Doing Without Money: A critical assessment of Woodford's analysis.

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Woodford employs an inter-temporal general equilibrium model to examine the properties of the monetary system as it evolves form the use of a physical medium of exchange to an electronic medium. He presents a structure in which cash as a means of payment can be made to vanish at the limit but where he claims the price level remains determinate. It is shown that Woodford's analysis involves fundamental but still widespread conceptual and methodological errors. His general equilibrium model does not map into the world of electronic money and his analysis has no implications for the art of central banking.

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I Introduction

The electronic revolution in banking has thrown up challenges for regulators and at the same time raised some interesting questions about the fundamentals of monetary theory. Of particular interest is Woodford's (1997, 1998)[2] model that is intended to shed light on the properties of a monetary economy as it evolves from the use of a physical medium of exchange (cash) to an electronic medium. The analysis presented in Woodford (1997, 1998) also provides the theoretical foundation for Woodford's (2003) neo-Wicksellian theory of monetary policy.

This is an important area of research and has obvious implications for the implementation of monetary policy in the face of the electronic revolution in the payments mechanism. Monetary theorists have recognised that we have to get this right if errors in policy and regulation are to be avoided (Green, 1999). Furthermore, the issue is of more than purely theoretical interest. Woodford's work has been cited with approval by Henckel, Ize and Kovanen (HIK) (1999, p. 31, fn 35, italics added). They make the claim that "…the path-breaking analysis in Woodford (1997), [….]

[1] I am grateful to Penny Neal and three anonymous referees for helpful comments on an earlier draft. Any remaining errors are mine.

[2] All page references are to Woodford (1997).

shows that *in a model where money vanishes* a stable local equilibrium continues to exist for the rate of inflation *even when money balances are zero*". It is this sort of claim with which I wish to take issue.

In my view the HIK interpretation of Woodford's results is incorrect. Woodford's analysis involves well documented, if not well known, conceptual errors. These errors originated with Patinkin (1965), if not earlier, and are embedded in modern cash-in-advance models of Clower, (1967) or Lucas (1980). Specifically, Woodford's analysis is based on a model in which money is an inessential addition in the sense of Hahn (1973a,b) and the analysis 'works' by sleight of hand[3] because it confounds the meaning of terms such as the price level (as measured by the CPI or GDP deflator) and nominal rates of interest, by attaching to them theoretical concepts that have fundamentally different properties. Consequently, nothing in Woodford's model maps to identifiable institutions or entities that would be recognisable to policy makers. A modern electronic monetary system is a monetary economy while Woodford's model is a non-monetary model of perfect or ideal barter based on an *unspecified clearing operation* that keeps track of net commodity trades. McCallum (1985, 2003, pp. 1-2) calls such a system an accounting system of exchange, a system in which: "there is no money but exchanges are conducted by means of signals to an accounting network, with debits and credits to the wealth accounts [endowments] of buyers and sellers being effected with each exchange. ….In effect, an accounting system of exchange is a highly efficient form of barter". A model of this type that produces an equilibrium vector of commodity relative prices by means of the computing power of the

[3] Apparently Woodford is not the first to commit this error. See the discussion of Fama (1980) in Hoover (1988).

auctioneer in a Walrasian or Arrow-Debreu auction should not be confused with an electronic money system[4].

In this paper I focus on the conceptual and theoretical confusions that result from Woodford's analysis. The remainder of the paper is arranged as follows. Section II presents the Hahn (1973a,b) property of some non-monetary models - money in whatever its form is an inessential addition to non-monetary models based on a Walrasian auction. Section III presents Woodford's analysis and section IV outlines the Hahn property of Woodford's model. Section V then briefly outlines the consequent conceptual errors that arise from Woodford's model while section VI considers Woodford's rationale for the use of a non-monetary model as a foundation for monetary theory and policy. Section VII concludes.

II **The Hahn property of some non-monetary models**

Essentially, the difficulties for Woodford arise because he does not specify what a monetary economy is. Usually it is said that a monetary economy is one that poses an entity that exhibits the three properties of money - medium of exchange, store of value and unit of account. Disagreement arises about which of these properties is paramount or if there is a symbiotic relationship between them (Hoover, 1988, p, 136; McCallum, 2003, p. 2). But at a more fundamental level there are differing views

[4] In an Arrow-Debreu auction all trades are determined at date 0. After date 0 these trades are executed using some *unspecified net clearing system*. See Ljungqvist and Sargent (2003, Ch 7) for a description. Such a system could be described as ideal or perfect barter as McCallum (2003) suggests. But as the clearing system is unspecified the model has nothing to say about how trades are executed. It is doubtful that such a non-monetary system could be made operational even with today's computing power.

about whether these three functions are sufficient to characterise a monetary economy. For a useful review see Smithin (2003 chapter2).

