Decomposing the co-movement of the business cycle: a time-frequency analysis of growth cycles in the eurozone*

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March 2005

Abstract

This article analyses the frequency components of European business cycles using real GDP by employing multiresolution decomposition (MRD) with the use of maximal overlap discrete wavelet transforms (MODWT). Static wavelet variance and correlation analysis is performed, and phasing is studied using co-correlation with the eurozone by scale. Lastly dynamic conditional correlation GARCH models are used to obtain dynamic correlation estimates by scale against the EU to evaluate synchronicity of cycles through time. The general findings are that eurozone members fall into one of three categories: i) high static and dynamic correlations at all frequency cycles (e.g. France, Belgium, Germany), ii) low static and dynamic correlations, with little sign of convergence occurring (e.g. Greece), and iii) low static correlation but convergent dynamic correlations (e.g. Finland and Ireland).

Keywords: Business cycles, growth cycles, European Union, multiresolution analysis, wavelets, co-correlation, dynamic correlation.

JEL Classification: C65, E32, O52

Acknowledgements: The Bank of Finland are to be thanked for their hospitality for Crowley during the 2004/05 academic year. Able research assistance was provided by Helinä Laakkonen, Emmi Martikainen and Tarja Yrjölä.

*Disclaimer: the views expressed in this paper are those of the authors, and do not reflect the views of the Bank of Finland.

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1 Introduction

All economies experience business cycles, and since the Second World War these cycles have been getting longer, but nonetheless, despite the occasional optimistic forecast that the phenomenon no longer exists[1], the cycle of economic expansion followed by recession persists. There is a long-standing interest in macroeconomics in the nature and origins of business cycles and, judging by certain undergraduate macroeconomics textbooks, some macroeconomists would maintain that macroeconomics and the study of the business cycle are mutually inclusive endevours. The causes of the business cycle are still largely a mystery in economics and for many constitute the sole *raison d'être* for a macroeconomist - indeed one of the major criticisms of mainstream economics[2] is that much of the focus in macroeconomics has moved away from trying to understand business cycles.to more technical aspects of models and econometrics. Nonetheless, over the last 15 years, with the introduction of real business cycle models into the mainstream literature, there has been a resurgence of interest in business cycles and what causes them.

Research on business cycles can be grouped into three different but overlapping strands - one strand looks at historical business cycles to try and ascertain the important factors driving the business cycle (and those that therefore should underpin any model), another strand looks at the international co-movement of business cycles, and the third strand looks at asymmetries in the business cycle itself. The original approach to business cycle research was made in the early part of the last century, and sought to uncover "stylized facts" by constructing datasets that are as long and as internationally broad as possible. This strand had its origins in early work done as far back as the 1920s by economists such as Kitchin (1923), Mitchell (1946), Kuznets (1958), and more recently Lucas (1977). The approach is particularly important for construction of theoretical models of the business cycle, as if they are to be relevant, models must replicate whatever regularities are observed in the data. This first strand of the literature is probably best summed up by Basu and Taylor (1999) who search for regularities in business cycles over more than a century of data from 18 countries. To motivate the second strand, one of the most noticeable trends of the 1980s and 1990s was the increasing regionalisation of the world economy along the line of trading blocs, and so in recognition of the fact that the likelihood of simultaneous economic downturns

[1] For example, to quote Rudiger Dornbusch in 1998, "Not to worry, this expansion will run forever; ... A slowdown is purely possible as is stock market correction, but not an old-fashioned recession; at most a banana".

[2] Such as those made by the Austrian School (see Garrison (2001) for example).

should be higher within a given regional trading block, economists started looking at the co-movement of business cycles across countries (see Backus and Kehoe (1992) for perhaps the seminal article here). The last strand of the literature perhaps probably had its genesis with Keynes (1936) but its latest variation began with Sichel (1993), who defined an asymmetric business cycle as "one in which some phase of the cycle is different from the mirror image of the opposite phase, so that contractions might be steeper, on average, than expansions". In this strand the advances in econometric methodology probably spurred the revival in interest in business cycles after the publication of the Hamilton (1989) Markov-switching model.

This paper is mainly concerned with the second strand of the business cycle literature referred to above, relating to the co-movement of business cycles, but it also takes in some of the concerns of the other strands as well. It takes a new unique approach, using new techniques in time-frequency analysis developed in the signal processing field of engineering to identify different periodicity cycles in real GDP. These cycles are then correlated at their various periodicities and then the dynamics of the cycle phases over time at these periodicities are studied.

The following section gives a brief review of the business cycle literature relating to the EU, then section 3 provides a very brief description of time-frequency analysis. Section 4 provides results for a static variance and correlation analysis of real GDP cycles using the wavelet approach while section 5 then explains and applies a dynamic correlation approach to the same data. Section 6 concludes and suggests further research.

2 A brief review of EU business cycle research

2.1 The co-movement of business cycles

As A'Hearn and Woitek (2001) note, Morgenstern (1959) was probably the first economist to observe and measure the comovement in business cycles on an international level. This observation was again picked up in the more recent literature by Backus and Kehoe (1992) and Backus, Kehoe, and Kydland (1992), who constructed a real business cycle model to examine how cyclical variations in output and other aggregates were correlated across countries. From their model, because of asymmetric supply shocks, they anticipated negative cross-correlation between output between countries, but in fact found quite strong positive correlations. Because of risk sharing giving rise to promotion of consumption smoothing, they expected quite high cross-correlations of consumption, but found only moderately high cross correlations. They also anticipated negative cross-correlations between investment and employment across countries (as asymmetric shocks cause capital flows between countries), and yet again found quite strongly positive correlations. These apparant anomolies are extensively discussed in Backus, Kehoe, and Kydland (1995), and since have become known in the literature as the *quantity anomoly*[3].

One criticism of this approach has been made by Canova and de Nicoló (2003), who point out that the models of the type used by Backus, Kehoe, and Kydland (1992) rely on temporary supply shocks to create the business cycles, and yet Canova and de Nicolo find that demand shocks are the most important source of output fluctuation. Canova and de Nicoló (2003) also found that structural disturbances appear uncorrelated across countries, with the exception of the US and Canada, and yet these types of disturbance have a key role in driving the business cycle in real business cycle models. Duarte and Holden (2003) took a slightly different approach by looking at the cyclical and trend components of real GDP for the G7 countries using various econometric specifications, to try and find similarities and differences, particularly in the cyclical components. They found that from around 1990 two separate cycles seem to be developing – one for the US, Canada, and the UK and the other for Germany, Italy, and France.

More recent research by Ambler, Cardia, and Zimmerman (2004) replicates the Backus et al results with a much larger dataset using a GMM methodology and notes that i) empirically, productivity correlations are greater than output correlations which in turn are greater than consumption correlations, and ii) that the empirical results tend not to support theoretical models that predict that comovements should either be negative (investment, output and employment) or high and positive (consumption). Clearly the inference from these results is that the underlying causes of business cycles needs to be better understood empirically, before theoretical models can properly fill in the modes and methods of transmission.

In an attempt to fill in these empirical gaps Baxter and Kouparitsas (2004) take a somewhat different approach to comovement in business cycles by noting that the main candidates for the comovement observation are trade[4], industrial structure[5], factor endow-

[3]This label appears to be somewhat of a misnomer, as there is little connection to quantities in an economics sense in their observation, and something is usually defined as anomolous if it is " a deviation or departure from the normal or common order, form, or rule". What was observed is probably better labelled an *ordering reversal*!

[4]See Frankel and Rose (2002)

[5]See Helpman and Krugman (1985)

ments[6], and gravity variables[7]. Using Leamer's "extreme bounds analysis" (see Leamer (1983)) so that measurement error can be taken into account.(- called robustness analysis), they find that higher bilateral trade is correlated with higher business cycle correlation, as is the stage of development of both countries and the distance between the two countries. Other variables (such as greater similarity in industrial structure belonging to a currency union and factor endowments) which have been thought to have an influence on business cycles in the literature, were all found to be fragile.

2.2 EU Business cycle research

The creation of a single currency area in Europe has prompted in a lot of business cycle research focused on the eurozone and the European Union. Studies by Artis and Zhang (1997), Artis and Zhang (1999) and Sensier, Artis, Osborn, and Birchenhall (2004) establish that since the inception of the exchange rate mechanism (ERM) of the European Monetary System (EMS), business cycles in the European Union have been on a convergent trend. This is an important issue, as economic theory doesn't offer any definitive guidance as to whether shocks should become more symmetric and cycles more synchronous. As Altavilla (2004) notes, one view maintains that monetary union, through increasing trade intensity, and co-committant economic and financial integration would yield both less asymmetric shock propogation and also greater business cycle synchronization. The other view, which stems from work by Krugman (1991), states that agglomeration effects would create more asymmetric shocks and therefore cause business cycles to be less synchronous. Indeed, although studies which stress the propagation of shocks tend to show that the former view dominates, those that put more emphasis on the synchronicity of business cycles (for example, De Haan, Inklaar, and Sleijpen (2002)) tend not to show a great degree of synchronicity in cycles between EU member states. Altavilla (2004) extends results by Agresti and Mojon (2001) using a variety of methodologies, mainly sourced from articles by James Hamilton (for example, Hamilton (1989)) and Harding and Pagan (see for example Harding and A. (2002)), and finds that although turning points for eurozone member states were similar, the time-path of output between these points was less so (but dependent on the filtering technique used). The average duration of the EU business cycle was calculated at around 3 years, which was equivalent in length to that of the US. In addition tests

[6] Standard trade theory (Ricardian and Heckscher-Ohlin, for example) would predict that this would influence the business cycle.

[7] Variables such as whether two countries are members of a currency union, the distance between the two countries, and at what stage in their development the two countries are, etc.

for synchronicity against the EU business cycle showed that although for some eurozone countries (notably Spain and Italy) this was quite low, but Germany, Belgium and France (in roughly that order) tended to have the strongest degrees of synchronization against the EU business cycle.

A completely different approach to EU business cycles originated in work done by Granger (1966)[8] and has been continued by those such as Croux, Forni, and Reichlin (2001), Valle e Azevedo (2002) and Levy and Dezhbakhsh (2003a), who use spectral analysis to study the properties of business cycle variables in the frequency domain. Croux, Forni, and Reichlin (2001) derive a measure of dynamic correlation which they label "cohesion"[9] for log differenced annual measures of GDP for European countries and annual personal income for the US states and Federal Reserve regions from 1962-97, and find that as expected, the US regions or states are far more cohesive than Europe, but that eurozone member states are a little more cohesive than the EU taken as a whole. They claim to identify the business cycle frequency as approximately 4 years in both cases, and find that cohesion at this frequency is at its highest for the US but this is not the case for the EU. In another paper in this strand, Valle e Azevedo (2002), using annual data from 1960-99, finds that the modal duration of the business cycle is about 9.25 years and the mean duration is 8.79 years. Using a co-spectrum, he goes on to estimate the dynamic correlation for each country vs the EU11, and arrives at high correlations for France and Germany, middling correlations for Italy and Spain and low correlations for other countries (- Finland, Sweden, UK and USA). Although the dynamic correlation results appear to be qualitatively similar between the two studies, note that the estimate of the business cycle duration in the study by Valle e Azevedo (2002) was significantly different from that obtained by Croux, Forni, and Reichlin (2001). Levy and Dezhbakhsh (2003a) estimate the output growth rate spectra for a group of 58 countries using both annual and quarterly data, and find that these spectra significantly between countries, but they find that for most of the OECD countries the mass of the spectrum lies in the business cycle frequency band (estimated at between 12 and 32 quarters). But in their results for quarterly data there are many exceptions[10], and for middle-developed and lesser-developed countries this general assertion about the location of the mass of the spectra is not true at all.

[8] Updated by Levy and Dezhbakhsh (2003b) for a variety of countries.

[9] In the paper they show that their measure at zero frequency is equivalent to the notion of stochastic co-integration.

[10] Those countries where the mass of the spectrum was located in i) longer cycles - Canada, Austria and Japan; ii) business cycles - Switzerland, France, UK, Finland; and iii) shorter cycles - US, Australia, Sweden, Denmark, Netherlands, Austria.

The traditional view, embodied in the work of Bergman, Bordo, and Jonung (1998) is that the business cycle lasts for 4.8 years on average during the post-war period, with countries like Finland having longer average cycles (5.8 years) and countries such as Norway having shorter cycles (3.6 years)[11]. Clearly different studies use different datasets, and different methodologies select different business cycle lengths in the data, and perhaps one could posit the existence of cycles occurring at different medium term frequencies[12], as mirrored in the approach taken by Comin and Gertler (2003) where they look for cycles in macroeconomic data in two frequency bands: from 2 to 32 quarters (0.5 to 8 years) and from 32 to 200 quarters (8 to 50 years), over the period from 1948 to 2001. Although their results and model focus on the longer medium term cycles, they suggest a considerably higher variance for these medium term cycles, and attach much greater importance to this cycle.

3 Multiresolution analysis

3.1 Why wavelets?

In all the research done to date on business cycles, it is has been impossible to satisfactorily simultaneously separate out different frequencies in output data so as to identify cycles in the time domain at different frequencies. Traditionally, spectral analysis suffers from the problem of its inability to deal with time-varying cycles, lack of regularity in the business cycle, and reliance on stationary data and therefore the usage of appropriate detrending methods. Thus the work horse of spectral analysis, namely the Fourier transform and its variants, has not always yielded interesting results when applied to actual economic data, as the Fourier method assumes that series are homogeneous in their characteristics, so that periodicity is regular and no shocks or other exogenous events exist. Further, any shift in periodicities would appear as peaks in the spectrum at two different frequencies, when in fact only a single process might have been at work. An example of typical spectrograms for EU and US quarterly GDP data are shown in figure 1.

The spectrograms do highlight several general points though, and these carry through for nearly all the data used in this study:

- there are two sets of frequencies, one at higher frequencies (cycles less than one year

[11] See table 1 on page 74 of Bergman, Bordo, and Jonung (1998)
[12] They define a long term cycle as anything greater than 50 years.

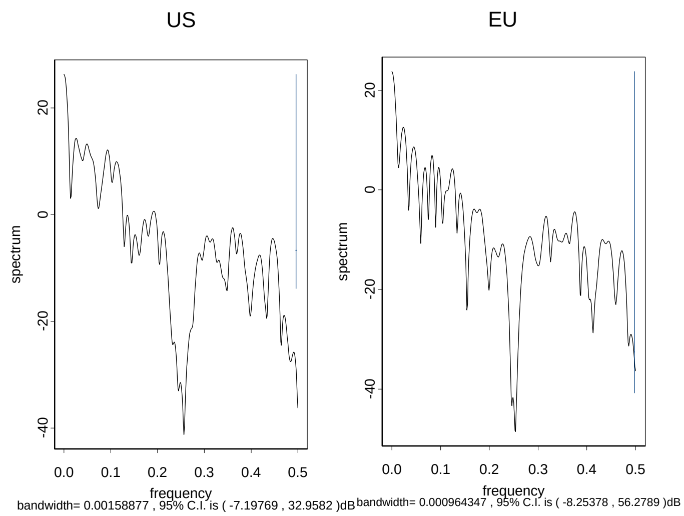

Figure 1: Spectrograms for US and EU quarterly GDP data: 1971-2003

long), and another at lower frequencies (greater than one year), with the lower frequency set being stronger than the higher frequency cycles;

- there is no obvious modal peak in the spectrum, which might denote the conventional conception of the business cycle, but several different peaks in each frequency range;
- the suggestion from the spectrum that there are very long frequencies that are significant in the data as the left hand part of the spectrum is the highest point in the spectra;
- a gap in the spectrum at around frequency of 0.25, which implies lack of a one year cycle - this is likely due to the fact that seasonally adjusted data is being used..

If we ignore the higher frequencies, then as Granger (1966) first showed (and more recently confirmed by Levy and Dezhbakhsh (2003b)) with level data, this yields a downward sloping spectrum with longer frequencies being more prominent in the data. As Levy and Dezhbakhsh (2003a) show with quarterly growth data, this general results carries through to many of the spectra for GDP data as well. In one sense this resolves the contradiction in the estimates of the duration of the business cycle, as the spectral methods point to strong medium term cycles influencing the conventional business cycle. But if the spectrogram is not smoothed, we also obtain the empirical observation that although as economists we measure the business cycle in terms of when recessions occur, the GDP series suggests that there are potentially many other cycles with different periodicities at work in the data[13]. Of course the business cycle itself has various phases, as Kontolemis (1997) makes clear, but if they are of the four phase variety originally described by Hicks (1950), then they could occur at different frequencies to the business cycle itself, given that accelerating and decelerating growth cycles might not be in concordance with the conventional business cycle. Zarnovitz (1985) first suggested that these more frequent "growth cycles" might have an important role to play in the business cycle itself, and set about studying them. In terms of dating these growth cycles, Zarnowitz and Ozyildirim (2002) conduct various time series decomposition approaches to identify the cycles, and construct a "growth cycle" chronology for the US.

Wavelet analysis sets itself apart from both the frequency and time domain approaches by combining elements of both[14]. Wavelets have the ability to decompose a series into various frequency components, albeit with constraints on how these frequencies are defined, at any given point in time. They therefore can be categorised as time-frequency analysis. The advantages of using wavelets to analyse business cycles are immediately clear: unlike frequency domain analysis, they can identify which frequencies are present in the data at any given point in time. Once a series has been decomposed into these different frequencies, time series can then be extracted for further analysis.

Only one previous contribution to the business cycle literature has been made using wavelet analysis, that of Crivellini, Gallegati, Gallegati, and Palestrini (2004). They use industrial production data for several EU countries, and decompose the data into different frequencies and then analyse each different frequency separately in terms of duration, amplitude, phasing and possible cause. The approach taken here is complementary to their study, but the focus is instead on the co-movement of growth cycles in the EU using quarterly data.

3.2 Data

The data in this study was provided by the Bank of Finland, which in turn was sourced from the OECD with the exception of the US data (which was obtained from the US

[13] So as not to confuse the generally accepted notion of a business cycle with also the more general definition of a medium term cycle used by Comin and Gertler (2003), from this point onwards the term "growth cycles" is used to describe cycles at different frequencies present in GDP growth data.

[14] Crowley (2005) provides a comprehensive overview of wavelets and reviews existing and potential applications in the economics literature.

Bureau of Economic Analysis) and the Swiss data (- which was obtained from the Bank for International Settlements)[15]. The eurozone (EU12) aggregate was sourced from the ECB's euro area wide model (AWM)[16]. The data frequency is quarterly, the span is from 1970-2004Q2 and the data is seasonally adjusted[17]. So as to identify growth cycles, the data is annually log-differenced, which should then also neutralise any differences in methods of seasonal adjustment. Summary statistics for the log-differenced GDP data are presented in table 1 and 2, including basic Pearson correlations with the eurozone GDP growth data and with the US growth data.

[15] In all cases sourced data are seasonally adjusted, with the exception of Switzerland, where the sourced Swiss data was seasonally adjusted by the Bank of Finland using the STAMP program. Adjustments were also made to the German data to eliminate the "jump" in the data after reunification in 1991.

[16] See Fagan, Henry, and Mestre (2001).

[17] Seasonal adjustment in business cycle studies continues to be controversial as Mir and Osborn (2004) show. The analysis here was repeated for several seasonally adjusted series, and only small differences were apparent.

	EU	Belgium	Finland	France	Germany	Greece	Ireland	Italy	Lux	NL	Portugal	Spain
Mean (10^{-3})	10.33	10.24	11.58	10.26	9.18	11.50	21.90	9.93	17.60	10.56	13.86	12.82
Variance (10^{-3})	0.048	0.073	0.183	0.048	0.053	0.300	0.170	0.091	0.228	0.072	0.201	0.086
Skewness	-0.22	-0.09	-1.06	-0.138	-0.04	-0.68	0.32	0.52	-0.58	-0.52	-0.20	0.41
Kurtosis	-0.07	-0.16	1.59	-0.60	-0.46	2.10	-0.23	1.34	1.33	0.04	0.28	0.25
Correlation with eurozone	1.00	0.85	0.34	0.89	0.88	0.41	0.16	0.81	0.56	0.75	0.80	0.70
Correlation with US	0.45	0.34	0.24	0.35	0.36	0.39	0.18	0.30	0.39	0.41	0.26	0.25

Table 1: Summary statistics for eurozone countries

	Denmark	Iceland	Japan	Sweden	Switzerland	UK	US
Mean (10^{-3})	8.01	15.91	13.28	8.40	7.06	10.28	13.44
Variance (10^{-3})	0.088	0.222	0.114	0.072	0.166	0.079	0.090
Skewness	-0.13	0.07	0.06	-0.80	-0.32	-0.46	-0.532
Kurtosis	0.18	-0.04	0.16	0.89	4.29	1.54	0.354
Correlation with eurozone	0.50	0.37	0.51	0.33	0.50	0.44	0.45
Correlation with US	0.50	0.22	0.32	0.21	0.21	0.58	1.00

Table 2: Summary statistics for non-eurozone countries

In table 1, unsurprisingly, the member states that were previously known as the "hard core" of EMU member states (France, Germany, Belgium, Luxembourg and the Netherlands) all have high correlations against the eurozone[18], but interestingly Italy and Portugal also now appear to have relatively high correlations against the eurozone. The low correlations for Finland, Greece and Ireland must give some cause for concern for policymakers, particularly as all of these member states are on the periphery of the EU. In table 2 for the non-eurozone countries, all of the correlations with the eurozone are relatively low but all positive, and interestingly for both Denmark and the UK, two countries with opt-outs from EMU, correlations with the US are greater or equal to those with the eurozone. These simple correlations though are in line with previous studies with respect to the groupings of countries before the inception of EMU.

