A Dynamic General Equilibrium Framework of Investment with Financing Constraint

Danyang Xie and Chi-wa Yuen[1]

July 2002

Abstract

In this paper, we provide a dynamic general equilibrium framework with an explicit investment-financing constraint. The constraint is intended as a reduced form to capture the balance sheet effects that have been widely regarded as an important determinant of financial crises. We derive a link between the value of the firm and social welfare. We find that the value of the firm can be greater with the constraint. Our model also sheds light on how the effects of productivity shocks and investors' misperception of productivity shocks may be amplified by the financing constraint.

JEL Classification Numbers: C61, D92
Keywords: Investment Constraint, Value of the Firm

[1]International Monetary Fund and University of Hong Kong, respectively. We thank the Hong Kong Research Grants Councils for financial support. We appreciate comments from Andrew Feltenstein, Eduardo Ley, Rodney Ramcharan, Reza Vaez-Zadeh, an anonymous referee and the Editor, Robert Flood.

1 Introduction

At the very beginning of the Asian financial crisis (AFC), most people took it as yet another currency crisis and many viewed it to belong to the second generation (self-fulfilling) type *à la* Obstfeld (1996) rather than first generation (fundamental) type *à la* Krugman (1979). As the crisis unfolded, however, it became obvious that, unlike exchange rate crises, the AFC was more related to banking and financial problems in the process of financing business investment. Since then, quite a few theories (so-called 'third generation' models) have been proposed to understand its sources—moral hazard or guaranteed bailouts (Krugman 1998), financial fragility (Chang and Velasco 2000), and balance sheet effects (Krugman 1999).[2]

As Krugman (2001) concludes, balance sheet effects are now believed to be the most crucial element behind the AFC. In particular, if firms are highly leveraged with debt denominated in foreign currency, then anything that triggers a massive capital outflow will result in a depreciation of the domestic currency and thus an increase in the firms' debt burden. As a consequence, net worth of the firms will be reduced, limiting their ability to borrow to finance their new investment. The resulting investment and output collapse will validate the capital flight and make the crisis self-fulfilling.

Despite its general acceptance by the profession as an important determinant of financial crises, the balance sheet effect has been studied mostly in models with complicated banking structure and multiple types of agents. For studies of firms' balance sheet effect on business cycle, see Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1999). For a growth analysis that incorporates banks' balance sheet effect, see Chakraborty and Ray (2001). The balance sheet effect has also been embedded in the study of the bank capital channel of monetary policy (see Van den Heuvel 2001, Kashyap and Stein 1995, Chami and Cosimano 2001). A related set of papers that emphasize the role of durable assets as collateral include Kiyotaki and Moore (1997) and Chen (2001).

In this paper, we provide a dynamic general equilibrium framework with an infinitely long-lived representative agent. We impose an explicit investment-financing constraint that is intended as a reduced form to capture the balance sheet effects. At the expense of microfoundations, our approach has

[2] See Schneider and Tornell (2000) for an attempt to synthesize some of these effects.

the advantage of simplicity. We think of our contribution as similar to that of money-in-utility-function (MIUF). The MIUF complements the cash-in-advance (CIA) and the overlapping-generation (OLG) models of money with more microfoundation. The lasting influence of MIUF is clearly seen in its wide adoption in the recent open economy literature (Obstfeld and Rogoff 1996). It is certainly our hope to see a future adaptation of our investment-financing constraint to real business cycle models, but as a first step, we focus on a continuous time and deterministic setting.

In this setup, we derive a link between the value of the firm and social welfare. We find that the value of the firm can be greater with the constraint. Our model also sheds light on how the effects of productivity shocks and investors' misperception of productivity shocks may be amplified by the financing constraint. We also discuss shocks such as accounting scandals that worsen the information asymmetry and shocks that add to transparency such as improved accounting standards and disclosure rules.

The organization of the paper is as follows. Sections 2.1-2.4 lay out the model and characterize solutions to the firm's value maximization and the consumer's utility maximization problems without the financing constraint. The constraint is introduced in Section 2.5, and numerical solutions reported in Section 3. Section 4 discusses implications from the model and possible extensions.

2 The model

Consider an infinite horizon economy where capital is the only factor of production. The representative household is endowed with some initial stock of capital, k_0. Using this capital stock, the household sets up a representative firm to produce output and to invest in new capital. The firm's output net of investment will be distributed back to the household to support its consumption.

2.1 The Firm's Value Maximization Problem

At any time t, the firm uses capital k_t to produce output $f(k_t)$ and invests an amount $\dot{k} + \delta k$ (where δ is the depreciation rate). The firm's problem is to choose $\dot{k}$ to maximize the present value of output net of investment, i.e.,

$$V^o(k_0) = \max \int_0^\infty e^{-\int_0^t r_s ds} \left[f(k) - \delta k - z\right] dt$$

$$\text{subject to} \quad : \quad \dot{k} = z,$$
$$k_0 \text{ given.}$$

where z is net investment. The superscript o stands for original, emphasizing the situation without an investment-financing constraint. Implicitly, we are assuming that the firm borrows funds from banks at a competitive interest rate r_t to finance its investment. A more explicit discussion about the role of the banking sector in this model economy is contained in the Appendix.

The first order conditions of this problem imply the familiar interest rate expression as follows:[3]

$$r_t = f'(k) - \delta. \tag{1}$$

2.2 The Consumer's Utility Maximization Problem

The consumer's problem is simply to choose consumption, c, to maximize his utility subject to the budget constraint that the present value of his consumption cannot exceed the value of the firm he owns, i.e.,

$$U^o(k_0) = \max \int_0^\infty e^{-\rho t} \left(\frac{c^{1-\sigma} - 1}{1 - \sigma} \right) dt$$
$$\text{subject to: } \int_0^\infty e^{-\int_0^t r_s ds} c dt \leq V(k_0).$$

[3] The first order conditions are given by

$$e^{-\int_0^t r_s ds} = \lambda,$$

and

$$\dot{\lambda} = -e^{-\int_0^t r_s ds} \left[f'(k) - \delta\right],$$

where λ is the multiplier associated with $\dot{k} = z$. The interest rate relation can be obtained by taking time derivative of the former and equating the resulting expression to the latter.

Implicit in the budget constraint is the assumption that the household is the supplier of loanable funds (via the bank at the competitive interest rate r) to help finance the firm's investment. (See Appendix for details.)

The first order condition[4] implies that

$$\frac{\dot{c}}{c} = \frac{r_t - \rho}{\sigma} \tag{2}$$

2.3 Equilibrium Firm Value and Consumer Utility

In equilibrium,

$$\dot{k} = f(k) - \delta k - c \tag{3}$$

Substituting $\dot{k}$ from (3) and r_s from (2) into the firm's value function, we have

$$V^o(k_0) = c_0^\sigma \int_0^\infty e^{-\rho t} c^{1-\sigma} dt, \tag{4}$$

which turns out to be equal to

$$V^o(k_0) = c_0^\sigma \left[(1-\sigma) U^o(k_0) + \frac{1}{\rho} \right]. \tag{5}$$

To our knowledge, this is the first time that an explicit link is established between the value of the representative firm and the welfare of the representative agent.

