A closer look at the gap. A comment on Cooper and Willis' "mind the gap" paper

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ABSTRACT

Recently, there has been a lively debate between Cooper and Willis (2001,2002,2003a, 2003b) and Caballero and Engel (2004) about the apropriateness of the so-called "gap approach" to labor adjustment. Cooper and Willis claim that the gap approach is unable to identify non-convex adjustment costs because of a measurement error under the alternative hypothesis of convex costs.

This comment assesses the validity of Cooper and Willis' claim by providing evidence from a number of Monte-Carlo experiments. In contrast to Cooper and Willis findings from single simulations, the experiments reveal no tendency to falsely reject the convex-cost hypothesis if one uses the correct one-sided test for non-convexities. In fact, the parameter estimates are typically biased against the hypothesis of non-convex costs. Consequently, there is no tendency to falsely reject although the estimates show substantial excess dispersion as a result of a spurious regression problem.

Keywords: Monte-Carlo Experiments, Employment Adjustment, Non-Convex Adjustment Costs

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1 Introduction

Recently, there has been a lively debate between Cooper and Willis (2001,2002,2003a, 2003b) and Caballero and Engel (2004) about the usefulness of the so-called "gap approach" to investment and labor adjustment. The gap approach is essentially an error-correction model for employment decisions. It has itself attracted considerable attention especially to describe lumpy adjustment in the presence of non-convex adjustment costs.[1] As an error-correction model, the gap model first identifies the desired level of employment and then in a second step it describes the adjustment of employment as a process that closes the gap between the desired and actual level of employment over time. This error-correction formulation of employment adjustment is especially attractive in the presence of non-convex adjustment costs, since non-convex costs imply, in contrast to convex costs, that the adjustment speed increases when the gap gets larger. As a result, higher order moments of the firm-level gap-distribution have explanatory power for aggregate employment adjustment only if adjustment costs are non-convex.

Yet, Cooper and Willis (2001, 2002, 2003a) argue that the gap approach is nevertheless unable to identify non-convexities in adjustment costs. They claim that measurement errors systematically arise under the alternative hypothesis of convex adjustment costs and consequently bias the estimates. Therefore, higher order moments of the gap become significant although they should not be so under the convex adjustment-cost alternative.[2] As aresult, the gap-approach regressions of Caballero and Engel (1993) and Caballero, Engel, and Haltiwanger (1997) if considered as a test for non-convex costs,[3] lack power. The measurement error induces a size distortion under the alternative hypothesis.

However, Cooper and Willis only provide evidence for this claim on the basis of a small number of simulations. This comment now tries to exactely quantify the claimed size distortion on the basis of a large number of Monte-Carlo experiments. While the Monte Carlo experiments confirm that the estimates are substantially biased and excessively dispersed, they reveal effectively no tendency to falsely rejecet the convex cost model if only the appropriate test is employed. A two-sided test that rejects the convex-

[1] See e.g. Caballero and Engel (1993,1999), or Caballero, Engel, and Haltiwanger (1995, 1997).

[2] Note that the terms "null" and "alternative" are used somewhat sloppily in this comment. When one tests for non-convexities, the null is the convex cost model which has to be rejected. However, we term the convex cost model the "alternative" since the prior typically is the non-convex model.

This semantic problem just as the econometric problem is similar to the situation with unit-root tests when one wishes to show stationarity with an ADF or similar test.

[3] Caballero and Engel (2004) emphasize that their procedure (1993) is not meant to test for non-convexities, but wants to investigate the aggregate implications of non-convexities. Nevertheless, their results would be even more striking, if their procedure also provides a test for non-convex adjustment costs.

cost hypothesis whenever higher order moments of the gap influence aggregate investment is size distorted. Yet, if the test rejects the convex-costs hypothesis only if the hazard-rate of adjustment is increasing in the (absolute size of the) gap, the test is (almost) free of any size distortion.

