Limited Commitment, Inaction and Optimal Monetary Policy

Tokhir Mirzoev*

September 29, 2004

JEL Codes: E50, E52, E58

Keywords: central bank, monetary policy, commitment, stabilization bias.

Abstract

This paper examines the optimal frequency of monetary policy meetings when their schedule is pre-announced. Our contribution is twofold. First, we show that in the standard New Keynesian framework infrequent but periodic revision of monetary policy may be desirable even when there are no explicit costs of policy adjustment. Adjustment of policy on a pre-announced schedule *de facto* acts as a commitment not to adjust in intermediate periods. We find that at short horizons gains from such commitment outweigh welfare costs of central bank's inaction. Second, we solve for the optimal frequency of policy adjustment and characterize its determinants. When applied to the U.S. economy, our analysis suggests that the Federal Open Markets Committee should revise the federal funds target rate no more than twice a year.

1 Introduction

As Clarida et. al. (1999) point out, no major central bank has announced a life-time commitment to a specific monetary policy rule. Thus, theoretical research has devoted a

*Department of Economics, The Ohio State University, 1945 North High Street, Columbus, OH 43210, Tel: (614) 292-6701, Email: mirzoev.1@osu.edu. I'm indebted to Bill Dupor and Paul Evans for invaluable advice and suggestions, Joseph Kaboski, Huston McCulloch, and seminar participants at the Ohio State University for constructive comments on earlier drafts. All remaining errors are mine. MATLAB codes used in the paper are available upon request.

great deal of attention to designing policies that could in one way or another mimic long-term commitment. In this paper we consider a simple policy that to some extent is already in place: the practice of holding infrequent and periodic monetary policy meetings.

Our motivation comes from two observations. First, central banks around the world make monetary policy decisions at discrete times and with differing frequency. The Bank of England's recent survey of over ninety central banks found that seven central banks held policy making meetings less than monthly, about thirty six had monthly meetings, while the rest made policy decisions more frequently, some even on a daily basis[1].

Second, in the absence of major shocks to the economy most major central banks hold policy meetings regularly. For example, the Bank of Japan's monetary policy meetings take place twice a month, the Governing Council of the European Central Bank meets monthly, while the Federal Open Markets Committee in the U.S. revises the federal funds target rate eight times a year. Moreover, most monetary authorities in developed countries announce the schedule of policy meetings in advance.

A natural question is whether there are benefits of infrequent policy adjustment in the absence of explicit commitment to a particular policy rule. By analogy with the sticky price literature, it is tempting to justify infrequent policy meetings by appealing to administrative difficulties, or other policy adjustment costs. It would then follow that the optimal frequency of policy meetings should depend on the tradeoff between central banks' internal cost of adjustment and social losses arising from policy makers' inaction. However, the adjustment cost analogy is unlikely to provide a complete story. For example, it is not useful in explaining the fact that some major central banks (e.g. the Bank of Japan, the ECB and the Bank of England) have policy meetings more often than some smaller ones (e.g. the Bank of Canada or the Riksbank). More importantly, the analogy does not exploit the external effects of central banks' actions. Note that when policy is adjusted on a pre-announced schedule, then

[1]See Mahadeva and Sterne (2000), chart 7.5. *How often do policy-makers meet to decide on the setting of policy instruments?* In the rest of the paper by policy adjustment we shall assume changes of the main policy instrument/target at the policy makers' level such as the revision of federal funds target rate at FOMC meetings.

following each policy meeting central bankers not only announce a new target, but also *de facto* promise to keep it fixed until the next meeting. Such implicit promises can be viewed as sequential short-term commitments. Therefore, the appropriate tradeoff in choosing the frequency of policy meetings is between the volatility of inflation and output arising from inaction and the benefits of short-term sequential commitments.

The contribution of this paper is twofold. First, we show that in the standard New Keynesian model and for most plausible parameter values infrequent but periodic policy adjustment is preferable to pure discretion *even without any adjustment costs*. Second, we solve for the optimal frequency of monetary policy meetings. Applied to the U.S. economy, our analysis suggests that the FOMC should meet no more than twice a year.

The rest of the paper is organized as follows. First, we consider a central bank in the Clarida et. al. (1999) world that is not able (or not willing) to make a life-time commitment to a policy rule (i.e. operates under pure discretion). As they show, even in the absence of the Barro and Gordon (1983) problem of inflationary bias, commitment is welfare improving because the impact of policy decisions on private sector forecasts improves the inflation-output variability tradeoff. To this we add that a welfare improving commitment need not be life-time: central banks that are unwilling to make long-term promises could instead offer short-term sequential commitments and also improve welfare relative to pure discretion. Furthermore, we solve for the optimal monetary policy under limited time commitment and find significant diminishing marginal returns from lengthier commitment. In the benchmark model announcing a new policy rule every year allows the central bank to realize about 90 percent of the total possible gains from life-time commitment. This is discussed in section 2.

Next, in section 3 we characterize a simple policy of infrequent adjustment where the central bank vows to revise the interest rate only every other period. One can think of this scenario as a policy of sequential short-term commitments to a degenerate non-state contingent rule - a commitment not adjust the policy. We find that infrequent adjustment is preferred to pure discretion for most plausible parameter values. The intuition behind this finding is straightforward. A discretionary policy allows a timely response to exogenous disturbances, but features higher output costs of reducing inflation. On the other hand, under infrequent adjustment, the central bank acts as if under commitment every second period, but leaves some shocks in non-meeting months unanswered. While the latter contributes to the volatility of target variables, the effects of commitment are the opposite. In particular, commitment reduces the cost disinflation at the time of adjustment and prompts a more aggressive response to inflationary pressures. Quick disinflations make the effects of exogenous shocks on inflation die out faster under periodic adjustment than they do under period-by-period adjustment. This effect contains inflationary expectations and inflation itself in non-meeting months, producing lower volatility of inflation in *all* periods. We find that for many plausible parameter values these benefits from commitment dominate the destabilizing effects of inaction. Section 4 discusses the optimal frequency of policy meetings, its determinants and the application of our analysis to the U.S. case. Section 5 concludes.

2 Short-term Sequential Commitments

2.1 The Model

We consider a central bank in the world of Clarida et. al. (1999), hereafter CGG99. It seeks to minimize the expected present value of quadratic losses of the form:

$$E_0\left(L\right) = \frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left(\alpha x_t^2 + \pi_t^2\right) \tag{1}$$

The economy is described by the usual New Keynesian IS and Phillips curves:

$$x_t = -\varphi(i_t - E_t\pi_{t+1}) + E_t x_{t+1} + g_t \tag{2}$$

$$\pi_t = \lambda x_t + \beta E_t\pi_{t+1} + u_t \tag{3}$$

where i_t is the nominal interest rate, π_t is the rate of inflation, x_t captures log deviations of real output from its natural level (output gap) and the two exogenous state variables u_t (cost-push shifts) and g_t (demand shifts) evolve according to:

$$g_t = \mu g_{t-1} + \hat{g}_t \tag{4}$$

$$u_t = \rho u_{t-1} + \hat{u}_t \tag{5}$$

The exact interpretation of the cost-push and demand shifts depends on the level of generality in the underlying nonlinear model. With government spending, variable markups and technology shocks each shift can be a combination of various types of disturbances: g_t usually incorporates government spending (as in Clarida et. al. (1999)) and shocks to the growth rate of natural output (as in Woodford (1999)). Similarly, in the general setup u_t captures exogenous variations in deviations between the marginal disutility of labor and the marginal product of labor.[2]

Note that the formulation of equation (1) assumes the absence of the inflationary bias, which occurs when the central bank targets a level of output above its natural level.[3] This is done for two reasons. First, the inflationary bias results in higher average inflation. The recent experience of the U.S. and most European countries does not suggest the presence of such bias. Secondly, inflationary bias is likely to lower welfare regardless of how often the policy makers meet and would not affect the rankings of the policies we consider. The primary focus of this paper is the role of commitment in removing the stabilization bias: over-stabilization of output under discretionary policies. Unlike the inflationary bias, the stabilization bias results in excessive volatility of inflation. Volatility of inflation is likely to be a more serious problem than its average level.

In the exercises below we base our choice of parameter values on the micro-foundations

[2] We do not argue that the model in (1)-(5) is a good representation of reality. This framework was chosen due to its popularity and in order to better relate to existing studies. In section 4.3. we briefly discuss some alternative model assumptions that could be of interest for future research.

[3] See Kydland and Prescott (1977) and Barro and Gordon (1983).

behind the model. The IS equation is derived from the consumption Euler equation, where φ is the inter-temporal elasticity of substitution. The plausible range of φ suggested in many studies lies between 0.5 and 1 (log utility). In the benchmark model we use an intermediate value: $\varphi = 0.67$. We choose $\beta = 0.997$ since the focus is on the monthly frequency. The Phillips curve is commonly derived from Calvo pricing equation, in which λ takes the form:

$$\lambda = \frac{(1-\beta\theta)(1-\theta)}{\theta}\left(\frac{1}{\varphi} + \frac{1-\eta+\vartheta}{\eta}\right) \tag{6}$$

where θ is the probability that a firm will not change its price in any given period, η is the weight of labor in the Cobb-Douglas production function, ϑ is the inverse of the Frisch elasticity of the labor supply. Common assumptions in the literature suggest $\theta = \frac{11}{12}$ and $\eta = \frac{2}{3}$. There is little agreement about the appropriate value for the labor supply elasticity. We use a high value of elasticity ($\frac{1}{\vartheta} = 5$, or $\vartheta = 0.2$) as prescribed by Prescott (2003) and used in Rotemberg and Woodford (1992, 1997) and Gali et. al. (2003). The optimal weight of output in the loss function under mild regularity conditions can be expressed as follows: [4]

$$\alpha = \frac{\lambda}{q} \tag{7}$$

where q is the demand elasticity. We set the latter at 6, which implies a steady state markup over marginal costs of 20%.

Innovations to cost and demand shifts are assumed uncorrelated with each other with standard deviations of: $\sigma_{\hat{u}} = \sigma_{\hat{g}} = 0.001$. Finally, the persistence parameters are: $\rho = \mu = 0.8$. In section 4 we consider an alternative calibration of the stochastic processes based on the U.S. data.

[4] See Woodford (2003), chapter 6, proposition 6.2.

2.2 Discretion vs. Life-Time Commitment

We begin by discussing the optimal policy under discretion and under life-time commitment. While the optimality of the latter is well known, here we re-state the main argument in order to motivate further discussion of alternative policies.

In the absence of commitment, which we will refer to as pure discretion, the central bank does not make any promises as to how it will adjust the interest rates in the future. Thus every period it minimizes contemporaneous losses while taking all forecasts as given. To obtain the solution it is convenient to first choose inflation and output that maximize the objective function subject to (3) and then obtain the interest rate from (2). The first order condition (F.O.C.) is given by:

$$\pi_t^{nc} = -\frac{\alpha}{\lambda} x_t^{nc}$$

Using this in (3), we obtain:

$$x_t = \frac{\alpha\beta}{\alpha + \lambda^2} E_t x_{t+1} - \frac{\lambda}{\alpha + \lambda^2} u_t$$

Recursive substitution together with (5) yields a solution:[5]

$$\begin{aligned} x_t^{nc} &= -\tfrac{\lambda}{\lambda^2+\alpha(1-\beta\rho)} u_t; \\ \pi_t^{nc} &= \tfrac{\alpha}{\lambda^2+\alpha(1-\beta\rho)} u_t; \end{aligned} \tag{8}$$

Note that the resulting interest rate policy always neutralizes demand shocks in the sense that it makes inflation and output react to supply shocks only.

Next, consider the case of commitment. For simplicity, as in CGG99 we begin by examining policies that that have the same functional form as under pure discretion, i.e. $\pi = \omega_1 \cdot u_t$ and $x_t = \omega_2 u_t$. Then future target variables can be expressed as multiples of their current period values: $\pi_{t+j} = \pi_t \frac{u_{t+j}}{u_t}$ and $x_{t+j} = x_t \frac{u_{t+j}}{u_t}$. The objective function (1) can therefore be

[5] The interest rate that implements the solution can be recovered from (2). Using the notation of Clarida et. al (1999), it can be represented as: $i_t = \gamma_\pi E_t \pi_{t+1} + \frac{1}{\varphi} g_t$, where $\gamma_\pi = 1 + \frac{(1-\rho)\lambda}{\rho\varphi\alpha}$.

re-written as:

$$L_t = \frac{1}{2}\left(\alpha x_t^2 + \pi_t^2\right) E_t \sum_{i=0}^{T} \beta^i \left(\frac{u_{t+i}}{u_t}\right)^2 \tag{9}$$

where T is the policy horizon (∞ in the standard case). Note that since the summation term in the last equation is exogenous, minimizing (9) is equivalent to minimizing current period losses $(\alpha x_t^2 + \pi_t^2)$. The first order condition, obtained by minimizing the objective function w.r.t. x_t and subject to the Phillips curve (3), is:

$$\alpha x_t + \frac{\partial \pi_t}{\partial x_t}\pi_t = 0 \tag{10}$$

The term $\frac{\partial \pi_t}{\partial x_t}$ is crucial in determining the tradeoff between inflation and output. Under discretion the central bank is unable to affect forecasts of the future. Hence from (3) we have $\frac{\partial \pi_t}{\partial x_t} = \lambda$, i.e. the output cost of reducing inflation by one unit is $\frac{1}{\lambda}$. On the other hand, under life-time commitment the central bank can count on the private sector to expect $E_t\pi_{t+1} = \rho\pi_t$. This ability to credibly influence private sector forecasts modifies the Phillips curve equation to be:

$$\pi_t = \frac{\lambda}{1-\beta\rho}x_t + \frac{1}{1-\beta\rho}u_t \tag{11}$$

Hence, under commitment we have: $\frac{\partial \pi_t}{\partial x_t} = \frac{\lambda}{1-\beta\rho}$, so that only $\frac{1-\beta\rho}{\lambda}$ units of output need to be sacrificed in order to bring inflation down by one unit. The multiplicative constant $(1-\beta\rho)$ captures 'savings' in the form of lower output costs of reducing inflation that arise from the central bank's ability to affect private sector forecasts. A complete solution is obtained by combining (10) and (11):[6]

$$\begin{aligned} \pi_t^c &= -\tfrac{\alpha k}{\lambda}x_t^c \\ x_t^c &= -\tfrac{\lambda}{\lambda^2+\alpha k(1-\beta\rho)}u_t \\ \pi_t^c &= \tfrac{\alpha k}{\lambda^2+\alpha k(1-\beta\rho)}u_t \end{aligned} \tag{12}$$

where $k = 1-\beta\rho$. Note that output (inflation) under discretion is less (more) volatile than

[6]The interest rate rule implied by the solution is given by $i_t = \gamma_\pi^c E_t\pi_{t+1} + \frac{1}{\varphi}g_t$, where $\gamma_\pi = 1 + \frac{(1-\rho)\lambda}{\rho\varphi\alpha(1-\beta\rho)}$.

under commitment. This is the stabilization bias, which is primarily due to high costs of reducing inflation under discretion. Under commitment, lower costs of reducing inflation prompt the central to 'buy' more inflation reduction at the cost of some extra output volatility ($k < 1$) thus alleviating the problem of stabilization bias. To see the welfare gains from commitment note that from (12) the unconditional expectation of the loss function can be expressed as: $E(L) = \frac{\alpha\lambda^2+\alpha^2 k}{[\lambda^2+\alpha k(1-\beta\rho)]^2}\frac{1}{1-\beta}E(u^2)$. It is easy to show that the loss is minimized when $k = (1-\beta\rho)$ and that any policy taking the form of (12) with $(1-\beta\rho) < k < 1$ is preferred to the case of pure discretion ($k = 1$).

