Social Norms and Monetary Trading*

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Abstract

Random matching models have been used in Monetary Economics to argue that money can increase the well being of all agents in the economy. If the model features a finite number of agents it will be shown that there is an equilibrium, analogous to the contagious equilibria described in Kandori (1992), that Pareto dominates the monetary one. However it will be shown also that monetaty equilibria have two important advantages: firstly, they are more plausible in large economies in the sense that the lowest discount factor compatible with monetary equilibria doesn't depend on the population size, which is not the case with contagious equilibria; secondly, it is more stable to finite deviations in the following sense: no matter what the past has been, future play of the equilibrium strategies will give players the same payoff as if the equilibrium strategies were always followed.

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I. Introduction

A question that has been approached by the recent literature in monetary theory is why an intrinsically useless object such as fiat money is widely used as a medium of exchange. Specifically, this question has been posed in random matching models (with a continuum of agents) where the introduction of money can take society from a situation of autarchy to a situation that exhibits frequent trade between different economic agents. In this context, the usual interpretation is that the role of money is to serve as a way of keeping track of the past. In this line of research, Kocherlakota (1998) states that money is "a (typically imperfect) form of memory".

The paradigm of random matching models has also been used in a different context – the context of the prisoners dilemma infinitely repeated. Here, the question that has been posed is: to what extent can society obtain cooperation in every stage when the available information is limited? It's a standard result that cooperation can be achieved when information about past histories is available to all players; since in some situations this assumption cannot be made, it is of interest to know if there is any hope for cooperation. Work by Okuno-Fujiwara and Postlewaite (1995), Kandori (1992), Ellison (1994) and Gata (1996) suggest that the answer is yes.

The objectives of my work are several. First, I will try to show a relation between these two literatures. More precisely, I think that in the framework of the infinitely repeated random matching prisoners' dilemma game the stage game can be modified to capture the spirit of the random matching models of money. In the terminology of the latest literature, the unique and non-cooperative Nash equilibrium of the stage game will be interpreted as autarchy; cooperation will be identified with a situation where one player produces the consumption good and his opponent does not. The usefulness of doing this exercise is that the intuition obtained by studying the infinitely repeated random matching prisoners' dilemma can be used to understand better the role of money in our society. Moreover, I think that the games studied by each of the two literatures belong to the same general class of games because, in essence, they differ only in the stage game and in the assumptions regarding the set of players; they have in common the fact that both games are a denumerable repetition of a given stage game, they have the same information restrictions resulting from random matching and also the same equilibrium concept has been used in both.

Second, I hope to get rid of the conventional assumption in monetary economics that there is a continuum of agents. This assumption together with the assumption that in each period, each player observes only the outcome in his specific match implies that in the monetary economics models, in the absence of money, the unique subgame perfect equilibrium has autarchy as outcome in every period. But, if the assumption of a continuum of agents is abandoned, it might be possible to construct strategies that implement cooperation in every match as an equilibrium, adapting the contagious strategies used by Kandori, Ellison and Gata, since this is the case in the infinitely repeated random matching prisoners' dilemma game. This is possible because they assume a finite number of players.

Third, I would also like to ask if there exists an equilibrium where the players use an intrinsically useless object ("money") to achieve cooperation. If there exist such an equilibrium and if the contagious strategies also constitute an equilibrium in this game, these equilibria could be compared. I think that this comparison might be interesting because the contagious strategies would have at least one advantage (for the players) over the use of money: there would then be trade (the only Pareto optimal outcome of the stage game) in every match, which is not necessarily the case in a monetary equilibrium (if the consumer in a given match doesn't have any unit of money, there isn't trade).

Given, then, that the use of money might have some limitation as a medium of exchange, I think that a natural question is: what are the advantages that it has over other alternatives that society has for implementing exchange? A possible approach to answer this question is to argue that money is the most efficient institution in some class of mechanisms, defined in term of some "desirable" properties (see Hurwicz (1980)). It is the purpose of this work to identify some of those "desirable" properties with respect to which money "behaves better" than other possible mechanisms. This exercise might also help to clarify the role that money plays in society and to explain why it is that money is so widely used, when there are several alternatives available.

Summarizing, I intend to study a game that only differs from the infinitely repeated random matching prisoners' dilemma in the stage game, the stage game being a particular model of exchange similar to the one used in the standard monetary random matching model (e.g. Kiyotaki-Wright). This will get rid of the assumption of a continuum of agents. Finally, I will try to compare monetary equilibria with contagious equilibria that have the property of implementing exchange in every match.

II. A Particular Model of Exchange

Consider the following payoff matrix, which wil be the general payoff matrix for the game described in section (III):

$i\backslash j$	P	NP
P	$-d, u-d$	$-d, u$
NP	$0, -d$	$0, 0$

Assume $u, d > 0$ and $u - d > 0$. Player i represents the producer and player j the consumer; action P is interpreted as "produce" and NP as "does not produce". Finally, u represents the utility of consuming and d the desutility of producing. The assumption that $u - d > 0$ then means that there are potential benefits from trade.