In this paper I am concerned with the mistreatment of the three functions of money in what Smithin (2003) labels the mainstream view. I point out that, in general, attempts at monetary theory that attach a quantity equation, cash-in-advance or cash-in-arrears constraint to accounting systems of exchange based on a Walrasian or Arrow-Debreu auction are non-monetary models and that Woodford's model is of this type. Models of this type suffer from what I call the Hahn property.

The Hahn (1973a,b) property arises when non-monetary models, accounting systems of exchange based on a Walrasian or Arrow-Debreu auction, have attached to them some or all of the characteristics of money, credit and debt. Hahn calls models of this type, inessential economies because, although money can always be added, there is nothing that can be said about the equilibrium of the monetary version of the model that cannot also be said about the equilibrium of the non-monetary version. Hahn (1973b, p231):

'The main content of the monetary theory of an inessential economy is implicit in its construction: there is nothing we can say about the equilibrium of an economy with 'money' that one cannot say about the equilibrium of a non-monetary economy."

Examples of models with this property include Patinkin (1967), Clower (1967) and now Woodford (1997, 2003). A corollary of the Hahn property of these models is that money appears to be a welfare reducing innovation when conventional wisdom

and monetary history suggest the opposite is true. This feature was exposed by what I call the Clower puzzle[5].

In mainstream monetary theory since at least Patinkin (1965) the emphasis has been on the medium of exchange function of money – by appending a quantity equation or a cash-in-advance constraint to a set of equations determining real relative prices. The real relative prices are generated by a Walrasian or Arrow-Debreu auction. But this approach to monetary theory leads to a puzzle recognised by Clower (1984, p. 267):

"…the choice alternatives confronting households were more restrictive in a money than in a barter economy, which meant that monetary exchange is less efficient that barter exchange, contrary to both common sense and two hundred years of conventional wisdom. Something obviously was wrong. But what?"

What is wrong is that the model of efficient potential barter does not need money. Adding money to such a model will then appear to be a welfare reducing friction.

Models based on accounting systems of exchange could be characterised as models of ideal or perfect barter because money is not required to resolve problems arising from the double co-incidence of wants or the extinguishing of debt, and they have nothing to say about how the equilibrium trades are actually executed. Adding a quantity equation (Patinkin 1965), cash-in-advance (Clower, 1967), or cash-in-arrears constraint (Hoover, 1996) to such models amounts to the imposition of an additional

[5] The Clower puzzle does not apply to the store-of-value function of money in overlapping-generations models. However, Hoover (1988, Ch. 6) points out that the store-of-value function is an inessential addition to such models.

constraint on the choice set of agents and is for that reason welfare reducing. In such models monetary exchange appears to be less efficient than barter, as Clower (1984) realised.

But the Clower puzzle disappears immediately once it is realised that an inappropriate comparison is made. The comparison is between inessential monetary exchange and a form of ideal barter or an accounting system of exchange based on an unspecified clearing system. Money or cash-in-advance is a welfare reducing innovation in such a model because it imposes a constraint on the equilibrium solution generated by the Walrasian auction. The comparison should be between real world barter, constrained by the double-coincidence of wants and the efficiencies afforded by money. Such real world barter is less efficient than a monetary economy that employs a common medium of exchange as conventional wisdom suggested and as the search theories of Kiyotaki and Wright (1989) formally demonstrated. In the real world money is welfare enhancing. The Clower puzzle is thus a symptom of the Hahn property of these models.

That Woodford's (1997, 1998, 2003, p. 35) model has the Hahn property is suggested by his description of money as a friction, or distortion and his intention to provide a theoretical foundation for monetary policy in a context in which the friction generated by money is eliminated. To achieve his aim Woodford introduces a version of the cash-in-advance constraint but amends it to allow for debt, or credit, that is extinguished later (at the beginning of the next period in a multi-period setting). Woodford's model appears to include two forms of payment- something called informal credit and deferred cash settlement that attracts interest. We could call this

formal credit as it does capture the concept of credit, as the term is generally understood, and the need for a means of final settlement to extinguish debt. But this treatment of credit turns out to be another example of an inessential addition to a non-monetary accounting system of exchange. Formal credit is extinguished in the cashless limit leaving only informal credit. On inspection informal credit turns out to be code for an accounting system of exchange- to see this consider Woodford's post-monetary world.

III **Woodford's post-monetary world**

The model is based on an infinitely lived representative household-producer, a self-employed artisan, for example, and each artisan seeks to maximize lifetime utility. To introduce a role for money Woodford draws his inspiration from Lucas (1980) for the use of Clower's cash-in-advance constraint and from Lucas and Stokey (1987) for the variation on this theme based on the distinction between what he calls cash and credit goods. As is well known the cash-in-advance feature gives money a role in the model because in this sort of set-up artisans must hold cash even though it is dominated in rate of return by bonds (or capital).