2.3 Discretion vs. Short-Term Commitment

The commitment described above was life-time. Such arrangement may be both unrealistic (e.g. because chairmen of central banks have limited terms in office) and undesirable (e.g. because of model uncertainty or because of extraordinary circumstances requiring deviations from the announced rule). As an alternative, central banks could use short-term commitments, i.e. announce policy rules that are valid for a pre-determined period of time. Such short-term promises allow the monetary authority to credibly influence private sector forecasts in the periods when the commitment is valid, but not in the long-run. Intuitively, we should expect such policy to be suboptimal relative to life-time commitment, but perform better than pure discretion.

As a simple verification, consider a monetary authority that announces a new commitment every other period[7]. Thus, its powers are restricted to affecting only one period ahead forecasts. It can be shown that under a class of linear interest rate rules considered above, the equilibrium with one-period commitments is also described by (12)[8] but with $k = \left(1+\frac{\beta\rho}{1+\varphi\lambda\rho+\rho}\right)^{-1}$. Note that in this case $(1-\beta\rho) < k < 1$, implying that even though life-time commitment is still the first best, central banks that are unwilling to commit forever can achieve a better outcome than pure discretion by offering short-term commitments.

[7] This is similar to the analysis of partial commitment in fiscal policy in Klein and Ríos-Rull (2003).

[8] A complete solution is provided in Appendix A.

2.4 Short-Term vs. Life-Time Commitment Under Unconstrained Optimum

Next, we evaluate relative welfare gains under life-time and short-term commitments when the policy under each scenario is globally optimal. Under life–time commitment the central bank chooses inflation and output to maximize the following Lagrangean:

$$\begin{aligned} \Im_1 = -\tfrac{1}{2}E_t\Bigg\{\sum_{i=0}^{\infty}\beta^i\Big[\alpha x_{t+i}^2 + \pi_{t+i}^2 + \\ \gamma_{t+i}\left(\pi_{t+i} - \lambda x_{t+i} - \beta E_t \pi_{t+1+i} - u_{t+i}\right)\Big]\Bigg\} \end{aligned} \tag{13}$$

The optimal policy sets inflation in proportion to the *change* output (see CGG99):

$$\begin{aligned} & x_{t+i} - x_{t+i-1} = -\tfrac{\lambda}{\alpha}\pi_{t+i}, \qquad i = 1,2,3..... \\ & \text{and} \quad x_{t+i} = -\tfrac{\lambda}{\alpha}\pi_{t+i}, \qquad i = 0 \end{aligned} \tag{14}$$

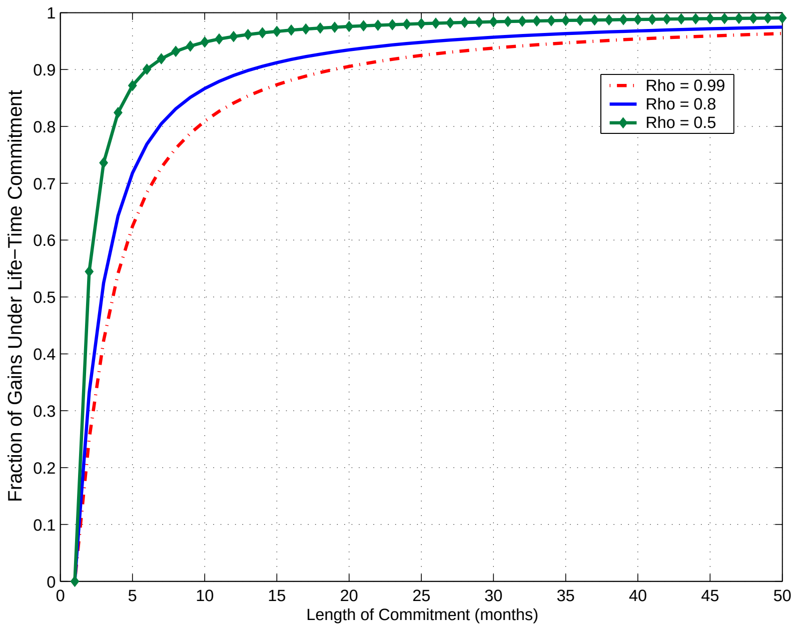
As an alternative, suppose that every $T+1$ periods the central bank announces a new commitment that is valid for T periods in the future. It can be shown that the optimal policy announced at date t and valid until $t+T$ (the 'commitment cycle') closely resembles (14) [9]:

$$\begin{aligned} & x_{t+i} - x_{t+i-1} = -\tfrac{\lambda}{\alpha}\pi_{t+i}, \qquad i = 1,2,3.....T \\ & \text{and} \quad x_{t+i} = -\tfrac{\lambda}{\alpha}\pi_{t+i}, \qquad i = 0 \end{aligned} \tag{15}$$

The only difference between (14) and (15) is that the short-term commitment expires at $t+T$ and at $t+T+1$ a new announcement is made. Model stationarity implies that at the beginning of each commitment cycle the central bank will always announce the same policy.

[9] Appendix B provides a complete solution. Note that this model resembles those of Schaumburg and Tambalotti (2002) and Kara (2003). Both of these studies consider a similar setting. Their models examine 'Calvo type' central bankers who offer life-time commitments, but each period face a constant probability of being replaced. The central bankers in our model are more of the 'Taylor type' - they are being replaced after serving fixed terms in office. We consider the latter to be a more plausible scenario for developed countries. In any case, they also find that most of the gains from commitment occur at short horizons.

Figure 1: Gains From Limited Commitment



[a]

[a]Gains from limited commitment relative to pure discretion as a fraction of total gains obtained under life-time commitment

To evaluate the relative performance of the two arrangements, we solve the model above under various durations of commitment (T) and measure welfare gains implied by each policy[10]. Figure (1) plots welfare gains under sequential commitments of various lengths as a fraction of the total gains obtained under life-time commitment. A striking feature in Figure (1) is the presence of strong diminishing returns from lengthier commitment. For example, in the benchmark specification sequential 12-month commitments allow the central bank to realize about 90% of the total gains obtained under life-time commitment. When the persistence of the cost-push shocks is reduced to 0.5, then the same fraction of the gains can be obtained by committing to a new policy every six months. Intuitively, under commitment the optimal policy seeks not only to eliminate the contemporaneous effects of current exogenous shocks, but also to neutralize the predetermined component of future expected effects coming from persistence ($E_t(u_{t+1}) = \rho u_t$). When persistence is large, then current innovations affect the forecasts of inflation and output far into the future making long-term commitment more

[10]Welfare gains were measured by the difference of the unconditional expectation of the loss function (1) under each case of commitment and under pure discretion.

important.[11]

Another reason for substantial gains from short-term commitments lies in their sequential nature. In any announcement period we expect the optimal policy to influence short-term private sector forecasts in a way that allows greater stabilization of inflation within the 'commitment' cycle. Moreover, in equilibrium agents expect the same policy to be re-announced after the expiration of the current commitment. Hence, the stabilization properties of the current policy are also embedded into the agents' long-term forecasts of future policy decisions. More stable long-term forecasts of inflation, in turn, further contain current inflation, thus multiplying the stabilizing effects of the announced commitment. This extra effect produces large overall welfare gains even when the commitment is set to expire soon.

To this end we have shown that the ability of short-term sequential commitments to affect private sector forecasts both in the short-run and in the long-run, makes them preferable to pure discretion and provides a useful alternative to life-time commitment when the latter is unfeasible. The prescription of this section is useful for central banks that possess at least a short-term commitment technology. Next, we consider a case when commitment to state-contingent rules in not feasible and examine a simple alternative - infrequent policy adjustment.

3 Infrequent Monetary Policy Adjustment

Central banks' unwillingness (or inability) to commit to explicit policy rules prompts researchers to look for implementable arrangements that somehow resemble commitment. Here we consider a simple policy of infrequent but periodic monetary policy adjustment. An attractive feature of such policy is its implementability: to some extent it is already in place since central banks around the world make monetary policy decisions at discrete points in time and with a stable frequency. For example, the Bank of Japan's monetary

[11] Note, that when $\rho = 0$ equation (12) reduces to (8), i.e. under the simple policy of section 2.1. gains from commitment disappear when shocks are completely unpredictable.

policy meetings take place twice a month, the Governing Council of the European Central Bank meets monthly, while the Federal Open Markets Committee in the U.S. revises the target federal funds rate eight times a year. In addition, monetary authorities in developed countries typically announce the schedule of policy meetings well in advance.

When monetary policy is adjusted infrequently and on a pre-announced schedule, then after each adjustment the central bank not only declares a new target, but also *de facto* promises to leave it unchanged until the next policy meeting. This closely resembles short-term sequential commitments of the previous section. The difference is that by fixing the target for several periods, the central bank commits to a non-state-contingent rule (or, alternatively, it commits not to adjust). This implies a trade-off between gains from commitment and losses arising from central bank's inaction. Moreover, the trade-off implicitly defines the optimal frequency of policy meetings - an important issue in monetary policy design.

In this section we consider a central bank that is unwilling or unable to commit to an explicit policy rule but holds policy meetings at pre-announced dates. We will assume that the economic agents believe that the monetary authority will adhere to the announced schedule of the meetings. [12] First, we examine a simple case where the central bank adjusts the interest rate every other period and establish the superiority of this policy relative to period-by-period adjustment under pure discretion. Then we solve for the optimal frequency of policy meetings and characterize its determinants. Finally, we examine the implications of our analysis to the case of the Federal Open Markets Committee.

3.1 Central Bank's Problem Under Infrequent Adjustment

To keep things tractable, we begin with a central bank that commits to holding policy meetings every other period. In every meeting period (denote t) it sets a new interest rate and promises to keep it fixed for two periods. If the promise is credible, the private sector

[12] In other words, we abstract from the possibility of unscheduled meetings. This is a reasonable assumption for major economies. For example, in the last decade only in 2001 has the FOMC had more than eight meetings a year. As long as unscheduled meetings represent true emergencies and have a small unconditional probability, their existence should not affect our main results.

forecast of the next period's output is given by:

$$E_t x_{t+1} = -\varphi i_t + \varphi E_t \pi_{t+2} + E_t x_{t+2} + \mu g_t \tag{16}$$

Similarly, expected next period inflation can be written as:

$$E_t \pi_{t+1} = -\lambda \varphi i_t + \lambda \mu g_t + (\lambda \varphi + \beta) E_t \pi_{t+2} + \lambda E_t x_{t+2} + \rho u_t \tag{17}$$

Using (16) and (17) in (2) and (3) allows to express the IS and Phillips curve equations as follows:

$$\begin{aligned} x_t = -i_t \left(2\varphi + \varphi^2 \lambda\right) + \varphi \left(\lambda \varphi + \beta + 1\right) E_t \pi_{t+2} + \left(\lambda \varphi + 1\right) E_t x_{t+2} + \\ + \varphi \rho u_t + \left(\varphi \lambda \mu + \mu + 1\right) g_t \end{aligned} \tag{18}$$

$$\pi_t = \lambda x_t - \beta \lambda \varphi i_t + \left(\lambda \beta \varphi + \beta^2\right) E_t \pi_{t+2} + \beta \lambda E_t x_{t+2} + \left(\beta \rho + 1\right) u_t + \beta \lambda \mu g_t \tag{19}$$

In contrast to the case of pure discretion, short-term commitments expand the policy horizon to the duration of the interest rate fixity. At the time of interest rate adjustment the central bank's problem is:

$$\max_{i_t} -\tfrac{1}{2}\left[\left(\alpha x_t^2 + \pi_t^2\right) + \beta E_t \left(\alpha x_{t+1}^2 + \pi_{t+1}^2\right) + F_t\right] \tag{20}$$

where F_t represents expected losses beyond $t+1$, which are taken as given. The constraints to the problem include the modified IS and Phillips curves at dates t (equations (18) and (19)) and $t+1$:

$$x_{t+1} = -\varphi(i_t - E_{t+1}\pi_{t+2}) + E_{t+1} x_{t+2} + g_{t+1} \tag{21}$$

$$\pi_{t+1} = \lambda x_{t+1} + \beta E_{t+1} \pi_{t+2} + u_{t+1} \tag{22}$$

where forecasts of π_{t+2} and x_{t+2} cannot be manipulated by the central bank and are taken as given. Inserting the constraints into the objective function and maximizing w.r.t. i_t yields the following first order condition:

$$\alpha x_t \frac{\partial x_t}{\partial i_t} + \pi_t \left(\frac{\partial \pi_t}{\partial i_t} + \frac{\partial \pi_t}{\partial x_t} \frac{\partial x_t}{\partial i_t} \right) + \beta E_t \left(\alpha x_{t+1} \frac{\partial x_{t+1}}{\partial i_{t+1}} + \pi_{t+1} \frac{\partial \pi_{t+1}}{\partial x_{t+1}} \frac{\partial x_{t+1}}{\partial i_{t+1}} \right) = 0.$$

Or:

$$\alpha \left(2 + \varphi\lambda\right) x_t + \lambda(2 + \varphi\lambda + \beta)\pi_t = -\beta \left(\alpha E_t x_{t+1} + \lambda E_t \pi_{t+1}\right) \tag{23}$$

Since the central bank is unable to respond to shocks in the next period, it sets the the instrument at the level that minimizes average expected losses between policy meetings[13]. This is similar to what one obtains in the Taylor model of price-setting where each firm adjusts its price periodically. However, this analogy is not as close as it seems. Firms' infrequent revision of prices is typically justified by the existence of price adjustment costs. In the absence of such costs firms would always do better by revising prices every period. On the other hand, central bank's actions have important external effects that define a different trade-off. Although changing the interest rate on a period-by-period basis allows a timely response to exogenous shocks, it features high output costs of reducing inflation. On the other hand, infrequent policy adjustment, while forcing the central bank to put up with extra volatility arising from inaction, creates commitment gains from the ability to affect short-term forecasts. As long as the gains from commitment exceed losses from inaction, the central bank would choose infrequent policy meetings even when there are no explicit costs of interest rate adjustment. Next, we define the equilibrium and evaluate welfare under the two alternatives.