Essentially, these findings result from the fact that the parameter of the third moment of gap is biased downwards under the convex-cost alternative and not upwards. Moreover, the parameter estimate for the third moment seems to suffer from a substantial spurious regression problem due to the serial correlation of both, the labor-adjustment rate and the series of the third moments of the gap. As a result, the finite sample variance of the estimate is much larger than the variance calculated from asymptotic theory, which also explains the sharp contrast of our results and Cooper and Willis' findings.

The remainder of this paper proceeds as follows: Section 2 gives a brief overview over Cooper and Willis central results and introduces our simulation analysis. Section 3 provides the results from the Monte-Carlo experiments and gives a potential explanation for our findings. Finally section 4 concludes.

2 A dynamic optimization framework for investment

Analyzing employment (and investment) data using the gap approach has been introduced by Caballero and Engel (1993, 1999) and Caballero et al. (1995, 1997). All three papers show that aggregate employment adjustment also depends significantly on higher order moments of the distribution of the gap between desired and actual employment. While Cooper and Willis (2003a) acknowledge that under the null hypothesis of non-convex adjustment costs the gap approach may be valid if firm productivity follows a random walk, they argue that the procedures used to measure the gap will result in a severe measurement error under the alternative hypothesis that adjustment costs are convex and productivity has below unit-root serial correlation.

At the core of their analysis is the value maximization problem of a firm that employs the production factor E, e.g. labor. The firm is subject to a random productivity shock A and when it wants to adjust the employment level of E it has to pay quadratic adjustment costs $\frac{\nu}{2}\left(\frac{E-E_{-1}}{E_{-1}}\right)^2 E_{-1}$, where E_{-1} denotes employment at the beginning of the period (or at the end of the previous period if one likes). The wage rate is w and the instantaneous payoff is given by AE^{α}. Consequently, the following Bellmann equation describes the firm's objective:

$$V(A, E_{-1}) = \max_{E} AE^{\alpha} - wE - \frac{\nu}{2}\left(\frac{E-E_{-1}}{E_{-1}}\right)^2 E_{-1} + \beta \mathsf{E}\left[V\left(A', E\right)|A\right]. \qquad (1)$$

In this equation β denotes the discount factor and $\mathsf{E}[V(A', E)|A]$ denotes expected firm value assuming that current productivity equals A. Cooper and Willis (2003a, p. 17) attribute a minor role to the decision on working hours per employee and find the main source of measurement error and "(t)he key to the nonlinearity seems to be the substitution of the static for the frictional gap [...]". Therefore, in this comment the choice of hours per worker is neglected (fixed) and the optimization problem is reduced to the choice of optimal employment levels.

The optimization problem (1) and the gap approach are closely related. If (log) productivity follows a random walk, the optimal policy $\Delta e(A, E_{-1}) := \frac{E-E_{-1}}{E_{-1}}$ can represented as a linear function of the gap z between the current level of employment E_{-1} and the level of employment E^* that the firm chooses if adjustment costs are zero for the current period.[4] The "gap" (defined in log terms) is $z := \ln(E^*/E_{-1})$. Moreover, if (log) productivity is a random walk the desired level of employment is proportional to the optimal static employment level E^{**} which is given by

$$E^{**} = \left(\frac{\alpha A}{w}\right)^{\frac{1}{1-\alpha}}. \tag{2}$$

Unlike the dynamic target E^*, the static target level E^{**} can be easily computed from (2) on the basis of a production-function estimate, so that it is very helpful if one can use E^{**} instead of E^*.

