Universidade Federal do Rio de Janeiro
Instituto de Economia

Estimating potential output: a survey of the alternative methods and their applications to Brazil

TD. 016/2004

Nelson H. Barbosa Filho

Série
Textos para Discussão

December 20, 2004

Estimating potential output: a survey of the alternative methods and their applications to Brazil[*]

Nelson H. Barbosa-Filho[**]

Abstract: This paper presents the main issues involved in estimating potential output. The objective is to describe the alternative methods and analyze their application and implications for growth forecasts and macroeconomic policy in Brazil. The text emphasizes the determinants of potential output under fixed and flexible coefficients of production. Given the wide use of aggregate measures of total factor productivity in growth accounting, and the sensitivity of such a variable to economic assumptions and errors of measurement, the text also presents the main applied critiques and alternatives to aggregate growth-accounting exercises. The main conclusions are: (1) the annual potential growth rate of Brazil's GDP varies substantially depending on the method and hypotheses adopted and, what is most important, potential GDP is not separable from effective GDP in the long-run; (2) growth-accounting and time-series studies of Brazil result in low potential-output growth rates because they extrapolate the slow growth of 1981-2003 to the future; (3) capital seems to be the main constraint on growth in Brazil and, therefore, a demand-led increase in investment can raise both its effective and potential output levels; (4) however, because of the slow adjustment of the capital stock, an investment boom can also hit a supply constraint before the stock of capital has time to adjust to the growth rate of investment; and (5) aggregate measures of potential output do not carry much information about the economy and, therefore, they should be complemented by sectoral estimates of capacity utilization to identify the bottlenecks in inter-industry flows and the corresponding demand pressures on inflation.

JEL codes: O110, O470, O490, O540

Keywords: growth, potential output, effective demand, Brazil

[*] Paper prepared for the joint research program of the *Comisión Economica para América Latina y el Caribe* (CEPAL) of the United Nations and the *Instituto de Economia Aplicada* (IPEA), of the Brazilian Government. The views expressed in this work are those of the author and do not reflect those of CEPAL, IPEA or its members. The author would like to thank Lance Taylor, Duncan Foley, Carlos Mussi, Renato Baumann, Ricardo Bielschowsky, Franklin Serrano, Fabio Freitas, Paulo Levy and Estevão Kopschitz for comments and suggestions. The usual disclaimer applies.

[**] Institute of Economics, Federal University of Rio de Janeiro (E-mail address: nhbarbosa@ie.ufrj.br).

1 – INTRODUCTION

Potential output is the capacity of production at a "high" level of resource use. In macroeconomics this corresponds to the level of GDP consistent with a stable rate of inflation, so that the difference between potential and effective output measures the demand pressures on prices. In mainstream growth theory potential output is usually estimated from the supply side, that is, from the long-run or equilibrium values of the capital and labor inputs and their corresponding productivity levels. In non-mainstream growth theories demand factors also enter in the calculation through the impact of investment on capital accumulation and productivity.

One of the main objectives of Brazil's current macroeconomic policy is to increase income and employment without raising inflation. In order to analyze the impacts of monetary and fiscal policy on growth, it is therefore necessary to estimate the potential growth rate of the economy. This paper presents the main issues involved in such a task. The objective is to describe the alternative methods of estimating potential output, as well as to analyze their application and implications for growth forecasts and macroeconomic policy in Brazil.

The text is organized in seven sections including this introduction. Section two presents the basic definitions used in growth accounting and the methods used for measuring labor, capital and the output gap. Section three analyzes the determinants of potential output under the assumption of fixed coefficients of production and describes the disaggregated input-output estimates of capacity utilization and employment rates derived from a Leontief production function. Section four analyzes the determinants of potential output under the assumption of flexible coefficients of production and describes the basic steps involved in growth-accounting exercises based on a Cobb-Douglas production function. Given the wide use of aggregate measures of multifactor productivity in growth accounting and the sensitivity of such a variable to economic assumptions and errors of measurement, section four also presents the main applied critiques and alternatives to aggregate growth-accounting exercises. Section five shifts the investigation to the main statistical filters used to estimate trends and cycles in univariate economic time series and discusses how this a-theoretical approach can substitute, or be combined with, theoretical approaches based on production functions. Section six merges theory and econometrics in a comparative analysis of recent estimates of the potential growth rate of Brazil. The main objective is to show how and why the estimates differ, as well as how sensitive each estimate is to small changes in its underlying assumptions and initial conditions. Section seven concludes with a summary of the main points of the analysis and some comments on how Brazil can achieve high and sustainable growth rates.

2 – BASIC DEFINITIONS

A production function maps a set of inputs to the output of one or more goods and it is the basis of economic estimates of potential output. The intuitive idea is to express the real output of a firm, a sector or an economy as a function of the quantity of inputs necessary for its production. More formally, production is represented by a function F, such that

$$Y_t = F(\mathbf{x}_t, t), \tag{2.1}$$

where Y_t is real output, $\mathbf{x}_t$ is a vector of containing all inputs necessary to produce Y_t, and t represents time. The inclusion of time as an independent input aims to capture the increase in output due to productivity growth, that is, the increase in output when we hold all other inputs constant.

The partial derivative of F in relation to time is an index of the total factor productivity (TFP) and, when we apply (2.1) to aggregate variables, such an index depends not only on technological factors, but also on relative prices and on the composition of outputs and inputs. For the moment, let us ignore these accounting complications and assume that (2.1) is in fact a good proxy of the technology of production. From the total derivative of $\ln Y_t$ in relation to time we have

$$y_t = a_t + \sum_{j=1}^{J_I} \alpha_j x_{jt} \tag{2.2}$$

where y_t and x_{jt} are the growth rates of Y_t and X_{jt}, respectively, a_t is the growth rate of TFP, α_j is the elasticity of Y_t in relation to X_{jt}, and J_I is the total number of inputs.[1] In economic terms, (2.2) allows us to decompose growth from the supply side, that is, a_t represents the amount of growth generated by TFP growth, whereas $\alpha_j x_{jt}$ represents the amount of growth generated by input j.

In the growth-accounting literature it is common practice to impose constant returns to scale, profit maximization (or cost minimization) and perfect competition on the unit of production to facilitate the application of (2.2) to real-world economies. To see why, note that, if the production function exhibits constant returns to scale, the corresponding elasticity parameters add up to one and the value of output equals the value of all inputs. Next, because profit maximization under perfect competition implies that the real price of each input corresponds to its marginal product, the share of each input in total output is necessarily equal to its elasticity parameter. Put together, these three assumptions mean that we can use the functional distribution of income, or the cost decomposition of gross product, as a guide for the elasticity parameters of the production function.

Perfect competition and constant returns to scale are obviously very strong assumptions to impose on real-world economies. Given the applied focus of this paper, section four will analyze only the applied implications of the theoretical debate. Harcourt (1969) provides a survey of the literature and, in response to the fragile theoretical foundations of growth accounting, the most common defense follows the "as-if" principle of Friedman (1953) and states that, even though the assumptions seem unrealistic, the model

[1] Unless stated otherwise, all growth rates are exponential growth rates.

gives good results when applied to real-world economies. As we shall see in section five, this is not a surprise because the national income and product accounting (NIPA) identities can also be used to derive (2.2).

To use a production function we have to define and measure its output and input variables. The two issues are obviously related and there are two alternatives in the growth-accounting literature: the "net-value" approach and the "gross-value" approach. [2] The first approach is based on the work of Solow (1956 and 1957) and it concentrates the analysis on the value added by capital and labor on the assumption that intermediate inputs tend to be a fixed proportion of gross output. In other words, the net-value approach assumes that there is limited or no substitution between intermediate inputs on the one side, and capital and labor on the other side. The second approach is based on the work of Jorgenson and Grilliches (1967) and it decomposes real gross output in terms of all inputs used in its production, that is, capital, labor and all intermediate inputs. The basic idea is that changes in the use and quality of intermediate inputs may also have an important impact on the net value of output. As we shall see in section four, this second approach is usually implemented in a disaggregated way and, therefore, it is much more complex in terms of data collection than the first one.

Because of its easy application and intuitive interpretation, most exercises in growth accounting follow the net-value approach and impose an aggregate production function on the data. The other common step is to follow the "as-if" principle and assume that both the capital and the labor elasticities of net output correspond to the average share of these factors in income. Put together, these two assumptions reduce the analysis to calculating TFP growth from the observed growth rates of income, labor and capital. The growth rate of income is usually obtained directly from the NIPA data. The measurement of labor and capital is not so straightforward and depends on a series of economic assumptions.

2.1 - Measuring labor

The two main methodological issues in measuring labor are how to define it and how to control for heterogeneity. From the perspective of production, the labor input should be measured by the number of hours worked in production. However, because data on work hours are not usually available for many sectors and economies, the standard practice in macroeconomic studies is to define labor as the number of people employed in production. The implicit assumption is that the average number of work hours per worker does not vary much and, when this is not the case, the variations are captured by the TFP residual.

[2] Unless stated otherwise, both growth-accounting strategies refer to the value of output at factors' cost, that is, they exclude net taxes and subsidies from the analysis. For a survey of the history of thought on growth accounting and TFP, see, for instance, Jorgenson (1990), Denison (1993), Griliches (1996) and Hulten (2000).

In other words, if we do not adjust the employment index for variations in the length of work shifts, we end up introducing a strong cyclical behavior in TFP.

The second issue is how to control for labor heterogeneity. To use Jorgenson's (1990, p.24) example, should one work hour of an electrical engineer count the same as one work hour of a truck driver? There are two direct answers and, not surprisingly, there are basically two strategies to deal with the problem. First, we can construct a weighted index of work hours where the wage rate functions as a proxy of the quality of work. For instance, if the wage paid for one hour of work A is two times the wage paid for one hour of work B, then one hour of work A should be equivalent to two hours of work B. To construct an index of labor input we have to choose a "unit of account" (usually the unskilled work or a composite index of all types of work), and then weight all types work according to the deviation of their corresponding wages from the reference wage. In this approach a change in the composition of employment toward high-paying jobs appears as an increase in the labor input.

To illustrate the above point, let N_t be the quality-adjusted Fisher index of work hours. Assuming that there are J_N types of work in the economy, each of which receives a wage W_{jt} per hour, the growth rate of the labor input is

$$n_t = \sum_{j=1}^{J_N} \beta_{jt}^N n_{jt} \,, \tag{2.3}$$

where n_{jt} is the growth rate of labor input j and β_{jt}^N is its average participation in the total wage bill of the current and previous periods, that is

$$\beta_{jt}^N = \frac{1}{2}(\omega_{jt} + \omega_{jt-1}) \tag{2.4}$$

and

$$\omega_{jt} = \frac{W_{jt} N_{jt}}{\sum_{j=1}^{J_N} W_{jt} N_{jt}} \tag{2.5}$$

for j=1,…, J_N.

Equations (2.4) and (2.5) produce a chained labor index, so that labor measurement does not depend on the prices of a reference year. Only the prices of the previous and current year enter in the calculation and, whenever we want to change the reference date, the levels of the series change, but their growth rates remain the same.

The second and most usual strategy to deal with heterogeneity is to ignore it and let TFP capture any change in the quality of work hours. The basic idea is to sum all work hours in a given period and put the result into the production function. Any change in the quality of work is interpreted as a productivity gain or loss, in the sense that when a worker moves up in the occupational ladder, his or her productivity is usually increased. In contrast to the previous approach, any change in the composition of employment toward more qualified jobs appears as an increase in TFP.

Formally, the second approach can be represented by the growth rate of another Fisher index of work hours, that is

$$\tilde{n}_t = \sum_{j=1}^{J_N} \tilde{\beta}_{jt}^N n_{jt} . \tag{2.6}$$

where the weight $\tilde{\beta}_{jt}^N$ depends only on the participation of labor input j in the total work hours of the current and previous periods, that is

$$\tilde{\beta}_{jt}^N = \frac{1}{2}(\tilde{\omega}_{jt} + \tilde{\omega}_{jt-1}), \tag{2.7}$$

where

$$\tilde{\omega}_{jt} = \frac{N_{jt}}{\sum_{j=1}^{J_N} N_{jt}} \tag{2.8}$$

Because the weights in (2.3) and (2.6) may differ, the two methodologies may result in different estimates of the labor input. In economic terms, the dividing line is whether to attribute differences in labor productivity to occupations or to workers. If one thinks that the same worker may have a different productivity depending on his or her occupation, then the most appropriate approach is to measure labor as the quality-adjusted index given in (2.3). The logic is that the same unit of analysis (the worker) can produce "skilled" and "unskilled" work depending on where it is employed. In this case it would be erroneous to associate an increase in productivity to changes in occupations because the "quality", or "human capital", of the unit "producing" the work hour remains the same. On the other hand, if one thinks the same worker has the same productivity independently of his or her occupation, then changes in occupations reflect changes in productivity and, therefore, they should be included in TFP rather than in the labor input.

The choice of methodology has important effects on the estimated growth of TFP, and we can measure the contribution of labor quality to growth from the difference between (2.3) and (2.6).

2.2 – Measuring capital

The definition and estimation of the capital stock is one of the most controversial issues in economic theory. Once again, given the applied orientation of this paper, we will concentrate the analysis on the assumptions and definitions behind the estimates rather than on their theoretical implications.

In the growth-accounting literature it is standard procedure to construct a real index for capital based on the perpetual inventory method (PIM). The logic is clear and intuitive: the current capital stock is the cumulated sum of past investment flows adjusted for depreciation. The crucial issue is how to define the service life and the depreciation rate of each type of capital good. To facilitate the analysis, consider the stock of one type of capital good (say, producer equipment). Following the polynomial benchmark approach, the dynamics of capital accumulation can be represented simply as

$$K_t = K_{t-1}(1-\delta) + I_{t-1} \tag{2.9}$$

where K_t is the stock of the capital good at the beginning of period t, I_{t-1} the gross investment of period t-1 and, for simplicity, δ is the constant rate of capital depreciation.[3] By recursive substitution we have

$$K_t = \left[\sum_{j=1}^{S} I_{t-j}(1-\delta)^{j-1}\right] + (1-\delta)^{S+1} K_{t-(S+1)} \tag{2.10}$$

and, therefore, we can estimate the current capital stock from the previous S investment flows and the capital stock at the "initial" period t-(S+1).

How does one estimate the capital stock at initial period? Here enters the hypothesis about the service life: if we assume that the asset under analysis has a lifetime of S periods, meaning that any good with an age of S+1 is automatically retired from production, we can ignore the second term on the right-hand side of (2.10) and estimate the current capital stock just from the past S investment flows.[4]

The service life and the depreciation rate are usually estimated through price surveys of used equipment and structures in resale markets. The basic assumption is that the reduction in the price of an asset is a good proxy of its rate of economic depreciation, which in its turn can be used to estimate its average service life. The longer the lifetime, the longer the series of investment flows needed to estimate the current capital stock.

The service life varies substantially across assets. For instance, according to the methodology of the US Bureau of Economic Analysis (BEA), the service life ranges from 4 to 33 years for private nonresidential equipment, 16 to 54 years for nonresidential structures, and 11 to 80 years for residential

[3] The calculation is slightly more complex with a variable rate of depreciation but the logic is the same. A detailed survey of the alternative methods for estimating depreciation can be found in Hulten and Wykoff (1981). Also to simplify the analysis, (2.9) assumes that new assets are placed in service at the end of the year, so that no depreciation rate should be applied to the previous investment. One possible alternative is to assume that placement occurs at midyear (see USDC BEA 2003), so that half of the depreciation rate is applied to the previous investment.
[4] Note that this approach implies an instantaneous ("sudden death") retirement at age S. An alternative and more realistic approach would be to assume that retirement is a probabilistic function of age.

structures.[5] Given the lack of long and disaggregated investment series for most economies, a simpler rule is to attribute an average and shorter service life to just three capital categories: residential capital, nonresidential structures, and machinery and equipment. For instance, Hoffman (1992 and 2003) attributed a service life of 50, 40 and 15 years to these three categories, respectively, and obtained capital-stock estimates for Latin-American countries starting in the 1950s. Marquetti (2000) and Reis and Morandi (2003) followed a similar approach to obtain their capital-stock estimates for Brazil.[6]

Because NIPA data became available only in the postwar period for most economies in the world, a 50-year service life means that capital estimates would be available only for the 21st century. To overcome this problem, some studies take an *ad hoc* shortcut and assume that capital and output grew at the same rate in the period preceding the first observation of investment (the "balanced-growth" assumption). In formal terms this reduces (2.10) to

$$K_t = \frac{I_{t+1}}{g+\delta}, \qquad (2.11)$$

where g is the average discrete-time growth rate of GDP in the years before t.[7] After estimating the capital stock for period t, the values for the subsequent periods can be constructed from the observed investment flows.

Given the fragility of capital-stock estimates in the absence of good data on investment, it is important to bear in mind how sensitive these estimates are to errors of measurement. From (2.10) we have

$$\frac{\partial K_t}{\partial K_{t-S}} = (1-\delta)^S > 0 \qquad (2.12)$$

and

$$\frac{\partial K}{\partial \delta} = -\sum_{j=1}^{S} I_{t-j}(j-1)(1-\delta)^{j-2} < 0. \qquad (2.13)$$

In words, the higher the initial estimate of the capital stock, the higher the current estimate of the capital stock and, the higher the depreciation rate, the lower the current estimate of the capital stock. The errors of measurement due to the initial capital stock tend to die out with time, whereas the errors due to the depreciation rate tend to grow with time. To obtain an alternative measure of the latter, we can use a modified version of (2.11) to obtain

$$\frac{\partial K_t}{\partial \delta} = -\frac{I_t}{(g_I+\delta)^2}, \qquad (2.14)$$

---

[5] Fraumeni (1997) presents the detailed numbers and the corresponding depreciation rates for each type of asset.
[6] Differently from Hoffman, Reis and Morandi attributed a service life of 20 years for machinery and equipment. Marquetti attributed 50 years to residential capital and nonresidential structures, and 14 years to machinery and equipment.
[7] Usually the average is taken over 5 to 10 years. See, for instance, Bernanke and Gurkaynak (2001).

where $g_I > 0$ represents the average discrete-time growth rate of investment between the initial and the current periods. Since investment usually grows through time, the errors due to a miscalculation of the depreciation rate also grow through time.

By analogy with labor, the other crucial methodological issue regarding capital is how to control for heterogeneity. In principle the PIM should be applied to each type of capital good, so that the availability of investment series determines the level of aggregation of the analysis. After obtaining the disaggregated estimates, an aggregated Fisher index can be calculated by attributing prices to each type of asset.[8]

Similar to labor measurement, the standard approach in the growth-accounting literature is to define the aggregated estimate as quality-adjusted average of the disaggregated estimates, where the weight of each type of asset corresponds to its participation in the total value of the capital stock.[9] The intuitive idea is that, in the same way that wages are a good proxy of the quality of each type of labor input, rental prices of fixed assets are a good proxy of the quality of each type of capital good. Assuming that there exist J_K types of assets, the growth rate of the aggregate capital stock is

$$k_t = \sum_{j=1}^{J_K} \beta_{jt}^K k_{jt} \ , \qquad (2.15)$$

where, by analogy with the labor index, the capital weights are defined to produce a Fisher quantity index, that is

$$\beta_{jt}^K = \frac{1}{2}(\kappa_{jt} + \kappa_{jt-1}) \, , \qquad (2.16)$$

where

$$\kappa_{jt} = \frac{R_{jt} K_{jt}}{\sum_{j=1}^{J_K} R_{jt} K_{jt}} \qquad (2.17)$$

is the share asset j in the total value of capital income of period t, and R_{jt} represents its rental price.

As mentioned above, the rental price of each asset is usually estimated from surveys of asset prices in resale markets. When this is not possible, the rental price has to be imputed from the expected capital income and the corresponding depreciation and interest rates. Similar to the labor input, we can construct an alternative measure of the capital stock from the simple sum of the disaggregated indexes, so

[8] Here lies one on the major theoretical critiques to growth-accounting exercises: in neoclassical growth theory one cannot determine asset prices without knowing the interest rate and one cannot know the interest rates without knowing asset prices. The mainstream solution is to substitute a general-equilibrium approach for aggregate growth accounting.

[9] See Jorgenson (1990) and CBO (2001).

that the difference between the quality-adjusted and the non-quality-adjusted indexes can be used as a proxy of the change in the quality of capital.

2.3 – The output gap

The previous analysis described how to estimate the input series necessary to construct a net-value production function. The next two sections will describe how to use such a function to estimate potential output, but, before we do that, it is worthy to analyze the implications of an output gap for growth. In some situations it is possible for income to grow much faster (or slower) than potential output while still converging to the latter.

In macroeconomic terms potential output is usually defined as sustainable output, that is, "the level of real GDP that is consistent with a stable rate of inflation" (CBO 2003). Potential output is not maximum output and, in any given period, effective output may be above or below it. When this occurs, the long-run growth rate of potential output is not a good indicator of the short-run growth potential of the economy. For instance, take the case where output is below its potential level, because of this gap, it is possible for effective output to grow faster than potential output in the next period without necessarily reaching the latter. How fast? It obviously depends on the output gap: the larger the gap the faster the possible growth rate.

In order to measure the possible deviations between short-run and long-run potential-output growth rates, let g_t^* be the growth rate necessary for output to reach its potential level in period *t*. In discrete time we have

$$g_t^* = \left(\frac{1 + g_{Pot,t}}{1 - h_{t-1}} \right) - 1 \qquad (2.18)$$

where $g_{Pot,t}$ is the growth rate of potential output in period t and h_{t-1} is gap between potential and effective output, expressed in terms of the latter, in period *t*-1. Table 1 presents some simulations of (2.18) and shows that, for instance, if the current output gap is 2% and potential output is expected to grow at 5%, effective output can grow at approximately 7% without rising above its potential level in the next period. Even though g_t^* is a nonlinear function of h_t, a linear approximation indicates that every percentage point of the output gap adds approximately one percentage point to g_t^*. The opposite holds for a "negative" output gap (when effective output is higher than potential output).

See table 1.

3 – FIXED COEFFICIENTS AND INPUT-OUTPUT SIMULATIONS

In the previous section we specified the production function in a general way to emphasize that (2.2) is consistent with alternative functional forms. To avoid cluttering the analysis with many examples of production functions, in this and the next sections we will concentrate the investigation on the implications of fixed and flexible technological coefficients of production for growth accounting.[10]

Starting with fixed coefficients and following the net-value approach outlined earlier, assume that F takes a Leontief form, that is

$$Y_t = Min\left[\frac{K_t}{V_t}, \frac{N_t}{B_t}\right], \tag{3.1}$$

where V_t and B_t represent the fixed capital-income and labor-income ratios, respectively. In words, (3.1) means that production uses capital and labor in a fixed proportion, so that output can be limited by a shortage of any of these two inputs. When $K_t / V_t < N_t / B_t$, capital is utilized fully and there are idle labor resources in the economy. When the opposite happens labor is utilized fully and there are idle capital resources in the economy.

The fixed-coefficient hypothesis can obviously be extended to any number of inputs. The basic idea is that we have as many constraints on output as the number of inputs, but only the lowest constraint can be binding in any given period. In terms of the growth-accounting decomposition given in (2.2), this implies setting one of the elasticity parameters equal to one and the remaining equal to zero. The TFP term is reduced to the growth rate of the productivity of the scarcest factor.

In (3.1) we defined production in the simplest possible way to have just two possible constraints on output: labor and capital. The reason is that most heterodox studies on growth and distribution use a net-value Leontief function and assume that either capital or labor is the scarcest factor in capitalist economies.[11] Let us see the implications of each of these two constraints separately before moving to general case.

3.1 – The labor constraint

If labor is the binding constraint, the growth rate of potential output depends on the growth rates of labor productivity and the labor force. Formally, let $Y^N_{\max t}$ be the labor-constrained output level, by definition

[10] The most used forms in the literature are the Cobb-Douglas (CD) function, the Constant-Elasticty-of-Substitution (CES) function, the Transcendental Logarithmic ("translog") function, and the Leontief function. The CD, CES, and Leontief functions are pretty standard and can be found in most graduate microeconomic textbooks. The basic reference for the translog function is Christensen, Jorgenson and Lau (1973), and a summary of its applications can be found in Kim (1992).