In the particular case when $f(k) = Ak\sigma$, $c(k) = [\rho + (1-\sigma)\delta]k/\sigma$ (see Xie 1991). We can show (see Appendix) that,

$$U^o(k_0) = \left(\frac{1}{1-\sigma} \right) \left\{ \frac{k_0^{1-\sigma} + \left(\frac{1-\sigma}{\rho} \right) A}{\left[\frac{\rho + (1-\sigma)\delta}{\sigma} \right]^\sigma} - \frac{1}{\rho} \right\}.$$

Hence,

$$V^o(k_0) = k_0 + \left(\frac{1-\sigma}{\rho} \right) A k_0^\sigma.$$

[4] The FOC is given by

$$c^{-\sigma} e^{-\rho t} = \mu e^{-\int_0^t r_s ds},$$

where μ is the multiplier associated with the budget constraint.

2.4 Policy Functions and Numerical Algorithms

For more general production functions, say, $f(k) = Ak\alpha$ where $\alpha \neq \sigma$, no analytical solution is available. Nonetheless we can still derive the differential equations governing the policy function, $c^o(k)$, and the firm's value function, $V^o(k)$.

The differential equation governing the policy function $c^o(k)$ can be obtained by substituting (1) and (3) into (2):

$$\frac{c^{o\prime}(k)}{c^o(k)}\left[Ak\alpha - \delta k - c^o(k)\right] = \frac{\alpha A k^{\alpha-1} - \delta - \rho}{\sigma}. \tag{6}$$

We can rewrite equation (4) at time τ:

$$V^o(k\tau) = c\tau\sigma \int \tau^{\infty} e^{-\rho(t-\tau)} c^{1-\sigma} dt, \tag{7}$$

and differentiate both sides with respect to τ to obtain,

$$\begin{aligned} V^{o\prime}(k)\dot{k} &= \sigma c^{\sigma-1}\dot{c}\int \tau^{\infty} e^{-\rho(t-\tau)} c^{1-\sigma} dt + c\sigma\left[-c^{1-\sigma} + \rho \int \tau^{\infty} e^{-\rho(t-\tau)} c^{1-\sigma} dt\right] \\ &= rV^o(k) - c^o(k). \end{aligned} \tag{8}$$

Substituting (1) and (3) into (8) and rearranging terms, we get:

$$V^{o'}(k) = \frac{\left(\alpha A k^{\alpha-1} - \delta\right) V^o(k) - c^o(k)}{Ak\alpha - \delta k - c^o(k)}.$$

To compute the solutions numerically, we need to shoot back from the steady state capital stock, k^*, where k^* is obtained by combining (1) and (2) and solving $\alpha A k^{\alpha-1} - \delta = \rho$,

$$k^* = \left(\frac{\alpha A}{\rho + \delta}\right)^{1/(1-\alpha)}.$$

Throughout the paper, we assume that the steady state capital stock, k^* is greater than the initial capital stock, k_0.

At the steady state (with $\dot{k} = 0$), consumption is given by

$$c^o(k^*) = Ak^{*\alpha} - \delta k^*$$

We need to compute $c^{o\prime}(k^*)$ in order to apply backward shooting methods. Applying the *L'Hopital* rule, we have:

$$\begin{aligned} c^{o\prime}(k^*) &= \left\{ \frac{\alpha A k^{\alpha-1} - \delta - \rho}{\sigma \left[A k \alpha - \delta k - c^o(k) \right]} \right\} \left(A k \alpha - \delta k \right) \bigg|_{k \to k^*} \\ &= \left\{ \frac{\alpha(\alpha-1) A k^{*\alpha-2}}{\sigma \left[\rho - c^{o\prime}(k^*) \right]} \right\} \left(A k^{*\alpha} - \delta k^* \right). \end{aligned}$$

The above is a quadratic equation in $c^{o\prime}(k^*)$, which can be solved to yield the following solution:

$$c^{o\prime}(k^*) = \frac{\sigma\rho + \sqrt{\sigma^2\rho^2 + 4\sigma\alpha(1-\alpha) A k^{*\alpha-2} \left(A k^{*\alpha} - \delta k^* \right)}}{2\sigma}, \tag{9}$$

where the negative root is ruled out by the assumptions of free disposal and no satiation. Using $c^{o\prime}(k^*)$, we can shoot backward from $c^o(k^*)$ to obtain $c^o(k)$.

As for the firm's value function, note that along the steady state path with $k = k^*$ and $r = \rho$, $V^o(k^*) = \left(A k^{*\alpha} - \delta k^* \right) / \rho$. Again, we use L'Hopital rule to compute $V^{o\prime}(k^*)$:

$$\begin{aligned} V^{o\prime}(k^*) &= \frac{\left(\alpha A k^{\alpha-1} - \delta \right) V^o(k) - c^o(k)}{A k \alpha - \delta k - c^o(k)} \bigg|_{k \to k^*} \\ &= \frac{\left[\alpha(\alpha-1) A k^{*\alpha-2} \right] V^o(k^*) + \left(\alpha A k^{*\alpha-1} - \delta \right) V^{o\prime}(k^*) - c^{o\prime}(k^*)}{\left[\alpha A k^{*\alpha-1} - \delta - c^{o\prime}(k^*) \right]} \\ &= \frac{\left[\alpha(\alpha-1) A k^{*\alpha-2} \right] \left(A k^{*\alpha} - \delta k^* \right) / \rho + \rho V^{o\prime}(k^*) - c^{o\prime}(k^*)}{\left[\rho - c^{o\prime}(k^*) \right]}, \end{aligned}$$

which implies that

$$V^{o\prime}(k^*) = 1 + \frac{\left[\alpha(1-\alpha) A k^{*\alpha-2} \right] \left(A k^{*\alpha} - \delta k^* \right)}{\rho c^{o\prime}(k^*)}. \tag{10}$$

Given $V^{o\prime}(k^*)$, we can shoot backward from $V^o(k^*)$ to obtain $V^o(k)$.

Lastly, we can compute $I^o(k)$ as follows:

$$I^o(k) = \dot{k} + \delta k = A k^\alpha - c^o(k).$$

2.5 Financing Constraint

In this paper, we examine the case where the representative firm's investment is limited by its ability to obtain financing. We assume that there is an implicit, competitive banking sector that provides loans (at the real interest rate r_t) to finance the firm's investment no greater than some fraction of its net present value, namely,

$$\dot{k} + \delta k_t \leq \gamma V(k_t) \text{ for any } t.$$

There could be many reasons why the firm may not be able to borrow any amount bigger than a fraction of its fundamental value, in particular, capital market imperfections such as default possibilities and asymmetric information problems. (See, e.g., Bernanke, Gertler, and Gilchrist 1999.) This financing constraint can be viewed as a reduced form representation of these imperfections that we do not explicitly model in this paper.