The property that E^* can be substituted by E^{**} also carries over to the case of non-convex adjustment costs. However, if adjustment costs are non-convex, the adjustment rate Δe is a non-linear function of the gap z, and the (conditionally expected) adjustment-rate $\mathsf{E}(\Delta e/z|z)$ will be (a convex) function of z. Therefore, both results jointly (linearity of Δe and proportionality of E^*) allow to construct a test for non-convexities. For this test, one firstly estimates the distribution of the gaps on the basis of the static target and then one regresses aggregate employment change ΔE_t on the first and third order moment of the gap distribution (m_t^1 and m_t^3 respectively)[5]

$$\Delta E_t = \mu + \lambda m_t^1 + \gamma m_t^3 + u_t. \tag{3}$$

For non-convex adjustment costs both moments have a positive parameter, $\lambda, \gamma > 0$, while under quadratic adjustment costs $\gamma = 0$ holds. Consequently, if γ is insignificant, one can reject the hypothesis of non-convex costs. However, what is important for our

[4] See Rotemberg (1987)

[5] This estimation equation corresponds to Cooper and Willis (2003a) estimation equation (18).

numerical simulations below, the correct test for non-convex costs is a one-sided test for $\gamma > 0$.[6]

Yet, if productivity A is stationary and hence has below unit-root serial correlation E^{**} and E^{*} are no longer proportional. Cooper and Willis (2003a) show that using E^{**} in this case for calculating the gap will introduce a severe measurement error that varies with the gap. Consequently, this measurement error biases all parameter estimates and the testing procedure may lack power under the alternative hypothesis of convex adjustment costs.

In order to quantify the bias and the resulting loss of power, we numerically simulate the model to obtain the policy function $\Delta e\,(A, E_{-1})$. Four model specifications are analyzed and for each of the four specification the Monte-Carlo experiment is replicated 1000 times. In each experiment, first a series is generated for 1010 aggregate productivity realizations from an AR(1) process with serial correlation parameter ρ_{agg} and variance σ^2_{agg}. This aggregate productivity is added to idiosyncratic productivity, for which a series is generated (with parameters ρ_{idio} and σ^2_{idio}) for 1000 firms and again 1010 periods. For simplicity, $\rho := \rho_{agg} = \rho_{idio}$ and $\sigma^2 = \sigma^2_{agg} = \sigma^2_{idio}$ is chosen, so that we have for each single firm-productivity series

$$\ln A_{it} = \rho \ln A_{it-1} + \varepsilon^{agg}_{t} + \varepsilon^{idio}_{it}. \tag{4}$$

These productivity series are used to simulate a series for the gap z between static optimal employment E^{**} and current employment E_{-1} and a series of employment changes, using the previously generated policy function Δe. Finally, we generate the moments of the gap distribution based on the static employment target and regress the mean relative employment change ΔE_t on the first and third moment of this gap distribution using (3) as the estimation equation. To avoid an influence of the choice of of initial values, we drop the first 10 observations in each of these regressions.

3 Results from the Monte Carlo Experiments

In all simulations we use a discount factor of $\beta = 0.95$, a labor share of 65% and a markup of 25%, so that $\alpha = 0.7222$. We choose the standard deviation of the shocks such that the stationary distribution has a variance of 0.008, i.e. $\sigma = 0.02\sqrt{1-\rho^2}$. We try two specifications for the adjustment cost parameter ν, $\nu = 0.025$ and $\nu = 0.25$ which correspond to $\nu = 1$ and $\nu = 10$ in Cooper and Willis (2003a) paper due to a different normalization of E. The wage rate w we set equal to 0.06 and for the serial correlation

[6] See Caballero and Engel (2004, p. 1).

Table 1: Simulation parameters

parameter	value
β	0.95
α	0.72
w	0.06
ν	0.025
	0.25
ρ	0.95
	0.99
σ	$0.02\sqrt{1-\rho^2}$

term we try $\rho = 0.95$ and $\rho = 0.99$. The mean (log) productivity is set to 0.