But Woodford moves beyond the traditional cash-in-advance specification because his intention is to set up a payments technology that will allow him to shrink to zero the number of payments requiring cash settlement. It is my contention the Woodford has slipped into confusion at this point. Specifically he has confused the existence of an Arrow-Debreu auction that has the computing power to determine all commodity inter-temporal relative prices, with the computing power required to run an electronic money system.

To make the point I consider Woodford's payments system and then examine its implications under the cashless limit in two settings: (a) when something called money is treated as exogenous and a determinant of the price level (one version of the traditional quantity theoretic view of monetary theory and policy – a Patinkin-type world), and, (b) a Wicksellian world, without money where Woodford attempts to analyse how the interest rate is manipulated to achieve a price level or inflation target.

The payments system

Although there is an infinite planning horizon, time is divided into discrete periods. Each period, artisans must purchase two types of good each associated with it's own payment technology. One category of goods can be purchased on credit during the period and settled at the end of the period without the need for the buyer to hold cash balances. Woodford calls this informal credit. The other category of goods is purchased with the second technology and requires the buyer to hold cash balances at the end of the period sufficient to cover all charges against his account (for the second category of goods only) during the period. Cash must then be used to discharge any outstanding debt at the beginning of the next period. In Woodford it is not the cash-in-advance of Clower (1967) but what Hoover (1996) called cash-in-arrears.

Note that only the second type of payments technology requires the artisan to hold cash balances during the period. As Woodford (1997, p. 10) explains, "Thus the second type of transactions but not the first, require the buyer to maintain cash balances". In addition, the second technology is specified so that it contains a

parameter that can be adjusted to zero. Setting the parameter to zero then eliminates both the second category of goods and the second payments technology. It also obviously eliminates the cash-in-arrears constraint and Woodford describes this case as the cashless limit. Thus in the cashless limit the only payments system which remains is informal credit. Hence, it is essential to have a clear understanding of what informal credit is because it is the only element of the payments technology that remains when Woodford eliminates money from his model in what he calls the cashless limit.

But on this crucial issue Woodford is ambiguous to say the least. We know that in a monetary economy transactions can be settled in spot cash or deferred cash. In the latter case we say that purchases are made on credit and the associated debt must be serviced and/or settled using cash (money) at some future date. By contrast, in Woodford's model both types of transaction are said to be a form of credit but only one requires cash settlement. Woodford (1997, pp. 10) notes simply that informal credit must be settled by the end of the period but he does not specify how this is to be achieved. But how is informal credit settled if not by cash?[6] If there is no cash there can be no means of final settlement. It is my contention that what Woodford calls informal credit is code for what McCallum (1985, 2003) calls an accounting system of exchange -a non-monetary model- in which the unit of account function is represented by an arbitrarily selected *numeraire*. Woodford's cash-in-arrears constraint is then an inessential addition to an accounting system of exchange.

[6] In this regard, is the Lucas and Stokey distinction between credit and cash goods any assistance? Lucas and Stokey (1987) define 'credit goods' (goods which do not need to be paid for in cash) as 'non-market goods', such as leisure. This does not seem to be what Woodford has in mind. When he sets up the model we are left with the distinct impression that both types of goods are market goods that are distinguished only by the form of payments technology that must be used for their payment. But without further explanation we are left to surmise that these goods are exchanged somehow without the need for money (cash) or credit (as those two terms are usually understood).

A quantity-theoretic world in a real general equilibrium setting

Woodford's quantity-theoretic version of the model - the version, in which *M* is treated as exogenous- can be described in terms of the following system of equations. (I follow Woodford's (1997) numbering for ease of comparison)

$$R_t \geq 1 \tag{2.16}$$

$$\frac{M_t}{p_t} \geq \mathsf{m}(R_t) y_t \tag{2.23}$$

$$\frac{u'(\mathsf{w}(R_t) y_t)}{p_t} \mathsf{q}_1(R_t)^{\mathsf{g}-1} = \mathsf{b} R_t \frac{u'(\mathsf{w}(R_{t+1}) y_{t+1})}{p_{t+1}} \mathsf{q}_1(R_{t+1})^{\mathsf{g}-1} \tag{2.24}$$

$$\sum_{T=t}^{\infty} \mathsf{b}^T u'(\mathsf{w}(R_T) y_T) \mathsf{w}(R_T) y_T < \infty \tag{2.26}$$

$$\lim_{T \to \infty} \mathsf{b}^T u'(\mathsf{w}(R_T) y_T) \mathsf{q}_1(R_T)^{\mathsf{g}-1} \frac{M_{T-1}}{p_T} = 0 \tag{2.27}$$

Woodford presents this system of equations as a complete set of equilibrium conditions that can be used to determine the sequence $\{p_t, R_t\}$. He requires, in addition that at least one of (2.16) and (2.23) holds with equality. A brief explanation of each equation follows.

