# Using Instrumental Variables to Estimate the Share of Backward-Looking Firms<sup>\*</sup>

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#### Abstract

This paper examines the small-sample distribution of the instrumental variables (IV) estimation procedure employed by Gali and Gertler (1999) to assess the empirical fit of the New Keynesian Phillips Curve (NKPC) and the *hybrid* Phillips Curve (HPC). Their estimation method is now widely used to assess the importance of firms that act in a backward-looking manner. Unfortunately, the IV method is highly sensitive to the way the hybrid model is normalized. Using Monte Carlo simulations, I find that one normalization used by Gali and Gertler (and others) finds evidence of backward-looking firms even when there is none by construction. In addition, the IV estimates are also sensitive to the choice of normalization in a broader range of specifications. Using Monte Carlo experiments, I identify which normalizations work better than others. Finally, I find that the bootstrapped standard errors are, not surprisingly, bigger than the asymptotic ones reported by Gali and Gertler. When using my preferred normalization, I find that the NKPC is rejected at the 5 percent but not at the 1 percent level.

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### 1 Introduction

Recent empirical research on inflation dynamics has focused on the importance of forward-looking price setting behavior. Sbordone's (2002) calibration exercise provides considerable support for the (purely forward-looking) New Keynesian Phillips Curve-henceforth, NKPC. In particular, even though the NKPC does not postulate any structural inertia in the inflation process, it can generate a persistent time series of inflation, given the apparent persistence of marginal cost in US data- as Goodfriend and King (2001) have emphasized. Gali and Gertler's (1999) instrumental variables regressions also suggest predominantly forward-looking behavior. More precisely, Gali and Gertler find support for a hybrid Phillips Curve- henceforth, HPC- with most firms following the NKPC, but a fraction following a backwardlooking rule of thumb. However, Gali and Gertler report that this fraction is imprecisely estimated. Specifically, their estimate ranges from 27 to almost 50 percent depending on the chosen normalization. In one case, the NKPC seems largely consistent with a purely forward-looking model; in the other case, it does not.

The inability of the instrumental variables regression to determine whether US inflation dynamics is consistent with a largely forward-looking model or not is potentially cause for concern. In theory, the alternative models have very different policy implications. The purely forward-looking model (NKPC) implies that a fully credible disinflation has no output cost– i.e., has a sacrifice ratio equal to zero. By contrast HPCs imply a positive sacrifice ratio that increases with the fraction of backward looking firms. Moreover, as Goodfriend and King (2001) emphasize the appropriate conduct of monetary policy hinges on whether inflation is purely forward-looking or not.

At the heart of the imprecision in the estimates of the fraction of backwardlooking agents is a seemingly inconsequential choice of how to normalize the HPC. Specifically, Gali and Gertler consider two different normalizations: one "appears to minimize the non-linearities, while the second normalizes the inflation coefficient to unity" (Gali and Gertler, p. 207). Asymptotically, it should not matter which normalizations is used but in small samples it can (see, for example, Fuhrer, Moore and Schuh (1995) for a discussion). In Gali and Gertler's case, it seems to matter a lot. In this paper, I use Monte Carlo experiments to explore the sensitivity of the estimates of the share of backward-looking firms to the choice of normalization further.

My paper is similar in spirit to Guerreri (2001). However, this paper differs from his in important aspects. First, I point out that the only error to the inflation equation is an expectational error. That expectational error reflects innovations to the real marginal cost innovation. Therefore, a Monte Carlo that samples only from the innovations to inflation, like Guerreri does, seems inappropriate. By contrast, in my Monte Carlo experiment, I sample both from the innovations to inflation and real marginal cost. Despite our different approches, Guerreri and I share the same overall conclusion: the normalization that normalizes the coefficient on current inflation to unity overestimates the share of backward-looking firms. Moreover, I show that the choice of normalization is an issue in a broader range of specifications than those considered by Guerreri. For example, I show that results obtained by Gali and Gertler (1999) and others under "reduced-form" and "present-value" estimation are also sensitive to the choice of normalization.

In what follows, Section 2 presents the relevant theory I address. Section 3

presents the empirical specifications and Gali and Gertler's results. Section 4 presents my Monte Carlo experiment and results. To anticipate my results, I find that one commonly used normalization overestimates the fraction of backward-looking agents.<sup>1</sup> Indeed, my results suggest that Gali and Gertler's estimate of 27 percent seem more reliable than their estimate of 50 percent. My findings have implications for a number of papers that report that the NKPC has to be augmented by lags of inflation to fit the data. I conclude in Section 5 by noting areas for further research.

## 2 Theory and Empirical Evidence

#### 2.1 The New Keynesian Phillips Curve

Unlike the traditional Phillips curve, the new Keynesian Phillips curve is derived from the pricing behavior of a monopolistic competitor that faces some price stickiness in the spirit of the seminal work of Taylor (1980). Most commonly, following Calvo (1983), it is assumed in every time period a firm has a fixed probability,  $1 - \alpha$ , that it gets to reset its price. Using this convenient, albeit unrealistic, tool to model price stickiness, a pricing equation of the following form can be derived:<sup>2</sup>

$$\hat{\pi}_t = \lambda \widehat{mc_t} + \beta E_t \hat{\pi}_{t+1} \tag{1}$$

<sup>&</sup>lt;sup>1</sup>As I will show, the normalization that overestimates the fraction of backward-looking agents coincide with the estimation method used in a number of recent papers. Examples of such papers include Jondeau and Le Bihan (2001), Rudd and Whelan (2001), Benigno and López-Salido (2001), and Galí, Gertler and López-Salido (2001).

 $<sup>^{2}</sup>$ See Woodford (2001) for a derivation. The NKPC can also be derived by assuming that firms have price adjustment costs (Rotemberg, 1982, 1987). See Sbordone (1999) for a comparison of the two methods.

where  $\hat{\pi}_t$  is current inflation,  $\widehat{mc}_t$  is real marginal cost,  $\hat{}$  denotes the percentage deviation of a variable from its steady-state value,  $\beta$  is the subjective discount rate,  $\lambda$  is a function of the structural parameters of the model,  $\lambda = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$ , and  $E_t \hat{\pi}_{t+1}$  is the mathematical expectation of next period's inflation  $(\hat{\pi}_{t+1})$  given information available in period t.

Under certain conditions, marginal cost is proportional to the output gap,  $x_t$ , and the pricing equation can be rewritten to get:

$$\hat{\pi}_t = \lambda \kappa x_t + \beta E_t \hat{\pi}_{t+1} \tag{2}$$

where  $\kappa$  is the output elasticity of marginal cost. Work by Fuhrer and Moore (1995), Fuhrer (1997) and Roberts (1998) examined equation (2), where marginal cost has been replaced by a measure of the output gap. They find that the purely forwardlooking NKPC, relating inflation and a measure of the output gap, is unable to generate the type of persistence observed in actual US data.