In the standard random matching models of money it is usually assumed that there are K types of agents specialized in production and consumption in the following way: a type k person consumes good k and produces good $k+1$ (modulo K), for $k = 1, 2, ..., K$, where $K \geq 3$. With the assumption of independent random matching and equal proportion of types among the population, the probability of an agent to be a producer is $\frac{1}{K}$, which is also the probability of being a consumer; the probability of being none of these is $1 - \frac{2}{K}$. More generally, it could be assumed that there is a probability α of being a producer and a probability of β of being a consumer, $\alpha + \beta \leq 1$. I will assume $\alpha = \beta = \frac{1}{2}$; this assumption is made in order for the game to be symmetric, and easier to analyse.

III. Random Matching Model

The game has I players indexed by $i \in \mathcal{I} = \{1, 2, ..., I\}$ where $I \geq 4$ is an even number. In each period $t \in \mathbb{N}$, the players are randomly matched into pairs with player i facing player $o_i(t)$. It is assumed that the pairings are independent over time and uniform so that

$$\Pr\{o_i(t) = j \,|h_{t-1}\} = \frac{1}{I-1} \; \forall j \neq i$$

for all possible histories h_{t-1}. At time t, each pair of players $(k, o_k(t))$ play the exchange game described in section (II). All players have discount factor $\delta \in (0,1)$ and their payoffs are the discounted sum of the payoffs in each stage game. At the end of period t, each player observes only the outcome of the exchange game he and his opponent played. He does not observe the identity of his opponent and does not observe the outcome of any of the games played by other pairs of players.

IV. Contagious Equilibria

The above formulation follows closely the one in Ellison (1994) and so it can be expected that his results carry through to this framework. The following strategies are a natural modification of the ones supporting cooperation in Ellison's work:

In period 1, all players begin play according to phase I.

Phase I. In period t, play P if producer and NP if consumer.

If (P, NP) is the outcome for matched players i and j, both play according to phase I in period $t+1$.

If $(NP, P), (P, P)$ or (NP, NP) results in the game between players i and j, then at time $t+1$ both play according to phase II.

Phase II. In period t, play NP. In period $t+1$ play according to phase II.

This strategies will allow the following:

Proposition 1 $\exists \delta^* < 1$ *such that* $\forall \delta \in (\delta^*, 1)$ *there is a sequential equilibrium in which* (P, NP) *is the outcome in every match along the equilibrium path.*

Let $f(k, \delta)$ be player i's continuation payoff from period t on when all players are playing the above strategies and player i and the others are playing according to phase II. If a player is a consumer in a given match, he won't deviate from the prescribed strategies, since he will be playing a strictly dominant action. A producer won't deviate during phase I if

$$d \leq \delta\left(\frac{u-d}{2(1-\delta)} - f(2,\delta)\right).$$

This condition is sufficient because at information sets where he haven't observed any deviation he must believe that he is the first one deviating. It simply says that the cost of producing today should be no greater than the cost of starting the non-cooperative phase II.

In the phase II, if he meets a player in phase II, he won't deviate because if he does his continuation payoff is unaffected but he losses d today. If he meets a player in phase I, he will play NP if

$$d \geq \delta E_j[f(j,\delta) - f(j+1,\delta)]$$

where the expectation reflects player i's beliefs over the number of players who will play according to phase II at time $t+1$. A sufficient condition is

$$d \geq \delta(f(j,\delta) - f(j+1,\delta)) \ \forall j \geq 3.$$

The reason why it's sufficient is as follows: if player i is in phase II then his opponent in the match where he first saw something different than (P, NP) must be in phase II as well. In the case where player i meets a player in phase I, then that previous opponent must have met some other player.

The following lemma is analogous to lemma 1 in Ellison (1994); it expresses that the loss of utility provoked by an additional player in phase II decreases as the number of players already in phase II increases.

Lemma 1 *Let $k \geq 1$. Then $f(k,\delta) - f(k+1,\delta) \geq f(k+s,\delta) - f(k+1+s,\delta), \forall s \geq 1$.*

Proof. Note that $f(k,\delta) = E_\omega f(k,\delta,\omega)$. Let $h(k,\delta,\omega)$ be player i's continuation payoff when he and players $2, ..., k$ and player I are playing according to phase II. Clearly

$$E_\omega f(k+1,\delta,\omega) = E_\omega h(k,\delta,\omega).$$

I show that

$$E_\omega[f(k,\delta,\omega) - h(k,\delta,\omega)] \geq E_\omega[f(k+s,\delta,\omega) - h(k+s,\delta,\omega)]$$

by showing that the inequality holds for every realization of ω.

