Wavelet variance and correlation analyses of output in G7 countries

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October 24, 2005

Abstract

In this paper we apply the wavelets methodology to the analysis of the industrial production index of the G-7 countries between 1961:1-2005:5. The analysis is performed using a multi-scaling approach which decomposes the variance of the industrial production index and the co-variance between the industrial production indices of two countries on a scale-by-scale basis through a non-orthogonal variant of the classical discrete wavelet transform, i.e. the *maximal overlap discrete wavelet transform* (MODWT). Wavelet variance analysis does not provide evidence of an international patterns of moderation in output volatility, as the moderation of output volatility occurred after the early eighties is confirmed only for the Euro-area countries plus Japan. Moreover, wavelet correlation analysis different correlation patterns at the different time-scale components and, that, with some exceptions, the linkages between countries are mostly significant only at the business cycle time scales, with the strongest relationships between the Anglo countries (particularly Canada and US), France and Germany, Japan and the Euro-zone countries, with Italy displaying the closest links with France .

Keywords: Wavelets, Time-scale decomposition, Business cycles fluctuations

JEL codes: C01, E31, E32

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1 Introduction

Most macroeconomists share the view that the business cycles are characterised by co-movements among different aggregative time series which are common to all decentralised market economies with no restriction to particular countries or time period, and therefore they are all alike (Lucas, 1977: 217). This notion replaces NBERs view that the business cycle consists of expansion occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle (Burns and Mitchell, 1946). These opposite approaches share the view according to which the underlying cyclical component can be represented by a Data Generation Process of an approximate length between 2 and 8 (10) years. This view is not without exceptions. Recently, Blanchard (1997), and Comin and Gertler (2003) have pointed out the existence of a medium run cycle of length between 10 and 40 years, together with a shorter cycle below 10 years. This last approach can be traced back to Schumpeter (1939) who disentangled the business cycle movements in several minor components.[1] Morevoer, following the modern definition of business cycle as the repeated fluctuations about trend and the regularities observed in the co-movements among different aggregative time series (Lucas, 1977), the cyclical nature of business cycles has been accomplished using frequency domain representations based on Fourier transform, a useful mathematical tool for analysing the frequency content of periodic and stationary signals. The two main limitations of the Fourier analysis for business cycle analysis are the loss of the time information in the transformation to the frequency domain and the requirement that the moments of the signals should not change appreciably over time (represented by the assumption of covariance-stationarity).

All these limits of the current empirical analysis on business cycles may be overcome using a relatively new, at least for economists, technique, i.e. *wavelet analysis*. Wavelets are particular types of function f(x) that are localized both in time and frequency domain and used to decompose a function f(x), i.e. a signal, a surface, a series, etc.., in more elementary functions which include information about the same f(x). The main advantage of wavelet analysis is its ability to decompose macroeconomic time series, and data in general, into their time scale components. Several applications of

[1] Schumpeter described short cycles under the Kitchin-Crun terminology; then the good old business cycle of allegedly eight to ten years periodicity was labeled Juglar cycles; and of course there were also the long waves of Kondratieff []. In between Juglars and Kondratieffs came Kuznetss intermediate cycles in construction and immigration, with an alleged approximate periodicity of 18 to 20 years, Samuelson, 1998: 33-34. The wavelet approach is able to disetangle the components with different periodicity. Lets note that this methodology violates the individualistic methodology of the mainstream approach pro a holistic view which claims that the aggregate is the outcome of the interaction of several different components.

wavelet analysis in economics and finance have been recently provided by Ramsey and Lampart (1998a, 1998b), Ramsey (2002), and Kim and Haueck In (2003) among others, with some attempts to apply this methodology to the analysis of business cycle fluctuations (Jagric and Ovin, 2004; Raihan, Wen and Zeng, 2005).

There is no need to assume any a-priori about the underlying cause of the business cycle (impulse-response function versus a deterministic approach); rather one should recognise that the different *causae causantes* (supply of and demand for durable-, non-durable-, or investment-goods, as well different economic policies and shifts of factors revenues) systematically display their effects on very different time-scale. According to this view, in order to understand the business cycle behaviour, one has to fully understand what is going on at a disaggregated level.

The underlying assumption of this paper is that economic activity may undergo different time scale periods, and they coexist within the time period of interest to business cycle analysts. It follows that the aggregate cyclical movement is nothing but the sum of each single component, which are displaced at different time horizons. It is therefore possible to analyze the relative importance of the various components and appreciate why the cycle has changed.[2] In this paper we apply the wavelet methodology to the analysis of the output series of G7 countries between 1961:1 and 2005:5. For each time-scale components of the industrial production series of G7 countries, we explore two issues: a) the moderation of volatility, and b) the linkages between countries at different scales.