By contrast, two more recent papers by Gali and Gertler (1999) and Sbordone (2002) find that U.S. data seem to be largely consistent with a pricing equation, relating inflation and marginal cost, from a forward-looking model. Rather than using the output gap as a proxy for real marginal cost, they use (log) real unit labor cost or the (log) labor share in output ( $s_t$ ). Both papers are widely quoted and have triggered a number of papers that explore their findings further.<sup>3</sup> In Gali and

<sup>&</sup>lt;sup>3</sup>Galí, Gertler and Lopez-Salido (2001) examine euro-area inflation, Benigno and Lopez-Salido (2001) examine individual European countries and Jondeau and Le Bihan (2001) examine the sensitivity of Galí, Gertler and Lopez-Salido's (2001) results. Rudd and Whelan (2001) argue that Galí and Gertler's IV estimation procedure is sensitive to small specification errors. Lindé (2001) argues that full information maximum likelihood (FIML) estimation may be more robust than GMM. Finally, Galí, Gertler and Lopez-Salido's (2003) respond to Rudd and Whelan and Lindé's criticisms.

Gertler's paper, they develop a hybrid model that incorporates two types of firms: firms that behave in the forward-looking manner described in the Calvo model, and firms that behave according to a "rule of thumb". The latter type set prices based on the past evolution of prices, thereby incorporating structural inertia into the model explicitly.

#### 2.2 The hybrid model

Motivated largely by the empirical observation that inflation is persistent, a number of studies have suggested alternative theories to explain the persistence in inflation. For example, Fuhrer and Moore (1995) consider a hybrid version of the new and old:

$$\hat{\pi}_t = \delta x_t + (1 - \eta) E_t \hat{\pi}_{t+1} + \eta \hat{\pi}_{t-1} \tag{3}$$

with  $0 < \eta < 1$  and  $x_t$  some measure of the cyclical movement of GDP. However, Fuhrer and Moore (1995) had limited success fitting their specification to US data. By contrast, motived by their finding that past failures were due to using the output gap instead of real unit labor cost as a measure of real marginal cost, Gali and Gertler derive an alternative hybrid version of the form:

$$\hat{\pi}_t = \lambda \hat{s}_t + \gamma_f E_t \hat{\pi}_{t+1} + \gamma_b \hat{\pi}_{t-1} \tag{4}$$

where

$$\begin{split} \tilde{\lambda} &\equiv (1-\omega) (1-\alpha) (1-\alpha\beta) \phi^{-1} \\ \gamma_f &\equiv \alpha \beta \phi^{-1} \\ \gamma_b &\equiv \omega \phi^{-1} \end{split}$$
(5)

with  $\phi \equiv \alpha + \omega [1 - \alpha (1 - \beta)]$  and  $\omega$  measuring the share of backward-looking firms, i.e., firms that behave simply by setting prices based on the recent history of aggregate prices (see Gali and Gertler). One convenient feature of Gali and Gertler's specification is that the hybrid version collapses to the purely forward-looking model when there are no backward-looking firms, i.e., when  $\omega = 0$ .

## 3 Instrumental variables regressions

In this section, I will discuss the instrumental variables procedure used by Gali and Gertler to estimate  $\alpha, \beta$  and  $\omega$ . Their estimates seem consistent with the underlying theory: their estimate of  $\alpha$  implies reasonable average price contracts (around 5 quarters), and  $\beta$  is estimated close to one. Finally, their estimate of the slope coefficient on real marginal cost is always positive and significant. However, they always find the share of backward-looking firms to be positive and statistically significant. Thus, they reject the purely forward-looking model.

Unfortunately, their estimate of  $\omega$  is very sensitive to the chosen normalization which I will now discuss in more details.

#### 3.1 Reduced-form estimation

Following Gali and Gertler, let  $\Omega_{t-1}$  denote a vector of variables observed at time t-1 or earlier (in fact,  $\Omega_{t-1}$  only need to be dated t or earlier to be orthogonal to the inflation surprise in period t).<sup>4</sup> Then, under rational expectations, equation (4) can be used to write the set of orthogonality conditions as:

$$E\left\{\left(\hat{\pi}_t - c - \tilde{\lambda}\hat{s}_t - \gamma_f\hat{\pi}_{t+1} - \gamma_b\hat{\pi}_{t-1}\right)\Omega_{t-1}\right\} = 0$$
(6)

where  $\tilde{\lambda}, \gamma_f$ , and  $\gamma_b$  are defined as before,  $\hat{s}_t$  is the observable measure of real marginal cost (i.e., the real unit labor cost) and c is a constant included for estimation purposes. Equation (6) lends itself to be estimated using instrumental variables (i.e., one version of generalized method of moments (GMM)).

Indeed, this specification has been widely used. For example, Gali and Gertler (1999) report estimates of the NKPC version of (6) (i.e., where  $\gamma_b = 0$  and  $\gamma_f = \beta$ ), Bårdsen, Jansen, and Nymoen (2002) explore Norwegian data using this specification, and Jondeau and Le Bihan (2001) use the same equation to estimate a restricted version where  $\gamma_b + \gamma_f = 1$ . However, as highlighted by Fuhrer, Moore and Schuh (1995) GMM estimates are known to sensitive to "asymptotically irrelevant aspects of the econometric specification, such as parameter normalization" (p. 116). Alternatively, equation (6) can be re-written as

$$E\left\{\left(\frac{1}{\gamma_f}\left[\hat{\pi}_t - c - \tilde{\lambda}\hat{s}_t - \gamma_f\hat{\pi}_{t+1} - \gamma_b\hat{\pi}_{t-1}\right]\right)\Omega_{t-1}\right\} = 0$$
(7)

<sup>&</sup>lt;sup>4</sup>Following Galí and Gertler, I use a constant, and four lags of the following variables as instruments: the labor share, inflation, wage inflation, commodity price inflation, and the long-short interest spread.

Asymptotically, it does not matter which normalization is used but in small samples it may. As table 1 shows, the choice of normalization matters.<sup>5</sup> Oddly, this sensitivity is not raised by Gali and Gertler when estimating (6) and (7) which they refer to the reduced form.<sup>6</sup> However, they raise the issue when estimating the structural parameters directly (see discussion that follows).