[11] For a comparative survey of heterodox and orthodox growth theories see, for instance, Marglin (1984), Dutt (1990), and Foley and Michl (1999).

$$Y^N_{\max t} = \frac{N_t}{B_t} = N_t Z_t , \tag{3.2}$$

where $Z_t = 1/B_t$ is the average labor productivity to simplify the notation.

Potential output is usually defined as a long-run variable, that is, a variable adjusted to exclude cyclical variations. To obtain an estimate of the labor-constrained growth rate we have therefore to eliminate the cyclical components from the right-hand-side of (3.2). The standard procedure is to apply a statistical filter to the corresponding time series, so that productivity can be divided in a trend and a cyclical component and only the former enters in the definition of potential output. Section five will present the main univariate methods used to separate trends from cycles in economic time series, for the moment let us just assume that labor productivity can be defined as

$$Z_t = u_{N,t} Z_t^{trend} , \tag{3.3}$$

where naturally Z_t^{trend} is the trend component of labor productivity and $u_{N,t} = Z_t / Z_t^{trend}$ its cyclical component.

In a long-run analysis we can ignore the cycle component and use just the trend growth rate of labor productivity to estimate potential output. In a short-run analysis the cyclical component should be taken into consideration because labor productivity tends to vary substantially with income. The basic sources of this cyclical pattern are labor hoarding, economies of scale and changes in the composition of employment.

Labor hoarding occurs because firms do not automatically adjust employment to variations in output. In other words, the output-elasticity of employment is smaller than one, meaning that during an upswing output rises faster than employment and vice versa. In mainstream economic theory the foundations of labor hoarding are usually asymmetric information and adjustment costs. Because firms cannot monitor work effort perfectly, they may reduce the variation of employment in order to improve labor relations and, through this, give an incentive for workers to perform at the desired effort level.[12] In the same vein, firms can also reduce the variation of employment because of the sunken costs in labor training. These two hypotheses imply that labor productivity is pro-cyclical, especially at the turning points of the cycle.

Economies of scale are another source of cyclical behavior because not all employees are directly involved in production and because of other non-labor fixed costs. “White-collar” jobs are usually more stable than “blue-collar” jobs and, therefore, when output grows, the average cost of the former falls. By analogy, because any other average fixed cost also tends to vary in the opposite direction of output,

[12] The logic is basically the same of the efficient-wage hypothesis, but the incentive comes in the form of stable jobs rather than higher real wages.

during an expansion economies of scale make income grow faster than the labor input. The opposite holds during a recession.

The third source of cyclicality is the change in the composition of employment between high-productivity and low-productivity sectors. During an expansion the former tends to grow and absorb workers from the latter, which raises the average level of productivity in the economy. The intuition is that the economy can be divided into a "modern" and a "traditional" sector, with the latter functioning as the residual employer of those who cannot find jobs in the modern sector. In developing "low-income" economies, industry is usually the modern sector and agriculture the traditional sector. The same logic can be applied to an industrialized "middle-income" economy as Brazil where low-productivity urban jobs take the role of agriculture as the residual employer.[13] The bottom line is that, in a dual economy, labor productivity is a positive function of the size of the modern sector, which in its turn tends to be procyclical.

In addition to cyclical factors, labor-productivity can also be a function of economic growth in the long run. The intuition comes from the classical proposition that growth increases the division of labor, which in its turn accelerates growth and so on. In the modern growth literature, the endogenous determination of labor productivity is usually associated with the Kaldor-Verdoorn laws, according to which growth leads to faster productivity growth via increasing returns and changes in the composition of income and employment. As summarized by Thirlwall (1983), the original proposition of Kaldor (1966) consisted of three laws: (i) fast GDP growth is associated with fast growth of manufacturing; (ii) because of increasing returns, there is a positive relation between labor-productivity growth and output growth in manufacturing; and (iii) the faster the growth of manufacturing, the faster the transference of labor from non-manufacturing sectors to manufacturing. Overall, the idea is basically the same as outlined earlier, plus a long-run positive relation between growth and labor productivity.

Moving to the number of work hours available for production, the long-run component of the labor input is usually defined by the growth rate of the working-age population and some statistical or economic assumptions about the rate of participation, the rate of employment (or unemployment), and the average number of work hours per employee. More formally, the labor input can be defined as

$$N = \left(\frac{N}{L_E}\right)\left(\frac{L_E}{L_A}\right)\left(\frac{L_A}{L_W}\right)L_W \qquad (3.4)$$

where L_E, L_A, and L_W represent the number of people that are respectively employed, economically active (meaning in the labor force), and in the working-age population. By definition: $L_E \leq L_A \leq L_W$.

[13] The basic reference is Lewis's (1954) dual-economy model. Basu (2003) presents a survey of the more recent literature on the topic.

From (3.4) it is straightforward that the growth rate of the labor input depends not only on the growth rate of the working-age population, but also on changes in the rates of participation (L_A / L_w), employment (L_E / L_A), and in the number of work hours per employee (N / L_E). Because these variables are highly pro-cyclical, the growth rate of the labor force also tends to be highly pro-cyclical.

It should be noted that the cyclical behavior of the labor input does not mean that the labor constraint is completely endogenous. By definition the participation and employment rates have an upper bound at 100%, and the number of work hours per worker has a physical or institutional maximum. Because of this, in the long run the growth rate of the labor input cannot deviate permanently from the growth rate of the working-age population, which is usually a stable parameter determined by migration and fertility and mortality rates. The intuition is that cyclical variations tend to balance out during a sufficiently long interval of time, whereas the precise length of such an interval is a topic to be defined empirically. Long swings in the employment and participation rates are a common feature of economic development and should be taken into consideration in medium-run projections.[14]

In contrast, for a short-run or year-to-year analysis we have to take in consideration how far effective output is from the labor constraint. As discussed in the previous section, a gap between the two variables can alter the short-run sustainable growth rate of the economy. In the case of the labor constraint, most studies emphasize the role of the unemployment rate and focus the analysis on its deviations from the non-accelerating-inflation rate of unemployment (NAIRU). The basic assumption is that there exists one and only one rate of unemployment consistent with a stable inflation rate, which then becomes the most important determinant of the labor constraint.

The NAIRU is usually estimated through a single-equation or multiple-equation econometric model where inflation is a function of the unemployment rate and other variables. From the estimated coefficients and the assumptions about the other cost determinants of inflation, we can obtain the rate of unemployment consistent with a stable or target rate of inflation. However, precisely because inflation has many other determinants than the rate of unemployment, NAIRU estimates tend to be very sensitive to small changes in economic conditions, especially in small open economies. For instance, if the exchange rate falls more than implicit in the NAIRU estimate, the rate of unemployment can fall without increasing inflation. As long as the increase in wages is compensated by the reduction in the exchange rate, inflation remains stable. A similar reasoning can be applied to the other main determinants of inflation as, for instance, the price of energy, the interest rate, and labor productivity. The conclusion is

[14] As we shall see in section six, in the case of Brazil the short-run seems to be any interval between zero and two years, the medium run between two and eight years, and the long run more than eight years.

that the NAIRU is possibly endogenous and highly unstable and, as such, it is not a good guide for inflation targeting.[15]

Finally, it should be noted that because the labor constraint is highly cyclical in the short run and possibly endogenous in the long run, many analysts choose to ignore it on the assumption that income determines employment rather than the other way around. The logic is Keynesian but its implications can be analyzed from the equations above, provided that we change the direction of causality. Given the output level and some assumption about the behavior of labor productivity, we can derive the labor requirement from (3.2). Given the labor requirement and the growth rate of the labor force, we can use (3.4) to derive the employment rate for some given rate of participation and work-employee ratio. The final result is that aggregate demand determines income and employment on the assumption that the labor constraint is almost never binding in capitalist economies.[16]

The simplest way to represent the Keynesian view is to use a variant of Okun's law to model employment as a function of income. For instance, assume that the long-run GDP-elasticity of employment is 0.5. By construction the growth rate of labor-productivity is 50% the growth rate of GDP and, if the labor force is assumed to grow 1.5% per year, as it seems to be the case of Brazil, GDP has to grow at least 3% to keep the rate of unemployment constant.[17]

3.2 – The capital constraint

When labor is not the binding constraint the next suspect is the capital stock. By analogy with the previous analysis, let Y_t^K be the capital-constrained output level, that is

$$Y_t^K = \frac{K_t}{V_t} = u_t^K \frac{K_t}{V_t^{Trend}}, \qquad (3.5)$$

where $u_t^K = V_t^{Trend} / V_t$ represents the rate of capital or "capacity" utilization, that is, the ratio of the trend value to the effective value of V_t. During an upswing the rate of capacity utilization tends to rise because the capital-output ratio tends to fall below its long-run trend. The opposite holds during a downswing and, for most industrialized economies, the capital-output ratio seems to be either constant or rising.

15 The possibility of multiple equilibrium points cannot be discarded a priori. For a summary of the debate over the usefulness of the NAIRU for inflation targeting, see Gordon (1997), Blanchard (1997), Galbraith (1997) and Stiglitz (1997) in the special issue of the Journal of Economic Perspectives on the topic.

16 This does not mean that the labor market does not matter for inflation, but only that labor does not usually poses a quantitative constraint on output. The constraint is usually on wage inflation and it tends to be solved by macroeconomic policy, that is, the rate of unemployment is usually kept on the level necessary to control workers' claims on income. For an outline and the implications of social conflict and effective demand for growth models, see, for instance, Taylor (1991 and 2004).

17 The argument is basically the same as proposed by the Kaldor-Verdoorn laws, with the difference that productivity is modeled through the GDP elasticity of employment. For an analysis of the two demand-led approaches, see McCombie (1983).

Similar to the labor constraint, the capital constraint has an important cyclical component because of fluctuations in capacity utilization, but we can assume that such fluctuations balance out in a long-run analysis. The growth rate of potential output then becomes identical to the growth rate of the capital stock minus the growth rate of the trend capital-output ratio. Also by analogy with the labor constraint, the trend capital-output ratio is usually obtained by applying a statistical filter to the original series or by estimating a non-accelerating-inflation rate of capacity utilization (NAICU).[18]

As we saw in the previous section, the current capital stock can be expressed as a function of past investments flows. The same logic can be applied forwards, in which case the growth rate of the capital stock becomes a function of the investment-income, the capital-income, and the depreciation ratios. More formally,

$$k_t = \left(\frac{I_t}{K_t}\right) - \delta \tag{3.6}$$

where I_t is the gross investment in period t.[19] After some algebraic operations we have

$$k_t = \frac{s_t u_t^K}{\rho_t V_t^{Trend}} - \delta \tag{3.7}$$

where s_t is the investment-income ratio obtained from NIPA data, and ρ_t is the relative price of investment goods (the price of investment goods divided by the output deflator).[20]

Let us consider the economic interpretation of each variable in (3.7) separately. First, an increase in the relative price of capital reduces the growth rate of the capital stock because this means that the same amount of income "buys" a smaller amount of capital goods. In high-income economies the relative of price of capital tends to be a stationary variable and, therefore, it enters only in short-run growth forecasts. In middle and low-income economies the relative price of capital usually shows wide fluctuations and it tends to follow the behavior of the real exchange rate because of the high share of imports in the total supply of capital goods.

Second, an increase in the rate of capacity utilization increases the growth rate of the capital stock because it means that the existing stock of assets is producing more goods than usual and, therefore, the economy can accumulate more capital if the other variables in (3.7) remain constant. Similar to the labor

---

[18] See Corrado and Mattey (1997).

[19] Recall that the capital stock of period *t* is the capital stock at the beginning of period *t*, so that the growth rate of capital measures the change between the beginning and the end of *t*. One can alternatively measure capital at mid-year, but since this would complicate the formulation without adding much qualitative information, we prefer to work with the simpler version given in (3.6).

[20] Equation (3.7) is a modified version of the one presented by Bacha and Bonelli (2004). Most growth-accounting studies define *s* as the saving-income ratio and measure it by the investment-income ratio. Both variables are obviously identical ex-post, but we prefer to use the latter name because a reduction in consumption does not necessarily increase investment in absolute terms. In fact, attempts increase *s* by reducing total consumption may backfire by reducing investment in a higher proportion than income.

constraint, the existence of a "capacity" variable in (3.7) introduces a strong cyclical component in the growth rate of the capital stock.

Third, an increase in the trend component of the capital-income ratio reduces the growth rate of the capital stock because it means that we need a higher amount of capital to produce the same amount of output. An alternative way to say the same thing is to note that the income-capital ratio is the average productivity of capital. When the latter falls, the productive capacity of the economy falls (both for consumer and capital goods) and, therefore, the growth rate of the capital stock decelerates.

Finally, considering the investment-GDP ratio, the interpretation is clear and straightforward: the higher the share capital goods in total production, the higher the growth rate of the capital stock. Most studies of the capital constraint emphasize the central role of the investment-income ratio in the capital constraint on the assumption that the other terms in (3.7) are constant in the long run. From such a perspective, it is possible for effective demand to raise the supply constraints on output, provided that growth is generated or accompanied by an increase in investment. If investment grows faster than the other components of aggregate demand, the growth rate of the capital stock accelerates. To illustrate this, let us rewrite (3.7) as

$$k_r = \left(\frac{1+i_t}{1+k_{t-1}}\right)k_{t-1} + \delta\left(\frac{i_t - k_{t-1}}{1+k_{t-1}}\right) \qquad (3.8)$$

where i_t is the growth rate of gross investment in the previous period.[21]

From (3.8) we can see that the growth rate of the capital stock is stable when $k_{t-1} = i_t$ and, what is most important, given a change in the growth rate of investment, the growth rate of the capital stock follows residually. The adjustment is demand-led but not automatic. For instance, assume that the annual depreciation rate is 4% and that investment and capital have been growing at 3% in the previous years.[22] After a permanent one percentage-point increase in the growth rate of investment, the pace of capital accumulation slowly accelerates until it reaches 4%. In numbers, 35% of the adjustment is completed after five 5 years, 55% after 10 years, 69% after 15 years and so on. Overall it takes approximately 67 years for the adjustment to be completed.

Because of the slow dynamics of the capital stock, it is highly probable that an investment boom would make the economy hit a supply constraint before raising its capital stock substantially. The reason is that the share of investment in income is usually much higher than its share in the current capital stock. In other words, given a change in the growth rate of investment, the adjustment of income is much faster than the adjustment of the capital stock. If the growth rates of other expenditures (consumption and net

[21] See Barbosa-Filho (2000) and Freitas (2002), respectively, for the derivation of the continuous-time and discrete-time versions of (3.8).
[22] We set depreciation at 4% based on the estimate of Reis and Morandi (2003) for Brazil. As usual, the faster the depreciation, the faster the adjustment of the capital stock to investment.

exports) do not decelerate to accommodate the increase in investment, the capital constraint tends to bind very fast even during an investment boom. The slow adjustment of capital to investment is the main real constraint on a demand-led growth strategy.

It should be noted that expressing the growth rate of capital as a function of the growth rate of investment is an important methodological departure from the supply-driven growth models of mainstream theory. If one allows the possibility that an independent investment function determines the capital constraint on the economy, it is then possible to explain the dynamics of potential output as a consequence of effective demand. The causality runs from expenditures to supply rather than the other way around.[23]

3.3 – Input-output simulations and potential output

So far we assumed that only labor and capital could impose a supply constraint on output. In reality there are many other sources of constraints like, for instance, energy or imports.[24] In a multi-sector model with fixed coefficients of production, the insufficient supply of any basic intermediate good can originate a bottleneck in inter-industry relations and, through this, create a supply constraint on total output. In other words, output can be below the labor and capital constraints and still be constrained from the supply side.

In real-world economies growth is an unbalanced process. The growth rates of each sector in the economy hardly coincide but, as long as the differences are not large and fluctuate around a common value, growth can proceed without necessarily hitting a supply constraint. For instance, given a discrepancy between the demand and potential output growth rates in a sector, the rate of capacity utilization changes in the short run, while investment increases potential output to attend the increase in demand in the long run.

The aggregate models presented so far are inadequate to deal with inter-industry bottlenecks. The alternative is to use a multi-sector model and the simplest and most intuitive choice is the input-output (IO) model. The basic idea is to model the output of all sectors or goods of the economy as a function of their intermediate and final demands. To facilitate the exposition, we will organize the analysis in terms of goods on the assumption that each sector produces just one good.[25] Assuming that there exist *m* goods in the economy, the supply-demand equilibrium in all markets can be represent as

$$\mathbf{x}_t + \mathbf{m}_t = \mathbf{c}_t^d + \mathbf{c}_t^m + \mathbf{f}_t^d + \mathbf{f}_t^m \qquad (3.9)$$

[23] See Panico (2003) for an analysis of the role of effective demand in economic growth. Barbosa-Filho (2003 and 2004a) presents a dynamical-accounting model of demand-led growth.

[24] The "import-constraint" is the origin of gap models and the balance-of-payment (BoP) constraint on growth. For an outline of the two approaches, see Taylor (1994) and McCombie and Thirlwall (1997). In addition to real variables, the BoP constraint also involves financial variables and is beyond the scope of this paper. For, the link between trade and finance in the BoP constraint, see, for instance, Barbosa-Filho (2001 and 2004b).

[25] In the general case we need a "market-share" matrix to move from goods to sectors.

where all entries are $m\times 1$ vectors at factors' prices. The left-hand side of (3.9) represents the total supply of all goods in the economy, that is, $\mathbf{x_t}$ is the vector of gross domestic output and $\mathbf{m_t}$ the vector of total imports.[26] The right-hand side of (3.9) represents the total demand, which is divided in four components: the intermediate demand for domestic, $\mathbf{c_t^d}$, and imported, $\mathbf{c_t^m}$, goods; and the final demand for domestic, $\mathbf{f_t^d}$, and imported, $\mathbf{f_t^m}$, goods.

In most countries NIPA disaggregated data are organized in a table of sources and uses of resources similar to (3.9). The main difference is that the NIPA data usually come in market prices, whereas (3.9) should be expressed in terms of the costs of production, that is, it should exclude indirect taxes and allocate the commercial and transport costs of every inter-industry flow to the commercial and transport sectors, respectively.[27] In the case of Brazil we have an additional difference because the NIPA annual tables do not separate intermediate demand in a domestic and an imported component.

Since by definition $\mathbf{x_t} = \mathbf{c_t^d} + \mathbf{f_t^d}$, when the data come in the form of (3.9), we can separate imports from domestic output and concentrate the analysis on the latter. The next step is to assume that the intermediate-demand vector is a linear function of the gross-output vector, that is

$$\mathbf{c_t^d} = \mathbf{A_t^d x_t}, \tag{3.10}$$

where $\mathbf{A_t^d}$ is a $m\times m$ matrix of input-output coefficients, that is, the a_{hjt} element of $\mathbf{A_t^d}$ represents the amount of good *h* necessary to produce one unit of good *j* in period *t*. Assuming that $\mathbf{A_t^d}$ is nonsingular, we have

$$\mathbf{x_t} = (\mathbf{I} - \mathbf{A_t^d})^{-1}\mathbf{f_t^d} \tag{3.11}$$

where **I** is the $m\times m$ identity matrix.

In the literature on economic systems (3.11) corresponds to a static open IO model. The model is static because it considers only the demand-creating effects of investment and open because it does not express the final demand and imports as a function of gross or net output. In dynamic models investment is usually modeled as a function of the change in output (the accelerator principle), whereas in closed models consumption is usually modeled as a function of the total value added in the economy (the Keynesian multiplier). These extensions result in slightly more complicated calculations but they do not alter the basic logic of (3.11), that is: we can forecast the output of each good in the economy based on an estimate of the coefficient matrix and of the final-demand vector for domestic goods. Given the focus of this paper, we will concentrate the analysis on the implications of this result for potential output.

---

[26] Again to simplify the analysis, (3.9) is based on the assumption that all imports are competitive, that is, there exists domestic production of all *m* goods. Taylor (1975 and 1979) presents the case with non-competitive imports.

[27] For the basic definitions and hypotheses used to construct an IO matrix, see, for instance, Miller and Blair (1985).

The first issue to consider is how to estimate or forecast the right-hand side of (3.11). The coefficient matrix is usually constructed from a matrix of inter-industry relations and sectoral surveys of inter-industry flows, but it does not tend to be updated frequently in the case of Brazil. The standard approach is to use the coefficients of a given year as guide on the assumption that they did not change substantially between the reference and the forecast periods. An alternative approach is to estimate the coefficient matrix through a computable general equilibrium model where, theoretically, the demand of each input depends on the price and quantity of all inputs and outputs. For instance, we can assume that a Cobb-Douglas or CES function represents the technology of production of each good, and then use some hypotheses about firm behavior and relative prices to update the coefficient matrix to the current period.

The next issue is how to forecast the level and the composition of final demand and imports. This is obviously a nontrivial task and the methodology varies according to the type of expenditure under analysis. For private consumption, the most common strategy is to estimate the corresponding vector through a function that links the consumption of each good with the level and the distribution of income, or to use a target vector as reference. The former approach transforms (3.11) into a closed model for consumption.

For government expenditures the most usual approach is to use the public expenditures implicit in budget targets and plans. Since these numbers are hardly expressed in IO terms, the government budget has to be translated in terms of the m goods of the final-demand vector. In the same vein, aggregate exports and imports are usually obtained from macroeconomic or sectoral studies and based on some forecasts for the exchange rate and the domestic and foreign income levels. The resulting aggregate estimates are then translated in terms of the m goods in the economy. Private investment tends to be determined either by surveys of business' plans and expectations or by the level necessary for supply to grow at the same rate as demand.

It should be noted that the determination of the final demand for domestic goods is not a one-way process, that is, iterative simulations are necessary to adjust macroeconomic forecasts to microeconomic evidence. In fact, this is one of the main advantages of IO models over aggregate models, IO models provide an channel of discussion between macroeconomists and sector specialists to produce and check the consistency of aggregate and multi-sector forecasts of output levels and input requirements.

In order to estimate potential output, we can use (3.11) to check whether the macroeconomic output forecasts are consistent with the capacity of production of each sector. A similar reasoning can be applied to the labor and capital requirements. For instance, given the output vector and an estimate of the labor-output coefficient of each sector, we can calculate the total increase in employment necessary for the economy to reach the macroeconomic output forecast. By analogy, the capital stock necessary to produce the output vector can be estimated through the capital-output coefficients of each sector. The difference between the effective and the "required" capital stock can then be used to calculate the change in each sectors' capital-output ratio, as well as the investment necessary to increase the capital stock in line with demand.

All of the above procedures are implemented by pre-multiplying the output vector by a diagonal matrix containing the appropriate coefficients. More formally, let **D** be the "diagonal" operator, that is, a function that transforms a $m\times1$ vector **z** into a $m\times m$ diagonal matrix **D(z)**, in which the j-th entry of **z** is placed an the j-th diagonal entry of **D**(**z**). The labor input necessary to produce the output vector is given by

$$\mathbf{n}_t = \mathbf{D}(\mathbf{b}_t)(\mathbf{I} - \mathbf{A}_t^d)^{-1}\mathbf{f}_t^d \qquad (3.12)$$

where $\mathbf{n}_t$ and $\mathbf{b}_t$ are the column vectors containing the labor input and labor-output coefficients of all sectors in the economy, respectively.

By analogy, the vectors containing the capital requirements ($\mathbf{k}_t$) and rates of capacity utilization ($\mathbf{u}_t$) can be estimated as

$$\mathbf{k}_t = \mathbf{D}(\mathbf{v}_t)(\mathbf{I} - \mathbf{A}_t^d)^{-1}\mathbf{f}_t^d \qquad (3.13)$$

and

$$\mathbf{u}_t = \mathbf{D}(\mathbf{x}_t^{Pot})^{-1}(\mathbf{I} - \mathbf{A}_t^d)^{-1}\mathbf{f}_t^d \qquad (3.14)$$

respectively, where $\mathbf{v}_t$ is the vector of sectoral capital-output coefficients, and $\mathbf{x}_t^{Pot}$ is a vector whose j-th entry is the potential output of the j-th sector obtained from sectoral studies.