The main properties of the wavelets and the analytical differences with other filtering methods are dealt with in Section 2, where the characteristics of our data set are also illustrated. Section 3 and 4 present the results from wavelet variance and wavelet correlation analyses, while section 5 concludes the paper.

2 Methodology and data

The series were filtered with a non-decimated discrete wavelet transform that is a relatively new (at least for economists) statistical tool that, roughly speaking, decomposes a given series in orthogonal components, as in the Fourier approach, but according to scale (time components) instead of frequencies. The comparison with the Fourier analysis is useful first because wavelets use a similar strategy: find some orthogonal objects (wavelet functions instead of sines and cosines) and use them to decompose the series.

[2]Of course, since the different time periods merge one into each other, and the impulses of the different components propagate through time, exact identification would be impossible. Moreover, the inexistence on a single source of recession (such as oil prices, interest rates) is erroneous (Zarnovitz, 1998). A holistic, cliometric approach would be more appropriate (Temin, 1998).

Second, since the Fourier analysis is a common tool in economics, it may be useful in understanding the methodology and also in the interpretation of results. Saying that, we have to stress the main difference between the two tools. Wavelet analysis does not need stationary assumption in order to decompose the series. This is because the Fourier approach decomposes in frequencies space that may be interpreted as events of time-period T (where T is the number of observations). Put differently, spectral decomposition methods perform a global analysis whereas, on the other hand, wavelets methods act locally in time and so do not need stationary cyclical components. A way to relax the stationary frequencies assumption of the Fourier transform has been introduced by Gabor (1946), called the short-time Fourier Transform (STFT), through a windowing Fourier decomposition that essentially applies the Fourier transform to only a small section (the window) of the original time series at a time. The problems with this approach may be the right choice of the window and, more important, its constancy over time.

For a natural comparison of the wavelet filter the well known HP and BK filters are usually used in business cycles analysis. According to the BK definition business cycles are cyclical components with a frequency higher than 18 months and less than 96. Failure of the stationary assumption ends up with biased estimated business cycles. For example, the weight of the different time-scale components characterising the series cannot be, in general, considered constant causing a difficult interpretation of band-pass filter estimations that are mixed objects of such time-scale components. In fact, recent investigations (Guay and St-Amant , 1997) seem to show that HP and BK filters have a correct interpretation only when the frequencies that have a peak in the spectrum belong to the ideal filter band. Unfortunately, as shown by Granger (1966), this does not happen for many macroeconomic time series since they seem to have most of their power at low frequencies and decrease monotonically moving toward higher frequencies. The consequence of this problem is the extraction of spurious cyclical components as shown also by Harvey-Jaeger (1993) and Cogley-Nason (1995).

Coming back to wavelets and going into some mathematical detail we may note that there are two basic wavelet functions: the father-wavelet and the mother-wavelet. The formal definition of the father wavelets is the function

$$\phi_{J,k} = 2^{-\frac{J}{2}}\phi\left(\frac{t-2^J k}{2^J}\right) \tag{1}$$

defined as non-zero over a finite time length support that corresponds to given mother wavelets

$$\psi_{J,k} = 2^{-\frac{J}{2}}\psi\left(\frac{t-2^J k}{2^J}\right) \tag{2}$$

with $j = 1, \ldots, J$ in a *J-level* wavelets decomposition. The former integrates to 1 and reconstructs the longest time-scale component of the series (trend), while the latter integrates to 0 (similarly to sine and cosine) and is used to describe all deviations from trend. The mother wavelets, as said above, play a role similar to sins and cosines in the Fourier decomposition. They are compressed or dilated, in time domain, to generate cycles fitting actual data.

For a discrete signal or function $f_1, f_2, ..., f_n$, the wavelet representation of the signal or function $f(t)$ in $L^2(R)$ can be given by

$$f(t) = \sum_k s_{J,k} \Phi_{J,k}(t) + \sum_k d_{J,k} \Psi_{J,k}(t) + ... + \sum_k d_{j,k} \Psi_{j,k}(t) + ... + \sum_k d_{1,k} \Psi_{1,k}(t) \tag{3}$$

where J is the number of multiresolution components or scales, and k ranges from 1 to the number of coefficients in the specified components. The coefficients d_{jk} and s_{Jk} of the wavelet series approximations in [3] are the details and smooth wavelet transform coefficients representing, respectively, the projections of the time series onto the basic functions generated by the chosen family of wavelets, that is

$$\begin{array}{l} d_{j,k} = \int \psi_{j,k} f(t) dt \\ s_{J,k} = \int \phi_{J,k} f(t) dt \end{array}$$

for $j = 1, 2, ..., J$. The smooth coefficients s_{Jk} mainly capture the underlying smooth behaviour of the data at the coarsest scale, while the details coefficients $d_{1k}, .., d_{jk}, .., d_{Jk}$, representing deviations from the smooth behaviour, provide progressively finer scale deviations. Each of the sets of the coefficients $s_J, d_J, d_{J-1}, ..., d_1$ is called a crystal.