When using Gali and Gertler's choice (i.e., equation (6)), lagged inflation appears almost as important as future inflation. Indeed, this result suggest that the NKPC has to be augmented by lags of inflation to fit the data. By contrast, when using equation (7) as the basis for estimation, lagged inflation seems less important (see table 1). Unfortunately, though, the slope coefficient on real marginal cost is not significant when using the latter specification.<sup>7</sup>

#### 3.2 Structural estimation

When substituting in the definitions of  $\tilde{\lambda}$ ,  $\gamma_b$ , and  $\gamma_f$  into equation (7), it is clear that there are several ways of normalizing the HPC. In their paper, Gali and Gertler (1999) propose two different normalizations. Their first normalization minimize nonlinearities by multiplying through by  $\phi$  to get:

<sup>&</sup>lt;sup>5</sup>I thank Andrew Jackson for sending me a copy of Galí and Gertler's RATS program. This paper uses a Matlab code by Mike Cliff, available at my website. My estimates coincide with those reported by RATS (using the "nlls" function with "robust" errors). Specifically, in my Matlab program, I use a two-step GMM estimator, the first iteration uses  $(\Omega'\Omega)^{-1}$  (where  $\Omega$  is the matrix of instruments) as a weighting matrix, and a 12-lags NW weighting matrix in the second iteration. The standard errors for  $\gamma_b, \gamma_f, \tilde{\lambda}$  and the average price duration (D) differ from those reported in Galí and Gertler's table 2. I believe that Galí and Gertler made a mistake when calculating the standard errors based on the delta method.

<sup>&</sup>lt;sup>6</sup>Once again, Gali and Gertler only estimates the NKPC in reduced-form, not the HPC. However, the choice of normalization is an equally important issue in that case.

<sup>&</sup>lt;sup>7</sup>Moreover, when using a similar normalization for the estimation of the NKPC, the estimate of  $\beta$  exceeds one, and the estimate of  $\lambda$  is negative and insignificant.

$$E\{(\phi\hat{\pi}_{t} - c - (1 - \omega)(1 - \alpha)(1 - \alpha\beta)\hat{s}_{t} - \alpha\beta\hat{\pi}_{t+1} - \omega\hat{\pi}_{t-1})\Omega_{t-1}\} = 0$$
 (8)

while the other normalizes the coefficient on inflation to unity:

$$E\left\{\left(\hat{\pi}_{t}-c-\frac{(1-\omega)\left(1-\alpha\right)\left(1-\alpha\beta\right)}{\phi}\hat{s}_{t}-\frac{\alpha\beta}{\phi}\hat{\pi}_{t+1}-\frac{\omega}{\phi}\hat{\pi}_{t-1}\right)\Omega_{t-1}\right\}=0\qquad(9)$$

where, as before,  $\phi \equiv \alpha + \omega [1 - \alpha (1 - \beta)]$ . I will refer to equation (8) as "normalization 1" and equation (9) as "normalization 2".

In table 1, I replicate Gali and Gertler's results for the two specifications. As can be seen the choice of normalization makes a significant difference. In one case, the estimate of  $\alpha$  implies that the average duration of prices is close to 5 quarters while the other normalization suggests that the average price duration is more than 6 quarters. Even more striking, the estimated share of backward-looking firms range from 0.265 to 0.486 depending on the choice of normalization.

The implied estimates of  $\gamma_b, \gamma_f$  and  $\tilde{\lambda}$  obtained using equation (9) ("normalization 2") coincide with those obtained using Gali and Gertler's "reduced-form" estimation (see table 1). Thus, there is no additional information gained by reporting both Gali and Gertler's choice of normalizing the "reduced-form" and "structural" estimates based on normalization 2, as is commonly done in this literature.<sup>8</sup>

<sup>&</sup>lt;sup>8</sup>In Benigno and López-Salido (2001) and Gali, Gertler and López-Salido (2001), it appears as if the structural estimates from normalization 2 no longer imply an estimate of  $\tilde{\lambda}$  that coincide with the "reduced-form" estimate of  $\tilde{\lambda}$ . However, this is due to a mistake in both papers. In both papers, they report their structural estimates (which are correct) but proceed to calculate the implied  $\tilde{\lambda}$  as  $\frac{(1-\omega)(1-\alpha)(1-\alpha\beta)}{\phi}$ . However, both papers augment the baseline model so that this definition of  $\tilde{\lambda}$ 

is no longer the correct. In Gali, Gertler and López-Salido the new definition of  $\tilde{\lambda}$  is  $\frac{(1-\omega)(1-\alpha)(1-\alpha\beta)}{\phi}\xi$  where  $\xi \equiv \frac{1-b}{1+b(\theta-1)}$  with  $\theta$  being the elasticity of substitution among differen-

 $<sup>\</sup>phi$   $\zeta$  where  $\zeta = \frac{1}{1+b(\theta-1)}$  with b being the elasticity of substitution alloing different tiated goods and b is the share of capital in the Cobb-Douglas production function. Once  $\tilde{\lambda}$  has been correctly calculated, the implied estimate of  $\tilde{\lambda}$  (obtained from normalization 2) coincide with the reduced-form estimate.

Following Gali and Gertler, researchers generally try different normalizations when estimating the structural parameters ( $\alpha, \beta$  and  $\omega$ ). By contrast, the normalization issue has been ignored when using other specifications. For example, Rudd and Whelan (2001) propose estimating the hybrid model in its present-value representation. In appendix C, I examine this estimation procedure in more details and find that it is also sensitive to the choice of normalization.

#### 4 Monte Carlo evidence

Given that the IV estimation results seem highly sensitivity to the choice of normalization, it seems worthwhile to explore its small sample properties. To do so, I use two different Monte Carlo experiments. First, I generate simulated data from an unrestricted VAR to study the bootstrapped distribution of the point estimates under different normalizations. These results confirm that the estimates of the share of backward-looking firms vary largely across normalization. Moreover, this exercise allows me to assess whether the asymptotic standard errors reported in Gali and Gertler's paper (and above) are appropriate.

However, this exercise does not provide any guidance in terms of which normalization to use since I do not know what the true values of  $\alpha$ ,  $\beta$  and  $\omega$  are. Therefore, I simulate data under the null that the HPC (with my choice of values for  $\alpha$ ,  $\beta$  and  $\omega$ ) is the data generating process. In that case, I know the true parameter values and can examine if the IV estimation correctly retrieves these values. Equally importantly, I can explore which normalization (if either) is to be preferred. To anticipate my results, I show that the IV estimates of  $\alpha$  and  $\beta$  are somewhat robust across normalizations but only when using normalization 1, which multiplies through by  $\phi$ , do I correctly estimate  $\omega$  for small values of  $\omega$ .