In the presence of the financing constraint, the firm's problem becomes:

$$W(k_0) = \max \int_0^\infty e^{-\int_0^t r_s ds} \left(Ak^\alpha - \delta k - z\right) dt$$

$$\begin{aligned} \text{subject to:} \quad & \dot{k} = z, \\ & \delta k + z \leq \gamma V(k), \\ & k(0) = k_0 \text{ given,} \end{aligned}$$

and $W \equiv V$. This problem can be solved as follows. Given any continuous and almost everywhere differentiable function V, the maximization problem is well defined and a function W can be obtained. We can write $W = T(V)$, where T is a mapping. Our task is to find the fixed point of T.

A rigorous investigation of the problem is an entirely different paper. For instance, to prove the existence of a fixed point, we will need to show that T is a contraction mapping.[5] Instead, we approach the problem in an intuitive fashion. First, we use the special case of $\alpha = \sigma$ to derive a special feature of the fixed point. Then we will construct a value function V displaying the same feature in general cases.

To begin, note that in the absence of the financing constraint, explicit functional forms of $I^o(k)$ and $V^o(k)$ are available when $\alpha = \sigma$. We can see

[5] Modification of standard argument in dynamic programming is required, but what is essential in the proof of contraction mapping is the discounting: $\rho > 0$ in continuous time model and $0 < \beta < 1$ in discrete time model.

that $I^o(k)$ is hump-shaped starting with $I^o(0) = 0$, increasing and reaching a maximum, then declining and approaching negative infinity as k goes to infinity; $V^o(k)$ is an increasing function starting with $V^o(0) = 0$ and approaching infinity as k goes to infinity. Furthermore, we have,

$$\begin{aligned} I^o(k) - \gamma V^o(k) &= Ak^\alpha - \frac{\rho + (1-\sigma)\delta}{\sigma} k - \gamma k - \gamma \left(\frac{1-\sigma}{\rho}\right) Ak^\sigma \\ &= \left[1 - \gamma\left(\frac{1-\sigma}{\rho}\right)\right] Ak^\alpha - \left[\gamma + \frac{\rho + (1-\sigma)\delta}{\sigma}\right] k. \end{aligned}$$

Clearly, if γ is large, namely, $\gamma \geq \rho/(1-\sigma)$, then $I^o(k) - \gamma V^o(k) \leq 0$ hence the financing constraint will never be binding. Let us focus on small γ instead, $\gamma < \rho/(1-\sigma)$. With a small γ, we see that there exists a critical capital stock, k^c,

$$k^c = \left\{ \frac{\left[1 - \frac{\gamma(1-\sigma)}{\rho}\right] A}{\gamma + \frac{\rho + (1-\sigma)\delta}{\sigma}} \right\}^{1/(1-\sigma)} > 0,$$

such that $I^o(k^c) - \gamma V^o(k^c) = 0$ and the financing constraint is only binding when $k < k^c$.

We conjecture that the existence of a critical k^c is also true in general. In fact, we are able to construct such a fixed point, V, for the mapping T.

The first order conditions are given by:[6]

$$e^{-\int_0^t r_s ds} = \lambda - \theta,$$

and

$$\dot{\lambda} = -e^{-\int_0^t r_s ds} \left(\alpha A k^{\alpha-1} - \delta\right) - \theta[\gamma V'(k) - \delta],$$

where λ and θ are the multipliers associated with $\dot{k} = z$ and financing constraints respectively and θ satisfies the following complementary slackness condition:

$$\theta \left[\gamma V(k) - \delta k - \dot{k}\right] = 0.$$

[6]Taking derivative of the first condition with respect to t and combining the resulting expression with the second condition, we get

$$r_t [\lambda - \theta] = [\lambda - \theta] \left[\alpha A k^{\alpha-1} - \delta\right] + \theta [\gamma V'(k) - \delta] + \dot{\theta}$$

The consumer's problem remains the same as before. Therefore, (2) still holds and equations (4) and (8) hold with W in place of V:

$$W'(k\tau) = c\tau\sigma \int_0^\infty e^{-\rho t} c^{1-\sigma} dt \tag{11}$$

$$W'(k)\dot{k} = rW(k) - c(k) \tag{12}$$

When $k \geq k^c$, the financing constraint is not binding so $\theta = 0$ and the policy and value functions are the same as in the unconstrained case described in the previous subsections, with $c(k) = c^o(k)$, $W(k) = V(k) = V^o(k)$, etc.

In what follows, we shall focus on the case where the constraint is binding, i.e., $k < k^c$ and $\theta > 0$. Is there a differential equation similar to (6) that governs $c(k)$? From (3) and the binding constraint, we have

$$Ak^\alpha - c(k) = \gamma V(k).$$

Differentiate this with respect to t. Making use of (2) and (12) and by imposing $V(k) \equiv W(k)$, we obtain,

$$\left[\alpha A k^{\alpha-1} - c'(k)\right] \dot{k} = \left[\rho + \sigma \frac{c'(k)}{c(k)} \dot{k}\right] \left[Ak\alpha - c(k)\right] - \gamma c(k),$$

which implies that,

$$c'(k) = \frac{\alpha A k^{\alpha-1} c(k)}{[(1-\sigma)c(k) + \sigma Ak\alpha]} + \frac{(\gamma+\rho)c^2(k) - \rho c(k) Ak\alpha}{[(1-\sigma)c(k) + \sigma Ak\alpha]\,[Ak\alpha - \delta k - c(k)]}.$$

We can compute $c(k)$ by backward shooting starting from k^c and $c(k^c) = A(k^c)\alpha - \gamma V^o(k^c)$.

Once $c(k)$ is computed, $V(k)$ can be found from the financing constraint simply as

$$V(k) = \left[Ak^\alpha - c(k)\right]/\gamma.$$

The fact that we make use of (12) and impose $V(k) \equiv W(k)$ in our derivation of $c'(k)$ above ensures that this $V(k)$ is the fixed point of the mapping T. Hence the function V that we construct is the value function.

3 A numerical example

We surmise that the firm will invest at a slower rate and probably earn a lower net present value $V(k)$ with than without the financing constraint. In the absence of explicit analytical solutions, we shall resort to numerical simulations to better understand the economic effects of this constraint.

In our numerical solutions, we assume the following benchmark parameter values: $\alpha = 0.36$, $\sigma = 0.5$, $\gamma = 0.015$, $\rho = 0.03$, $\delta = 0.1$, $A = 12$, and $k_0 = 20$. We first compute the policy functions $c^o(k)$, $V^o(k)$, and $I^o(k)$ in the absence of the financing constraint by shooting backward from k^* to k_0. Then, we use $I^o(k) = \gamma V^o(k)$ to solve for the critical value k^c.[7] The corresponding functions $c(k)$, $V(k)$, and $I(k)$ in the presence of the investment constraint can be obtained by shooting backward from k^c to k_0 for $k \in [k_0, k^c]$ (when the constraint is binding) and combining it with $c^o(k)$, $V^o(k)$, and $I^o(k)$ for $k \in [k^c, k^*]$ (when the constraint is non-binding). The graphs for $\gamma V(k)$ with and without the financing constraint as well as the investment function $I^o(k)$ are displayed in Figure 1.

