Baby Boom, Asset Market Meltdown and Liquidity Trap*

Junning Cai

Abstract

A so-called "asset market meltdown hypothesis" predicts that baby boomers' large savings will drive asset market booms that will eventually collapse because of the boomers' large retirement dissavings. As good news to baby boomers, our analysis shows that this meltdown hypothesis is fundamentally flawed; and baby-boom-driven asset market booms may not necessarily collapse. However, bad news is that, in the case where meltdowns are about to happen, forward-looking baby boomers' attempts to escape them will be futile and may lead the economy into a "liquidity trap". (JEL E21, E22, E44, G12)

Keywords: baby boom; asset market meltdown; liquidity trap; investment elasticity.

* January 2004. Junning Cai. Department of Economics, University of Hawaii at Manoa. 1711 East-West Road, MSC#819, Honolulu, HI 96822, USA. junning@hawaii.edu.

1. Introduction

A so-called "asset market meltdown hypothesis" predicts that baby boomers' large amount of savings during their "prime saving years" will drive asset market booms that will eventually collapse as the boomers start dissaving during their retirements. Poterba (2001) provides a succinct explanation to the rationale behind this hypothesis by using the following equation:

$$qK = s(wN), \tag{1.1}$$

whose left and right-hand sides represent asset supply and demand respectively. Given constant asset supply (K), wage rate (w) and saving rate (s), equation (1.1) clearly indicates a positive relationship between asset price (q) and the number of savers (N). Thus, while a high N during baby boomers' prime saving ages tends to drive up q, a low N during their retirements will bring it down.

Such a simplified justification of the meltdown hypothesis certainly depends on strong assumptions on K, w and s. Yet more sophisticated studies in the literature also generally support the hypothesis (Abel, 2003; Brook, 2000; Yoo, 1994; among others).[1]

In this paper we take a deeper look at the meltdown hypothesis and provide some new insights that have not been well recognized by the existing literature.

First, it is worth clarifying that baby booms can affect asset returns through asset prices as well as asset earnings.[2] Yet, by assuming the so-called "putty-putty" investment technology, many studies (e.g. Brooks, 2000; Yoo, 1994) essentially abstract away the impact of baby booms on asset prices, and capture only their impacts on asset earnings. Without asset price fluctuations, baby-boom impacts on asset returns tend to be limited.

[1] See Poterba (2001) for a review and references therein.

[2] While the baby-boom impact on asset prices is explained by the aforementioned Poterba's (2001) interpretation, that on asset earnings is through capital-labor ratio.

For example, in simulating the impacts of a baby boom-bust cycle on asset returns based on a putty-putty model, Brooks (2000) finds that the rate of return to capital is peaked at 4.64 percent during the baby boom, and bottomed at 4.45 percent during the baby bust.

As an exception, Abel (2003), in studying the impacts of baby booms on asset prices and capital accumulation, assumes convex investment adjustment costs and hence takes into consideration of baby-boom impact on asset prices. Lim and Weil (2003) also consider convex "installation costs" in their study on the baby-boom impact on stock market booms, and point out that the magnitude of the impact is positively related to installation costs.

In this paper, instead of assuming convex investment adjustment costs, we consider the risk aversion of entrepreneurs as another "investment impediment" that reduces investment elasticity. We find that, not surprisingly, the more risk-averse the entrepreneurs are; or in general, the less elastic the investments are, the greater the baby-boom-driven asset market booms will be.

The surprising result of our analysis is with respect to the prediction that baby-boom-driven asset price booms will meltdown during baby boomers' retirement ages. Although there are some debates on whether some real-world factors (such as bequest motives or foreign savings) can help preventing the potential meltdown,[3] theoretical research in the literature generally supports the meltdown hypothesis *per se* (e.g. Abel, 2003). However,

[3] One argument against the meltdown hypothesis claims that people in the real world usually do not deplete their wealth before they die (Poterba, 2001). However, Abel (2001) shows that baby boomers' bequest motives may not help attenuating the potential meltdown. Another popular argument against the meltdown hypothesis claims that foreign demands on domestic assets can help preventing the meltdown. However, facing similar ageing problems, major developed countries (e.g. Japan) are not likely to provide such demands. Developing countries (e.g. China) may (or may not) be able to provide the demands; yet the resulting current account deficits may not be a pleasant side effect. Moreover, the so-called Feldstein-Horioka puzzle (Feldstein and Horioka, 1980) clouds any hope for relying on savings from abroad.

we find the meltdown hypothesis to be fundamentally flawed, and show that the meltdown is indeed state-contingent and may not necessarily happen.

Unfortunately, the meltdown is still possible. Then, an intriguing question is whether forward-looking baby boomers can escape the potential meltdown by holding assets free from price fluctuations? Or as specifically asked by Abel (2003, p.552), "if these investors are forward-looking in the first place, would they so eagerly buy stocks that are destined to fall in price eventually?" However, by using a model with capital being the only store of value, Abel (2003) gives those unfortunate investors no other choices.

Brooks (2000), on the other hand, does give baby boomers in his model a chance to hold riskless bonds that essentially represent their lendings to younger generations. Yet he finds that the option does not help baby boomers to avoid being hurt by a low rate of return during their retirement ages. This should not be surprised. After all, as baby-boom-induced asset market fluctuations are fundamentally driven by savings and dissavings, forward-looking baby boomers' attempts to avoid potential asset price meltdowns (by holding short-term or riskless assets) will tend to depress the general interest rate level for the entire asset markets.

However, a conjecture is that, as the existence of money creates a zero-interest bound, forward-looking investors should at least be able to protect themselves against severe future meltdowns that imply negative returns to capital.[4] Our analysis confirms this conjecture. Nevertheless, we find that the escape will be at the cost of a "liquidity trap".

The remainder of the paper is organized as follows. Section 2 analyzes the baby-boom impact on asset market performances. Section 3 discusses what would happen

[4] Negative returns are not possible in Brooks' (2000) model that assumes the putty-putty investment technology. Yet, we will show that they are possible when baby-boom impacts on asset prices are taken into consideration.

when forward-looking baby boomers foresee potential meltdowns. We conclude the paper in section 4.

2. Baby boom and asset market performances

During their prime saving ages, baby boomers' large amount of savings will put upward pressures on asset prices. Yet the magnitude of resulting asset price appreciation depends on investment elasticity—the higher the elasticity is, the more the pressures can be absorbed by increases in capital stock; hence the less the price appreciation will be. Notwithstanding, as long as investments are not perfectly elastic, asset price appreciation will happen.

It is tempting to apply a similar argument to hypothesize the following: With insufficient asset demand due to a small number of workers, baby boomers' large amount of dissavings in their retirements, representing massive asset supply, will cause asset market meltdowns. While such an argument is generally accepted by the existing literature, it has actually missed a crucial yet underappreciated point. That is, while asset supply tends to be high during baby boomers' retirement eras, so will be asset demand. This is because the large capital stock built up by baby boomers' savings will have positive influence on incomes and hence savings during baby boomers' retirement eras.

Therefore, a conjecture is that, while baby boomers' large savings tend to drive asset market booms, asset market meltdowns may not necessarily follow. We examine this conjecture in the following.