Finally, having constructed data under the null that the NKPC is true (i.e., when I impose that  $\omega = 0$ ), I can ask how likely it is that I mistakenly reject the null that  $\omega = 0$  when it is true by construction. That is, I can calculate the empirical rejection frequency.

#### 4.1 Boostrapped standard errors

I simulate 10,000 artificial data sets for inflation, the labor share, and the additional variables needed as instruments assuming that a simple unrestricted VAR is the true data generated process (see details in Appendix A). For each of the 10,000 simulated data sets, I estimate  $\alpha, \beta$  and  $\omega$  using both normalizations 1 and 2 and report their median values and 95 percent confidence intervals. As can be seen in table 2, the median value of the estimates of  $\alpha$  and  $\beta$  and the 95 percent confidence intervals are not very sensitive to the chosen normalization. By contrast, the estimates of  $\omega$  are very sensitive. In figure (2), I show the histograms of the estimates of  $\omega$  based on the two normalizations. As can be seen from the figure and table, when using normalization 1, the median value of  $\omega$  is 0.185 and the 95 percent confidence interval is (0.003-0.366). When using normalization 2, the estimates are significantly larger. The median value is 0.486 and the confidence interval is (0.342-0.613).<sup>9</sup>

Regardless of the chosen normalization, the bootstrapped standard errors are larger than the asymptotic ones reported by Gali and Gertler (and in table 1).

<sup>&</sup>lt;sup>9</sup>Surprisingly, the large differences in the estimate of  $\omega$  do not seem to be due to the number of observations. I simulated data that had up to 10,000 observations, and the sensitivity to the chosen normalization persisted (see graphs in appendix).

Indeed, when using normalization 1, the 95 percent confidence interval nearly reaches zero. Thus, although I can still reject the purely forward-looking NKPC at the 5 percent level, I cannot reject it at the 1 percent level. By contrast, when using normalization 2, the rejection of the NKPC seems sounder. Still, I am unable to conclude which of the two normalizations should be preferred since I do not know the true value of  $\omega$ . Therefore, in the next Monte Carlo experiment, I simulate data under the null that the NKPC (or a version of the HPC) is the data generating process (DGP). This experiment also allows me to ask how likely it is that the IV regressions incorrectly reject the null when it is true.

# 4.2 Simulating data under the null that NKPC (or HPC) is the DGP

In contrast to the previous section, I now simulate data under the null hypthesis that the NKPC is the data generating process process for inflation. Then I generate data under the null that the HPC is the data generating process with several values for  $\omega$ . Similar to above, I need to simulate data for all the variables I will be using in my estimation: inflation, the labor share, and the other instruments.<sup>10</sup>

Ideally, I would have liked to have simulated data for inflation, the labor share, and Gali and Gertler's instrument set, which includes four additional variables.<sup>11</sup> However, as discussed in Appendix B, when I include lags of all six variables as instruments and impose the null that the NKPC is DGP, the resulting system is

<sup>&</sup>lt;sup>10</sup> Again, this is where my paper differs from Guerreri (2001). We both impose the same restriction on the inflation equation but Guerreri keeps actual data for the labor share and the instruments in his Monte Carlo experiment. That is, he generates 100 simulated inflation series where he has imposed that the NKPC (or HPC) is the DGP. However, he does not simulate data for the other variables needed in his estimation (see Appendix B for a more detailed discussion).

<sup>&</sup>lt;sup>11</sup>The additional variables are wage inflation, commodity price inflation, the output gap and the long-short interest spread.

nonstationary. Instead, I re-ran Gali and Gertler's estimation using a smaller instrument set: a constant and four lags of the labor share, inflation, wage inflation and commodity price inflation.

As can be seen when comparing table 1 and 3, had Gali and Gertler chosen to use only this smaller instrument set, they would have gotten almost identical results. The NKPC is still resolutely rejected with  $\omega$  estimated at 0.289 and with a small standard error (using normalization 1). However, the rejection is not quite as sound as before since the standard error is larger with the smaller instrument set. While it is dissatisfying not being able to simulate data that would allow me to directly assess Gali and Gertler's results, given that the results do not seem to vary across the two instrument sets, I believe my Monte Carlo results are nevertheless informative.<sup>12</sup>

To simulate data under the null, I note that the NKPC (and HPC) only imposes two restrictions: first, defining  $\varepsilon_{t+1} \equiv \hat{\pi}_{t+1} - E_t \hat{\pi}_{t+1}$  and replacing  $\widehat{mc}_t$  by  $\hat{s}_t$ , equation (1) can be lagged and re-arranged to yield:

$$\hat{\pi}_t = -\frac{\lambda}{\beta}\hat{s}_{t-1} + \frac{1}{\beta}\hat{\pi}_{t-1} + \varepsilon_t \tag{10}$$

and, in the case where  $\omega > 0$ ,

$$\hat{\pi}_t = -\frac{\hat{\lambda}}{\gamma_f} \hat{s}_{t-1} + \frac{1}{\gamma_f} \hat{\pi}_{t-1} - \frac{\gamma_b}{\gamma_f} \hat{\pi}_{t-2} + \varepsilon_t \tag{11}$$

 $<sup>^{12}</sup>$ For details of the results reported in table 3, see notes under table 1. The instrument set includes a constant, and four lags of inflation, the labor share, wage inflation, and commodity price inflation.

The second restriction is that  $\hat{\pi}_t$  Granger causes the labor share. Therefore, to simulate data under the null, I proceed as follows: first, I estimate an unrestricted VAR using four variables: the labor share, wage inflation, commodity price inflation, and inflation.

$$Z_t = \begin{bmatrix} \hat{s}_t & dw_t & dc_t & \hat{\pi}_t \end{bmatrix}'$$

Second, I replace the unrestricted estimates in the inflation equation by the restrictions implied by equations (10) or (11), depending on whether I am interesting in generating data under the null of  $\omega = 0$  or  $\omega > 0$ . I choose  $\alpha = 0.8$  and  $\beta = 0.99$ as the "true" parameters. For  $\omega = 0$ , this implies the following reduced-form parameters:  $\gamma_b = 0, \gamma_f = \beta = 0.99$ , and  $\tilde{\lambda} = 0.052$ . I leave the other equations in the VAR, including the equation for  $\hat{s}_t$ , unrestricted. Since lags of inflation appear with non-zero coefficients in the labor share equation, the second restriction appears to be satisfied. Using this restricted VAR, I sample (with replacement) from the residuals to construct i = 1, ...10,000 artificial samples  $\{Z_t^i\}_{t=1}^T$ .