Not surprisingly, $I(k) < I^o(k)$ and, since contemporaneous output is unaffected by changes in investment, $c(k) > c^o(k)$ for $k < k^c$ (see Figure 2, panel 2). It is, however, surprising to find that $V(k) > V^o(k)$. In order to understand this, it is necessary to also compute the consumption path over time because the equilibrium value of the firm is simply the present value of equilibrium consumption in our model (without the labor-leisure choice; see budget constraint of the representative consumer).

From the time path of consumption (Figure 2, panel 1), we see that while consumption under the financing constraint initially exceeds its unconstrained counterpart, it grows at a slower rate and is soon surpassed by

[7] The time T required for $k(T) = k^c$ can be solved from the following differential equation:

$$\begin{aligned} \dot{k} &= Ak^\alpha - \delta k - c(k) \\ k(0) &= k_0 \text{ given,} \\ k(T) &= k^c. \end{aligned}$$

the latter.[8] As a result, consumer utility is lowered by the constraint, i.e., $U(k_0) < U^o(k_0)$. This may give the impression that $V(k) < V^o(k)$. However, the firm's value also depends on the effect of discounting. We thus have to consider how the financing constraint affects the behavior of interest rate over time. As shown in Figure 3, panel 2, initially interest rate is significantly lower with than without the investment constraint. The constraint induces a jump in the interest rate from 4% to 5% at the time when the capital stock hits its critical value and gradually converges to its steady state value (3%) thereafter. The discount rate at time t (given by $\int_0^t r_s ds$), represented by the area under the interest rate paths from 0 to t, will, at any rate, be smaller with than without the constraint despite the interest rate jump. It turns out that this discounting effect dominates the consumption growth effect to make $V(k) > V^o(k)$ under the set of parameter values we have chosen.

The equilibrium relation between the firm's value and consumer utility, $V(k_0) = c_0\sigma\left[(1-\sigma)U(k_0) + \frac{1}{\rho}\right]$, holds irrespective of the financing constraint. In terms of this relation, whether $\sigma > 1$ or $\sigma < 1$, it is possible that $V(k_0) > V^o(k_0)$ while $U(k_0) < U^o(k_0)$ provided that c_0 is sufficiently larger with than without the constraint. When $\sigma \to 0$, however, $V(k_0)$ and $U(k_0)$ will be positively correlated and will both be lowered by the constraint.

The interest rate behavior under the financing constraint may suggest a partial resolution to the Lucas (1990) puzzle why capital does not flow from rich to poor countries. In particular, the interest rate functions as portrayed in Figure 4 indicate that while a 10-fold difference in capital stocks between rich and poor countries (say, $k = 20$ versus $k = 200$) could induce a more than 13-fold difference in their interest rates ($r(20) = 0.535$ versus $r(200) = 0.0455$) in the absence of the constraint, the interest rate gap will be significantly reduced to 4-fold ($r(20) = 0.188$ versus $r(200) = 0.0455$) under the constraint. It may sound tautological that the presence of financing constraint reduces interest rate differential across countries. In fact, it could be given empirical content if one could calibrate parameter γ to obtain a quantitative measure of the reduction in interest rate differential. The re-

[8] Observe that while the "constrained" consumption *function* lies everywhere above its "unconstrained" counterpart, the same is not true for the consumption *paths*. This is because capital (of which consumption is a function) will grow more slowly with than without the constraint. The same logic applies to comparisons between policy functions of other variables and their corresponding time paths.

maining differential can then be attributed to other factors such as political risk, institutional and trade barriers.

4 Discussion and possible extensions

Our simple model can be easily extended to include labor as an additional input in the firm's production technology and the labor-leisure choice in the consumer's utility maximization problem. This extension would allow us to examine the effect of the financing constraint on employment as well — especially when the constraint does not apply just to investment-financing, but also to hiring workers and footing their wage bills. In the presence of this more severe constraint, employment and output could both be adversely affected so that consumption may not surge at the beginning despite the fall in investment.

This model is cast in a deterministic framework and therefore the following arguments on its potential applications to cases with uncertainty are only suggestive. Nevertheless, we list them here for discussions.

- An increase in total productivity, A, will shift both the $\gamma V(k)$ and $I(k)$ schedules upward. The impact on investment is not a monotonic function of the capital stock. As shown in Figure 5 based on our numerical computations, the impact on investment is hump-shaped. This suggests that in the emerging countries, broadly interpreted as countries with the size of capital between that of the less-developed countries and the developed, investment is more responsive to productivity shocks A than in the rest of the world.

- In a deterministic framework, it is easy to detect any discrepancy between V, the investors' perception of a firm's net worth, and W, the net worth based on fundamentals. In a stochastic world, there will always be a discrepancy between the two. Misperception can last for some time without being refuted by incoming data when capital stock is in a region where the investment constraint is binding. For example, when an important innovation such as internet raises the total productivity, captured by parameter A in our model, no one knows exactly the new value of A. If the market estimate A_M is higher than the true A, the increased valuation of the firm's net worth will allow the firm to raise the investment above equilibrium level due to a more relaxed constraint. As a result, output will be higher, which partially justifies the rise in market estimate A_M. Because it may take several periods of observations for the investors to gauge the real impact of the innovation and the calculation of the fundamental value W involves projection of future profits, the discrepancy between W and V may not be statistically detectable for a number of years. Only when the subsequent earnings reports of firms consistently indicate that V is unduly above W would A_M be revised downward. Again, no one knows the correct amount of downward revision of A_M. No doubt that investors' incentive to make profits in well-functioning markets ensures that in the long-run, A_M will settle around the true A, but in the short-term, over-shooting on the upside and downside can occur whenever an important innovation in general-purpose technology comes to the scene. The magnitude of the fluctuation in investment depends on the level of the capital stock. From Figure 6, investment fluctuation is more pronounced in emerging countries than the rest of the world. To be sure, a logically consistent model of business cycles would require an explicit probability specification of the magnitude and the dynamics of misperception.

- The recent episodes of accounting scandals in the US such as those involving Enron Corp. and Worldcom Inc. will make investors lower the value of γ, which will result in tighter financing constraint and hence lower aggregate investment. The congressional effort in tightening government regulations that raise the accounting standards and make CEO's action more accountable to shareholders will likely stop γ from sliding further, hence will stabilize investment. In a full-fledged RBC model with investment constraint, parameter γ should be calibrated to changes in government regulation and supervision in financial markets. For instance, China's increasing effort since 1996 to adapt its financial markets to international standards in preparation to its WTO entry will allow the banks to raise γ in their decision to lend to the private sector, thereby raising investment of private firms. Thus, even in the absence of technological change in goods production, institutional changes that ease the problem of asymmetric information in the financing process would mean a higher γ and a less stringent financing constraint. As a result, capital stock accumulates more rapidly and as capital stock approaches the financing threshold, the investment volatility eventually become more moderate as argued in the above paragraph.