Finally, the matrix version of (2.9) can be used to estimate the investment vector necessary to keep the capital-output ratios of all sectors constant, that is

$$\mathbf{i}_t = \mathbf{D}(\mathbf{v}_t)(\mathbf{I} - \mathbf{A}_t^d)^{-1}\mathbf{f}_t^d - [\mathbf{I} - \mathbf{D}(\boldsymbol{\delta}_t)]\mathbf{k}_{t-1}, \qquad (3.15)$$

where $\mathbf{i}_t$ and $\boldsymbol{\delta}_t$ are the column vectors containing each sectors' investment and depreciation rates, respectively.

From the above equations we can see that even though IO models are simple and intuitive, they are much more data-demanding than aggregate models. In order to forecast output, we need to estimate all inter-industry relations and the level and composition of final demand. To estimate the capital and labor requirements, we also have to estimate the labor-output and capital-output ratios of all sectors in the economy, whereas to estimate capacity utilization we have to estimate of their potential output levels. Because of a lack of good data and the many restrictive assumptions involved in IO simulations, most growth analysis tend to be done through aggregate models in which we need to estimate only a few variables and use just one technological parameter to represent all input-output coefficients of the economy.

4 – FLEXIBLE COEFFICIENTS AND GROWTH ACCOUNTING

Most growth-accounting studies assume that the coefficients of production are flexible and use some version of (2.2) to estimate the growth rate of potential output from the "normal" or long-run growth rates of TFP and the inputs used in production. The other common practice is to assume profit maximization (or cost minimization), perfect competition, and constant returns to scale in order to obtain the elasticity parameters directly from the share of each input in the total value of output. With a net-value production function these shares correspond to the participation of capital and labor in the valued added. With a gross-value function they correspond the participation of each input in the total cost of production.

The simplest and most common growth-accounting studies use a Cobb-Douglas function to decompose the growth rate of the net-value of production in terms of just three components: capital, labor and TFP. The basic reference is the Solow-Swan growth model, in which the growth rate of TFP is an exogenous parameter.[28]

4.1 – The Solow-Swan model and potential output

Assume that there is perfect competition, constant returns to scale, and that firms maximize profit or minimize cost. When the technology of production can be represented by a Cobb-Douglas function, we have

$$Y_t = A_t K_t^{\alpha} N_t^{1-a} \tag{4.1}$$

where A_t is the level of TFP and α is the average share of capital income in total income. From (4.1) the growth decomposition given in (2.2) can be reduced to

$$y_t = a_t + \alpha k_t + (1-\alpha) n_t , \tag{4.2}$$

and we can calculate the growth rate of TFP directly from the functional distribution of income and the growth rates of capital and labor.

There are two possible errors of measurement in calculating TFP from (4.1). First, when the capital stock is not fully utilized in production, changes in the rate of capacity utilization are measured as changes in TFP. Second, as we already saw in sections two and three, when labor is measured by the number of employees (or people in the work force), the TFP also captures the cyclical changes in labor markets. More formally, assume that $u_t^K K_t$ is the amount of capital effectively used in production, where u_t^K is the rate of capacity defined in section three. From (3.4) we can rewrite (4.1) as

[28] See, for instance, Barro and Sala-I-Martin (1995) for the origins and basic characteristics of the Solow-Swan model.

$$Y_t = A_t A_t^{Error} K_t^{\alpha} L_{Wt}^{1-\alpha}, \quad (4.3)$$

where

$$A_t^{Error} = (u_t^K)^{\alpha} \left[(N_t / L_{Et})(L_{Et} / L_{At})(L_{At} / L_{Wt})L_{Wt}\right]^{1-\alpha} \quad (4.4)$$

is the error of measurement implicit the estimates of TFP derived from (4.1). These errors respond for most of the variation in TFP growth in the short and medium run.

In the long-run the cyclical factors tend to balance out and, therefore, the growth rate of TFP is estimated from the long-run or equilibrium values of the capital and labor inputs. In the case of capital, the equilibrium value is obtained by multiplying the existing capital stock by the equilibrium rate of capacity utilization, which in its turn can be defined from the long-run component of the capital-income ratio, from surveys of utilization rates, or from the non-accelerating-inflation rate of capacity utilization. A fourth and simpler alternative is to assume that the potential and effective values of the capital stock are the same, so that only the deviations of the unemployment rate from the NAIRU have an impact on prices.[29]

In the case of labor, the equilibrium value is based on the average ratio of work hours to employees during business fluctuations and on the long-run or non-accelerating-inflation rates of participation and unemployment. Most of the studies emphasizes the latter concept and estimate the NAIRU through some version of the Phillips curve expanded to incorporate inflation expectations.

In addition to the cyclicality created by errors of measurement, TFP can also have a cyclical behavior itself. As we will see at the end of this section, TFP is actually an index of all output-input coefficients in the economy and, as such, it changes according to the changes in relative prices and in the output and input composition during the cycle. For the moment, let us follow the approach of the previous section and assume that A_t can be represented as the product of a cyclical and a trend component, that is

$$A_t = A_t^{Cycle} A_t^{Trend}, \quad (4.5)$$

where $A_t^{Cycle} = A_t / A_t^{Trend}$.

The trend component of TFP can be obtained by applying a statistical filter to the effective series or by a regression of its natural logarithm on a set of trends. These trends are usually defined by multiplying a linear trend by a group of dummy variables, which in their turn are determined by a quantitative or qualitative analysis of the history of the economy in question.[30]

Putting all equations together, the growth-accounting equation becomes

[29] This is the approach of the US Congressional Budget Office (CBO 2001).
[30] For instance, the CBO (2003) includes a "broken-trend" dummy at the peak of each the business cycle of the US economy. The idea is that long-run TFP growth may vary across cycles.

$$y_t = a_t^{Trend} + a_t^{Cycle} + a_t^{Error} + \alpha k_t + (1-\alpha) l_{Wt}, \qquad (4.6)$$

where a_t^{Trend}, a_t^{Cycle}, and a_t^{Error} are respectively the growth rates of the trend, cycle and error-of-measurement components of TFP. The growth rate of potential output can be obtained from (4.6) by assuming that the error and cycle components are zero in the long run, which in its turn is tautologically defined as the period of time necessary for these variables to be equal to zero.

Formally, the growth rate of potential output is given by

$$y_t^{Pot} = a_t^{Trend} + \alpha k_t^{LR}{}_t + (1-\alpha) l_{Wt}, \qquad (4.7)$$

where k_t^{LR} is the long-run growth rate of the capital stock analyzed in section three.

Finally, if we assume further that the economy reaches a steady state in the long run, then the growth rate of the capital stock converges to the growth rate of investment, income grows at the same rate of investment, and the growth rate of potential output is

$$y_t^{Pot} = \left(\frac{1}{1-\alpha}\right) a_t^{Trend} + l_{Wt}, \qquad (4.8)$$

that is, the ultimate sources of growth are TFP and the labor force.

4.2 – Human capital and the AK model

In contrast to the exogenous nature of productivity in the Solow-Swan model, the "New Growth Theory" aims to investigate the determinants of TFP by expanding the production function to include other technological and institutional variables. The list of candidates is as long as the economists' imagination but, for the purpose of estimating potential output, we will concentrate the analysis on the implications of human and physical capital for growth because these are the most widely used variables in growth-accounting studies.[31]

The simplest way to include human capital in the analysis is to rewrite (4.1) as

$$Y_t = K_t^{\alpha} (E_t N_t)^{1-\alpha} \qquad (4.9)$$

where E_t is an index of labor efficiency. By definition $E_t = A_t^{1/(1-\alpha)}$, that is, TFP growth augments the labor input.

The next step is to assume that labor efficiency is a positive function of the stock of human capital in the economy H_t, that is

[31] See, for instance, Barro and Sala-I-Martin (1995) for a survey of the mains aspects of the new growth theory. Sala-I-Martin's (1997) "four million regressions" test almost all hypotheses regarding the determinants of TFP.

$$E_t = \Phi_t (H_t)^{\zeta} \tag{4.10}$$

where Φ_t represents the non-human-capital determinants of labor efficiency, and ζ is a positive parameter that represents the human-capital elasticity of labor efficiency. From (4.10) and (4.9) we have

$$y_t = \phi_t + \zeta h_t + \alpha k_t + (1-\alpha) n_t \tag{4.11}$$

where ϕ_t and h_t are the growth rates of Φ_t and H_t, respectively.

In relation to the growth decomposition of the Solow-Swan model, (4.11) divides the growth rate of TFP in a human-capital and a non-human capital component. In economic terms, part of the increase in the efficiency of the economy can be explained by the increase in the quality of its labor input, which is usually measured by the average schooling of the its working-age population.

As before, all variables in (4.9) are subject to errors of measurement and can be divided in a trend and a cycle component. In the same vein, an estimate of potential output can be obtained from the trend or equilibrium values of each variable under the assumption that the errors of measurement and cycle components balance out in the long run.

The AK model is another popular alternative to the Solow-Swan model because of its simplicity and high predictive power.[32] The basic idea is that output depends only on the capital stock and, therefore, the determinants of potential output are the same as in the capital constraint analyzed in section three. The AK model can also be represented as a special case of the labor-efficiency model given in (4.9), provided that we assume that labor efficiency is proportional to the amount of capital per unit of labor. For instance, let us redefine E_t as

$$E_t = A_t^K \frac{K_t}{N_t} \tag{4.12}$$

where A_t^K the average the impact of the capital-labor ratio on labor efficiency. After substituting (4.12) in (4.9) we have

$$Y_t = (A_t^K)^{1-\alpha} K = \widetilde{A}_t K_t \tag{4.13}$$

where $\widetilde{A}_t$ represents the average productivity of the capital stock. In terms of growth rates,

$$y_t = (1-\alpha) a_t^K + k_t \tag{4.14}$$

where a_t^K is the growth rate of A_t^K.

[32] See Bernanke and Gurkaynak (2001).

It should be noted that, since $A_t = E_t^{1-\alpha}$, we can also obtain the growth rate of TFP from the AK model, that is

$$a_t = (1-\alpha)(a_t^K + k_t - n_t). \qquad (4.15)$$

The difference between (4.9) and (4.15) is that the AK growth-accounting equation models TFP as a function of the capital-labor ratio.

According to the AK model, potential output is proportional to the capital stock in the short run and, in the long run, its growth rate can be estimated from the long-run or equilibrium growth rates of capital productivity and the capital stock. By analogy with the Solow-Swan model, the variables in (4.13) are subject to errors of measurement and can be decomposed into a trend and a cycle component for long-run growth forecasts.

The main difference between the Solow-Swan and the AK models lies on the long-run impact of the investment-income ratio on growth. In the Solow-Swan model income and capital tend to converge to the same growth rate and variations of the investment-GDP ratio do not have any permanent impact on growth. In contrast, in the AK model, an increase in the investment-income ratio has a permanent effect on growth because it accelerates capital accumulation and raises labor efficiency and TFP. The opposite holds for a decrease in the investment-GDP ratio and, assuming that the long-run growth rate of capital productivity is exogenous and the relative price of investment goods is stable, the only way for an economy to accelerate the growth rate of its potential output is to increase its investment-income ratio.

When applied to real-world economies, the AK model generally obtains a similar or better statistical fit than the Solow-Swan or human-capital models, that is: fast-growing economies tend to have a high investment-GDP ratio. From the perspective of growth theory, this result can indicate either that the AK model is in fact a better representation of reality, or that the speed of convergence of the Solow-Swan or human-capital model is very slow. Independently of the interpretation, the stylized fact is that investment seems to be the key variable to increase potential output in the medium run, which in the case of Brazil seems to be two and eight years, as we shall see in section six.[33]

4.3 – Disaggregating TFP

So far we analyzed only aggregate growth models based on the net-value approach. The unifying point of these models is that, in the long run, growth depends on the growth rates of capital, labor and TFP. The latter was further decomposed in terms of a human-capital and a non-human component, or a physical-capital and a non-physical capital component. In both approaches we have an exogenous

[33] When we apply the band-pass filter for periods between two and eight years, we obtain business fluctuations that closely reproduce the evolution (exogenous shocks, stabilization plans, institutional crises, etc) of the Brazilian economy in 1980-2003.

technological parameter to represent the exogenous part of TFP and, in some applications of growth accounting, the TFP residual responds for most of the growth rate of income.

The importance of TFP is much reduced when we follow the gross-value multi-sector approach. The reasons are basically two. First, in real-world economies supply shocks are common and tend to alter the value added by production for the same level of input usage. For instance, the oil shocks of the 1970s increased the cost of energy and, through this, they reduced the value added by both capital and labor even when the usage of these factors did not change much. If we follow the net-value approach, changes in the relative price of a key input tend to appear as changes in TFP when the underlying mechanism may actually be market power rather than technology. One can obviously "solve" this problem by assuming that adverse supply shocks are a form negative productivity shocks, but this makes TFP too vague and highly uncertain. In fact, it is always possible to explain growth fluctuations in terms of a net-value growth-accounting equation ex post. However, when TFP includes any possible supply shock, it is very difficult to forecast it from past trends.

More formally, let Y_t^{Gross}, Y_t^{Net} and C_t be the value of gross output, net output and intermediate consumption, respectively. By definition

$$Y_t^{Gross} = Y_t^{Net} + C_t \tag{4.16}$$

and, therefore

$$y_t^{Gross} = \theta_{t-1} y_t + \theta_{t-1} c_{t-1}, \tag{4.17}$$

where $\theta_{t-1} = Y_{t-1} / Y_{t-1}^{Gross}$ is the share of the net value in the gross value of output in the previous period, and y_t^{Gross}, y_t and c_t are the discrete-time growth rates of the corresponding variables in (4.16).

Combining (4.17) with (2.2) we can rewrite the growth rate of TFP as

$$a_t = y_t^{Gross} - \alpha k_t - (1-\alpha) n_t + \left(\frac{1-\theta_{t-1}}{\theta_{t-1}} \right) (y_t^{Gross} - c_t), \tag{4.18}$$

which clearly shows that an adverse supply shock ($y_t^{Gross} < c_t$) decelerates the growth rate of TFP and vice versa. In contrast, when gross output and intermediate consumption grow at the same rate, (4.18) is reduced to net-value growth accounting given by (2.2). In the case of Brazil, supply shocks come often in the form of currency crises because devaluation tends to increase the relative price of intermediate tradable goods.

The second reason to reduce the importance of aggregate TFP is to control for heterogeneous inputs. The idea is basically the same for the net-value and gross-value approaches and we will present it for former to facilitate the exposition. So far we have been working with aggregate variables, which, in principle, represent the average of all sectors of the economy under the assumption that the composition of inputs and outputs remain the same through time. In reality, the growth and development of capitalist economies is marked by structural transformations.

For example, consider the case where growth is characterized by a change in employment from low-productivity to high-productivity sectors, as analyzed in section three. The average labor productivity changes and, if we measure labor just by the number of work hours, this change will be misrepresented as an increase in TFP. Even though aggregate productivity does increase, its ultimate cause is a change in labor allocation instead of a change in the technology of production at the sectoral or firm level. As before, one can rationalize this effect by saying that the aggregate technology changed, but once again this enlarges the definition of productivity to include many other variables than technology.

In order to control for changes in the composition and quality of the labor inputs, let us assume, as in section two, that there are J_N types of labor. In addition to this, let us also assume that there are J_m sectors in the economy. The growth rate of the labor index of sector *m* can be defined as

$$n_{mt} = \sum_{j=1}^{J_N} \beta_{jmt} n_{jmt} = n_{mt}^{Quantity} + n_{mt}^{Quality} , \tag{4.19}$$

where

$$n_{mt}^{Quantity} = \sum_{j=1}^{J_N} \widetilde{\beta}_{mjt-1} n_{jmt} \tag{4.20}$$

and

$$n_{mt}^{Quality} = \sum_{j=1}^{J_N} (\beta_{mjt} - \widetilde{\beta}_{mjt}) n_{jmt} , \tag{4.22}$$

where the weights β_{jt} and $\widetilde{\beta}_{jt-1}$ have the same definition as in section two, but are applied at the sectoral level, and n_{jmt} is the growth rate of labor input *j* used in sector *m* during period *t*.

In economic terms, the growth rate of the quantity index $n_{mt}^{Quantity}$ is just the sum of the growth rate of each type of labor weighted by its participation in the total number of work hours in sector *m*. The growth rate of the quality index $n_{mt}^{Quality}$ depends on the deviation of the quality-adjusted labor index from the quantity index, where the former is the sum of the growth rates of each type of labor weighted by its participation in the total wage bill of sector *m*. If we measure the sector labor index just by the quantity index, changes in the quality of labor will be counted as changes in TFP.

In addition to changes in labor quality within sectors, we can also control for changes in employment across sectors. By analogy with (4.19), the growth rate of the aggregate labor index can be defined as

$$n_t = \sum_{j=1}^{J_M} \beta_{jt} n_{jt} = n_t^{Level} + n_t^{Composition} \tag{4.22}$$

where

$$n_t^{Level} = \sum_{j=1}^{J_M} \beta_{jt-1} n_{jt} \tag{4.23}$$

and

$$n_t^{Composition} = \sum_{j=1}^{J_M} (\beta_{jt} - \beta_{jt-1}) n_{jmt} \ , \tag{4.24}$$

where the growth rate of the "level" index n_t^{Level} is the sum of the sector growth rates using the weights of the previous period, and the growth rate of the "composition" index $n_t^{Composition}$ measures the change in the composition of employment between periods.

Putting (4.19) and (4.22) together, we can decompose the growth rate of the labor input in terms of a quantity, a quality and a composition index. The same methodology can be applied to the capital and intermediate inputs and, through this, we can obtain a better estimate of the technological and non-technological determinants of aggregate TFP. To see why this is important, consider the disaggregated growth-accounting study of the US by Jorgenson (1990). Table 2 presents his results and can be interpreted as follows: the average annual growth rate of income was 3.28%, of which 1.46% came from the capital input, 1.12 % from the labor input and only 0.70% from TFP. In relative terms, the contribution of capital and labor represented 45% and 34% of the growth in the period, respectively, with TFP coming in third with 21%.

The contribution of capital can be further decomposed in terms of the changes in the quality and quantity of the capital stock, each of which responded for 0.58% and 0.88% of the growth of income, respectively. In the same vein, the contribution of labor can be divided in 0.39% from changes in quality and 0.73% from changes in the quantity indexes. The contribution of TFP can also be divided in two components, 0.88% from changes in productivity at the sectoral level, and -0.17% from changes in the composition of output and input usage across sectors. In other words, 27% of the growth rate can be attributed to productivity gains at the sectoral level. If we had ignored quality and composition effects, the total contribution of TFP would be 1.67%, making it by far the most important source of growth for the US in 1947-85.

See table 2.

For the purpose of decomposing growth ex-post, the multi-sector approach is much more accurate than the aggregate alternative. However, for the purpose of estimating potential output ex-ante, the multi-sector approach is much more difficult because it involves forecasting and solving a general equilibrium model with many parameters and hypotheses about technology, relative prices, and the composition of output and inputs. Because of this, most growth-accounting studies continue to follow the aggregate approach.[34]

4.4 – TFP and accounting identities

Growth accounting is usually based on an aggregate production function but it does not need such an assumption. The NIPA identities can also be used to obtain (2.2) without imposing any aggregate production function on the data. Despite the common result, the interpretation is substantially different. Instead of being a technological variable, TFP becomes a composite index of the real price of each factor of production. As analyzed by Fisher and Felipe (2003), this dual measurement of TFP was a well-known fact in the early literature on growth accounting, but somehow it disappeared in the new growth theory. In fact, most recent works on growth accounting continue to impose an aggregate production function on the data without even mentioning that they are actually estimating an accounting identity.

The accounting definition of TFP can be obtained either from the net-value or the gross-value approach. It can also be obtained at the macro, sectoral and firm level. To facilitate the exposition, we will present the net-value macroeconomic case and indicate the necessary modifications to apply it to the other levels of analysis. The starting point is the income decomposition of the value added by production, that is

$$Y_t = W_t N_t + R_t K_t \tag{4.25}$$

where W_t and R_t represent the real wage and the real rental price of capital, respectively. In terms of the NIPA definitions, (4.25) states that the value added by production is the sum of the total wage bill ($W_t N_t$) and the gross operating surplus ($R_t K_t$), with all variables in real terms.

The growth rate of income in discrete time is

$$y_t = \xi_{t-1}(r_t + k_t + r_t k_t) + (1 - \xi_{t-1})(w_t + n_t + w_t n_t) \tag{4.26}$$

where $\xi_{t-1} = R_{t-1}K_{t-1} / Y_{t-1}$ is the share of capital income in total income in the previous period, and w_t and r_t are the growth rates of the real wage and the real rental price of capital, respectively. For short intervals of time, say, a quarter or a year, we can ignore the high-order term $\xi_{t-1} r_t k_t + (1 - \xi_{t-1}) w_t n_t$ and work with

$$y_t = \xi_{t-1}(r_t + k_t) + (1 - \xi_{t-1})(w_t + n_t) \tag{4.27}$$

[34] For a defense of aggregate, net-value, growth accounting, see CBO (2003).

To obtain the growth-accounting decomposition, let ξ_A be the average share of capital income in total income, instead of the capital-elasticity of income as in (2.2). By construction

$$\xi_t = \xi_A + \varepsilon_t, \qquad (4.28)$$

where ε_t is the difference between the effective and average values of ξ_t. Substituting (4.28) in (4.27) we have

$$y_t = \xi_{t-1} r_t + (1-\xi_{t-1}) w_t + \varepsilon_{t-1}(k_t - n_t) + \xi_A k_t + (1-\xi_A) n_t. \qquad (4.29)$$

The last step is to define a total-factor-income (TFI) index

$$a_t = \xi_{t-1} r_t + (1-\xi_{t-1}) w_t + \varepsilon_{t-1}(k_t - n_t) r_t, \qquad (4.30)$$

such that (4.29) can be rewritten as

$$y_t = a_t + \xi_A k_t + (1-\xi_A) n_t \qquad (4.31)$$

In economic terms, without using any production function we can obtain a growth-accounting equation identical to (2.2). The TFI income index in (4.31) is a random variable and it corresponds to the TFP index in (2.2).[35] The interpretation is obviously different, that is, the TFI index is a distributional rather than a technological variable. Moreover, the coefficients multiplying the growth rates of capital and labor in (4.31) add up to one because the factors' share of income add up to one, not because there is profit maximization under perfect competition and constant returns to scale. What growth-accounting studies actually do is estimate an accounting identity. As long as the functional distribution of income does not vary much and the growth rate of TFI is modeled appropriately, the results will show a good fit because they are based on an accounting identity, not because firms behave "as if" they followed the results of old or new neoclassical growth theory.

It should also be noted that (4.32) allows an alternative interpretation of changes in TFP. For instance, a macroeconomic policy that reduces real wages and does not increase the real rental price of capital tends to result in a negative growth rate of the TFP index. Because it is always true, the NIPA approach is very flexible and can be used in either a demand-led or a supply-driven interpretation of economic growth, as we will see in section six.

Given the dual nature of the residual in growth-accounting studies, a natural point of investigation is whether we can still retain a technological interpretation of it. The answer is yes, provided that we define TFP as an aggregate index of the output-input coefficients at the sectoral or firm level. In this way we do not need to impose an aggregate production function on the data, but still retain the technological interpretation of the residual. To see why, note that a positive growth of TFP can be interpreted as an

[35] For long periods of time the high-order terms should be included in the TFI index.

increase in the efficiency of the economy because it means that the same amount of capital and labor can receive a higher real income. From (4.31):

$$a_t = \xi_A(y_t - k_t) + (1 - \xi_A)(y_t - n_t), \qquad (4.32)$$

that is, the TFI index is a weighted average of the growth rates of labor and capital productivity.