3.3 Basic wavelets

Wavelets are a relatively recent innovation in mathematics and originally stem from research by Mallat (1989) and Debauchies (1992). The main feature of wavelet analysis is that it enables the researcher to separate a variable or signal into its constituent frequency components. Consider a double sequence of functions:

$$\psi(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-u}{s}\right) \tag{1}$$

where s is a sequence of scales, where scale here corresponds to a particular frequency range. The term $\frac{1}{\sqrt{s}}$ ensures that the norm of $\psi(.)$ is equal to one. The function $\psi(.)$ is then centered at u with scale s. In the language of wavelets, the energy of $\psi(.)$ is concentrated in a neighbourhood of u with size proportional to s, so that as s increases the length of support in terms of t increases For example, when $u = 0$, the support of $\psi(.)$ for $s = 1$ is $[d, -d]$. As s is increased, the support widens to $[sd, -sd]$. Dilation (i.e. changing the scale) is particularly useful in the time domain, as the choice of scale indicates the "stretching" used to represent any given variable or signal. A broad support wavelet yields information on variable or signal variations on a large scale, whereas a small support wavelet yields information on signal variations on a small scale. The important point here is that as projections are orthogonal, wavelets at a given scale are not affected by features of a signal at scales that require narrower support. Lastly, if a wavelet is shifted on the time line, this is referred to as translation or shift of u. Any series $x(t)$ can be built up

[18] With the exception of Luxembourg.

as a sequence of projections onto father and mother wavelets indexed by both j, the scale, and k, the number of translations of the wavelet, where k is often assumed to be dyadic. As shown in Bruce and Gao (1996), if the wavelet coefficients are approximately given by the integrals:

$$s_{J,k} \approx \int x(t)\phi_{J,k}(t)dt \tag{2}$$

$$d_{j,k} \approx \int x(t)\psi_{j,k}(t)dt \tag{3}$$

$j = 1, 2, ...J$ such that J is the maximum scale sustainable with the data to hand, then a multiresolution representation of the signal $x(t)$ is can be given by:

$$x(t) = \sum_k s_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \sum_k d_{J-1,k}\psi_{J-1,k}(t) + ... + \sum_k d_{1,k}\psi_{1,k}(t) \tag{4}$$

where the basis functions $\phi_{J,k}(t)$ and $\psi_{J,k}(t)$ are assumed to be orthogonal, that is:

$$\begin{array}{c} \int \phi_{J,k}(t)\phi_{J,k'}(t) = \delta_{k,k'} \\ \int \psi_{J,k}(t)\phi_{J,k'}(t) = 0 \\ \int \psi_{J,k}(t)\psi_{J',k'}(t) = \delta_{k,k'}\delta_{j,j'} \end{array} \tag{5}$$

where $\delta_{i,j} = 1$ if $i = j$ and $\delta_{i,j} = 1$ if $i \neq j$. Note that when the number of observations is dyadic, the number of coefficients of each type is given by:

- at the finest scale 2^1 :there are $\frac{n}{2}$coefficients labelled $d_{1,k}$.
- at the next scale 2^2 :there are $\frac{n}{2^2}$coefficients labelled $d_{2,k}$.
- at the coarsest scale 2^J:there are $\frac{n}{2^J}$coefficients $d_{J,k}$ and $S_{J,k}$

In wavelet language, each of these coefficients is called an "atom" and the set of coefficients for each scale are termed "crystals"[19]. The multiresolution decomposition (MRD) of the variable or signal $x(t)$ is then given by the set of crystals:

$$\{S_J, D_J, D_{J-1}, ...D_1\} \tag{6}$$

The interpretation of the MRD using the DWT is of interest as it relates to the frequency at which activity in the time series occurs. For example with a quarterly time series table 3 shows the frequencies captured by each scale crystal:

[19] Hence the atoms make up the crystal for each scale of the wavelet resolution.

Scale crystals	Quarterly frequency resolution
d1	1-2
d2	2-4
d3	4-8=1-2yrs
d4	8-16=2-4yrs
d5	16-32=4-8ys
d6	64-128=8-16yrs
d7	128-256=16-32yrs
d8	etc

Table 3: Frequency interpretation of MRD scale levels

Note that as quarterly data is used in this study, to capture the conventional business cycle length scale crystals need to be obtained for 5 scales. This requires at least 64 observations, but as we have 132 observations this is easily accomplished[20]. The data are transformed into year over year changes in log real GDP. It should be noted at this juncture, that if conventional business cycles are usually assumed to range from 12 quarters (3 years) to 32 quarters (8 years)[21], then crystals d4 and d5 will be assumed to contain business cycle frequencies.

3.4 MODWT

In the wavelet literature there are various methods that can be used for decomposing a series or signal. The first transform to be used extensively in applications was the Discrete Wavelet Transform (DWT). Although extremely popular due to its intuitive approach, the DWT suffers from two drawbacks: dyadic length requirements for the series to be transformed and the fact that the DWT is non-shift invariant. In order to address these two drawbacks, the maximal-overlap DWT (MODWT)[22] was introduced by **?**) and a phase-corrected version was introduced and was compared and found superior to other methods

[20] A preliminary version of this paper used 6 scales, but has been dropped in this version of the paper, as for virtually all countries, the 6th scale carries very little energy.

[21] Given our discussion above, and as assumed by Levy and Dezhbakhsh (2003a), when evaluating output growth rate spectra.

[22] As Percival and Walden (2000) note, the MODWT is also commonly referred to by various other names in the wavelet literature such as non-decimated DWT, time-invariant DWT, undecimated DWT, translation-invariant DWT and stationary DWT. The term "maximal overlap" comes from its relationship with the literature on the Allan variance (the variation of time-keeping by atomic clocks) - see Greenhall (1991).

of frequency decomposition[23] by **?**). The MODWT gives up the orthogonality property of the DWT to gain other features, given in Percival and Mofjeld (1997) as:

- the ability to handle any sample size regardless of whether the series is dyadic or not;
- increased resolution at coarser scales as the MODWT oversamples the data;
- translation-invariance - in other words the MODWT crystal coefficients do not change if the time series is shifted in a "circular" fashion; and
- the MODWT produces a more asymptotically efficient wavelet variance estimator than the DWT.

Both Gençay, Selçuk, and Whicher (2001) and Percival and Walden (2000) give a thorough and accessible description of the MODWT using matrix algebra. With time series, one of the problems in using the MODWT is that the calculations of crystals occurs at roughly half the length of the wavelet basis into the series at any given scale. Thus the crystal coefficients start further and further along the time axis as the scale level increases. As the MODWT is shift invariant, the MRD will not change with a circular shift in the time series, so that each scale crystal can be appropriately shifted so that the coefficients line up with the original data. This is done by lagging the crystals by increasingly large amounts as the scale order increases. Figure 2 shows MODWTs for the eurozone using an s8 wavelet (a symmetric 8-tap filter), and figures 3 to 10 replicate the analysis for the other countries in the dataset.

Several things are immediately apparent from stackplots:

i) apart from d1 crystal, which appears to contain mostly noise, there appear to be several growth cycles of varying strength apparent in the data from the 2-4 quarter crystal to lower frequencies, all with fairly regular cycles;

ii) although the wavelet smooth (s5) appears to dip during recessions, a coincidence of downward turns in growth cycles at different frequencies coincides with recessions;.

iii) up until the early 1980s in the case of the EU, and the mid-1980s in the case of the US, lower-order scale crystals (higher frequencies) d1 to d3 appear to contain higher energy (stronger) cycles, whereas in the 1990s these lower-order scale cystals appeared to be

[23] The MODWT was found superior to both the cosine packet transform and the short-time Fourier transform.

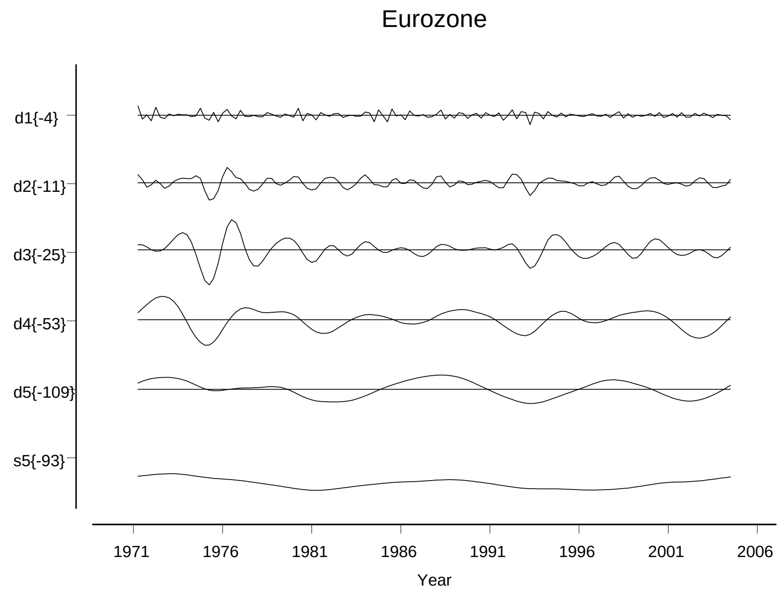

Figure 2: MODWT

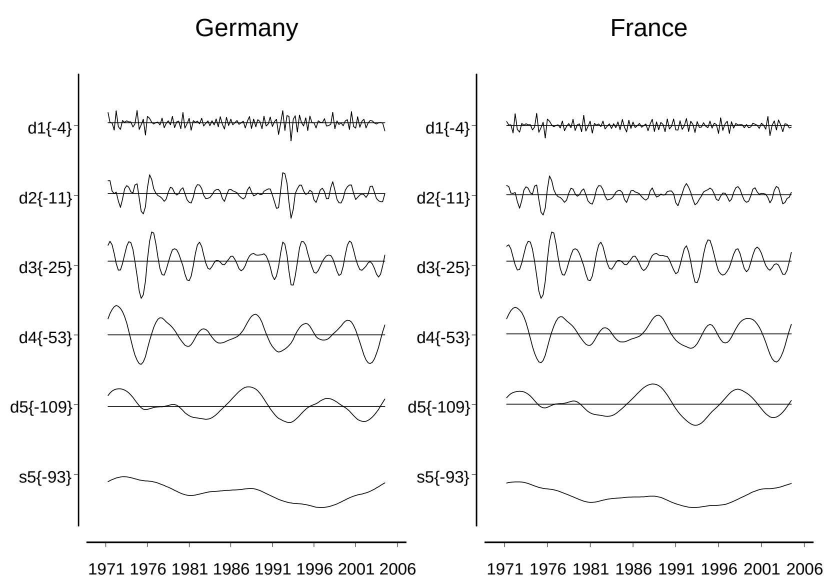

Figure 3: MODWT

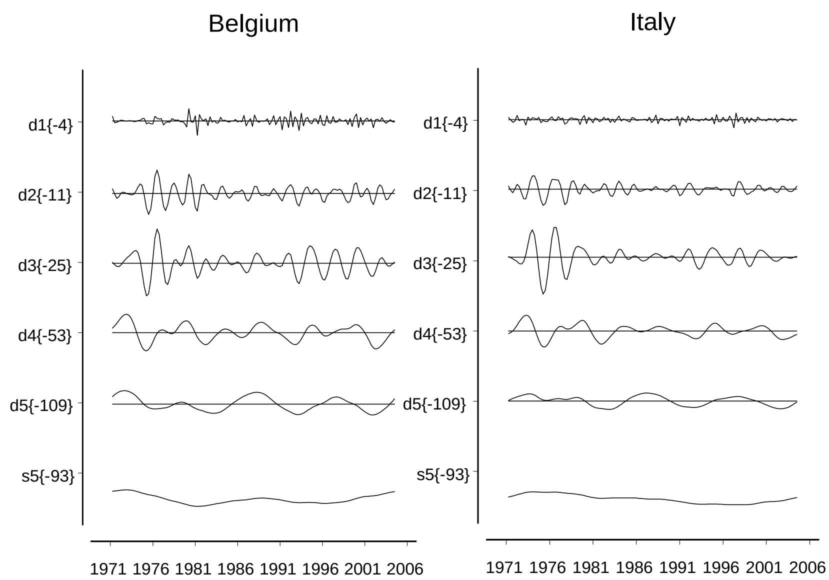

Figure 4: MODWT

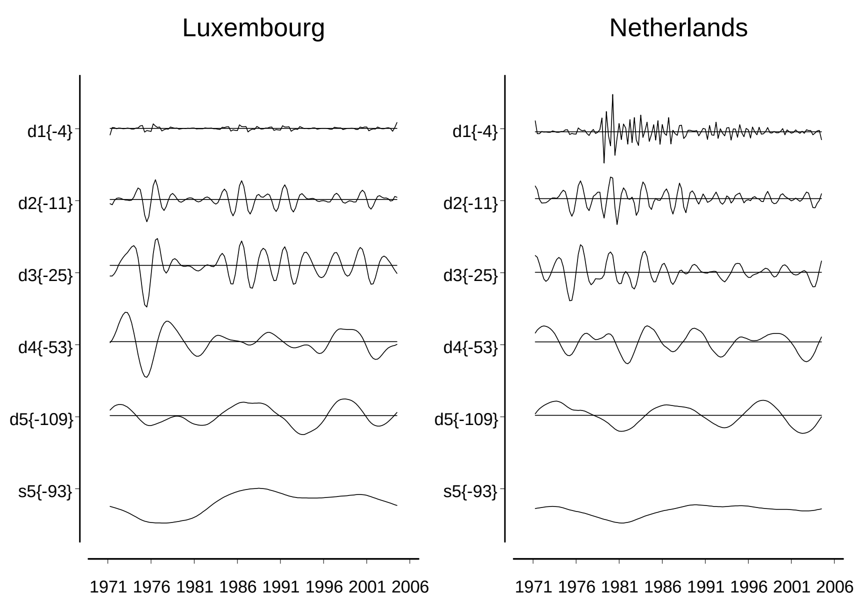

Figure 5: MODWT

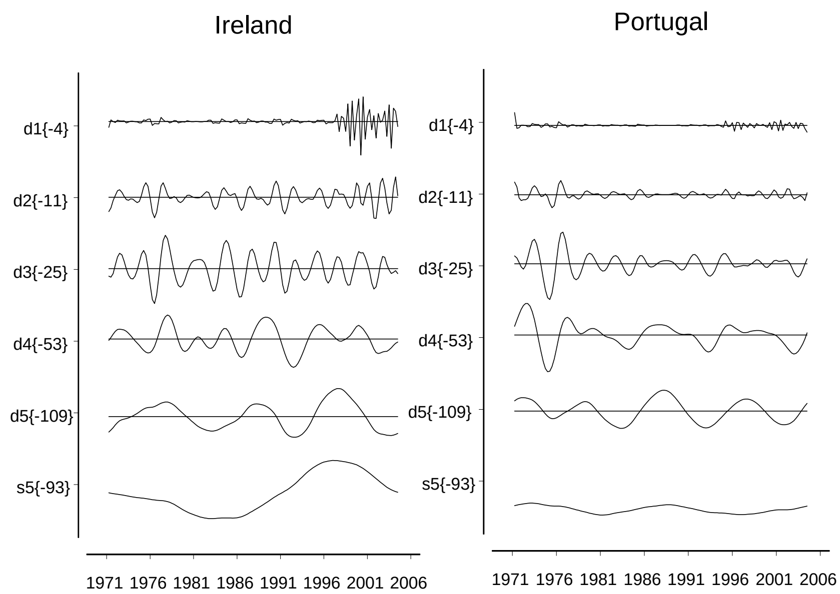

Figure 6: MODWT

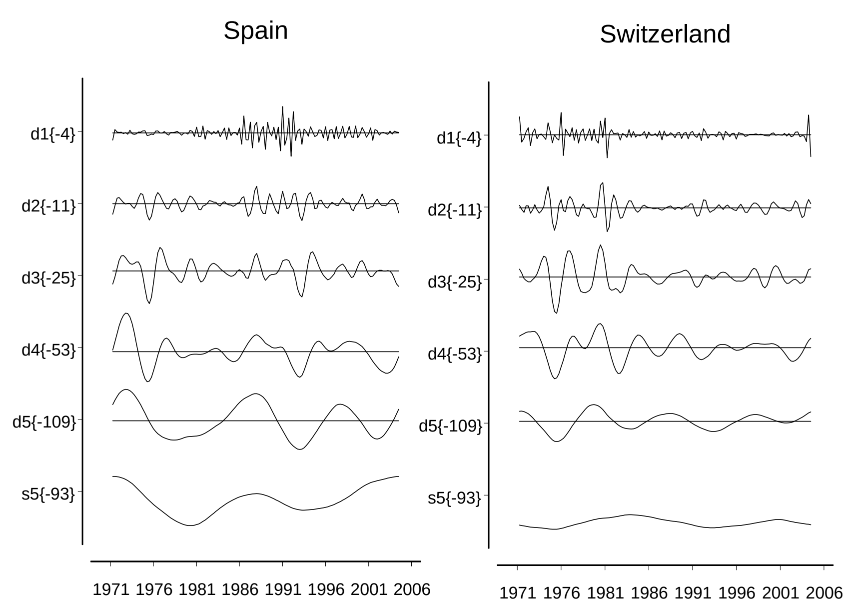

Figure 7: MODWT

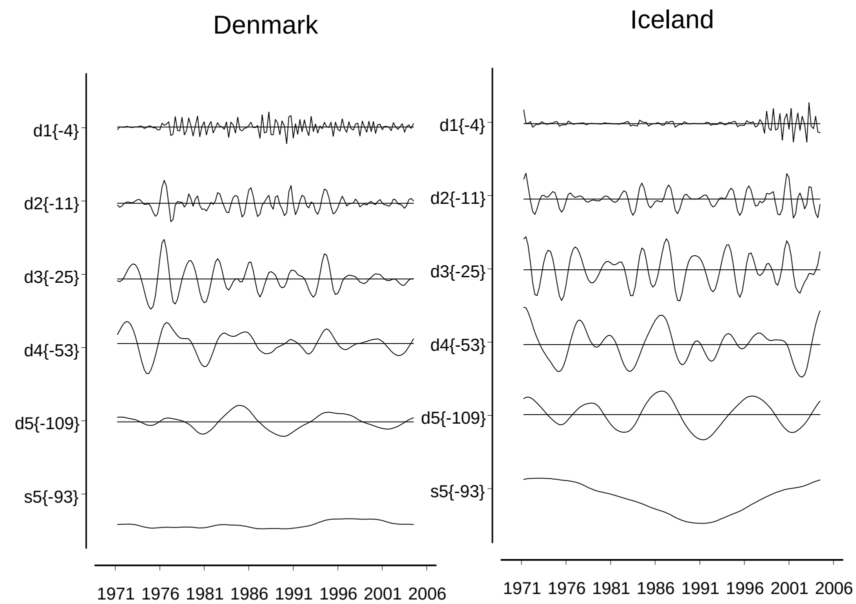

Figure 8: MODWT

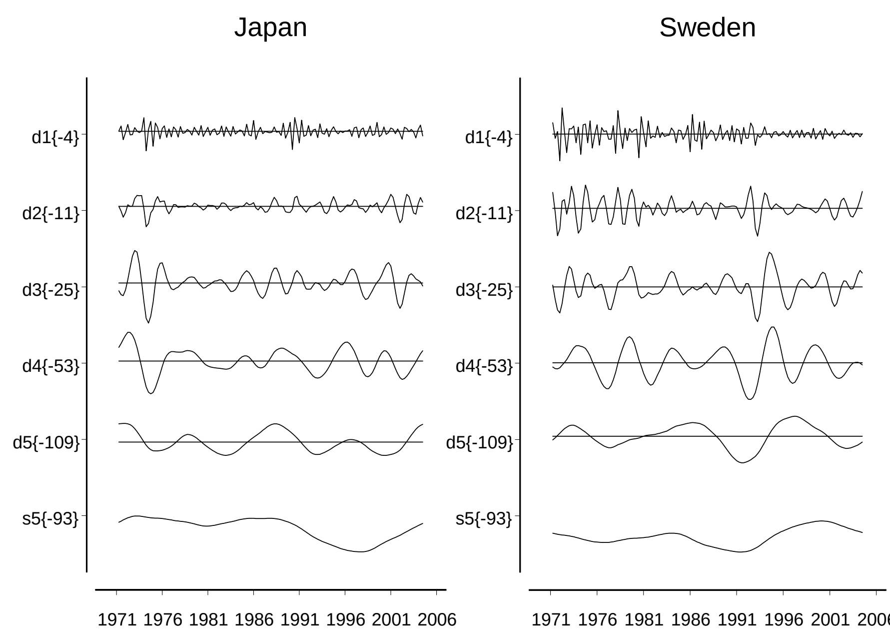

Figure 9: MODWT

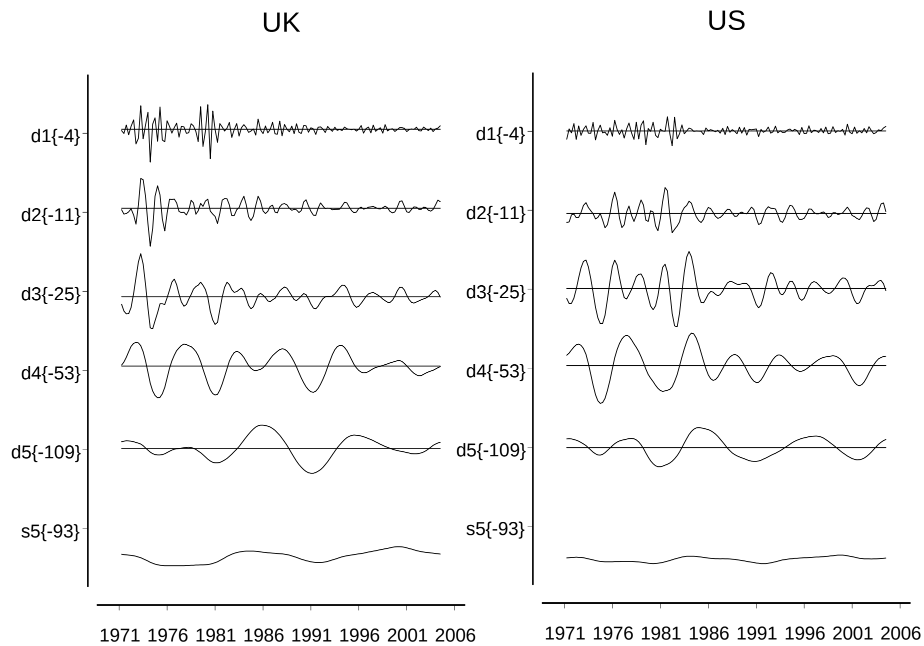

Figure 10: MODWT

less volatile - corroborating the widely recognized lower growth volatility observation (- particularly in the US).

The energy distribution for all the real GDP series considered in this paper are given in tables 4 and 5, where the rebased energy distribution is given as:

$$E_j^d = \frac{1}{E^d} \sum_{k=1}^{\frac{n}{2^j}} d_{j,k}^2 \tag{7}$$

where $E^d = \sum_j E_j^d$.represents the energy in the detail crystals E_j^d. This leads to two more noteworthy points about the different frequency cycles:

iv) in all cases the wavelet smooth (s5) contained most energy. This corroborates the result from spectral analysis that longer cycle freqencies appear to carry most energy.

v) the detail crystals containing the greatest energy are either the d3 or d4 crystal, and this is consistent across nearly all the countries considered[24]. These crystals correspond

[24]Except for Spain, where d5 possesses most energy.

to cycles of 1-2 and 2-4 years, respectively, which tend to be at a slightly shorter frequency than that of the conventional business cycle.