- As it can be seen from the discussions above, both a technological progress and an institutional improvement can raise investment and output. Traditional RBC models do not distinguish one change from another. To account separately the changes in A and in γ requires serious effort in gathering relevant empirical data, for instance the trust indicator reported in Knack and Keefer (1997) and the survey data on bank regulation and supervision in Barth, Caprio, and Levine (2001). For a developing country trapped in low investment, the priority for policy changes is likely to be the different depending on whether the low investment is caused by a low A, in which case importing advanced technologies and ideas is needed, or a low γ, in which case rules and regulations improving transparency of business transactions are called for. In the sense of Prescott (1985), here again "Theory is ahead of business cycle measurement."

Figure 1: Investment Function and Financing Constraint

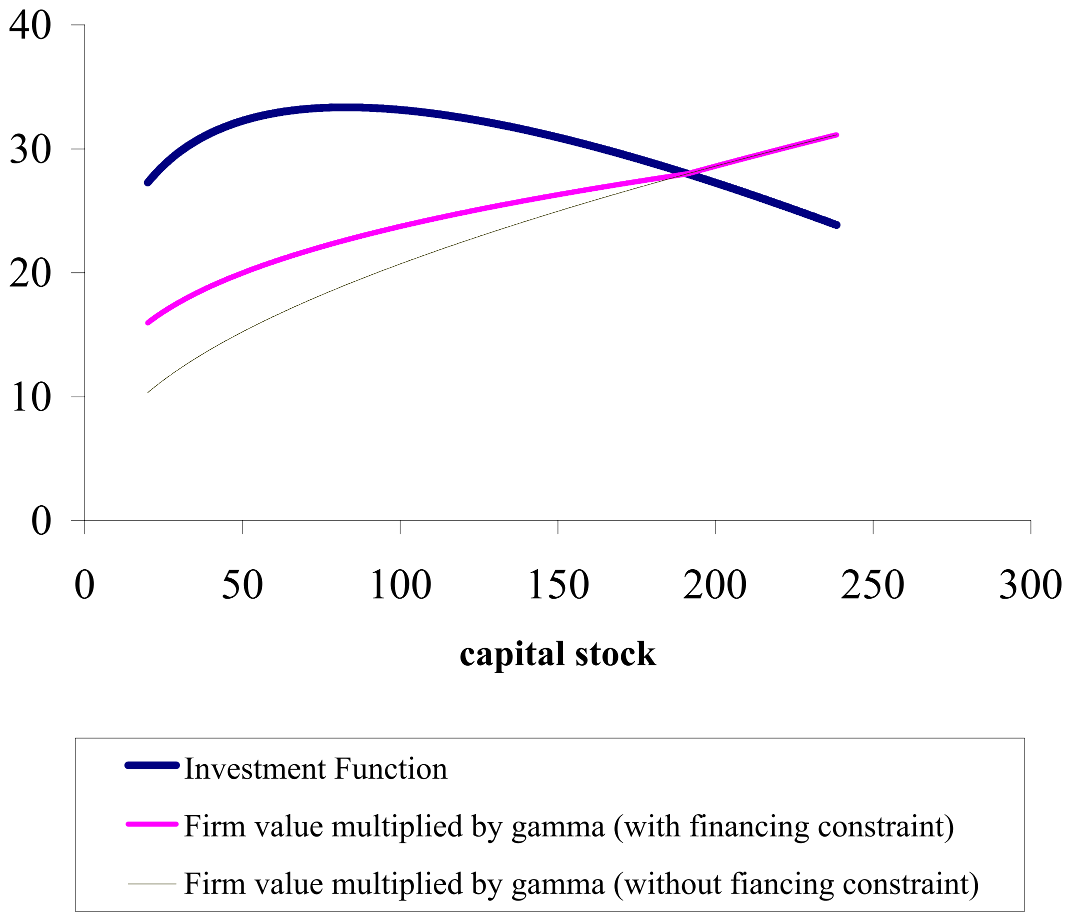
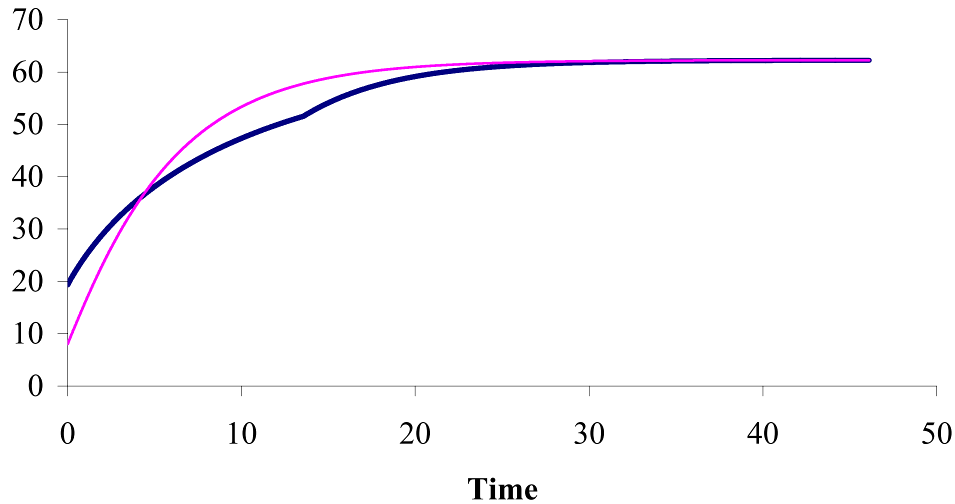