Expression (2.16) states that the endogenous nominal *gross* interest rate, $R_t \geq 1$. The condition that the nominal *gross* interest rate is always non-negative (and greater than one) is required to ensure an equilibrium exists and that artisans' debt is bounded (Woodford, 1997, 16). Expression (2.23) is the cash-in-arrears constraint where M_t and y_t are the exogenous money supply and known output in each period *t*. Expression (2.24) is the Euler equation which determines the optimal choice path for the representative artisan. Expression (2.24) includes the term $\mathsf{w}(R_t)$, which reflects the use of the payments system that requires cash settlement. Woodford introduces the parameter, a as the fraction of transactions requiring cash settlement. He then argues that as the cost of each informal credit good is p_t the cost of each good subject to cash settlement is $R_t p_t$. Those goods requiring cash settlement involve an additional interest charge because of the credit extended. The relationship $R_t p_t$ implies that $R_t = 1 + i_t$ where i_t is the net nominal rate of interest.

Artisans purchase the same quantities, c_{1t}, of each informal credit good and, c_{2t}, of each cash settlement good. By definition $c_t = (1-\mathsf{a})c_{1t} + \mathsf{a}c_{2t}$, and under market clearing conditions (assuming no government purchases) $(1-\mathsf{a})c_{1t} = y_{1t}$ and $\mathsf{a}c_{2t} = y_{2t}$. Expressions (2.26) and (2.27) are boundary conditions where $0 < \mathsf{b} < 1$ is a discount factor. Also, expression (2.26) requires that artisans' do not borrow more than they can possibly repay- based on their potential capacity to produce and maintain a minimum positive level of consumption (Woodford 1997, 11-12). Expression (2.27) is a transversality condition of a form required for an infinite planning horizon problem.

To model the post-monetary world Woodford generates the cashless limit by setting $\alpha = 0$ and examining whether it is still possible to determine what he calls the price level p_t. Accepting the analysis of the necessary mappings between infinite sequences, and that the required continuity conditions are met, it is possible to show that the first order non-linear difference equation which describes the equilibrium state of the model can determine p_t when $\alpha > 0$ (subject to the two boundary conditions). That is, writing the sequence of equilibrium conditions in general form as

$$\Phi(p; M, y; \alpha) = 0. \qquad (3.2)$$

Woodford shows that it is possible to find a function,

$$p = f(\alpha), \qquad (3.4)$$

such that (3.2) holds so long as $\alpha > 0$. However, it is not possible to express the perfect foresight equilibrium price sequence as a continuous function, $p = f(\alpha)$ on a domain that includes $\alpha = 0$. Woodford (1997, p. 25) explains the intuition behind this result as follows: "This result makes intuitive sense, of course, from the point of view of the quantity-theoretic approach to price level determination; as the fraction of transactions that require cash falls to zero, desired real money balances fall to zero as well, and so one would expect the equilibrium price level to cease to be defined. The failure of the solution to be continuous near $\alpha = 0$ simply means that one cannot [derive] a solution for the limiting case as an approximation to the small- α case;

instead the use of money in transactions is intrinsic to the model's ability to determine the price level".

All this is no doubt familiar to anyone acquainted with Patinkin (1965). The problems arise in Woodford's model when he attempts to circumvent this result. In that endeavour, Woodford is, quite rightly, not concerned with a world in which *M* is exogenous. He regards that case as of little practical importance today because central bankers make no attempt to render some *M* exogenous. Today's world of electronic banking has more in common with Wicksell than Friedman[7].

A "Wicksellian" world in a real general equilibrium setting

Woodford (1997, p. 27) suggests the following relationships as a complete set of necessary and sufficient conditions for sequences $\{p_t, R_t\}$ to constitute equilibrium in a model which is a proxy for a world in which central bankers manipulate a nominal interest rate to achieve a price level or inflation target.

$$R_t = f(p_t) \tag{3.7}$$

$$\frac{u'(w(R_t)y_t)}{p_t} q_1(R_t)^{g-1} = bR_t \frac{u'(w(R_{t+1})y_{t+1})}{p_{t+1}} q_1(R_{t+1})^{g-1} \tag{2.24}$$

$$\sum_{T=t}^{\infty} b^T u'(w(R_T)y_T)w(R_T)y_T < \infty \tag{2.26}$$

[7] At times Woodford (1997) gives the impression that by a post-monetary world he means a world in which the quantity of money is no longer relevant. But money and monetary theory is not synonymous with the quantity theory of money

$$\lim_{T\to\infty} \beta^{T} u'(\omega(R_T) y_T)\theta_1(R_T)^{\gamma-1} \frac{W}{p_T} = 0 \qquad (3.9)$$