In tables 4 and 5, one of the issues is how to characterise the wavelet smooth in economic terms. In theory, the wavelet smooth could incorporate longer term "trend" cycles, but it could also just include residual "drift" in the growth rate of GDP. Clearly for the US and the EU, the energy contained in the wavelet smooth is significant, but for Switzerland, an apparent outlier in this regard, the amount of energy contained in this crystal is not as large, so suggesting that long cycles are not an empirical "stylized fact". Unfortunately, though, in this study it is not possible to discern the nature of the content of this crystal, given the length of the GDP series used.

	EU	Belgium	Finland	France	Germany	Greece	Ireland	Italy	Lux	NL	Portugal	Spain
d1	3.07	4.57	8.74	3.90	4.98	14.72	8.17	2.72	0.54	13.80	1.16	6.28
d2	8.20	17.72	6.58	7.83	10.64	20.79	10.73	13.87	10.53	15.05	3.78	5.59
d3	30.54	39.22	16.08	29.23	29.24	34.63	31.49	50.03	34.67	22.87	26.40	15.32
d4	34.78	22.31	35.28	35.19	35.16	22.95	22.12	21.28	33.25	24.18	39.75	28.45
d5	23.41	16.18	33.31	23.84	19.98	6.91	27.50	12.09	21.02	24.10	28.92	44.36
s6	71.32	61.70	50.28	72.25	66.67	43.05	82.28	56.30	67.03	63.30	50.64	73.73

Table 4: Wavelet energy distribution for eurozone

	Denmark	Iceland	Japan	Sweden	Switzerland	UK	US
d1	7.41	3.16	6.57	9.83	9.05	10.29	3.30
d2	12.01	9.17	7.02	13.60	11.67	11.57	8.82
d3	37.65	31.63	30.15	21.94	29.67	21.09	31.79
d4	29.55	32.97	34.13	35.02	32.65	35.16	39.80
d5	13.38	23.08	22.14	19.61	16.87	21.88	16.29
s6	43.75	63.81	70.04	54.369	26.44	59.58	67.20

Table 5: Wavelet energy distribution for other countries

4 Wavelet Variance and Correlation Analysis

Given that wavelet analysis can decompose a series into sets of crystals at various scales, it is natural to then take each scale crystal and use it as a basis for decomposing the variance of a given series into variances at different scales. Here we follow a very simplified version according to Constantine and Percival (2003) which is originally based on Whitcher, Guttorp, and Percival (2000) (with mathematical background provided in Whitcher, Guttorp, and Percival (1999)).

Let x_t be a (stationary or non-stationary) stochastic process, then the time-varying wavelet variance is given by:

$$\sigma^2_{x,t}(\lambda_j) = \frac{1}{2\lambda_j} V(w_{j,t}) \tag{8}$$

where λ_j represents the jth scale level, and $w_{j,t}$ is the jth scale level crystal. The main complication here comes from making the wavelet variance time independent, the calculation of the variance for different scale levels (because of boundary problems) and accounting for when decimation occurs, as with the DWT. As we are dealing with an MODWT then we can write equation 8 as:

$$\widetilde{\sigma}^2_{x,t}(\lambda_j) = \frac{1}{M_j} \sum_{t=L_j-1}^{N-1} d^2_{j,t} \tag{9}$$

where M_j is the number of crystal coefficients left once the boundary coefficients have been discarded. These boundary coefficients are obtained by combining the beginning and end of the series to obtain the full set of MODWT coefficients, but if these are included in the calculation of the variance this would imply biasedness. If L_j is the width of the wavelet (filter) used at scale j, then we can calculate M_j as $(N - L_j + 1)$[25].

Calculation of confidence intervals is a little more tricky. Here the approach is to first assume that $d_j \sim iid(0, \widetilde{\sigma}^2_j)$ with a Gaussian distribution, so that the sum of squares of d_j is distributed as $\kappa\chi^2_\eta$, and then to approximate what the distribution would look like if the d_j are correlated with each other (- as they are likely to be). This is done by approximating η so that the random variable $(\sigma^2_{x,t}\chi^2_\eta)/\eta$ has roughly the same variance as $\widetilde{\sigma}^2_{x,t}$ - hence η is not an actual degrees of freedom parameter, but rather is known as an "equivalent degrees of freedom" or EDOF parameter. There are then three ways of estimating the EDOF,

[25] $L_j = [(2^j - 1)(L - 1) + 1]$ as an L tap filter will clearly have a larger base at larger scales.

based on i) large sample theory, ii) *a priori* knowledge of the spectral density function and iii) a band-pass approximation. Here large sample theory is used to estimate the EDOF.

Figures 11, 12 and 13 show the various sequences by scale for the real GDP data. The bands represent 95% confidence intervals for the estimate of the variance and the estimates for the EU are shown in a lighter shade in the background[26]. In figure 11, the variances by scale for Germany, France and the Netherlands are remarkably similar to those of the EU, those for Italy and Belgium appear similar for higher scales, but lie above those for the eurozone at lower scales (- sometimes significantly so), and Luxembourg appears to have significantly different wavelet variances at two scales. In figure 12 Spain's variances are similar to those of the eurozone and in some cases even lie below those of the eurozone, while Greek and Swiss wavelet variances are significantly different from eurozone wavelet variances at lower scales but appear similar at higher scales, and lastly Finnish, Irish and Portuguese variances tend to lie above those of the eurozone at most scales, but not significantly so. In figure 13 Iceland's variances lie significantly above those for the eurozone at lower scales, while apart from scale 1, most wavelet variances are similar although above those of the eurozone. As the US is a long-standing monetary union, it would be expected that the scale variances might lie below those of the EU - but as tables 1 and 2 also show, this is not the case either for the data as a whole, or for the scale cystals. This observation might be due to the fact that shocks could be less symmetric or cycles less synchronous in the EU member states cycles, thereby offsetting one another - in the US, closer synchronicity might cause greater crystal variances.

[26]For scale = 32, which represents the wavelet smooth, s5, the variance is usually higher than for most other scales and the confidence interval is small.

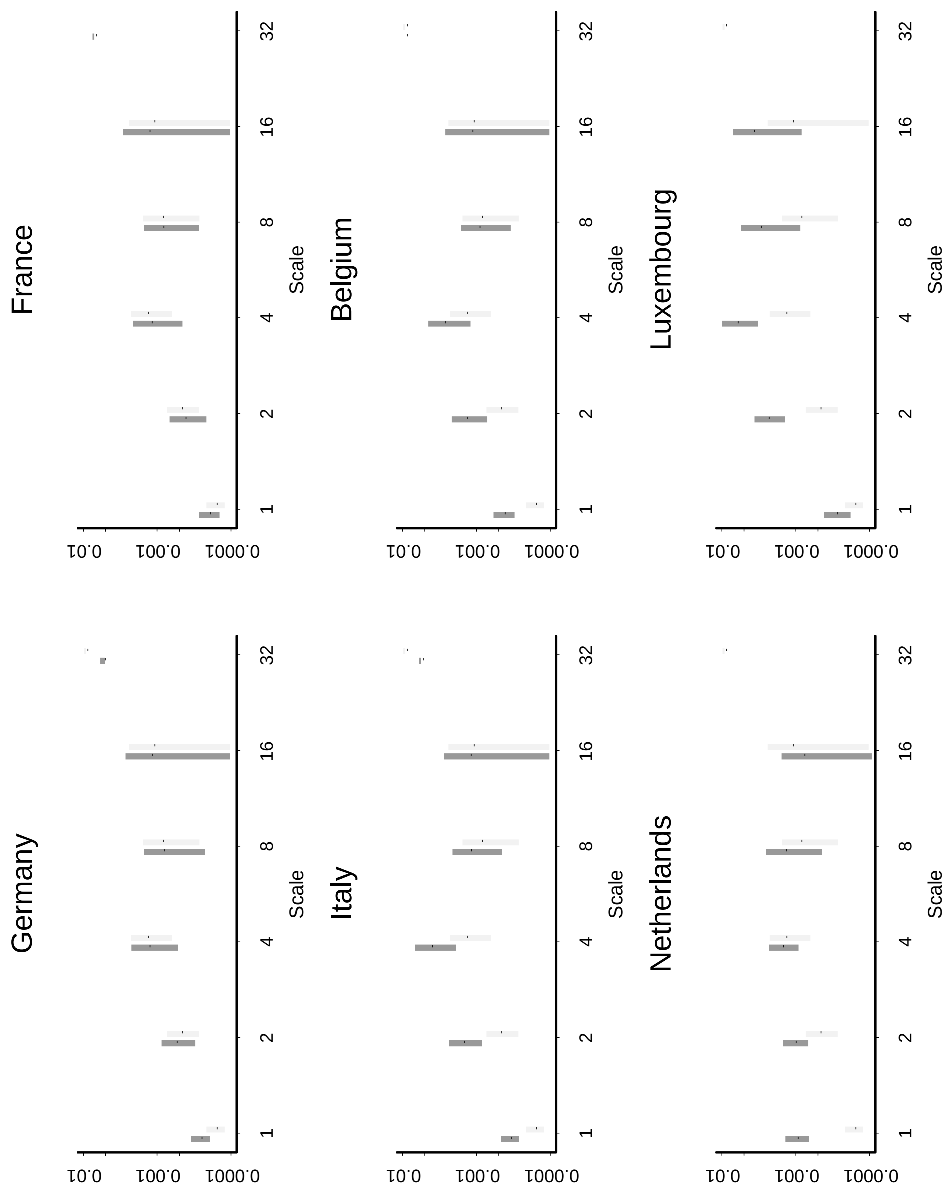

Figure 11: Wavelet variance: group 1

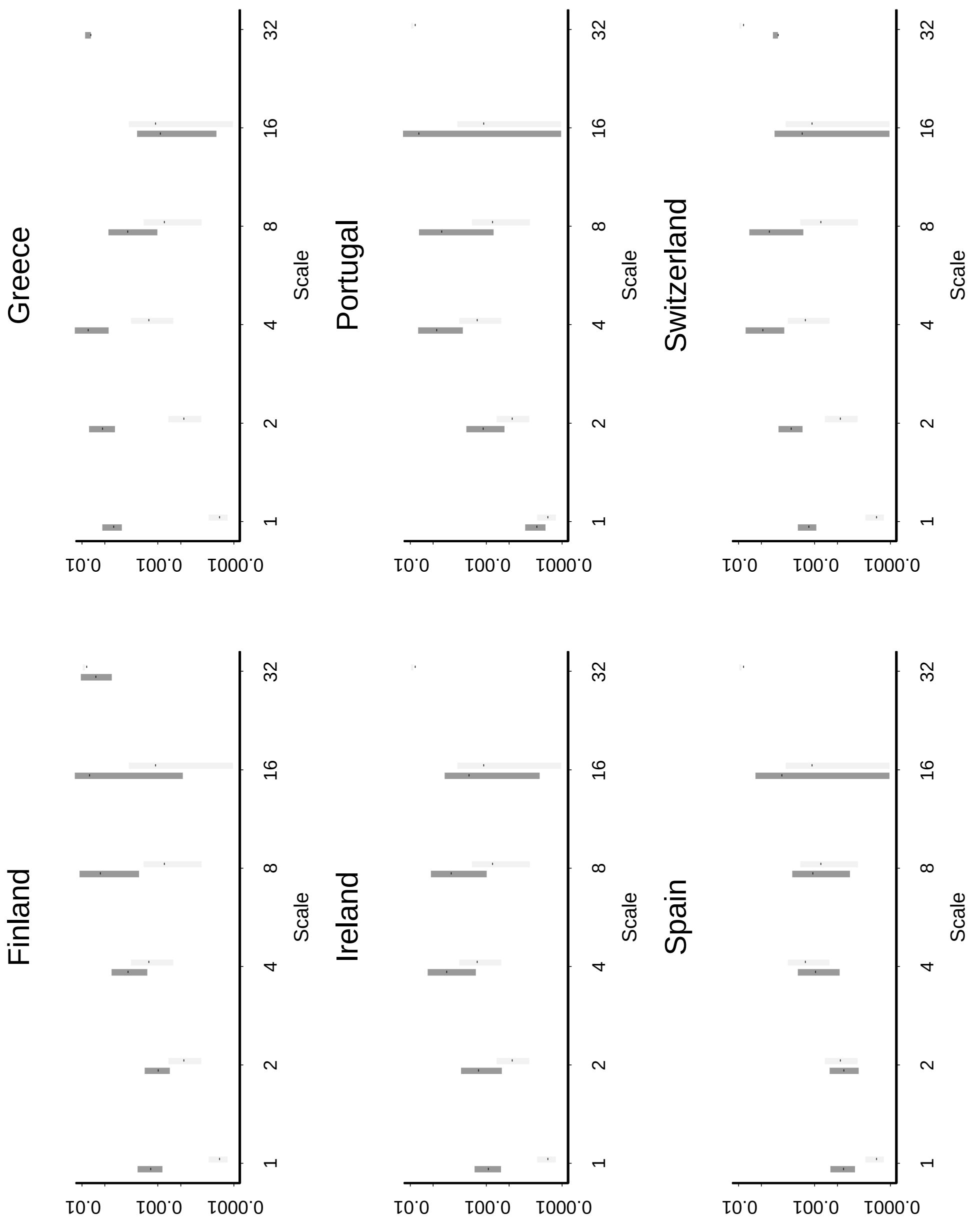

Figure 12: Wavelet variance: group 2

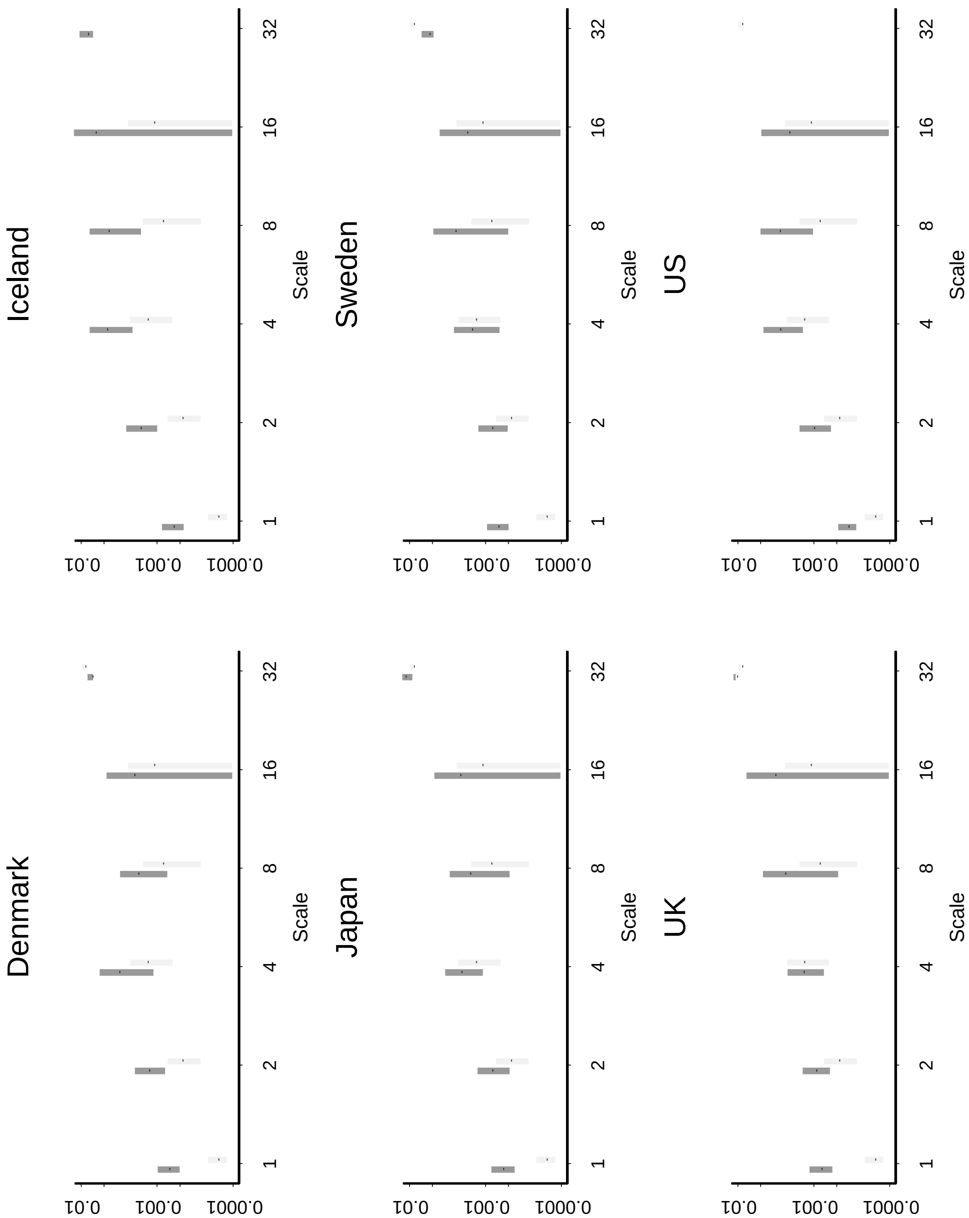

Figure 13: Wavelet variance: group 3

Whitcher, Byers, Guttorp, and Percival (1998) developed a framework for applying a test for homogeneity of variance on a scale-by-scale basis to long-memory processes[27]. The test relies on the usual econometric assumption that the crystals of coefficients, $w_{j,t}$ for scale j at time t have zero mean and variance $\sigma_t^2(\lambda_j)$. This allows us to formulate a null hypothesis of:

$$H_0 : \sigma^2_{L_j}(\lambda_j) = \sigma^2_{L_j+1}(\lambda_j) = ... = \sigma^2_{N-1}(\lambda_j) \tag{10}$$

against an alternative hypothesis of:

$$H_A : \sigma^2_{L_j}(\lambda_j) = ... = \sigma^2_{k}(\lambda_j) \neq \sigma^2_{k+1}(\lambda_j) = ... = \sigma^2_{N-1}(\lambda_j) \tag{11}$$

where k is an unknown change point and L_j represents the scale once the number of boundary coefficients have been discarded.. The assumption is that the energy throughout the series builds up linearly over time, so that for any crystal, if this is not the case, then the alternative hypothesis is true. The test statistic used to test this is the D statistic, which was introduced by Inclan and Tiao (1994) for the purpose of detecting a change in variance in time series. Define P_k as:

$$P_k = \frac{\sum_{j=1}^{k} w_j^2}{\sum_{j=1}^{N} w_j^2} \tag{12}$$

then define the D statistic as $D = \max(D^+, D^-)$ where $D^+ = (\widehat{L} - P_k)$ and $D^- = (P_k - \widehat{L})$ where $\widehat{L}$ is the cumulative fraction of a given crystal coefficient to the total number of coefficients in a given crystal. Tables 6 and 7 report the results of this test by scale[28].

Table 6 crystals d1, d2 and d3 are clearly least likely to have heterogeneity of variance, with perhaps Portugal being the most extreme example of this. In all cases, crystal d4 has homogeneity of variance, with only Finland, Luxembourg and Spain having any significant heterogeneity in the d5 crystal. In table 7 although none of the tests of the d5 crystal found any heterogeneity, the US d4 crystal was significantly heterogeneous at the 10% level. Most of the significant tests were found for the d1 to d3 crystals, with the UK notable as the most extreme example of this.

[27] A good summary of the procedure can be found in Gençay, Selçuk, and Whicher (2001).
[28] Test is significant at *=10%, **=5%, and ***=1%.

	EU	Belgium	Finland	France	Germany	Greece	Ireland	Italy	Lux	NL	Portugal	Spain
d1	0.216*	0.163	0.479***	0.141	0.111	0.226*	0.713***	0.180	0.201	0.357***	0.283***	0.365***
d2	0.415***	0.379**	0.277	0.473***	0.329**	0.508***	0.320*	0.568***	0.268	0.432***	0.545***	0.313*
d3	0.475*	0.286	0.248	0.235	0.217	0.631***	0.300	0.617***	0.189	0.437*	0.592***	0.357
d4	0.375	0.199	0.376	0.388	0.426	0.380	0.349	0.546	0.387	0.316	0.547	0.495
d5	0.533	0.560	0.999**	0.756	0.749	0.842	0.549	0.585	0.994*	0.918	0.627	0.999**

Table 6: Wavelet variance homogeneity test: eurozone

	Denmark	Iceland	Japan	Sweden	Switzerland	UK	US
d1	0.162	0.597***	0.197	0.288***	0.568***	0.481***	0.306***
d2	0.208	0.396***	0.285	0.486***	0.512***	0.500***	0.446***
d3	0.522**	0.333	0.592***	0.538**	0.490**	0.687***	0.543**
d4	0.630	0.408	0.257	0.433	0.510	0.612	0.446*
d5	0.876	0.805	0.685	0.948	0.525	0.923	0.306

Table 7: Wavelet variance homogeneity test: non-eurozone

Covariance by scale can also be obtained using similar methods to those described above, so that wavelet variances and covariances can be used together to obtain scale correlations between series and confidence intervals can be derived for the correlation coefficients by scale (these are derived in Whitcher, Guttorp, and Percival (2000)). The correlation between the various real GDP series and the eurozone are estimated and plotted in figures 14, 15 and 16 by scale, although for the wavelet smooth, s5, denoted scale=32 only point estimates of correlation are shown, as no correction can be made for boundary effects given that there is no defined frequency band for the wavelet smooth.

The difference between figure 14 and 15 is certainly quite striking. In figure 14, France appears to possess the highest correlations by scale, with Germany closely following, but Luxembourg possesses low correlations for the d1 crystal, and for d4 and d5 correlations, these are not significantly different from zero. In figure 15, Finland and Ireland have no wavelet correlations that are significantly different from zero, and for Portugal, Spain and Switzerland, only the d2 and d3 crystal correlations are significantly different from zero. In figure 16 only Denmark possesses 3 sets of crystals with significantly positive correlations, while Japan, Sweden, the UK and US only have one significantly crystal correlation, while Iceland possesses none.