On each of these samples, I use the IV estimation procedure under several different specification: first, I use the two "reduced-form" normalizations discussed earlier to estimate  $\gamma_b, \gamma_f$  and  $\tilde{\lambda}$ . Second, I estimate  $\alpha, \beta$  and  $\omega$  based on both normalization 1 and 2.<sup>13</sup> For each of these four specifications, I report the median values and the 95 percent confidence intervals in table 4.

When I impose that  $\omega = 0$ , I find the following: on the positive side, using normalization 1 (equation (8) generally yields estimates of  $\alpha$  and  $\beta$  that are close to

<sup>&</sup>lt;sup>13</sup>Again, the implied values of  $\gamma_b$ ,  $\gamma_f$  and  $\tilde{\lambda}$  obtained under specification 2 will coincide with the estimates obtained using Gali and Gertler's proposed reduced-form estimates.

the "true" parameters. The median estimate of  $\omega$  is also close to the truth,  $\omega = 0$ . Moreover, the "reduced-form" estimates based on equation (7) are also quite good.

Surprisingly, normalization 2 (and, therefore, Gali and Gertler's proposed normalization of the "reduced form") yields quite poor results. Although the median estimates of  $\alpha$  are  $\beta$  are close to target, a surprisingly high number of estimates were very far from the "true" values, resulting in the large confidence intervals reported in table 7.<sup>14</sup> More troubling, though, is that the estimates of  $\omega$  were far from the "true" parameter. Specifically, the median estimate was 0.878 and the 95 percent confidence interval ranged from 0.356 to 1.597. Similarly, when using Gali and Gertler's proposed normalization of the "reduced form", the median estimate of  $\gamma_b$  is 0.481 when, by construction, is should be zero.

My Monte Carlo experiment also allows me to increase the sample size of my simulated data. By increasing the sample sample and re-estimating the structural parameters on this new longer data set, I confirm that the normalization issue is a small-sample problem. In particular, for very large sample sizes, the estimates of  $\omega$  are virtually identical across the two choices of normalization.<sup>15</sup>

I also simulated data under the null hypothesis that the hybrid model is the DGP. For example, I constructed data when  $\omega = 0.25, \omega = 0.5$  and  $\omega = 0.7$ . Similar to the case when  $\omega = 0$ , I report the median values of my estimates and their 95 percent confidence intervals in tables 5-7. Interestingly, normalization 2 does equally poorly when  $\omega = 0.25$ . For example, the median estimate of  $\omega$  is 0.593, whereas normalization 1 is almost right on target. By contrast, when simulating data

<sup>&</sup>lt;sup>14</sup>Specifically, more than 4 and 5 percent of the estimates of  $\beta$  and  $\alpha$ , respectively, were larger than 1.5 with some estimates exceeding 100.

<sup>&</sup>lt;sup>15</sup>The results are available upon request.

from models with higher degrees of backward-lookingness ( $\omega = 0.5$  and  $\omega = 0.7$ ), normalization 2 seems to do as well as (if not better than) normalization 1. Indeed, when the true value of  $\omega$  is 0.7, the median value estimated is 0.621 and 0.720 when using normalization 1 and 2, respectively. The median estimates of both  $\alpha$  and  $\beta$ are closer to target when using normalization 2.

Still, the estimates obtained when using normalization 1 are never far from the true values. Thus, a good strategy to adopt when estimating hybrid Phillips curves seem to be the following: avoid normalization 2!

These results have implications for a number of recent papers, including Jondeau and Le Bihan (2001), Rudd and Whelan (2001) and Benigno and López-Salido (2001) who estimate the HPC in its reduced form.<sup>16</sup> In most cases, the authors soundly reject the NKPC in favor of a HPC with a large weight on lagged inflation.

For each of the 10,000 simulated data, I also ran my version of the presentvalue estimation (see appendix C for more details).<sup>17</sup> Not surprisingly, the same overall conclusion holds true: the normalization that leaves the coefficient on current inflation as unity, tends to overestimate the importance of backward-looking firms. In light of this, there are good reasons to suspect that Rudd and Whelan's (2001) proposed "present-value" procedure suffers from the same problems. Specifically, they ignore the issue of normalization altogether and decide to leave the coefficient on current inflation as unity.

<sup>&</sup>lt;sup>16</sup>In Benigno and Lopéz-Salido (2001), they use structural estimation but only use normalization 2. As discussed, this method yields estimates that are equivalent to Gali and Gertler's "reducedform" estimation.

<sup>&</sup>lt;sup>17</sup>That is, I used the following equations from appendix C: (13) and (14) to estimate  $\gamma_b, \gamma_f$  and  $\tilde{\lambda}$ .

Given the results in table 4, it seems plausible that the IV estimation method could find evidence of backward-looking behavior (and, thus, reject the NKPC) even when there is none by construction. To explore this possibility further, I simply choose to focus on normalization 1 and calculate the empirical rejection frequency as follows: first, for each of the 10,000 estimates reported in table 4, I calculate the *t*-statistic

$$t_{\omega}^{(i)} = \frac{\hat{\omega}^{(i)} - 0}{std\left(\hat{\omega}^{(i)}\right)}$$

where  $\hat{\omega}^{(i)}$  is the *i'th* estimate of  $\omega$  and  $std\left(\hat{\omega}^{(i)}\right)$  is its asymptotically-based standard error. Second, I calculate

$$\Pr(\text{type I error}) = \frac{sum\left(t_{\omega}^{(i)} > 1.96\right)}{\#\text{bootstrap samples}}$$

and find that the empirical rejection frequency is 17.8 percent when using normalization 1. That is, even when using the most favorable normalization, Gali and Gertler could mistakenly be rejecting the NKPC in 17.8 percent of the cases when using the asymptotically-based t-distribution.

Still, Gali and Gertler not only rejected the null that  $\omega = 0$ , they resolutely rejected it by finding  $\hat{\omega} = 0.266$  with a standard error of 0.032, implying a *t*-statistic of 8.19 when using normalization 1. However, since I am using a different instrument set, I cannot directly assess this result. However, focusing on their results for normalization 1, I ask the related question of how likely they would be to get the result reported in table 3, namely, a point estimate of  $\hat{\omega} = 0.289$ , and a related *t*-statistic of 6.54. To answer this, I ask how likely it is to get a *t*-statistic above 6.54 by calculating

$$\Pr(t\text{-stat} > 6.54) \frac{sum\left(t_{\omega}^{(i)} > 6.54\right)}{\#\text{bootstrap samples}}$$

The answer is: not very likely. Although, approximately 13 percent of the estimates of  $\omega$  exceed 0.289, their standard errors are larger than the ones obtained on actual data. Therefore, less than one percent of the estimates had a *t*-stat as large as the one obtained by Gali and Gertler.