In capitalist economies the capital-income ratio is usually stable or increasing in the long run, which means that the first term in the right-hand side of (4.32) is either zero or negative for long intervals of time. The growth rate of TFP depends therefore on whether or not the growth rate of labor productivity exceeds the rate of capital deepening of the economy.

It should be noted that, because we do not need a production function to obtain (4.32), the average income-labor and income-capital ratios are aggregate variables that include not only the technology at the micro level, but also the composition and scale effects at the macro level. For instance, in the short run growth can be achieved by increasing the utilization of capital, which will appear as a positive growth rate of capital productivity in (4.32). From the demand-led perspective, causality runs from the aggregate output-input indexes to the TFP index, that is, demand-led growth can increase potential output by increasing the average labor and capital productivities in the short run. Whether the same is valid for the long run is a point to be investigated empirically, but, since growth tends to happen together with structural changes during long intervals of time, it is theoretically possible and empirically likely for demand to have permanent effects on TFP.

The NIPA approach to growth accounting can be applied at any level of aggregation (sector, firm, lines of production within firms, etc) and to any definition of output. For instance, in the gross-value approach we can decompose growth from the income side because the total value of output equals the net-value plus intermediate consumption. The difference is that instead of having just two factors of production as in (4.32), we will have as many factors as the total number of inputs to production. As before, the TFI index can be expressed as a weighted average of all output-input coefficients and, through this, it provides an important link between macro and input-output models.

5 – TRENDS AND CYCLES

In the previous sections we assumed that the input and TFP series could be divided into a trend and a cycle component by some statistical methods. This section will present four methods that are commonly used to de-trend univariate economic time series, namely: linear trends, stochastic trends, the Hodrick-Prescott filter, and the band-pass filter.[36] All of these methods are focused on the statistical

[36] Given the mix of theory and econometrics of this paper, we choose to present only univariate methods to simplify the analysis. The intuition is that the multivariate methods apply the same logic of univariate methods to vectors. The most usual way to find long-run trends in multivariate economic time series is to test whether or not there exists a cointegrating vector for

properties of the series under investigation without making any economic assumption about its economic determinants. Because of this, these methods are a-theoretical and can be applied to any theory that separates the analysis into a trend and a cycle component.

In supply-driven theories, the trend component of income is a proxy of its potential level because it represents the value to which the effective series eventually converges. In contrast, in demand-led theories, the trend component of income is determined by the effective level of income, that is, given a permanent acceleration or deceleration of growth, the trend component changes accordingly. Both interpretations are correct statistically. In the one hand, the trend component of income does represent its long run or low-frequency component. In the other hand, the trend component is also a function (usually a weighted moving average) of the effective income series. The same logic can be applied to any of the other series involved in growth accounting.

5.1 – Linear and stochastic trends

Let z_t represent the level of the economic series under analysis. In the case of output and input series, z_t is usually the natural logarithm of the original variable. In the case of ratios (capital-income, employment, capacity utilization etc) z_t is usually the original variable. Because most economic time series exhibit auto-correlation and periodic fluctuations, the natural starting point is to model z_t as an autoregressive process with distributed-lags (ADL), that is

$$z_t = \phi_0 + \tau t + \sum_{j=1}^{J_A} \phi_j z_{t-j} + \sum_{j=0}^{J_M} \theta_j \varepsilon_t . \qquad (5.1)$$

In words, z_t depends on its previous values, on a linear time trend and on the previous values of the disturbance term ε_t.

The linear time trend is usually included for output and input series because these variables tend to grow with time. For "ratio" and "growth" variables the trend is usually omitted because these variables tend to fluctuate around a fixed value. When the evidence indicates it, the linear time trend can be substituted by a nonlinear function of the time index *t*. The usual candidates are polynomials or periodic functions of *t*. The linear time trend can also have structural breaks, which means that we should multiply it by a set of dummy variables.[37]

---

the variables under analysis. If so, their dynamics can be modeled through a vector-error-correction (VEC) model, as shown by Hamilton (1994).

[37] Structural breaks are usually determined by testing the stability of the residuals and the most common approach is the Chow breakpoint test. For the application of this test, see, for instance, Greene (2002).

The past values of z_t are included to represent the autocorrelation of the series. The number of lags is normally determined by specification tests and by the number of observations available. If the roots of

$$\left(1-\sum_{j=1}^{J_A}\phi_j z^j\right)=0 \tag{5.2}$$

lie outside the unit circle, z_t is trend-stationary if $\tau \neq 0$ and stationary if $\tau = 0$. In both cases it can be represented as an infinite moving-average process.[38]

If the roots of (5.2) fall on or inside the unit circle, z_t is nonstationary and statistical inference based on (5.1) is not valid because the variance of z_t is a linear function of time. When this happens we move to the first difference of the series and test it for the existence of a unit root. If this hypothesis is rejected, we say that z_t is I(1), that is, integrated of order one. If the first difference is also nonstationary, we move to second difference and so on. The order of integration of the series is the number of times we have to difference it to obtain a stationary series.

The moving-average component of (5.1) aims to capture the periodic fluctuations of z_t and, by assumption, the disturbance term ε_t is a white noise, that is, an identically and independently distributed (IID) random variable.

Most input and output time series are integrated of order one, so that we have to work with their first difference. To facilitate the exposition, assume that z_t is already the first difference of the natural logarithm of the original series. Because z_t represents a growth rate in this case, we do not include a time trend in (5.1) and the long-run growth rate of the underlying variable is given by the constant and auto-regressive coefficients, that is

$$z_{Long-run} = \frac{\phi_0}{1-\sum_{j=1}^{J_A}\phi_j}. \tag{5.3}$$

Possible structural breaks of the growth rate are represented by exogenous changes in the constant coefficient ϕ_0.

An alternative but similar approach is to divide a nonstationary series explicitly into a stationary cycle component and a nonstationary trend component.[39] For instance, assume that z_t is the natural logarithm of an annual variable. We can model it as

[38] The most common tests of the unit-root hypothesis are the Augmented-Dickey-Fueller (ADF) and the Phillips-Perron tests. For the econometrics of time series, see, for instance, Hamilton (1994).

$$z_t = z_t^{Trend} + z_t^{Cycle}, \qquad (5.4)$$

where

$$z_t^{Trend} = \phi_0 + z_{t-1}^{Trend} + \varepsilon_t^{Trend} \qquad (5.5)$$

and

$$z_t^{Cycle} = \sum_{j=0}^{J_M} \theta_j \varepsilon_t^{Cycle} \qquad (5.6)$$

where z_t^{Trend} and z_t^{Cycle} are the trend and cycle components of z_t, and ε_t^{Trend} and ε_t^{Cycle} are uncorrelated white-noise disturbance terms.

The trend component is stochastic because it includes a disturbance term and, even though this disturbance is not observable, we can estimate it by combining (5.4), (5.6) and (5.7), or by the state-space representation of the data-generating process. Taking the first approach, assume for simplicity that $J_M = 1$ and $\theta_0 = 1$. After substituting (5.4) into (5.5), and then (5.6) in the resulting expression, we have

$$\Delta z_t = \phi_0 - \varepsilon_t^{Cycle} + (1-\theta_1)\varepsilon_{t-1}^{Cycle} - \theta_1 \varepsilon_{t-2}^{Cycle} + \varepsilon_t^{Trend}, \qquad (5.7)$$

so that we can estimate the stochastic trend component from the residuals of (5.7).

In other words, the assumptions implicit in (5.4), (5.5) and (5.6) imply that the growth rate of z_t can be represented as a moving average process, and the residuals of such a regression are the estimates of the stochastic component of z_t^{Trend}. The complete stochastic trend can be constructed ex-post and, to forecast growth, we use the estimated value of ϕ_0.

5.2 – The Hodrick-Prescott and the band-pass filters

The Hodrick-Prescott (HP) filter separates trends from cycles based on a pre-specified smoothing parameter. As in the stochastic-trend approach, the starting point is to model z_t as

$$z_t = z_t^{HPT} + z_t^{HPC}, \qquad (5.8)$$

where z_t^{HPT} and z_t^{HPC} are respectively the HP trend and cycle components.

The innovation of the HP approach is to measure the smoothness of the trend component from the square of its second difference on the assumption that the average value of the cycle component is zero over long intervals of time. More formally, the trend component is defined by minimizing

[39] This approach is based on Watson (1986).

$$\left\{\sum_{t=1}^{T}(z_t - z_t^{HPT})^2 + \lambda\sum_{t=3}^{T}[(z_t^{HPT} - z_{t-1}^{HPT}) - (z_{t-1}^{HPT} - z_{t-2}^{HPT})]^2\right\} \quad (5.9)$$

for the sequence $\{z_t^{HPT}\}_{t=1}^{T}$, where λ is the smoothing parameter. The larger the value of λ, the smoother the value of the trend. From the partial derivative of (5.9) in relation to λ we can see that, for a sufficiently large λ, the trend component approaches a linear trend. In contrast, the lower the value of λ, the closer the trend component is to the original series. For $\lambda=0$, the two series are the same.

The intuitive idea of the HP filter is to define the trend component to minimize the sum of the square of the deviations of the original series from its trend, subject to the restriction that the sum of the square of the second differences of the trend series equals zero. The smoothing parameter is the weight attributed to reducing volatility in the constrained minimization exercise.

The value of λ varies according to the periodicity of the series in question and the assumption of what is a "moderate" variation. One possible rule, proposed by Hodrick and Prescott (1997), is to define λ as the square of the ratio of the "moderate" change in the cyclical component to the "moderate" change in the trend component. Assuming that z_t is a quarterly series expressed in logarithm terms, the most common rule is to define more that 5% as an excessive variation of the cycle component, and more than 0.125% as an excessive variation of the trend component. The resulting smoothing parameter is $\lambda = [5/(1/8)]^2 = 1600$. A similar reasoning can be applied to annual series, for which λ is usually set at 100.[40]

One of the main advantages of the HP filter is its easy application. As shown by Pedersen (1999), from the first-order conditions of minimizing (5.9) we have

$$z_t^{HPC} = \lambda(z_t^{HPT} - 2z_{t+1}^{HPT} + z_{t+2}^{HPT}) \quad (5.10)$$

for t=1;

$$z_t^{HPC} = \lambda(-2z_{t-1}^{HPT} + 5z_t^{HPT} - 4z_{t+1}^{HPT} + z_{t+2}^{HPT}) \quad (5.11)$$

for t=2;

$$z_t^{HPC} = \lambda(z_{t-2}^{HPT} - 4z_{t-1}^{HPT} + 6z_t^{HPT} - 4z_{t+1}^{HPT} + z_{t+2}^{HPT}) \quad (5.12)$$

for t=3,4,…,T-2;

$$z_t^{HPC} = \lambda(z_{t-3}^{HPT} - 4z_{t-2}^{HPT} + 5z_{t-1}^{HPT} - 2z_t^{HPT}) \quad (5.13)$$

for t=T-1; and

$$z_t^{HPC} = \lambda(z_{t-2}^{HPT} - 2z_{t-1}^{HPT} + z_t^{HPT}) \quad (5.14)$$

[40] Note that this value implies that the ratio of the excessive variations of the cycle to the trend components is 10 to 1.

for t=T.

In matrix notation:

$$\mathbf{z_t^{HPC}} = \lambda \mathbf{C} \mathbf{z_t^{HPT}} \tag{5.11}$$

where $\mathbf{z_t^{HPC}}$ and $\mathbf{z_t^{HPT}}$ are the vectors containing the cycle and trend components of the series, and $\mathbf{C}$ is a $T \times T$ block-diagonal matrix of fixed weights obtained from the first-order conditions above. Since by definition $\mathbf{z_t} = \mathbf{z_t^{HPT}} + \mathbf{z_t^{HPC}}$, we can obtain the trend series by a linear operation on original series $\mathbf{z_t}$, that is

$$\mathbf{z_t^{HPT}} = (\mathbf{I} + \lambda \mathbf{C})^{-1} \mathbf{z_t} \tag{5.12}$$

Despite its wide use in applied macroeconomics, the Hodrick-Prescott filter is not considered a good guide for cyclical fluctuations because, in some cases, it may create spurious cycles and distort unrestricted estimates of cyclical component. The main critique is that, for many economic time series, it is unreasonable to assume the existence of a smooth trend a priori.[41]

A second objection to the HP filter is that if filters only the low-frequency component of the data, leaving seasonal and cyclical effects together in the cycle component. One possible way to reduce this problem is to apply the HP filter to seasonally adjusted series, so that the HP cycle variable measures fluctuations with a "medium-run" periodicity, that is, longer than seasonal factors but smaller that long-run factors. Because this implies some arbitrary definition of what is seasonal and what is structural, a more direct approach is to substitute a band-pass filter for the HP filter, in which case the analyst can directly specify what are the lower and the upper bounds of the cycle.

More formally, the band-pass filter consists of applying two high-pass or two low-pass filters to a time series, so that the remaining component captures only the fluctuations whose periodicity falls within the pre-specified bounds. The high-pass filter excludes the low-frequency component of the data, leaving only short-run fluctuations, as, for instance, less than a year. The low-pass filter excludes the high-frequency component from the data, leaving only the long-run fluctuations as, for instance, more than ten years. The band-pass filter is the combination of two high-pass or two low-pass filters to exclude both the low and the high-frequency fluctuations from the data. The upper and lower bounds of the band-pass filter depends on the history of the economy in question and the objective of the analysis. For the US economy, the "NBER" approach defines cycles as fluctuations with a periodicity higher than six quarters

[41] In substitution to HP filter, Harvey and Jaeger (1993) propose fitting structural time series models to the data. Because their trend-cycle approach is similar to the one adopted by Watson (1986), we will not consider it here. For a detailed analysis of structural time-series models, see Harvey (1989).

and lower than eight years. As we will see in the next section, in the case of Brazil the quarterly GDP data indicate a periodicity between two and eight years.[42]

The theoretical foundations and derivation of the band-pass filter involve translating the series under analysis from the time domain to the spectral domain, but its application is simple and intuitive. The band-pass filter is simply an infinite weighted moving-average of the original series, where the weights of each component are defined by a trigonometric function of the pre-specified lower and upper frequencies.[43] More formally, let f_{lower} and f_{upper} be the lower and the higher frequency limits of the cycle component, that is

$$f_{lower} = \frac{2\pi}{p_{higher}} \tag{5.13}$$

and

$$f_{higher} = \frac{2\pi}{p_{lower}}, \tag{5.14}$$

where p_{higher} and p_{lower} are the higher and lower bounds of the periodicity of the cycle component in the time domain. For example, if we are working with quarterly series and the cycle is defined as the sum of all fluctuations with periodicity between 8 and 32 quarters, $f_{lower} = 2\pi/32$ and $f_{higher} = 2\pi/8$.

Given the upper and lower frequency limits, the band pass filter is defined as

$$z_t^{BPC} = \lim_{L\to\infty} \sum_{j=-L}^{L} \varphi_j z_{t-j}, \tag{5.15}$$

where the weights are

$$\varphi_j = \frac{f_{upper} - f_{lower}}{\pi}, \tag{5.16}$$

for $j = 0$, and

$$\varphi_j = \frac{\sin(f_{upper} j) - \sin(f_{lower} j)}{j\pi}, \tag{5.17}$$

for $j \neq 0$.

Since we cannot apply (5.15) to real-world economies because we do not have infinite time series, we have to use an approximation by limiting the number of terms in it. For quarterly series the common procedure is to use twelve observations in both directions, so that we loose the first and the last three

[42] The origin of the NBER approach is the seminal work of Burns and Mitchell (1946). An updated and recent version can be found in Zarnowitz (1992). For an analysis of the US cycles, see Stock and Watson (1998 and 2002).
[43] See Priestley (1981) for the foundations of low-pass and high-pass filters.

years of data. This choice is not without cost, that is, as we reduce the number of terms in (5.15), the precision of the band-pass filter diminishes because frequencies outside the pre-specified range are partially captured by the cycle component.

In addition to the above, when apply the BP filter to nonstationary series, the sum of the weights of the low-pass or (high-pass) filters in (5.16) and (5.17) has to be one for the resulting cycle component to be stationary. The standard procedure, proposed by Baxter and King (1995), is to compute the difference between one and the sum of the unrestricted coefficients, divide the result by the number of weights in the filter, and then add the resulting average to each unrestricted coefficient.

The band-pass filter gives us the cycle component of a time series for some pre-specified definition of the periodicity of the cycle. The same reasoning can be applied to obtain the trend component of the series through a low-pass filter. By analogy, the short-run component can be obtained from a high-pass filter, or simply from the difference between the effective series and its medium-run and long-run components.

6 – ESTIMATES OF POTENTIAL OUTPUT FOR BRAZIL

Most growth-accounting studies of Brazil follow the aggregate net-value approach and derive the potential growth rate of the economy from either a Cobb-Douglas function or from the growth rate of the capital stock. The labor constraint is not usually emphasized because of Brazil's fast population growth in the last decades, as well as because of its current high rate of unemployment. In addition to this, the high percentage of workers in informal low-productivity jobs also indicates that the labor input can respond very quickly to variations in labor demand.

In the next paragraphs we will analyze the most recent growth-accounting studies and, to facilitate the exposition, we will start with the labor and capital constraints, and then move to the estimates of TFP. In each subsection we will present some growth simulations for alternative values of the parameters and, in the last subsection, we will apply the statistical methods of section five to estimate the cycle and trend components of GDP. These methods can be applied to all of the time series under analysis but we will restrict our investigation to GDP to simplify the exposition.

6.1 – The labor constraint

Assume for the moment that labor is the scarcest factor in Brazil. In this case the growth rate of potential output depends solely on the growth rates of the labor input and labor productivity. As we saw in section three, the labor input grows at the same rate as the working-age population in the long run. In the case of Brazil this rate is between 1% and 2% per year depending on the evolution of the rate of participation. More specifically, the Brazilian population is expected to grow at 1.1% per year in 2005-

2010,[44] but the working-age population is expected to grow faster than that because of a change in its age composition.

In the short run it is important to calculate the impact of fluctuations of the rate of unemployment on the growth rate of the labor input. Assuming that the labor force grows 1.5% per year, table 3 presents the annual growth rate of the labor input based on alternative reductions of the rate of unemployment. For instance, if the current rate of unemployment is 10% and it remains at such a level in the subsequent year, the labor input grows at the exogenous rate of 1.5%. If the current rate of unemployment is 12% and it is expected to fall to 10% in the next year, the growth rate of the labor input accelerates to 3.8%. In general, each 1.0 reduction in the rate of unemployment adds 1.1 to 1.2 to the growth rate of the labor input, all numbers expressed in percentage points. If we assume that the "natural" or "long-run" unemployment rate of Brazil is 6%, the growth rate of the labor input can reach something between 6.0 and 8.4% in the short run without making the economy hit its labor constraint.[45]

See table 3.

Labor productivity is the other (and most important) component of the labor constraint. As we analyzed in section three, this variable is highly procyclical and, to illustrate this for Brazil, figure 1 shows the growth rates of income and labor productivity in 1950-2000 based on Heston et al (2002). There is clearly an almost perfect synchronicity between the two series and, based on the HP trend of labor productivity, its long-run growth rate was approximately 1% at the end of the sample period.

See figure 1.

The synchronized fluctuations in figure 1 also indicate that labor-productivity growth may actually be a consequence rather than a source of income growth, as analyzed in section three. To see whether the same pattern is valid for more disaggregated data, figure 2 presents the growth rates of labor productivity for the agricultural, industrial and service sectors of Brazil, in 1991-2002. Despite the limited number of observations, the growth rates show the same cyclical fluctuation observed in the longer aggregate series of Heston (2002). Agriculture had the highest and most volatile growth rate, whereas the service sector had the lowest and most stable one. For industry, labor-productivity growth was high during the expansion of 1993-97, and mostly negative after that. Considering the whole period, the cumulative growth was 18.4% for the whole economy, 77.8% for agriculture, 30.6% for industry and –0.4% for services. The corresponding annual average rates were 1.4%, 4.9%, 2.3% and zero, respectively.

[44] See Oliveira (1997).
[45] The current unemployment rate in Brazil is between 10 and 12%, depending on the survey adopted. Muinhos and Alves (2003) and Areosa (2004) estimate the Brazilian NAIRU to be between 5 and 6%.

The zero growth of labor productivity in the service sector should not be interpreted only in terms of the technology of production, but also as a result of the dual nature of the Brazilian economy, that is, in Brazil, low-skill activities in the service sector function as an employment of last resort for those who cannot find jobs in the modern sectors of the economy.

See figure 2.

Table 4 presents the growth rates of labor productivity for all sectors of the Brazilian economy. The average numbers indicate that the fastest growth occurred in the production of electrical equipment and in oil refinement (8.2% per year), followed closely by communications (8%). In contrast, the reduction of labor productivity was more severe in the clothing sector (-2.2%), followed by private nonmarket services and plastic processing and products (both sectors with -1.4%), and services to households (-1.2%). Since most of the fast growth happened in the production of tradable industrial goods, we can conclude the average labor-productivity growth may accelerate if economic growth comes together with an increase in the size of the "modern" tradable sector of the economy, as analyzed in section three.

See table 4.

Altogether, the long-run growth rates of population and labor productivity indicate that the most conservative estimate of potential-output growth is 3% according to the labor constraint.[46] However, given the increase in Brazil's unemployment rate in recent years and the large discrepancy between the sectoral productivity growth rates shown in table 4, output may actually growth much faster than 3% for several years as the rate of unemployment falls and labor-productivity growth accelerates along the Kaldor-Verdoorn laws. In short, the labor constraint is not an important limit to growth in Brazil.

6.2 – The capital constraint

The next and most important constraint comes from capital. From our previous analysis, assume that capacity utilization and the capital-income ratios are constant in the long run ($u_t^K = 1$ and $V_t = V_t^{trend}$). In this case the only way to speed up potential output growth is to increase the investment-GDP ratio and/or reduce the relative price of investment goods. For example, assume that the annual rate of depreciation is 3.9% and the capital-income ratio is 3.1, as it seems to be the current case of Brazil. Table 5 shows the growth rate of the capital stock for alternative values of the investment-GDP ratio and small changes in the relative price of investment goods. If the relative price is 1.00 and the investment-GDP ratio is 18%, the capital stock grows only 1.9% per year. If the investment-GDP ratio rises to 20%, the growth rate of the capital stock accelerates to 2.6%. In the same vein, if the investment-GDP ratio

[46] Assuming that the labor force and labor productivity grow 1.5% per year.

remains at 18% and the relative price of capital falls 5%, the growth rate of the capital stock also accelerates to 2.2%.

See table 5.

Comparing the two ways to accelerate capital accumulation, we can see that an increase of the growth rate of capital from 2.0% to 2.6% can be achieved by either a 10% reduction of the relative price of capital or by raising the investment-GDP ratio in 2 percentage points. This trade-off is obviously valid only in the short run because the relative price of capital cannot be reduced continuously. It should also be noticed that the argument run both ways, that is, given an increase in the relative price of capital because of, say, a devaluation of the domestic currency, capital accumulation tends to slow down.[47] In the long run the relative price of capital is stable and the determinants of the capital constraint are fundamentally the investment-GDP and the capital-income ratios.

In the case of Brazil, the capital-income ratio increased from 1.8, in 1950, to 3.1 in 2002, as shown in figure 3. Most of the increase occurred between 1973 and 1983 and it corresponded to investments in residential capital and nonresidential structures. As shown in figure 4, the participation of nonresidential structures in the total capital stock rose from 34%, in 1950, to 52%, in 2002. During the same period the share of residential capital fell from 38%, in 1950, to 20%, in 1976-78, and then rose to 30% in 2000. The counterpart of these movements was a reduction in share of machinery and equipment in the total capital stock, especially during the 1980s. In numbers, machinery and equipment responded for 28% of the capital stock in 1950, 33% in 1976, and only 18% in 2002.

See figures 3 and 4.