Expression (3.7) is simply the monetary policy rule that links interest rates to prices (or rates of change in prices in the case of an inflation target). There is no change to expressions (2.24) or (2.26) but (3.9) does involve a re-specification of fiscal policy to incorporate a target level for the total nominal value of government liabilities, $W > 0$. This accounts for the appearance of W in expression (3.9). Expression (2.23), which embodied the cash-in-arrears feature of the previous model, is still implicit in the model in the sense that it could be used to determine the equilibrium path of the now endogenous money supply once the equilibrium sequence $\{p_t, R_t\}$ has been determined. (Presumably this is true only for those cases where $\alpha > 0$ because M disappears in the cashless limit as all payment is by informal credit).

This model is essentially identical to the model with exogenous money (Woodford, 1997, pp. 28-29) but Woodford argues that it is *qualitatively* different in one important respect: in this version of the model the equilibrium equations remain well behaved in the cashless limit. In other words, the function f in expression (3.4) is continuous in the cashless limit; i.e., when $\alpha = 0$. Hence, the price sequence $p_t = \hat{p}(y_t, y_{t+1,}.....)$ defined by $\hat{p} = f(y,0)$ is a solution to the equilibrium conditions in the cashless limit ($\alpha = 0$). It also means that this solution approximates a perfect

foresight equilibrium in the case of any parameter sequence in which α_t is small enough for all t.

In the cashless limit the equilibrium conditions in the Wicksellian version become:

$$\frac{u'(y_t)}{p_t f(p_t)} = \beta \frac{u'(y_{t+1})}{p_{t+1}} \tag{3.13}$$

$$\sum_{T=t}^{\infty} \beta^T u'(y_T) y_T < \infty \tag{3.14}$$

$$\lim_{T\to\infty} \beta^T u'(y_T) p_t^{-1} = 0 \tag{3.15}$$

Clearly, the Euler equation (expression 3.13) makes no reference to any money or transactions technology. This is the crucial step in Woodford's (1997, p. 34) argument because he takes this as evidence ". that there is in fact no need to describe the way in which cash facilitates transactions, ..., in order to derive these equations.... One might instead have simply written down the equilibrium conditions:

$$u'(y_t) = \beta \frac{R_t p_t}{p_{t+1}} u'(y_{t+1}) . \tag{3.19}$$

And

$$\lim_{T\to\infty} \beta^T u'(y_T) \frac{W_T}{p_T} = 0 . \tag{3.20}$$

Combining these with the monetary and fiscal policy rules produces (3.13) and (3.15) directly[8].

IV The Hahn property of Woodford's model

What is wrong with Woodford's analysis? The simple answer is that Woodford's model exhibits the Hahn property - money or credit is an inessential addition to non-monetary accounting system of exchange. Appending a cash-in-arrears constraint, forces agents to hold money even though the medium of exchange function of money is a welfare reducing innovation, friction or distortion. But if the medium of exchange function of money is an inessential addition then transactions technologies are similarly redundant. The model cannot then be used to analyse the evolution of money.

The inessential nature of money in Woodford's analysis is straightforward to confirm because Woodford repeatedly draws attention to this property of his model. To begin with, in the version with exogenous *M*, Woodford derives the non-monetary version of the Euler equation (expression (2.24) by applying the condition $\alpha = 0$ and deriving the consequent results; $\theta_1(R) = 1$, $\theta_2(R) = R^{-1/1-\gamma}$, $\omega(R) = 1$ and $\mu(R) = 0$ for all $R \geq 1$. In that case the Euler equation (2.24) reduces to:

$$\frac{u'(y_t)}{p_t} = \beta \frac{u'(y_{t+1})}{p_{t+1}} \qquad (3.5)$$

[8] The fiscal policy rule is simply that the conventional budget deficit is zero in each period (Woodford, 1997, p. 27). The monetary policy rule is expression (3.7) given above.

which, as Woodford (1997, p. 24) notes, is the "..familiar asset-pricing equation for a non-monetary economy, applied to an asset which pays no dividend and has an exchange value (in terms of the consumption good) of $1/p_t$".

In terms of the Hahn property, money is obviously an inessential addition to this version of Woodford's model - as it was in Patinkin (1965).

The same conclusion also applies to Woodford's Wicksellian world in the cashless limit as he himself points out when he notes that there is no need to take account of any transactions technology when deriving the equilibrium solution. One might more conveniently simply write down the equilibrium conditions from the non-monetary version of the model. In this regard expression (3.13) is particularly interesting because it is apparent that it differs from expression (3.5) only by the inclusion of the term $\mathrm{f}(p_t)$. But in Woodford's cashless limit there is no money so by definition there is no nominal rate of interest either. If only informal credit remains then there is no deferred payment that attracts nominal interest. In the cashless limit informal credit is, I contend, what McCallum calls the accounting system of exchange and no means of final settlement is required. Such a system is a non-monetary economy that could be described as a form of efficient or perfect barter as components of the endowments are exchanged without any of the constraints encountered by real-world barter (Subject to the caveat in footnote 3).