Several conclusions can be drawn from the Monte Carlo results: first, normalization 2 tends to overestimate the share of backward-looking firms, and should be avoided. Second, "reduced-form" estimation is as vulnerable to the choice of normalization as "structural estimation." On this note, I show that the normalization chosen by Gali and Gertler is not to be preferred. Third, the empirical rejection frequency of a test with a 5 percent nominal size is considerably larger than 5 percent (i.e., when using t-stats based on asymptotic standard errors, the IV estimation method incorrectly rejects the NKPC too often). Fourth, even when using the correct normalization, Gali and Gertler soundly rejected the NKPC (with a t-stat well about 6). Examining data constructed from a purely forward looking model (i.e., when  $\omega = 0$ ), I was only able to reject the NKPC as soundly as Gali and Gertler were in less than 1 percent of my simulations.

## 5 Conclusion

Monetary theory models tell us that the appropriate conduct of monetary policy hinges on the specification of inflation. Not surprisingly, empirical interest in testing whether the NKPC – a key aspect of the theoretical literature –fits actual inflation dynamics has recently flourished. Recently, Gali and Gertler developed a *hybrid* model which nests the purely forward-looking NKPC. This model provides for a convenient way of testing the NKPC against an alternative model that explicitly incorporates structural inertia. Specifically, their model allows for a share of firms to act in a backward-looking (rule-of-thumb) manner. Unfortunately, as Gali and Gertler note in their conclusion: "there is some imprecision in our estimates of the importance of backward looking behavior" (p. 219).

In fact, when employing their IV estimation procedure, there is quite a lot of imprecision in the estimated share of backward-looking firms. Unfortunately, the imprecision hinges on an asymptotically irrelevant choice of how to normalize the hybrid model. According to one normalization, the share of backward-looking firms is small, and the NKPC seems like a reasonable approximation to reality. According to another normalization, almost half the firms act in a rule-of-thumb manner.

The debate regarding the importance of backward-looking versus forward-looking agents has flourished since Gali and Gertler's paper. In my paper, find that one normalization used by Gali and Gertler overestimates the share of backward-looking firms. Moreover, I also point out that the IV estimates are also sensitive to the choice of normalization when estimating the hybrid model in "reduced-form" or in its "present value" form.<sup>18</sup> Similarly, Gali, Gertler and López-Salido's (2003) most

 $<sup>^{18}</sup>$ The latter specification has recently been proposed by Rudd and Whelan (2001) as a superior

recent paper, "Robustness of the Estimates of the Hybrid New Keynesian Phillips Curve" also ignores the issue of normalization.

In conclusion, when using my preferred normalization and bootstrapped standard errors, I can still reject the NKPC (by finding evidence of backward-looking firms) at the 5 percent but not at the 1 percent level. The sensitivity to the choice of normalization documented in this paper can be seen as yet another example of the types of problems that occur when using instrumental variables.

way of testing for the importance of backward-looking firms (see appendix C). Unfortunately, the sensitivity to normalization plays as important a role in their estimation as it does in Galí and Gertler's original work. And, unfortunately, their choice of normalization is likely to find evidence of backward-looking behavior even in models where there is none, by construction.

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## 6 Appendix A - Boostrapped standard errors

1. Consider the estimated unrestricted VAR process  $Z_t = \hat{A}Z_{t-1} + \hat{V}_t$  as the data generating process (DGP) and save the residuals  $\{\hat{V}_1, \hat{V}_2, ..., \hat{V}_T\}$ , where T is the observed sample size (T = 146), with

$$Z_t = \begin{bmatrix} s_t \\ dw_t \\ ygap_t \\ spr_t \\ dc_t \\ \hat{\pi}_t \end{bmatrix}$$

where  $s_t$  is the labor share,  $dw_t$  is wage inflation,  $ygap_t$  is the output gap,  $spr_t$  is the long-short interest spread,  $dc_t$  is commodity price inflation, and  $\hat{\pi}_t$  is inflation.

2. Simulate i = 1, ...10,000 artificial samples  $\{Z_t^i\}_{t=1}^T$  by taking random draws with replacement from the estimated residual vector and inserting them into the assumed DGP.

3. Compute Gali and Gertler's GMM coefficient estimates for  $\alpha, \beta$  and  $\omega$  for each of the 10,000 artificial samples.

4. Based on the 10,000 estimates for  $\alpha, \beta$  and  $\omega$  calculate the 95 percent confidence interval.



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Figure 1: 5,000 estimates of  $\omega$  with a sample size of 2000 (5.8 hours)

## 7 Appendix B - The empirical rejection frequency

Simulating data under the null that the NKPC is the DGP for the inflation equation.

Step 1: Use actual data for 
$$Z_t = \begin{bmatrix} s_t \\ dw_t \\ dc_t \\ \hat{\pi}_t \end{bmatrix}$$
 to estimate the companion matrix,  $A$ , of the system

$$Z_t = AZ_{t-1} + \varepsilon_t$$

Step 2: Throw out the equation corresponding to inflation and replace it by  $\hat{\pi}_t = -\frac{\tilde{\lambda}}{\gamma_f}s_{t-1} + \frac{1}{\gamma_f}\hat{\pi}_{t-1} - \frac{\gamma_b}{\gamma_f}\hat{\pi}_{t-2}$  (the restriction on the inflation equation implied by the HPC). That is, I will get a restricted companion matrix,  $A^R$  which will look like (with two lag in the VAR):

$$A^{R} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} & a_{28} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} & a_{37} & a_{38} \\ -\frac{\tilde{\lambda}}{\gamma_{f}} & 0 & 0 & \frac{1}{\gamma_{f}} & 0 & 0 & 0 & -\frac{\gamma_{b}}{\gamma_{f}} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where  $\tilde{\lambda}, \gamma_b$  and  $\gamma_f$  are as defined in the text. Note that when  $\omega = 0 \Rightarrow \tilde{\lambda} = \lambda, \gamma_f = \beta$ and  $\gamma_b = 0$  which means that the only restrictions imposed by the (purely forwardlooking) NKPC are:  $a_{41} = -\frac{\lambda}{\beta}$  and  $a_{44} = \frac{1}{\beta}$ . An additional restriction is that inflation has to Granger cause real marginal cost. Thus, the coefficient on inflation in the real marginal cost equation  $(a_{14})$  has to be non-zero. When estimating  $a_{14}$ using actual data, it is non-zero. It is not obvious that the resulting  $A^R$  necessarily give rise to non-explosive paths of the simulated series of inflation. In fact, I found that when I use an instrument set that includes the output gap,  $A^R$  always gave rise to explosive paths of inflation.