Define the set $C(t,k,\omega)$ by

$$\begin{aligned} C(1,k,\omega) &= \{k+1, k+2, ..., I\} \\ C(t+1,k,\omega) &= \{i \in C(t,k,\omega) : o_i(t,\omega) \in C(t,k,\omega)\}. \end{aligned}$$

$C(t, k, \omega)$ will be the set of all players who will be playing according to phase I in period t when players $1, ..., k$ begin in phase II in period 1.

Define the set $D(t, \omega)$ by

$$\begin{aligned} D(1, \omega) &= \{I\} \\ D(t+1, \omega) &= D(t, \omega) \cup \{i : o_i(t, \omega) \in D(t, \omega)\} \end{aligned}$$

$D(t, \omega)$ gives the set of all players who will be playing according to phase II in period t when player I begins in phase II in period 1.

Finally, let $P(t, \omega)$ be the set of players that are producers in t.

Note that the payoff to player 1 in period t differs between the situations of $f(k, \delta, \omega)$ and $h(k, \delta, \omega)$ only if his opponent is a producer and plays P when players $1, ..., k$ start in phase II but plays NP when players $1, ..., k$ and player I start in phase II. Thus,

$$f(k, \delta, \omega) - h(k, \delta, \omega) = \sum_{t=1}^{\infty} \delta^t u I(o_i(t) \in C(t, k, \omega) \cap D(t, \omega) \cap P(t, \omega))$$

The definition of C implies that

$$C(t, k+s, \omega) \subseteq C(t, k, \omega)$$

and so

$$C(t, k+s, \omega) \cap D(t, \omega) \cap P(t, \omega) \subseteq C(t, k, \omega) \cap D(t, \omega) \cap P(t, \omega).$$

$\blacksquare$

It follows that in order to prove proposition 1, it suffices to show that $d \leq \delta(\frac{u-d}{2(1-\delta)} - f(2, \delta))$ and $\delta(f(2, \delta) - f(3, \delta)) \leq d$. The strategy will be to find an open interval (δ_0, δ_1) in which this two inequalities hold; this will allow me to apply a result of Ellison (1994) that will prove proposition 1 for $\delta \geq \frac{\delta_0}{\delta_1}$.

Proof. *of proposition 1.*

Step1: $\exists \underline{\delta} \in (0, 1)$ *such that* $\underline{\delta}(\frac{u-d}{2(1-\underline{\delta})} - f(2, \underline{\delta})) = d$.

We may write

$$f(2, \delta) = \sum_{t=1}^{\infty} \delta^t a_t$$

where a_t is the expected payoff in the t-th period after phase II begins. With probability 1 all players will eventually be infected and start playing D and so $a_t \rightarrow 0$. Also, $|f(2,\delta)| \leq \frac{u}{1-\delta}$ and $f(2,\delta) < \frac{u-d}{2(1-\delta)}$ for δ big enough. We then have

$$\begin{aligned}\lim_{\delta\rightarrow 1}\delta(\frac{u-d}{2(1-\delta)} - f(2,\delta)) &= \infty, \\ \lim_{\delta\rightarrow 0}\delta(\frac{u-d}{2(1-\delta)} - f(2,\delta)) &= 0.\end{aligned}$$

By continuity we can choose $\underline{\delta}\in (0,1)$ so that

$$\underline{\delta}(\frac{u-d}{2(1-\delta)} - f(2,\underline{\delta})) = d.$$

Step2: $\exists\gamma > 0$ *such that* $\delta(f(0,\delta) - f(2,\delta) - (f(2,\delta) - f(3,\delta))) \geq \gamma$ *for* $\delta > \underline{\delta}$.

Define the set $D'(t,\omega)$ by

$$\begin{aligned}D'(1,\omega) &= \{I-1, I\} \\ D'(t+1,\omega) &= D'(t,\omega) \cup \{i : o_i(t,\omega) \in D'(t,\omega)\}\end{aligned}$$

$D'(t,\omega)$ gives the set of all players who will be playing according to phase II in period t when players $I-1$ and I begins in phase II in period 1.

Then (for a player different than $I-1$ and I),

$$\begin{aligned}f(0,\delta) - f(2,\delta) &= \sum_{t=1}^{\infty}\delta^t u\mu(D'(t) \cap P(t)) \\ &\geq \sum_{t=1}^{\infty}\delta^t u\mu(D(t) \cap P(t))\end{aligned}$$

since $D(t) \subseteq D'(t), \forall t$. Hence,

$$\begin{aligned}& f(0,\delta) - f(2,\delta) - (f(2,\delta) - f(3,\delta)) \\ \geq\ & \sum_{t=1}^{\infty}\delta^t u\mu((D(t)\backslash C(t,2)) \cap P(t)).\end{aligned}$$

The second term of this sum is $\delta u\mu((D(1)\backslash C(1,2)) \cap P(1))$. If player 2 is matched with player M in period 0 and is the producer under ω we have

$$\{2, I\} \subset I\backslash C(1,2,\omega) \text{ and } D(1,\omega) = \{2, I\}.$$

Together, these imply

$$D(1,\omega)\backslash C(1,2,\omega) = \{2, I\}.$$

From this, we know that the probability term $\mu((D(1)\backslash C(1,2)) \cap P(1))$ is at least the probability that $(2, I)$ is a match in period 0 and that $(2, 1)$ or $(I, 1)$ is a match in period 1. This probability is $\frac{1}{2(I-1)^2}$.