The Model

We use a two-period OLG model similar to the one used by Abel (2003). One major difference is that, while Abel (2003) models convex investment adjustment costs as an "investment impediment" responsible for less than perfectly elastic investments, the impediment modeled here is risk-averse investment behaviors. In addition, we introduce government bond as an alternative asset (to capital) to facilitate discussion in the next section.

Consumers

At the beginning of period *t*, $N_{y,t}$ numbers of identical young consumers are born, each of whom will supply inelastically one unit of labor during the period and receives wage income (w_t) at the end of which. After paying tax (T_t), an individual young consumer consumes $c_{y,t}$ and saves in capital ($k_{y,t}$) and/or government bond ($d_{y,t}$). She carries over her assets into and retires during the next period *t*+1; and at the end of which she finishes her life cycle by using the gross returns to her savings to finance her old-age consumption ($c_{o,t+1}$).

At the end of period *t*, a period-*t* young consumer faces the following optimizing problem:

$$Max\ E_t[\log(c_{y,\,t}) + (1+\theta)^{-1}\log(c_{o,t+1})],$$

subject to:

$$c_{y,t} + q_t k_{y,t} + d_{y,t} = w_t - T_t \tag{2.1}$$

and

$$c_{o,t+1} = k_{y,t}(q_{t+1} + R_{t+1}) + d_{y,t}(1 + r_{t+1}) , \qquad (2.2)$$

where variables R_t , r_t , and q_t are, respectively, the period-*t* (per unit) capital income (in terms of consumption), the period-*t* (bond) interest rate, and the capital price (in terms of consumption goods) at the end of period *t*; and parameter θ represents time preference.

The (government) bond is a one-period coupon bond denominated in consumption goods. Capital is free from default risks and hence a perfect substitute of bond. Thus, the equilibrium (expected) return to one unit of capital should be equal to the return to equivalent bond investment. With capital prices at q_t, one unit of capital is equivalent (in return) to q_t units of bond; thus,

$$R_{t+1} + E_t q_{t+1} - q_t = r_{t+1} q_t , \qquad (2.3)$$

according to which the budget constraints (2.1) and (2.2) can be combined into

$$c_{y,t} + c_{o,t+1}(1 + r_{t+1})^{-1} = w_t - T_t \ .$$

Therefore, first order conditions give the individual young consumption function

$$c_{y,t} = (1 + \theta)(2 + \theta)^{-1}(w_t - T_t \ ) ,$$

which, with $N_{y,t}$ number of identical young consumers, gives the (aggregate) young consumption function:

$$C_{y,t} = (1 + \theta)(2 + \theta)^{-1}(w_t - T_t)N_{y,t} . \qquad (2.4)$$

As period-*t* old consumers finance their consumption via the gross returns to their assets, the (aggregate) period-*t* old consumption function is given by

$$C_{o,t} = K_{y,t-1}(q_t + R_t) + D_{y,t-1}(1 + r_t) \qquad (2.5)$$

where $K_{y,t-1} = N_{y,t-1} k_{y,t-1}$ and $D_{y,t-1} = N_{y,t-1} d_{y,t-1}$.

Firms

In every period, identical, profit-maximizing, and perfectly competitive firms hire capital and labor to produce perishable consumption goods with the standard Cobb-Douglas technology:

$$Y_t = F(K_t, L_t) = \lambda K_t^{\alpha} L_t^{1-\alpha}, \tag{2.6}$$

where Y_t, K_t, L_t, and λ denote, respectively, output, capital stock, labor, and technical coefficient.

Entrepreneurs

In every period, identical entrepreneurs engage in investing activities that transforms consumption goods into new capital.[5] Note that capital is irreversible; and its value depends on capital income and capital price (both in terms of consumption).

An individual entrepreneur *j* chooses the amount of investment (I_t^j) to maximize her expected utility:[6]

$$Max\, EU(\Pi_t^j)$$

where $\Pi_t = q_t I_t - c(I_t)$ represents investment profits—$c(I)$ is the investment cost (function) in terms of consumption.

[5] The separation of investment activities from production activities here is similar to the "two sector" modeling framework adopted by Abel (2003). As opposed to firms being the producers of consumption goods, entrepreneurs here represent activities devoted to capital formation. We do not explicitly model who these entrepreneurs are, but simply assume they always exist. Indeed, we can let some of the young consumers be workers and the rest be entrepreneurs. Yet the results will not be different.

[6] The utility rather than profit maximization is for the purpose of modeling risk-averse investment behaviors; otherwise, utility and profit maximizations are equivalent

If any, entrepreneurs will hold investment profits earned at the end of period *t* in form of capital and sell them at the end of period *t+1* for consumption. Thus, period-*t* entrepreneurs' consumption is given by

$$C_{e,t} = (q_t + R_t)(\Pi_{t-1} / q_{t-1}) . \tag{2.7}$$

Investments are risky with a stochastic cost function:

$$c(I_t^j) = I_t^j (1 + z_t^j) , \tag{2.8}$$

where $z_t \sim N(0, \sigma^2)$ is a normally distributed random variable. Entrepreneurs are risk-averse with utility function:

$$U(\Pi) = -e^{-\varphi\Pi} , \tag{2.9}$$

where parameter φ measures (constant) absolute risk aversion.

According to equations (2.8) and (2.9), entrepreneur *j*'s maximizing problem becomes

$$\underset{I_t^j}{Max} \;\; E_t U(\Pi_t^j) = -\int e^{-\varphi\Pi_t^j} f(\Pi_t^j) d\Pi_t^j = -e^{-\varphi[(q_t - 1)I_t^j - \varphi I_t^{j2} \sigma^2 / 2]} ,$$

the solution to which gives the individual investment function: $q_t = 1 + \varphi\sigma^2 I_t^j$. Then, the aggregate investment function (with *n* identical entrepreneurs) would be

$$q_t = 1 + \frac{\varphi\sigma^2}{n} I_t , \tag{2.10}$$

where $I_t = nI_t^j$ represents the aggregate investment.[7]

Equation (2.10) implies that under risky investments (σ > 0) and risk-averse entrepreneurs (φ > 0), the aggregate investment is not perfectly elastic; and its elasticity

[7] Notwithstanding based on risk aversion, equation (2.10) is similar to Abel and Eberly's (1997) investment function (equation 15 in their paper) based on quadratic adjustment costs.

is negatively correlated with the riskiness of investments and the risk aversion of entrepreneurs.

Given a large number of entrepreneurs ($n >> 0$) and according to the law of large numbers, the aggregate investment cost function is

$$c(I_t) = \sum_j c(I_t^j) = I_t^j (n + \sum_{j=1}^{n} z_t^j) = nI_t^j = I_t, \qquad (2.11)$$

which implies constant marginal cost of investment in aggregate. Note that, notwithstanding the constant marginal investment cost in aggregate, aggregate investment is not perfectly elastic because of increasing (marginal) risk premia demanded by risk-averse entrepreneurs.

Government

Government uses tax revenues and bond issuance to finance its expenditures including (bond) interest payments and government consumption (assumed to be zero for simplicity). Thus,

$$T_t N_{y,t} + \dot{D}_t = r_t D_t , \qquad (2.12)$$

with the left and right hand sides representing government's revenues and expenditures respectively.