To create simulated data, I proceed as follows: I use the residuals from the VAR from the first three equations (call them  $\hat{\varepsilon}_t^s, \hat{\varepsilon}_t^{dw}$  and  $\hat{\varepsilon}_t^{dc}$ ) and construct residuals for the inflation equation as implied by the restricted  $A^R$ :<sup>19</sup>

$$\hat{\varepsilon}_t^{\pi} = \hat{\pi}_t - (-0.0525) \, s_{t-1} - 1.0101 \hat{\pi}_{t-1}$$

Using these residuals, I proceed to step 3.

Step 3: Draw 10,000 random samples (with replacement) from  $\hat{\varepsilon}_t = \begin{bmatrix} \hat{\varepsilon}_t^s \\ \hat{\varepsilon}_t^{dw} \\ \hat{\varepsilon}_t^{dc} \\ \hat{\varepsilon}_t^{dc} \\ \hat{\varepsilon}_t^{\pi} \end{bmatrix}$ and use  $A^R$  to construct simulated data,  $\{Z_t^R\}^{(rep)}$ 

Step 4: Estimate equations (8) and (9) using each of the 10,000 simulated data and get  $\hat{\omega}^{(rep)}$  (and, also  $\hat{\alpha}^{(rep)}, \hat{\beta}^{(rep)}$  and the constant).

<sup>&</sup>lt;sup>19</sup>With  $\beta = 0.99$  and  $\alpha = 0.8$ , the coefficients on  $s_t$  and  $\pi_{t-1}$  are as reported here.

## 8 Appendix C - Present-value estimation

Given the recent popularity in using IV estimation to explore the importance of lagged inflation, it seems worthwhile stressing that the estimated coefficient on lagged inflation is always sensitive to the choice of normalization. Following Gali and Gertler, researchers generally try different normalizations when estimating the structural parameters ( $\alpha, \beta$  and  $\omega$ ). By contrast, the normalization issue has been ignored when using other specifications. For example, Rudd and Whelan (2001) propose estimating the hybrid model in its present-value representation:<sup>20</sup>

$$E\left\{\left(\hat{\pi}_t - c - \frac{\tilde{\lambda}}{\delta_2 \gamma_f} \sum_{k=0}^{\infty} \delta_2^{-k} \hat{s}_{t+k} - \delta_1 \hat{\pi}_{t-1}\right) \Omega_{t-1}\right\} = 0$$
(12)

By truncating the infinite sum at, say 12, Gali, Gertler and Lopéz-Salido (2003) show that they get virtually identical results to those reported in the first row table 1. However, the present-value representation is also sensitive to normalization. To illustrate this point, consider simply iterating the HPC one period forward to get:

$$E\left\{\left(\hat{\pi}_{t}-c-\tilde{\lambda}\hat{s}_{t}-\gamma_{f}\tilde{\lambda}\hat{s}_{t+1}-\left(\gamma_{f}\right)^{2}\hat{\pi}_{t+1}-\gamma_{f}\gamma_{b}\hat{\pi}_{t}-\gamma_{b}\hat{\pi}_{t-1}\right)\Omega_{t-1}\right\}=0$$
 (13)

The estimates based on equation (6) resemble very closely the results reported in Gali, Gertler and López-Salido (2003). Moreover, as Gali, Gertler and López-Salido (2003) point out, these results look very similar to the reduced-form estimates based

<sup>&</sup>lt;sup>20</sup>Gali, Gertler and López-Salido (2003) point out a mistake in the way Rudd and Whelan (2001) derive the present-value representation of the hybrid model. Therefore, the shown equation is Gali, Gertler and López-Salido's version.

on equation (6). However, alternatively, I can estimate:

$$E\left\{\left(\frac{1}{\gamma_f}\left[\hat{\pi}_t - c - \tilde{\lambda}\hat{s}_t - \gamma_f\tilde{\lambda}\hat{s}_{t+1} - (\gamma_f)^2\hat{\pi}_{t+1} - \gamma_f\gamma_b\hat{\pi}_t - \gamma_b\hat{\pi}_{t-1}\right]\right)\Omega_{t-1}\right\} = 0$$
(14)

Again, estimates based on equation (14) resemble the estimates based on equation (7) rather than the ones based on equation (6). Since I am not estimating  $\omega$ directly, I cannot draw any direct implications between the choice of normalization and the estimated share of backward-looking agents. However, it does seem to be the case that whenever the coefficient on current inflation is normalized to unity, the estimate of lagged inflation is larger.<sup>21</sup> This result was confirmed when I eaxmined the structural estimates.

<sup>&</sup>lt;sup>21</sup>For a given value of  $\alpha$  and  $\beta$ , this will imply that the share of backward-looking agents is larger.

	α	β	ω	$\widetilde{\lambda}$	$\gamma_{\rm b}$	$\gamma_{\rm f}$	Duration	J-stat (p-value)
Reduced-form estimatio	n							
Gali and Gertler (GG) (eq	. 6)			0.016 ** (0.005)	0.379 <b>**</b> (0.021)	0.591 <b>**</b> (0.023)		9.75 (0.982)
Dividing through by $\gamma_f$ (ec	į. 7)			0.003 (0.006)	0.186 ** (0.025)	0.837 <b>**</b> (0.029)		9.77 (0.982)
Iterated one-step ahead	(see appendix	к C)						
Resembling GG (eq. 13)				0.011 **	0.424 **	0.553 **		10.14
				(0.004)	(0.017)	(0.019)		(0.977)
Dividing through by $\gamma_f$ (ec	ı. 14)			0.005 (0.004)	0.296 ** (0.019)	0.710 <b>**</b> (0.020)		10.27 (0.975)
Structural estimation								
Normalization 1 (eq. 8)	0.809 ** (0.015)	0.885 ** (0.031)	0.266 ** (0.032)	0.038 ** (0.008)	0.253 ** (0.024)	0.682 <b>**</b> (0.026)	5.879 ** (0.458)	9.84 (0.981)
Normalization 2 (eq. 9)	0.834 ** (0.021)	0.910 ** (0.032)	0.486 <b>**</b> (0.041)	0.016 ** (0.005)	0.379 <b>**</b> (0.021)	0.591 <b>**</b> (0.023)	8.635 ** (1.501)	9.75 (0.982)

#### Table 1: IV estimates based on actual US data

*Note:* Asymptotic standard errors based on a Newey-West covariance matrix robust to serial correlation up to 8 lags \*\*, \* and \*/ denotes significant at the 1, 5 and 10 percent level, respectively. Gali and Gertler's data run from 1960:1 to 1997:4 (152 observations) but in the process of constructing instruments (using four lags of all variables), and by constructing inflation as the log difference in prices, the first five observations are lost, and since  $\pi_{t+1}$  is used as a regressor the last observation is also lost. Therefore, the estimation is made on data from 1961:2-1997:3 (146 observations).