Figure 3 also shows that that Brazil's effective capital-income ratio has a strong anti-cyclical component. For instance, it increased substantially during the recession of the early 1980s, decreased during the recovery of the mid-1980s, and increased again during the recession of the early 1990s. In order to obtain a measure of capital utilization, we calculated the Hodrick-Prescott trend of the ratio of non-residential capital to income, and then obtained the corresponding rate of capacity utilization as described in section three. Figure 5 shows the results, which not surprisingly coincide with the growth fluctuations observed in Brazil. For example, the rate of "capital" utilization fell to approximately 91% in the debt-crisis of early 1980s, and rose to 105% in the subsequent export-led recovery.

In addition to the capital-income estimate of capacity utilization, we can also use an index obtained through multi-sector surveys of business decisions and expectations. In Brazil, the longest series is the one calculated by the *Fundação Getulio Vargas* (FGV) for the industrial sector since 1970. Figure

[47] Bacha and Bonelli (2004) find an increase of the relative price of capital to be the main cause of the capital-growth deceleration since the early 1970s. Even though the authors don't emphasize it, most of the changes in the relative price of capital coincide with changes in Brazil's real exchange rate, that is, one can blame the foreign financial fragility of the economy for its slow capital accumulation.

5 compares it with the capital-income estimate and, during most of the period, both variables moved in the same direction. Using the last ten years as a guide, average rate of capacity utilization seems to be approximately 82% according to the FGV measure. Considering the whole sample, the highest value is 90%, in 1973, and the lowest value is 72%, in 1990.

See figure 5.

Figure 6 shows the effective and HP trend values of the growth rate of Brazil's capital stock in 1950-2002. For the three types of capital, there is a clear "wave" pattern starting in the mid-1960s and ending in the early 1990s, with a peak in the mid-1970s. At the end of the sample, the long-run annual growth rate of the total capital stock was approximately 2.3%. The corresponding numbers for residential capital, non-residential structures and machine and equipment were 3.7%, 1.7%, and 1.3%, respectively.

Assuming that the capital-income ratios will remain approximately the same, and that nonresidential structures and machinery and equipment are the most relevant capital inputs to production, the low growth rates at the end of the sample means that potential-income growth is less than 2% at Brazil's current pace of capital accumulation. On the good side, as we saw in section three, by definition the growth rate of the capital stock converges to the growth rate of investment in the long run. The low growth rates of recent years should be interpreted as a result of the low investment-GDP ratio of the 1980s and 1990s rather than an inevitable constraint. If and when the economy resumes growing and the growth of investment accelerates, the pace of capital accumulation will also accelerate.[48]

See figure 6.

6.3 – Input-output simulations

There are many ways for an economy to grow at some given aggregate rate. For instance, Brazil can grow 5% through an investment or a consumption boom. With input-output models we can simulate these alternative hypotheses to check how different growth strategies impact on each sector, and then compare the results with estimates of the productive capacity derived from sectoral studies.

In the case of Brazil the most recent IO matrix available was estimated for 1996, that is, eight years ago and during a period of an appreciated currency. Despite the unreality of the assumption that IO coefficients remained the same since then, table 6 presents some simulations to illustrate the method described in section three. The underlying assumption is that the composition of final demand and imports remain the same as in 1996, as well as that the output-elasticity of employment is one for every

[48] It should also be noted that, because machinery and equipment and nonresidential structures have a shorter service life than residential capital, the growth rate of the capital stock effectively used in production responds more rapidly to changes in investment than the aggregate measure.

sector of the economy. The analysis can be easily updated when a more recent IO matrix becomes available for Brazil.

See table 6.

In economic terms the results indicate that, given an isolated 1% growth in investment, the output of the agricultural sector grows 0.09%, the output of the mining sector grows 0.22%, the output of manufacturing grows 0.21% and so on. Comparing each expenditure category, table 6 shows that an increase in investment has a high impact on the output of the construction sector, followed by mining and manufacturing. The impact of investment on the service and agricultural sectors in almost zero, whereas its impact on imports is almost two times its impact on income. In contrast, an increase in government consumption has a high impact on the service sector and a negligible impact on agriculture and industry. Government consumption and investment have almost the same impact on employment, but government consumption has a much lower impact on imports than investment.

An increase in exports tends to pull agriculture and industry almost at the same rate, having a little impact on the service sector. Its impact on employment is also low, and it tends to increase GDP and imports in the same proportion. Because it is the largest expenditure, consumption has a high impact on the output of almost all sectors of the economy, as well as on total employment. Moreover, the impact on consumption on GDP and total imports is almost the same.

What about capacity utilization? If we assume that potential output does not change much in one year, we can use the numbers in table 6 to obtain a conservative estimate of the change of capacity utilization in the industrial sector. According to the FGV index, capacity utilization was 82% in the second quarter of 2004. Given an 1% increase in investment, industrial output is expected to increase 0.33%, and capacity is expected to increase to (1+0.33%)×82%=82.3%. In other words, it would take an approximately 3.6% growth of investment to increase capacity utilization in one percentage point. The corresponding numbers for exports, government consumption, and private consumption are 9.2%, 31.2% and 2.5%, respectively. In short, moderate growth rates of consumption and investment have a high impact on industrial capacity utilization, only a high export growth rate would have a similar impact, and government consumption have a negligible impact. The same calculations can be done for each industrial sector, provided that we obtain their initial rates of capacity utilization from sectoral studies.

6.3 – Growth-accounting studies

Most growth accounting studies of Brazil follow the net-value aggregate approach and impose a Cobb-Douglas function on the data. The disaggregated approach is not possible because of the lack of data for most of the previous 50 years. Because the Cobb-Douglas function actually mimics an accounting identity, when income growth accelerated, Brazil's TFP growth also accelerated and most growth-accounting studies interpret the latter as the cause of the former. A demand-led interpretation is also possible and much more plausible, as we will se below.

The first study to impose an aggregate Cobb-Douglas function on the Brazilian quarterly NIPA data was Silva Filho (2001). His basic assumption was that output could be modeled as a net-value Cobb-Douglas function of capital and labor with constant return to scales, as analyzed in section five. To obtain the factors' "elasticity" parameters, Silva-Filho used the functional distribution of income as guide and set the capital share of income at 49%. To construct the capital-stock series, he applied the PIM with a depreciation rate of 5% and assumed that growth was balanced prior to 1970. Through these assumptions Silva-Filho was the able to construct an index of the capital stock starting in 1970, which he used to explain growth fluctuations in 1980-2000.

To control for cyclical changes, Silva-Filho multiplied his estimate of the capital stock by the rate of capacity utilization in the industrial sector and obtained the capital input. In the same vein, Silva-Filho multiplied the labor force by the rate of unemployment to estimate the labor input. The TFP index followed residually from the Cobb-Douglas identity and, not surprisingly, the fluctuations of Silva-Filho's TFP estimate coincide with the fluctuations of Brazil's income during the period. Table 7 presents his results, which indicate the negative growth rate of TFP as the main responsible for the lost "decade" of the 1980-92.

See table 7.

A demand-led interpretation of Silva-Filho's results is that the slow income growth of the 1980s was the main responsible for the negative TFP growth. Brazil's 1982 debt crisis and the subsequent transfer of a high percentage of its real resources to rest of the world reduced the value-added by the Brazilian capital and labor inputs in the early and mid-1980s. After that, the "stagflation" of the late 1980s reduced the average productivity of labor, and the low investment-GDP ratio kept the average productivity of capital relatively stable. As also shown in table 7, Silva Filho's estimate of TFP growth can be decomposed as follows: in 1980-92 the reduction in capacity utilization and the slow growth of the economy reduced the average productivities of capital and labor and, through this, resulted in a negative TFP growth rate. In 1993-2000 the opposite happened and TFP growth became positive.

Following the same approach of Silva-Filho, Pinheiro *et al* (2001) also imposed an aggregate production function on the annual Brazilian data. Using data from the IBGE, the IMF and Maddison (2001), the authors managed to construct annual indexes for the physical and human capital stocks of the Brazilian economy starting in 1930. Similar to Silva Filho, Pinheiro *et al* also measured the labor input by the number of people employed and set the capital share of income at 50%. From these assumptions the authors derived the growth rate of MFP and, since they did not find the contribution of human capital to be statistically significant in explaining Brazil's growth rate, we will consider only their two-factor growth accounting. Table 8 presents their results and, for the period after 1980, they replicate the signs but not the levels found by Silva Filho. Overall, capital appears as the main source of growth in 1931-2000.

See table 8.

Similar to Silva Filho's results, the estimates of Pinheiro *et al* can also be interpreted in a demand-led way. From the NIPA perspective, table 8 shows that the contribution of capital productivity to TFP growth was negative during most of the period under analysis. This reflects the increase in the capital-income ratio of Brazil presented earlier, that is, when capital deepening was more than compensated by labor-productivity growth, the final result was a positive growth rate of TFP. When capital deepening proceeded but labor-productivity growth decelerated, the growth rate of TFP became negative. Because of this, TFP can also be interpreted as an endogenous variable determined by the fluctuations of the labor-productivity and capacity-utilization indexes.

Gomes *et al* (2003) did another growth-accounting analysis of Brazil based on a Cobb-Douglas function. Differently from Silva Filho and Pinheiro *et al*, they included human capital in their investigation by assuming that the labor-efficiency index presented earlier is a function of the average schooling of the labor force. Their model is basically the same as the one presented in section four.[49] Their capital stock estimate was obtained from Reis and Morandi (2002) and, similar to the previous two studies, Gomes *et al* defined the labor index as the number of people employed because of the lack of data on the total number of hours worked. Differently from the previous studies, Gomes *et al* set the capital share of income at 40%. Table 9 presents their results for the growth rate of per capita income.

See table 9.

Similar to the results obtained by Silva Filho and Pinheiro et al., table 9 indicates a negative growth rate of TFP in the 1980s and a positive rate afterwards. Also similar to the previous two studies, the results of Gomes et al can also be interpreted in the reverse direction that is, the slowdown of growth led to a reduction in the average productivity of labor and an increase in the capital-income ratio because of idle capacity in industry. These two effects resulted in the reduction in the TFP index in the 1980s. The difference between table 9 and the previous growth accounting is the inclusion of human capital as an independent input. In tables 7 and 8 the contribution of human capital is implicit in the growth rate of TFP.

Bacha and Bonelli (2004) also estimated the growth rate of TFP for Brazil. However, differently from the previous authors, they modified the Cobb-Douglas function by assuming that labor efficiency is proportional to the capital-labor ratio, and then modeled growth through the AK model. The mechanics

[49] The main difference is that Gomes *et al* divided MFP in a frontier and a discounted component. The frontier component is the average growth of rate of labor productivity in the US economy, adjusted to exclude the contribution of human capital. The discounted component is the difference between the effective and the frontier components of TFP growth. The basic idea is that part of the technical progress is global, so that the discounted component measures only the share of it generated locally.

of their model are basically the same as presented in section four. Bacha and Bonelli used the capital-stock estimates of Reis and Morandi (2002) and estimated the global rate of capacity utilization as a weighted average, where 35% of the economy is assumed to operate at full capacity, and the remaining 65% is assumed to operate at the capacity level indicated by the FGV index mentioned earlier. In order to obtain an estimate of the long-run capital-income ratio, the authors set 87% as full capacity utilization. The depreciation rate was set at approximately 4% and the capital share of income at 50%. Table 10 presents the resulting growth decomposition and the economic conclusions are basically the same of the previous studies.

See table 10.

In two recent studies, Muinhos and Alves (2003) and Areosa (2004) used a Cobb-Douglas aggregate function to estimate the output gap in Brazil. Their main objective was to estimate a short-run index of the demand pressures on inflation rather than a long-run estimate of potential-output growth. Despite this difference, their theoretical approach is basically the same as the net-value aggregated growth accounting outlined in section four.

Muinhos and Alves used a Cobb-Douglas function to estimate the growth rate of TFP from the capital and labor inputs. To exclude cyclical variations, both inputs were multiplied by their corresponding "utilization" rates, and the labor and capital elasticity parameters were obtained by a smoothing method similar to the HP filter. [50] In relation to the previous studies, their innovation was to estimate the NAIRU and NAICU by minimizing the square of the first difference of potential output. In other words, they estimated potential output from the observed TFP and capacity-utilization and unemployment-rates series under the constraint that potential output is not too volatile. Their results indicate a NAIRU and NAICU of 5.3% and 84.9%, respectively.

Areosa used a Cobb-Douglas function to estimate potential output without direct measures of the capital and labor inputs. Her strategy can be summarized in three steps. First, as done by Muinhos and Alves, she modeled the output gap as a function of the deviations of the rates of unemployment and capacity utilization from the NAIRU and NAICU, respectively. Second, because the NAIRU and NAICU are not observable, she estimated both of them through a multivariate version of the HP filter constrained to produce trend values that satisfy the Cobb-Douglas constraint on output. Third, given the estimated NAIRU and NAICU, she estimated potential output from the observed income growth and unemployment and capacity-utilization rates. Unfortunately, Areosa did not present her estimate of the long-run growth of the economy or the NAIRU and NAICU rates. From the graphs of her paper we can conclude that, at the end of 2002, the NAIRU and the NAICU were approximately 5% and 76%, respectively. The low NAICU is probably a result of the calibration done by Areosa and should not be taken as a long-run

[50] Given the lack of quarterly data, Alves and Muinhos minimized the square of the second difference of the unobserved quarterly series subject to the constraints that each quarterly measure is positive, and the average quarterly measure is equal to the corresponding annual number.

constraint. In fact, because the NAIRU and NAICU are very sensitive to the weights used in calibrating the minimization exercises done by Muinhos and Alves and by Areosa, the resulting output gap is very sensitive to the authors' assumptions.

Given the centrality of the functional distribution of income for traditional growth accounting, it is worthy to check what the Brazilian data tell us. Table 11 shows the Brazilian functional distribution of income in 1991-2002. The average participations of labor and capital income in total income are 47% and 46%, respectively. The remaining 7% correspond to the income of self-employed persons, which contains both wages and profits. If we assume that self-employed income is distributed in the same way as non-self-employed income, the average labor and capital shares become 50%.

See table 11.

Finally, to estimate the growth potential of Brazil and based on the labor constraint analyzed earlier, assume that employment can grow 1.5% per year in the next ten years. Assume also that the capital share adjusted to include part of self-employed income is 50%. Altogether, these assumptions imply that the long-run growth rate of potential output can be modeled as TFP growth + 0.5×0.015 + $0.5\times$capital-stock growth. For instance, if TFP and the capital stock grow 0.5% and 2% per year, respectively, potential output grows 2.3% per year. If the capital growth rate accelerates to 3%, potential-output growth accelerates to 2.8% and so on. Table 12 presents other estimates based on alternative values of the growth rates of TFP and capital. The corresponding growth rates of labor and capital productivities can be obtained from the difference between potential-output growth and the corresponding input rates.

See table 12.

The natural question is what is the most appropriate number for Brazil? From our analysis of the capital constraint we can conclude that the economy can sustain a capital growth rate of 3% with an investment-GDP ratio of approximately 21%. From the growth accounting studies analyzed above we can conclude that the average TFP growth ranges from 1% to 2%.[51] Combining these assumptions, the long-run potential growth rate is between 3.3% and 4.3%, whereas the short-run rate is probably two or more percentage points higher because of the low capacity utilization and high unemployment of the previous years.

In summary, Brazil can start growing much faster than in last 20 years, provided that growth is led or accompanied by an increase in investment. The intuition is that if investment grows faster, the

[51] Recalling the TFP growth numbers: Gomez *et al* estimated 0.4% for 1990-2000, Silva Filho 0.9% for 1993-2000, Bacha and Bonelli 1.2% for 1994-2002, and Pinheiro *et al* 2.1% for 1994-2000. Because of the endogenous nature of TFP growth analyzed earlier, we believe that something between 1% and 2% is more likely to occur because of scale economies and demand-led labor productivity growth.

economy will probably enter in a virtuous cycle where productivity gains increase profits and wages, which raises aggregate demand, which leads to another increase in income and investment.

6.4 – Statistical estimates of potential-output growth

So far we saw how to estimate of potential-output growth from alternative theoretical definitions of the supply constraint. The remaining task is to check whether these theoretical estimates are consistent with statistical estimates. Considering the annual data first, the ADF test indicates that Brazil's GDP is an I(1) variable and, therefore, its growth rate is I(0).[52]

The first method is to model GDP growth as an ARMA process and, after testing alternative specifications, the results indicate that an ARMA(1,2) model is a good proxy of the underlying data generating process. The results also indicate a structural break after 1980, which marks the beginning of Brazil's current low-growth phase. Table 13 presents the econometric results and, based on the estimated coefficients, the expected long-run growth rate of Brazil was [0.119/(1+0.644)]=7.2% in 1949-80, and just [(0.119-0.089)/(1+0.644]=1.8% in 1981-2003.

See table 13.

The second statistical method is similar to previous one, except that it constraints the GDP series to be an ARMA (1,2) process with a unitary AR coefficient. Given that the ADF tests do indicate the existence of a unit root, this assumption is also a reasonable approximation of the data generating process. In order to obtain the stochastic component of the trend we have to estimate the residuals of the restricted ARMA(1,2) specification of GDP, and then add each residual to previous level of GDP. Table 14 presents the results of the ARMA(1,2) model under the constraint that the AR coefficient equals one.[53] Similar to the results of the unconstrained ARMA model, the long-run growth rate was approximately 7.2% until 1980, and just 2.0% since then. Figure 7 shows the residuals obtained from the restricted model.

See table 14.

See figure 7.

Based on the trend-cycle decomposition presented in section five, we modeled the residuals shown in figure 8 as a MA(2) process. Table 15 presents the econometric results and figure 8 shows the corresponding effective and stochastic-trend components of GDP. Because the two series are basically the same, the stochastic rate of capacity utilization stays within 1% of its equilibrium value of 100 during the whole period, as shown in figure 9. Despite this limited volatility, the fluctuations do replicate the

[52] Because of the limited number of observations, the ADF test was applied to an AR(2) and an AR(1) specification of GDP for the 1949-2003 sample.
[53] Not surprisingly, the Durbin-Watson statistic indicates autocorrelation of the residuals, which we expect to be a MA process.

historical growth fluctuations of Brazil's GDP, that is, they indicate eight trough-to-trough cycles: 1951-57, 1957-64, 1964-69, 1**969-76, 1976-83, 1983-90, 1990-96, and 1996-2001.**

See table 15**.**

See figures 8 and 9

The third method is to apply the HP filter to the data. Starting with the annual series, figure 10 shows the trend and effective levels of GDP when the smoothing parameter equals 100, and figure 11 shows the corresponding HP estimate of the global rate of capacity utilization. Most of the fluctuations stay within 2% of the HP trend and indicate cycles that are less frequent, but more irregular, than the ones obtained from the stochastic trend. In general, we can identify seven trough-trough cycles in 1947-2002, that is: 1947-53, 1953-56, 1956-67, 1967-83, 1983-92,1992-99 and 1999-2003. Even though the turning points and the volatility of the HP cycles do not coincide with the cycles obtained from the stochastic trend, the HP estimate of the trend growth rate is 2.1% at the end of the sample period, that is, almost the same obtained from the stochastic trend.

See figures 10 and 11**.**

Given the irregular and short cycles in figure 11, figure 12 shows the effective and HP-trend growth rates of Brazil's GDP in 1947-2003. Based on the growth instead of the level of GDP, we can identify one long-run growth "wave" in the Brazilian economy since 1947, that is: during the 1950s the trend growth rate was approximately 6.8% and, after decelerating to approximately 6% in the early 1960s, it accelerated and reached the peak value of 8.7% in 1972. Thereafter, the trend growth rate decelerated continuously until it reached the trough value of 1.9% in 1990. Since then, it has remained at approximately 2.1%.

See figure 12.

The results are basically the same when we apply the HP filter to the quarterly seasonally-adjusted 1980-2003 data, figures 13 and 14 show the corresponding level and capacity-utilization variables. At the end of the sample period the HP-trend growth rate is 1.6% per year, but this result is highly influenced by the low growth rates since 1981. In other words, because the HP income trend is a weighted moving average of effective income, its growth rate follows the long-run fluctuations of the latter. To illustrate this, figure 15 shows the effective and HP-trend growth rates of income for 1980-2003. The quarterly data clearly reveal two medium-run growth waves since 1980, with peaks in 1985 and 1994, and a trough in 1990. At the peaks, the value of the growth rate was 1.1% and 0.9% per quarter (4.5% and 3.6% in annual terms), respectively, whereas at the trough value it was only 0.14% per quarter (0.6% in annual terms). Since 2002 the HP-trend growth rate has remained at 0.4% per quarter, that is, 1.6% per year.

See figures 13, 14 and 15.

Finally, to obtain cycles through the band-pass filter, let us define the latter to capture any fluctuation with a periodicity between two and eight years. We chose to set the lower bound at eight quarters instead of six quarters, as done for the US, because this makes the turning points of the cycles coincide with major events in the recent economic history of Brazil.[54] In the other hand, we chose to set the upper bound at eight years because this currently represents two presidential terms in Brazil and, therefore, it best reflects the political business cycle of the economy. Anyway, the results are basically the same when we increase the upper bound to nine, ten years or even twenty years.

Figure 16 shows the corresponding decomposition of Brazil's GDP in terms of a short-run (seasonal), a medium-run (cycle) and a long-run (trend) component. Because we have to use the first and last 12 quarterly observations to apply the BP filter to the data, the filtered series are presented for the 1983-2001 period.

See figure 16.

The short-run component of GDP contains its high-frequency fluctuations and remains within ±2% of the estimated trend during most of the period under analysis. In addition to this, there is a small increase in the volatility of short-run fluctuations during the stagnation and inflation acceleration of 1988-92. In contrast to the short-run component of GDP, the medium-run component fluctuates within ±1% of the trend component, as shown in figure 17. In addition to this, figure 18 shows that the capacity utilization derived from the BP filter also moves in the same direction as the estimate derived from the HP filter, but with smoother variations.

See figures 17 and 18.

The BP quarterly filter reveals five well-defined cycles since 1980. Using the business trough as a reference, the first cycle started in the last quarter of 1983, peaked in the first quarter of 1987, and ended in the third quarter of 1988. The upswing represents the recovery of the Brazilian economy from its 1982 debt crisis and the expansionary effects of the Cruzado stabilization plan (in 1986). The downswing corresponds to the failure of the latter and the growth deceleration caused by the acceleration of inflation. The total duration of the cycle was 19 quarters, that is, almost five years.

The second cycle was very short and it corresponded to the expansion of 1989 and the contraction of 1990. The peak occurred in the third quarter of 1989, and the trough in the fourth quarter of 1990. The total duration of was just 9 quarters (approximately two years) and the underlying mechanisms were the unsuccessful Summer and Collor stabilization plans of 1989 and 1990, respectively. The third cycle was also short because the recovery from the 1990 recession was quickly halted by the political crisis

[54] In addition to this, except for the early 1990s, the cycle component is basically the same with the lower bound at six or eight quarters. We did not set the lower bound at one year because this introduces high-frequency fluctuations in the medium-run component of GDP.

associated with the impeachment of President Collor in 1992. Its peak and trough occurred in the last quarter of 1991 and the second quarter of 1993, respectively. The total duration was just 10 quarters (2.5 year).

The fourth cycle started at the end of 1993 and corresponded to the expansionary effects of the *real* plan on the Brazilian economy. The peak value was reached in the first quarter of 1995, and the trough value in the second quarter of 1996. The underlying determinants were the consumption boom that followed the real *plan* and the restrictive macro policy adopted in the beginning of 1995 to defend the Brazilian currency from the contagion of the Mexican 1994-95 crisis. Altogether the cycle lasted twelve quarters (3 years).

The fifth cycle started in the second quarter of 1996 and corresponded to the loosening of the Brazilian macro policy after international financial markets absorbed the Mexican crisis. The peak occurred in the third quarter of 1997, which was marked by the East Asian crisis. To defend value of the *real*, the government once again resorted to restrictive macroeconomic measures but devaluation was unavoidable. The trough occurred in the second quarter of 1999, that is, shortly after Brazil's 1999 currency crisis. Similar to the fourth cycle, the fifth cycle also lasted twelve quarters (3 years).