Thus, in the cashless limit only the accounting system of exchange, an unspecified clearing system, and the *numeraire* remain. Hence it must be the case that $R=1$

because the net rate of interest on cash is not defined in a world without cash. That means expressions (3.5) and (3.13) or (3.19) are identical in the cashless limit. Both versions of Woodford's model have the same non-monetary solution. The distinction in price between cash and informal credit goods also disappears in the cashless limit – the price of cash goods was $R_t p_t$ but is only p_t, the same as the price of informal credit goods, when cash vanishes and $R_t = 1$.

Furthermore, the same non-monetary equilibrium solution can be derived for the model even when $a > 0$ if we consider the case of $R = 1$. Expression (2.16) may hold with equality in this model (which Woodford allows) so this case also generates the non-monetary Euler equation (3.5). The condition $R_t \geq 1$ is required for equilibrium to exist but the equilibrium that exists when $R_t = 1$ is a non-monetary equilibrium. This is the Hahn property of Woodford's model. My point here is that these results are simply examples of Hahn's (1973a,b) conclusion that it is possible to show that a non-monetary equilibrium exists when a non-essential addition of money is made to an accounting system of exchange in which an equilibrium is generated by a Walrasian or Arrow-Debreu auction.

Woodford's strategy of appending a cash-in-arrears transactions technology to an accounting system of exchange then appears for what it really is; an inessential addition that in no way compromises the integrity of the underlying accounting system of exchange. Consequently cash can be made to vanish without undermining the existence of a real equilibrium. But we should not be surprised that appending a redundant transactions technology, or money in any form, to the model causes nary a

ripple when it is removed. It does, however, throw up some interesting conceptual issues.

V Conceptual problems raised by Woodford's analysis

Woodford clearly intends his analysis to have policy relevance in view of Rotemburg and Woodford (1997) or Woodford (2003). To be of relevance to policy makers the variables in Woodford's model must be proxies for their real world counterparts. For example, a short-term nominal interest rate is the instrument of choice in modern monetary systems, and policy is directed to achieve price level stability defined in terms of the CPI (or nominal GDP). In Woodford's cashless limit, however, neither of these concepts can be given any meaningful interpretation. That leaves no role for central bankers.

Consider first the nominal rate of interest. Whenever $a > 0$ the gross and net nominal rates of interest are defined. In each case there exists some *M* by virtue of the cash-in-arrears constraint which is imposed on artisans. But in the cashless limit, where $a = 0$ there is no money and hence the nominal rate of interest, the rate charged for borrowing money, is conceptually undefined. The same is true of the inflation adjusted nominal rate. The Fisherian real rate is also not defined in Woodford's cashless limit. In any event, no artisans use the second technology in the cashless limit and any arbitrarily selected commodity can be labelled the *numeraire*. But even if the $ was selected as the *numeraire*, and all prices quoted in $s, how is actual payment to be made in this world? The model circumvents this question.

What then, are we to make of the role of central bankers? If central bankers use a nominal interest rate as the instrument of policy, albeit that they have an inflation-adjusted rate in mind, how are they to implement monetary policy in Woodford's post-monetary world where no nominal instrument and no means of final settlement exists? In Woodford's post-monetary world of the cashless limit expression (3.7) collapses as only commodity interest rates exist –there is no nominal rate and no Fisherian real rate either. Furthermore, as only informal credit remains in the cashless limit there is no means of final settlement and therefore no means by which central bankers can manipulate an interest rate instrument.

Could central bankers not use a commodity rate directly? It seems not. Commodity interest rates in Woodford's cashless limit are implicit in the sequence of commodity inter- and intra-temporal relative prices –the quantity of jam today vs the quantity of jam tomorrow or the oil price of a new car[9]. The set of commodity interest rates is simply another way of presenting the set of commodity relative prices. The set of commodity relative prices is generated given the tastes, endowment and technology as a solution to the dynamic optimising behaviour of the artisans. There is simply no way in which someone called a central banker can interfere with this process. If the model has no essential role for money it certainly has no essential role for central bankers. What purpose would central bankers serve in this world? It seems to me that there is a form of cognitive dissonance at work here. Consequently, because the model is one in which any transactions technology is redundant it really makes no sense to use it to examine the properties of a world in which the transactions technology is evolving.

[9] The use of a numeraire would simplify the computations required but it is not clear that a numeraire is an *essential property* of a model with the computing power and foresight of an Arrow-Debreu auction backed by a highly efficient clearing system.