Hence, for $\underline{\delta}$ as defined in step 1, we have for any $\delta >\underline{\delta}$,

$$\delta(f(0,\delta) - f(2,\delta) - (f(2,\delta) - f(3,\delta))) \geq \frac{\underline{\delta}^2 u}{2(I-1)^2} := \gamma$$

Step 3: Let $\eta < \frac{\gamma}{2}$. Then $\exists \delta_0 \in (0, 1)$ such that $\delta_0(f(0, \delta_0) - f(2, \delta_0)) = d + \eta$ and $\delta_0(f(2, \delta_0) - f(3, \delta_0)) < d - \eta$.

From step 1 we know that

$$d = \underline{\delta}(f(0,\underline{\delta}) - f(2,\underline{\delta}))$$

Since

$$f(0,\delta) - f(2,\delta) = \sum_{t=1}^{\infty} \delta^t u\mu(D'(t) \cap P(t)).$$

it follows that

$$\frac{\partial}{\partial\delta}(f(0,\delta) - f(2,\delta))\bigg|_{\underline{\delta}} > 0$$

Thus we can choose $\delta_0 \in (\underline{\delta}, 1)$ so that

$$\delta_0(f(0,\delta_0) - f(2,\delta_0)) = d + \eta.$$

Because $\delta(f(0,\delta) - f(2,\delta) - (f(2,\delta) - f(3,\delta))) \geq \gamma$ for $\delta > \underline{\delta}$ and $\eta < \frac{\gamma}{2}$, it follows that

$$\delta_0(f(2,\delta_0) - f(3,\delta_0)) < d - \eta.$$

Step 4: Apply Ellison's lemma 2: Let $G(\delta)$ be any repeated game of complete information, and suppose that there is a non-empty interval (δ_0, δ_1) such that $G(\delta)$ has a sequential equilibrium $s^*(\delta)$ with outcome a for all $\delta \in (\delta_0, \delta_1)$. Then there is $\underline{\delta} < 1$ such that $\forall\delta \in (\underline{\delta}, 1)$ we can also define a strategy profile $s^{**}(\delta)$ which is a sequential equilibrium of $G(\delta)$ with outcome a.

By step 3, $\delta_0(f(0,\delta_0)-f(2,\delta_0)) > d+\frac{\eta}{2}$ and $\delta_0(f(2,\delta_0)-f(3,\delta_0)) < d-\frac{\eta}{2}$. By continuity, $\exists \delta_1 > \delta_0$ such that for $\delta \in [\delta_0, \delta_1]$

$$\delta(f(0,\delta)-f(2,\delta)) > d+\frac{\eta}{4} \text{ and } \delta(f(2,\delta)-f(3,\delta)) < d-\frac{\eta}{4}.$$

Define $\delta^* = \frac{\delta_0}{\delta_1}$. Now, the construction of Ellison's lemma 2 gives an equilibrium for all $\delta \in [\delta^*, 1)$. ■

The above result shows that, for sufficiently high discount factor, the efficient outcome can be attained in every match as an equilibrium outcome. This is a good property of the contagious equilibrium but it relies on the threat of a rather severe punishment for deviators: if a player deviates, he might expect that sooner or later every one in the society will stop cooperating. Because of this extreme feature, this equilibrium also has some unpleasant properties, that will be discussed below.

Proposition 1 gives a sufficient condition in term of the discount factor so that the "contagious equilibrium" exists in this particular model. It is conceivable that it depends on the number of player; moreover it might be the case that as the number of players increases the cutoff discount factor approaches 1. This is indeed true, but a stronger result would be to consider the limiting behavior of a discount factor with the property that for any smaller δ the contagious equilibrium wouldn't exist. In other words, my plan is: for every I find a discount factor $\underline{\delta}(I)$ with the property that if $\delta < \underline{\delta}(I)$, then the contagious strategies are not an equilibrium. After $\underline{\delta}(I)$ has been defined, I will try to show that $\lim_{I\to\infty} \underline{\delta}(I) = 1$.