This analysis of correlation at different growth cycle periodicities demonstrates that although there is a high degree of correlation in longer cycles, for shorter cycles, these correlations are not always significantly different from zero. The sample of EU member states roughly divides into 4 groups:

a) those member states that have significantly positive correlations at all cycle frequencies (Germany, France and Belgium);

b) those member states that have significantly positive correlations except for the d5 crystal, representing 4-8 year cycles (Italy, Netherlands);

c) those member states that have two or more significantly positive crystal correlations (Luxembourg, Portugal, Spain, Switzerland and Denmark); and

d) those member states that have one or less signficantly positive crystal correlations (Finland, Greece, Ireland, Sweden, UK)

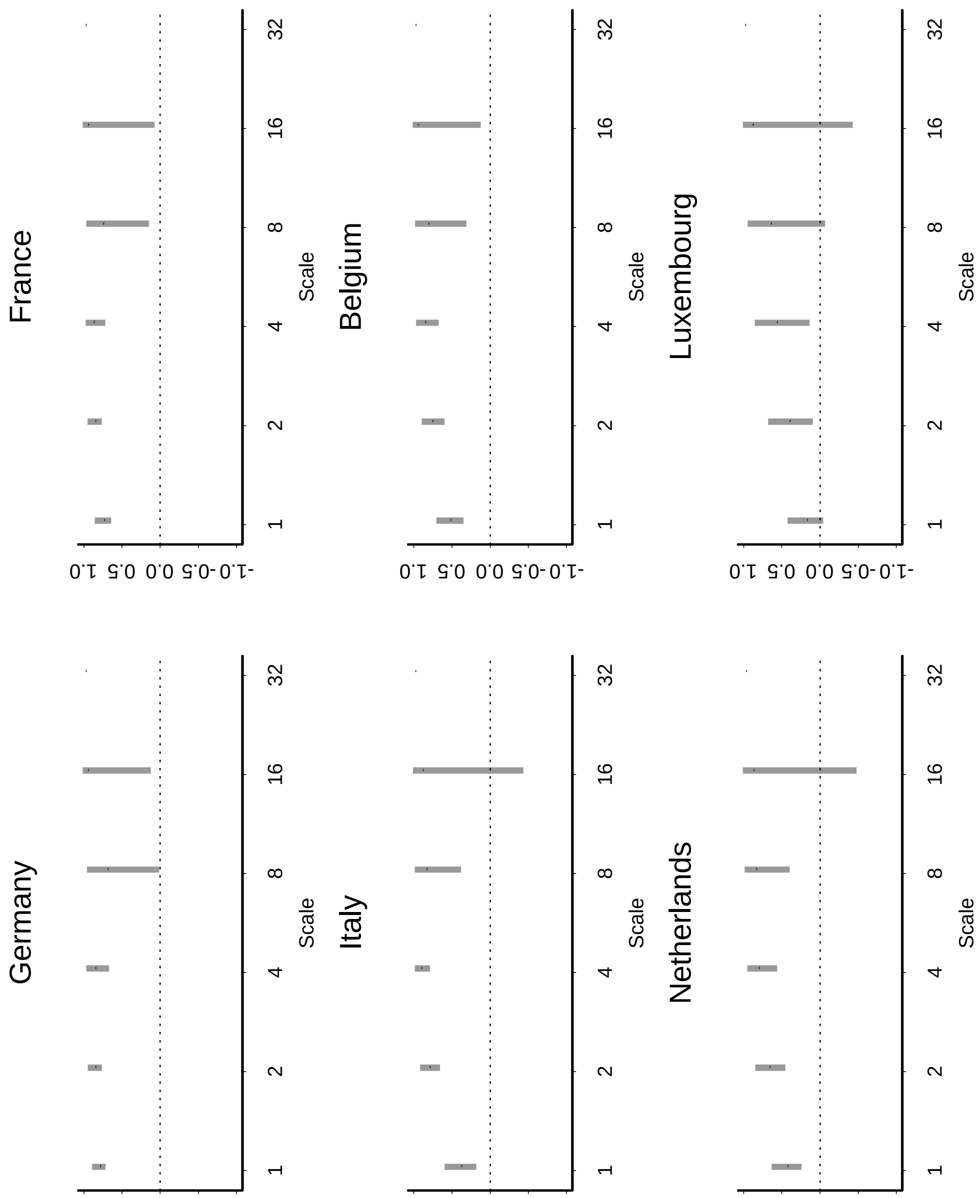

Figure 14: Wavelet correlation: group 1

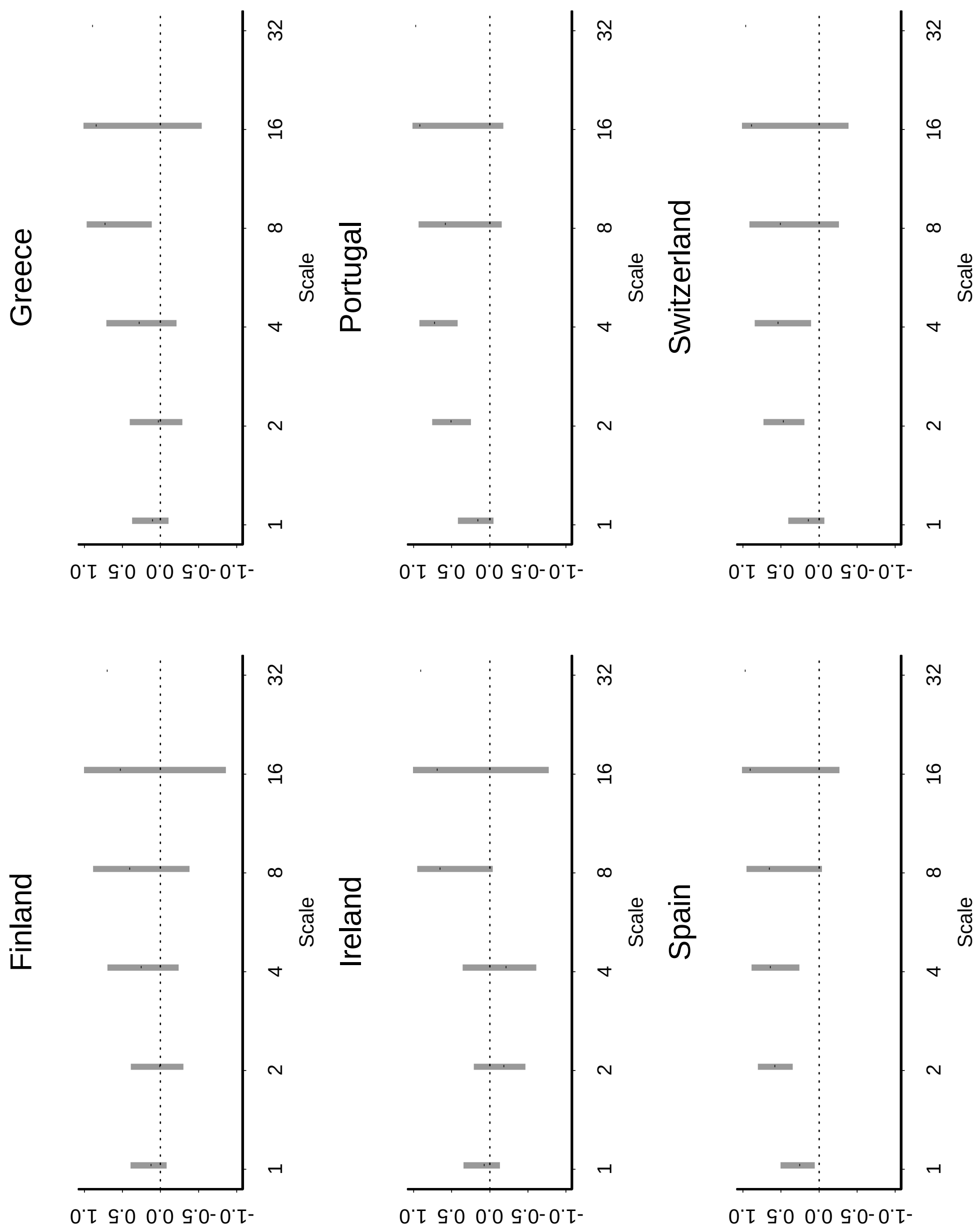

Figure 15: Wavelet correlation: group 2

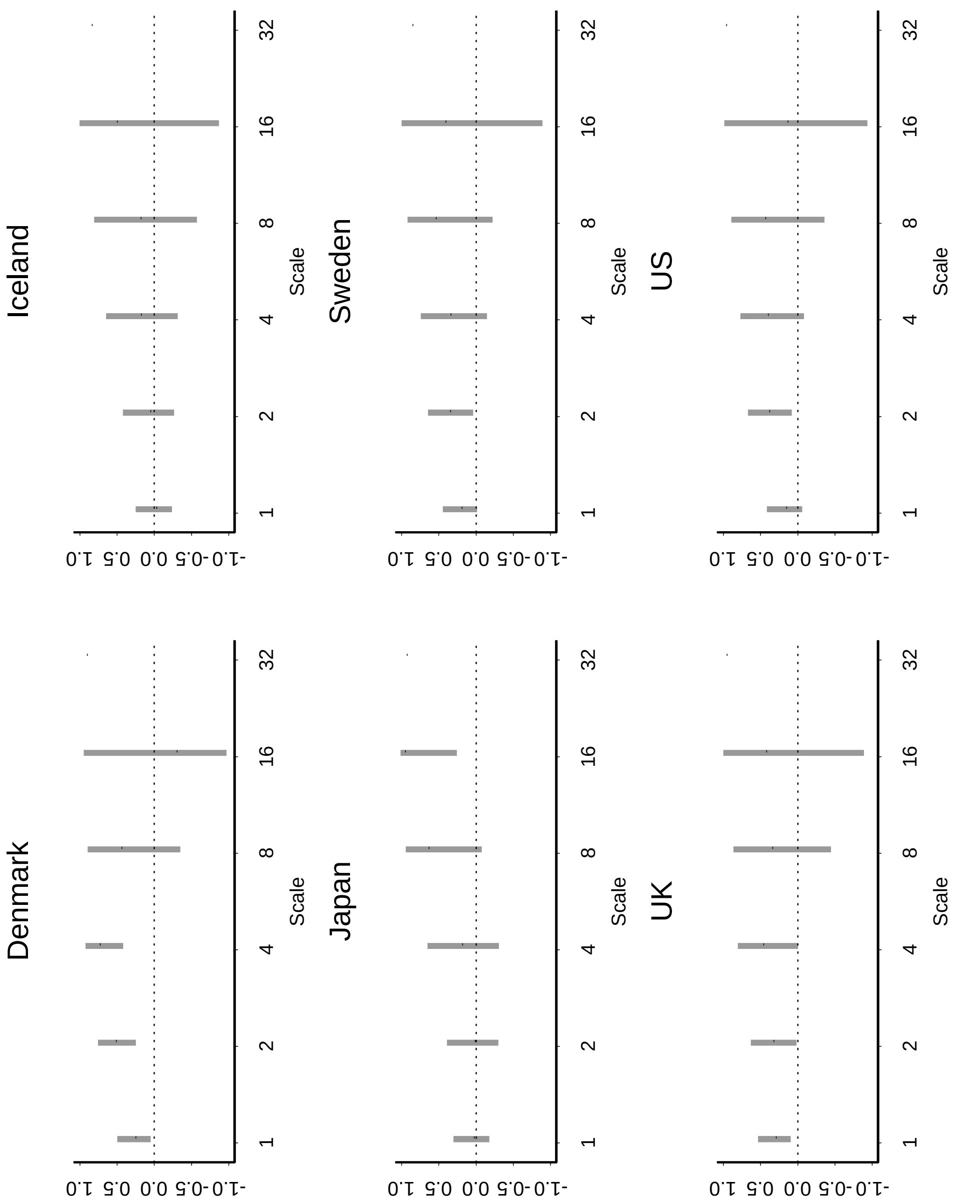

Figure 16: Wavelet correlation: group 3

Once scale correlations have been obtained, co-correlations can be calculated so as to study the phase relationship versus the eurozone. Each crystal is lagged against its eurozone equivalent and figures 17, 18 and 19 show co-correlations[29] . These co-correlations measure only how the correlations change by lagging the country series against the equivalent eurozone series, so they are able to study phasing of cycles rather than the magnitude of the correlations themselves. The co-correlations against the eurozone in figure 17 indicate that all these eurozone countries have particularly synchronous growth cycles to those of the EU, with no phasing issue at any cycle except perhaps for the d4 crystal in the case of Germany and France (where there is a slight lead) and the d5 crystal in the case of Italy and the Netherlands (again, a lead). In figure 18, Finland, Portugal and Spain tend to have relatively synchronous cycles with the eurozone, but Greece and Ireland only have synchronicity in the higher order crystals, with Ireland having negative contemporaneous correlations for d2 and d3 crystals, as could be seen also in figure 15. Switzerland also appears not to be have zero phase against the eurozone, but here it appears that there is a lag relationship of roughly 2 quarters in the d3 and d4 crystals. Turning to figure 19, what is striking here is the similarity between the co-correlation plots for the UK, Sweden and the US. In all cases there is a lead relationship with the eurozone, particularly at longer cycles, with the d5 crystal indicating roughly a 10 quarter lead against the eurozone for the US, and roughly an 8 quarter lead for the UK. Denmark is synchronous in shorter cycles, but verging on being asynchronous in its d5 crystal. Iceland appears not to have any high correlations against the eurozone at shorter frequencies, but has a lead against the eurozone in its d5 crystal, and Japan is only synchronous in longer term cycles.

In terms of synchronicity of cycles, then, the eurozone members roughly fall into the following groupings:

a) those member states that are relatively well synchronised against the eurozone (Germany, France, Italy, Belgium, Netherlands, Luxembourg, Finland, Portugal and Spain);

b) those member states that are synchronised at high frequency cycles, but not at low frequency cycles (Denmark, Sweden and the UK); and

c) those member states that are synchronised at low frequency cycles, but not at high frequency cycles (Greece and Ireland).

[29]The x-axis refers to the lag of the country detail crystal against the eurozone equivalent. Hence a high correlation at -5 indicates the correlation value if the country series is lagged by 5 quarters against the eurozone aggregate.

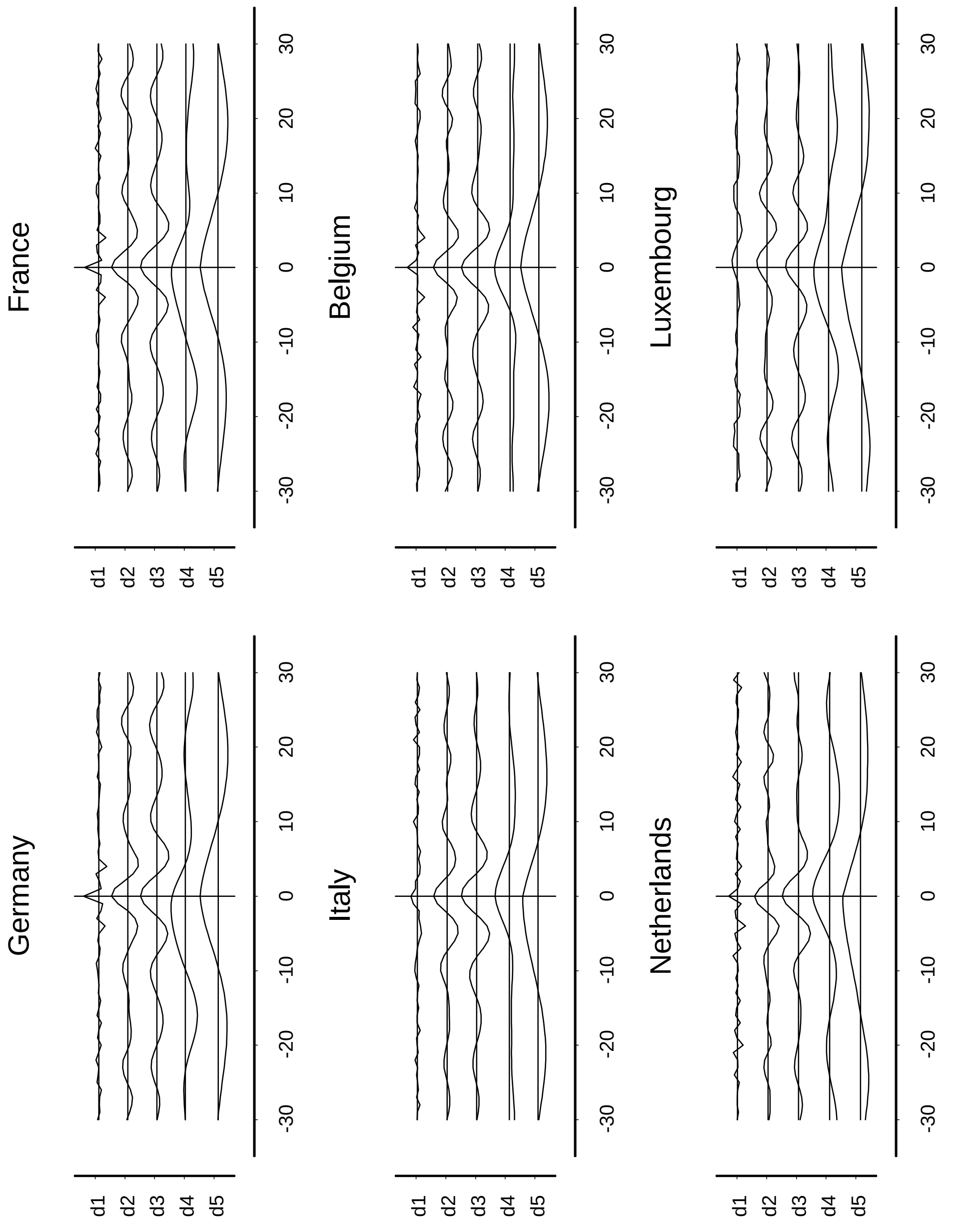

Figure 17: Wavelet co-correlation: group 1

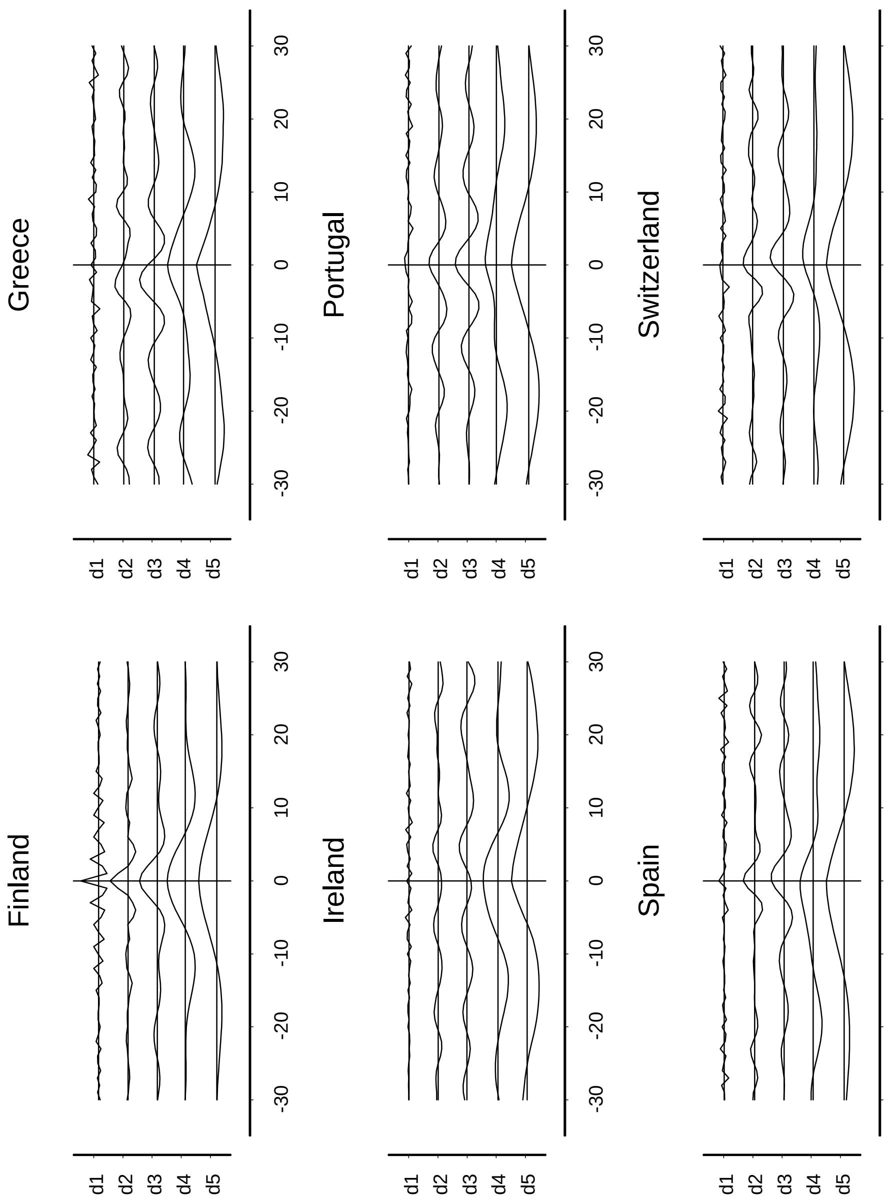

Figure 18: Wavelet co-correlation: group 2

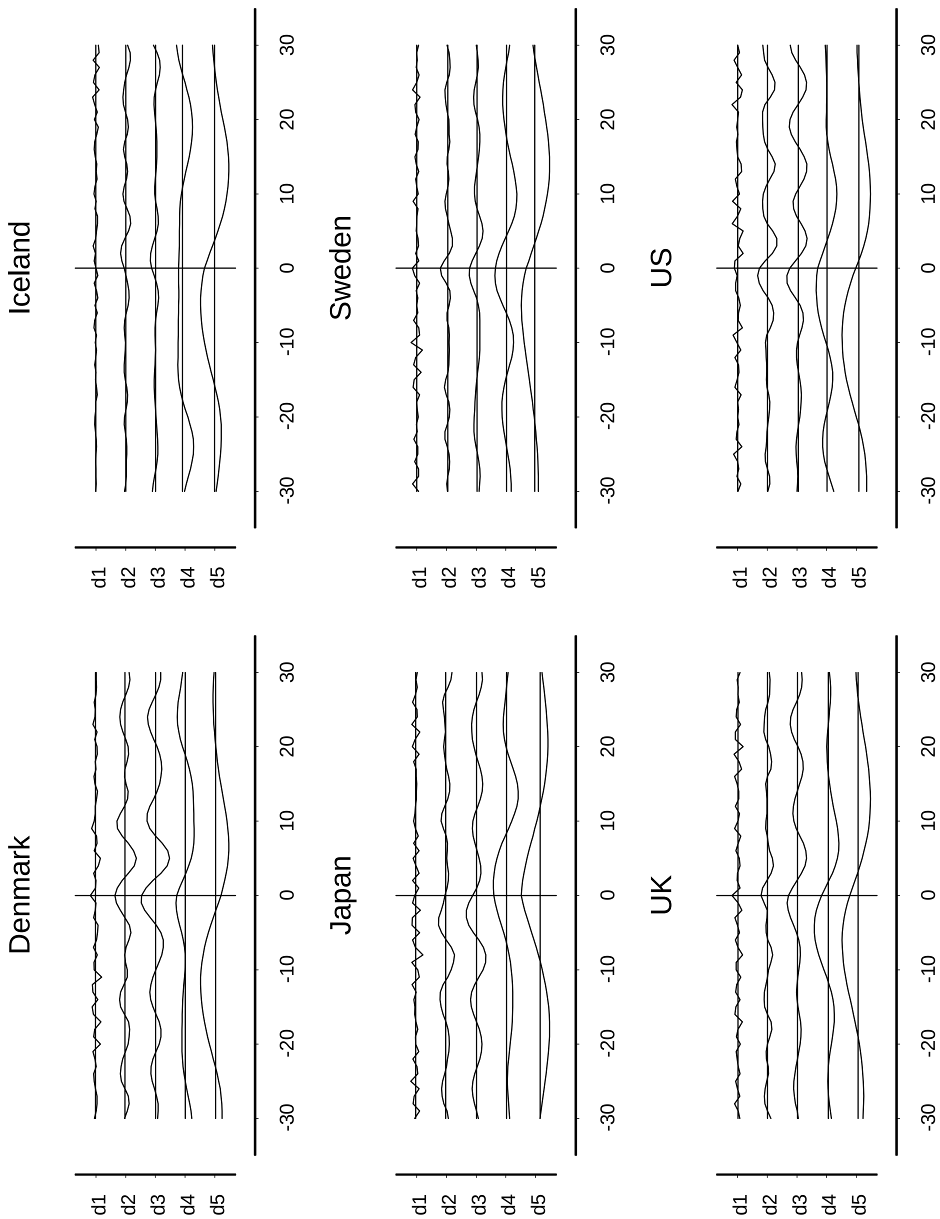

Figure 19: Wavelet co-correlation: group 3

5 Dynamic conditional correlation (DCC) analysis

5.1 Methodology

The analysis so far has been of a static nature, but it is useful to have some idea of how these wavlet correlations change through time. In order to do this, Engle's DCC analysis is used[30]. Although the notion of dynamic correlation has traditionally been implemented by use of rolling regressions, Engle (2002) introduced the notion of DCC using the GARCH framework of time series analysis, so as to incorporate conditional correlations into a dynamic framework. Full details of this approach are available in Engle and Sheppard (2001) - below only an abridged version of the approach is presented.