Identity

The period-*t* capital stock (K_t) is equal to period-*t* old consumers' capital holding plus entrepreneurs' investment profits earned in period *t*-1 and held in form of capital in period *t*. Thus,

$$K_t = K_{y,t-1} + \Pi_{t-1} / q_{t-1} \tag{2.13}$$

The period-*t* bond stock (D_t) is solely held by period-*t* old consumers. Thus,

$$D_t = D_{y,t-1} \tag{2.14}$$

Equilibrium

There are five markets: (consumption) goods, labor, rental capital, capital, and bond. Take consumption as the numeraire.

The goods market is in equilibrium when consumption output is completely absorbed by the consumption of young consumers, old consumers, entrepreneurs, plus the costs for investments; i.e., $Y_t = C_{y,t} + C_{o,t} + C_{e,t} + c(I_t)$, which, according to equations (2.4), (2.5), (2.6), (2.7), (2.11), (2.13) and (2.14), can be transformed into

$$F(K_t,\ L_t) = (1+\theta)(2+\theta)^{-1}(w_t - T_t)N_{y,t} + K_t\,(q_t + R_t) + D_t\,(1+r_t) + I_t\ . \tag{2.15}$$

As each of young consumers (as the only source of labor) inelastically supplies one unit of labor, the labor supply function is given by

$$L_t = N_{y,t}\ . \tag{2.16}$$

The demand for labor comes from firms, who (under perfect competition) will pay factors by their marginal products. Thus, according to equation (2.6), the labor demand function is given by

$$w_t = F_2(K_t\,,\, L_t), \tag{2.17}$$

which, with inelastic labor supply, determines the labor market-clearing wage rate.

Similarly, as the supply of rental-capital is inelastic and equal to the existing capital stock, the market-clearing rental rate is determined by the rental-capital demand function

$$R_t = F_1(K_t, L_t)\,. \tag{2.18}$$

As the supply of existing capital stock is inelastic, the supply-side equilibrium condition for the capital market is determined by the aggregate investment function [equation (2.10)], which can be notationally summarized into

$$q_t = 1 + \eta I_t\,, \tag{2.19}$$

where coefficient $\eta = \varphi\sigma^2 / n$ is negatively correlated with the q-elasticity of investments ("investment elasticity" in short).

Equation (2.3), which is essentially a capital demand function, can be rearranged into

$$r_{t+1} = \frac{R_{t+1} + E_t q_{t+1} - q_t}{q_t} \tag{2.20}$$

where, according to equations (2.16) and (2.18),

$$R_{t+1} = F_1(K_{t+1}, N_{y,t+1})\,. \tag{2.21}$$

Suppose government keeps its debt level constant at $\overline{D}$ via balancing its budget in every period (i.e. $\dot{D}_t = 0$), then the supply-side bond market equilibrium condition is given by

$$D_t = \overline{D}\,, \tag{2.22}$$

and, according to equation (2.12)

$$T_t\, N_{y,t} = r_t D_t\,, \tag{2.23}$$

According to Walras' Law, the demand-side bond market equilibrium is implied by equilibria in the other markets.

Finally, assume no capital depreciation for simplicity; then the capital accumulation is governed by

$$K_{t+1} = K_t + I_t \tag{2.24}$$

Summary

At the end of period *t*, the equilibrium of the economy is characterized by the simultaneous system composed of equations (2.15)-(2.24), in which variables $N_{y,t}$ and $N_{y,t+1}$ are exogenous demographic features; variables K_t and r_t are initial conditions exogenously determined by history; variables L_t, w_t, T_t, q_t, R_t, D_t, I_t, r_{t+1}, R_{t+1}, and K_{t+1} are endogenously determined; and variable $E_t q_{t+1}$ depends on agents' expectations that are assumed to be rational in this paper.

Dynamics of capital stock and capital price

According to the simultaneous system (2.15)-(2.24), the dynamics of capital accumulation can be characterized by

$$\dot{K}_t = K_{t+1} - K_t = \frac{S_t^g - K_t - \Lambda\overline{D}}{1 + \eta K_t} \tag{2.25}$$

where $S_t^g = (2+\theta)^{-1}(1-\alpha)\lambda K_t^{\alpha} N_{y,t}^{1-\alpha}$ measures the gross saving of the economy;[8] and $\Lambda = (2+\theta)^{-1}(2 + r_t + \theta)$ is a summarizing notation.

For analytical convenience, let $\overline{D} = 0$.[9] Then, according to equation (2.25), the steady state ($\dot{K}_t = 0$) capital stock with constant population ($N_{y,\,t} = \overline{N}$) can be determined by the following equation:

$$S^{g*} - K^* = 0, \tag{2.26}$$

where $S_t^{g*} = (2+\theta)^{-1}(1-\alpha)\lambda K^{*\alpha}\overline{N}^{1-\alpha}$. Equation (2.25) implies that

[8] It is not difficult to verify that $S_t^g = w_t - C_{y,t}$

[9] The inclusion of government bond in the model is to facilitate analysis in the next section.

$$\partial \dot{K}_t / \partial K_t = [(\alpha S_t^g / K_t - 1)(1 + \eta K_t) - \eta (S_t^g - K_t)](1 + \eta K_t)^{-2} . \tag{2.27}$$

According to equations (2.26) and (2.27),

$$\partial \dot{K}_t / \partial K_t \Big|_{K_t = K^*} < 0 . \tag{2.28}$$

Thus, we have the following proposition.

Proposition 2.1 *The steady state K* is unique and stable.*

Corollary 2.1 $\forall K_t < K^*, \ \dot{K}_t > 0$.

Proof: According to equation (2.26), *K** is unique. According to inequality (2.28), *K** is stable. With a unique and stable *K**, Corollary 2.1 is self-evident.

Put plainly, given initial K_0 less than the steady-state *K**—which is what we consider here—capital stock will be on a monotonic upward trend until it reaches the steady state.

The dynamics of *K* convergence can be characterized by the following proposition.

Proposition 2.2 $\exists \widetilde{K} \in (0, K^*] : \partial \dot{K}_t / \partial K_t \Big|_{K_t = \widetilde{K}} = 0$.

Corollary 2.2 $\forall K_t \in (0, \widetilde{K}) : \partial \dot{K}_t / \partial K_t > 0$.

Corollary 2.3 $\forall K_t \in (\widetilde{K}, K^*] : \partial \dot{K}_t / \partial K_t < 0$.

Proof: According to equation (2.27), it is not difficult to verify that $\partial \dot{K}_t / \partial K_t$ is a monotonically decreasing function of K_t, and $\lim_{K_t \to 0} \partial \dot{K}_t / \partial K_t \to \infty$. Thus, with inequality (2.28), $\widetilde{K}$ must exist; and Corollaries 2.2 and 2.3 must hold.

According to Propositions 2.1 and 2.2., the growth path of K_t can be graphically depicted by Figure 1, with the $K_t < \widetilde{K}$ portion being convex and the $K_t > \widetilde{K}$ portion concave.

Accordingly, the dynamic of capital price can be characterized by the following proposition.