3.2 Equilibrium

[13] Note also that in the absence of next period considerations $(\beta = 0)$, the optimality condition reduces to the standard solution without commitment.

The presence of an endogenous state variable i_{t-1} in periods of central bank's inaction implies that the minimum state vector is different across periods. In particular, in periods of adjustment the relevant state is summarized by contemporaneous shocks $e_t = \{g_t, u_t\}$, whereas in periods of inaction the relevant states can be summarized as $s_t = \{e_{t-1}, e_t\}$. We use the following notion of equilibrium:

Definition 1 *A rational expectations infrequent policy adjustment equilibrium is described by a set of policy functions* $\{i^a(e_t), x^a(e_t), \pi^a(e_t)\}$ *and* $\{i^n(s_t), x^n(s_t), \pi^n(s_t)\ \}$ *such that:*

1. *In periods of adjustment the policy is described by* $\{i^a(\cdot), x^a(\cdot), \pi^a(\cdot)\}$ *which satisfy (18), (19) and (23).*

2. *In periods of central bank's inaction the policy is described by* $\{i^n(\cdot), x^n(\cdot), \pi^n(\cdot)\}$*, where:* $i^n(s_t) = i^a_{t-1}(e_{t-1})$ *and* $x^n(\cdot)$ *and* $\pi^n(\cdot)$ *satisfy (2)-(3).*

3. *Private sector forecasts are consistent with the policy. That is:*

   - *In periods of inaction, forecasts in (2) and (3) are given by*

   $$E_t x_{t+1} = E_t x^a(e_{t+1}) \qquad E_t \pi_{t+1} = E_t \pi^a(e_{t+1})$$

   - *In periods of adjustment forecasts in (23) are given by (16) and (17), while the forecasts in (16) and (17) are given by:*

   $$E_t x_{t+2} = E_t x^a(e_{t+2}) \qquad E_t \pi_{t+2} = E_t \pi^a(e_{t+2})$$

Here we'll describe the solution in general terms[14]. First, start with periods of adjustment. The equilibrium is described by a system of three expectational equations: (18), (19), and (23). To obtain the solution, we can first substitute the interest rate out using one of the

[14]A complete solution is provided in Appendix C.

equations. Second, since the system is linear and because all state variables in adjustment periods are exogenous, we seek a solution of the form:

$$y_t^a = De_t \tag{24}$$

where $y_t = [x_t, \pi_t]'$, $e_t = [g_t, u_t]'$ and superscript ^{a} indicates periods of adjustment. Next, note that since at $t + 2$ the central bank faces exactly the same problem as at time t, rational expectations imply $E_t y_{t+2} = DP^2 e_t$, where P is a diagonal matrix with persistence parameters (μ and ρ). Using this forecasting rule leaves us with a deterministic system of 4 linear equations in the unknown coefficients of D, which is straightforward to solve[15]. Given the solution (24), we can back out the interest rate rule, which is also linear in exogenous states:

$$i_t^a = \Psi e_t \tag{25}$$

In periods when the central bank rests, equilibrium is described (2) and (3) where $i_t^n = i_{t-1}^a = \Psi e_{t-1}$ and $E_t \pi_{t+1}$ and $E_t x_{t+1}$ must be consistent with the adjustment policy, i.e. $E_t y_{t+1} = DPe_t$. The solution of this system takes the form:

$$y_t^n = D_1 e_{t-1} + D_2 e_t \tag{26}$$

where the superscript ^{n} indicates no-adjustment periods.

[15]Note that unlike in the case of pure discretion, the matrix D does not generally have zeros in the first column. This is because with the interest rate being fixed for two periods, it is no longer optimal to neutralize demand shocks at the time of adjustment. Instead, a policy of minimizing the average effects of demand and supply shocks over two periods is preferred.

3.3 Welfare Measures

We use the unconditional expectation of the loss function (1) to measure social loss. In the case of infrequent adjustment welfare costs can be expressed as:

$$E(L^d) = \frac{0.5}{1-\beta}\left(0.5E\left(L^a\right) + 0.5E\left(L^n\right)\right)$$

where $E\left(L^a\right)$ and $E\left(L^n\right)$ represent unconditional expectations of losses in times of adjustment and inaction, respectively and are given by:

$$\begin{aligned} E\left(L^a\right) &= D\Omega D' \\ E(L^n) &= D_1\Omega D_1' + D_2\Omega D_2' + 2D_1\Sigma D_2' \end{aligned} \tag{27}$$

where Ω and Σ are, respectively, covariance and autocovariance matrices of e_t ($\Omega = E(e_t e_t')$ and $\Sigma = (e_{t-1}e_t')$).

3.4 Discreteness vs. Discretion

As was mentioned above, the desirability of a policy of infrequent interventions depends on the size of the gains from short-term commitment relative to losses arising from the inability to respond to exogenous shocks in a timely fashion. In the benchmark model welfare loss under the discrete adjustment policy (0.0045) is smaller than under period-by-period adjustment with discretion (0.0053) [16]. To provide a better intuition behind this result we examine exact numerical solutions. In the benchmark model the equilibrium in periods of intervention is given by (see eq. (24)):

$$\begin{pmatrix} x_t^{da} \\ \pi_t^{da} \end{pmatrix} = \begin{pmatrix} 0.0445 & -23.0357 \\ -0.0009 & 2.8620 \end{pmatrix} \begin{pmatrix} g_t \\ u_t \end{pmatrix} \tag{28}$$

[16] As a reference point, the loss under life-time commitment is 0.0019.

where the superscript ^{da} indicates periods of adjustment under discrete policy.

Under pure discretion (superscript ^{pd}) the variables in all periods evolve according to:

$$\begin{pmatrix} x_t^{pd} \\ \pi_t^{pd} \end{pmatrix} = \begin{pmatrix} 0.0000 & -19.3301 \\ 0.0000 & 3.2217 \end{pmatrix} \begin{pmatrix} g_t \\ u_t \end{pmatrix} \tag{29}$$

The expressions reveal that even though the optimal policy under infrequent adjustment allows inflation and output to react to demand shocks, their contemporaneous effect is small, and more so in the case of inflation. Thus, both inflation and output are mostly driven by the supply shocks. The response of inflation to supply shocks is smaller and the response of output is larger under infrequent adjustment. This is a 'substitution effect' of lower output costs of reducing inflation: the central bank 'buys' more inflation reduction at the expense of output.

Next, consider central bank holidays. To characterize the equilibrium, note that the interest rate adjustment policy implied by (28) takes the form[17]:

$$i_t^{da} = \begin{pmatrix} 1.3368 & 8.2772 \end{pmatrix} \begin{pmatrix} g_t \\ u_t \end{pmatrix} \tag{30}$$

Inflation and output are described by (21) and (22) where next period expectations must be consistent with (28). Combining these equations yields equilibrium in periods of inaction (superscript ^{dn}):

$$\begin{pmatrix} x_t^{dn} \\ \pi_t^{dn} \end{pmatrix} = \begin{pmatrix} -0.8912 & -5.5181 \\ -0.0160 & -0.0993 \end{pmatrix} \begin{pmatrix} g_{t-1} \\ u_{t-1} \end{pmatrix} + \begin{pmatrix} 1.0351 & -16.9022 \\ 0.0179 & 2.9785 \end{pmatrix} \begin{pmatrix} g_t \\ u_t \end{pmatrix} \tag{31}$$

Although target variables in periods of inaction are functions of both current and past shocks, in most cases past shocks enter with the opposing sign thus reducing the impact of

[17]The interest rate rule can be obtain from IS equation (18) where x_t and π_t are given by (28) and $E_t x_{t+2} = \mu^2 D_{11} g_t + \rho^2 D_{12} u_t$ and $E_t \pi_{t+2} = \mu^2 D_{21} g_t + \rho^2 D_{22} u_t$.

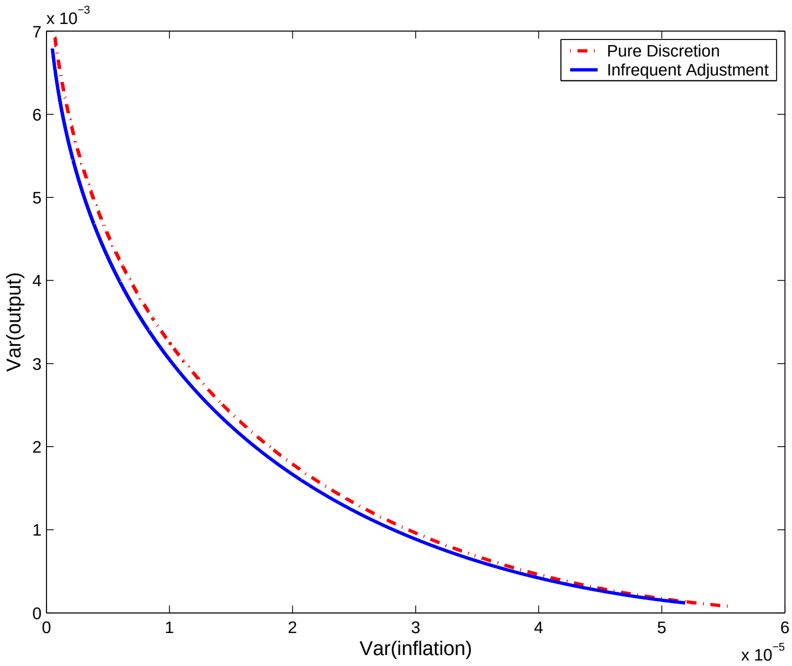
current shocks. The magnitude of this offsetting effect is large when shocks are persistent. Intuitively, when persistence is high future shocks have a large predetermined component. Since the central bank at the time of adjustment seeks to balance current and future expected targets, the interest rate reacts to both contemporaneous shocks and to the predetermined component of the next period's shocks. Thus, the larger the predetermined component, the more effective is the policy makers' ability to neutralize shocks in two periods. An interesting finding implied by the solution is that despite infrequent adjustment, inflation is less responsive to supply shocks (and less volatile overall) in all periods, not only in periods of intervention. This is due to the 'spillover' effect of short-term commitments: since agents expect a more aggressive response to inflationary pressures at the next policy meeting, the effects of exogenous shocks on inflation are not expected to persist for too long. Lower inflationary expectations, in turn, contain inflation itself.

The solution further reveals that the impact of demand shocks on inflation continues to be small in periods of inaction: although the coefficient on g_t is larger than in (28), a negative coefficient on g_{t-1} together with high persistence in g_t imply a smaller overall effect. The effect on output is also small, although larger than in the case of inflation. On the other hand, average impact of supply shocks on output for the most part is expected to be larger than under pure discretion (because current and past shocks enter with the same sign).

Overall we conclude that the infrequent adjustment policy generates less volatile inflation and more volatile output in all periods, thus reducing the stabilization bias of the discretionary policy. Table 1 summarizes standard deviations of inflation and output across periods and between policies in the baseline specification. Finally, Figure 2 illustrates gains in inflation-output variability tradeoff by plotting the efficient policy frontier for the two alternatives. The frontier is constructed by measuring unconditional variances of inflation and output for various values of α[18]. The figure shows that under the simple policy of infrequent

[18] The output weight (α) was changed by picking different values of demand elasticity (q) in equation (27), so as to keep λ unchanged. In the case of infrequent adjustment policy the depicted frontier corresponds to average variances across periods of action and inaction

Figure 2: Efficient Policy Frontier Under the Two Policies



interventions the central bank faces a more favorable choice between inflation and output variability.

Table 1: Unconditional Volatility of Target Variables

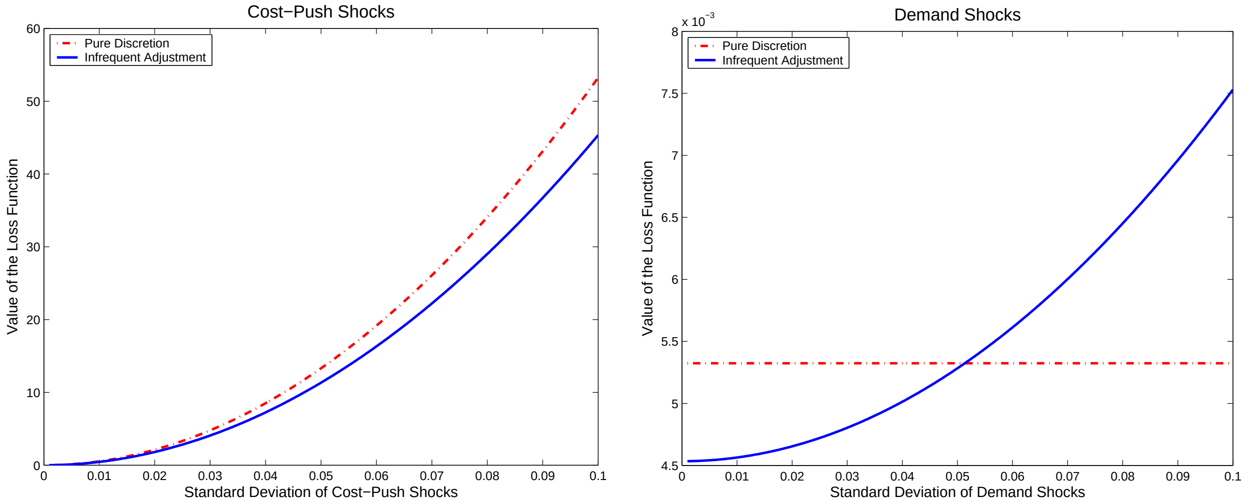
Policy (Periods)	$St.Dev.(x)$	$St.Dev.(\pi)$
Infrequent adjustment (work)	0.0383	0.00477
Infrequent adjustment (holiday)	0.0359	0.00483
Infrequent adjustment, average	0.0371	0.00480
Pure Discretion	0.0323	0.00537

To this end we have established that in the benchmark model the policy of infrequent adjustments is preferable to pure discretion. Below we show that this conclusion holds for a wide range of plausible parameter values.