Figure 2

Consumption Paths



Consumption Functions

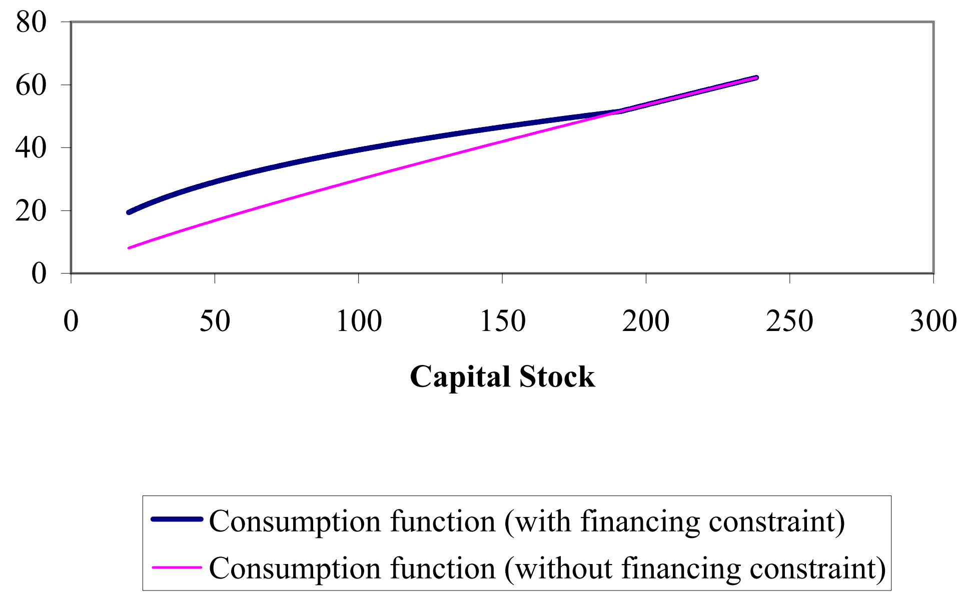
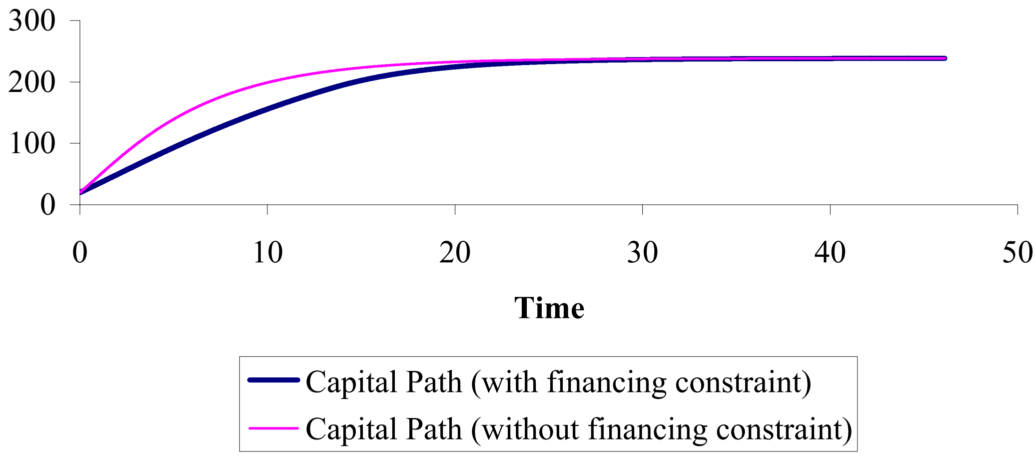