Let $\mathbf{y}_{jt} = \begin{bmatrix} y_{1jt} & y_{2jt} \end{bmatrix}'$ be a vector containing the two crystals from the real GDP series, one for the country concerned and the other for the EU series. Dropping the j subscript denoting the crystal scale for ease of exposition, then a conditional mean equation in reduced form VAR format can be written as:

$$A(L)\mathbf{y}_t = \varepsilon_t \tag{13}$$

where $A(L)$ is a polynomial matrix in the lag operator, L, and $\boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \mathbf{H}_t)$,for all t, where $\mathbf{H}_t$ is a conditional variance-covariance matrix. The DCC-GARCH framework can be best understood by re-writing $\mathbf{H}_t$ as:

$$\mathbf{H}_t = D_t R_t D_t \tag{14}$$

where $D_t = diag\left\{\sqrt{h_{it}}\right\}$ is a 2x2 diagonal matrix of time-varying standard deviations from univariate GARCH models and $R_t \equiv \left\{\rho_{ij}\right\}$ for $i, j = 1, 2,$.where ρ_{ij} are conditional correlation coefficients. The elements in D_t follow a univariate GARCH(p,q) process in the following manner:

$$h_{it} = \omega_i + \sum_{p=1}^{P_i} \alpha_{ip}\varepsilon_{it-p}^2 + \sum_{q=1}^{Q_i} \beta_{iq} h_{it-q} \tag{15}$$

Engle (2002) had a specific structure for the DCC(M,N) process, and it can be reproduced as:

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1} \tag{16}$$

[30] This was implemented using the RATS program.

where:

$$Q_t = (1 - \sum_{m=1}^{M} a_m - \sum_{n=1}^{N} b_n)\overline{Q} + \sum_{m=1}^{M} a_m(\xi_{t-m}\xi_{t-m}^{'}) + \sum_{n=1}^{N} b_n Q_{t-n} \tag{17}$$

where $\xi_t = \varepsilon_{it}/\sqrt{h_{it}}$, which is a vector containing standardized errors, $Q_t \equiv \{q_{ij}\}_t$ is the conditional variance-covariance matrix of standardized errors with its unconditional variance covariance matrix, $\overline{Q}$,obtained from the first stage of estimation, and Q_t^* is a diagonal matrix containing the square root of the diagonal elements of Q_t:

$$Q_t^* = \begin{bmatrix} \sqrt{q_{11}} & 0 \\ 0 & \sqrt{q_{22}} \end{bmatrix} \tag{18}$$

From R_t, the conditional correlation between y_{1t} and y_{2t} can be obtained, namely:

$$\rho_{12,t} = \frac{q_{12,t}}{\sqrt{q_{11,t}q_{22,t}}} \tag{19}$$

The system is estimated using the maximum likelihood method in which the log-likelihood can be expressed as:

$$\mathcal{L} = -\frac{1}{2}\sum_{t=1}^{T}\left\{2\log(2\pi) + 2\log|D_t| + \log|R_t| + \xi_t^{'}R_t^{-1}\xi_t\right\} \tag{20}$$

The estimation approach is then as follows:

i) obtain one period ahead residuals from the estimation of a bivariate VAR of y_{1t} on y_{2t} with lag length selected by the Bayesian information criterion;

ii) estimate the univariate GARCH processes for the output and price residuals; then

iii) estimate the conditional correlation matrix (R_t) using the log-likelihood function (eqn. 20) conditional on the GARCH parameter estimates in ii).

5.2 Empirical results

Table 7 provides some diagnostics for the series as a whole. The first panel of the table reports the Ljung-Box test for serial correlation using the squares of the residuals. The Q-statistics for an order of 20 clearly indicate the presence of serial correlations in both the original series and the d4 crystal at the 1% level of significance. The second column of both tables shows test results for Engle's LM test for ARCH with 20 lags, and indicates the presence of conditional heteroskedasticity at the 1% level of significance. The following three columns conduct various tests of normality of the residuals, and indicate that several of the series are non-normal. This suggests the precautionary use of Bollerslev and Wooldridge's quasi-maximum likelihood method to generate consistent standard errors that are robust to non-normality. A comparison of the log-likelihood values among alternative lag specifications of the DCC-GARCH model suggests that the data are best represented by a DCC(1,1) with each of the conditional variances captured by a GARCH(1,1) model.

	Ljung-Box Q(20)	ARCH LM(20)	Skewness	Kurtosis	Bera-Jarque
EU	322.95*	86.31*	-0.19	-0.11	0.87
France	312.72*	92.13*	-0.11	-0.64	2.53
Germany	144.08*	71.09*	-0.02	-0.22	0.27
Italy	446.89*	67.10*	0.45**	0.22	4.92*
Spain	225.29*	65.85*	0.60*	1.42*	19.13*
Finland	175.08*	77.64*	-0.94*	1.31*	29.01*
Sweden	170.90*	74.82*	-0.71*	0.71***	13.81*
Switzerland	139.81*	68.54*	-0.11	4.15*	95.04*
UK	118.65*	61.63*	-0.33	1.56*	15.84*
US	205.11*	83.98*	-0.44*	0.31	4.74*

Table 8: Diagnostic tests for GDP series

Figures 20 through 22 now provide stack plots of the dynamic conditional correlations, first for the original data, with shaded areas corresponding to 95% confidence intervals, and then figures 23 to 28 show dynamic conditional correlations for the d4 and d5 scale crystals, as these roughly align with the conventional business cycle. The red lines denote the beginning of recessions as announced by the euro area business cycle dating committee (see appendix A for more details). The dynamic correlation plots for other crystals are relegated to an appendix.

5.3 Discussion and analysis

5.3.1 Discussion

To make the most effective use of the results graphically displayed in the previous section, the conditional correlations for the data as a whole are first analysed, and then each crystal is separately analysed in turn.

In terms of the GDP data as a whole (Figures 20 to 22), in figure 20, it is clear that France and German dynamic correlations are very similar, which suggests similar movements in