Lemma 2 *$\forall I\ \exists \underline{\delta}(I)$ such that for $\delta < \underline{\delta}(I)$ the contagious strategies are not an equilibrium.*

Proof. Define $\underline{\delta}(I) := \inf\{\delta \in [0,1) : \delta(\frac{u-d}{2(1-\delta)} - f(2,\delta,I)) = d\}$ where $f(2,\delta,I) = \sum \delta^t a_{tI}$, where a_t is the expected payoff in the t-th period after phase II begins. The set $\{\delta : \delta(\frac{u-d}{2(1-\delta)} - f(2,\delta,I)) = d\}$ is non empty (step 1 of proposition 1) and closed; hence $\underline{\delta}(I)$ is well defined and belongs to $\{\delta : \delta(\frac{u-d}{2(1-\delta)} - f(2,\delta,I)) = d\}$. Since for $\delta = 0$ one has $\delta(\frac{u-d}{2(1-\delta)} - f(2,\delta,I)) = 0$, it follows that $\underline{\delta}(I) > 0$. Finally, if $\delta < \underline{\delta}(I)$, then $\delta(\frac{u-d}{2(1-\delta)} - f(2,\delta,I)) < d$ and so the contagious strategies are not an equilibrium. ■

Proposition 2 *$\underline{\delta}(I) \to 1$ as $I \to \infty$.*

Proof. First note that $a_{tI} = \frac{u}{2}\Pr(\text{find a player in phase I in period } t) \leq \frac{u}{2}, \forall t$. Since $\Pr(\text{find a player in phase I in period } t) \geq 1 - \frac{2^{t+1}}{I-1}$, it follows that $\lim_{I\to\infty} a_{tI} = \frac{u}{2}$.

This implies that $\lim_{I\to\infty} f(2,\delta,I) = \frac{u/2}{1-\delta}$ because

$$\begin{aligned}
\left|\sum_{t=0}^{\infty}\delta^t a_{tI} - \frac{u/2}{1-\delta}\right| &= \sum_{t=0}^{\infty}\delta^t\frac{u}{2} - \sum_{t=0}^{\infty}\delta^t a_{tI} \leq \frac{u}{2}\sum_{t=0}^{\infty}\min\{\frac{2^{t+1}}{I-1},1\}\delta^t \\
&\leq \frac{u}{2}\left(\sum_{t=0}^{I}\frac{2^{t+1}}{I-1}\delta^t + \frac{\delta^{I+1}}{1-\delta}\right) \\
&= \frac{u}{2}\left(\frac{2}{I-1}\frac{1-\delta^I}{1-\delta} + \frac{\delta^{I+1}}{1-\delta}\right) \to 0 \text{ as } I \to \infty.
\end{aligned}$$

.

Define $h(\delta, I) := \delta(\frac{u-d}{2(1-\delta)} - f(2,\delta,I))$.

Since $\underline{\delta}(I) < 1 \forall I$, then $\limsup_{I\to\infty}\underline{\delta}(I) \leq 1$. Suppose, by way of contradiction, that $\liminf_{I\to\infty}\underline{\delta}(I) < 1$. Then, given $\varepsilon > 0$, there is N such that $\inf_{I\geq N}\underline{\delta}(I) < 1-\varepsilon$. Thus, there is a convergent sequence $\{\underline{\delta}_n\}_{n=1}^{\infty} \subset [0, 1-\varepsilon]$ such that $h(\underline{\delta}_n, n) = d, \forall n$ and $\delta := \lim\underline{\delta}_n \in [0, 1-\varepsilon]$.

It follows that $\lim_{n\to\infty} h(\underline{\delta}_n, n) = d > 0$; but also $\lim_{n\to\infty} h(\underline{\delta}_n, n) = \delta(\frac{u-d}{2(1-\delta)} - \frac{u/2}{1-\delta}) = \frac{\delta}{1-\delta}(-\frac{d}{2}) < 0$, a contradiction.

Thus $\lim_{I\to\infty}\underline{\delta}(I) = \limsup_{I\to\infty}\underline{\delta}(I) = \liminf_{I\to\infty}\underline{\delta}(I) = 1$. ■

Proposition 2 says that in large economies it is unlikely to observe peoples behaving as prescribed by contagious strategies. An additional drawback of the equilibrium using this strategies is that it isn't global stable, in the sense of Kandori (1992). Roughly speaking, an equilibrium is globally stable if no matter what the past has been, future play of the equilibrium strategies will give players the same payoff as if the equilibrium strategies were always followed. Not surprisingly, contagious equilibrium doesn't satisfy this property because if someone deviates then, conditional on such history, payoffs will approach zero. In fact, there is only one history consistent with such a property.

Definition 1 *An equilibrium sustaining payoffs $v = (v_i)_{i\in I}$ is globally stable if for any given finite history of actions h,*

$$\lim_{t\to\infty} E(\,f_i(t)|\,h) = v_i,\ \ \forall i \in I,$$

where $f_i(t)$ is player i's continuation payoff at t.

Proposition 3 *The contagious equilibrium is not globally stable.*

Proof. Consider the one period history where every player played NP in the first period. Then $f_i(t) = 0$, $\forall t > 1$ ■

V. Money

Since we observe frequent trade between people in the 'real world', it is important for economic theory that (P, NP) be a stage game outcome in many matches. The only Nash equilibrium of the game represented by the payoff matrix in (II) has (NP, NP) as outcome, which is clearly not Pareto optimal. Indeed, the only Pareto optimal outcome is (P, NP). This implies that to achieve Pareto optimality society must have a way to achieve "cooperation". Therefore, an objective of a recent literature in monetary theory is to argue that the introduction of money in an economy helps to implement the Pareto optimal equilibrium (P, NP).