The sixth and last cycle started in the second half of 1999 and it seems to have reached a peak in the last quarter of 2000. After that growth decelerated because of the negative impact of the energy rationing and the Argentine crisis of 2001. Given the duration of the previous cycles and the observed growth deceleration of 2001-03, it is probable that the trough of the sixth cycle occurred in the second quarter of 2003, making it 16 quarters long (4 years). If so, a seventh cycle may be under way and, if it follows the 3 to 4-year pattern of the previous three cycles, the Brazilian economy may reach a peak between the end of 2004 and the middle of 2005, and a trough between the middle of 2006 and 2007. However, it should be noted that the peak turning points of the three previous cycles were associated with exogenous shocks (the Mexican crisis of 1995, the East Asian crises of 1997 and the energy rationing and Argentine crisis of 2001). In the absence of adverse exogenous shocks, we can expect the current cycle to last longer than the previous three.

Figure 19 shows the long-run component of GDP according to the BP and HP filters. Because the HP quarterly filter with a smoothing parameter of 1600 is very similar to a low-pass filter for fluctuations of 32 quarters or more, the two trend variables are basically the same. The growth rate of the BP trend variable shows the same two long waves already derived from the HP cycle and, at the end of the sample, it indicates a long-run growth rate of 0.4% per quarter, that is, 1.6% per year.

See figure 19.

Overall the statistical methods allow us to identify the cycles and structural breaks in the evolution of Brazil's GDP. Their estimates of the current long-run growth rate of Brazil vary between 1.6% and

2.1% per year, but because most of these estimates were derived from a moving-average of the effective series, they should not be interpreted as constant. In fact, an increase in the effective growth rate would eventually raise the long-run trend estimates, provided that it lasts, say, more than four years. Contrary to the supply-driven approach of most growth accounting studies, causality runs from the effective to the trend GDP series in the statistical approach.

7 – CONCLUSION

There are many ways to estimate potential output. Economic theory does not give us a unique or right answer, it tells us what to look for. From the previous sections we can conclude that we should look for the capital and labor constraints, as well as for the possible bottlenecks in inter-industry flows. We can also conclude that PTF is an aggregate index of productive efficiency and income distribution derived from an accounting identity, not necessarily from an aggregate production function. In fact, aggregate PTF is a weighted average of all output-input coefficients of the economy and, therefore, it can be interpreted from a supply-driven or a demand-led perspective. From the mainstream supply-driven perspective, long-run TFP growth is usually an exogenous variable that limits the effective growth of the economy for some given growth rates of labor and capital. In contrast, from the heterodox demand-led perspective, TFP growth is an endogenous variable generated by the effective growth of the economy. The recent economic history of Brazil indicates that the latter is a much more plausible explanation for the slow growth since 1981.

Potential output is highly procyclical in the short run. For one or two-year forecasts we have to take in consideration the cyclical behavior of labor and capital productivity, the changes in the composition of output and employment, the variations in the ratio of intermediate consumption to income, and, what is most important, we have to measure the various sources of output gaps to estimate how fast an economy can grow without surpassing its short-run potential output level.

In the long run potential output is endogenous. It depends fundamentally on the labor and the capital constraints, that is, on the growth of population, on capital accumulation and on the increase in the long-run capital and labor productivities. The growth rate of population is the exogenous and most stable parameter. The growth rate of capital is endogenous in the sense that it converges to the growth rate of investment over long intervals of time. The growth rate of the capital stock is also connected with labor and capital productivity because, during an investment boom, capital deepening tends to happen together with an increase in labor productivity and, depending on what is higher, TFP growth can accelerate of decelerate. In short, and not surprisingly, investment is the key variable for long-run growth.

In the case of Brazil labor does not seem to be an important constraint. Because of the high unemployment rates of previous years and the large share of the labor force in informal low-productivity activities, both employment and labor productivity tend to respond very fast to demand variations without generating significant wage-cost pressures on inflation. In other words, employment and labor productivity can be considered endogenous variables rather than a constraint on growth.

The most important short-run constraint on Brazil seems to be the capital stock. Because of the low investment-GDP ratios of the last two decades, Brazil can only grow something between 2% and 3% per year at the current pace of capital accumulation. The statistical estimates indicate that an annual rate of 2% is more likely, but they are heavily biased downward because of the low growth rates of the past 20 years. Statistical estimates tend to extrapolate the past average to the future and, therefore, are not a good guide to potential-output growth if some structural change is under way or expected to happen.

In the case of Brazil the structural change needed is clear and straightforward: the country has to increase its investment-GDP ratio. With a higher investment-GDP ratio labor productivity will increase because of technological gains, scale economies and changes in the composition of employment. With a higher investment-GDP ratio the capital stock will also grow faster if there are no major adverse changes in the relative price of investment goods. TFP growth will follow residually from the changes in labor productivity and the capital-income ratio.

It should be noted that, in the short run, the natural limits to capacity utilization and the possible bottlenecks in inter-industry flows may obstacle an acceleration of income growth before the increase in investment has sufficient time to raise potential output.[55] If one or more key inputs become scarce, inflation is likely to accelerate even when there are still idle labor and capital resources in the economy. Because of this, it is not sufficient to increase investment, it is necessary to increase and plan investment to eliminate or alleviate the main bottlenecks of the economy.

From 1950 through 1980 the government was the main agent behind the investment drive and income expansion of Brazil. After the 1982 debt crisis the investment capacity of the Brazilian government was much reduced and the private sector has not yet taken its role in leading a long-run expansion. The challenge for the near future is to coordinate government and private actions to promote another sustainable expansion of investment.

Finally and most importantly, growth is never a balanced process. During an expansion the economy is bound to experience some bottlenecks. Localized demand pressures are normal and necessary for sustainable growth. Changes in relative prices and in capacity utilization are the main channels through which firms identify profitable opportunities for investment, which in its turn generates another round of effective and potential output growth necessary to sustain the expansion. Because it takes some time for investment to increase potential output, any sustainable growth process is likely to

[55] Assuming obviously that the economy keeps its current account compatible with the availability of foreign finance, so that the BoP constraint does not bind while growth accelerates. When the BoP constraint becomes a problem, the capital and labor constraints become secondary, as indicated by the recent history of Brazil.

experience high capacity-utilization rates in its first years. In fact, if we interpret potential-output growth as a long-run average, the economy must be above it during some time. When macroeconomic policy is managed to reduce growth at the first sign of high capacity utilization, as it seems to be the current case of Brazil, the result is a reduction of both effective and potential output. Precisely when high capacity utilization should lead to an increase in investment, macroeconomic policy enters the stage to slow down the economy. If the government does not allow potential output to grow, the fear of inflation becomes self-fulfilling and the economy tends to experiences a series of short cycles.

The obvious and rational alternative is to give growth a chance. If anything, economic history shows us that sustainable growth experiences are based on a virtuous cumulative process where an increase in aggregate demand induces an increase in investment, which in its turn raises wages and profits and, through this, generates another round of expansion. Our analysis shows that Brazil is ready to start such a process, provided that the government supports it. This does not mean that the government should be careless with inflation, but only that macroeconomic policy should be guided by longer horizons than one year. It also means that government authorities should complement aggregate measures of the output gap with a disaggregated analysis of inter-industry flows in order to avoid killing an expansion because of an unfounded fear of accelerating inflation.

References:

Alves, S.A. and M.K. Muinhos (2003), "Medium-Size Macroeconomic Model for the Brazilian Economy," Texto para Discussão 64, Brasília: Banco Central do Brasil.

Areosa, M.B.M. (2004), Combining Hodrick-Prescott Filtering with a Production function Approach to Estimate Output Gap, paper presented at the conference Seminário 5 Anos de Metas de Inflação, Banco Central do Brasil, (www.bcb.gov.br/Pec/Seminario5Anos/Port/carosmmarPaper.pdf).

Bacha, E.L. and Bonelli, R. (2004). "Accounting for Brazil's Growth Experience – 1940-2002." Texto para Discussão 1018. Rio de Janeiro: IPEA.

Barbosa-Filho, N.H. (2000), "A note on the theory of demand-led growth," Contributions to Political Economy 19.

Barbosa-Filho, N.H. (2001), "The balance-of-payments constraint: from balanced trade to sustainable debt." Banca Nazionale Del Lavoro Quarterly Review, n.219, p.381-99.

Barbosa-Filho, N.H. (2003). "Effective demand and growth in a one-sector Keynesian model." In Salvadori, N. (ed.) Old and New Growth Theories: An Assessment. Cheltenham, UK: Edward Elgar.

Barbosa-Filho, N.H. (2004a), A simple model of demand-led growth and income distribution, Paper presented at the conference "Economic Growth and Distribution: On the Nature and Causes of the Wealth of Nations," Lucca, Italy (http://growth-distribution.ec.unipi.it/).

Barbosa-Filho, N.H. (2004b). "International Liquidity and Growth in Brazil." In G. Epstein (ed.): Financialization and the World Economy, Cheltenham, UK: Edward Elgar (forthcoming).

Barro, R. and X. Sala-I-Martin (1995), Economic Growth, Englewood Cliffs: New York: McGraw-Hill.

Basu, K. (1997), Analytical Development Economics: The Less Developed Economy Revisited, Cambridge, MA: The MIT Press.

Bernanke, B. and Gurkaynak, R. (2001), "Is growth exogenous? Taking Mankiw, Romer and Weil seriously," NBER Working Paper 8365.

Blanchard, O. (1997), "What do we know and do not know about the natural rate of unemployment," Journal of Economic Perspectives 10, pp.51-72.

Burns, A.F. and W.C. Mitchell (1946), Measuring Business Cycles, New York: NBER.

Christensen, L., D.W. Jorgenson and L.J. Lau (1973), "Transcendental logarithmic production frontiers," Review of Economic and Statistics 55, pp-29-45.

Congressional Budget Office (2001). CBO's Method for Estimating Potential Output: An Update. Washington, DC: The Congress of the United States.

Congressional Budget Office (2003). A Summary of Alternative Methods for Estimating Potential GDP, Washington, DC: The Congress of the United States.

Corrado, C. and J. Mattey (1997), “Capacity utilization,” Journal of Economic Perspectives 10, pp.151-167.

Dale W. Jorgenson (1990). “Productivity and Economic Growth.” In: E. Berndt and J. Triplett (ed.), Fifty years of economic measurement, Studies in Income and Wealth, vol. 54. Chicago, IL: University of Chicago Press (reprinted in D.W. Jorgenson, International Comparisons of Economic Growth, Harvard, MA: The MIT Press).

Denison, E.F. (1993). “The Growth Accounting Tradition and Proximate Sources of Growth.” In: A. Szimai, B. van Ark and D. Pilat (eds.) Explaining Economic Growth, Amsterdam: Elsevier, pp.37-64.

Dutt, A.K. (1990), Growth, Distribution and Uneven Development, Cambridge, UK: Cambridge University Press.

Felipe, J. and F. Fisher (2003). “Aggregation in production functions: what applied economists should know.” Metroeconomica 54, pp.208-262.

Foley, D. and T. Michl (1999). Growth and Distribution. Cambridge, MA: Harvard University Press.

Fraumeni, B.M. (1997). “The Measurement of Depreciation in the US National Income and Production Accounts.” US Department of Commerce, Bureau of Economic Analysis, Survey of Current Business, July, pp.7-23

Freitas, F.N.P. (2002). Uma análise da evolução das idéias de Kaldor sobre o processo de desenvolvimento econômico. (Unpublished Ph.D. Dissertation) Institute of Economics: Federal University of Rio de Janeiro.

Friedman. M. (1953), “The methodology of positive economics,” In: M. Friedman, Essays in Positive Economics, Chicago, IL: The University of Chicago Press.

Galbraith, J.K. (1997), “Time to ditch the NAIRU,” Journal of Economic Perspectives 10, pp.93-108.

Gomes, V., Pessôa, S. de A. and Veloso, F.A. (2003). “Evolução da produtividade total dos fatores na economia brasileira: Uma análise comparativa.” Ensaios Econômicos EPGE n.o 483, Rio de Janeiro: EPGE, Fundação Getútlio Vargas.

Gordon, R.J. (1997), “Time-varying NAIRU and its implications for economic policy,” Journal of Economic Perspectives 10, pp.11-32.

Griliches, Z. (1996), “The Discovery of the Residual: A Historical Note,” Journal of Economic Literature 34, pp 1324-30.

Harcourt, G.C. (1969), "Some Cambridge Controversies in the Theory of Capital," Journal of Economic Literature, 7(2), pp.369-405.

Harvey, A.C. (1989), Forecasting, Structural Time Series Models and the Kalman Filter, Cambridge, UK: Cambridge University Press.

Harvey. A.C. and A. Jaeger (1993), "Detrending, Stylized Facts and the Business Cycle," Journal of Applied Econometrics 8, pp231-247.

Heston, A., Summers, R. and Aten, B (2002), Penn World Table Version 6.1, Center for International Comparisons at the University of Pennsylvania (CICUP).

Hodrick, R.J. and E.C. Prescott (1997), "Postwar US business cycles: An empirical investigation," Journal of Money, Credit and Banking 29, pp-1-16.

Hofman, A.A. (1992). "Capital Accumulation in Latin America: A Six Country Comparison for 1950-89." Review of Income and Wealth, Series 38, No.4, pp.365-401.

Hofman, A.A. (2003), "Standardised capital stock estimates in Latin America: a 1950-94 update," Cambridge Journal of Economics, pp.45-86.

Hulten, C.R. (2000), "Total factor productivity: A Short Biography," NBER Working Paper 7471.

Hulten, C.R. and F.C. Wykoff (1981), "The Measurement of Economic Depreciation." In: C.R. Hulten (ed.) Depreciation, Inflation, and the Taxation of Income from Capital, Washington, DC: The Urban Institute, pp.81-125.

Jorgenson, D.W. and Z. Griliches (1967), "The Explanation of Productivity Change," Review of Economic Studies 34(3), pp.249-83.

Kaldor, N. (1966), Causes of the Slow Rate of Economic Growth of the United Kingdom, Cambridge, UK: Cambridge University Press.

Kim, H.Y. (1992), "The translog production function and variable returns to scale," Review of Economic and Statistics, pp.546-52.

Lewis, A.C. (1954), "Economic Development with Unlimited Supplies of Labor," Manchester School of Economic and Social Studies 22, pp.139-191.

Marglin, S.A. (1984), Growth and Distribution, Cambridge, MA: Harvard University Press.

Marquetti, A. (2000), "Estimativa do Estoque de Riqueza Tangível no Brasil, 1950-1998," Nova Economia 10:2, pp.11-37.

McCombie, J.S.L. (1983), "Kaldor's laws in retrospect," Journal of Post Keynesian Economics 5, pp.414-429.

McCombie, J.S.L. and A.P. Thirlwall (1997), "Economic Growth and the Balance of Payments Constraint Revisited." Em: Arestis, P., Palma, G. e M. Sawyer (eds). Markets, Unemployment and Economic Policy: Essays in Honour of Geoffrey Harcourt, vol.2. London: Routledge.

Miller, R. and P. Blair (1985), Input-Output Economics, Foundations and Extensions, Englewood Cliffs: Prentice-Hall.

Morandi, L. and E.J. Reis (2003), O Estoque de Capital Fixo no Brasil, 1950-2000, Mimeo, Rio de Janeiro: IPEA.

Oliveira, J. de C. (1997), "Tendências populacionais no Brasil e pressão sobre o mercado de trabalho futuro," In: Velloso, J.P.d.R. (coord.), Brasil – desafios de um país em transformação, José Olympio Editora.

Panico, C. (2003), "Old and New Growth Theory: what role for aggregate demand?" Em: Salvadori, N. (ed.), Old and New Growth Theories, An Assessment. Cheltenham, UK: Edward Elgar.

Pedersen, M. (1999), Spectral Analysis, Business Cycles, and Filtering of Economic Time Series: A Survey, Institute of Economics, University of Copenhagen, Denmark.

Pinheiro, A.C., I.S. Gill, L. Servén, and M.R. Thomas (2001), Brazilian Economic Growth, 1900-2000, Lessons and Policy Implications, Mimeo.

Priestley, M.B. (1981), Spectral Analysis and Time Series, London: Academic Press.

Sala-I-Martin, X. (1997), "I just ran four million regressions," NBER Working Paper 6252.

Silva-Filho, T.N.T. (2001). "Estimando o Produto Potencial Brasileiro: Uma Abordagem de Função de Produção." Texto para Discussão 17, Brasília: Banco Central do Brasil.

Solow, R. (1956), "A contribution to the theory of economic growth," Quarterly Journal of Economics 70, pp.65-94.

Solow, R. (1957), "Technical Change and the Aggregate Production Function." Review of Economics and Statistics 39(3), pp.312-20.

Stiglitz, J. (1997), "Reflections on the Natural Rate Hypothesis," Journal of Economic Perspectives 10, pp.3-10.

Stock, J.H. and M.W. Watson (1998), "Business Fluctuations in US Macroeconomic Series," NBER Working Paper 6528.

Stock, J.H. and M.W. Watson (2002), "Has the Business Cycle Changed and Why?," NBER Working Paper 9127.

Taylor, L. (1975), “Theoretical foundations and technical implications,”in: Blitzer, C.R., P.B. Clark and L. Taylor (ed.) Economy-Wide Models and Development Planning, New York: Oxford University Press.

Taylor, L. (1979), Macro models for developing countries, New York: McGraw-Hill.

Taylor, L. (1991), Income Distribution, Inflation and Growth: Lectures on Structuralist Macroeconomic Theory, Cambridge, MA: The MIT Press.

Taylor, L. (2004), Reconstructing Macroeconomics : Structuralist Proposals and Critiques of the Mainstream, Cambridge, MA: Harvard University Press.

Thirlwall, A.P. (1983), “A plain man’s guide to Kaldor’s growth laws,” Journal of Post Keynesian Economics 5, pp.345-56.

US Department of Commerce, Bureau of Economic Analysis (2003), Fixed Assets and Consumer Durable Goods in the United States, 1925-97, Washington DC: Government Printing Office.

Watson, M.W. (1986), “Univariate Detrending Methods with Stochastic Trends,” Journal of Monetary Economics 18, pp.49-75.

Zarnowitz, V. (1992), Business Cycles: Theory, History, Indicators, and Forecasting, Chicago: The University of Chicago Press.

Table 1: annual growth rate necessary for output to reach its potential level in the next period given the current gap between effective and potential output and the growth rate of potential output.

Output Gap*	Potential output growth			
	4.0%	5.0%	6.0%	7.0%
-2%	2.0%	2.9%	3.9%	4.9%
-1%	3.0%	4.0%	5.0%	5.9%
0%	4.0%	5.0%	6.0%	7.0%
1%	5.1%	6.1%	7.1%	8.1%
2%	6.1%	7.1%	8.2%	9.2%

*Difference between the potential and effective output levels as a percentage of the former. A positive gap means that effective output is below potential output and vice versa.

Table 2: aggregated and disaggregated growth accounting for the US economy, 1947-85.

Variable	Rate*	%
Average growth rate	3.28	100
Disaggregated accounting		
Contribution of capital	1.46	45
Contribution of capital quality	0.58	18
Contribution of capital stock	0.88	27
Contribution of Labor	1.12	34
Contribution of labor quality	0.39	12
Contribution of hours worked	0.73	22
Contribution of productivity	0.70	21
Sectoral productivity growth	0.88	27
Composition effects	-0.18	-5
Reallocation of value added	-0.19	-6
Reallocation of capital	0.04	1
Reallocation of labor	-0.03	-1
Aggregated accounting		
Average growth rate	3.28	100
Contribution of capital stock	0.88	27
Contribution of hours worked	0.73	22
Contribution of productivity	1.67	51

*Rates in percentage points.
Source: Jorgenson (1990, p.4).

Table 3: annual growth rate of employment for alternative changes in the rate of unemployment and a 1.5% growth rate of the labor force.

Final rate of unemployment	Initial rate of unemployment		
	10%	11%	12%
10%	1.5%	2.6%	3.8%
9%	2.6%	3.8%	5.0%
8%	3.8%	4.9%	6.1%
7%	4.9%	6.1%	7.3%
6%	6.0%	7.2%	8.4%

Table 4: cumulative and average labor-productivity growth rates for Brazil, 1991-2002.

	Cumulative	Average
GDP growth	34.1%	**2.5%**
Labor-productivity growth	18.4%	**1.4%**
Agriculture and cattle	77.8%	**4.9%**
Industry	30.6%	**2.3%**
Mining excluding fuels	63.3%	4.2%
Oil, natural gas, coal and other fuels	10.4%	0.8%
Non-metallic mineral products	45.8%	3.2%

Siderurgy	136.9%	7.5%
Metallurgy of non-iron products	55.3%	3.7%
Other metallic products	29.2%	2.2%
Production and maintenance of machines and tractors	35.8%	2.6%
Electrical machines and equipments	**157.7%**	**8.2%**
Electronic machines and equipments	46.7%	3.2%
Automobile industry	114.6%	6.6%
Other vehicles and vehicles parts	74.4%	4.7%
Wood and furniture products	6.9%	0.6%
Paper and printing	50.5%	3.5%
Rubber industry	86.4%	5.3%
Chemical industry excluding petrochemical products	71.6%	4.6%
Oil refinement and petrochemical products	**158.0%**	**8.2%**
Miscellaneous chemical products	62.1%	4.1%
Pharmaceutical industry	27.3%	2.0%
Plastic industry	**-15.1%**	**-1.4%**
Textile industry	27.8%	2.1%
Clothing industry	**-23.5%**	**-2.2%**
Shoe and leather products	-0.7%	-0.1%
Coffee industry	29.0%	2.1%
Tobacco and other processed vegetable products	33.6%	2.4%
Meat industry	10.0%	0.8%
Dairy industry	19.2%	1.5%
Sugar industry	34.5%	2.5%
Production and refinement of vegetable oil and fat for food products	92.5%	5.6%
Other food and beverage products	40.3%	2.9%
Miscellaneous industrial sectors	19.5%	1.5%
Public utility services	120.3%	6.8%
Construction	8.4%	0.7%
Services	-0.4%	0.0%
Commerce	-9.8%	-0.9%
Transport	-0.8%	-0.1%
Communications	**150.9%**	**8.0%**
Banking and finance	14.7%	1.1%
Services to households	-13.5%	-1.2%
Services to firms	-1.2%	-0.1%
Rents	61.8%	4.1%
Public administration	10.5%	0.8%
Non-market private services	**-15.8%**	**-1.4%**

Source: IBGE, national income and product accounts, 1991-2002 (www.ibge.gov.br).

Table 5: annual growth rate of the capital stock when the capital-output ratio is 3.1, capacity utilization is at its long-run value, and the rate of capital depreciation is 3.9%.

Investment /GDP	*Relative price of capital*				
Ratio*	***0.90***	***0.95***	***1.00***	***1.05***	***1.10***
18%	2.6%	2.2%	1.9%	1.6%	1.4%
20%	3.3%	2.9%	2.6%	2.2%	2.0%
22%	4.0%	3.6%	3.2%	2.9%	2.6%
24%	4.7%	4.2%	3.8%	3.5%	3.1%
26%	5.4%	4.9%	4.5%	4.1%	3.7%

*Excluding changes in inventories.

Table 6: input-output simulations of the impact of a 1% annual growth of investment (INV), exports (EXP), government consumption (GOV), and private consumption (CON) in Brazil.