Figure 3

Capital Paths



Real Interest Rate Paths

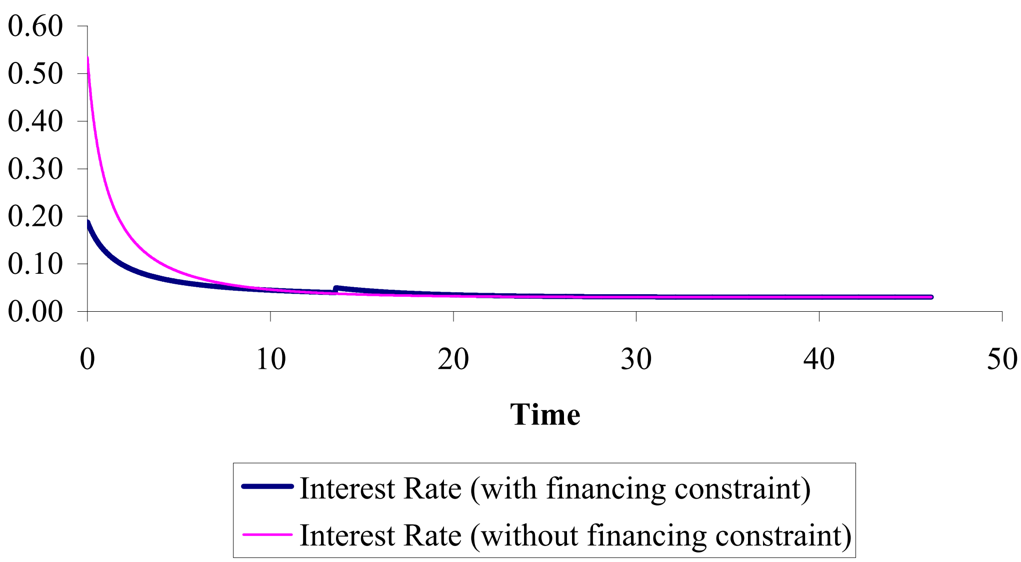


Figure 4: Interest Rate Functions

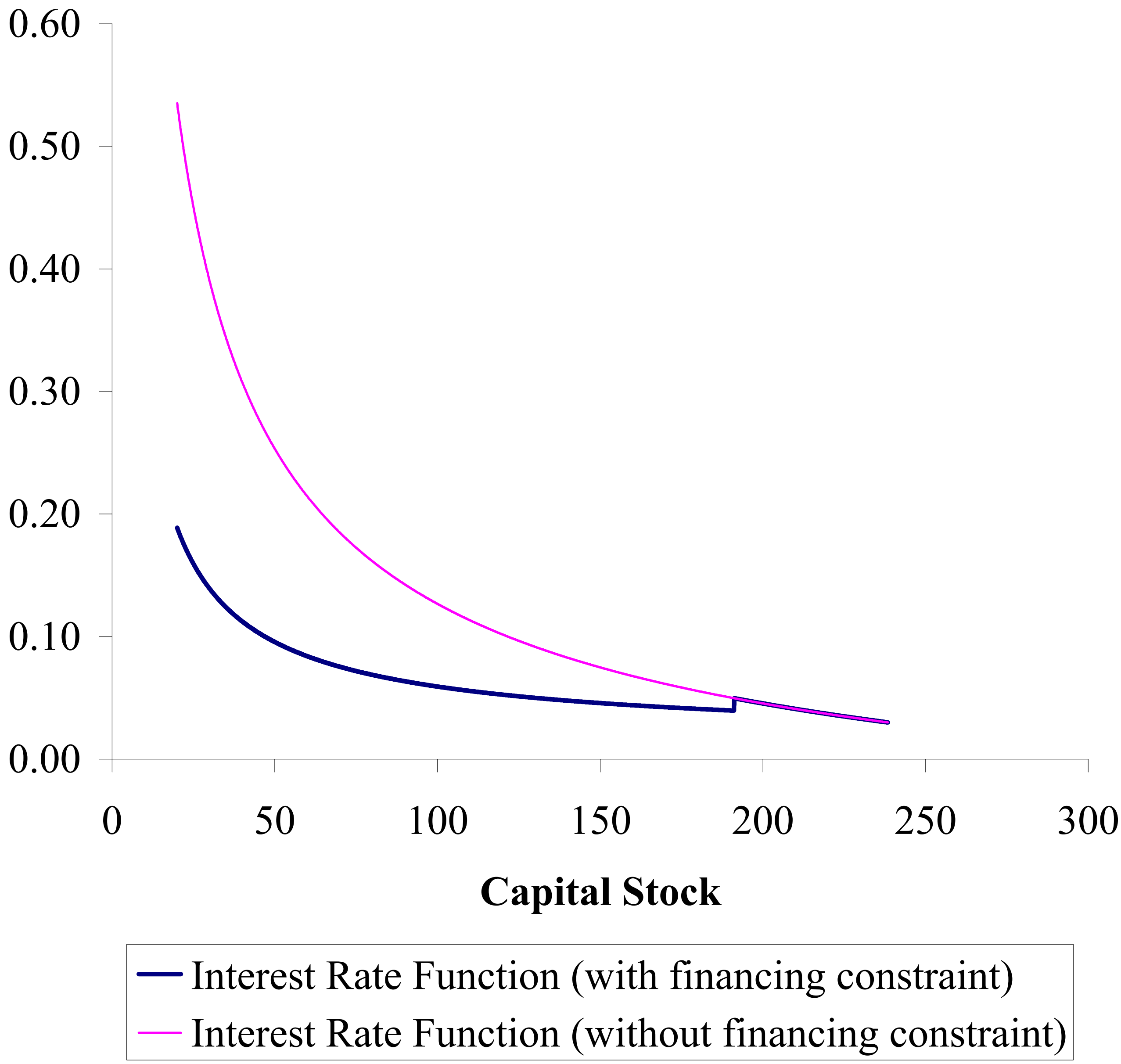


Figure 5: Change in Productivity

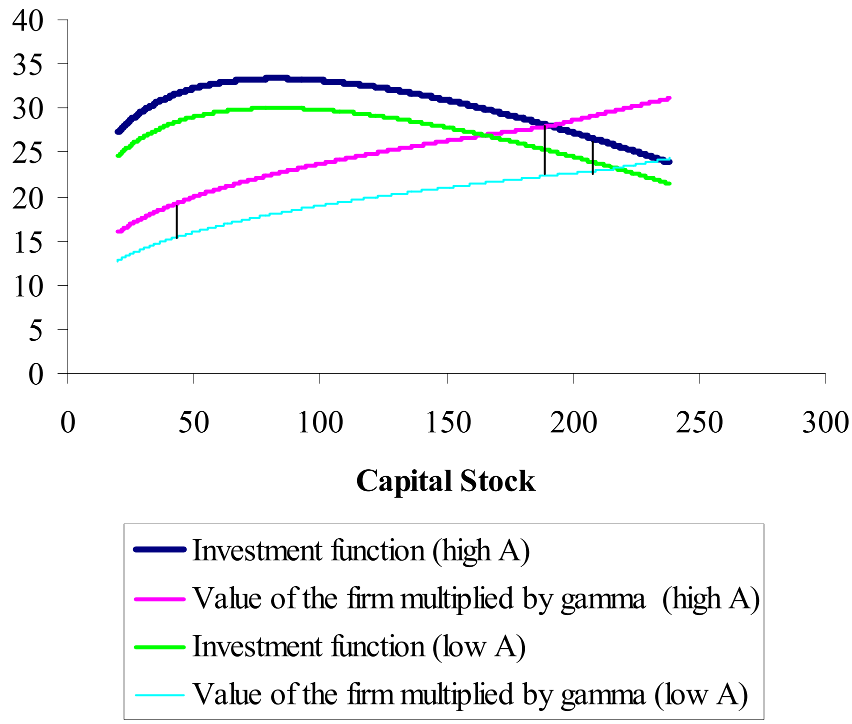
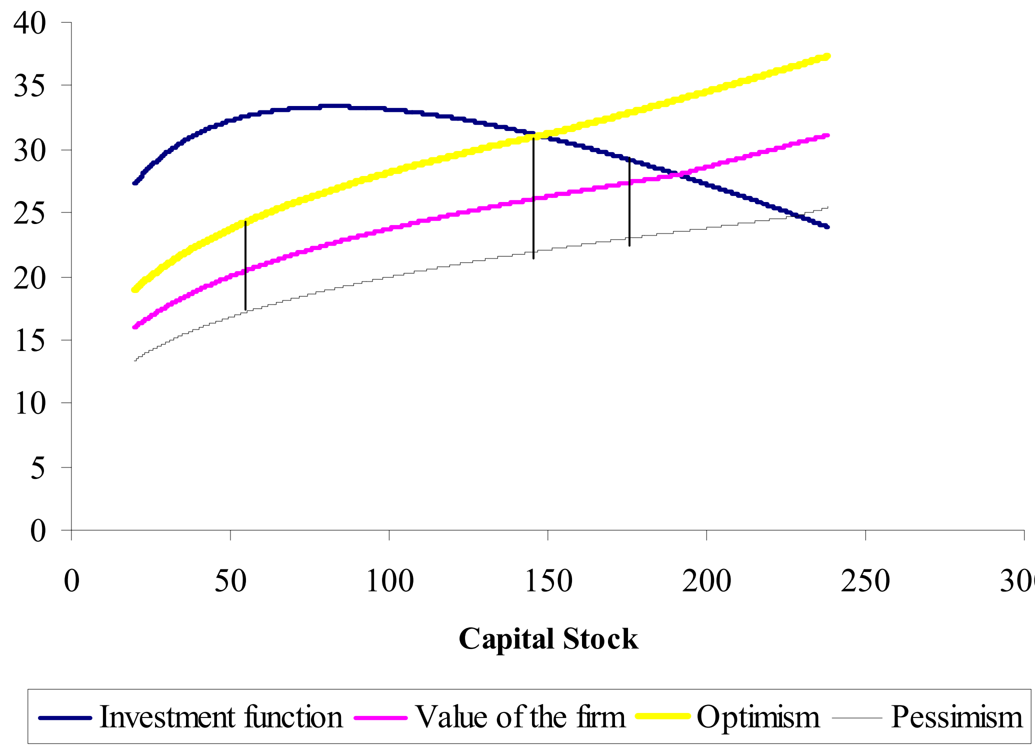


Figure 6: Misperception: Optimism and Pessimism



Appendix

4.1 A more detailed description of the banking sector in discrete time

At time 0, the household uses its initial capital k_0 to purchase shares of the firm and thus becomes its owner. With k_0, the firm produces $f(k_0)$, which it pays to the household as dividends. It then borrows I_0 from the bank at the competitive interest rate r_0 to finance its investment. When time 1 comes around, the capital stock grows to $k_1(= I_0 + (1-\delta)k_0)$, yielding output $f(k_1)$. After repaying principal and interest to the bank, the residual $f(k_1) - I_0(1+r_0)$ is paid out to the household. A new loan is then raised to finance investment I_1 at interest rate r_1. At time 2, the capital stock $k_2(= I_1 + (1-\delta)k_1)$ generates output $f(k_2)$ and dividend $f(k_2) - I_1(1+r_1)$. So on and so forth.