In this section, it is supposed that players can use money, which, in the context of the model, may be described as a physical good that can be stored but doesn't affect players payoffs. I will be interested in monetary exchange, which can be interpreted as a social mechanism that works as follows: when a consumer meets a producer, if they both agree to trade, then the producer produces the good for the consumer while the latter gives one unit of money to the producer.

To be more precise about the meaning of the monetary exchange mechanism, I will describe, for each match, the actions available to the players, the outcome function (mapping players actions into outcomes) and the evolution of players' money holdings. Regarding the actions available to the players, I assume that in every match (i, j), they can choose from the set $\{Yes,\ No\}$. If $Q : \{Yes,\ No\} \times \{Yes, No\} \to \{P, NP\} \times \{P,\ NP\}$ denotes the outcome function, then I assume that Q satisfies: $Q((Yes, Yes)) = (P, NP)$ and $Q((Yes, No)) = Q((No, Yes)) = Q((No, No)) = (NP, NP)$. Finally, regarding the evolution of players' money holdings, if $m' : \{Yes, No\} \times \{Yes, No\} \to \{0, ..., M\} \times \{0, ..., M\}$ denotes next period vector of money holdings it is assumed that

$$m'((Yes, Yes)) = \begin{cases} (m_i + 1, m_j - 1) \text{ if } m_j \geq 1 \\ (m_i, m_j) \text{ otherwise} \end{cases}$$

$m'((Yes, No)) = m'((No, Yes)) = m'((No, No)) = (m_i, m_j)$.

I will define a *monetary strategy* for a given player as follows: in every information set where he is a consumer chose "Yes"; in every information set where he is a producer and is opponent has money chose "Yes"; finally, in every information set where he is a producer and is opponent has no money chose "No". If this strategies form a sequential equilibrium, it will be referred as a *monetary equilibrium*.

In order to keep the model tractable, I assume that there is only one unit of money. Furthermore, I assume that each player starts the game with that unit of money with probability $\frac{1}{I}$. This assumption will be used when dealing with global stability.

As in section (IV), I will show that the monetary strategies constitute an equilibrium if the discount factor is high enough. Note first that when a given player is a producer and is matched with a consumer without money he will choose "No". This is because if he choses "Yes" he would receive $-d$ in that period but his continuation payoff would be the same as if he had chosen "No". Also, in the same situation, the consumer is indifferent between "Yes" and "No" and so it is (weakly) optimal to chose "Yes". It is then sufficient to concentrate on those matches in which the consumer has money, since in those where the consumer doesn't have money players are chosing optimally.

Let (i^*, j^*) be a given match and assume that player j^* (the consumer) has money. For $l = i^*, j^*$ and for $\delta \in (0, 1)$, player l's continuation payoff when he holds one unit of money will be denoted by $V(l, 1, \delta)$ and $V(l, 0, \delta)$ will denote his continuation payoff when he holds zero units of money. Because of the symmetric structure of the game, both $V(l, 1, \delta)$ and $V(l, 0, \delta)$ don't depend on l, and so in what follows I suppress l as an argument. Then, player i^* will chose "Yes" if

$$d \leq \delta(V(1, \delta) - V(0, \delta))$$

and player j^* will chose "Yes" if

$$u \geq \delta(V(1, \delta) - V(0, \delta)).$$

That is, the discounted value of money, which is measured by $\delta(V(1, \delta) - V(0, \delta))$, has to be high enough in order to compensate for the cost of acquiring it (here, the cost of producing the consumption good), but not too high.

Proposition 4 $\exists \underline{\delta}' < 1$ *such that* $\forall \delta \in [\underline{\delta}', 1)$ *there is a monetary equilibrium.*

Proof. If all the players are following the monetary strategy, a player that starts a given period with one unit of money will, if he gets to be the consumer in his match, receive u in that period (since $Q((Yes, Yes)) = (P, NP)$ and the payoff for the consumer of this action pair is u as described in figure 1) and start next one with zero units of money (by the definition of m') . If he gets to be the producer then his current utility will be zero (since $Q((No, Yes)) = (NP, NP)$ and the payoff for the producer of this action pair is 0 as described in figure 1) and he will start next period again with one unit of money ($m'((No, Yes)) = (m_i, m_j) = (1, 0)$). Hence,

$$V(1,\delta) = \frac{1}{2}(u + \delta V(0,\delta)) + \frac{\delta}{2}V(1,\delta).$$