Proposition 2.3 *For $K_t \in (0, \widetilde{K})$, q_t will be positively correlated with K_t and hence on an upward trend; whereas, for $K_t \in (\widetilde{K}, K^*]$, q_t will be negatively correlated with K_t and hence on a downward trend.*

Proof: According to equation (2.19), $dq_t / d\dot{K}_t > 0$—note that $\dot{K}_t = I_t$—which, together with Corollaries 2.2 and 2.3, implies that $\partial q_t / \partial K_t \big|_{K_t \in (0, \widetilde{K})} > 0$ and $\partial q_t / \partial K_t \big|_{K_t \in (\widetilde{K},\, K^*]} < 0$. Accordingly, as $\dot{K}_t \big|_{K_t \in (0, K^*)} > 0$, we have $\dot{q}_t \big|_{K_t \in (0, \widetilde{K})} > 0$ and $\dot{q}_t \big|_{K_t \in (\widetilde{K},\, K^*]} < 0$.

Intuitively, the sign of *q*-*K* correlation depends on the balance between two opposite influences of *K* on *q*. On the one hand, *K per se* represents capital supply and hence has a negative influence over *q* directly. On the other hand, *K* also has a positive influence over *q* indirectly through savings that represent asset demand—note that a large *K* can help generating large wage incomes. As the balance of the two effects cannot be determined *a priori*, the sign of the *q*-*K* correlation is state contingent. With diminishing marginal product of capital, the positive (indirect) effect tends to prevail when *K* is small, and be dominated when *K* is large.

As will be shown later, when *q* is positively correlated with *K* and on an upward trend, the meltdown may not necessarily happen.

Baby-boom effect on capital accumulation

Suppose a baby boom occurs in period $t = 0$, which can be described by

$$N_{y,t} = \begin{cases} N^b, & t = 0 \\ \overline{N}, & t \neq 0 \end{cases},$$

where $N^b > \overline{N}$ measures the magnitude of the baby boom. With this assumption, we consider a situation with a constant number of newborns over time except a spike in period zero caused by a baby boom.[10] Thus, while period zero represents the baby boomers' saving ages, period one represents their retirement ages.

As it is not difficult to verify that $\partial S_t^g / \partial N_{y,t} > 0$, equation (2.25) implies that a *permanent* population growth (from $\overline{N}$ to N^b) in time t = 0 will shift up the capital growth path from $K_0\overline{N}$ to $K_0 N^b$ in Figure 2. Yet, as the high newborn level N^b is temporary and will drop back to $\overline{N}$ in time t = 1, the baby-boom effect on capital accumulation will be characterized by the path $K_0 N^b \overline{N}$ in Figure 2.

Specifically, we have the following proposition regarding the effect of the period-zero baby boom on K_1.

Proposition 2.4 *A period-zero baby boom has a positive effect on capital stock in period one; i.e.,* $dK_1 / dN^b > 0$.

[10] Admittedly being a special case, this simplified modeling captures the feature of baby booms that motivates the meltdown hypothesis. That is, a low dependency ratio during boomers' saving ages and a high ratio during their retirements. Similar modeling has been used in the literature: In a model with zero population growth in general, Brooks (2000) models a baby-boom-baby-bust cycle by assuming two periods of 2% population growth followed by two periods of -2% growth. Note that the assumption of a "baby bust" that reduces the number of newborns to the pre-baby-boom level may not be realistic; yet it is innocuous to our main point that meltdown may not necessarily happen.

Proof: According to equation (2.25), $dK_1 / dN^b = (1+\eta K_0)^{-1}(dS_0^g / dN^b) > 0$.

When q and K are positively correlated—recall Proposition 2.3 for its possibility, that $dK_1 / dN^b > 0$ will imply $dq_1 / dN^b > 0$; i.e., a positive impact of period-zero baby boom on period-one capital price. This is the key for the meltdown not to happen. We will come back to this point later. First let us examine the baby-boom impact on q_0.

Baby boom and capital price boom

According to the simultaneous system (2.15)-(2.24), the capital price at the end of period t is given by

$$q_t = (1+\eta K_t)^{-1}\left(1+\eta\lambda(1-\alpha)(2+\theta)^{-1}K_t^{\alpha}N_{y,t}^{1-\alpha}\right). \tag{2.29}$$

Thus, the period-zero capital price will be

$$q_0 = (1+\eta K_0)^{-1}\left(1+\eta\lambda(1-\alpha)(2+\theta)^{-1}K_0^{\alpha}N^{b^{1-\alpha}}\right), \tag{2.30}$$

which implies

$$dq_0 / dN^b = (1+\eta K_0)^{-1}\left(\eta\lambda(1-\alpha)^2(2+\theta)^{-1}K_0^{\alpha}N^{b^{-\alpha}}\right). \tag{2.31}$$

Given $\eta > 0$, equation (2.31) implies

$$\mathrm{d}q_0 / \mathrm{d}N^b > 0 \tag{2.32}$$

and

$$\partial(\mathrm{d}q_0 / \mathrm{d}N^b)/\partial\eta > 0,$$

which give the following propositions.

Proposition 2.5 *If investments are not perfectly elastic (i.e. $\eta > 0$), a period-zero baby boom will have a positive impact on q_0.*

Corollary 2.4 *Ceteris paribus, the lower the investment elasticity (i.e., the higher the η) is, the larger the baby-boom impact on q_0 will be.*

Since investments in reality can hardly be perfectly elastic, Proposition 2.5 supports the first part of the meltdown hypothesis. That is, a baby boom tends to drive capital market boom during baby boomers' saving ages. Nevertheless, Corollary 2.4 suggests that the magnitude of the boom negatively depends on investment elasticity.

Baby boom and capital market meltdown

We proceed to examine whether a period-zero baby boom will cause a capital market meltdown in period one. We first examine the impact of the baby boom on q_1.

According to equation (2.29),

$$dq_1 / dK_1 = \eta\Gamma(K_1) \text{ ,} \tag{2.33}$$

where $\Gamma(K_1) = \lambda\alpha(1-\alpha)(2+\theta)^{-1} K_1^{\alpha-1} \overline{N}^{1-\alpha} - 1 - \eta\lambda(1-\alpha)^2 (2+\theta)^{-1} K_1^{\alpha} \overline{N}^{1-\alpha}$. Equation (2.33), together with $dK_1 / dN^b = dI_0 / dN^b = \eta^{-1} dq_0 / dN^b$, implies that

$$dq_1 / dN^b = \Gamma(K_1)\, dq_0 / dN^b \text{ .} \tag{2.34}$$

With $\alpha < 1$, it is not difficult to verify that, $\partial\Gamma / \partial K_1 < 0$, $\lim_{K_1 \to 0} \Gamma(K_1) = \infty$, and $\Gamma(K^*) < 0$.

Thus, $\exists \hat{K} \in (0, K^*]: \ \Gamma(K_1) \begin{cases} > 0; \ K_1 \in (0, \hat{K}) \\ = 0; \ K_1 = \hat{K} \\ < 0; \ K_1 \in (\hat{K}, K^*] \end{cases}$.