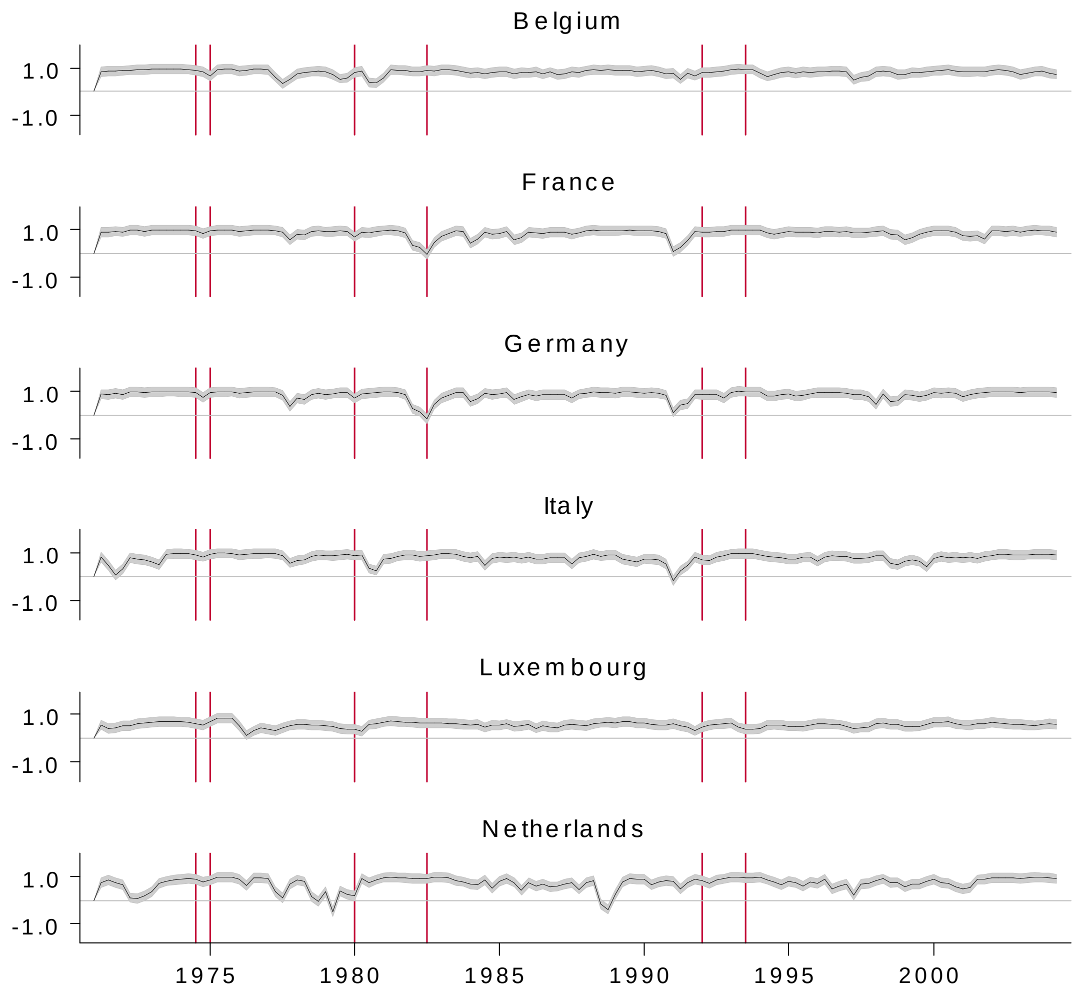

Figure 20: DCC for original data: group 1

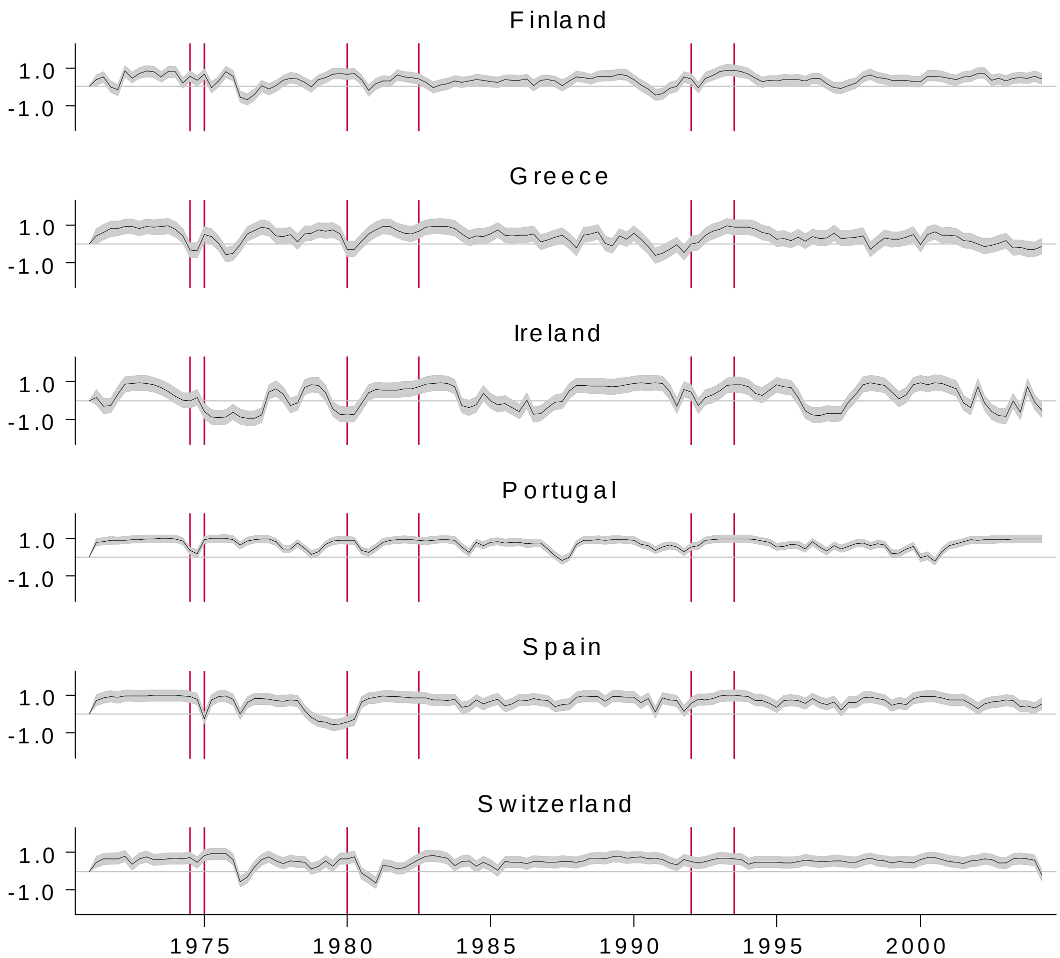

Figure 21: DCC for original data: group 2

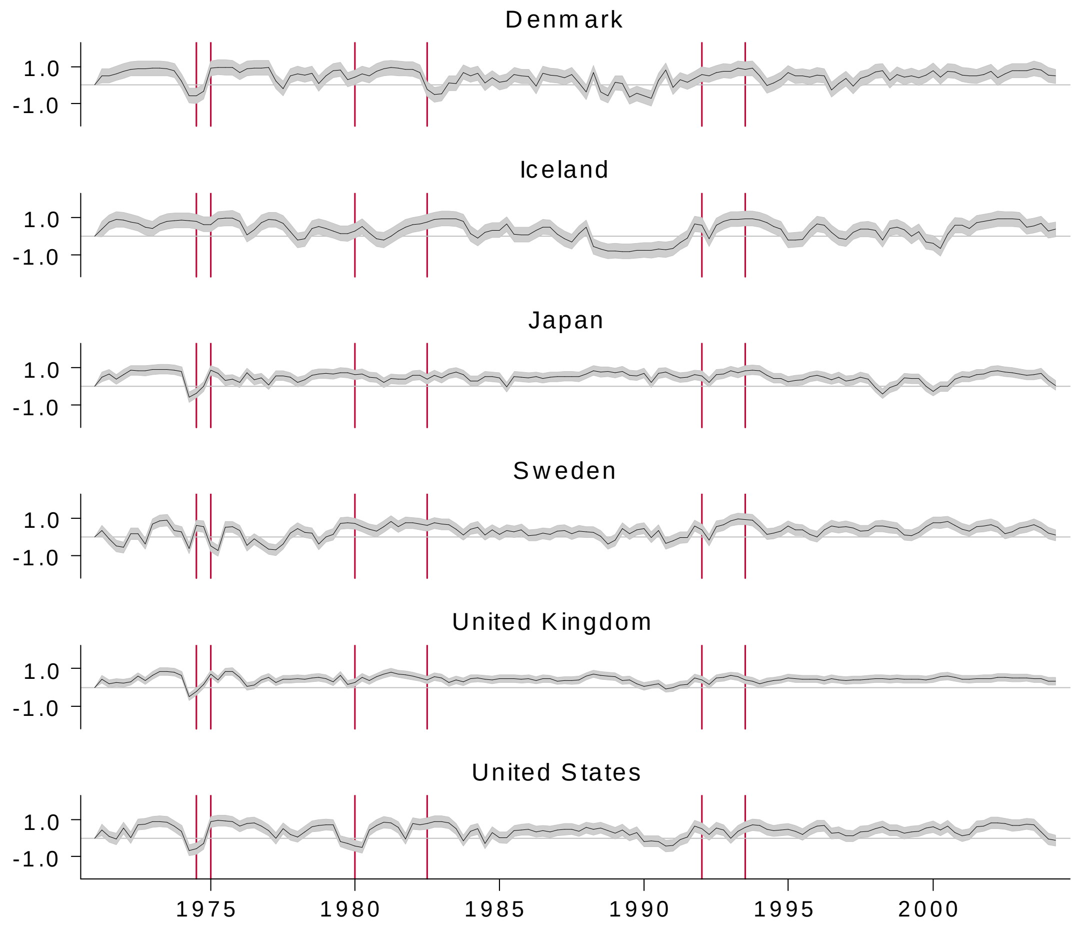

Figure 22: DCC for original data: group 3

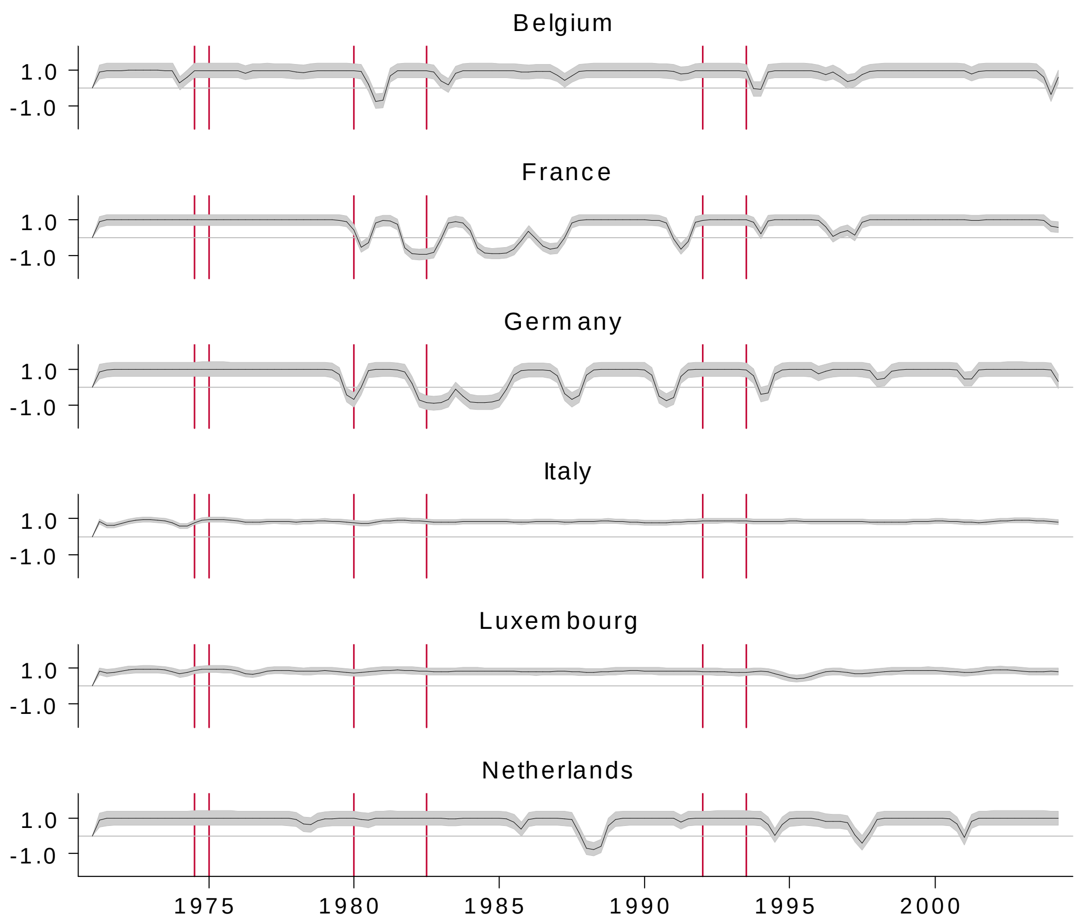

Figure 23: DCC for d4: group 1

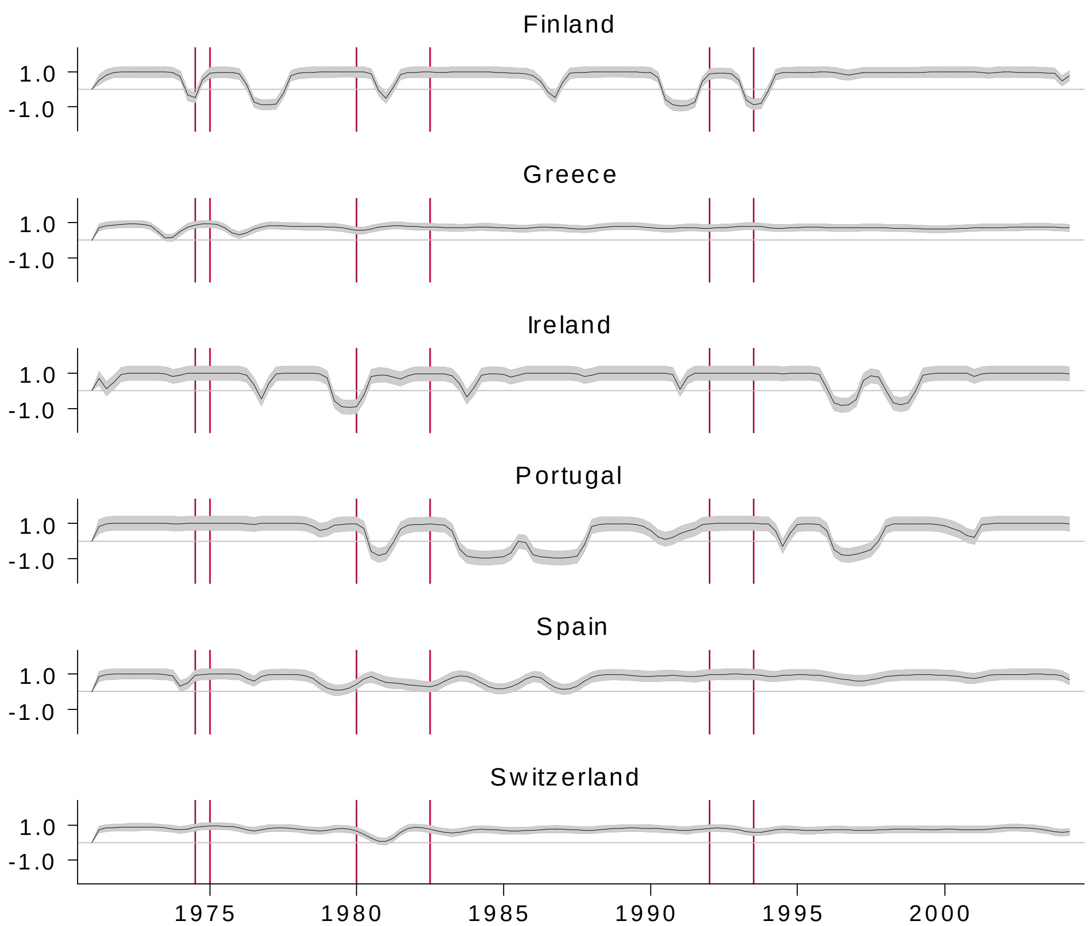

Figure 24: DCC for d4: group 2

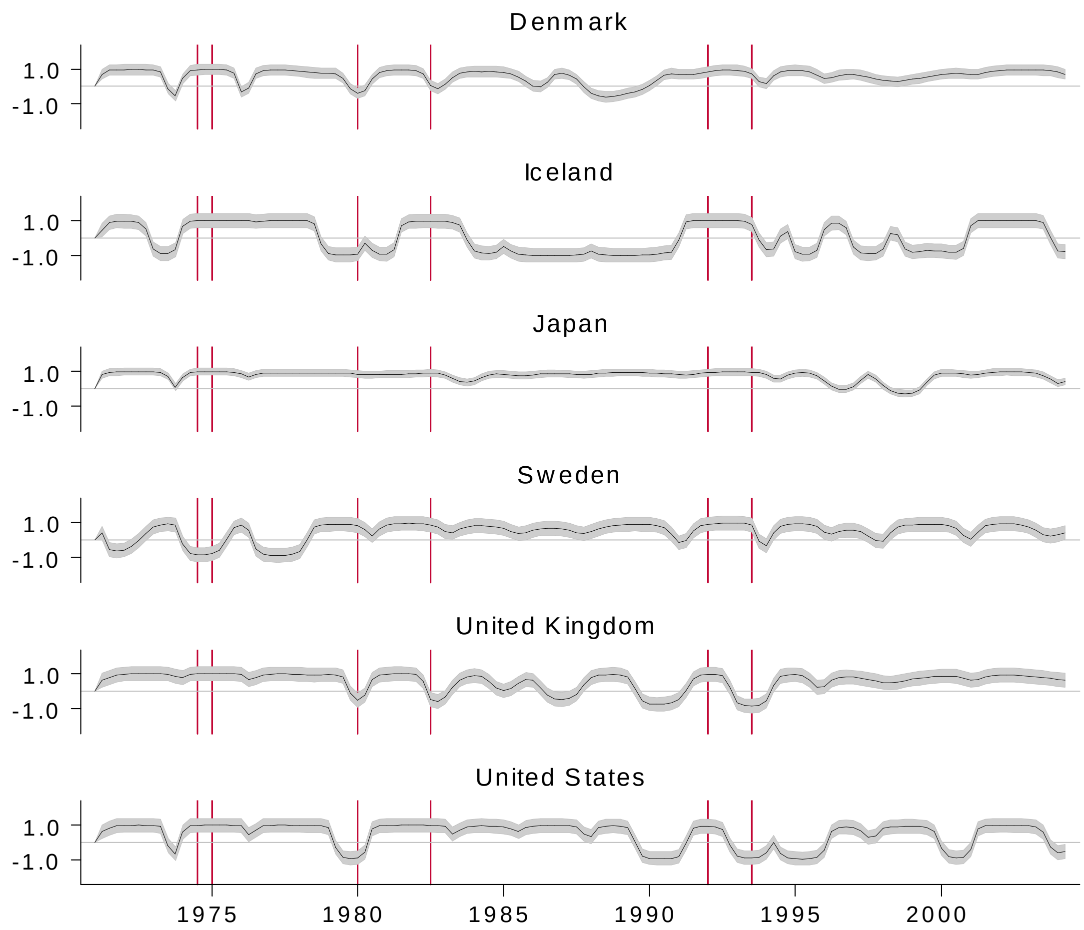

Figure 25: DCC for d4: group 3

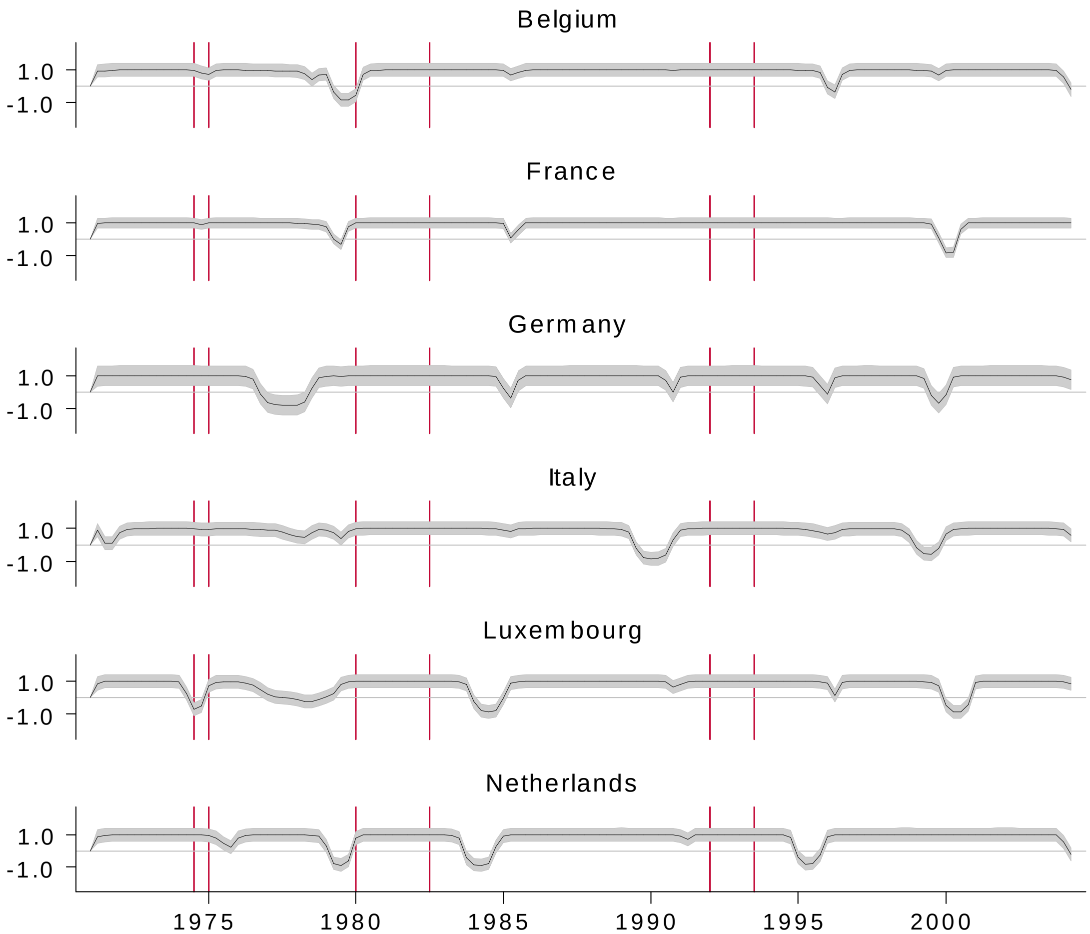

Figure 26: DCC for d5: group 1

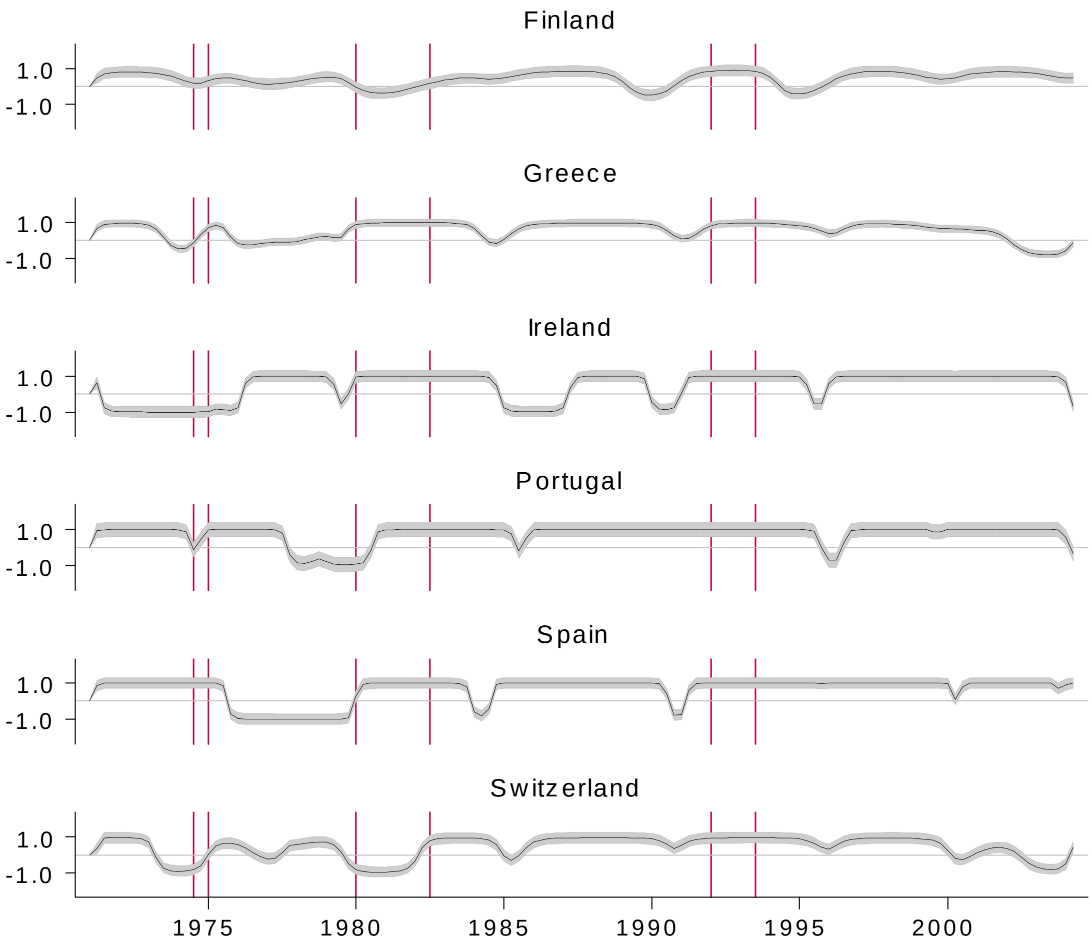

Figure 27: DCC for d5: group 2

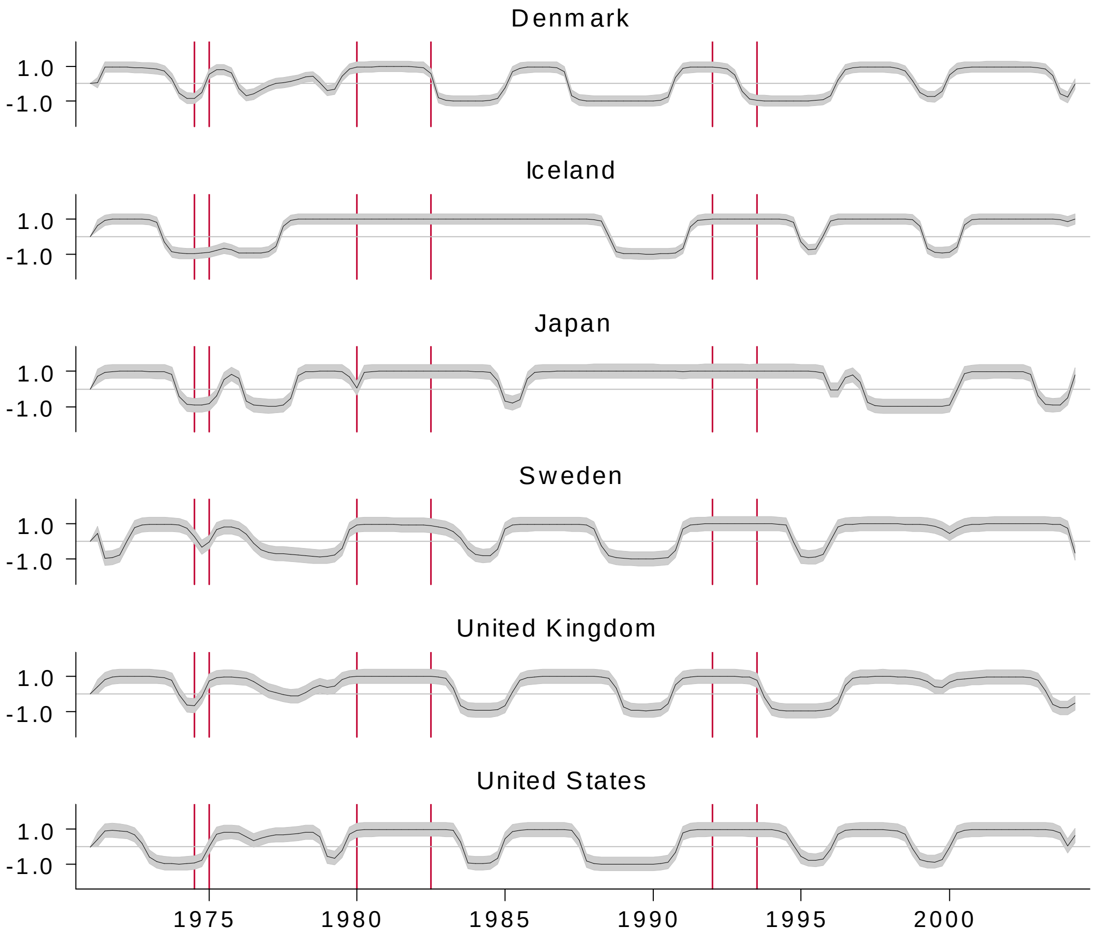

Figure 28: DCC for d5: group 3

real GDP[31]. Nevertheless, correlations against the eurozone suggest that at the end of the early 1980s recession, both Germany and France grew differently from the rest of the eurozone, and also before the 1990s recession in 1991-2, both countries, along with Italy, had different growth paths from the rest of the eurozone[32]. Reassuringly, through the latter part of the 1990s and the 2000s, dynamic correlations have been significantly positive. In figure 21, among new eurozone members, Ireland seems to have least consistently positive correlations against the rest of the eurozone, and the recent fall in correlation in Greece must be some cause for concern as well. One of the noticeable things in figure 21 is that several member states have higher correlations at the end of recessions than at the beginning of recessions, suggesting that turning points are more correlated than growth in between the recessions. Portugal and Spain have largely significantly positive correlations, but Finland less so. In figure 22, dynamic correlations for Denmark, Sweden and the UK are more often significantly positive in the 1990s, compared with the 1980s, which corroborates the notion of a convergence in European business cycles, but Iceland, Japan, and the US, as might be expected, do not exhibit convergence.

i) **Crystal d1 (3m-6m cycles; see appendix B).** In the MODWT analysis above, this crystal seemed to contain noise rather than any growth cycles. In figure 29, rather unexpectedly, French and German (very) short-term dynamic correlations are high, and particularly those for France. The Belgian and Netherlands correlations at this frequency are very similar, suggesting similar short term growth cycles, but Luxembourg's correlations have been negative throughout the 1990s and 2000s. No other countries exhibit positive correlations at this frequency of cycle, except for Ireland which did so from 1997 until late 2003.

ii) **Crystal d2 (6m-1yr cycles; see appendix B).** In figure 32, again France and Germany have consistently positive and nearly always significant dynamic correlation coefficients. Italy's correlations have been almost always significantly positive over the period 1992 until 2004, but the Netherlands appears to have the highest and most consistently positive correlations with the eurozone at this frequency. In figure 33, Spain's correlations appear to have been positive over the period 1992 until 2002, but apart from the observation that Switzerland appears to be mostly significantly

[31] In fact the correlation between France and Germany in this dataset was 0.921, the highest correlation between any two countries in the dataset.

[32] This is likely an ERM effect, as "hard core" ERM members took whatever steps were necessary to stay in the ERM (- by shadowing Bundesbank policy), following the crises that took place that year.

postively correlated with the eurozone at this frequency, there appears to be no other consistent pattern for other countries.

iii) **Crystal d3 (1yr-2yrs cycles; see appendix B).** One of the interesting features of this frequency cycle is that France and Germany have different correlations with the eurozone aggregate, as figure 35 shows. Correlations in both Belgium and the Netherlands appear to have the same profile in 2002, however, suggesting a short-term departure from the average growth profile of the rest of the eurozone. In figure 36, Portugal, Spain and Switzerland all appear to have had significantly positive correlations against the eurozone from the late 1980s onwards, with Greece also having a positive but mostly insignificant correlation throughout from the 1980s to date. At this frequency Denmark appears to have had significantly positive correlations with the eurozone throughout the entire period.

iv) **Crystal d4 (2-4yrs cycles; figures 23 - 25).** As by some measures this is the frequency of the conventionally measured business cycle, these are perhaps important crystals to analyse. In figure 23 Italy and Luxembourg have consistently positive correlations throughout the entire period, but here cycles between France and Germany and the rest of the eurozone appear to have been significantly negative, particularly in the early 1980s, although at this frequency it is noticeable that since the mid 1980s German and the Netherlands correlations appear to be quite closely linked. As monetary policy for the two member states was very closely linked during this period, it suggests that this might be the cycle at which monetary policy begins to impact growth cycles. In figure 24, Greece, Spain and Switzerland appear to possess positive and mostly significant correlations against the eurozone, and Finland, Ireland and Portugal's correlations do not seem to follow any consistent pattern. In figure 25 it is surprising to see such a consistently positive correlation for Japan against the EU, apart from 2 episodes in 1996 and 1998, which probably coincides with events before and after the East Asian crisis. Also notable is the fact that correlations for both Denmark and the UK have been mostly significantly positive since the mid-1990s.

1. **Crystal d5 (4-8yrs cycles; figures 26 - 28).** Once again, this crystal is important to analyse, as it represents the higher part of the frequencies at which the conventional business cycle occurs. In figure 26 and 27 it is interesting to note the alignment of variations in correlations, none of which occur during recessions, indicating that turning points are relatively well aligned across member states in the eurozone. De-

viations from high correlations with the eurozone appear to have occurred among different sets of member states but in a relatively synchronised manner, in 1984 (Luxembourg, the Netherlands, Spain, with smaller aftershocks in France, Germany and Greece), 1990 (Italy, Germany, Finland, Ireland and Spain), 1996 (Netherlands, Belgium, Germany, Ireland and Portugal) and then in 1999/2000 (France, Germany, Italy and Luxembourg). Apart from these brief departures, correlations in these member states remained significantly positive, with perhaps the exception of Greece, Finland and Ireland. In figure 28 it is once again striking to see the way that correlations with the eurozone are high during the eurozone recessions in the 1980s and 1990s, suggesting that there is a high degree of correlation between business cycle turning points internationally as well as within the EU. As might be expected given previous comments, the correlation profiles for Iceland, Sweden, the UK and the US appear quite similar.

vi) **Crystal s5 (8yrs cycles and above; figures 38 - ??).** Given that this crystal possesses most energy, it is intriguing to see that the there is little pattern in these dynamic correlations. This either implies that these cycles need to be properly resolved as detail crystals before any patterns can be discerned, or that this crystal contains mostly idiosyncratic trends or residual "drift".

5.3.2 Analysis

It is now interesting to ask whether correlations in growth cycles with the eurozone at different frequency cycles are converging. To this end, the data was split into two samples 1989-1999[33] and after 1999, and average DCCs were calculated for the data as a whole, and each crystal. The results appear in tables 9 and 10 with upper figures in box representing the average dynamic conditional correlation for 1989Q1 to 1998Q4, and the lower figure in each box representing the average dynamic conditional correlation for 1999Q1 to 2004Q2. Using a standard difference in correlations test, significant differences at 10%, 5% and 1% levels are denoted by 1, 2, and 3 asterisks respectively, and are attached to the lower of the two figures in each cell[34].

[33] As the comparison was supposed to isolate the most recent changes in the EU due to EMU, the period 10 years prior to the inception of EMU was used as a benchmark.

[34] The test was for significant differences in the correlations from two randomly selected samples and was a two-sided test using a Fisher transform.