Therefore, value of the firm equals the present value of the net cash flow, i.e.,

$$\begin{aligned} V(k_0) &= f(k_0) + \left(\frac{1}{1+r_0}\right)[f(k_1) - I_0(1+r_0)] \\ &\quad + \left(\frac{1}{1+r_0}\right)\left(\frac{1}{1+r_1}\right)[f(k_2) - I_1(1+r_1)] + ... \\ &= [f(k_0) - I_0] + \left(\frac{1}{1+r_0}\right)[f(k_1) - I_1] \\ &\quad + \left(\frac{1}{1+r_0}\right)\left(\frac{1}{1+r_1}\right)[f(k_2) - I_2] + ... \end{aligned}$$

Regarding the household, she receives $f(k_t) - I_{t-1}(1+r_{t-1})$ from the firm as its shareholder and $I_{t-1}(1+r_{t-1})$ from the firm as its debt-holder, consumes $c_t = f(k_t) - S_t$, and deposits her savings S_t with the bank.

In equilibrium, supply of loans by the household (S_t) equals demand by loans by the firm (I_t), so that $c_t = f(k_t) - I_t$ and the present value of consumption simply equals the firm's value.

4.2 Derivation of $U(k_0)$ when $\alpha = \sigma$

Given $c = [\rho + (1-\sigma)\delta]\, k/\sigma$,

$$\begin{aligned} U(k_0) &= \frac{1}{1-\sigma}\int_0^\infty e^{-\rho t} c^{1-\sigma} dt - \frac{1}{(1-\sigma)\rho} \\ &= \frac{1}{1-\sigma}\left[\frac{\rho+(1-\sigma)\delta}{\sigma}\right]^{1-\sigma}\int_0^\infty e^{-\rho t} k^{1-\sigma} dt - \frac{1}{(1-\sigma)\rho}, \end{aligned}$$

where, with $\dot{k}/k = [\sigma A k^{\sigma-1} - (\rho+\delta)]/\sigma$,

$$\begin{aligned} \int_0^\infty e^{-\rho t} k^{1-\sigma} dt &= -\frac{e^{-\rho t}}{\rho} k^{1-\sigma}\Big|_0^\infty + \int_0^\infty \frac{e^{-\rho t}}{\rho}(1-\sigma)k^{-\sigma}\dot{k}dt \\ &= \frac{1}{\rho}k_0^{1-\sigma} + \int_0^\infty \frac{e^{-\rho t}}{\rho}(1-\sigma)\left[\frac{\sigma A - (\rho+\delta)k^{1-\sigma}}{\sigma}\right]dt \\ &= \frac{1}{\rho}k_0^{1-\sigma} + \frac{(1-\sigma)A}{\rho^2} - \frac{(1-\sigma)(\rho+\delta)}{\rho\sigma}\int_0^\infty e^{-\rho t}k^{1-\sigma}dt \end{aligned}$$

Hence,

$$\int_0^\infty e^{-\rho t} k^{1-\sigma} dt = \frac{k_0^{1-\sigma} + \left(\frac{1-\sigma}{\rho}\right)A}{\frac{\rho+(1-\sigma)\delta}{\sigma}},$$

implying

$$U(k_0) = \left(\frac{1}{1-\sigma}\right)\left\{\frac{k_0^{1-\sigma} + \left(\frac{1-\sigma}{\rho}\right)A}{\left[\frac{\rho+(1-\sigma)\delta}{\sigma}\right]^{\sigma}} - \frac{1}{\rho}\right\},$$

and

$$V(k_0) = k_0 + \left(\frac{1-\sigma}{\rho}\right)Ak_0^{\sigma}.$$

References

[1] Barth, James R., Gerard Caprio, Jr., and Ross Levine, "Bank Regulation and Supervision, What Works Best," Manuscript, World Bank, 2001.

[2] Bernanke, Ben S., Mark Gertler, and Simon Gilchrist, "The financial accelerator in a quantitative business cycle framework," in John B. Taylor and Michael Woodford (eds.), Handbook of Macroeconomics, volume 1, North-Holland, 1999.

[3] Carlstrom, Charles and Timothy Fuerst, "Agency cost, net worth, and business fluctuations: a computable general equilibrium analysis," American Economic Review, Vol. 87, 1997: 893–910.

[4] Chakraborty, Shankha and Tridip Ray, "Bank-based versus market-based financial systems: a growth-theoretic analysis," mimeo, Hong Kong University of Science and Technology, 2001.

[5] Chami, Ralph and Thomas F. Cosimano, "Monetary policy with a touch of Basel," IMF Working Paper No. 01/151, 2001.

[6] Chang, Roberto, and Andres Velasco, "Financial fragility and exchange rate regimes," Journal of Economic Theory 92 (2000), 1-34.

[7] Chen, Nan-Kuang, "Bank net worth, asset prices and economic activity," Journal of Monetary Economics, Vol. 48, 2001: 415–436.

[8] Kashyap, Anil K., and Jeremy C. Stein, "The impact of monetary policy on bank balance sheets," Carnegie-Rochester Conference Series on Public Policy, Vol 42, 1995: 151–195.

[9] Kiyotaki, Nobuhiro and John Moore, "Credit cycles," Journal of Political Economy, Vol. 105, 1997: 211–248.

[10] Knack S. and P. Keefer,"Does Social Capital Have a Payoff? A Cross Country Investigation," Quarterly Journal of Economics, Vol. 112, 1997: 1251-88.

[11] Krugman, Paul, "Crises: The next generation," paper presented at the Razin conference, Tel Aviv University, March 25-26, 2001.

[12] _________, "Balance sheet effects, the transfer problem, and financial crises," International Tax and Public Finance 6 (1999), 459-472.

[13] _________, "Bubble, boom, crash: Theoretical notes on Asia's crisis," mimeo, January 1998.

[14] _________, "A model of balance of payment crises," Journal of Money, Credit, and Banking 11 (1979), 311-325.

[15] Lucas, Robert E., Jr., "Why doesn't capital flow from rich to poor countries?" American Economic Review Papers and Proceedings 80 (1990), 92-96.

[16] Obstfeld, Maurice, "Models of currency crises with self-fulfilling features," European Economic Review 40 (1996), 1037-1048.

[17] Obstfeld, Maurice and Kenneth Rogoff, "Foundations of international macroeconomics," The MIT Press, 1996.

[18] Prescott Edward "Theory Ahead of Business-Cycle Measurement," Carnegie-Rochester Conference Series on Public Policy, Vol. 25, Autumn, 1985: 11–44

[19] Schneider, Martin, and Aaron Tornell, "Balance sheet effects, bailout guarantees, and financial crises," working paper, UCLA, November 2000.

[20] Van den Heuvel, Skander J., "The bank capital channel of monetary policy," mimeo, University of Pennsylvania, 2001.