Similarly we can derive an expression for $V(0,\delta)$: consider a player that starts a given period without money. If she is a consumer then she won't be able to consume and so she will start next period without money; if she is a producer and meets a player without money, she won't produce and she will again start next period without money. Finally if she is a producer and meets the players with money, she will produce and will start next period with one unit of money. Therefore,

$$V(0,\delta) = \frac{\delta}{2}V(0,\delta) + \frac{1}{2}\left(\frac{1}{I-1}(-d + \delta V(1,\delta)) + \left(1 - \frac{1}{I-1}\right)\delta V(0,\delta)\right).$$

It follows that,

$$V(1,\delta) - V(0,\delta) = \frac{1}{(2-\delta)I + 2(\delta - 1)}(d + (I-1)u).$$

For $\delta \leq 1$, then $\delta(V(1,\delta) - V(0,\delta)) < u$. If $\delta \geq \frac{2d}{u+d}$, then $\delta(V(1,\delta) - V(0,\delta)) \geq d$. Defining $\underline{\delta}' := \frac{2d}{u+d}$, it follows then that for $\delta \geq \frac{2d}{u+d}$ the monetary strategies are a sequential equilibrium. ■

From the proof, we see that $\underline{\delta}'$ doesn't depend on the number of players. This is in sharp contrast with section (IV), where the set of discount factor for which there might be contagious equilibrium becomes very small when the number of players is big. Moreover, it can be shown that the value of money increases with the size of the economy, suggesting that the benefits of holding money are bigger in large economies.

Corollary 1 *(a)* $\lim\limits_{I\to\infty}\underline{\delta}'(I) = \frac{2d}{u+d}$
(b) for $\delta > \underline{\delta}'$, $\delta(V(1,\delta,I) - V(0,\delta,I))$ *increases with* I.

Proof. Part (a) follows immediately from the proof of proposition 7. For (b), it's enough to show that $\frac{d(V(1,\delta,I)-V(0,\delta,I))}{dI} > 0$. Since

$$\frac{d(V(1,\delta,I) - V(0,\delta,I))}{dI} = \frac{\delta(u+d) - 2d}{((2-\delta)I + 2(\delta-1))^2}$$

the result follows. ■

The intuition for the last result is clearer in the limiting case, when δ approaches 1. One has $\lim\limits_{\delta\to 1}\delta(V(1,\delta) - V(0,\delta)) = \frac{1}{I}d + \frac{I-1}{I}u$, which is a linear combination of u and d. This expression illustrates the potential benefits of money: by obtaining one unit of money a player will be able to consume in the first period where he happens to be a consumer afterwards. Also, since by accepting money he reduces the amount the player he meet has, if he meets him again, he will be excused to produce. However, the latter just gives him utility of d, while the first gives him u. As I increases, it is less likely that he receives only d because is less likely that he meets a particular player.

In section (IV) it was shown that the contagious equilibrium was not globally stable. The next proposition shows that, on the contrary, the monetary equilibrium is. This is somewhat expected because monetary strategies depend on the past only by the amount of money a player brings from the previous period. The expected payoff at a given period is either $V(1,\delta)$ or $V(0,\delta)$, depending on whether he has money or not. However, asymptotically, the probability of having money will be equal to $\frac{1}{I}$, regardless of the initial condition. Hence,

Proposition 5 *The monetary equilibrium is globally stable.*

Proof. Note first that the payoff sustained by the monetary equilibrium is $\frac{1}{I}V(1,\delta) + \frac{I-1}{I}V(0,\delta)$, for every player.

Let h be a given history and let t_h be the length of h. That is, h gives the initial vector of money holdings, who met who and the actions players played from $t=1$ until $t=t_h$. So, it will be possible to determine for each player if he will start period t_h+1 with money or not.

Let $i \in I$ and let $p_t^i, t \geq 1$ denote the probability that player i has money at the beginning of period $t_h + t$, given history h. As remarked above p_1^i is either zero or one. Then,

$$E[v_i(t_h + t)|\, h] = p_t^i V(1, \delta) + (1 - p_t^i) V(0, \delta),$$

and so it suffices to show that $\lim_{t \to \infty} p_t^i = \frac{1}{I}$.

Since

$$p_t^i = \frac{1}{2} p_{t-1}^i + \frac{1}{2(I-1)} (1 - p_{t-1}^i), t \geq 2$$

it follows by induction that,

$$p_t^i = \frac{1}{2(I-1)} \sum_{n=0}^{t-2} \left(\frac{1}{2} - \frac{1}{2(I-1)} \right)^n + p_1^i \left(\frac{1}{2} - \frac{1}{2(I-1)} \right)^{t-1}.$$

Hence,

$$\lim_{t \to \infty} p_t^i = \frac{1}{I}.$$

■

VI. Discussion

Random matching model were introduced in monetary economics in order to derive a role for money endogenously. Given the friction imbedded in the model, it was shown that the introduction of money in the system would help people to exchange their goods and to specialize in production (Kiotaki and Wright (1989), (1993)). The emphasis of this approach was on efficiency properties of money, that is, on the welfare gains that economic agents would benefit in moving from a nonmonetary to a monetarized economy, and at least implicitly it was the reason justifying the widespread use of fiat money.