	INV	EXP	GOV	CON
Agriculture	0.09%	0.11%	0.04%	0.76%
Mining	0.22%	0.34%	0.04%	0.40%
Manufacturing	0.21%	0.16%	0.04%	0.59%
Public utilities	0.09%	0.07%	0.12%	0.72%
Construction	0.94%	0.00%	0.01%	0.04%
Commerce	0.12%	0.05%	0.06%	0.77%
Transport	0.11%	0.16%	0.06%	0.68%
Communications	0.09%	0.06%	0.11%	0.74%
Finance	0.14%	0.07%	0.14%	0.66%
Rents	0.01%	0.01%	0.01%	0.97%
Government services	0.01%	0.01%	0.94%	0.04%
Other services	0.06%	0.04%	0.14%	0.77%
Agriculture	0.09%	0.11%	0.04%	0.76%
Industry	0.33%	0.13%	0.04%	0.49%
Services	0.06%	0.04%	0.30%	0.60%
GDP	0.17%	0.07%	0.20%	0.56%
Employment	0.14%	0.07%	0.13%	0.66%
Imports	0.30%	0.08%	0.06%	0.56%

Source: 1996 input-output matrix of Brazil, IBGE (www.ibge.gov.br).

Table 7: growth accounting for Brazil, 1980-2000.

	1980-1992	1993-2000
Income growth	1.9%	3.2%
Capital growth	2.4%	2.8%
Labor growth	2.9%	2.0%
Growth accounting from a Cobb-Douglas function		
MFP growth	-0.7%	0.9%
Capital contribution	1.2%	1.4%
Labor contribution	1.5%	1.0%
Sources of TFP growth from the NIPA identities		
Contribution of capital productivity	-0.2%	0.2%
Contribution of labor productivity	-0.5%	0.6%

Source: Silva Filho (2001), capital share of income=49%.

Table 8: growth accounting for Brazil, 1930-2000.

	1931-1950	1950-1963	1964-1980	1981-1993	1994-2000
Income growth	5.1%	6.9%	7.8%	1.6%	3.1%
Capital growth	5.3%	8.7%	9.0%	2.6%	2.3%
Labor growth	1.8%	2.8%	3.3%	2.2%	-0.4%
Growth accounting from a Cobb-Douglas function					
MFP growth	1.6%	1.1%	1.7%	-0.7%	2.1%
Capital contribution	2.7%	4.3%	4.5%	1.3%	1.2%
Labor contribution	0.9%	1.4%	1.6%	1.1%	-0.2%
Sources of TFP growth from the NIPA identities					
Capital productivity	-0.1%	-0.9%	-0.6%	-0.5%	0.4%
Labor productivity	1.7%	2.0%	2.3%	-0.3%	1.7%

Source: Pinheiro et al (2001), capital share of income = 50%.

Table 9: decomposition of the growth rate of per capita income.

	1950-59	1960-69	1970-79	1980-89	1990-2000
Per capita income growth	3.8%	3.4%	3.3%	-1.4%	1.7%
TFP growth	1.8%	1.8%	1.7%	-2.6%	0.4%
Contribution of Physical capital	1.8%	1.1%	1.9%	0.4%	0.5%
Contribution of Human capital	0.2%	0.5%	-0.2%	0.8%	0.8%

Source: Gomes et al (2003), capital share of income = 40%.

Table 10: growth accounting for Brazil according to the AK model, 1950-2002.*

	1942-52	1952-65	1965-74	1974-84	1984-93	1994-2002
Income growth	6.9%	6.4%	9.5%	3.9%	2.5%	2.7%
Capital growth	2.3%	1.6%	0.4%	5.1%	1.6%	-0.2%
Labor growth	4.3%	4.9%	6.9%	6.0%	5.0%	3.2%
Growth accounting from a Cobb-Douglas function						
TFP growth	3.6%	3.1%	5.8%	-1.7%	-0.8%	1.2%
Capital contribution	1.1%	0.8%	0.2%	2.5%	0.8%	-0.1%
Labor contribution	2.2%	2.4%	3.5%	3.0%	2.5%	1.6%
Sources of MFP growth from the NIPA identities						
Capital productivity	2.3%	2.4%	4.5%	-0.6%	0.5%	1.5%
Labor productivity	1.3%	0.8%	1.3%	-1.1%	-1.3%	-0.3%

Source: Bacha and Bonelli (2004)

Table 11: functional distribution of income in Brazil, 1990-2002.

Year	Labor compensation	Self-employed	Operational surplus
1990	53%	8%	38%
1991	48%	8%	44%
1992	50%	7%	43%
1993	52%	7%	41%
1994	48%	7%	46%
1995	45%	7%	48%
1996	45%	7%	48%
1997	44%	7%	50%
1998	45%	6%	48%
1999	45%	7%	48%
2000	45%	6%	48%
2001	45%	6%	49%
2002	44%	6%	51%
Average	47%	7%	46%

Source: IBGE (www.ibge.gov.br).

Table 12: annual potential-output growth when the labor input grows 1.5%, the capital share of income is 50%.

Capital-stock growth rate	TFP growth rate			
	0.5%	1.0%	1.5%	2.0%
2%	2.3%	2.8%	3.3%	3.8%
3%	2.8%	3.3%	3.8%	4.3%
4%	3.3%	3.8%	4.3%	4.8%
5%	3.8%	4.3%	4.8%	5.3%

Table 13: ARMA(1,2) model of Brazil's annual growth rate, 1949-2003.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Constant	0.119	0.011	10.58	0.000
Dummy 1980-2003	-0.089	0.016	-5.73	0.000
Growth in t-1	-0.644	0.089	-7.21	0.000
MA coefficient 1	0.996	0.156	6.40	0.000
MA coefficient 2	0.391	0.144	2.72	0.009
R-squared	0.554	Mean dependent var		0.050
Adjusted R-squared	0.519	S.D. dependent var		0.039
Log likelihood	122.529	F-statistic		15.550
Durbin-Watson stat	1.910	Prob(F-statistic)		0.000

Table 14: stochastic trend-cycle decomposition of Brazil's annual growth rate, 1948-2003.

	Coefficient	Std. Error	t-Statistic	Prob.
Constant	0.072	0.005	13.753	0.000
Dummy 1981-2003	-0.052	0.008	-6.417	0.000
R-squared	0.999	Mean dependent var		3.997
Adjusted R-squared	0.999	S.D. dependent var		0.881
Durbin-Watson stat	1.333	Sum squared resid		0.049

Table 15: MA(2) model of the GDP residuals obtained from the model of table 14.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000	0.006	-0.028	0.978
MA(1)	0.295	0.131	2.252	0.029
MA(2)	0.277	0.131	2.106	0.040
R-squared	0.150	Mean dependent var		0.000
Adjusted R-squared	0.118	S.D. dependent var		0.030
Log likelihood	122.469	F-statistic		4.695
Durbin-Watson stat	1.976	Prob(F-statistic)		0.013

Figure 1: annual growth rate of Brazil's real GDP and real GDP per worker. Source: Heston *et al* (2002).

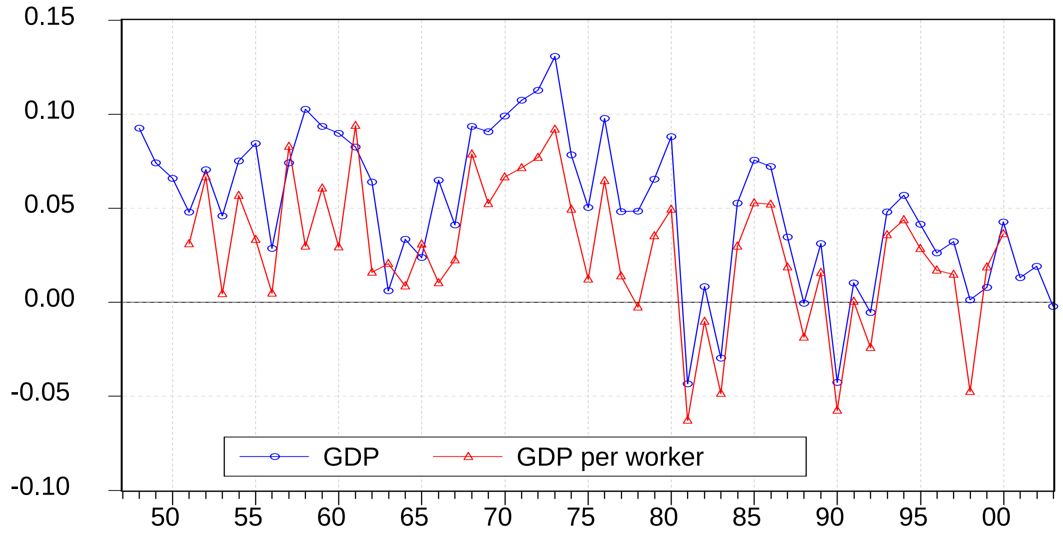

Figure 2: annual growth rate of the average labor productivity in Brazil's agricultural (AGR), industrial (IND), and service (SER) sectors. Source: IBGE (www.ibge.gov.br).

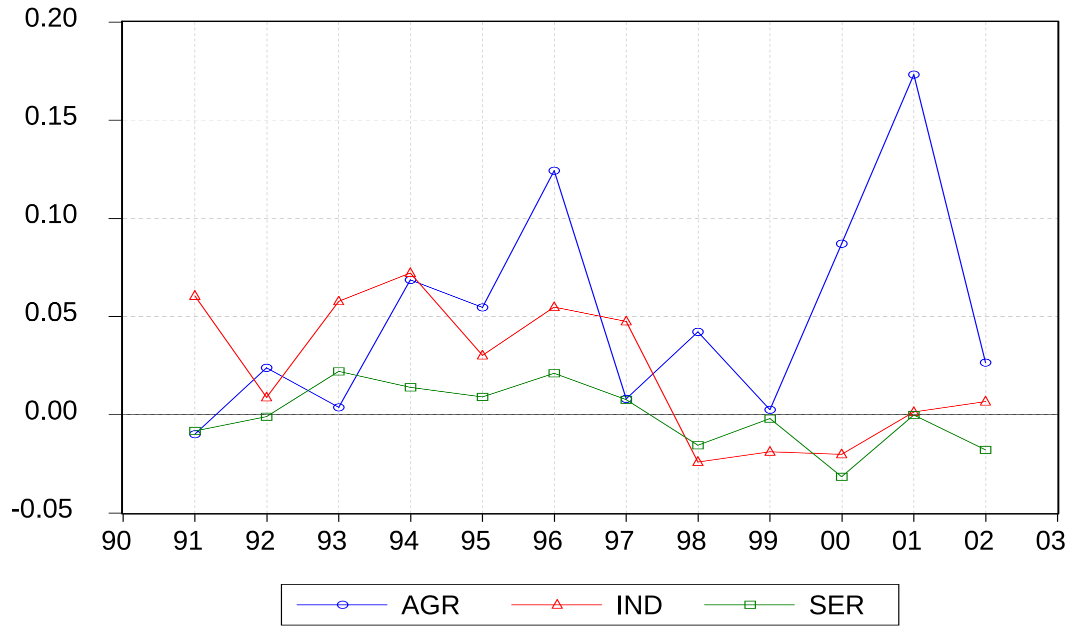

Figure 3: Brazil's capital–income ratio, 1950-2002: RES=residential capital; NRS=Non-Residential Structures; and ME=Machinery and Equipment. Source: IPEADATA (www.ipeadata,gov.br).

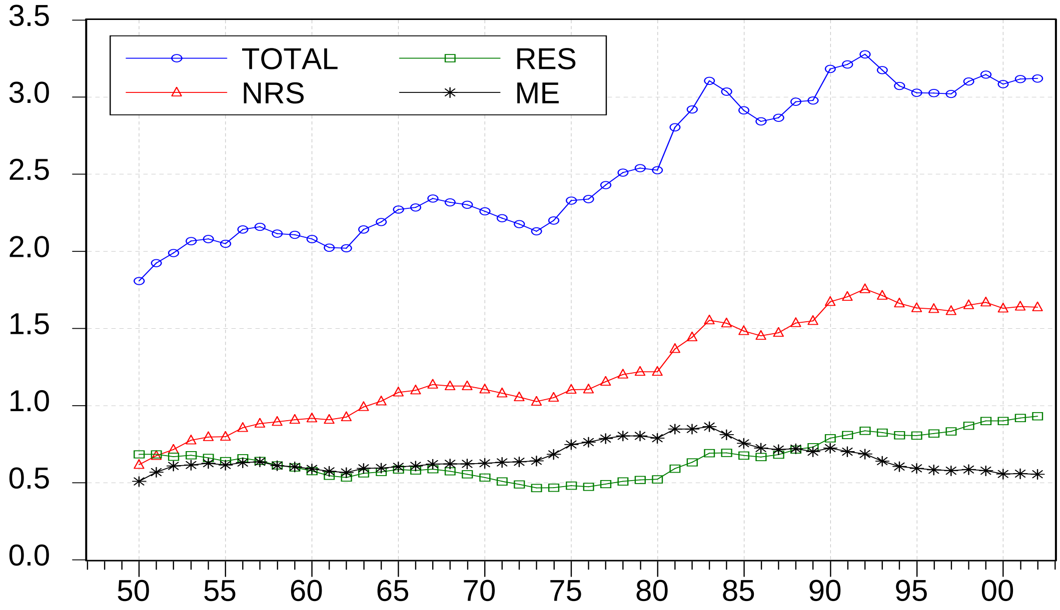

Figure 4: Brazil's capital–stock composition, 1950-2002: RES=residential capital; NRS=Non-Residential Structures; and ME=Machinery and Equipment. Source: IPEADATA (www.ipeadata,gov.br).

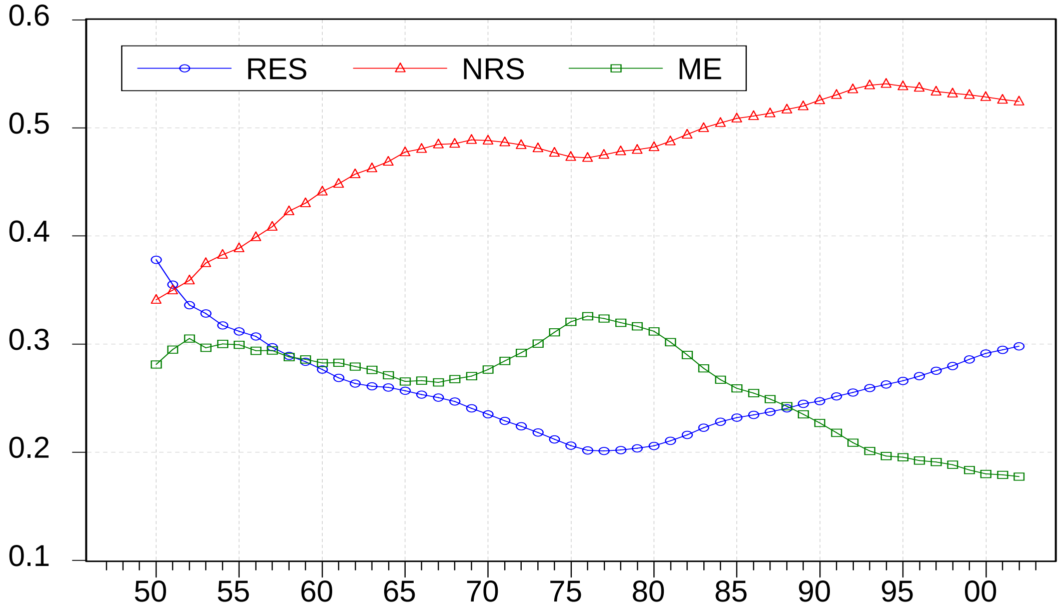

Figure 5: Rate of capacity utilization in Brazil, 1950-2002 (according to the deviations to the capital-income ratio from its trend* and the FGV industrial index). Source: Source: IPEADATA (www.ipeadata.gov.br) and author's calculation.
*Capital-income estimate on the left scale.

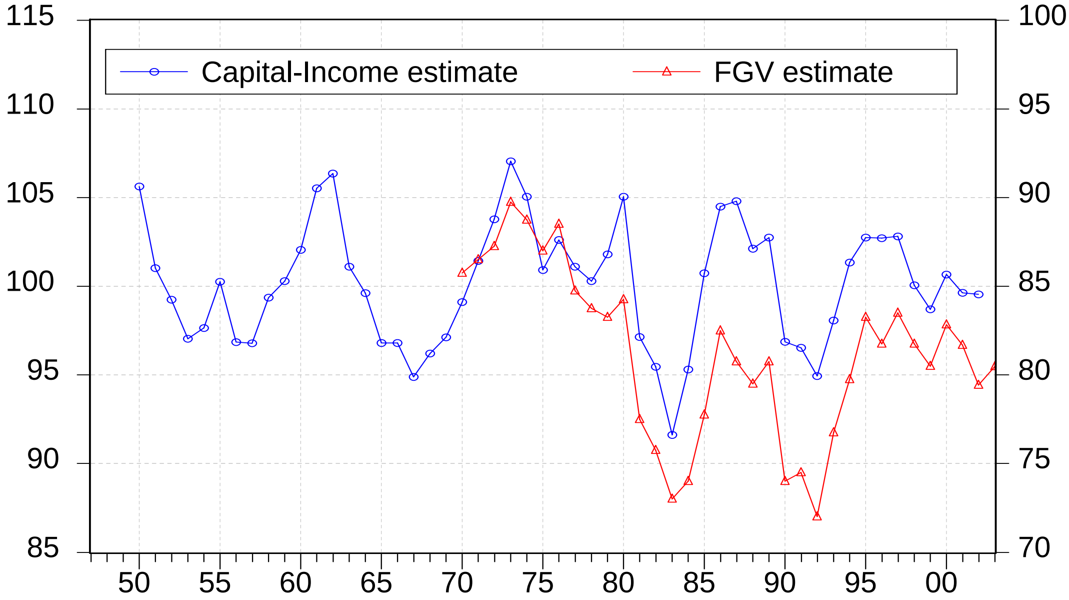

Figure 6: growth rate of the capital stock of Brazil, 1950-2002: RES=residential capital; NRS=Non-Residential Structures; and ME=Machinery and Equipment. Source: IPEADATA (www.ipeadata,gov.br) and author's calculation.

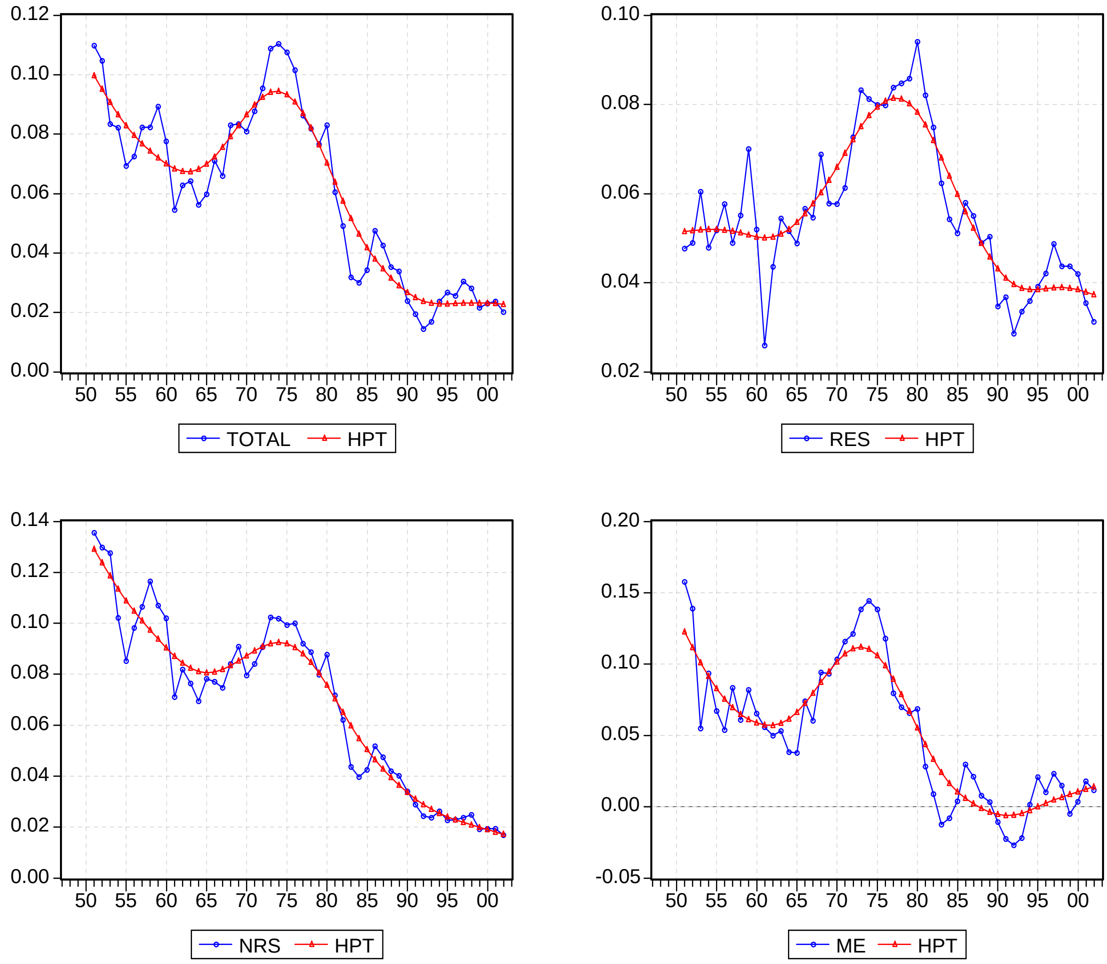

Figure 7: residual terms from a regression of the log of Brazil's GDP on a constant, its previous value, and a structural break in 1980. Period: 1948-2003. Source: IPEADATA (www.ipeadata.gov.br) and author's calculation.

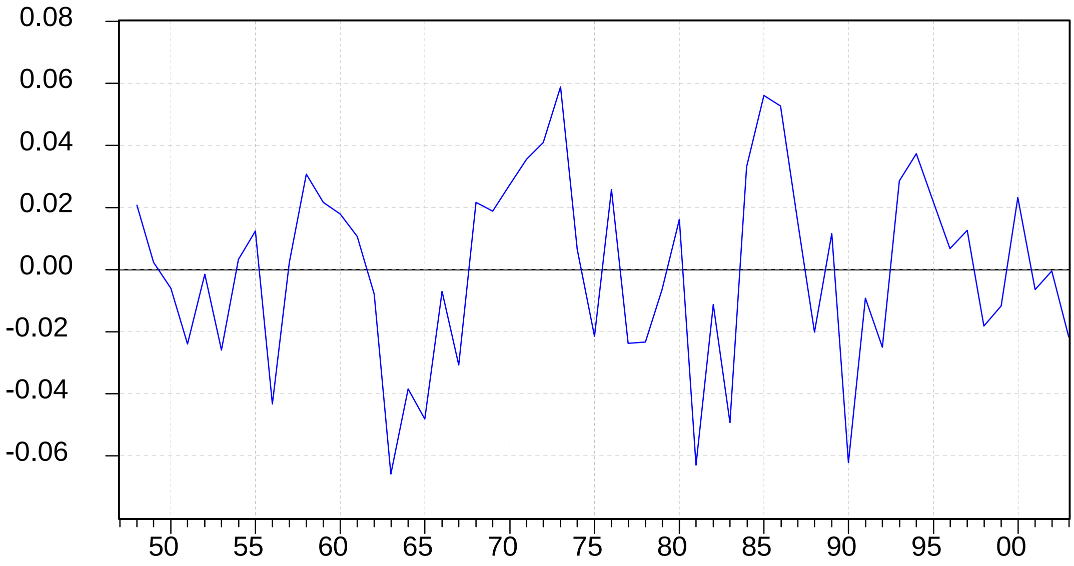

Figure 8: effective and stochastic trend of Brazil's real GDP (in log terms), 1947-2003. Source: IPEADATA (www.ipeadata.gov.br) and author's calculation

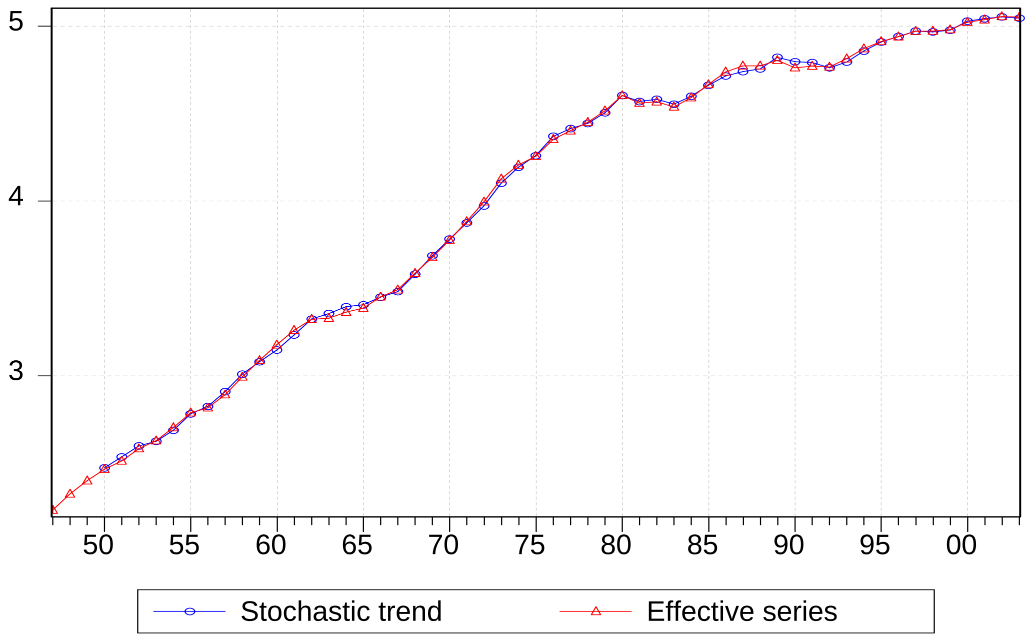

Figure 9: rate of capacity utilization measures as the ratio of the effective to the stochastic trend of Brazil's real GDP (in log terms), 1947-2003. Source: IPEADATA (www.ipeadata,gov.br) and author's calculation.