3.4.1 Volatility of Exogenous Shocks

Figure 3 plots social losses for various standard deviations of demand and cost-push innovations ($\sigma_{\hat{g}}$ and $\sigma_{\hat{u}}$). The left panel suggests that infrequent adjustment is more preferable

Figure 3: Comparison of Policies: Role of Shock Volatility



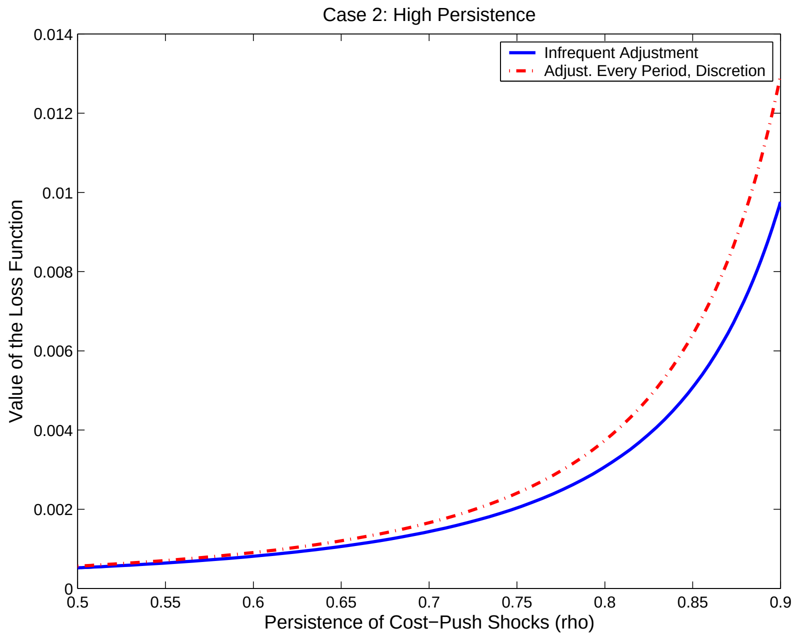
when cost-push innovations are more volatile. This is because more volatile supply shocks call for greater attention to inflation stabilization. The latter (in light of the previous discussion) is easier to achieve when the central has periodic holidays. The right panel tells us that the volatility of demand shocks works against the case for more holidays. However, since the impact of demand shocks on target variables is small, in the benchmark model $\sigma_{\hat{g}}$ must be 50 times larger than $\sigma_{\hat{u}}$ for the central bank to choose pure discretion over periodic holidays.

3.4.2 Persistence of Supply Shocks

As is evident from Figure 4, the desirability of infrequent interventions is increasing in the persistence of the cost-push shocks[19]. Intuitively, the more persistent they are, the greater is the pre-determined component of the conditional short-term forecasts of inflation and output, which, in turn, raises the importance of the central bank's ability to affect those forecasts. A straightforward way to see this is to examine the simple commitment rules of section 2. Note that k under commitment (see eq. (12)) reduces to unity when $\rho = 0$, and is larger when it is large. Hence, gains from commitment are increasing in ρ. In the

[19]Persistence of g_t was not found to affect the ranking of the policies.

Figure 4: Comparison of Policies: Role of Persistence in Supply Shocks



benchmark model with infrequent interventions, when ρ is roughly less than 0.17, benefits from commitment are not sufficient to outweigh the effects of demand shocks. However, this threshold is far below what is empirically plausible.

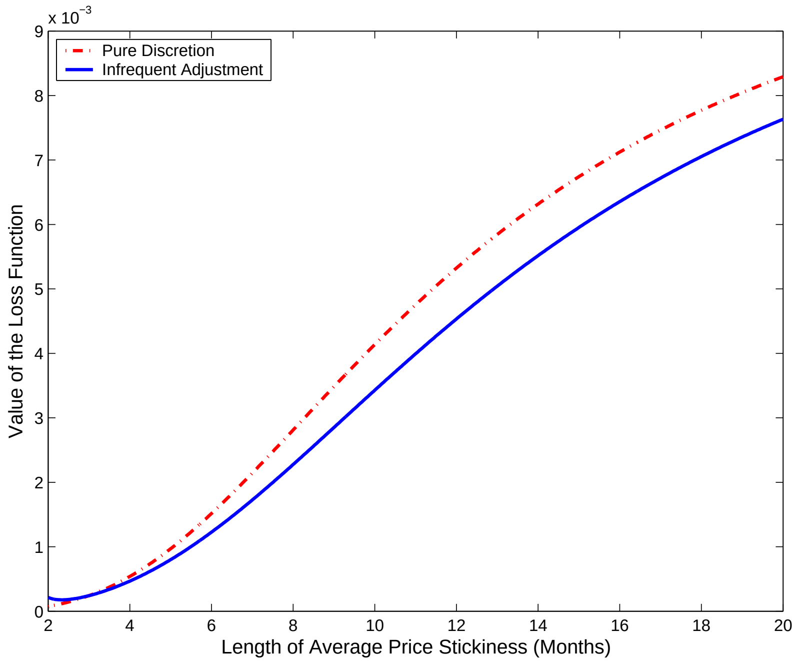
3.4.3 Length of Price Stickiness

Duration of average price stickiness in the Calvo model is determined by the firms' probability of not adjusting prices $\left(duration = \frac{1}{1-\theta}\right)$. Thus, longer price stickiness decreases λ (see eq. 6), resulting in i) smaller sensitivity of inflation to output fluctuations (hence a smaller effect of demand shocks on inflation); and ii) less weight of output in the social loss function (see eq. 7). In light of the discussion above, both effects work to increase the desirability of the discrete adjustment policy (see Figure 5).

3.4.4 Labor Supply Elasticity

Perhaps the most controversial parameter in macroeconomics is the elasticity of labor supply, $\frac{1}{\vartheta}$. Most empirical estimates based on micro level data suggest values of elasticity in the

Figure 5: Comparison of Policies: Role of Price Stickiness



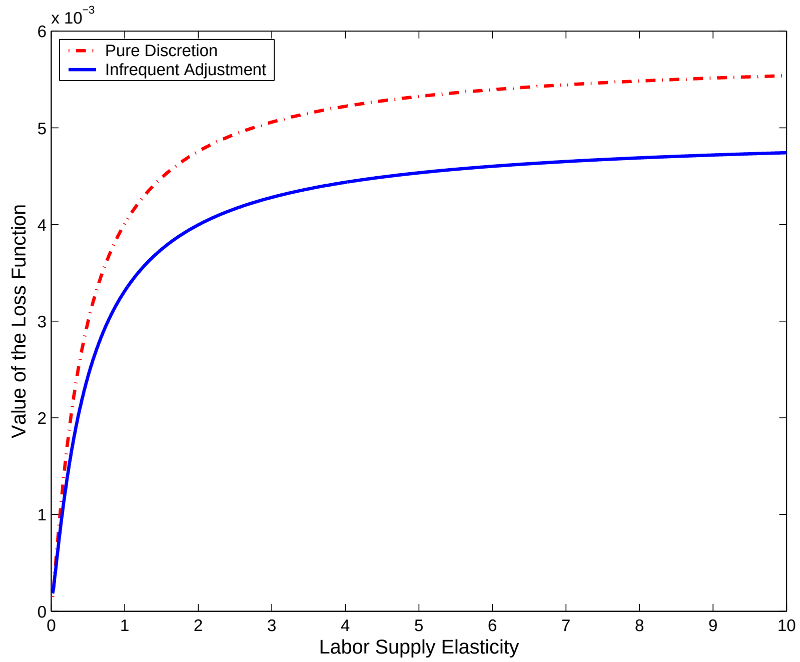
range of near zero to 0.5 (See, for example, Altonji (1986) or Domeij and Floden (2002)). On the other hand, most of the macro studies suggest quite the opposite (e.g. Woodford and Rotemberg (1992)). Prescott (2003) argues that a highly elastic labor supply is more plausible to account for cross-country variations in labor effort[20]. In the context of our model, when labor is inelastic (high ϑ), firms' marginal cost schedules are very steep, raising the impact of exogenous shocks on inflation and making output stability a priority. The opposite is true when the labor supply is highly elastic. In our baseline model infrequent adjustment is preferred when the elasticity exceeds roughly 0.03 (see Figure 6), which is plausible from both camps' perspectives.

4 Optimal Frequency of Policy Meetings

Next, we seek to characterize the optimal frequency of adjustment. This is done in two steps. First we develop a solution to a more general model where the central bank revises

[20]In a more general setting other phenomena, such as sticky or efficiency wages, can also affect output sensitivity w.r.t. inflation. A low ϑ may be partly a metaphor for them.

Figure 6: Comparison of Policies: Role of Labor Supply Elasticity

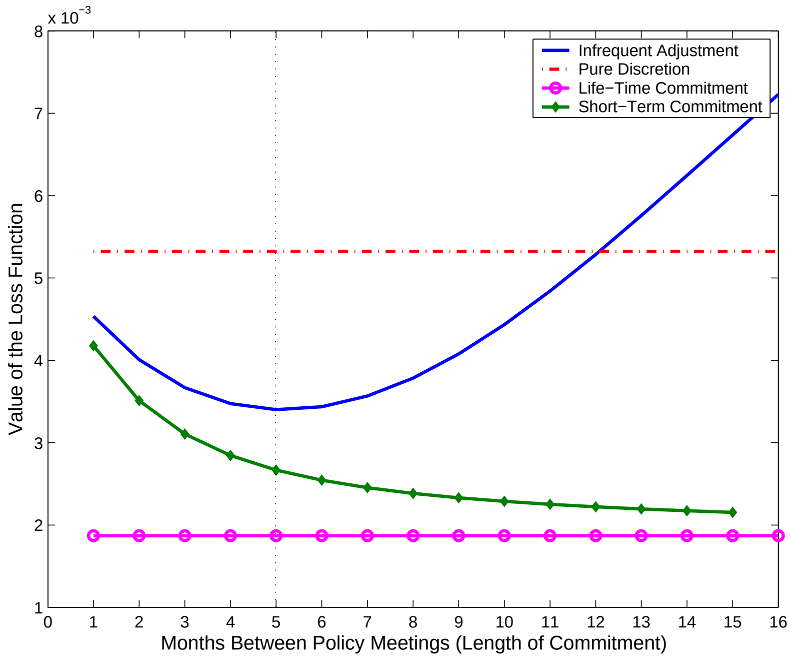


the interest rate every $T+1$ periods. Then we choose the optimal T that minimizes expected social loss. A complete solution to this model is presented in Appendix C .

4.1 Baseline Model

Figure 7 plots values of the social loss function for various T in the benchmark model. The optimal frequency of policy meetings is once every six months ($T = 5$) or twice a year. With the optimal frequency of adjustment the central bank can realize over 50 percent of the gains obtainable under life-time commitment. To further illustrate the gains from moving to the optimal frequency of adjustment, Figure 8 presents simulated series under the optimal frequency of policy meetings. It shows that choosing the frequency optimally can achieve sizable gains in stabilizing inflation. Another observation is that the interest rate is more stable. This is interesting in light of the recent research on interest rate smoothing. Interest rate stability in this model stems in part from lengthier duration of interest rate fixity and in part from the central bank's ability to affect longer-term forecasts. The latter is related to the discussion in Woodford (1999): when monetary authority can affect longer term forecasts,

Figure 7: Social Losses and Length of Commitment



it takes a smaller change in the interest rate to achieve the desired effect on output and inflation. Note, however, that smoothing in this model is different from what is implied by Woodford's analysis. He examines the case of life-time commitment and therefore calls for frequent, but very small adjustment of the interest rate. The main message in Figure 8 is that when explicit long-term commitment is not feasible, smoothing occurs as a result of infrequent but somewhat larger (although not too large) changes in the interest rate.

Finally, Figure 9 explores how the optimal time between policy meetings varies with model parameters. Its interpretation is directly related to the discussion in section 3. Factors that decrease λ (see eq. 6) lower the importance of demand shocks and output stabilization. Thus they raise the benefits of infrequent adjustment and imply longer optimal duration of inaction. More persistent supply shocks raise the importance of the predetermined component of long-term forecasts and also increase the optimal duration between the meetings. Interestingly, with higher persistence the optimal length of inaction appears to rise exponentially. Finally, higher volatility of demand shocks relative to supply shocks raises expected losses from inaction and works to increase the desirable frequency of monetary policy meetings.

