative. It may hold if $\gamma > 1$, depending on parameters ε, γ and θ.

[94] The constraint requires $(s - s_t) + (s - s_{t+1})/R > 0$ when $\rho = 0$. Observe that $\gamma < 1$ results in $\bar{\varepsilon}^{\gamma} > \bar{\varepsilon}$, hence s_{t+1} and s_t both exceed s, see (12).

[95] The solution for $\gamma = 2$ is tenuous - it allows a sunspot solution (the negative root) and a real solution may fail to exist for large ρ.

B3. Fixed indebtedness in general equilibrium

The behaviour of firms observing constant indebtedness was derived in section 3.2(3) and appendix A3. We now look at the equilibrium implications. In t, asset market equilibrium is given in (38), goods market equilibrium (29) remains unchanged, and the failure of old firms implies $p_t = \underline{p}$. In $t+1$, goods market equilibrium can be derived from (25),

$$s_{t+1} - s = -RL\,\Omega' + \left(qh - q_{t+1}h'_t\right),$$

and the failure of mid-age firms would similarly imply $p_{t+1} = \underline{p}$.

From here onwards, the equilibrium equations become invariant over time, since all future generations have asset and goods demands analogous to young firms of generation t. All asset markets $T \geq t+1$ can therefore be found by substituting into $h_T + h'_T = 2h$ the results of appendix A3, namely $q_T h_T = qh\,p_{T+2}$ and $h'_T = h_{T-1}$, and solving for q_T,

$$q_T = \frac{p_{T+2}}{2/q - p_{T+1}/q_{T-1}} \qquad \forall T \geq t+1. \tag{58}$$

Iterating forward gives

$$q_T = q_{T-2}\frac{p_{T+2}}{p_T} \qquad \forall T \geq t+2. \tag{59}$$

This asset price guarantees that each generation keeps constant indebtedness equal to (17), buying the assets of old firms when young, and passing on this quantity of assets to young firms when old (see figure 2).

Similarly, all goods markets $T \geq t+2$ can be found by substituting into $p_{t+2}y = s_{t+2} + p_{t+2}c_{t+2}$ the results of appendix A3, to obtain

$$s_T = p_T\left[y - \Pi + qh\left(R^2 - \frac{q_T}{q_{T-2}}\right)\right].$$

Using this expression in (55) yields, after simplification,[96] the generic version of (40),

$$p_T = \left[1 + \left(1 - \frac{q_T}{q_{T-2}}\right)\frac{qh}{s}\right]^{-\gamma} \qquad \forall T \geq t+2. \tag{60}$$

The equilibrium sequence of asset and goods prices, suggests a periodicity of 2. If any two even-dated price levels are equal, $p_{T+2} = p_T$ with T arbitrarily large, it follows from (59) that even-dated asset prices are equal, which in turn implies that even-dated price levels in (60) equal $p_T = 1$. The same argument can be made for odd-dated prices. One obtains $p_T = 1 \;\; \forall T > t+1$, and

$$\begin{aligned} q_T &= q_t && \forall \text{ even } T > t \\ q_T &= q_{t+1} && \forall \text{ odd } T > t. \end{aligned}$$

While the price level reverts to 1, the asset price enters a periodic orbit around q.[97] Constant indebtedness has asset prices cycle indefinitely around their steady state value. This reflects a cycle in asset holding, as each generation spends exactly qh. Following the distress selling by mid-age agents, young agents in t absorb more assets than in steady state, and hold them until $t+2$. Hence,

[96] The simplification follows from using the steady state relations $y = w + \Pi$, $s - w = (R-1)\,D$, and $D = (1+R)\,qh$.

[97] The periodicity of 2 can be recognized from the map in (58), namely

$$q_{T+1} = \frac{1}{2/q - 1/q_T} \equiv G(q_T),$$

whose second iterate returns the fixed point, $q_{T+2} = G(q_{T+1}) = G(G(q_T)) = q_T$.

the new generation in $t+1$ must, in equilibrium, purchase less assets than in steady state, holding these until $t+3$, and so on.

B4. Wage rigidity, employment, and output

This appendix shows that wage rigidity reduces employment and output to the extent that the price level remains low. Recall that firms hire at t to produce output sold in $t+2$ (page 11). Due to the delay in production, the wage set in t and paid out in $t+2$, w_{t+2}, corresponds to p_{t+2}. Firms' first-order condition was given in (1) as $y'(n_t^d) = w_{t+2}/p_{t+2}$. Using a specific production function that satisfies $y(1) = y$, for example $y(n) = yn^{\varepsilon}$, the condition becomes

$$\varepsilon p_{t+2} y n_t^{\varepsilon-1} = w_{t+2}. \tag{61}$$

In assuming wage flexibility, we have so far imposed the market clearing condition $n_t^d = 1$ to determine the competitive wage. Now, introducing wage rigidity, we impose a fixed wage and determine equilibrium unemployment. Assuming that wages do not immediately adjust to the shock, the wage set at t remains unchanged from the previous period, $w_{t+2} = w = \varepsilon y$. From (61) one easily obtains the equilibrium values in the two regimes:

Equilibrium Value		Wage Flexibility	Wage Rigidity
Wage	$w_{t+2} =$	$\varepsilon p_{t+2} y$	εy
Unemployment	$1 - n_t =$	0	$1 - p_{t+2}^{1/(1-\varepsilon)}$
Output	$y(n_t) =$	y	$y p_{t+2}^{\varepsilon/(1-\varepsilon)}$
Profits	$\Pi_{t+2} =$	$(1-\varepsilon)\, p_{t+2} y$	$(1-\varepsilon)\, p_{t+2}^{1/(1-\varepsilon)} y.$

To the extent that $p_{t+2} < 1$, wage rigidity leads to higher wages, positive unemployment, reduced output, and lower profits. Lower profits in turn imply reduced spending on goods and assets by firms. The mass product of wages is also reduced, $n_t w = \varepsilon p_{t+2}^{1/(1-\varepsilon)} y < \varepsilon y$. This all suggests that the equilibrium price level under wage rigidity should indeed be lower than under wage flexibility.

However, reduced aggregate spending must be set against reduced aggregate output. Goods market equilibrium again requires $p_{t+2} y\,(n_t) = s_{t+2} + (1-\theta)\,\Pi_{t+2}$. Using the equilibrium values associated with wage rigidity, one obtains $s_{t+2} = \bar{\varepsilon} p_{t+2} y$, as was the case in (12). Since nothing invalidates the Euler equation in (27), it follows that $p_{t+2} = 1$, as was the case under wage flexibility (appendix B2). In this example, aggregate supply falls by as much as aggregate spending, leaving the price level unchanged.