For similar reasons I find the description of the model as Wicksellian in the cashless limit to be problematic. The essence of Wicksell's analysis is the distinction between the market and natural rates of interest. The market rate is the rate on money loans charged by banks and the natural rate represents the return on real capital investment earned by firms. (The two rates can be compared in nominal or inflation adjusted real terms) Assuming full employment, Wicksell argued that inflationary (deflationary) pressure resulted whenever the market rate lay below (above) the natural rate. The discrepancy between the two rates could occur as a result of monetary and/or productivity shocks. Furthermore, Wicksell also had a model that bears some relationship to the cashless world Woodford has in mind- the pure credit model.

Wicksell's pure credit model is a world in which no cash exists and all transactions are conducted by book entry. Today he would no doubt say by electronic transfer. Nevertheless, Wicksell's theory was clearly intended to apply to a monetary economy in which all three functions attached to the book entries (or in a modern setting the electronic medium of exchange) and final settlement by the central bank is required to tie down the price level. For example, even though the $ bill does not exist (there is no cash) in Wicksell's pure credit economy all prices are expressed in $'s and book (electronic) entries expressed in $ units are the medium of exchange and store of value. Woodford's model does not have either of the last two functions in the cashless limit and the *numeraire* can be selected arbitrarily from the list of commodities[10]. In a monetary economy the *numeraire* is not arbitrary. Finally, because there is no nominal rate of interest in Woodford's cashless limit there is nothing in the model

[10] Buiter (1999) stresses this point.

equivalent to Wicksell's market rate. For these reasons I don't see any connection between Wicksell's pure credit economy and Woodford's cashless limit. Woodford's neo-Wicksellian monetary theory is a caricature of Wicksell.

Additional conceptual difficulties arise when we begin to inquire into the meaning of the term price level in Woodford's post-monetary world. In the cashless limit there is no *M*, so there is no sense in describing the price sequence, $p_t = \hat{p}(y_t, y_{t+1,}.....)$ as a price level (even if we are dealing with a single commodity). The price sequence is simply a set of inter-temporal commodity relative prices and bears no resemblance to the CPI or the GDP deflator. In that case Woodford's model cannot provide the basis for sensible advice about the process of inflation or price level targeting undertaken by central bankers. As a matter of interest, Hoover pointed out some time ago that a model by Fama (1980) involved just this conceptual error[11]. Hoover (1988, p. 99) alerted us to the confusion when he observed that: "Fama's use of the term 'price level' is somewhat misleading. The barrel-of-oil price of each good is a *relative* price while the price level is usually taken to refer to *absolute* or *nominal* prices." The same conceptual error is found in Woodford's analysis when he suggests that inflation of the price level can meaningfully be analysed by a model in which only an inter-temporal relative commodity price vector exists. Buiter (1999, p. 36, fn 30) makes the same point when he concludes that: "In an economy without money the price of money is not just indeterminate. It is conceptually undefined."

[11] Fischer Black (1974) makes a similar conceptual error. It seems that these authors confuse the existence of an auctioneer (with amazing computing ability) with the use of electronic means of payment and the widespread adoption of computers in the banking sector. There is an obvious conceptual distinction.

All this suggests that the conceptual errors inherent in Woodford's analysis are nothing new. In addition to Hahn (1973a, b)), whom, it might be thought, was making a rather esoteric point, monetary theorists from across the spectrum have reached essentially the same conclusion. In particular, Laidler (1990) or Clower and Howitt (1993) have drawn attention in the literature to this form of conceptual error: real general equilibrium models are exactly that- real general equilibrium models that have nothing to say about how transactions are actually carried out. The existence of the Walrasian auctioneer is usually identified as the assumption that precludes the need for money or to model any transactions technology. Clower and Howitt (1993, p. 2) make the point rather forcefully:

"Although in every case some attempt is made to motivate the holding of money by invoking some kind of transactions cost, the very existence of such costs is ruled out by the assumption (explicit or implicit) that 'the auctioneer' establishes the terms of all planned trades without cost to any 'agent'. In all such theories, transactions cost are ruled out by the assumption that trading plans can be executed whenever for each good the sum of everyone's planned purchases equals the sum of their planned sales, without regard to the question of who trades what for which or with whom".

Laidler (1990) raises the same issue when he stresses that in any model the Walrasian auction and money are substitutes not complements. The existence of a Walrasian auction obviates the need for money. Simply put, a model with an explicit, or implicit, Walrasian auction precludes the need for money and hence a transactions technology (electronic money); the model is simply not designed to address the question of how transactions will be executed. Using it to address such questions or as a basis for a

theory of monetary policy leads to the sort of conceptual confusions exhibited by Woodford's analysis.