However, society has available some forms of community enforcement and it can use them to sustain a full efficient outcome as an equilibrium situation. An example of such a form of community enforcement is the contagious equilibrium described above[1], where the failure of exchange between two

[1] To my knowledge, the strategies supporting such equilibrium were first introduced by Kandori (1992). Corbae et al.(1999) also use them in a monetary economics model that is close to the one presented here, but they use a different equilibrium concept.

people will trigger a return to autarchy for society as a whole. This suggests that the rationale for money should at least include some other features besides efficiency.

The question that motivated this present work was: why did monetary trading settled as the main trading mechanism in developed economies? This question is far from being answered here, but nevertheless I would like to refer two properties that monetary trading mechanism possesses, and that might be important in justifying it as a customary practice.

A first property is that it seems to work as well in small as in large economies. When a person refuses to exchange his goods for money, this information will be transmitted when this person runs out of money and he will be punished at that time since he won't be able to consume. So far as it doesn't become more likely for a person to be able to sell the goods he produces or to buy the goods he consumes as the number of people in the economy, then this implicit punishment does not depend on the size of the economy. This is, however, in sharp contrast with the case of contagion. Here, when a person refuses to give the commodities he produced to some other person, he can expect that cooperation will break down in society. But, if the economy is large, chances are that will not get punished in the near future. That is, when he is one of many agents in society, he can reasonably expect to fail to abide by the social norm and get by with it for a long time. Thus, it seems to be more natural that money circulates in large economies and in some sense this seems to be in agreement with the evidence that only in this century did fiat money become the general medium of exchange.

A second property is that monetary exchange seems to be somewhat "safe". What I mean is that if people make some mistakes, or if they fail to understand or to accept the mechanism at the beginning, even in this case all the agents in the economy will tend to be as well off as if nothing of that had occurred. In this sense, monetary exchange is almost independent of the past which allow society some form of forgetfulness. On the contrary, this is not the case with contagion. In this case, a single mistake by someone will make all the trading collapse. So, it also seems that money has a more important role in more complex and unstable economies, which again seems to match some simple empirical evidence.

Although these properties seem to be important, are they sufficient to make money the "best" exchange mechanism? Again this question will not be answered here but it is worthwhile to mention some other mechanisms that have been studied and compare the monetary exchange mechanism with them. In Okuno-Fujiwara and Postlewaite (1995) study of social norms, they specify a class of mechanisms in which a label is attached to each person and those labels completely determine players' behavior. Kandori (1992) states that it is possible to design such mechanism in order to allow for frequent trade in a way that is globally stable and independent of population size. Kocherlakota (1998b) describes a mechanism that in particular can implement trade using either a physical object that is divisible, nondisposable and observable or two divisible objects, not necessarily concealable or disposable, object that essentially play the role of labels[2]. Finally, Townsend (1987) comparing the relative efficiency of portable concealable objects as record-keeping devices with electronic telecommunications, concludes that the latter is a dominant technology in the sense that it allows a bigger value for weighted average across the agents in the economy of their ex-ante expected utility.

Do monetary trading mechanisms have some advantages when compares with those mechanisms? As described above, "label" mechanisms appear to be quite appealing, but they make the assumption that trading partners can observe each other's labels. An attractive feature that money has is that, if we think of labeling people as having money or not, then it is true that trading partners can observe each other's labels. However, the general "labels" mechanisms described in Okuno-Fujiwara and Postlewaite (1995) and Kandori (1992) simply assume that; furthermore, their mechanism has the property that if the agents could choose to conceal their label, some would choose to do so.

This last problem, regarding the observability of the labels, is overcome in Kocherlakota's two-money mechanism. The potential benefit of the monetary trading mechanism over that mechanism, is that the former is quite complex, involving a mapping from individual histories into the decimal expansion of money holdings.

Finally, money seems to be a much "cheaper" alternative than electronic telecommunication proposed by Townsend (1989). In the context of finding a mechanism to allow exchange in a given society, his mechanism would require that after ever exchange, the people involved in it had to communicate its outcome to some centralized bureau that everyone in the economy could access. In a large economy, this would probably be very expensive.

[2]At first sight those mechanisms seem to be independent of the population size, but not globally stable.

VII. Conclusions

I have considered a particular model of trade that has as its defining features a finite number of agents and limited information resulting from random matching. This two elements have been used in the context of the infinitely repeated prisoners' dilemma game to study social norms and also in monetary economics. I argued that both literatures have common elements and as a result I considered a model that allow for a direct comparison of the results of both.

The motivation for this approach was the existence in the infinitely repeated random matching prisoners' dilemma game of an equilibrium that implements the efficient outcome in every match. I believe that this opens the question of why monetary trade is customary in actual economies instead of the mechanism of that efficient outcome. The model I considered provided an example of a framework where money does have at least two advantages: it behaves relatively better in large economies and it is more stable to finite deviations.

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