Therefore, according to equations (2.32) and (2.34),

$$dq_1 / dN^b \begin{cases} > 0; \ K_1 \in (0, \hat{K}) \\ = 0; \ K_1 = \hat{K} \\ < 0; \ K_1 \in (\hat{K}, K^*] \end{cases},$$

which implies the following propositions.

Proposition 2.6 *The effect of a period-zero baby boom on period-one capital price is state contingent.*

Corollary 2.5 *In the case of* $K_1 > \hat{K}$*, a period-zero baby boom will have a negative impact on* q_1*.*

Corollary 2.6 *In the case of* $K_1 < \hat{K}$, *a period-zero baby boom will have a positive impact on* q_1*.*

The result in Corollary 2.6 may seem counterintuitive: As there are not enough workers (savers) to demand baby boomers' large asset supply in period one, the baby boom should have a negative impact on q_1. However, a crucial yet underappreciated point in the spirit of "Say's Law" is that baby boomer's large savings can create its own demand. This is because the large K_1 built up by baby boomers' large savings will have a positive impact on period-one income (and hence saving) that represents a demand-side force on q_1. When the marginal product of capital (*MPK*) is sufficiently large,[11] this demand-side force can be strong enough to prevail over the downward pressure (on q_1) produced by the large K_1 together with the small $N_{y,1}(=\overline{N})$.

[11] This is why the positive baby-boom impact on q_1 tends to happen when K is small.

As shown in Appendix, dq_1 / dN^b will be positive when $sMPK_1 > q_1$, where $s = (2+\theta)^{-1}(1-\alpha)$ represents the saving rate of the economy; and $MPK_1 = \alpha\lambda K_1^{\alpha-1}\overline{N}^{1-\alpha}$. Put plainly, when the "marginal saving" of capital is more than enough to purchase one unit of capital, the more the K_1 built up by baby boomers' large savings, the higher the q_1 will be.

It should be noted that, even when a period-zero baby boom has a positive impact on q_1, asset market meltdown can still happen when the impact is less than the positive baby-boom impact on q_0; i.e., if $\mathrm{d}(\Delta q_{1\text{-}0}) / \mathrm{d}N^b < 0$, where $\Delta q_{1\text{-}0} \equiv q_1 - q_0$. Yet we will show that $\mathrm{d}(\Delta q_{1\text{-}0}) / \mathrm{d}N^b > 0$ is possible; i.e., a period-zero baby boom can have a positive impact on the capital price movement in period one. Put plainly, the meltdown may not necessarily happen.

According equation (2.34), $d\Delta q_{1-0} / dN^b = (\Gamma - 1)\, dq_0 / dN^b$. As $\partial\Gamma / \partial K_1 < 0$, $\lim_{K_1 \to 0} \Gamma(K_1) = \infty$ and $\Gamma(K^*) < 0$, $\exists \hat{\hat{K}} \in (0,\, K^*]:\ \Gamma(\hat{\hat{K}}) = 1$. Thus,

$$d\Delta q_{1-0} / dN^b \begin{cases} > 0;\ K_1 \in (0,\, \hat{\hat{K}}) \\ = 0;\ K_1 = \hat{K} \\ < 0;\ K_1 \in (\hat{\hat{K}},\, K^*] \end{cases},$$

which implies that the following proposition.

Proposition 2.7 *The impact of a period-zero baby boom on period-one capital price variation is state contingent.*

Corollary 2.7 *In the case of* $K_1 > \hat{\hat{K}}$, *a period-zero baby boom will have a negative impact on capital price movement in period one.*

Corollary 2.8 *In the case of* $K_1 < \hat{\hat{K}}$ *, a period-zero baby boom will have a positive impact on capital price movement in period one.*

Proposition 2.7 implies that the widely-accepted meltdown hypothesis is flawed—baby-boom-driven asset market booms may not necessarily collapse but rather keep booming during baby boomers retirement ages.

Intuitively, the higher the baby-boom-driven q_0 is, the higher the K_1 will be. When K and q are positive correlated, a higher K_1 will imply a higher q_1. As the magnitude of the impact of K on q can be very large—it is not difficult to verify from equation (2.33) that $\lim_{K_1 \to 0} dq_1 / dK_1 = \infty$—the positive baby-boom impact on q_1 can outweigh its impact on q_0 so that the meltdown will not happen.

Graphically, in Figure 3, the upward and downward trends of the hump-shaped q path correspond respectively to the portions of $K_t < \widetilde{K}$ and $K_t > \widetilde{K}$ in Figure 1. In a situation where the q-path is downward-sloping, a period-zero baby boom will drive q_0 up to point g, higher than point e where q_0 would have been without the baby boom. Similar to the capital price dynamics in Abel's (2003) model, q_1 will be mean-reverting. As the baby boom will increase K_1 relative to its "non-baby-boom" level, q_1 will drop to the point h, which is lower than it would have been without the baby boom (i.e., the point f). Although q is on a downward trend even without the baby boom (from e to f), the q depreciation with the baby boom (from g to h) is clearly of a greater magnitude. In this sense, the baby boom has caused a capital market meltdown in period one.

However, the situation where the q-path is upward-sloping is less straightforward. Similarly, a period-zero baby boom will drive q_0 up to point c; and q_1 will be mean reverting. Yet, as q is on an upward trend, its mean reversion can have different implications. If q_1 only reverts to point b, we can say the baby boom has caused a "meltdown" in the sense the q appreciation with baby boom (i.e. from c to b) is less than it would have been without the baby boom (i.e., from a to b). However, as the period-zero baby boom will increase K_1, mean-reverting q_1 tends to be higher than point b. Indeed, it could reach point d (or higher), where the q appreciation from c to d is greater than that from a to b. In this situation, as the period-zero baby boom has a positive impact on q movement in period one, the meltdown does not happen.

Baby boom and the rate of return to capital market

Even when a period-zero baby boom has a positive impact on capital price movement in period one (i.e. $\mathrm{d}\Delta q_{1\text{-}0} / \mathrm{d}N^b > 0$), it can still have a negative impact on the rate of return to capital in period one, because of its negative impact on capital income through capital-labor ratio. Indeed, we find that in our model the baby boom will definitely have a negative impact on the period-one rate of return to capital; i.e., $dRR_1 / dN^b < 0$, where

$$RR_1 = \frac{R_1 + q_1 - q_0}{q_0} \tag{2.35}$$

represents the rate of return to capital in period one.

According to equations (2.21) and (2.29), we have

$$q_1 + R_1 = (1+\eta K_1)^{-1}\left(1 + \eta\lambda(1+\alpha+\theta\alpha)(2+\theta)^{-1} K_1^{\alpha}\overline{N}^{1-\alpha} + \alpha\lambda K_1^{\alpha-1}\overline{N}^{1-\alpha}\right),$$

which implies

$$d(q_1 + R_1)/dK_1 = \left[(\alpha-1)\Theta - \eta\right](1+\eta K_1)^{-2} < 0, \tag{2.36}$$

where

$$\Theta = \eta\lambda\alpha(3+2\theta)(2+\theta)^{-1}K_1^{\alpha-1}\overline{N}^{1-\alpha} + \alpha\lambda K_1^{\alpha-2}\overline{N}^{1-\alpha} + \eta^2\lambda(1+\alpha+\theta\alpha)(2+\theta)^{-1}K_1^{\alpha}\overline{N}^{1-\alpha} > 0 .$$

According to Proposition (2.4), inequality (2.36) implies

$$d(q_1 + R_1)/dN^b < 0, \tag{2.37}$$

which, together with inequality (2.32), implies

$$dRR_1 / dN^b < 0. \tag{2.38}$$

Inequality (2.38) implies the following proposition.