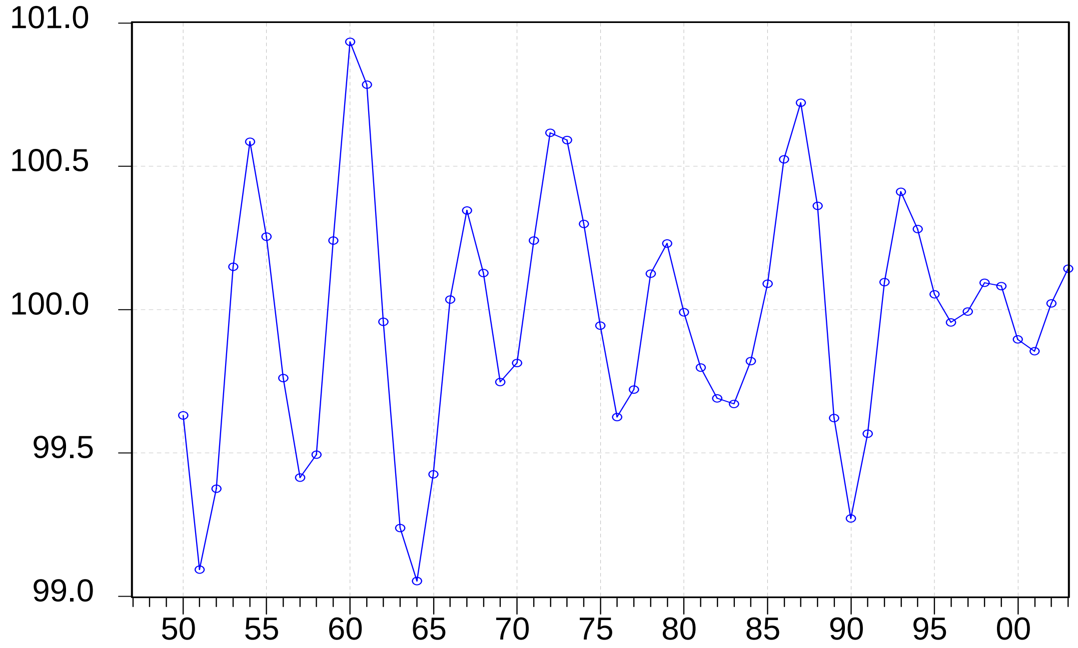

Figure 10: effective and Hodrick-Prescott-trend (HP) value of Brazil's annual GDP, in log terms, in 1947-2003. The HP estimate corresponds to a smoothing parameter of 100.

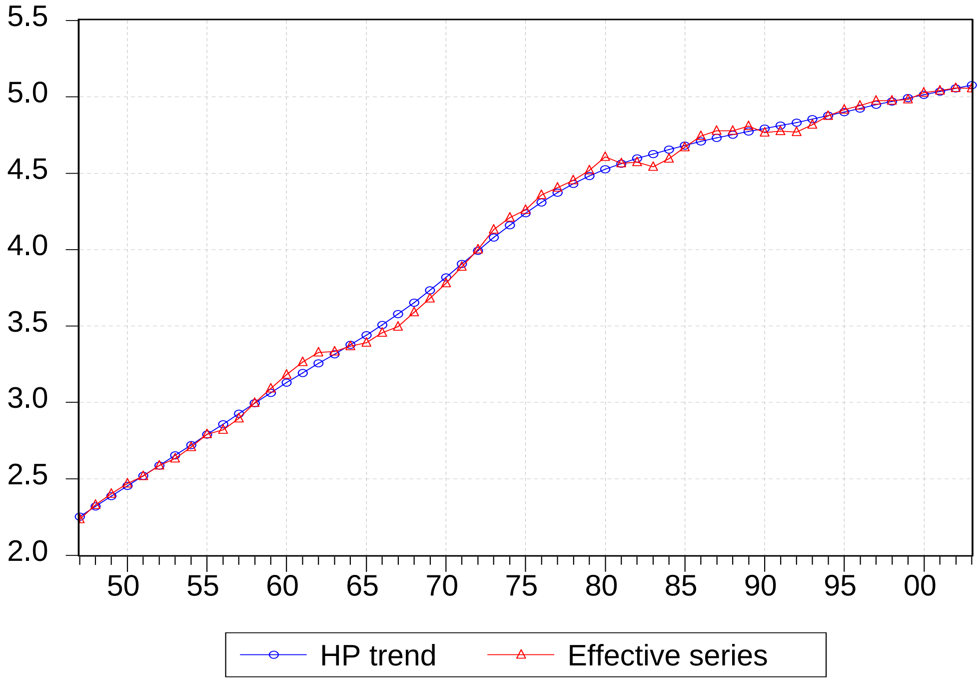

Figure 11: rate of capacity utilization according to Hodrick-Prescott (HP) trend of Brazil's annual GDP, in log terms, in 1947-2003. The HP estimate corresponds to a smoothing parameter of 100.

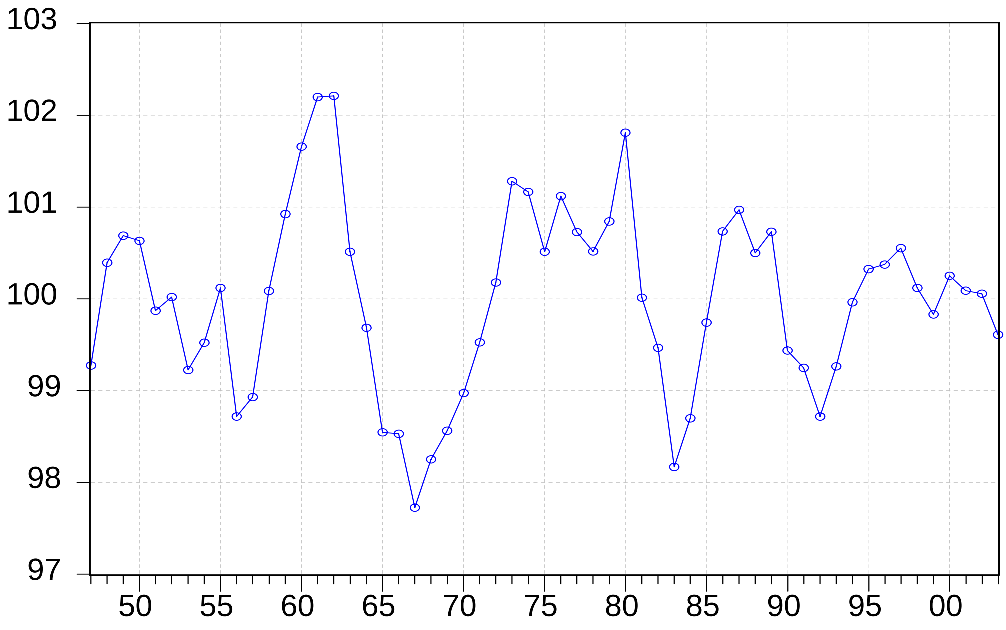

Figure 12: annual growth rate of the Hodrick-Prescott (HP) trend of Brazil's real GDP. The HP trend corresponds to a smoothing parameter of 100.

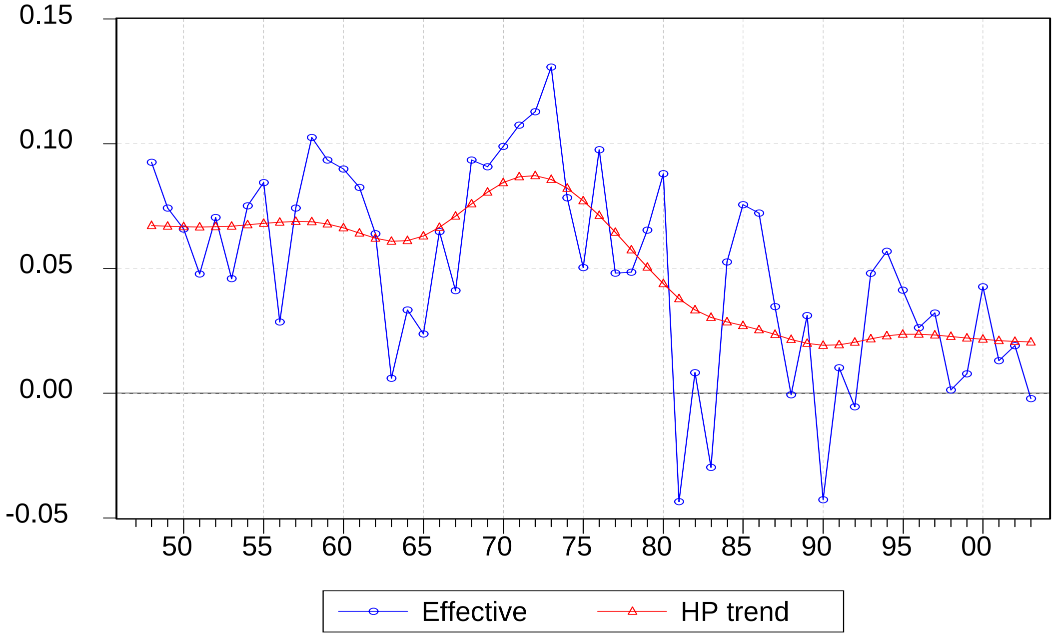

Figure 13: Hodrick-Prescott trend of Brazil's real quarterly GDP, seasonally adjusted and in log terms. The HP trend corresponds to a smoothing parameter of 1600.

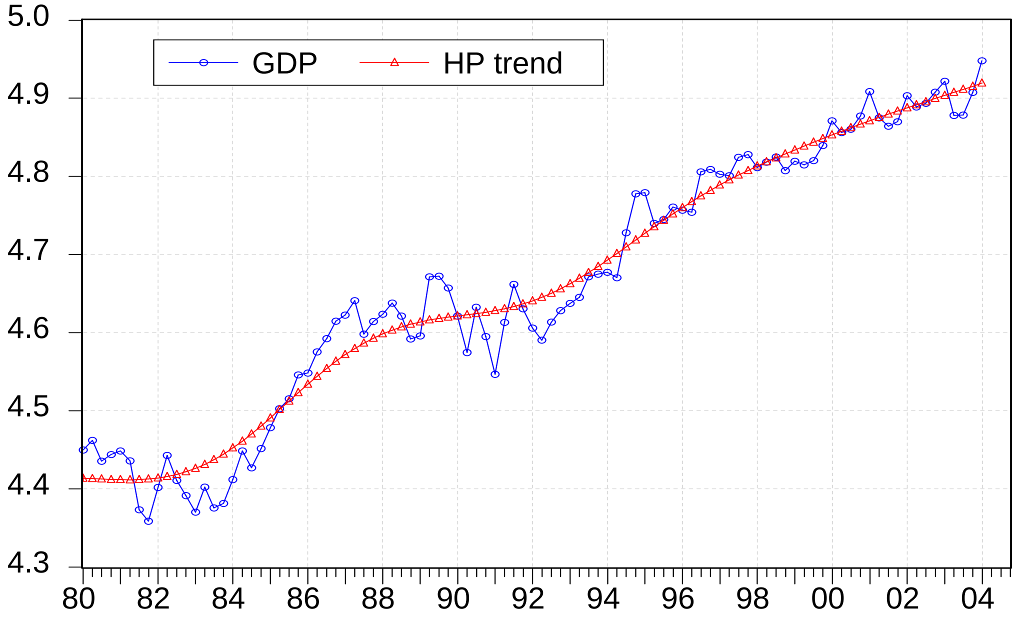

Figure 14: Brazil's global rate of capacity utilization, in percentage points, based on the Hodrick-Prescott trend of the quarterly real GDP, in log terms, and with a smoothing parameter of 1600.

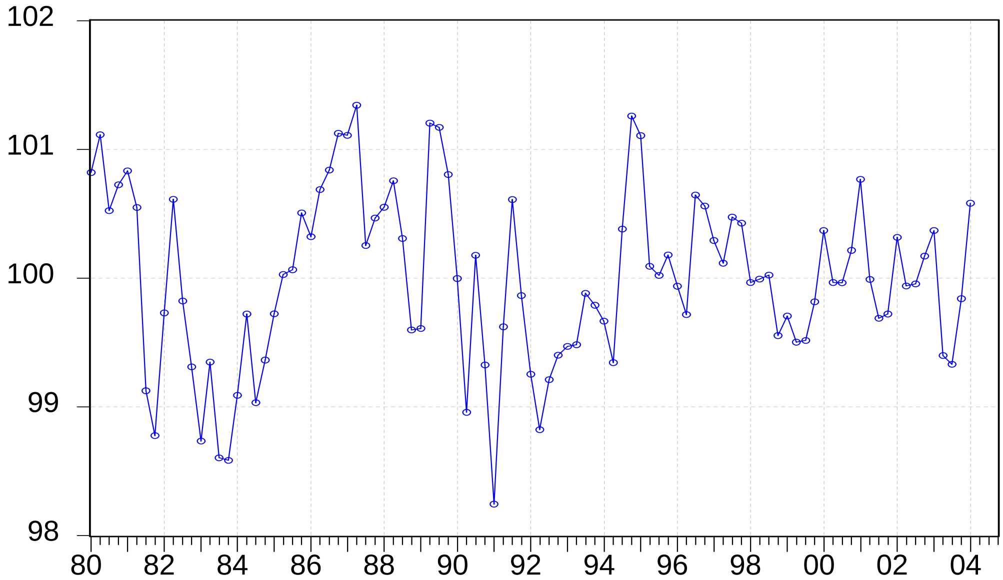

Figure 15: quarterly growth rate of the Hodrick-Prescott trend of Brazil’s quarterly real GDP, 1980-2003. The HP measure corresponds to a smoothing parameter of 1600.

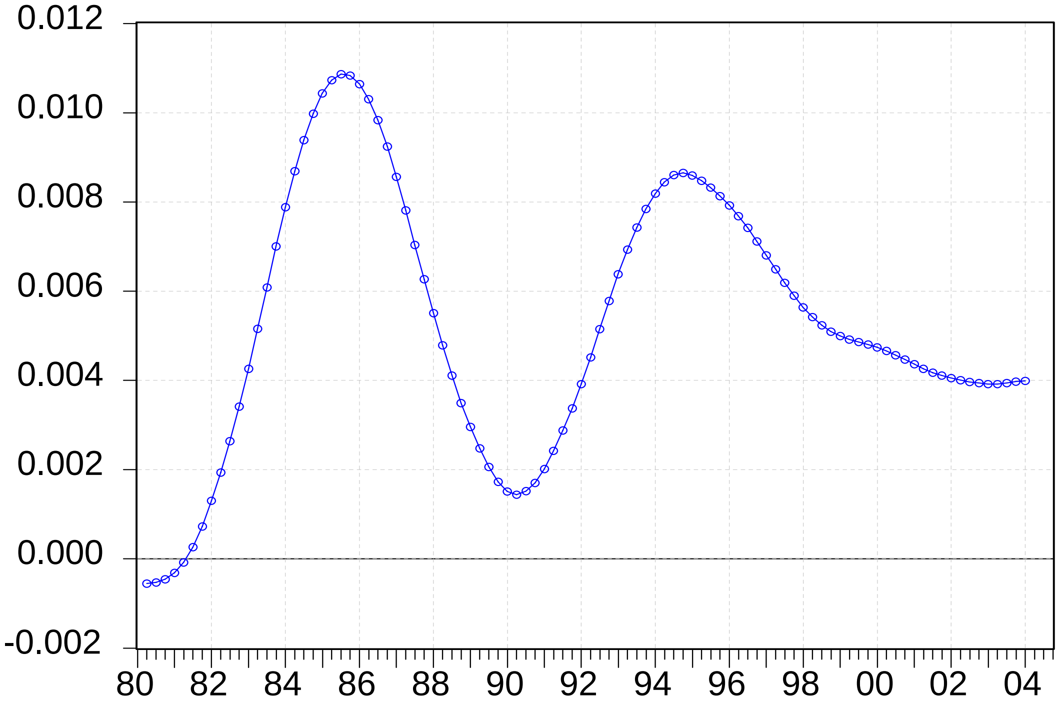

Figure 16: decomposition of the log of Brazil's real GDP in a short-run (less than 8 quarters), medium-run (between 8 and 32 quarters), and long-run (more than 32 quarters) component.

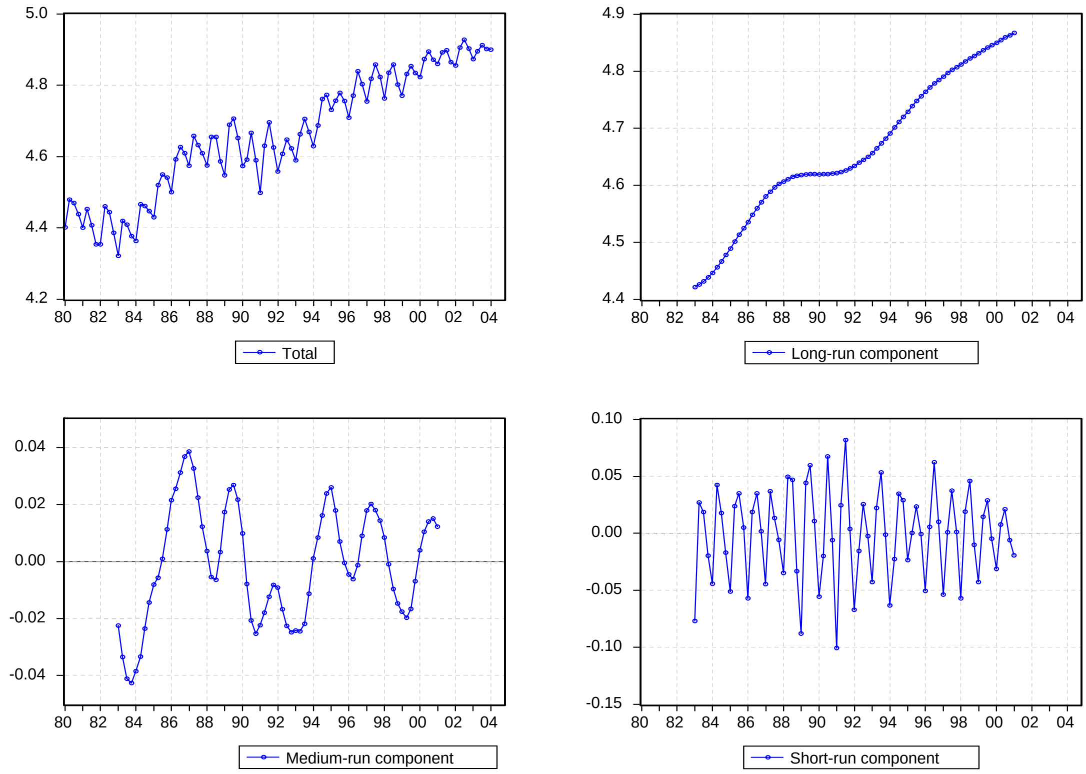

Figure 17: medium and short-run fluctuations of Brazil's real GDP, in log terms, normalized by its long-run trend. All variables are expressed in percentage points of the trend and the medium-run component measures all fluctuations with periodicity between 8 and 32 quarters

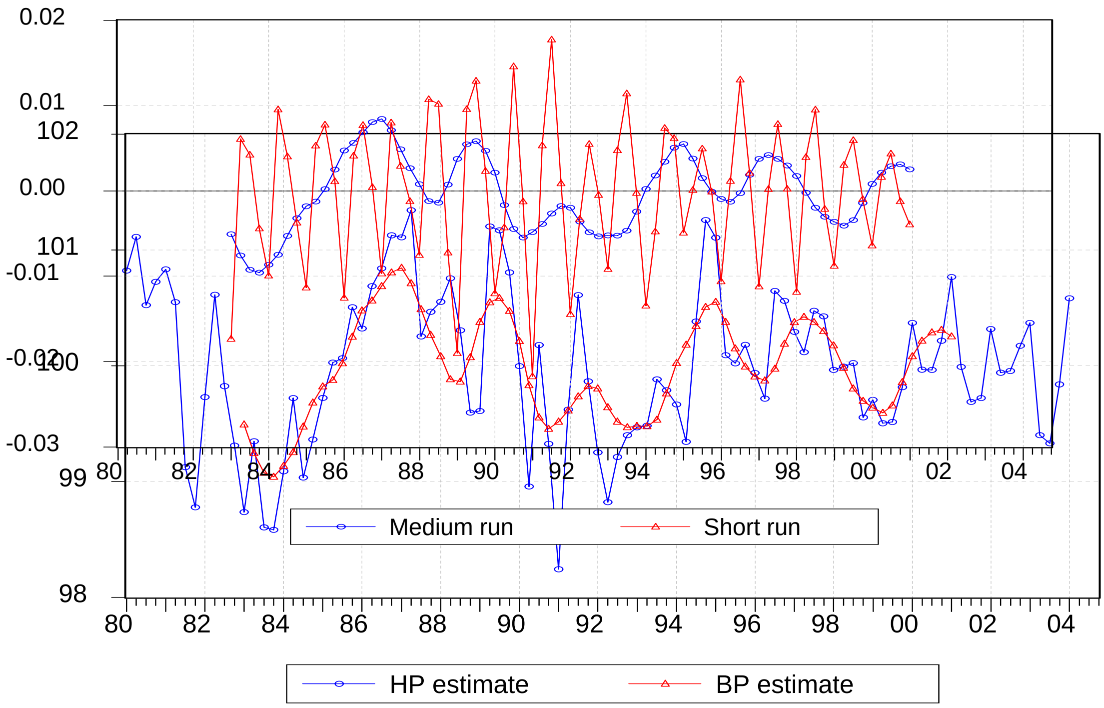

Figure 18: Hodrick-Prescott (HP) and Band-Pass (BP) estimates of business cycles in Brail's real GDP, 1980-2003. The HP measure corresponds to a smoothing parameter of 1600, and the BP measure corresponds to fluctuations between 8 and 32 quarters.

Figure 19: Hodrick-Prescott (HP) and Low-Pass (LP) estimates of the long-run trend of Brazil's real GDP, in log terms, for 1980-2003. The HP measure corresponds to a smoothing parameter of 1600, and the LP measure corresponds to fluctuations of 32 quarters or more.