The multiresolution decomposition of the original signal $f(t)$ is given by the following expression

$$f(t) = S_J + D_J + D_{J-1} + ... + D_j + ... + D_1 \tag{4}$$

where $S_J = \sum_k s_{J,k} \Phi_{J,k}(t)$ and $D_j = \sum_k d_{J,k} \Psi_{J,k}(t)$ with $j = 1, \ldots, J$. The sequence of terms $S_J, D_J, ..D_j, ..., D_1$ in (4) represent a set of signals components that provide representations of the signal at the different resolution levels 1 to J, and the detail signals $\mathbf{D}_j$ provide the increments at each individual scale, or resolution, level.

In addition to the features stated above Whitcher *et al.* (1999, 2000) have extended the notion of wavelet covariance for the maximal overlap DWT (MODWT) and defined the wavelet cross covariance and wavelet cross correlation between two processes. The maximal overlap DWT (MODWT) may be regarded as a modified version of the discrete wavelet transform (DWT), but as MODWT employs circular convolution the coefficients generated by both beginning and ending data could be spurious.[3] For a signal $f(t)$ the MODWT applying the Daubechies compactly supported wavelet produces J vectors of wavelet coefficients $\widehat{w}_1, \widehat{w}_2, ..., \widehat{w}_J$ and one vector of scaling coefficients, $\widehat{s}_J$. The wavelet variance for a signal $f(t)$ is defined to be the variance of the wavelet coefficients at scale 2^{j-1} and an unbiased estimator using the MODWT after removing all coefficients affected by the periodic boundary conditions through

$$\widehat{v}^2_{f(t)}\left(2^{j-1}\right) = \frac{1}{\widehat{N}_j}\sum_{t=L_{J-1}}^{N-1} \widehat{\mathbf{w}}^2_{j,t} \tag{5}$$

where $\widehat{N}_j = \frac{N}{2^j - L_j}$ with $L_j = \left[(L-2)\left(1-2^j\right)\right]$ being the lenght of the scale 2^{j-1} wavelet filter, and the vector $\widehat{\mathbf{w}}$ are *n-dimension* vectors containing the coefficients s_J, d_J,......, d_1 of the wavelet series approximations. Thus level j wavelet variance is simply the variance of the wavelet coefficients at that level (Gencay *et al.,* 2002).

In order to analyse the linkages among the G-7 economies over different time scales it is necessary to generalise to multiple time series the wavelet scale analysis of univariate time series. To determine the magnitude of the association between two series of observations X and Y on a scale-by-scale basis the notion of wavelet covariance has to be used. Following Gencay *et al.* (2001) the wavelet covariance at wavelet scale j may be defined as the covariance between the scale j wavelet coefficients of X and Y, that is $Cov_{XY}\left(2^{j-1}\right) = Cov[\widehat{\mathbf{w}}^X_{j,t}\widehat{\mathbf{w}}^Y_{j,t}]$. Again, after removing all wavelet coefficients affected by the boundary conditions, an unbiased estimator of the wavelet covariance using the MODWT may be given by:

$$\widehat{Cov}_{XY}\left(2^{j-1}\right) = \frac{1}{\widehat{N}_j}\sum_{t=L_{J-1}}^{N-1} \widehat{\mathbf{w}}^{f(t)}_{j,t}\widehat{\mathbf{w}}^{g(t)}_{j,t} \tag{6}$$

Analogously to the usual unconditional correlation coefficients, the MODWT estimator of the wavelet cross correlation coefficients may then be obtained making use of the wavelet covariance $\widehat{Cov}_{f(t)g(t)}$ and the square root of their wavelet variances $\widehat{v}^2_{f(t)}$ and $\widehat{v}^2_{g(t)}$ as follows:

$$\widehat{\rho}_{f(t)g(t)}\left(2^{j-1}\right) = \frac{\widehat{Cov}_{f(t)g(t)}\left(2^{j-1}\right)}{\widehat{v}_{f(t)}\left(2^{j-1}\right)\widehat{v}_{g(t)}\left(2^{j-1}\right)} \tag{7}$$

[3] If the lenght of the filter is L, there are $\left(2^j-1\right)(L-1)$ coefficients affected for 2^{j-1}-scale wavelet and scaling coefficients, while $\left(2^j-1\right)(L-1)-1$ beginning and $\left(2^j-1\right)(L-1)$ ending components in 2^{j-1}-scale details and smooth would be affected (Perival and Walden, 2000).