Figure 8: Model Simulation Under Optimal Frequency of Adjustment

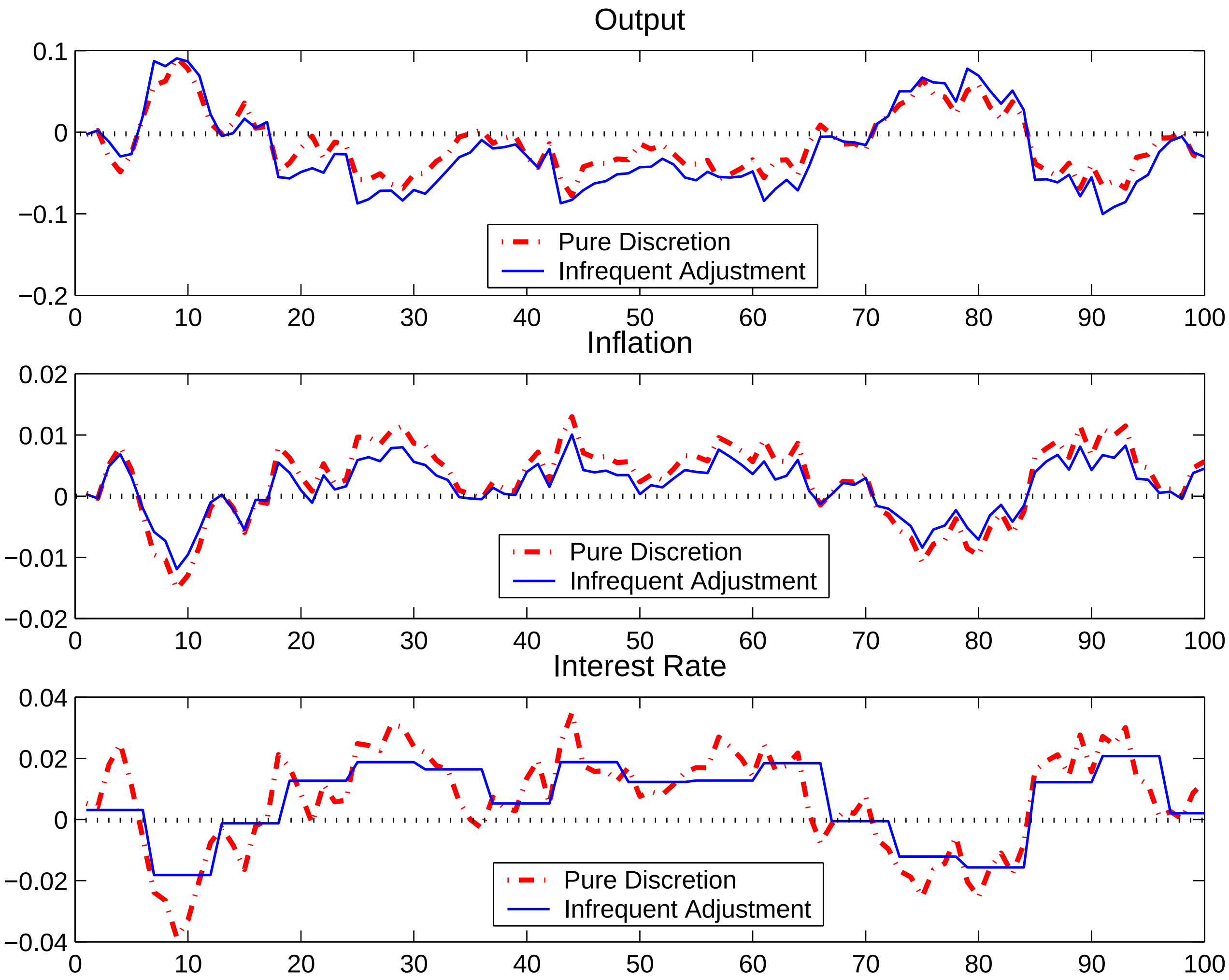
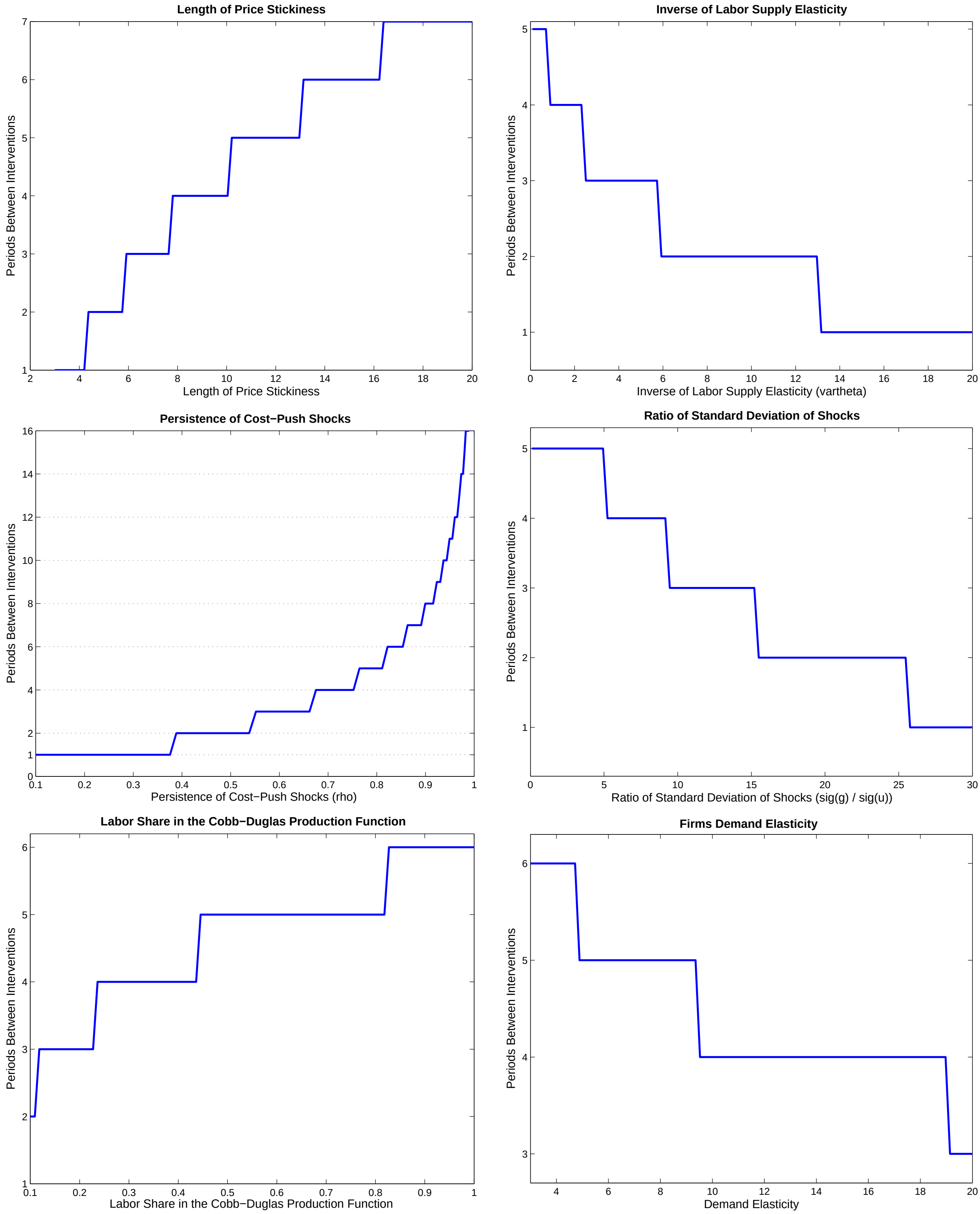


Figure 9: Optimal Frequency of Adjustment and Parameter Values



4.2 More Holidays for the FOMC?

To tailor our analysis to the case of the U.S. we consider an alternative calibration of some model parameters, the most important of which are those describing stochastic processes of exogenous shocks. We interpret cost-push shocks as exogenous markup variations arising from labor market imperfections. As in Clarida, Gali and Gertler (2001) we define the wage markup as the wedge between between the consumers' marginal disutility of labor and their marginal return from labor[21]. The generalized wage markup can be expressed as:

$$\mu_t^w = -\frac{U_{C_t}}{U_{L_t}} w_t \tag{32}$$

where $-\frac{U_{C_t}}{U_{L_t}}$ is the inverse of the marginal rate of substitution between consumption and labor and w_t is the real wage rate. Assuming a period utility function of the form:

$$U(C_t, L_t) = \frac{C_t^{1-\frac{1}{\varphi}}}{1-\frac{1}{\varphi}} - \frac{L_t^{1+\vartheta}}{1+\vartheta}$$

we have:

$$\ln(\mu_t^w) = -\frac{1}{\varphi}\ln(C_t) - \vartheta \ln(L_t) + \ln(w_t) \tag{33}$$

We construct the $\ln(\mu_t^w)$ series from the previous equation using benchmark parameter values and quarterly U.S. data covering 1947:1-2004:2[22] . The cost push shock u_t is taken to be the Hodrick-Prescott (HP) filtered wage markup series. The AR(1) fit of the resulting u_t is:

$$u_t = \underset{(0.0008)}{-0.00003} + \underset{(0.041)}{0.78}\, u_{t-1} + \hat{u}_t \tag{34}$$

[21] See also Gali et. al. (2001)

[22] The data series are: C_t - real personal consumption expenditures (from BEA), L_t - hours in the nonfarm business sector (from BLS), w_t - real compensation per hour in the nonfarm business sector (BLS). Consumption and labor series were transformed into per capita levels using a measure of population obtained from GDP and GDP per capita series (BEA).

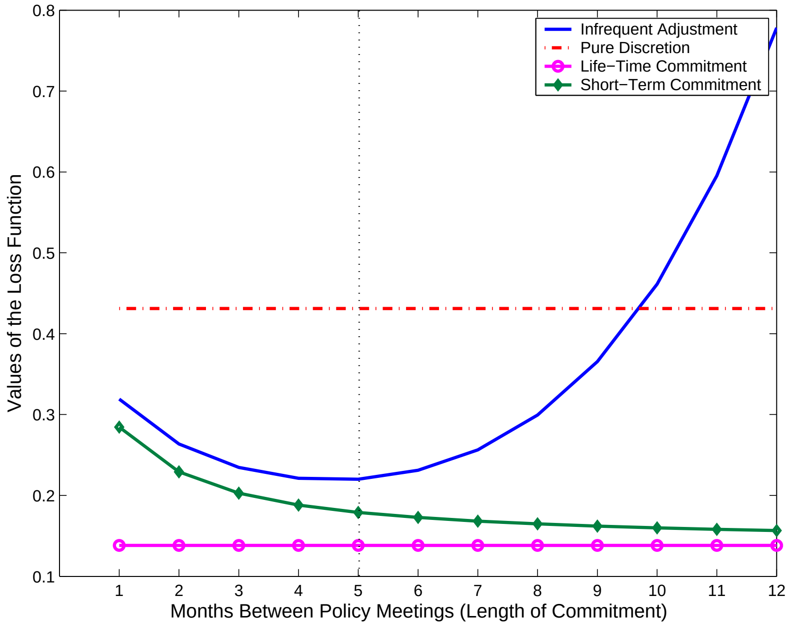
where the numbers in parenthesis are standard errors, and the standard deviation of the innovation ($\sigma_{\hat{u}}$) is 0.0125. Ignoring the intercept term, and translating to monthly frequency, the persistence parameter and the standard deviation of innovations are approximately 0.92 and 0.0072 respectively.

The demand shocks g_t were constructed using the deviations of the share of government spending in GDP[23] from the HP trend. The fitted process at the quarterly frequency is:

$$g_t = \underset{(0.0016)}{-0.0004} + \underset{(0.044)}{0.74}\, g_{t-1} + \hat{g}_t \tag{35}$$

with $\sigma_{\hat{g}} = 0.0243$. Similarly, approximate values of shock persistence and standard devia-

Figure 10: Social Loss and Length of Commitment: Alternative Calibration



tion of innovations are 0.9 and 0.0140 at the monthly frequency. Finally, we consider the possibility of lower duration of average price stickiness. At the lower end we take the average estimate in Gali et. al. (2001) of 2.35 quarters, or 7 months, and at the high end we take the commonly used value of 12 months. First, consider the lower-end value. Figure 10 plots loss

[23]The exact measure uses deviations of $gg_t = -\ln\left(1 - \frac{G_t}{Y_t}\right)$ from the HP trend. Here $G-$ government current expenditures, and Y - nominal GDP (both series taken from the BEA).

functions under the four policies. It reveals that under alternative calibration the optimal choice of the frequency of policy meetings (once every six months) the central bank can gain about 75 percent of the total gains available under life-time commitment.

As an alternative, we also quantify these gains using the "inflation equivalent" measure: a permanent deviation of inflation from its target that generates the same welfare loss as a move from commitment to discretion. Dennis and Söderström (2002) use this measure to quantify gains from commitment in New Keynesian models. As they point out, the correspondence between "inflation equivalent" and the percentage reduction in the value of the loss function is not one-to-one, making this a useful alternative. The measure can be expressed as follows:

$$\pi = \sqrt{(1-\beta)\left(L^{discretion} - L^{alternative}\right)}$$

Under our calibration welfare gains from life-time commitment are equivalent to a permanent reduction in the deviation of inflation from its target by 2.96% [24].On the other hand, a move from pure discretion to the optimal frequency of policy meetings is equivalent to a permanent reduction by 2.52%, generating 85% of the total possible gains measured using the "inflation equivalent".

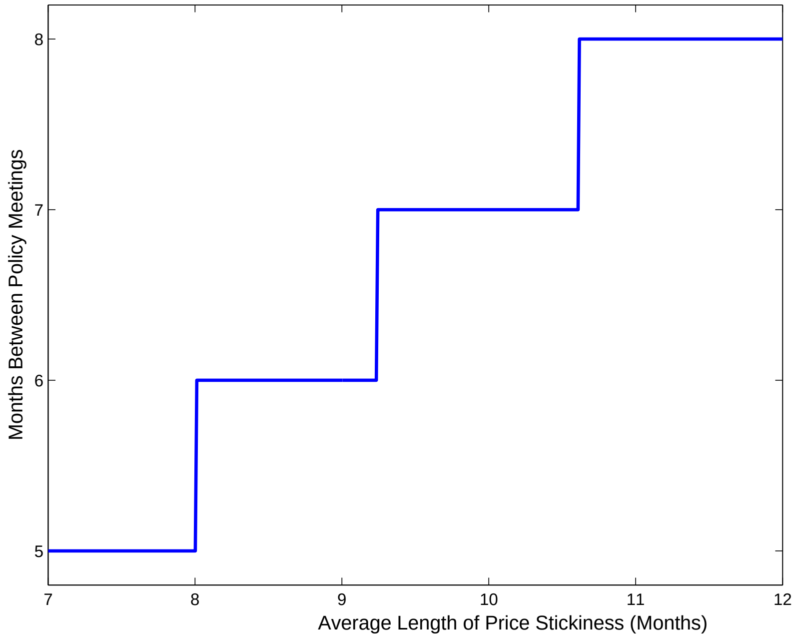
Finally, the optimal duration between FOMC meetings under the alternative calibration and for various durations of price stickiness is presented in Figure 10. The result suggests that the FOMC should allow at least five months of no adjustment or, put differently, it should meet no more than twice a year.