Proposition 2.8 *A period-zero baby boom will definitely reduce the rate of return to capital in period one.*

This result shows that in our model, even when a period-zero baby boom has a positive impact on period-one capital price movement, the impact will be dominated by the negative baby-boom effect on capital income.

3. **Baby boom and liquidity trap**

Although the meltdown may not necessarily happen, it is still possible. Then what if baby boomers foresee potential meltdowns; or more specifically, can forward-looking baby boomers protect themselves against potential meltdowns? We examine this question in the following.

In the above model, to escape from potential capital market meltdowns, baby boomers can choose to hold the one-period government bond. However, notwithstanding free from price variations, bond may not be a "safe haven" either, because its market-determined interest rate can be reduced by the flight from capital to bond.

In general, baby boomers' attempts to flee from capital to short-term and/or riskless assets will not be able to shelter them against potential capital market meltdowns, but rather tend to drag the general interest rate down to such a level that all assets become as "unattractive" as capital.

With a zero bound on its interest rate, bond will be a safe haven when potential meltdowns are so severe that the rate of return to capital becomes negative. Nevertheless, with a fixed bond supply[12], the safe haven will be too small to shelter all of baby boomers' wealth. Therefore, a possible scenario will be the following: Baby boomers still have to hold capital with negative returns; the bond interest rate is zero; and there is excessive demand for bond.

In sum, even with perfect foresights, baby boomers may have to bear with potential capital market (or asset market in general) meltdowns in their old ages as an ill-fated consequence of the family plans of their parents and them as parents.

Nevertheless, a little reflection on reality indicates that baby boomers should at least be able to guarantee non-negative returns to their savings, because they can always choose to hold on their wage incomes, which tend to be paid in form of money—shoe workers in reality are seldom paid in shoes.

Based on this observation, we have the following conjecture. Should baby boomers during their saving ages plan to hold on their monetary wages in order to protect themselves against negative returns implied by potential capital market meltdowns during their dissaving ages, they may not be able to earn the wages in the first place. That firms are willing to pay factors in money is because they expect to recover it via selling the

[12] In general, the supply of government bond is driven by government's fiscal policies and tends not to be perfectly elastic with respect to the interest rate. Therefore, I assume a fixed bond supply for simplicity.

goods produced by the factors. Yet, baby boomers' "hoarding" behaviors will make firms unable to sell all the goods and hence incur negative profits. Expecting such a situation, firms may not want to produce as much. Then, a "liquidity-trap" scenario could happen, in which the return to capital as well as the general interest rate is on its zero bound; and some baby boomers are unemployed. Based on the above model, we examine this conjecture in the following.

We first modify the above model by assuming that instead of consumption goods, firms pay factors with "money", which is a default-free instrument that promises (by firms) to pay its bearer one unit of consumption whenever presented.[13] Accordingly, we assume firms accept only money as payments for their goods. To differentiate, we refer to the original model (without money) as the "real" model, and the modified model (with money) as the "monetary" model.

Money so modeled is an asset with zero rate of return. Nevertheless, when the rate of return to capital is positive, money will be an inferior asset and hence not used as store of value. Thus, firms will be able to recover all the monetary factor payments. In this situation, money essentially plays the role of medium of exchange, which will not be captured by the equilibrium of the economy. In another word, equilibrium will be the same in the monetary model as the real model.

When potential future capital market meltdowns are so severe that the rate of return to capital is expected to be negative, money will nonetheless become a relatively attractive asset; and its zero rate of interest may provide a zero bound for the rate of return to

[13] The assumption of "money" issued by firms is a convenience way to capture the feature of "inside" money without explicitly modeling financial intermediation. Another alternative is to assume that firm can borrow money from government, pay it as wages, and then recover it from selling goods. Since in situations under our consideration firms will recover all the money they pay out, then whether the money is issued by firms or borrowed from government will not matter.

capital. With respect to this situation, we in the following examine the equilibrium (or lack of which) in the real and monetary models.

According to the simultaneous system (2.15)-(2.24), a period-zero full-employment equilibrium can be characterized by $\{q_0^e, q_1^e, R_1^e, K_1^e, L_0^e, RR_1^e; N^b\}$ that satisfies the following simultaneous system.

$$L_0 = N^b \tag{3.1}$$

$$q_0 = (1+\eta K_0)^{-1}\left(1+\eta\lambda(1-\alpha)(2+\theta)^{-1}K_0^{\alpha}L_0^{1-\alpha}\right) \tag{3.2}$$

$$K_1 = K_0 + (q_0 - 1)\eta^{-1} \tag{3.3}$$

$$q_1 = (1+\eta K_1)^{-1}\left(1+\eta\lambda(1-\alpha)(2+\theta)^{-1}K_1^{\alpha}\overline{N}^{1-\alpha}\right) \tag{3.4}$$

$$R_1 = \alpha\lambda K_1^{\alpha-1}\overline{N}^{1-\alpha} \tag{3.5}$$

$$RR_1 = \frac{R_1 + q_1 - q_0}{q_0} \tag{3.6}$$

Equation (3.1) represents the full-employment condition in period zero. Equation (3.2), derived from equations (2.6), (2.15), (2.17), (2.18) and (2.19), is a necessary condition for all the markets being simultaneously cleared. According to this equation, the following proposition is self-evident.

Proposition 3.1 *Given capital stock K_0 and employment L_0, the market-clearing period-zero capital price q_0^e is uniquely determined.*

Equation (3.3) captures the period-zero capital accumulation. Similar to equation (3.2), equation (3.4) is the (rationally expected) period-one market clearing condition. Finally, equations (3.5) and (3.6) represent the determinations of period-one capital income and rate of return to capital respectively.

With respect to the period-one (equilibrium) rate of return to capital in the real model, we have the following proposition.

Proposition 3.2 *In the real model*, $\exists N^{b*}:\ RR_1^e = 0$.

Corollary 3.1 *In the real model,* $\forall N^b < N^{b*}:\ RR_1^e > 0$

Corollary 3.2 *In the real model,* $\forall N^b > N^{b*}:\ RR_1^e < 0$

Proof: Equation (2.30) implies that $\lim_{N^b \to \infty} q_0 = \infty$, which, together with inequality (2.37), implies that $\lim_{N^b \to \infty} RR_1 = -1$. Then, according to inequality (2.38), a unique N^{b*} must exist; and Corollaries 3.1 and 3.2 are self-evident.

Proposition 3.2 verifies that a large enough period-zero baby boom can lead to potential negative rate of return to capital in period one.

According to Proposition 3.2, when $N^b \le N^{b*}$, RR_1^e will be non-negative; hence the zero-interest bound in the monetary model will not be binding. Thus, we have the following proposition.