The analysis was conducted using the monthly production index (source; OECD MEI) between 1961:1-2005:5 for Canada, France, Germany, Italy, Japan, UK and US.[4] We perform a *J-level* decomposition of the aggregate monthly industrial production series for each G7 country using the *maximal overlap discrete wavelet transform* (*MODWT*) which is a non-orthogonal variant of the classical discrete wavelet transform that, unlike the orthogonal discrete wavelet transform, is translation invariant, as shifts in the signal do not change the pattern of coefficients. The wavelet filter used in the decomposition is the Daubechies least asymmetric (LA) wavelet filter of lenght $L = 8$, that is $LA(8)$, based on eight non-zero coefficients (Daubechies, 1992), with reflecting boundary conditions. The application of the translation invariant wavelet transform with a number of scales $J = 7$ produces eight wavelet and scaling filter coefficients $v_7, w_7, w_6, w_5, w_4, w_3, w_2, w_1$. Each wavelet scale is associated with a particular time period. Thus, since we use monthly data scale 1 represents 2-4 months period dynamics, while scales 2, 3, 4, 5, 6 and 7 correpond to 4-8, 8-16, 16-32, 32-64, 64-128 and 128-256 months period dynamics, respectively. The first three time scales correspond the very short run elements, scales 4 to 6 roughly correspond to the standard business cycle time period (Stock and Watson, 2000)[5] , while scale 7 and the trend correspond, respectively, to the medium frequency variation in business activity and the long run elements.[6]

3 Wavelet variance analysis

In this section we explore the issue the moderation of volatility over time and across countries at different time-scales using the time-scale components of the industrial production series of G7 countries. The wavelet variance decomposes the variance of a time series on a scale-by-scale basis through a wavelet multiresolution analysis. Figure 1 reports the estimated wavelet variances at different wavelet scales, from 1 to 7, for the G7 countries over the whole sample, where the level j wavelet variance is obtained using the wavelet coefficients w_1 to w_7 not affected by the boundary (Gencay *et al.* 2001). The analysis of the change in wavelet variance by scale for the G7 industrial production series displayed in Figure 1 reveals that wavelet variances tend to increase and to have a more similar pattern as the wavelet scale increases. In particular, the main differences among G7 countries refer to France, Italy and US (and partly Germany), as their wavelet variances at the lower wavelet scales appear to be particularly high (low in the case

[4]

[5]According to the modern definition of business cycle, business cycles fluctuations consist of frequencies between 6 to 32 quarters (18 and 96 months), which roughly correspond to scales 4, 5 and partly 6

[6]See Blanchard (1997) and Caballero and Hammour (1998) for a notion of the medium term business cycle.

of US) as compared to the other G7 countries.