4.3 Directions for Future Research

Considerations of clarity and simplicity led us to conduct the analysis above within a very simple and a purely forward looking model. This leaves a number of interesting extensions for future research. First, in light of Fuhrer (1997), Mankiw and Reis (2002) and many others it

[24] This is consistent with Dennis and Söderström (2002). In their models/calibrations life-time commitment reduces inflation by 0.05% to 3.6%

Figure 11: Optimal Duration Between FOMC Meetings: Alternative Calibration



would be interesting to examine the optimal frequency of policy decisions in an environment where some agents are either backward-looking or do not update their information set. Second, an analysis of the effects of model uncertainty on the optimal frequency of policy meetings would certainly increase our understanding of the proper policy design. Thirdly, central banks around the world use targets at different horizons. The exact horizon of the target is likely to affect the optimal frequency of policy meetings. Fourth, our analysis could be extended by explicitly introducing emergency/unscheduled meetings. Their presence is likely to lower inflationary expectations, requiring fewer scheduled meetings. Fifth, many central banks, and the Federal Reserve in particular, often use additional tools in affecting private sector expectations, such as FOMC bias announcements[25]. Private sector and the media perceive bias announcements as indications of future policy changes. The latter clearly gives the Fed an extra leverage in influencing private sector forecasts.

The analysis of the optimal frequency of policy decisions from the standpoint of commit-

[25]Conley, Dupor and Mirzoev (2004) discuss the usefulness of bias announcements in the estimation of monetary policy rules.

ment could also be applied in other areas of macroeconomics, most importantly the fiscal policy. Klein and Rios-Rull (2003) show that partial commitment has important implications for the optimal fiscal policy. Our analysis could be applied to their problem to study the optimal length of commitment. More generally, examining the optimal frequency of adjusting various income, trade and other taxes could be of great interest to macroeconomists.

5 Conclusion

In this paper we have examined the issue of optimal frequency of monetary policy meetings. Viewing infrequent adjustment of monetary policy as simple short-term sequential commitments, we showed that it is preferred to period by period adjustment under discretion. Crucial in our argument is the finding that benefits from commitment spread to periods of central bank's inaction. This happens because expectations of aggressive inflation stabilization at the time of policy adjustment contain inflation and mute effects of exogenous shocks in times when the central bank is on holiday. In addition we have provided a solution for the optimal frequency of policy meetings. Under a sensible calibration describing the U.S. economy the model prescribes holding FOMC meetings twice a year or less.

References

[1] Altonji, Joseph G., 1986, Intertemporal Substitution in Labor Supply: Evidence From Micro Data, Journal of Political Economy, 94(2), S176-215

[2] Barro, Robert J. and David B. Gordon, 1983, A Positive Theory of Monetary Policy in a Natural Rate Model, Journal of Political Economy, 91:4, pp. 589-610.

[3] Calvo, Guillermo, 1983, Staggered Prices in a Utility Maximizing Framework, Journal of Monetary Economics, 12:3, pp. 383-98.

[4] Clarida, Richard, Jordi Galí, and Mark Gertler, 1999, The Science of Monetary Policy: A New Keynesian Perspective, Journal of Economic Literature, 37, pp. 1661-1707

[5] Clarida, Richard, Jordi Galí, and Mark Gertler, 2001, Optimal Monetary Policy in Open vs. Closed Economies: An Integrated Approach, American Economic Review, vol. 91, no.2, pp. 248-252.

[6] Conley, Timothy, Bill Dupor, and Tokhir Mirzoev, 2004, Improving Monetary Policy Estimates, mimeo, Ohio State University.

[7] Dennis, Richard, The Policy Preferences of the US Federal Reserve, 2001, Working Paper 2001-08, Federal Reserve Bank of San Fransisco.

[8] Dennis, Richard and Ulf Söderström, How Important Is Precommitment for Monetary Policy?, 2002, Sveriges Riksbank Working Paper No. 139, September 2002.

[9] Domeij, David, and Martin Floden, 2002, Labor Supply Elasticity And Borrowing Constraints: Why Estimates Are Biased, mimeo, Stockholm School of Economics.

[10] Favero, Carlo A. and Riccardo Rovelli, "Macroeconomic stability and the preferences of the Fed. A formal analysis, 1961-98", forthcoming, Journal of Money, Credit and Banking.

[11] Fuhrer, Jeffrey, 1997, The (Un)importance of Forward-Looking Behavior in Price Specifications, Journal of Money, Credit and Banking, 29(3), pp. 338-350.

[12] Galí, Jordi, David López-Salido and Javier Vallés, Understanding the Effects of Government Spending on Consumption, 2003, mimeo, Universitat Pompeu Fabra.

[13] Galí, Jordi, Mark Gertler and David López-Salido, 2001, European Inflation Dynamics, NBER Working Paper 8218

[14] Ireland, Peter N., 2002, Technology Shocks in the New Keynesian Model, Boston College, Economics Department Working Paper 536.

[15] Klein, Paul and José-Víctor Ríos-Rull, 2003, Time-Consistent Optimal Fiscal Policy, International Economic Review 44(4), November 2003, pp. 1217-1245.

[16] Kydland, Finn E. and Edward C. Prescott, 1977, Rules Rather Than Discretion: The Inconsistency of Optimal Plans, Journal of Political Economy, 85, pp. 473-91.

[17] Kara, Hakan, 2003, Optimal Monetary Policy, Commitment and Imperfect Credibility, mimeo, New York University.

[18] Mahadeva, Lavan, and Gabriel Sterne, eds., 2000, Monetary Policy Frameworks In a Global Context, Bank of England.

[19] Mankiw, Gregory N. and Ricardo Reis, 2002, Sticky Information Versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve, Quarterly Journal of Economics, vol. 117 (4), pp. 1295-1328.

[20] Miranda, Mario J. and Paul L. Fackler, 2002, Applied Computational Economics and Finance, MIT Press.

[21] Prescott, Edward, C., 2003, Why Do Americans Work So Much More Than Europeans?, Federal Reserve Bank of Minneapolis, Staff Report, 321.

[22] Rotemberg, Julio and Michael Woodford, 1992, Oligopolistic Pricing and The Effects of Aggregate Demand on Economic Activity, Journal of Political Economy, 100, 1153-1297.

[23] Rotemberg, Julio and Michael Woodford, 1997, Interest Rate Rules in an Estimated Sticky Price Model, in Monetart Policy Rules, John Taylor, ed.

[24] Schaumburg, Ernst, and Andrea Tambalotti, 2002, An Investigation of the Gains from Commitment in Monetary Policy, mimeo, Northwestern University.

[25] Söderstrom, Ulf, Paul Söderling and Anders Vredin, 2002, Can A Calibrated New Keynesian Model of Monetary Policy Fit the Facts?, Sveriges Ricksbank, Working Paper, 140.

[26] Woodford, Michael, 1999, Optimal Monetary Policy Inertia, NBER working paper no. 7261

[27] Woodford, Michael, 2003, Interest and Prices: Foundations of a Theory of Monetary Policy, Princeton University Press.

A Equilibrium in a Simple Model of One-Period Sequential Commitments

Here we describe a solution to a model where the central bank announces commitment to a new policy every other period. Assuming that the interest rate policy also neutralizes demand shocks ($i_t = \gamma u_t + \frac{1}{\varphi} g_t$.) implies that the IS equation at the time of announcement is:

$$x_t = -\varphi(\gamma u_t - E_t \pi_{t+1}) + E_t x_{t+1} \tag{36}$$

Since the rule is valid in the next period, rational expectation of the next period's output is:

$$E_t x_{t+1} = -\varphi\gamma\rho u_t + \varphi E_t \pi_{t+2} + E_t x_{t+2} \equiv -\varphi\gamma\rho u_t + f_{2t}$$

where the two period ahead forecasts (summarized in f_{2t}) are taken by the CB as given. Similarly, next period's inflation forecast is:

$$E_t \pi_{t+1} = \lambda E_t x_{t+1} + \beta E_t \pi_{t+2} + \rho u_t \equiv -\lambda\varphi\gamma\rho u_t + f_{3t}$$

where $f_{3t} = \beta E_t \pi_{t+2} + \lambda f_{2t} + \rho u_t$. The last two expressions can be combined with the IS curve to obtain output as a function of exogenous shocks and long-term forecasts. Using the expression for output to modify the Phillips curve yields:

$$\pi_t = \lambda \left(1 + \frac{\beta\rho}{1 + \varphi\lambda\rho + \rho}\right) x_t + f_{4t} + u_t \tag{37}$$

where f_{4t} summarizes two period ahead forecasts. The extra output term appearing in (37) represents the effect of short-term commitment on next period's inflation forecast (compare to equation 11). The output cost of reducing inflation falls by a factor of $k_2 = \left(1 + \frac{\beta\rho}{1+\varphi\lambda\rho+\rho}\right)^{-1}$ relative to pure discretion.

At the time of announcement, the central bank's problem is to minimize (9) with $T = 1$ subject to (37). The first order condition (also given by 10) can be expressed as:

$$\pi_t^{1c} = -\frac{\alpha k_2}{\lambda} x_t^{1c} \tag{38}$$

Note that $k_1 < k_2 < 1$, i.e. under short-term commitment output cost of reducing inflation is higher than under long-term commitments but lower than under pure discretion, as hypothesized earlier.

The minimum aggregate state vector in this model is the vector of exogenous shocks $e_t = \{g_t, u_t\}$. The restriction that the interest rate policy must neutralize the demand shocks implies that inflation and output must be functions of supply shocks only. With these in mind, we use the following definition of a rational expectations equilibrium.

Definition 2 *A rational expectations equilibrium is described by policy functions* $i_t(e_t), x_t(u_t)$, *and* $\pi_t(u_t)$ *such that:*

1. *Equations (2) and (3) are satisfied in all periods.*
2. *In periods of policy announcement, policies* x *and* π *must also satisfy (38).*

3. In periods when no announcement is made, inflation and output are given by[26]*:*

$$x_t(u_t) = x_{t-1}(u_t), \qquad \pi_t(u_t) = \pi_{t-1}(u_t) \qquad i_t(e_t) = i_{t-1}(e_t).$$

4. Private sector forecasts in (2) and (3) are consistent with the central bank's policy:

$$E_t x_{t+1} = E_t x_{t+1}(u_{t+1}), \qquad E_t \pi_{t+1} = E_t \pi_{t+1}(u_{t+1}).$$

Given the linear-quadratic structure of the problem, the last condition requires that in announcement periods expected inflation and output are given by:

$$E_t \pi_{t+1} = \rho \pi_t \quad \text{and} \quad E_t x_{t+1} = \rho x_t. \tag{39}$$

Moreover, since the central bank solves the same problem every announcement period it will always announce the same rule. Hence in equilibrium (38) and (39) must hold in all periods and policy rules $i_t(\cdot), x_t(\cdot)$, and $\pi_t(\cdot)$ are time-invariant. The solution is obtained by combining (38), (39) and (3):

$$\begin{aligned} x_t^{1c} &= -\tfrac{\lambda}{\alpha k_2(1-\beta\rho)+\lambda^2} u_t \\ \pi_t^{1c} &= \tfrac{\alpha k_2}{\alpha k_2(1-\beta\rho)+\lambda^2} u_t \end{aligned} \tag{40}$$

[26]This is equivalent to saying that the pre-announced policy is valid in non-announcement periods.

B Unconstrained Optimum Under Limited Commitment

Here we describe the unconstrained optimum under long- and short-term commitment. The case of life-time commitment has been examined in CGG99. Recall that in that case the central bank chooses inflation and output to maximize the following Lagrangean:

$$\begin{array}{l} \Im_1 = -\frac{1}{2} E_t \left\{ \sum\limits_{i=0}^{\infty} \beta^i \left[\alpha x_{t+i}^2 + \pi_{t+i}^2 + \right.\right. \\ \left.\left. \gamma_{t+i} \left(\pi_{t+i} - \lambda x_{t+i} - \beta E_t \pi_{t+1+i} - u_{t+i} \right) \right] \right\} \end{array} \tag{41}$$

The optimal policy (see CGG99) is described by:

$$\begin{array}{l} x_{t+i} - x_{t+i-1} = -\frac{\lambda}{\alpha} \pi_{t+i}, \qquad i = 1, 2, 3..... \\ \text{and} \quad x_{t+i} = -\frac{\lambda}{\alpha} \pi_{t+i}, \qquad i = 0 \end{array} \tag{42}$$

Next we turn to the case of partial commitment.

B.1 Partial (Short-Term) Commitment

Suppose that at time t the central bank announces a rule that is valid until $t + T$. Before analyzing this case, note that the Phillips curve (3) can be re-written as:

$$\pi_t = E_t \sum_{i=0}^{\infty} \beta^i \left(\lambda x_{t+i} + u_{t+i} \right)$$

Since the commitment is set to expire at a finite future date, the central can only manipulate private sector forecasts until the expiration date of the announced policy rule. Hence, the sequence of constraints from the date of the announcement until $t + T$ can be presented as:

$$\pi_t = E_t \sum_{i=0}^{T} \beta^i \left(\lambda x_{t+i} + u_{t+i} \right) + \beta^{T+1} E_t \pi_{t+T+1}$$

....

$$\pi_{t+j} = E_{t+j} \sum_{i=j}^{T} \beta^{i-j} \left(\lambda x_{t+i} + u_{t+i} \right) + \beta^{T+1-j} E_{t+j} \pi_{t+T+1}$$

....

$$\pi_{t+T} = \lambda x_{t+T} + u_{t+T} + \beta E_{t+T} \pi_{t+T+1}$$

where all forecasts of π_{t+T+1} are taken by the central bank as given. The policy for inflation and output is chosen by maximizing the following Lagrangean:

$$
\begin{aligned}
&\Im_2 = -\tfrac{1}{2}E_t\Bigg\{\left(\sum_{i=0}^{T}\beta^i\left[\alpha x_{t+i}^2+\pi_{t+i}^2\right]+F_{1t}\right)+\\
&\gamma_t\left(-\pi_t+\sum_{i=0}^{T}\beta^i\left(\lambda x_{t+i}+u_{t+i}\right)+\beta^{T+1}F_{2t}\right)+\\
&+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+\\
&+\gamma_{t+j}\beta^j\left(-\pi_{t+j}+\sum_{i=j}^{T}\beta^{i-j}\left(\lambda x_{t+i}+u_{t+i}\right)+\beta^{T+1-j}F_{2t}\right)+\\
&+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+\\
&+\gamma_{t+T}\beta^T\left(-\pi_{t+T}+\lambda x_{t+T}+u_{t+T}+\beta F_{2t}\right)\Bigg\}.
\end{aligned}
\tag{43}
$$

where F_{1t} summarizes expected losses beyond $t+T$ and $F_{2t}=E_t\pi_{t+T+1}$. The first order conditions are given by:

for $i=0$:

$$
\begin{aligned}
\alpha x_{t+i}+\lambda\gamma_{t+i}&=0\\
\pi_{t+i}-\gamma_{t+i}&=0
\end{aligned}
\tag{44}
$$

for $T\geq i>0$:

$$
\begin{aligned}
&\alpha x_{t+i}+\lambda\left(\gamma_t+\gamma_{t+1}+\ldots+\gamma_{t+i}\right)=0\\
&\pi_{t+i}-\gamma_{t+i}=0
\end{aligned}
\tag{45}
$$

Re-arranging, yields the solution of the same form as under full commitment:

$$x_{t+i}=-\frac{\lambda}{\alpha}\pi_{t+i},\qquad i=0 \tag{46}$$

$$x_{t+i}-x_{t+i-1}=-\frac{\lambda}{\alpha}\pi_{t+i},\qquad i=1,2,3.....T \tag{47}$$

The only difference is that the commitment expires at time $t+T$, at which point the central bank re-optimizes and announces a new commitment. Since the structure of the problem faced by the "new" central bank at time $t+T+1$ is the same as at time t, it will announce the same policy. We use the following notion of equilibrium.