Proposition 3.3 $\forall N^b \le N^{b*}$, *equilibria in the monetary and real models are identical.*

On the other hand, the existence of the zero-interest bound in the monetary model implies that, if $RR_1 < 0$, consumers will have incentives to hold money as store of value. Then firms will not be able to recover all their factor payments, which implies that the goods market will not be cleared. Therefore, we have the following proposition.

Proposition 3.4 *In the monetary model, a necessary condition for goods market equilibrium is* $RR_1 \geq 0$.

Corollary 3.3 *In the monetary model,* $\forall N^b > N^{b*}$ *, equilibrium with all markets simultaneously cleared does not exist.*

Proof (by contradiction): Suppose equilibrium $\{q_0^e, q_1^e, R_1^e, K_1^e, L_0^e, RR_1^e; N^b > N^{b*}\}$ exists; then, according to Corollary 3.2, $RR_1^e < 0$, which is in contradiction with Proposition 3.4.

Now it should be clear that, although the zero-interest bound provides forward-looking baby boomers an option to protect the value of their wealth, it will be at the cost of market equilibrium.

While equilibrium is usually well defined, "disequilibrium" states are not, depending on which market (or markets) is in disequilibrium. Recall that there are five markets in the model: labor, goods, capital, rental capital, and bond, among which the capital and bond markets are the least likely to be in disequilibrium because of the efficiency of asset markets. Disequilibrium in the goods market, which implies negative profits for firms' production, is also not likely to sustain.

Arguably, the most likely scenario is as follows. Expecting a potential future capital market meltdown, baby boomers will avoid holding capital, which will cause low capital price and hence lead to insufficient aggregate demand. The impact of the insufficient demand will be eventually felt by factor markets as firms reduce production accordingly.

While the rental rate for capital tends to be flexible, the wage rate for labor is likely to be rigid. Thus, a disequilibrium state could be such that all other four markets are in equilibrium except the labor market. We call such a state as "labor-market disequilibrium", in which there exists (involuntary) unemployment.

Many factors (e.g. contract or union) can cause wage rigidity, which we will not model explicitly but simply assume the following. Firms will pay employed baby boomers by their marginal products; and the rest of baby boomers will stay unemployed even though they are also willing to work under the current wage rate.

With respect to such a labor-market disequilibrium state, we have the following proposition.

Proposition 3.5 *Denote a labor-market disequilibrium as* $\{q_0^{de}, q_1^{de}, R_1^{de}, K_1^{de}, L_0^{de}, RR_1^{de}; N^b\}$. *Then,* $\forall N^b > N^{b*}$, $\{q_0^{de}, q_1^{de}, R_1^{de}, K_1^{de}, L_0^{de}, RR_1^{de}; N^b\} = \{q_0^e, q_1^e, R_1^e, K_1^e, L_0^e, RR_1^e; N^{b*}\}$.

Proposition 3.5 says that, for any baby-boom magnitude greater than N^{b*}, the corresponding labor-market *disequilibrium* state will be "equivalent" to the *equilibrium* state when the baby-boom magnitude is equal to N^{b*}. It should be noted that the "equivalent" is from an aggregate point of view—with different numbers of baby boomers, the two states will certainly not equivalent from individual baby boomers' perspectives. The proof of this proposition is straightforward. As $N^b > N^{b*}$, the zero-interest bound is binding. Thus, the labor-market disequilibrium state needs to satisfy equations (3.2)-(3.5) together with $RR_1 = 0$. Then, according to Proposition (3.2), the disequilibrium state (with $N^b > N^{b*}$) can be uniquely characterized by the equilibrium state (with $N^b = N^{b*}$).

A self-evident corollary of Proposition 3.5 is as follows.

Corollary 3.4 *Denote the period-zero unemployment rate as* $u(N^b) = (N^b - L_0)/N^b$*; then,* $u(N^b)\big|_{N^b > N^{b*}} > 0$ *and* $du/dN^b\big|_{N^b > N^{b*}} > 0$.

That is, a labor-market disequilibrium state with unemployment will occur when the magnitude of baby boom exceeds N^{b*}; and the larger the baby boom is, the higher the unemployment will be.

Summary

As the baby-boom impact on asset price fluctuations is driven by saving/dissaving patterns, forward-looking baby boomers' attempts to escape from potential capital market meltdown will merely drag down the general interest rate level for the entire asset markets. The zero-interest bound due to the existence of money can protect baby boomers from negative returns in the future; yet that will be at the cost of current unemployment.

4. Conclusion

When baby boomers' large savings cannot be effectively turned into investments due to investment impediments, they tend to drive asset price booms. However, whether baby-boom-driven asset price booms will meltdown (as commonly hypothesized) during baby boomers' retirement eras is state contingent, depending on whether large capital stock built up by baby boomers' large savings can generate enough asset demand (indirectly through high incomes) to sustain the asset price booms. Whether baby boomers in the United States need to worry about the meltdown hypothesis is certainly an empirical question. Yet, they may not need to worry too much if they believe that the U.S. asset prices are on an upward trend, because our analysis shows that the meltdown tends not to happen when asset prices are increasing. To the question of "Sell? Sell to whom?" that succinctly captures the essence of the meltdown hypothesis (Siegel, 1999, p.41), our analysis provides a comforting answer: "Sell to a richer generation."

However, when the meltdown is unfortunately about to happen, baby boomers' attempts to escape it will be futile and merely drag down the general interest rate level for the entire asset markets. Although a zero-interest bound (thanks to the existence of money) can protect baby boomers against negative returns in the future, it would nevertheless be at the cost of current unemployment in a liquidity trap. It is interesting to point out that, even when there is no potential meltdown, the liquidity trap can still be caused by baby boomers' misguided belief in the meltdown hypothesis.

Appendix: The condition for positive baby boom impact on q_1.

According to Proposition 2.4, $dK_1 / dN^b > 0$. Thus, $dq_1 / dN^b > 0$ if $dq_1 / dK_1 > 0$. Then, to determine the condition for $dq_1 / dN^b > 0$, we in the following examine the condition for the *q*-*K* correlation to be positive.

Abstracted from government bond and tax, equation (2.15) will give the following goods market equilibrium condition:

$$F(K_t, \overline{N}) = (1+\theta)(2+\theta)^{-1} w_t \overline{N} + K_t (q_t + R_t) + I_t \ , \tag{2.15}$$

in which output, wage, capital income, and investment are given respectively by

$$F(K_t, \overline{N}) = \lambda K_t^{\alpha} \overline{N}^{1-\alpha} ,$$

$$w_t = (1-\alpha)\lambda K_t^{\alpha} \overline{N}^{-\alpha} ,$$

$$R_t = \alpha\lambda K_t^{\alpha-1} \overline{N}^{1-\alpha} ,$$

$$I_t = \eta^{-1}(q_t - 1) .$$

Substituting them into equation (2.15), we have

$$\lambda K_t^{\alpha} \overline{N}^{1-\alpha} = (1+\theta)(2+\theta)^{-1}(1-\alpha)\lambda K_t^{\alpha} \overline{N}^{1-\alpha} + q_t K_t + \alpha\lambda K_t^{\alpha} \overline{N}^{1-\alpha} + \eta^{-1}(q_t - 1) ,$$

which can be simplified into

$$s\lambda K_t^{\alpha} \overline{N}^{1-\alpha} = q_t K_t + \eta^{-1} q_t - \eta^{-1} \tag{A.1}$$

where $s = (2+\theta)^{-1}(1-\alpha)$ represents the saving rate of the economy.