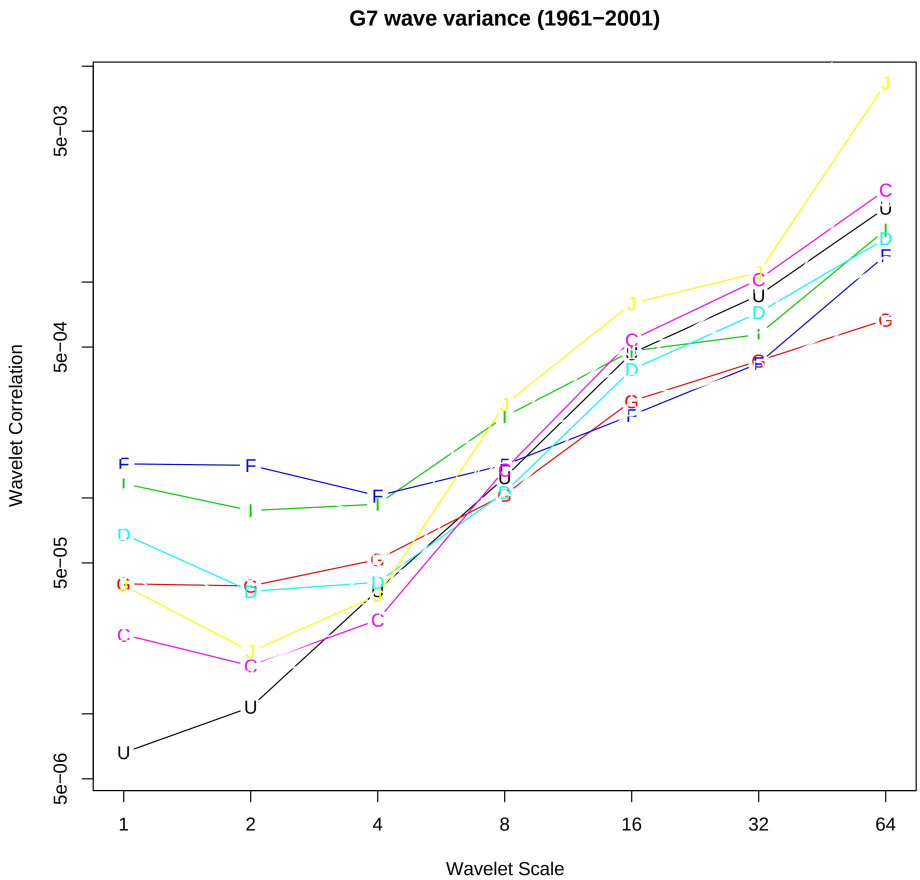

Figure 1: G-7 multiscale variance

Heterogeneity in business cycles is not a country specific characteristic, as it may involve different periods of time as well. Thus, as the overall sample analysis may mask some important differences over different time periods, we perform the multiscale wavelet variance analysis splitting the overall sample into four distinct selected sub-periods: period I *before Bretton Woods collapse* (1961-1971), period II the *oil shocks* (1972-1982), period III the *slow recovery* (1983-1992) and period IV the *irrational euphoric years* (1993-2005). In Figure 2 we plot the estimated wavelet variances at different time scales for the four subsamples. [7] A rapid glance at the results in Figure 2 indicates that no uniform behaviour in volatility seems to emerge

[7] In Figure 2 the numbered lines 1, 2 3, and 4 represent the estimated multiscale wavelet variances for sub-periods I, II, II and IV, respectively.

both over time and across countries (the only exception seems to be Canada, whose differences in multiscale wavelet variances among subsamples are not particularly remarkable). In particular, both the Euro-area countries and Japan display values of the wavelet variances that are higher before than after the early eighties at almost all scales (at the last three wavelet scale in the case of Germany and at all scales except the first one in the case of Japan), while in the case of the Anglo countries the highest values of the wavelet variances at any scale characterize only the *oil-shocks* period. Moreover, the analysis of the change in wavelet variance by scale for different time periods reveals that the high values of the wavelet variance of the Euro-area countries at the highest scales have their roots in different time periods . In particular, the sub-periods analysis of the wavelet variances suggests that the whole sample high values of France, Italy and Germany at the smallest scales depends on the *before Bretton Woods collapse*, the *oil-shocks* and the *irrational euphoric years* periods, respectively (probably as a consequence of shocks like the French May, the two oil-shocks and the German Reunification).

In recent years there has been a renewed interest in the volatility of economic activity due to the reduction in the volatility of output growth, and a concomitant moderation of business cycle fluctuations in the past two decades (Stock and Watson, 2003). The results stemming from the recent wide literature on the evidence and the cause of the moderation of the business cycle (Mc Connel and Perez Quiros, 2000; Kim and Nelson, 1999; Stock and Watson, 2002; Chauvet and Potter 2001) provide a twofold evidence: the international patterns of moderation differ among countries;[8] there is evidence of lower (higher) volatility at business cycle (lower) frequencies. These empirical results are only partially confirmed by our wavelet variance analysis. Indeed, if the finding that the estimated wavelet variances increase going from small to long time scales tends to confirm the evidence of lower (higher) volatility at business cycle (lower) frequencies, the results of the wavelet variance analysis for different time periods suggest a different conclusion as regards the moderation claimed by the literature on the volatility of economic acitivity in most G-7 economies. Once we estimate the multiscale wavelet variances for different time periods the stabilization occurred after the 1983 trough seems to be, at least for UK and US, an artefact of the aggregation procedure determined by splitting the whole sample in a *before oil shocks* and an *after oil shocks* period. Thus, wavelet variance analysis does not provide evidence of an international patterns of moderation in output volatility.

All in all, we may say that the consensus on the moderation of output

[8]The stabilization occurred after the 1983 trough for Canada and US, from the early 70ies for Germany and Japan (but increased recently), and from the mid 70ies for Italy and UK.