Definition 3 *A stationary limited commitment economic equilibrium is described by a set of policy rules* $\{i_j\left(s_t^j\right),x_j\left(s_t^j\right),\pi_j\left(s_t^j\right)\}_{j=0}^{T}$, *where j indicates position of period t within the commitment cycle ($0-$time of announcement, $T-$ commitment expiration date) and s_t^j summarizes the vector of states at time t relevant at j'th stage of the commitment cycle*[27]*, such that:*

1. *Equations (2) and (3) always hold.*
2. *In periods of announcement the policy also satisfies (46).*
3. *In intermediate periods the policy also satisfies (47).*

[27] As will become clear below for $j=0$, $s_t^j=u_t$, while for $j>1$, $s_t^j=\{s_t^{j-1},u_t\}$.

4. Private sector forecasts are consistent with the policy. That is:

- *In periods of announcement, forecasts in (2) and (3) are given by*

$$E_t x_{t+1} = E_t x_1(s^1_{t+1}) \qquad E_t \pi_{t+1} = E_t \pi_1(s^1_{t+1})$$

- *In intermediate periods for $j = 1, ..., T-1$, the forecasts in (2) and (3) are given by:*

$$E_{t+j} x_{t+j+1} = E_{t+j} x_{j+1}(s^{j+1}_{t+j+1}) \qquad E_{t+j} \pi_{t+j+1} = E_{t+j} \pi_{j+1}(s^{j+1}_{t+j+1})$$

- *In the period of policy expiration, the forecasts in (2) and (3) are given by:*

$$E_t x_{t+1} = E_t x_0(s^0_{t+1}) \qquad E_t \pi_{t+1} = E_t \pi_0(s^0_{t+1})$$

The cyclical nature of commitments implies different processes describing output and inflation in three types of periods: periods of policy announcement, periods of policy expiration and intermediate periods. In times of policy announcement, the following 3 equations describe the evolution of inflation, contemporaneous policy and the expected policy respectively:

$$\pi_t = \lambda x_t + \beta E_t \pi_{t+1} + u_t; \qquad x_t = -\frac{\lambda}{\alpha}\pi_t; \qquad E_t \pi_{t+1} = -\frac{\alpha}{\lambda} x_{t+1} + \frac{\alpha}{\lambda} x_t$$

Combining the three, we obtain the following difference equation in output:

$$x_t = \beta \frac{\alpha}{\alpha(1+\beta)+\lambda^2} E_t x_{t+1} - \frac{\lambda}{\alpha(1+\beta)+\lambda^2} u_t \tag{48}$$

Similarly, in intermediate periods, the following 3 equations are valid:

$$\pi_t = \lambda x_t + \beta E_t \pi_{t+1} + u_t; \qquad x_t = x_{t-1} - \frac{\lambda}{\alpha}\pi_t; \qquad E_t \pi_{t+1} = -\frac{\alpha}{\lambda} x_{t+1} + \frac{\alpha}{\lambda} x_t$$

Combining them yields:

$$x_t = \frac{\alpha}{\alpha(1+\beta)+\lambda^2} x_{t-1} + \beta \frac{\alpha}{\alpha(1+\beta)+\lambda^2} E_t x_{t+1} - \frac{\lambda}{\alpha(1+\beta)+\lambda^2} u_t \tag{49}$$

Finally, in the last period, when the commitment expires, we have:

$$\pi_t = \lambda x_t + \beta E_t \pi_{t+1} + u_t; \qquad x_t = x_{t-1} - \frac{\lambda}{\alpha}\pi_t; \qquad E_t \pi_{t+1} = -\frac{\alpha}{\lambda} x_{t+1}$$

Hence:

$$x_t = \frac{\alpha}{\alpha+\lambda^2} x_{t-1} + \beta \frac{\alpha}{\alpha+\lambda^2} E_t x_{t+1} - \frac{\lambda}{\alpha+\lambda^2} u_t \tag{50}$$

To summarize, if we start with period t when a new commitment is announced, then output between periods t and $t+T$ evolves according to:

$$\begin{array}{l} x_t = \beta\delta_1 E_t x_{t+1} - \frac{\lambda}{\alpha}\delta_1 u_t \\ x_{t+1} = \delta_1 x_t + \beta\delta_1 E_{t+1} x_{t+2} - \frac{\lambda}{\alpha}\delta_1 u_{t+1} \\ \dots\dots\dots\dots\dots\dots \\ x_{t+T-1} = \delta_1 x_{t+T-2} + \beta\delta_1 E_{t+T-1} x_{t+T} - \frac{\lambda}{\alpha}\delta_1 u_{t+T-1} \\ x_{t+T} = \delta_2 x_{t+T-1} + \beta\delta_2 E_{t+T} x_{t+T+1} - \frac{\lambda}{\alpha}\delta_2 u_{t+T} \end{array} \tag{51}$$

where $\delta_1 = \frac{\alpha}{\alpha(1+\beta)+\lambda^2}$ and $\delta_2 = \frac{\alpha}{\alpha+\lambda^2}$. The system of equations above fully characterizes the evolution of output within each commitment 'cycle'.

The solution presented below proceeds as follows. First, note that under the optimal policy output does not respond to demand shocks (i.e. the interest rate policy neutralizes demand shocks as it does under pure discretion). Given the linear-quadratic nature of the problem, we can guess that at time t (beginning of the commitment cycle) the policy takes the following form: $x_t = \omega_1 u_t$. Model stationarity implies that the policy at the beginning of the next cycle will have the same form: $x_{t+T+1} = \omega_1 u_{t+T+1}$. Hence, at time t rational agents expect $E_t x_{t+T+1} = \omega_1 E_t u_{t+T+1} = \omega_1 \rho^{T+1} u_t$. To solve the model, we can start with $t+T$ and through backward substitution of the equations in (51) express each x_{t+j}, $j = 0, 1...T$ as a function of $E_{t+j} x_{t+T+1}$, past outputs and shocks. Note that the resulting equation for x_t will not depend on past output and therefore can be solved for ω_1, which gives us the equilibrium output policy in periods of announcement. Output response in all other periods can be obtained recursively. Finally, inflation is obtained from the central bank's F.O.C.'s.

B.2 Equilibrium in Announcement Periods

Starting with period $t+T$ in (51), we can recursively substitute out all forecasts between t and $t+T$:

at $t+T$:

$$x_{t+T} = \delta_2 x_{t+T-1} + \beta\delta_2 E_{t+T} x_{t+T+1} - \frac{\lambda}{\alpha}\delta_2 u_{t+T}$$

which implies that $E_{t+T-1} x_{t+T} = \delta_2 x_{t+T-1} + \beta\delta_2 E_{t+T-1} x_{t+T+1} - \frac{\lambda}{\alpha}\delta_2 \rho u_{t+T-1}$. Using this in the equation for $t+T-1$ yields:

$$x_{t+T-1} = \frac{1}{1-\beta\delta_1\delta_2}\left(\delta_1 x_{t+T-2} + (\beta\delta_1)(\beta\delta_2) E_{t+T-1} x_{t+T+1} - \frac{\lambda}{\alpha}\delta_1 (1+\beta\rho\delta_1)\delta_2 u_{t+T-1}\right)$$

Continuing in this fashion, it is easy to see, that output throughout the commitment cycle can be represented as follows:

For $j \in [1, T]$:

$$x_{t+j} = A_j \left[\delta_1 x_{t+j-1} + C_j E_{t+j} x_{t+T+1} - \frac{\lambda}{\alpha}\delta_1 B_j u_{t+j}\right] \tag{52}$$

and in the initial period ($j = 0$):

$$x_t = A_0 \left[C_0 E_t x_{t+T+1} - \frac{\lambda}{\alpha}\delta_1 B_0 u_t\right] \tag{53}$$

where the coefficients can be computed recursively as follows:

$$\begin{aligned} A_j &= \tfrac{1}{1-\beta\delta_1^2 A_{j+1}}; \\ C_j &= (\beta\delta_1)\, C_{j+1} A_{j+1}; \\ B_j &= 1+\beta\rho\delta_1 A_{j+1} B_{j+1} \end{aligned} \tag{54}$$

with the terminal values of:

$$A_T = \frac{\delta_2}{\delta_1}; C_T = \beta\delta_1; B_T = 1$$

Having solved for the coefficients, we can obtain the equilibrium in periods of announcement. Using $x_t = \omega_1 u_t$ and $E_t x_{t+T+1} = \omega_1 E_t u_{t+T+1} = \omega_1 \rho^{T+1} u_t$ in (53) for $j = 0$ yields:

$$\omega_1 = \left(1 - A_0 C_0 \rho^{T+1}\right)^{-1} \left(-\frac{\lambda}{\alpha}\delta_1 A_0 B_0\right) \tag{55}$$

This gives a solution for output in periods of announcement. Inflation is obtained from the central bank's first order condition. Since both inflation and output both depend on contemporaneous shocks only, their unconditional variances and expected social losses in periods of announcement are straightforward to compute.

B.3 Equilibrium in Non-Announcement Periods

To obtain equilibrium in other periods we exploit the functional form of the solution in periods of announcement. Note that for any j, $E_{t+j} x_{t+T+1} = \rho^{T+1-j} \omega_1 u_{t+j}$. Use this to iterate (52) forward and express output in each period as a function of exogenous shocks only.

At time $t+1$:

$$x_{t+1} = A_1 \delta_1 \omega_1 u_t + A_1 \left[C_1 \rho^T \omega_1 - \frac{\lambda}{\alpha}\delta_1 B_1\right] u_{t+1}$$

at $t+2$:

$$\begin{aligned} x_{t+2} = & \left(A_2\delta_1\right)\left(A_1\delta_1\right)\omega_1 u_t + \\ & + \left(A_2\delta_1\right) A_1 \left[C_1\rho^T\omega_1 - \tfrac{\lambda}{\alpha}\delta_1 B_1\right] u_{t+1} + \\ & + A_2 \left[C_2\rho^{T-1}\omega_1 - \tfrac{\lambda}{\alpha}\delta_1 B_2\right] u_{t+2} \end{aligned}$$

Continuing further we obtain the following representation for output for $j > 0$:

$$\begin{aligned} x_{t+j} = & \delta_1^j \left(\prod_{i=1}^{j} A_i\right) \omega_1 u_t + \\ & + \delta_1^{j-1} \left(\prod_{i=1}^{j} A_i\right) \left[C_1\rho^T\omega_1 - \tfrac{\lambda}{\alpha}\delta_1 B_1\right] u_{t+1} + \\ & + \delta_1^{j-2} \left(\prod_{i=2}^{j} A_i\right) \left[C_2\rho^{T-1}\omega_1 - \tfrac{\lambda}{\alpha}\delta_1 B_2\right] u_{t+2} + \\ & + \dots\dots\dots\dots\dots\dots\dots\dots \\ & + A_j \left[C_j\rho^{T-j+1}\omega_1 - \tfrac{\lambda}{\alpha}\delta_1 B_j\right] u_{t+j} \end{aligned} \tag{56}$$

or:

$$x_{t+j} = a_0^{(j)} u_t + a_1^{(j)} u_{t+1} + \dots + a_j^{(j)} u_{t+j} = \sum_{k=0}^{j} a_k^{(j)} u_{t+k} \tag{57}$$

where the upperscript $^{(j)}$ indicates the coefficients' dependence on j. Using the central bank's first order condition, we can express inflation as:

$$
\begin{aligned}
\pi_{t+j} &= -\tfrac{\alpha}{\lambda}(x_{t+j} - x_{t+j-1}) = \\
&= -\tfrac{\alpha}{\lambda}\sum_{k=0}^{j-1}\left(a_k^{(j)} - a_k^{(j-1)}\right)u_{t+k} - \tfrac{\alpha}{\lambda}a_j^{(j)}u_{t+j} = \\
&= \sum_{k=0}^{j} c_k^{(j)} u_{t+k}
\end{aligned} \tag{58}
$$

Using these representations of inflation and output, computing unconditional variances of inflation and output in each intermediate period is straightforward. They are given by:

$$
Var(x_{t+j}) = \left(\sum_{k=0}^{j}\left(a_k^{(j)}\right)^2\right)\gamma_0 + 2\left[\sum_{i=1}^{j}\left(\sum_{k=i}^{j} a_k^{(j)} a_{k-i}^{(j)}\right)\gamma_i\right] \tag{59}
$$

$$
Var(\pi_{t+j}) = \left(\sum_{k=0}^{j}\left(c_k^{(j)}\right)^2\right)\gamma_0 + 2\left[\sum_{i=1}^{j}\left(\sum_{k=i}^{j} c_k^{(j)} c_{k-i}^{(j)}\right)\gamma_i\right] \tag{60}
$$

where $\gamma_0 = E(u^2)$ and γ_i's are $i-$th order autocovariances of u_t.