	Belgium	Finland	France	Germany	Greece	Ireland	Italy	Lux	NL	Portugal	Spain
Data	0.82	0.35	0.87	0.83	0.28	0.40	0.74	0.54	0.76	0.71	0.70
	0.85	0.46	0.86	0.92*	0.14	0.19	0.82	0.59	0.81	0.64	0.63
d1	0.46	0.15	0.71	0.80	0.13	0.20	0.16	-0.19	0.43	0.10	0.23
	0.47	0.18	0.72	0.76	0.13	0.46	0.23	-0.27	0.41	-0.17	0.23
d2	0.669	0.38	0.78	0.81	-0.18	0.20	0.57	-0.38	0.67	0.13	0.62
	0.756	0.35	0.89*	0.89	-0.07	0.31	0.80*	0.31***	0.69	-0.16	0.30*
d3	0.790	0.26	0.87	0.61	0.32	0.00	0.86	0.48	0.71	0.75	0.65
	0.762	0.37	0.89	0.99***	0.31	0.16	0.80	0.57	0.81	0.71	0.64
d4	0.870	0.55	0.78	0.73	0.72	0.60	0.83	0.75	0.84	0.54	0.87
	0.887	0.95***	0.96***	0.91**	0.70	0.93**	0.85	0.83	0.92*	0.88***	0.91
d5	0.926	0.38	1.00	0.91	0.77	0.69	0.72	0.95	0.83	0.86	0.88
	0.901	0.65*	0.77***	0.79**	0.11***	0.90**	0.71	0.67***	0.92**	0.91	0.93
s5	0.92	0.15	0.36	1.00	-0.40	-0.676	0.89	0.97	-0.67	0.84	0.73
	0.58***	0.20	0.93***	1.00	0.99***	-0.460	1.00***	-0.18***	0.14***	0.89	0.46*

Table 9: Average DCC for 1989-1998 and 1999-2004Q2: eurozone

	Denmark	Iceland	Japan	Sweden	Switzerland	UK	US
Data	0.314 0.612*	0.093 0.407	0.483 0.446	0.362 0.448	0.572 0.529	0.358 0.466	0.318 0.494
d1	0.144 0.222	0.000 0.030	-0.250 0.090	-0.069 0.312*	0.091 0.080	0.283 0.291	0.123 0.114
d2	0.402 0.363	-0.022 -0.269	-0.045 0.029	0.578 0.381	0.593 0.674	0.302 0.307	0.372 0.337
d3	0.766 0.762	0.123 0.235	0.158 0.222	0.630 0.305*	0.538 0.576	0.360 0.519	0.369 0.389
d4	0.542 0.785*	-0.091 0.137	0.667 0.712	0.602 0.662	0.735 0.745	0.267 0.784***	-0.008 0.363*
d5	-0.057 0.434**	0.393 0.545	0.576 0.091**	0.364 0.836***	0.849 0.060***	0.124 0.515*	0.336 0.541
s5	-0.776 -0.193***	-0.176 1.000***	0.996 0.672***	-0.455 0.677***	0.397 0.135	-0.053 0.984***	0.076 -0.651***

Table 10: Average DCC for 1989-1998 and 1999-2004Q2: non-eurozone

Tables 9 and 10 yield some striking and intriguing results. First, there has been no significant increase in correlation for eurozone member states between the two periods, with the exception of Germany, which perhaps might be expected given that the shock of reunification falls into the prior period. In fact, correlations for Greece, Ireland, Portugal and Spain fell in the post-EMU inception period. Outside the eurozone, Denmark was the only country to experience a significant increase in correlation, although correlations did rise for all other countries outside the eurozone with the exception of Japan, again which might be anticipated. This is likely once again due to the international synchronisation of turning points in business cycles observed earlier, given the generalised slowdown in economic growth for industrialised countries in the period since 1999.

Once the data is decomposed by frequency of growth cycle though, a somewhat different picture emerges. Finland, for example, has an insignificant increase in its overall correlation, but the 2-4 year growth cycles have become much more correlated with the eurozone, and the 4-8 year growth cycles are also more correlated. France also has some intriguing results, as there is virtually no overall correlation change, but the 2-4 year growth cycle and cycles longer than 8 years are now significantly more correlated, while the 4-8 year cycle is significantly less correlated with the eurozone. Given the similarity of the French and German business cycles, the directions in the movement of correlations for the German growth cycle correlations are similar to those of France, except that now it is obvious that the increase in overall German correlations have been caused by an increase in the correlations of cycles at a 1-2 year growth cycle frequency. In the case of Greece, we know that the energy is not as heavily concentrated in the wavelet smooth as for other countries (see table 4), so that even though the s5 crystal is much more highly correlated with the eurozone, the fall in the 4-8 year growth cycle correlation dominates, giving an overall fall in correlation.

As business cycle frequencies are to be found in the d4 and d5 crystals, it is interesting to note that in all member states in the eurozone (the exception being Greece) the d4 correlation increased, and significantly so in Finland, France, Germany, Ireland, the Netherlands and Portugal. For non-eurozone countries, this increase in correlation for the d4 crystal is also true in all cases, but only significantly so for Denmark, the UK and the US. For the d5 crystal, correlation increased significantly for Finland, Ireland, the Netherlands for the eurozone member states, although it decreased significantly for France, Germany, Greece and Luxembourg. For the non-eurozone member states, there was a significant increase in correlation for Denmark, Sweden and the UK, which suggests that the establishment of the euro has had an impact on economies outside of the eurozone area. Clearly unless we know the frequency at which business cycles occurred in each member state, it is difficult to make any judgement about whether there has been further convergence since the inception of EMU, but even without knowing this, we can state[35] that there appears to have been a significant convergence of business cycles with the eurozone for Finland, Ireland, and the Netherlands, and for non-eurozone countries, there has only been significant convergence with the eurozone business cycle for Denmark.

6 Conclusions

In this paper we have studied the co-movement of GDP growth cycles for a set of European member states and other industrialized countries against that of the eurozone using wavelet time-frequency analysis. This decomposed quarterly real GDP data to obtain series according to different ranges of cycle frequencies (or crystals), which correspond to growth cycles at different frequency ranges. Using these crystals a static variance and correlation analysis of the crystals was conducted, and then the crystals were used in a GARCH-DCC model to obtain dynamic conditional correlations for growth cycles against their eurozone counterpart.

The conclusions are as follows:

a) as most of the variables contained higher energy in the wavelet smooth than in the detail crystals, the multiresolution decomposition analysis confirmed the findings of Granger (1966) and Levy and Dezhbakhsh (2003b) that most energy in economic series can be found in longer term fluctuations, based on the shape of the generalized spectrum for economic variables.

b) confining the analysis to cycles of less than 8 years, several different growth cycles consistently appear in the data, notably at several frequencies above the 2 quarter periodicity, some of which contain significant energy as compared with the conventionally measured business cycle. This confirms and extends the notion of growth cycles originally proposed by Zarnovitz (1985) and analysed by Kontolemis (1997) for the G7 and Zarnowitz and Ozyildirim (2002) for the US. To date, most business cycle research has concentrated on the conventionally-measured business cycle, but clearly several growth cycles at different frequencies are also at work.

[35] This is by assuming that if there is a significant increase in correlation for both the d4 and d5 crystals, then there must be convergence in growth cycles at this frequency.

c) the MODWT analysis showed that since the 1970s and 1980s the energy contained in the d1 and d2 crystals (cycles lasting less than one year) has waned, but that relatively more energy is now located in the d3 and d4 (1-4 year cycles) crystals.

d) the MODWT analysis also suggested that recessions appear to occur because of a coincident dip in growth cycles at all frequencies;

e) the static wavelet variance analysis showed that apart from Germany and France, most EU member states have variances for different frequency cycles that are above eurozone variances. Also, it was noted that eurozone variances by scale lie above those for the US, which may be due to offsetting asymmetric shocks. A test was also conducted for homogeneity of variance through time for each crystal, and this was found not to be the case for the high frequency detail crystals, but most low frequency detail crystals possessed variance homogeneity.

f) the static wavelet correlation analysis showed that EU member states roughly fall into one of 4 categories, with high and significant correlations for France, Germany and Belgium, but a low number of significantly positive correlations for Finland, Greece, Ireland, Sweden and the UK).

g) The static co-correlation analysis categorized the EU member states into three groups, with most eurozone member states being highly synchronised against the eurozone aggregate, but some member states were not synchronised against the eurozone either at higher (Greece and Ireland) or lower frequencies (Denmark, Sweden, UK).

h) dynamic conditional correlation coefficients showed that for all crystals, correlations were significantly positive for most of the time for France, Germany and Belgium, but that for other crystals, correlations were not always significantly positive. The correlations for Greece and Ireland were particularly worrysome, as there were periods during which the correlations were significantly negative against the eurozone.

i) analysis of convergence in the dynamic correlation of the growth cycles revealed that there has been no significant increase in the overall correlation of eurozone member states before and after EMU, with the exception of Germany. However, when the analysis is repeated at business cycle frequencies, it appears that significant convergence with the eurozone can be confirmed since 1999 for Finland, Ireland, and the Netherlands; and for non-eurozone countries, only Denmark has converged.

Further research is obviously needed here, as this methodology clearly opens up new avenues of research. One of the mysteries of the frequency domain literature is the existence of very long cycles in the data, that are obviously being picked up by spectral analysis. With long enough series, wavelet analysis might be able to separately "resolve" these long cycles in the data, and identify their approximate periodicity. The second question that springs to mind is "what is driving these growth cycles?" Clearly, more analysis could be done with the crystals for each country to shed some light on the factors driving these growth cycles, now that they have been detected and properly resolved by the methodology used here.

Appendices

A Euro area business cycle chronology

The euro area business cycle dating committee only make an announcement when they think there has been a definite new trough. They have not recently meet formally, but are in contact on a regular basis to review economic data as it appears, both leading (such as http://www.cepr.org/data/EuroCOIN/latest/) and trailing indicators.

The decision the committee reached when it met in 2003 was as follows:

" Euro area GDP has slowed down since the first quarter of 2001. A weak resurgence of positive growth at the beginning of 2002 seems to have come to a new halt. Employment has grown somewhat, while industrial production, after having fallen sharply in 2001, shows weak signs of recovery. Investment has been declining for more than two years, but government consumption rose 2.2% in 2001 and 2.7% in 2002.

Qualitatively, the recent behavior of GDP resembles that of the 1980s recession. Employment, however, is not declining. Based on currently available data, our current judgment is therefore that the euro area has been experiencing a prolonged pause in the growth of economic activity, rather than a full-fledged recession. "

So for the purpose of this study, euro area recessions are given as: 1974q3 to 1975q1, 1980q1 to1982q3, and 1992q1 to 1993q3.