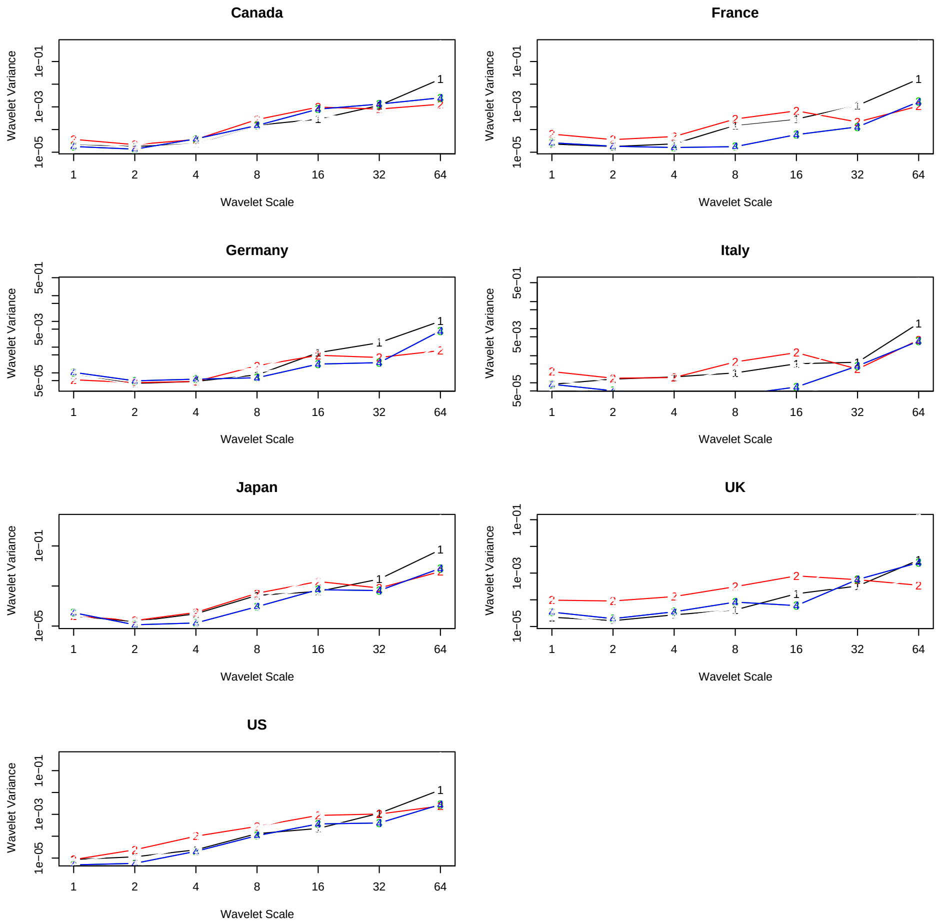

Figure 2: G-7 countries multiscale variance over different sub-periods

volatility occurred after the early eighties is confirmed only for the Euro-area countries (for Germany only at the longest scales) plus Japan, while the evidence for the Anglo conutries (with the exception of Canada) stemming from wavelet variance analysis suggests that such a a result may be the effect of an incorrect aggregation procedure which include the *oil shocks* period (characterized by higher values of the estimated wavelet variance at any scale) into the wider *before oil shocks* period.

4 Wavelet correlation analysis

The overwhelming majority of empirical studies on business cycle has focussed on the linkages among major advanced economies, investigating if business cycle characteristics and co-movements in aggregative time series are robust, *i.e.* if they are common across countries and over time. In particular, the issue of international synchronization of business cycles has been largely explored in recent years using different methodologies, sample, countries and estimation methods (Stock and Watson, 2003, Bordo and Helbing, 2003, De Haan *et al.* 2002, Kose *et al.* 2003, among the others). These papers analyze cross-country differences and similarities of macroeconomic fluctuations among countries generally employs pairwise correlations coefficients of the (filtered) cyclical components as a measure of co-movement.

The results of the wavelet correlation analysis for the G-7 countries are reported in Figures 3 to 9. These figures display the wavelet correlation coefficients between each country and the other G-7 countries at different wavelet scales. Lower wavelet scales 1 to 3 (note that lower scales correspond to higher frequency bands), do not generally exhibit wavelet correlations significantly different from zero, even if there are some exceptions represented by wavelet correlations at the third scale (8-16 month periods) of UK with Germany, Italy and US (.24, .20 and .20, respectively), of Japan with Canada and US (.27 and .31, respectively), of France with Germany (.35) and of Canada and US (.26, .31 and .39, at wavelet scales 1, 2 and 3 respectively).

On the contrary, at scales roughly corresponding to business cycle frequencies,[9] *i.e.* scales 4 to 6, wavelet correlations are always positive and significantly different from zero. In particular, the highest values of wavelet correlation coefficients, more than .80, are between US and Canada at the longest wavelet scales (.84, .91 and .90, from fourth to sixth scale respectively), and at the longest scale, *i.e.* scale 6, between US and UK (.86) and between France and Germany (.86). Thus, at the longest wavelet scale, scale 6, there is evidence of a very high positive contemporaneous relationship

[9]According to the modern definition of business cycle, business cycles fluctuations consist of frequencies between 6 to 32 quarters (18 and 96 months), which roughly correspond to scales 4, 5 and partly 6

between Canada, UK and US, between France and Germany and between Japan and the Euro-zone countries. Moreover, as regards the shortest business cycle time-scale components, *i.e.* scales 4 and 5, high positive wavelet correlation coefficients, between .60 and .80, characterize the relationships i) between G-7 Euro-area countries at scales corresponding to business cycle frequencies, ii) between Japan and the G-7 Euro-area countries (at scales 5 and 6), as well as with Canada and US at scale 5, iii) between the Euro-zone (except Germany) and the Anglo-countries (at scales 5 and 6 for France and at scale 6 for Italy).