Finally, in each period within the commitment cycle, expected value of the loss function is given by:

$$
E(L_j) = \frac{1}{2}\left[\alpha Var(x_{t+j}) + Var(\pi_{t+j})\right] \tag{61}
$$

And total unconditional expectation of social losses is a simple average:

$$
E(L) = \frac{1}{1-\beta}\left(\frac{1}{T+1}\sum_{j=0}^{T} E(L_j)\right) \tag{62}
$$

C Solution to a Model of Infrequent Policy Adjustment

Here we assume that the Central Bank fixes the interest rate at t until some future date $t+T$. The logic of the solution is the same as before. First we would like to eliminate all endogenous forecasts until date $t+T$. Then, we solve for the optimal policy at date t and back up the equilibrium in periods of inaction. Note, that by taking all forecasts beyond $t+T$ as given, the central bank essentially minimizes:

$$E_t\left(L\right) = \frac{1}{2}E_t\sum_{j=0}^{T}\beta^j\left(\alpha x_{t+j}^2 + \pi_{t+j}^2\right)$$

subject to a sequence of $T+1$ Phillips curve and IS constraints. To obtain the first order condition, we first obtain the impact of the current interest rate on each future forecasts of inflation and output. Since the forecasts beyond date $t+T$ when the commitment expires are taken by the CB as given, we start from date $t+T$ where we have:

$$\frac{\partial E_t x_{t+T}}{\partial i_t} = -\varphi; \qquad \text{and} \qquad \frac{\partial E_t \pi_{t+T}}{\partial i_t} = \lambda\frac{\partial E_t x_{t+T}}{\partial i_t} = -\lambda\varphi$$

At date $T-1$ the impact is:

$$\begin{aligned}\frac{\partial E_t x_{t+T-1}}{\partial i_t} &= -\varphi + \frac{\partial E_t x_{t+T}}{\partial i_t} + \varphi\frac{\partial E_t \pi_{t+T}}{\partial i_t} = \\ &= -\varphi + \frac{\partial E_t x_{t+T}}{\partial i_t}\left(1+\varphi\lambda\right);\end{aligned}$$

and:

$$\frac{\partial E_t \pi_{t+T-1}}{\partial i_t} = \lambda\frac{\partial E_t x_{t+T-1}}{\partial i_t} + \lambda\beta\frac{\partial E_t x_{t+T}}{\partial i_t}$$

Continuing in this fashion and noting that $\frac{\partial E_t \pi_{t+j}}{\partial i_t} = \frac{\partial \pi_{t+j}}{\partial i_t} \equiv B_j^{\pi}$ and $\frac{\partial E_t x_{t+j}}{\partial i_t} = \frac{\partial x_{t+j}}{\partial i_t} \equiv B_j^{x}$, $T \geq j \geq 0$ we can see that the impact can be expressed in a recursive form:

$$\begin{aligned}B_j^x &= -\varphi + B_{j+1}^x + \lambda\varphi\sum_{k=1}^{T-j}\beta^{k-1}B_{j+k}^x \\ B_j^{\pi} &= \lambda\sum_{k=0}^{T-j}\beta^k B_{j+k}^x = \lambda B_j^x + \beta B_{j+1}^{\pi}\end{aligned} \tag{63}$$

for any $j \in [0,T)$ and for $j=T$, the impact is

$$B_{t+T}^x = -\varphi; \qquad \text{and} \qquad B_{t+T}^{\pi} = -\lambda\varphi \tag{64}$$

Then, the Central Bank's first order condition can be expressed as:

$$E_t\sum_{j=0}^{T}\beta^j\left(\alpha x_{t+j}B_{t+j}^x + \pi_{t+j}B_{t+j}^{\pi}\right) = 0 \tag{65}$$

C.1 Modified Structural Equations

To solve for the equilibrium we perform repeated substitutions to express each $E_t x_{t+j}$ and $E_t \pi_{t+j},\ j \in [0, T]$ above as a function of the current period interest rate and the variables which the Central Bank takes as given: exogenous states u_t and g_t and time $t+T+1$ forecasts of inflation and output. Each forecast in the equation above can be represented as:

$$E_t x_{t+j} = B_j^x i_t + A_j^{xx} E_t x_{t+T+1} + A_j^{x\pi} E_t \pi_{t+T+1} + A_j^{xg} g_t + A_j^{xu} u_t \tag{66}$$

$$E_t \pi_{t+j} = B_j^\pi i_t + A_j^{\pi x} E_t x_{t+T+1} + A_j^{\pi\pi} E_t \pi_{t+T+1} + A_j^{\pi g} g_t + A_j^{\pi u} u_t \tag{67}$$

where coefficients B and A are constants. In the same way as we derived the coefficients on i_t is possible to show that other coefficients can be computed recursively as follows:

Expected Inflation:

$$\begin{aligned} A_j^{x\pi} &= A_{j+1}^{x\pi} + \lambda\varphi \sum_{k=1}^{T-j} \beta^{k-1} A_{j+k}^{x\pi} + \varphi\beta^{T-j} \\ A_j^{\pi\pi} &= \lambda \sum_{k=0}^{T-j} \beta^k A_{j+k}^{x\pi} + \beta^{T-j+1} = \lambda A_j^{x\pi} + \lambda\beta \sum_{k=1}^{T-j} \beta^{k-1} A_{j+k}^{x\pi} + \beta^{T-j+1} \end{aligned} \tag{68}$$

Expected output:

$$\begin{aligned} A_j^{xx} &= A_{j+1}^{xx} + \lambda\varphi \sum_{k=1}^{T-j} \beta^{k-1} A_{j+k}^{xx} \\ A_j^{\pi x} &= \lambda \sum_{k=0}^{T-j} \beta^k A_{j+k}^{xx} = \lambda A_j^{xx} + \beta A_{j+1}^{\pi x} \end{aligned} \tag{69}$$

Demand Shocks:

$$\begin{aligned} A_j^{xg} &= \mu^j + A_{j+1}^{xg} + \lambda\varphi \sum_{k=1}^{T-j} \beta^{k-1} A_{j+k}^{xg} \\ A_j^{\pi g} &= \lambda \sum_{k=0}^{T-j} \beta^k A_{j+k}^{xg} \end{aligned} \tag{70}$$

Cost-Push Shocks:

$$\begin{aligned} A_j^{xu} &= A_{j+1}^{xu} + \varphi A_{j+1}^{\pi u} \\ A_j^{\pi u} &= \lambda A_j^{xu} + \beta A_{j+1}^{\pi u} + \rho^j \end{aligned} \tag{71}$$

for any $j \in [0, T)$ and for $j = T$ the initial values are:

$$\begin{aligned} &A_{t+T}^{x\pi} = \varphi; \quad \text{and} \quad A_{t+T}^{\pi\pi} = \beta + \lambda\varphi \\ &A_{t+T}^{xx} = 1; \quad \text{and} \quad A_{t+T}^{\pi x} = \lambda \\ &A_{t+T}^{xg} = \mu^T; \quad \text{and} \quad A_{t+T}^{\pi g} = \lambda\mu^T \\ &A_{t+T}^{xu} = 0; \quad \text{and} \quad A_{t+T}^{\pi u} = \rho^T \end{aligned} \tag{72}$$

C.2 Equilibrium in Period of Adjustment

The first order condition can be written as:

$$\sum_{j=0}^{T} \beta^j \begin{pmatrix} \alpha B_j^x & B_j^\pi \end{pmatrix} \begin{pmatrix} E_t x_{t+j} \\ E_t \pi_{t+j} \end{pmatrix} = 0 \tag{73}$$

As before, we guess that at time t the response of inflation and output takes the form:

$$\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = D \begin{pmatrix} g_t \\ u_t \end{pmatrix} \tag{74}$$

So that:

$$\begin{pmatrix} E_t x_{t+T+1} \\ E_t \pi_{t+T+1} \end{pmatrix} = DP^{T+1} \begin{pmatrix} g_t \\ u_t \end{pmatrix}$$

It follows that:

$$\begin{pmatrix} E_t x_{t+j} \\ E_t \pi_{t+j} \end{pmatrix} = \begin{pmatrix} B_j^x \\ B_j^\pi \end{pmatrix} i_t + A_j \begin{pmatrix} g_t \\ u_t \end{pmatrix} \tag{75}$$

where:

$$A_j = \underbrace{\begin{pmatrix} A_j^{xx} & A_j^{x\pi} \\ A_j^{\pi x} & A_j^{\pi\pi} \end{pmatrix}}_{\equiv A_{1j}} DP^{T+1} + \underbrace{\begin{pmatrix} A_j^{xg} & A_j^{xu} \\ A_j^{\pi g} & A_j^{\pi u} \end{pmatrix}}_{\equiv A_{2j}} \tag{76}$$

The first order condition becomes:

$$\left[\sum_{j=0}^{T} \beta^j \left(\alpha \left(B_j^x\right)^2 + \left(B_j^\pi\right)^2\right)\right] i_t + \left[\sum_{j=0}^{T} \beta^j \begin{pmatrix} \alpha B_j^x & B_j^\pi \end{pmatrix} A_j\right] \begin{pmatrix} g_t \\ u_t \end{pmatrix} = 0$$

We can express the optimal interest rate as:

$$i_t = \underbrace{-\left[\sum_{j=0}^{T} \beta^j \left(\alpha \left(B_j^x\right)^2 + \left(B_j^\pi\right)^2\right)\right]^{-1}}_{\equiv C_1} \cdot \left[\sum_{j=0}^{T} \beta^j \begin{pmatrix} \alpha B_j^x & B_j^\pi \end{pmatrix} A_j\right] \begin{pmatrix} g_t \\ u_t \end{pmatrix} \tag{77}$$

$$i_t = C_1 \cdot \left(A_3 DP^{T+1} + A_4\right) \begin{pmatrix} g_t \\ u_t \end{pmatrix} \tag{78}$$

where

$$A_3 = \left[\sum_{j=0}^{T} \beta^j \begin{pmatrix} \alpha B_j^x & B_j^\pi \end{pmatrix} A_{1j}\right]$$

$$A_4 = \sum_{j=0}^{T} \beta^j \begin{pmatrix} \alpha B_j^x & B_j^\pi \end{pmatrix} A_{2j}$$

To solve for equilibrium in periods when the interest rate is adjusted, we combine the previous equation with forecast equation when $j = 0$[28]:

$$D \begin{pmatrix} g_t \\ u_t \end{pmatrix} = \underbrace{\begin{pmatrix} B_t^x \\ B_t^\pi \end{pmatrix}}_{B_0^x} i_t + A_0 \begin{pmatrix} g_t \\ u_t \end{pmatrix}$$

Hence, D must satisfy:

$$D = B_0^x C_1 \cdot \left(A_3 D P^{T+1} + A_4\right) + \left(A_{1_0} D P^{T+1} + A_{2_0}\right)$$

or:

$$D - \underbrace{(B_0^x C_1 A_3 + A_{1_0})}_{\equiv C_2} D P^{T+1} = \underbrace{B_0^x C_1 A_4 + A_{2_0}}_{\equiv C_3}$$

This is a system of 4 linear equations in coefficients of D which are straightforward to solve.

C.3 Equilibrium in Periods of Non-Adjustment

In periods when the interest rate is fixed ($0 < j \leq T$), equilibrium is described by:

$$x_{t+j} = -\varphi i_t + E_{t+j} x_{t+j+1} + \varphi E_{t+j} \pi_{t+j+1} + g_{t+j} \tag{79}$$

$$\pi_{t+j} = \lambda x_{t+j} + \beta E_{t+j} \pi_{t+j+1} + u_{t+j} \tag{80}$$

with $T \geq j > 0$. These equations can be also expressed as:

$$x_{t+j} = B_j^x i_t + A_j^{xx} E_{t+j} x_{t+T+1} + A_j^{x\pi} E_{t+j} \pi_{t+T+1} + A_j^{xg} \frac{g_{t+j}}{\mu^j} + A_j^{xu} \frac{u_{t+j}}{\rho^j} \tag{81}$$

$$\pi_{t+j} = B_j^\pi i_t + A_j^{\pi x} E_{t+j} x_{t+T+1} + A_j^{\pi\pi} E_{t+j} \pi_{t+T+1} + A_j^{\pi g} \frac{g_{t+j}}{\mu^j} + A_j^{\pi u} \frac{u_{t+j}}{\rho^j} \tag{82}$$

where the forecasts of $(t + T + 1)$ variables must be consistent with interest rate adjustment policy:

$$\begin{pmatrix} E_{t+j} x_{t+T+1} \\ E_{t+j} \pi_{t+T+1} \end{pmatrix} = D P^{T+1-j} \begin{pmatrix} g_{t+j} \\ u_{t+j} \end{pmatrix}$$

Solution for the interest rate can be expressed as $i_t = \Psi e_t$, where $e_t = \begin{pmatrix} g_{t+j} & u_{t+j} \end{pmatrix}'$. Collecting terms, we obtain:

$$\begin{pmatrix} x_{t+j} \\ \pi_{t+j} \end{pmatrix} = \begin{pmatrix} B_j^x \\ B_j^\pi \end{pmatrix} \Psi e_t + \left[A_{1j} D P^{T+1-j} + A_{2j} \left(P^j\right)^{-1}\right] e_{t+j}$$

Let:

$$C_{5j} = \begin{pmatrix} B_j^x \\ B_j^\pi \end{pmatrix}$$

[28] $A_0 = A_{1_0} D P^{T+1} + A_{2_0}$

$$C_{6j} = A_{1j} D P^{T+1-j} + A_{2j} \left(P^{j} \right)^{-1}$$

Hence:

$$\begin{pmatrix} x_{t+j} \\ \pi_{t+j} \end{pmatrix} = C_{5j} e_t + C_{6j} e_{t+j}$$

Unconditional variances can be expressed as:

$$E x_{t+j}^{2} = \left(c_{5j_1} \Psi \right) \Omega \left(c_{5j_1} \Psi \right)' + \left(c_{6j_1} \right) \Omega \left(c_{6j_1} \right)' + 2 \left(c_{5j_1} \Psi \right) \Sigma_j \left(c_{6j_1} \right)' \tag{83}$$

$$E \pi_{t+j}^{2} = \left(c_{5j_2} \Psi \right) \Omega \left(c_{5j_2} \Psi \right)' + \left(c_{6j_2} \right) \Omega \left(c_{6j_2} \right)' + 2 \left(c_{5j_2} \Psi \right) \Sigma_j \left(c_{6j_2} \right)' \tag{84}$$

where c_{5j_1} and c_{5j_2} are first and second elements of C_{5j}, c_{6j_1} and c_{6j_2} are rows of C_{6j}, Ω is unconditional covariance matrix of e_t and Σ_j is the unconditional correlation matrix $E(e_t e_{t+j})$.