By totally differentiating equation (A.1) we can obtain

$$s(MPK_t) dK_t = K_t dq_t + q_t dK_t + \eta^{-1} dq_t \ , \tag{A.2}$$

where $MPK_t = \alpha\lambda K_t^{\alpha-1} \overline{N}^{1-\alpha}$ is the marginal product of capital. Equation (A.2) can be rearranged into

$$dq_t / dK_t = (sMPK_t - q_t)(K_t + \eta^{-1})^{-1} . \tag{A.3}$$

According equation (A.3), the condition for $dq_t / dK_t > 0$ is $sMPK_t > q_t$.

References

Abel, Andrew (2001). "Will bequests attenuate the predicted meltdown in stock prices when baby boomers retire?," *Review of Economics and Statistics* (83): 589-595.

Abel, Andrew (2003). "The effects of a Baby Boom on Stock Prices and Capital Accumulation in the Presence of Social Security." *Econometrica* (71): 551-578.

Abel, Andrew. and Janice. Eberly (1997). "An Exact Solution for the Investment and Value of a Firm Facing Uncertainty, Adjustment Costs, and Irreversibility," *Journal of Economic Dynamics and Control* (21(: 831-852.

Brooks, Robin (2000). "What Will Happen to Financial Markets When the Baby Boomers Retire?" *International Monetary Fund Working Paper*: WP/00/18.

Feldstein, Martin and Charles Horioka (1980). "Domestic saving and international capital flows," *Economic Journal* 90: 314-329.

Lim, Kyung-Mook and David Weil (2003). "The Baby Boom and the Stock Market Boom," *Scandinavian Journal of Economics* (105): 359-378.

Poterba, James (2001). "Demographic Structure and Asset Returns," *Review of Economics and Statistics* (83): 565-584.

Siegel, Jeremy (1998*). Stocks for the Long Run*, 2nd ed. (New York: McGraw Hill, 1998).

Yoo, Peter (1994). "Age Distributions and Returns of Financial assets," *Federal Reserve Bank of St. Louis Working Paper* no. 94-002B.

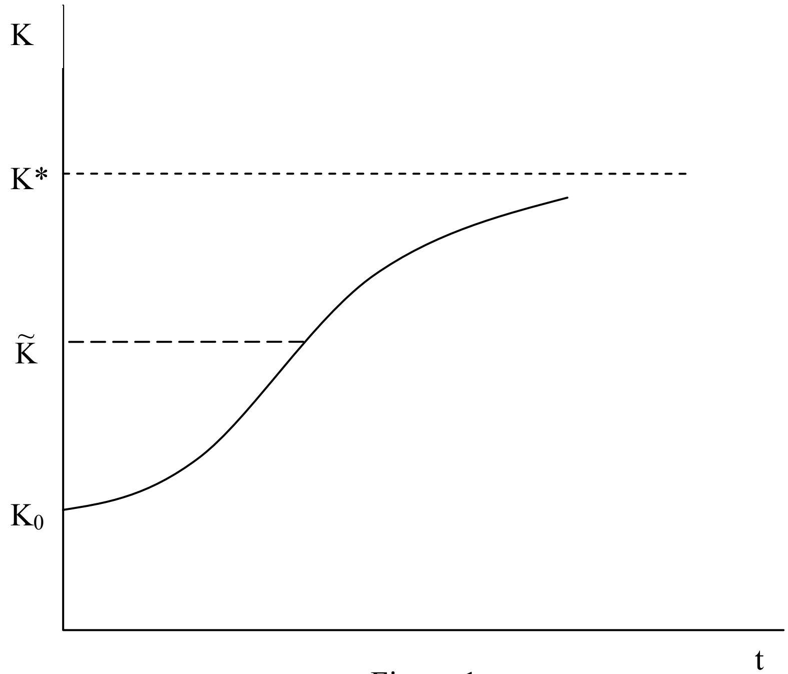

Figure 1

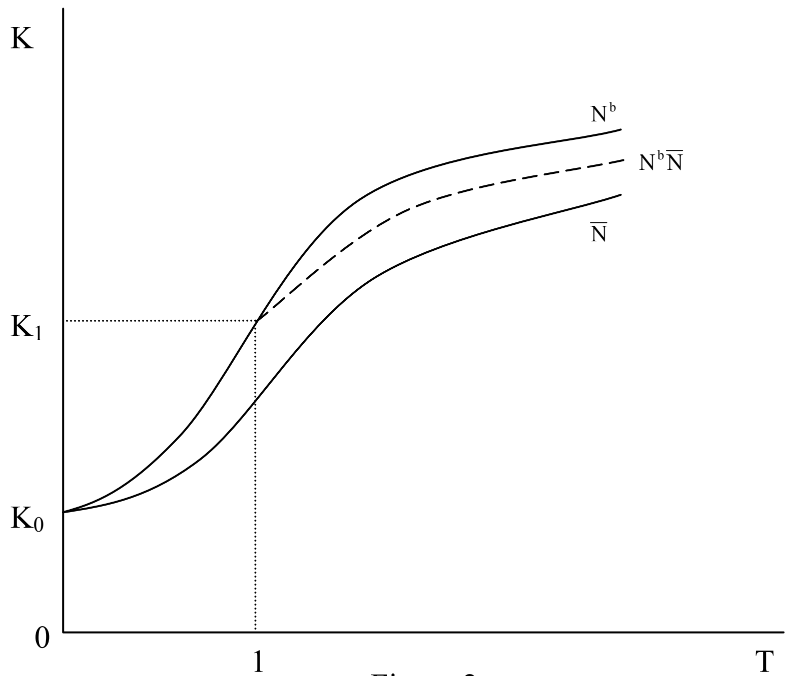

Figure 2

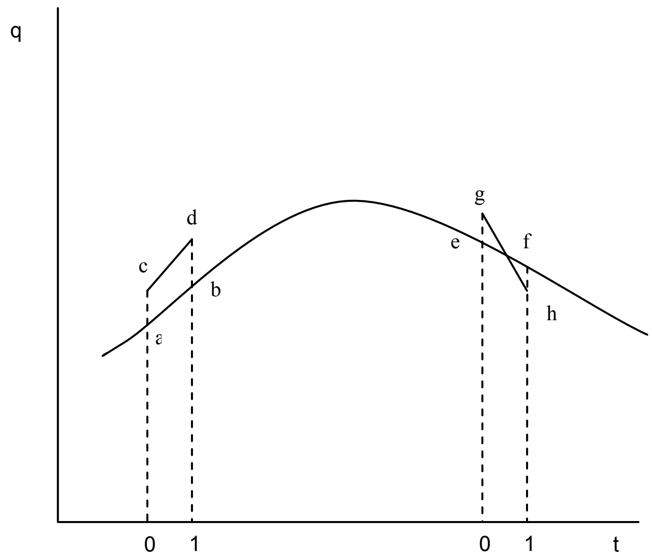

Figure 3