To summarize, wavelet correlation analysis between G7 countries at different time scales shows that:

- the links among countries differ at different wavelet scales, as wavelet correlations are low at the lowest and high at the highest scales;

- at the business cycle time scales emerge some strong and stable relationships (in the sense that they hold across time-scales) among the Anglo countries (particularly between Canada and US), and between France and Germany, with Italy displaying the closest links with France and Japan more with the Euro-zone countries than with the Anglo countries;

5 Conclusions

In this paper we apply the wavelets methodology to the analysis of the industrial production index of the G-7 countries between 1961:1-2005:5. The analysis is performed using a multi-scaling approach which decomposes the variance of the industrial production index and the covariance between the industrial production indices of two countries on a scale-by-scale basis through a non-orthogonal variant of the classical discrete wavelet transform, that is the *maximal overlap discrete wavelet transform* (MODWT).

The analysis of volatility and synchronization of G7 countries across different time scales suggests that the consensus on the moderation of output volatility occurred after the early eighties is confirmed only for the Euro-area countries plus Japan, while the evidence for the Anglo conutries (with the exception of Canada) suggests that such a a result may be the effect of an incorrect aggregation procedure which include the *oil shocks* period (characterized by higher values of the estimated wavelet variance at any scale) into the wider *before oil shocks* period.

Morevoer, wavelet correlation analysis indicates that, with some exceptions, the linkages between countries are generally significant only at the business cycle time scales, that is from scale 4 to 6, and that at these scales there is evidence of a high positive relationship between Canada, UK and US (particularly Canada and US), between France and Germany and between

Japan and the Euro-zone countries, with Italy displaying the closest links with France;

In this way our results confirm, on one hand, previous business cycle results on G-7 countries, but on the other one indicate that wavelet wavelet analysis may provide useful information in analyzing economic relationships.

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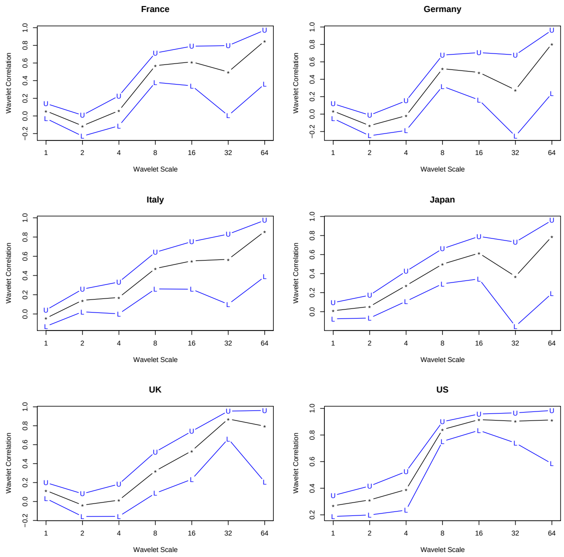

Figure 3: Wavelet correlation between Canada and the other G7 countries at different scales

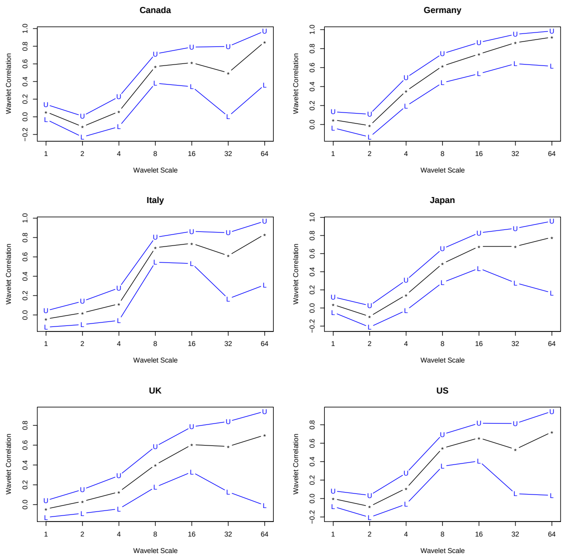

Figure 4: Wavelet correlation between France and the other G7 countries at different scales

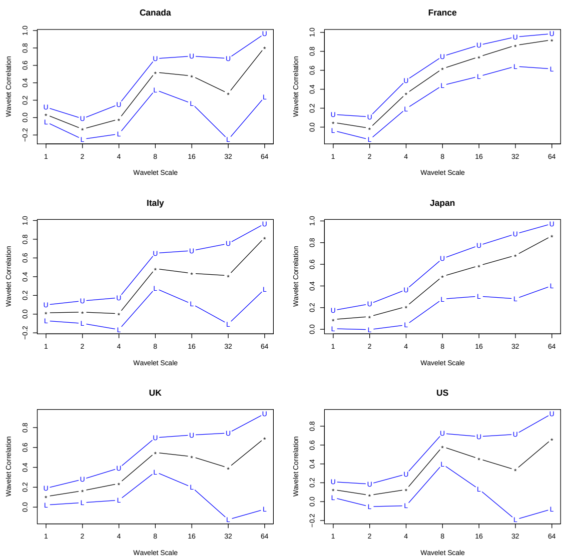

Figure 5: Wavelet correlation between Germany and the other G7 countries at different scales