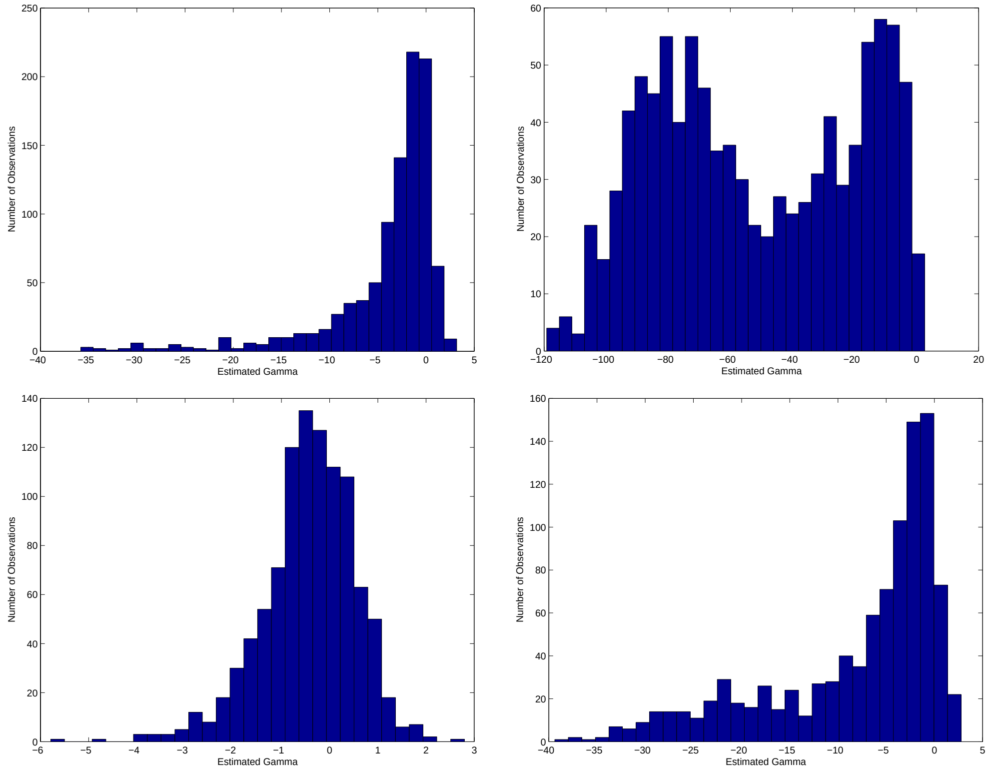
For the numerical simulation of the optimization problem we use a grid of 256 points for both, productivity and employment and perform value function iteration. The productivity grid is generated following the procedure outlined in Tauchen (1986) and Adda and Cooper (2003), by which we also approximate the AR(1) process by a Markov chain for the optimization problem. The grid for employment is an equispaced grid between the static optimal employment level for the second highest and second lowest point on the productivity grid. Table 1 summarizes our parameter choices.

Table 2 reports the main results of our Monte-Carlo experiments. Figure 1 displays the distribution of estimates for γ, the parameter of the third moment of the gap, in all four specifications. The mean of λ, the parameter of the first moment, is close to the values reported in Cooper and Willis (2003a) and the estimate of λ has a relatively small variance. For γ however, in all four specifications we obtain a negative mean and median. Consequently, we substantially over-reject the hypothesis $\gamma = 0$. Yet, the number of rejections using the better suited one-sided test for $\gamma > 0$ is relatively close to or even smaller than the value of 5%, which would result if γ was normally distributed around zero with the standard deviation that is estimated. On the contrary, there is a slight tendency for underrejection, but there is no tendency to accept the non-convex cost model although costs are convex.

However, when we compare the standard deviation of parameter estimates to the average standard deviation that is calculated from asymptotic theory, we find that the estimates are overly dispersed. Especially for γ this problem is substantial. Potentially this could be the result of a spurious regression problem. The serial correlations of both, ΔE_t and z_t is approximately equal and vary between 0.4 and 0.75, see table 3. Granger (2001) shows that the overrejection-rate for the null of no correlation between these two series should be approximately 10% and 25% respectively.