Figure 2: Monte Carlo results: 10,000 estimates of  $\omega$  using normalization 1 and 2

**Table 2:** Median values and 95 percent bootstrapped CI $\alpha$  $\beta$  $\omega$ 

		10		
(1)	0.785 (0.724-0.831)	0.862 (0.648-0.976)	0.185 (0.003-0.366)	
(2)	0.809 (0.748-0.871)	0.864 (0.705-0.962)	0.486 (0.342-0.613)	

 Table 3: HPC estimates using a smaller instrument set

	$\alpha$	eta	ω	$ ilde{\lambda}$	$\gamma_b$	$\gamma_f$	D	J-stat
(1)	$0.812^{**}$ (0.019)	$0.900^{**}$ (0.037)	0.289** (0.044)	$0.033^{**}$ (0.009)	$0.268^{**}$ (0.032)	$0.679^{**}$ (0.032)	$5.325^{**}$ (0.544)	(p-value) 9.58 (0.544)
(2)	$0.834^{**}$ (0.026)	$0.926^{**}$ (0.036)	$0.459^{**}$ (0.047)	$0.016^{**}$ (0.005)	$0.363^{**}$ (0.025)	$0.611^{**}$ (0.027)	$6.041^{**}$ (0.939)	9.49 (0.735)

"True" parameters:	α=0.8	β=0.99	ω=0	$\widetilde{\lambda} = 0.052$	$\gamma_b = 0$	$\gamma_{\rm f} = 0.99$	Duration = 5	
Reduced-form estim	ation							
GG				0.000 (-0.021- 0.026)	0.482 (0.308- 0.646)	0.521 (0.356- 0.692)		
Dividing through by 7	Ŷf			0.048 (0.007- 0.182)	0.059 (-1.009- 0.381)	0.945 (0.621- 2.026)		35
Structural estimatio	n							
Normalization 1	0.780 (0.730- 0.832)	0.996 (0.965- 1.061)	0.029 (0.000- 0.445)	0.058 (0.016- 0.202)	0.036 (-1.706- 0.361)	0.964 (0.638- 2.076)	4.555 (3.702- 5.939)	
Normalization 2	0.847 (0.449- 2.423)	0.999 (0.331- 2.011)	0.878 (0.356- 1.597)	0.000 (-0.017- 0.026)	0.481 (0.308- 0.652)	0.521 (0.347- 0.692)	4.950 (-13.103- 51.310)	

# Table 4: IV estimates when null is NKPC (ω=0)

"True" parameters:	α=0.8	β=0.99	ω=0.25	$\widetilde{\lambda} = 0.030$	$\gamma_{b} = 0.239$	$\gamma_{\rm f} = 0.756$	Duration = 5	
Reduced-form estim	ation							
GG				0.009 (-0.010- 0.038)	0.417 (0.261- 0.529)	0.589 (0.478- 0.744)		
Dividing through by $\gamma$	ſſ			0.031 (0.001- 0.097)	0.249 (-1.009- 0.411)	0.754 (0.596- 1.133)		36
Structural estimatio	n							
Normalization 1	0.771 (0.716- 0.833)	0.996 (0.962- 1.065)	0.234 (0.000- 0.513)	0.040 (0.013- 0.109)	0.232 (-1.706- 0.395)	0.767 (0.607- 1.156)	4.372 (3.520- 5.983)	
Normalization 2	0.809 (0.381- 1.286)	1.019 (0.950- 2.754)	0.593 (0.275- 1.259)	0.009 (-0.017- 0.038)	0.417 (0.260- 0.529)	0.589 (0.475- 0.744)	5.050 (-13.103- 43.858)	

# Table 5: IV estimates when null is HPC ( $\omega$ =0.25)

"True" parameters:	α=0.8	β=0.99	ω=0.50	$\widetilde{\lambda} = 0.016$	$\gamma_{b} = 0.386$	γ <sub>f</sub> =0.611	Duration = 5	
Reduced-form estim	ation							
GG				0.011 (-0.007- 0.036)	0.420 (0.333- 0.475)	0.583 (0.530- 0.667)		
Dividing through by $\gamma$	ſ			0.018 (-0.005- 0.052)	0.382 (-1.009- 0.450)	0.619 (0.554- 0.734)		37
Structural estimation	n							
Normalization 1	0.763 (0.710- 0.832)	0.991 (0.961- 1.051)	0.445 (0.000- 0.615)	0.026 (0.008- 0.061)	0.369 (-1.706- 0.436)	0.629 (0.566- 0.746)	4.226 (3.443- 5.941)	
Normalization 2	0.800 (0.686- 1.044)	1.009 (0.968- 1.531)	0.588 (0.380- 0.993)	0.011 (-0.017- 0.036)	0.420 (0.333- 0.476)	0.583 (0.528- 0.667)	4.913 (-13.103- 53.358)	

# Table 6: IV estimates when null is HPC (ω=0.5)

"True" parameters:	α=0.8	β=0.99	ω=0.70	$\widetilde{\lambda} = 0.008$	$\gamma_b = 0.468$	$\gamma_{f} = 0.530$	Duration = 5	
Reduced-form estin	mation							
GG				0.007 (-0.005- 0.025)	0.471 (0.420- 0.505)	0.528 (0.499- 0.574)		
Dividing through by	$\gamma \gamma_{\rm f}$			0.010	0.460	0.538		
				(-0.003- 0.031)	(-1.009- 0.497)	(0.505- 0.592)		38
Structural estimati	on							
Normalization 1	0.772 (0.729- 0.831)	0.983 (0.961- 1.015)	0.621 (0.000- 0.765)	0.015 (0.004- 0.036)	0.448 (-1.706- 0.482)	0.548 (0.518- 0.603)	4.386 (3.686- 5.910)	
Normalization 2	0.803 (0.654- 1.251)	0.995 (0.957- 1.579)	0.720 (0.537- 1.198)	0.007 (-0.017- 0.025)	0.471 (0.420- 0.504)	0.528 (0.500- 0.574)	4.906 (-13.103- 13.167)	

# Table 7: IV estimates when null is HPC (ω=0.7)