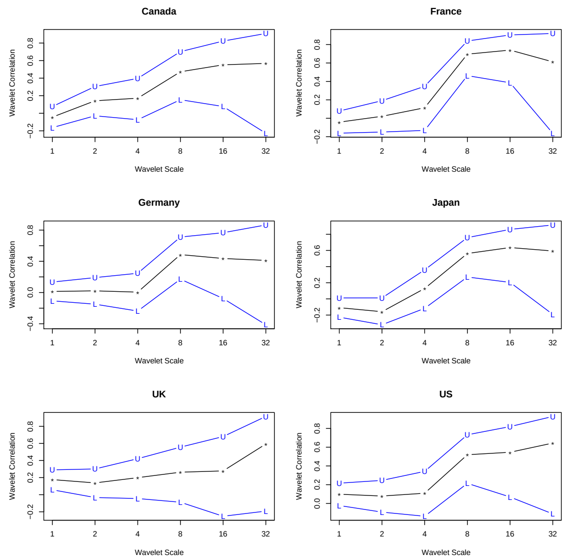

Figure 6: Wavelet correlation between Italy and the other G7 countries at different scales

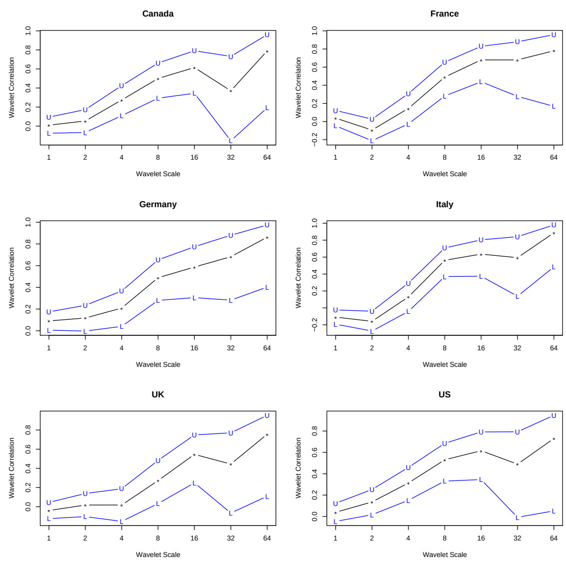

Figure 7: Wavelet correlation between Japan and the other G7 countries at different scales

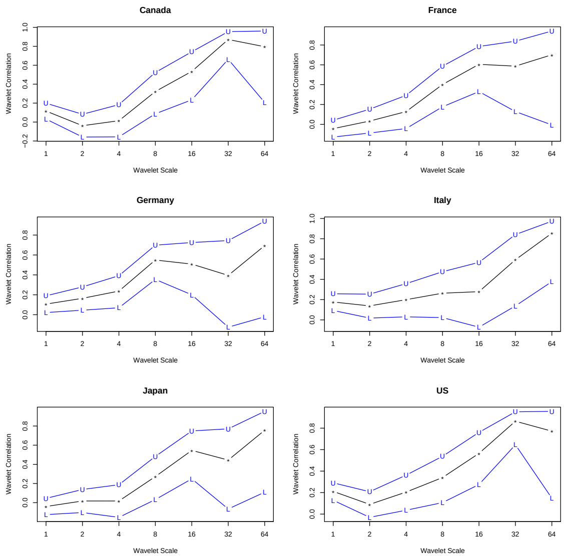

Figure 8: Wavelet correlation between UK and the other G7 countries at different scales

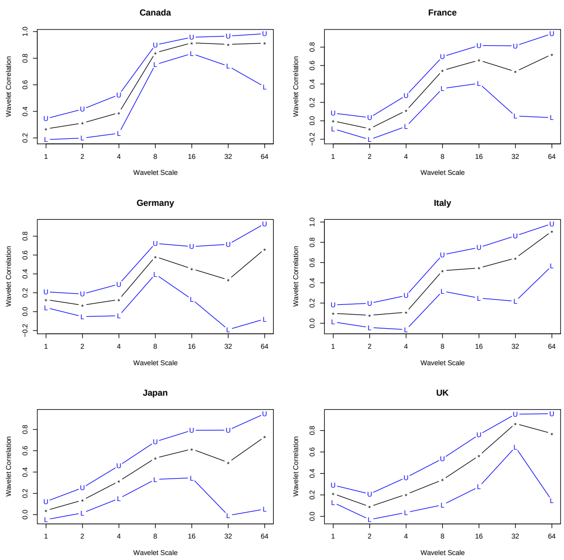

Figure 9: Wavelet correlation between US and the other G7 countries at different scales