Figure 1: Distribution of the estimates of gamma

$\nu \backslash \rho$ $\quad$ $\rho = 0.95$ $\quad$ $\rho = 0.99$



ν=0.025

ν=0.25

That we find a negative estimate for γ on average can be explained as follows: Figure 1 in Cooper and Willis (2003a) shows that the static gap is typically larger in absolute terms than the frictionless gap, i.e. the correctly measured gap. Therefore, from this figure, one would conclude

$$E^{**} = E^{*} - K(A) + \theta E_{-1}, 0 < \theta < 1. \tag{5}$$

Therefore, when productivity is large, most of the firms will have employment below the static target level and $E^{**} - E^{*} < 0$ results. In the same way, when productivity is small, most firms will have employment above the static target level, and $E^{**} - E^{*} > 0$. The log-transformation now gives the non-linear relation and the negative parameter for m_t^3.

Table 2: Distribution of parameter estimates for (3), Monte-Carlo experiments 1000 replications

ρ	ν		mean	median	P(|t|>1.96)	P(t>1.65)	$\frac{std.err\ estimate}{aver.\ est.\ std.\ err}$
0.95	0.025	$\hat{\lambda}$	0.532	0.526	1.000	1.000	3.73
		$\hat{\gamma}$	-4.051	-2.00	0.436	0.009	4.66
	0.25	$\hat{\lambda}$	0.178	0.177	1.000	1.000	1.77
		$\hat{\gamma}$	-0.428	-0.360	0.298	0.078	1.98
0.99	0.025	$\hat{\lambda}$	0.575	0.577	1.000	1.000	9.78
		$\hat{\gamma}$	-51.58	-55.34	0.956	0.000	12.66
	0.25	$\hat{\lambda}$	0.214	0.209	1.000	1.000	6.34
		$\hat{\gamma}$	-7.988	-4.271	0.701	0.015	7.81

Table 3: Serial correlation of employment changes and gap measures

ρ	ν	serial correlation ΔE_t	serial correlation m_t^3	average R^2
0.95	0.025	0.41	0.37	0.9847
	0.25	0.75	0.72	0.9402
0.99	0.025	0.48	0.47	0.9947
	0.25	0.75	0.72	0.9835

4 Conclusion

The present paper extends Cooper and Willis (2003a) approach and quantifies the bias of the gap approach to labor adjustment and investment if productivity shocks have below unit-root serial correlation. Cooper and Willis provide evidence only on the basis of a small number of simulations for their central claim that one might falsely accept the hypothesis of non-convex adjustment costs although adjustment costs are convex if one runs regressions of the type studied by Caballero and Engel (1993). We now run a large number of Monte Carlo experiments to obtain the distribution of parameter estimates of the gap model based on the static target of employment. These experimets reveal no substantial tendency to overreject the convex adjustment cost model. Nevertheless, the parameter estimate of the higher order moments of the gap distribution are biased as Cooper and Willis claim. However, this bias negative.

Moreover, our Monte-Carlo experiments reveal a substantial spurious correlation problem for the gap-approach regressions if adjustment costs are convex. This spurious correlation problem is found to be very strong in particular if adjustment costs are low or if the serial correlation of productivity is large. This spurious regression result is especially disappointing for attempts to recover structural parameters on the basis of the gap regressions by indirect inference, see e.g. Cooper and Willis (2003b). If the simulated data for indirect inference comes from a model with convex costs, the estimates heavily vary due to spurious correlation. Therefore, even without any change in the parameters of the underlying microeconomic model, the simulated model will match a large variety of regressions from real data simply by replicating the simulation over and over again. This effect may be strong enough to render the attempt to match gap regressions from real data with the regressions generated from a simulated convex adjustment-cost model spurious itself. However, when the adjustment costs are non-convex in reality and one excludes the convex cost model from the estimation beforehand (e.g. on the basis of direct plant level observations), the spurious regression problem can be expected to be less pronounced as there is no serial correlated measurement error involved then.

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