The most telling illustration of how the conceptual errors embedded in Woodford's analysis come home to roost has been provided by Buiter's (1998, 2002, 2004) critique of Woodford's fiscal theory of the price level. With respect to Woodford's treatment of money in that context Buiter raises precisely the conceptual issues raised in this paper. In particular, he warns that the claim to be pricing something that doesn't exist -the price of money in an accounting system of exchange–should have set off some alarm bells[12].

VI **Woodford's rationale for non-monetary models as a foundation for monetary theory and policy**

Despite the conceptual problems raised in the previous section Woodford (1997, pp. 35-36) argues that non-monetary models provide a sound basis for the foundations of monetary theory and policy and he gives three reasons:

(i) Many economists consider it sensible to neglect money when writing the Euler equations.

(ii) It is sensible to ignore monetary variables in the Euler equations, at least when analysing an economy with a highly developed financial system.

[12] Buiter (1999, 2002, 2004) refers to 'money' in Woodford's model as phlogiston –the mythical substance once thought to cause combustion.

(iii) The cashless limit provides a better justification for excluding money in the Euler equation than the use of an additively separable utility function.

None of these reasons is convincing.

In the first instance many sensible economists are wrong. In addition to those puzzled by the treatment of money as a friction in traditional general equilibrium models, examples of two representative sensible economists are Fisher Black (1970) and Eugene Fama (1980). As noted above, Hoover (1988) illustrated how the Hahn property of their models led them into confusion of the type exhibited by Woodford's analysis.

The second reason involves an obvious confusion between the properties of a modern electronic money using economy and the computing power of the auctioneer in the Walrasian or Arrow-Debreu auctions that underpin non-monetary accounting systems of exchange. The two issues are conceptually distinct. Accounting systems of exchange have nothing to say about how trades are to be executed so they are not concerned with the feasibility of the set of commodity relative prices that is generated. Hence they have nothing to say about money or the evolution of the monetary system. The Arrow-Debreu auction applies to a non-operational thought experiment because the trades required to achieve such equilibrium would not be practical. It is simply not practical to trade 1000 haircuts to the suppliers of an overseas holiday. That is why using money would expand the practical choice set, stimulate specialisation, and be a welfare improving innovation.

The third reason reflects Woodford's (1998, p. 173 italics added) view that an important contribution is "…an 'approximation' theorem, showing the existence, for a simple *monetary model,* of a well-behaved 'cashless limit' in which money balances held to facilitate transactions become negligible". Money in the utility function is, of course, subject to the Hahn (1973a,b) property. However, it has been demonstrated above that the Hahn property applies equally to Woodford's analysis of the cashless limit. Cash-in-arrears is an inessential addition to Woodford's model- whether it is an approximation to the cashless limit or any other state[13]. The approximation theorem therefore offers no advantages over the use of separable utility functions. The conceptual problem is common to both and lies much deeper.

VII **Conclusion**

Woodford presents an analysis that purports to model the evolution of the monetary system from a world of cash to a world of electronic money and to provide a Wicksellian foundation for a theory of monetary policy. The analysis is unconvincing because it proceeds by a well documented, but apparently not well-known, sleight of hand that obscures fundamental conceptual errors. On the face of it Woodford's treatment of cash-in-arrears would appear to capture an important feature of modern electronic money systems in which accounts at the central bank provide the means of final settlement. But on closer inspection Woodford's cash-in-arrears constraint turns

[13] In similar vein, Buiter (2002, fn 30) argues that the limit should be taken in such a way that the mathematics adequately captures the process of demonetisation. This Woodford's analysis of the cashless limit fails to do. Buiter (2002, fn30) concludes that:
" Searching for limiting behaviour of the equilibrium price of money sequence in a demonetising monetary economy, in the hope of finding one which converges to an equilibrium price of money sequence in a non-monetary economy, is therefore bound to be a will-o'-the wisp". It is a will-o'-the wisp because Woodford's concept of a cashless limit does not capture the process of evolution of the monetary system from cash to electronic money. Woodford's cashless limit is simply the real equilibrium solution to an accounting system of exchange to which money or credit, be it cash (in-advance or in-arrears) or electronic transfer, is an inessential addition in the sense of Hahn

out to be an inessential addition to his model. Removing formal credit in the cashless limit leaves only informal credit in the guise of a model of ideal barter based on an unspecified clearing system – a non-monetary accounting system of exchange of the Walrasian or Arrow-Debreu type. Such a model cannot be employed to examine the evolution of money, be it electronic or otherwise. Neither can it be employed as a foundation for a theory of monetary policy. In short, there is nothing in Woodford's model that corresponds with anything that would be recognizable to a central banker. It cannot then provide a basis for sensible advice to central bankers or provide a basis for empirical analysis of real economies. Woodford mistakes a non-monetary accounting system of exchange for the real world of electronic money.

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