B Dynamic conditional correlation plots

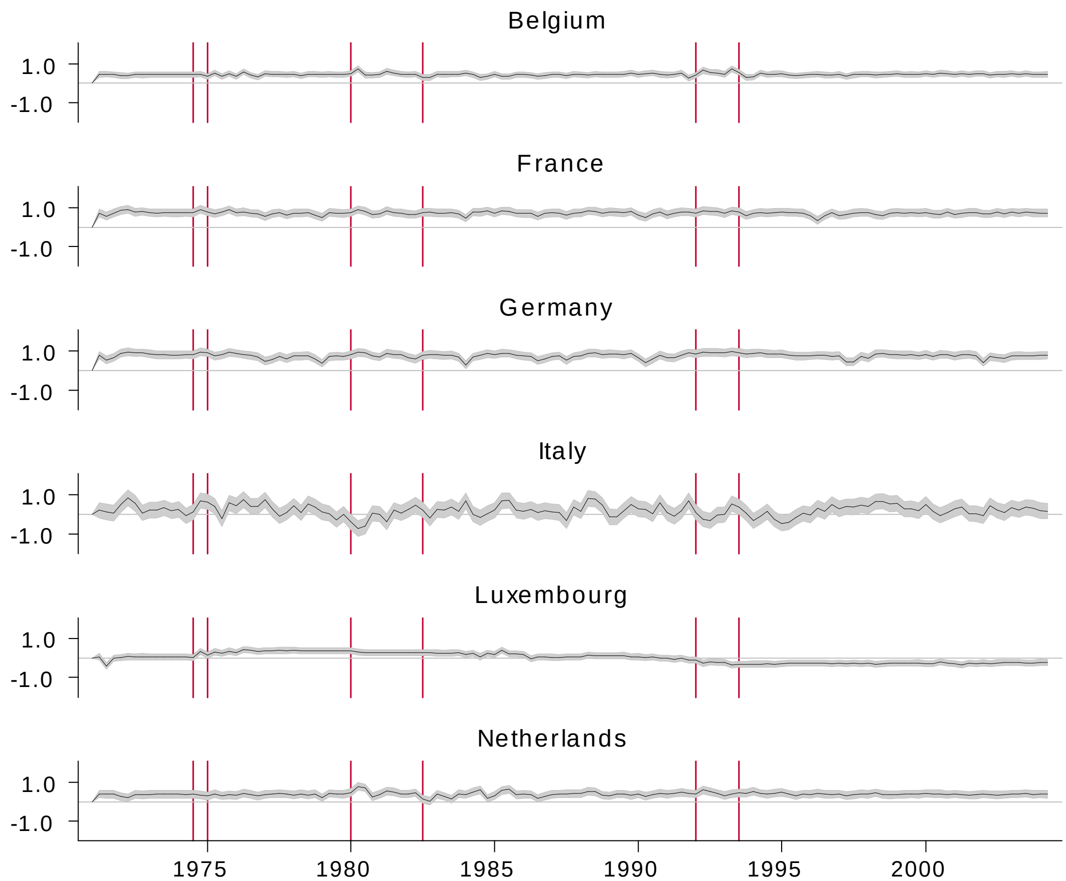

Figure 29: DCC for d1: group 1

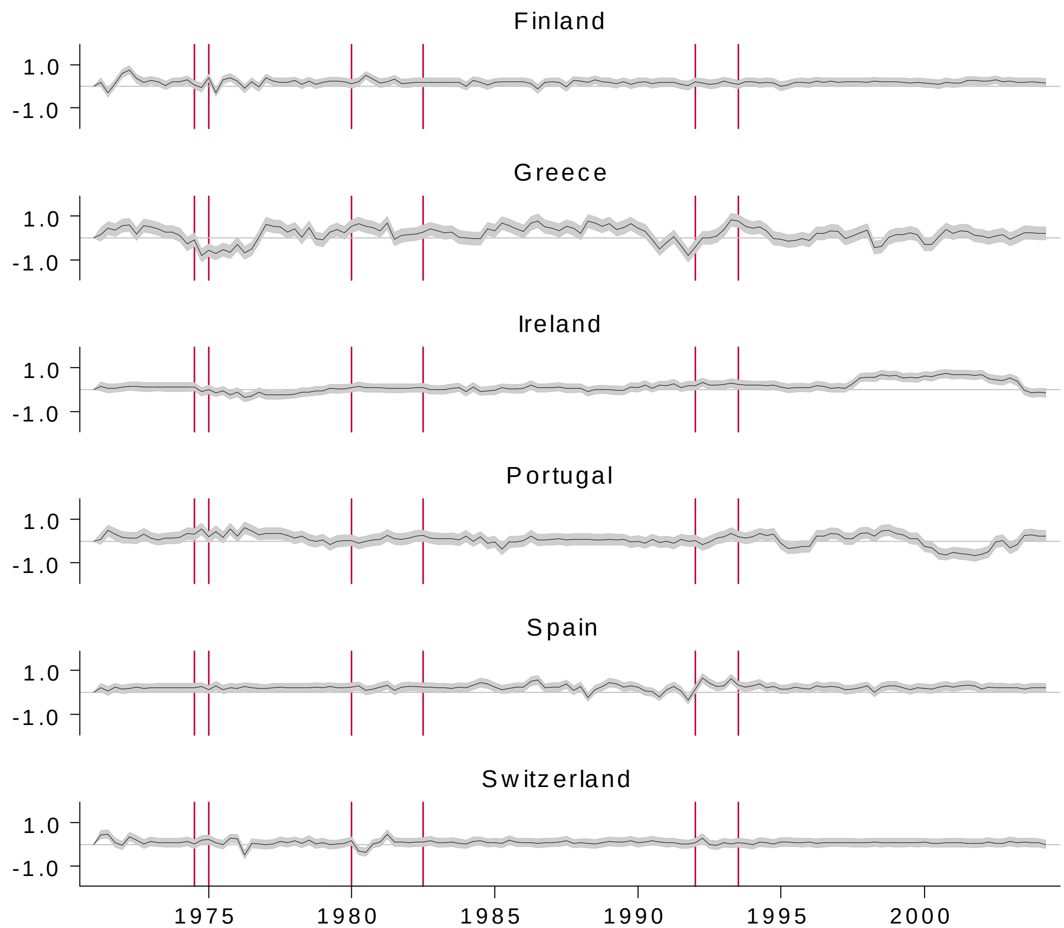

Figure 30: DCC for d1: group 2

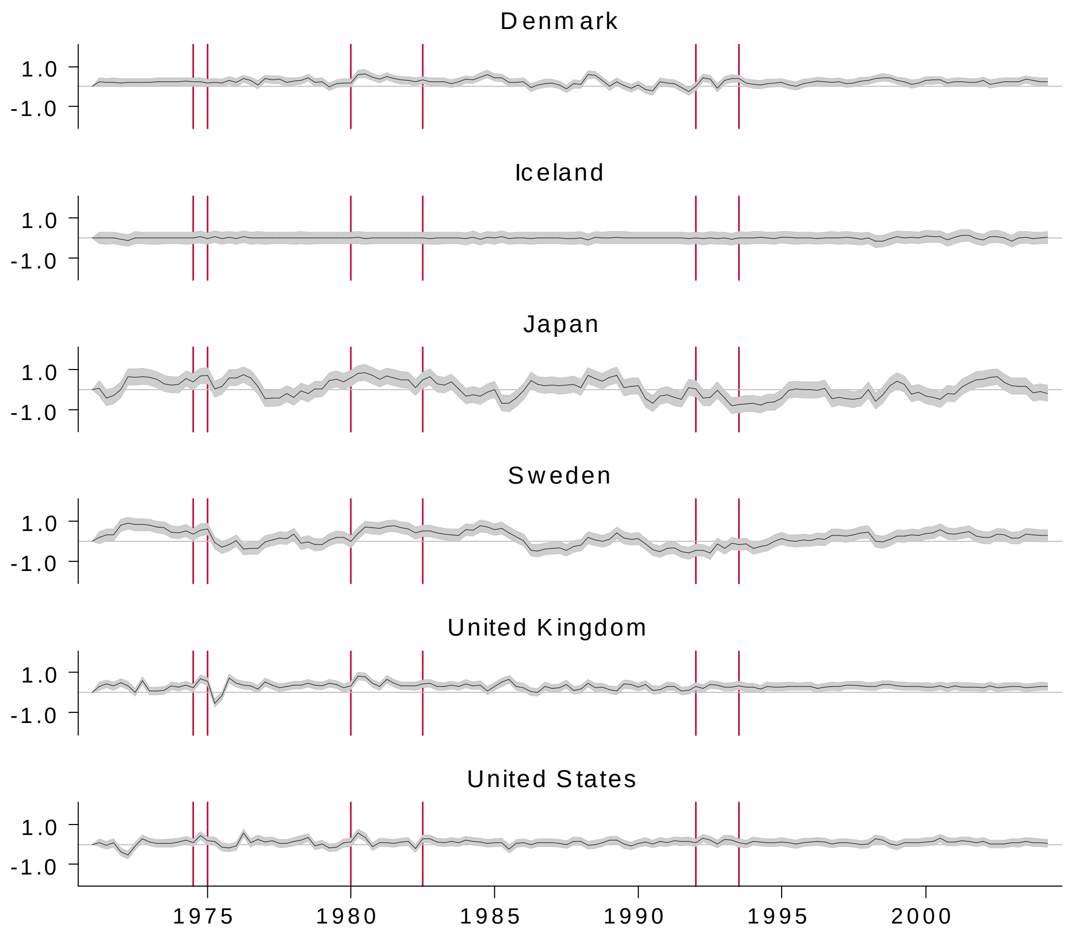

Figure 31: DCC for d1: group 3

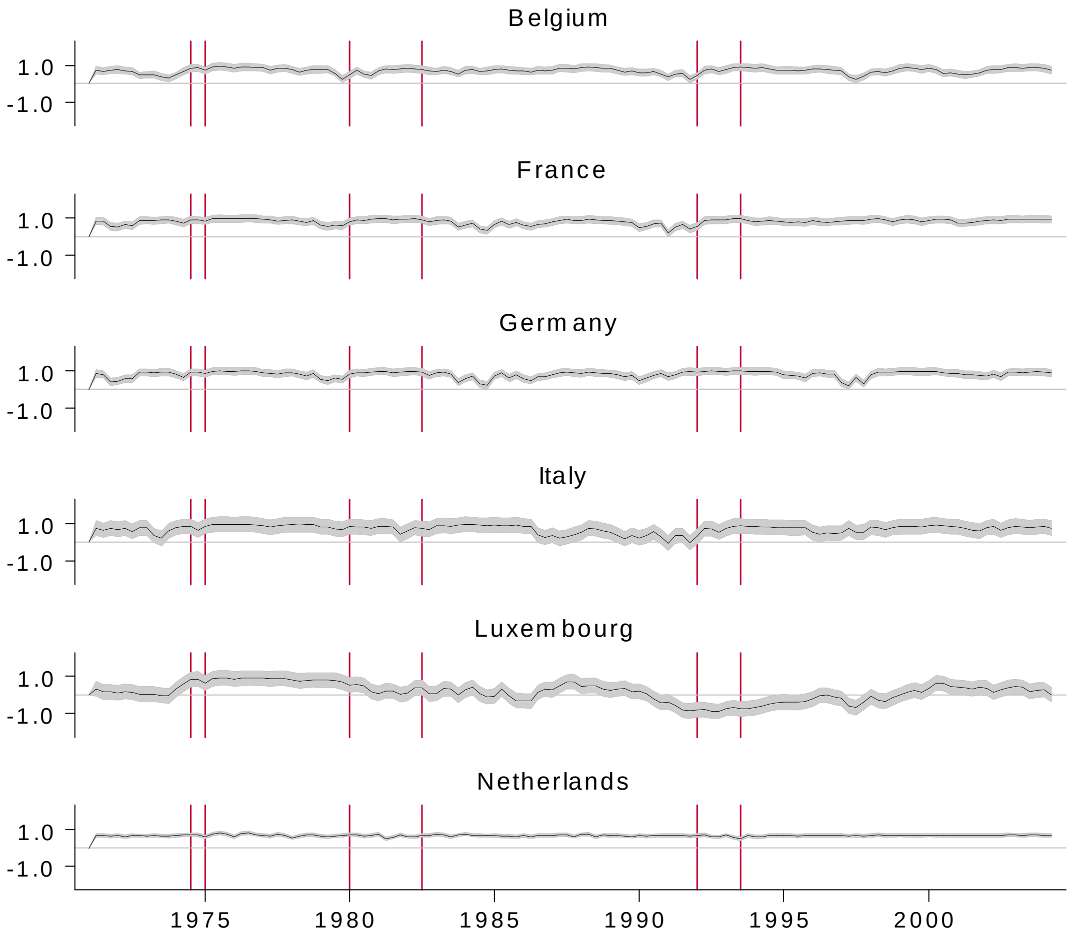

Figure 32: DCC for d2: group 1

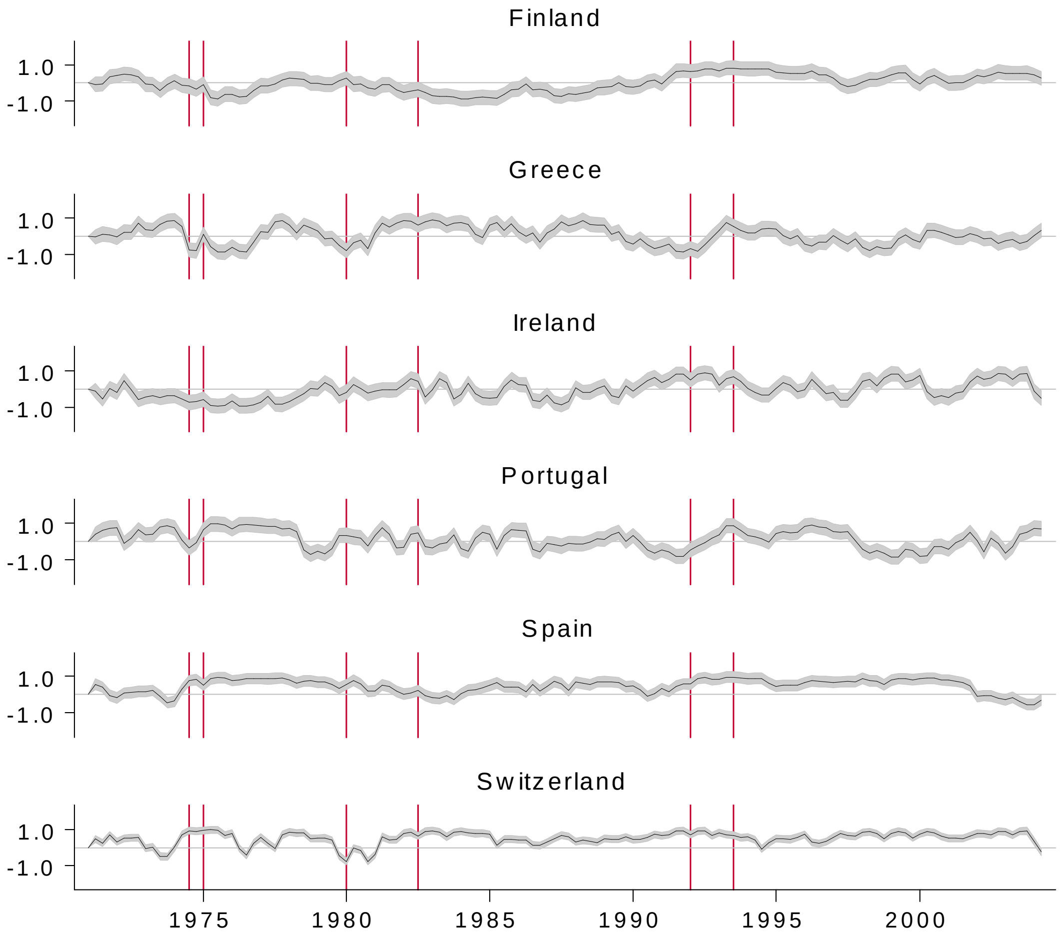

Figure 33: DCC for d2: group 2

Figure 34: DCC for d2: group 3

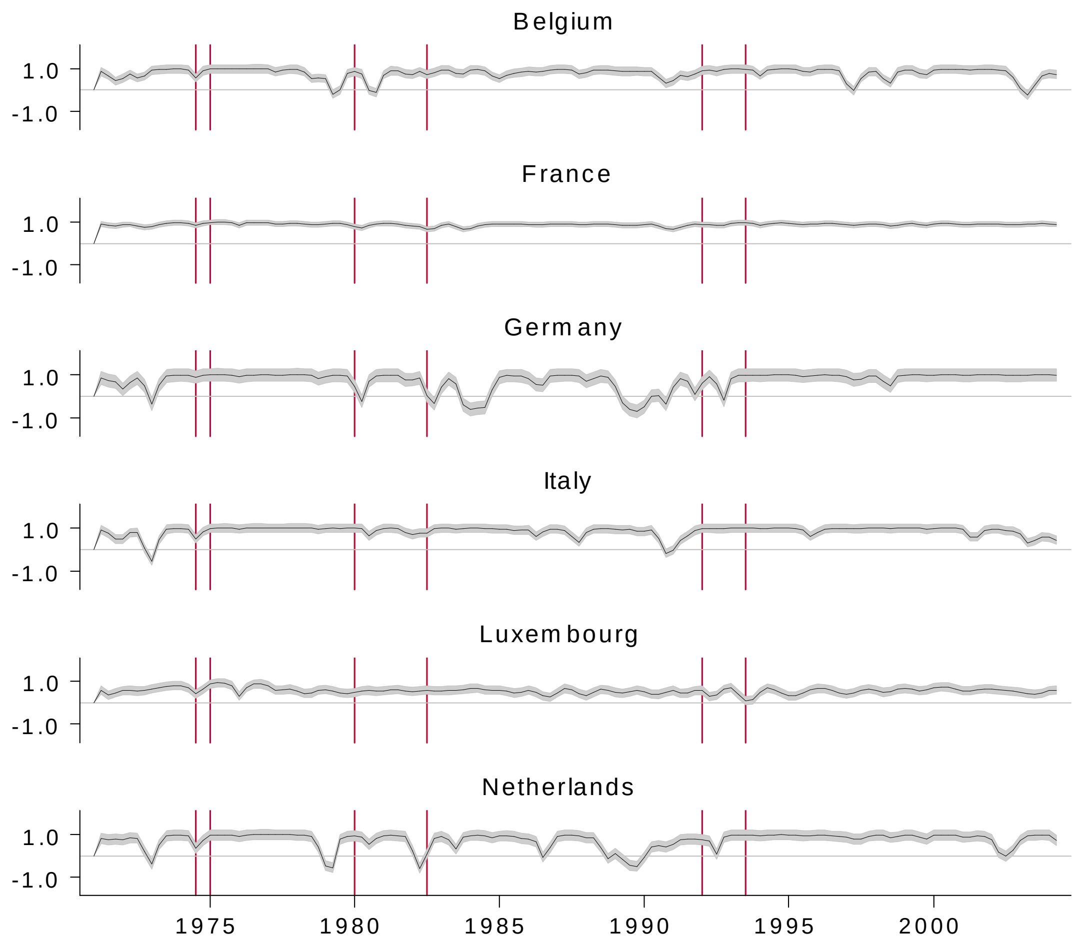

Figure 35: DCC for d3: group 1

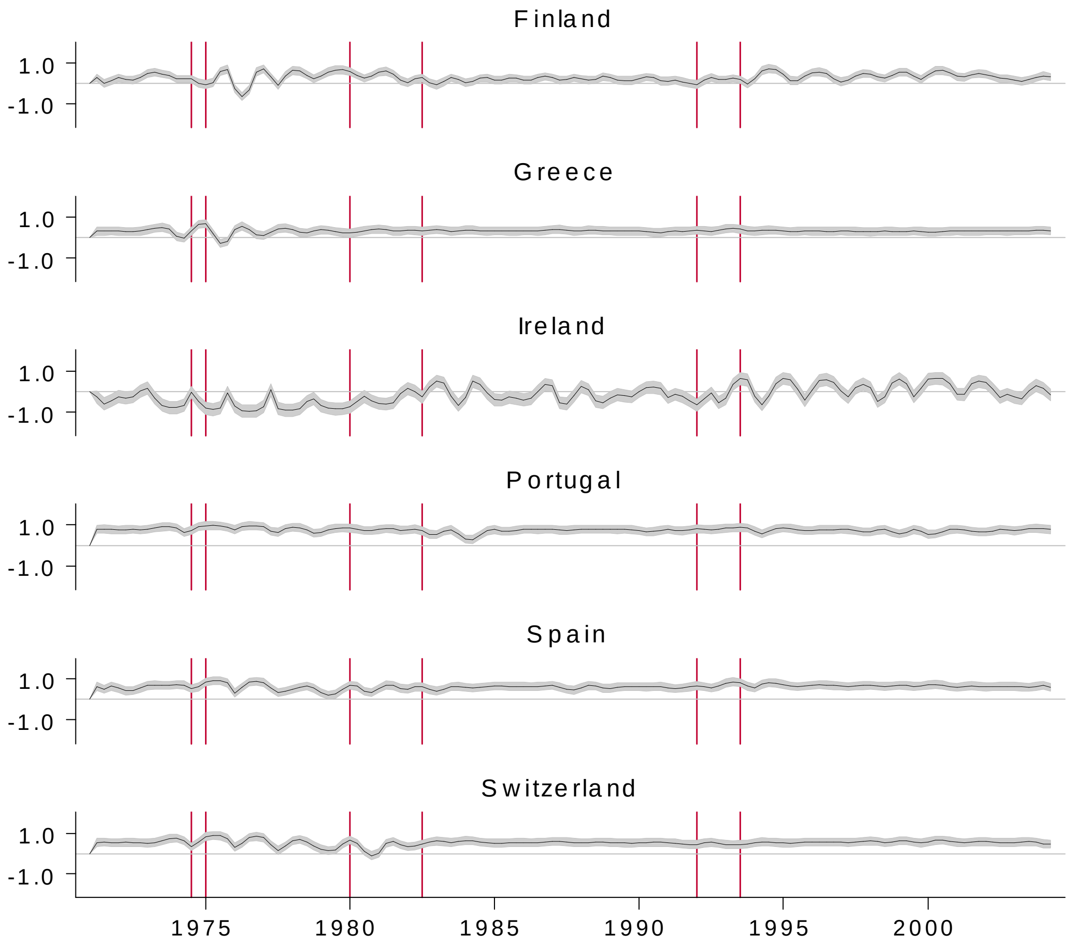

Figure 36: DCC for d3: group 2

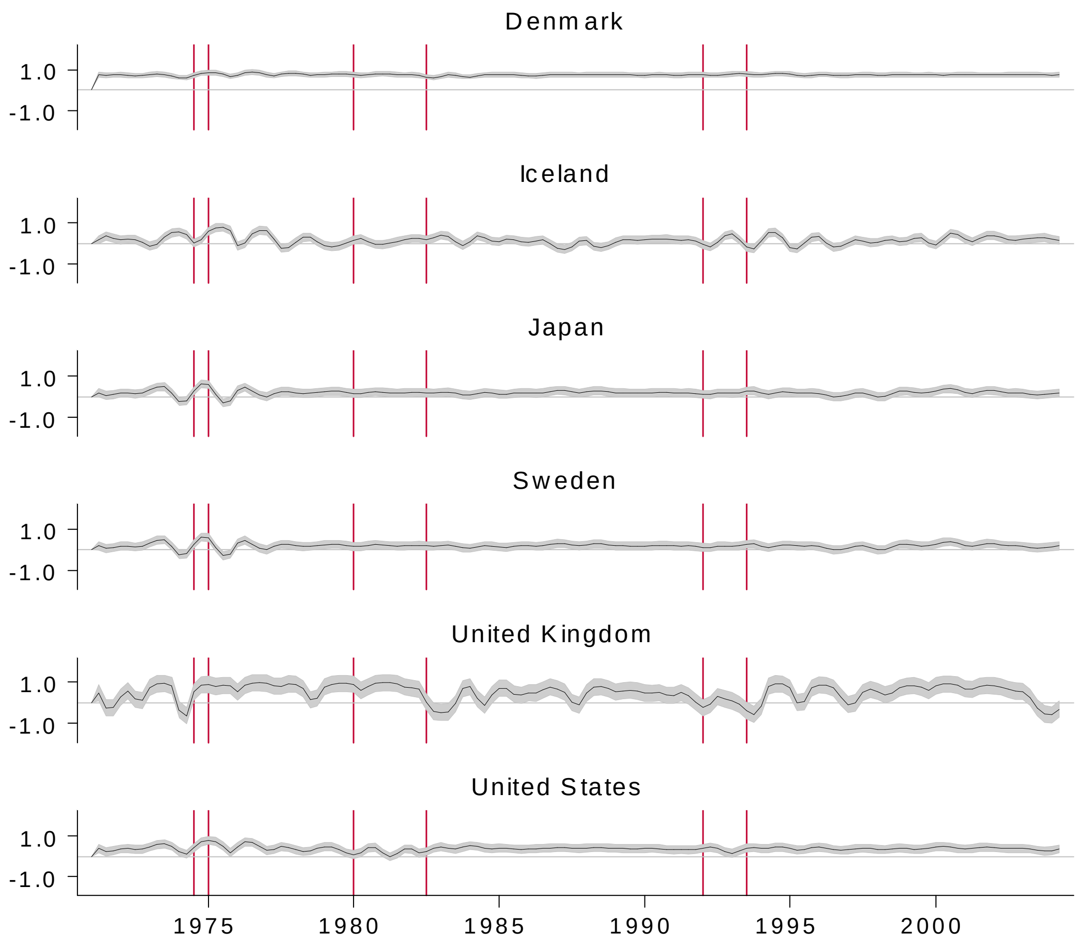

Figure 37: DCC for d3: group 3

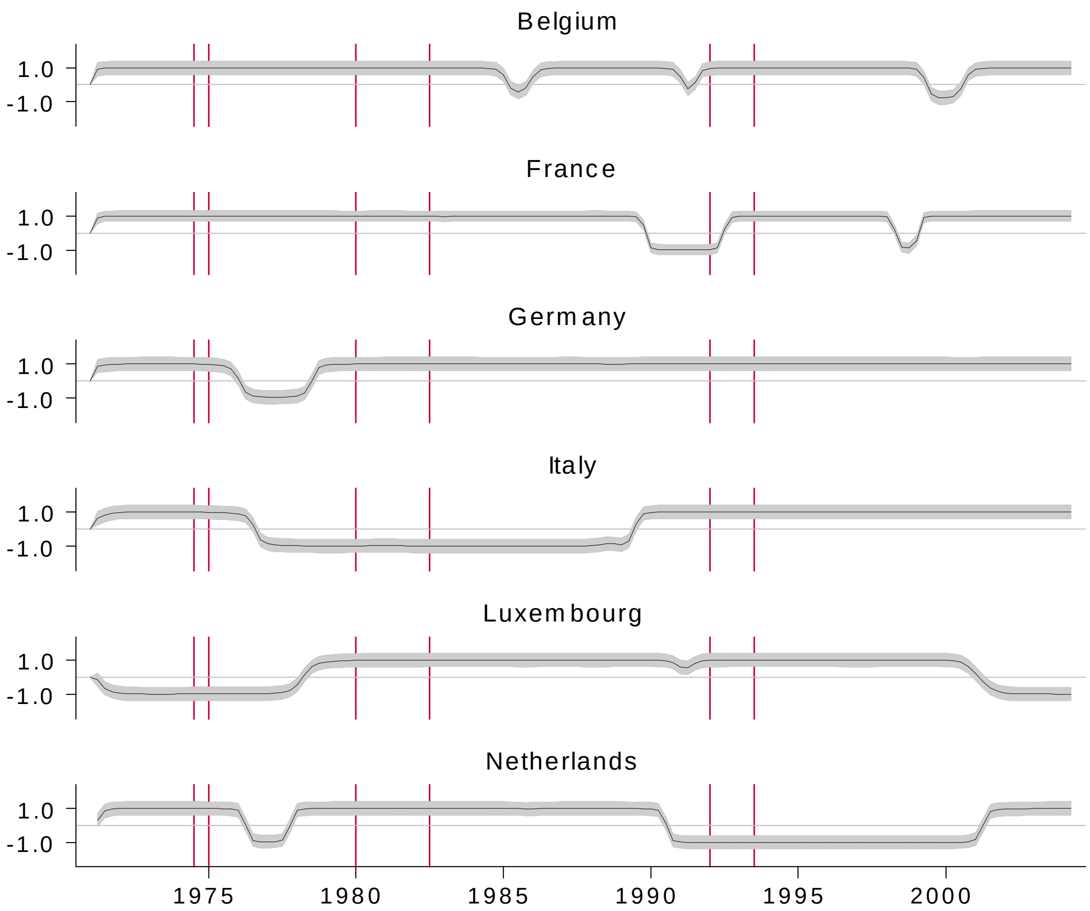

Figure 38: DCC for s5: group 1

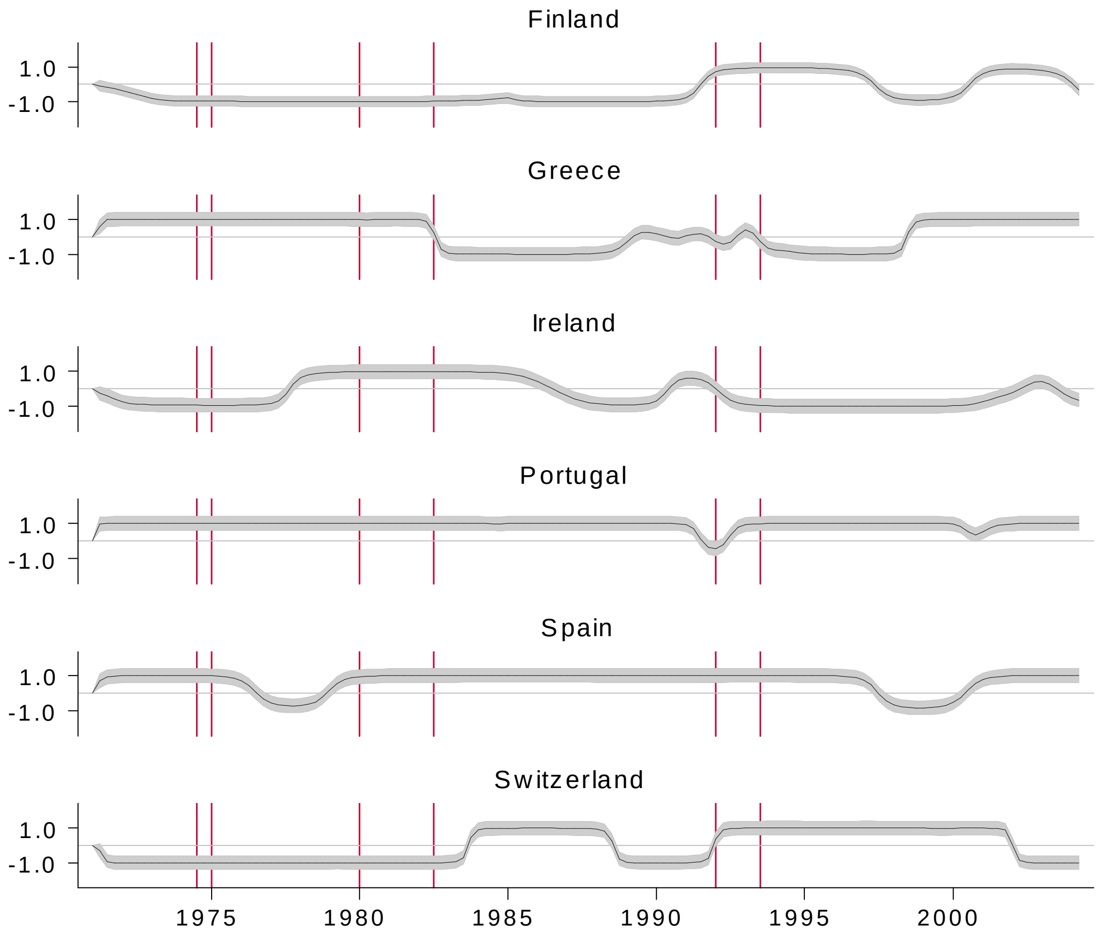

Figure 39: DCC for s5: group 2

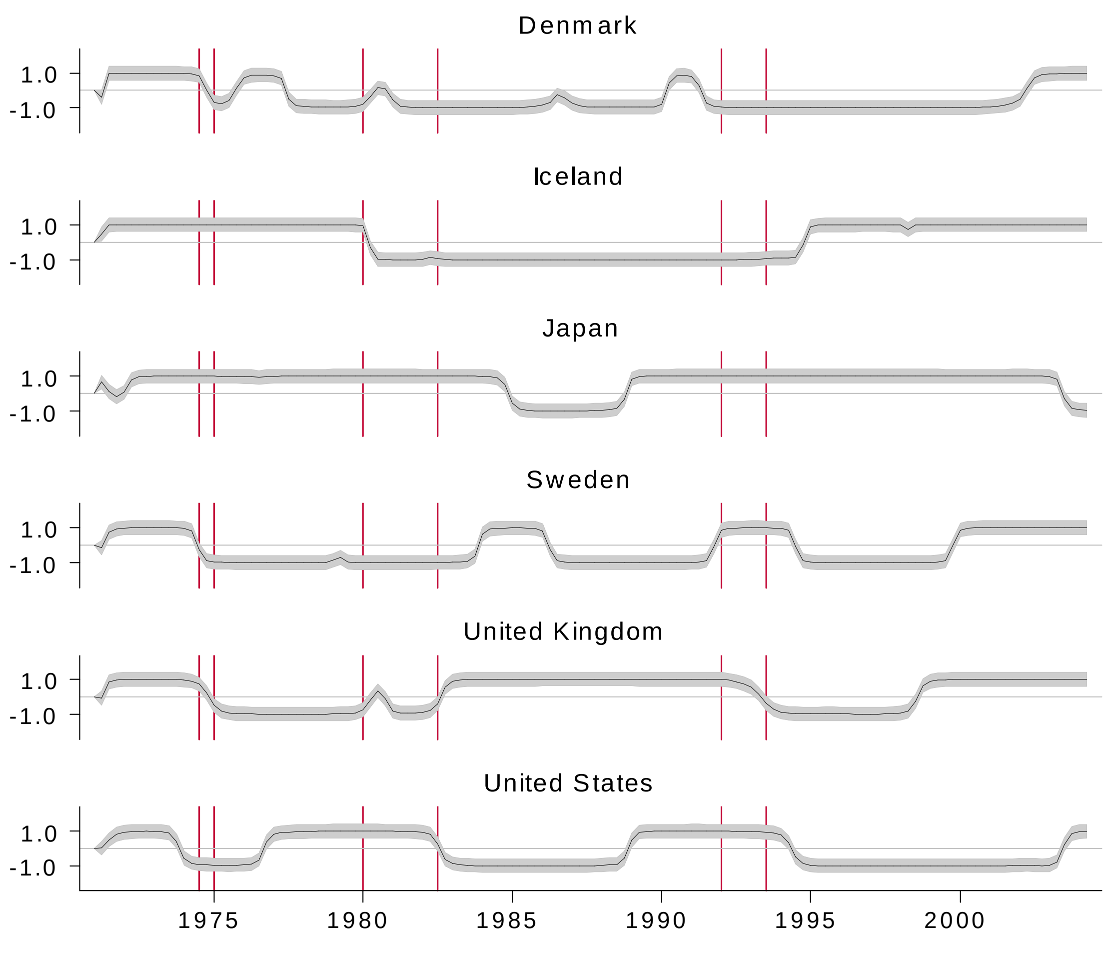

Figure 40: DCC for s5: group 3

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