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Abstract

This paper re-examines the individual income tax evasion decision in the simple framework introduced by Allingham and Sandmo (1972), where the individual taxpayer decides how much of his income is invested in a safe asset (reported income) and in a risky asset (concealed income). These early models could not convincingly reproduce the empirically observed positive influence of higher tax rates and higher gross income on tax evasion simultaneously. We replace the standard assumption that risk aversion is the factor limiting the extent of evasion by assuming risk neutral taxpayers and argue that this is a reasonable approximation. The observation that concealing income is costly leads to the conclusion that, instead of risk aversion, evasion costs (such as concealment expenses and moral cost) might be the factors that limit tax evasion. We reproduce the stylized facts not explained by older models for very general tax and penalty schemes, including those where the standard model definitely fails to do so.

JEL-Classification: H26

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1 Introduction

The still widely used neoclassical framework for the analysis of income tax evasion was set out within the seminal papers of Allingham and Sandmo (1972) and Yitzhaki (1974). One of the most important questions in these early papers were: How do taxpayers (and evaders) react to changes in the tax rate, and do people evade more when they get richer? Without doubt, the intuitive answers are: Higher tax rates lead to more evasion and richer taxpayers will ceteris paribus evade more. And in fact, econometric studies suggest that in this case intuition is a reliable guide (see e.g. Clotfelter (1983), Dubin et al. (1987), or Feinstein (1991) for influential econometric studies, Andreoni et al. (1998) and Bayer and Reichl (1997) contain more recent surveys). Unfortunately, the early models couldn't simultaneously reproduce the empirically observed relations in a convincing manner. Furthermore, the comparative static results were not very robust against small changes in the tax and penalty schemes. In a setting where the penalty depends on the evaded tax (Yitzhaki, 1974) and risk-averse taxpayers maximize expected utility the two effects even unambiguously point in different directions. When tax evasion increases with the gross income it decreases with the tax rate or vice versa.

The neoclassical attempts to solve this puzzle led into two different directions. Many authors endogenized variables such as public good provision, labour income and wages (see Cowell (1990) for a comprehensive survey of these attempts). Others tried to incorporate personal perception variables like equity into the utility function (e.g. Cowell, 1992; Bordignon, 1993). The former approach did not lead to plausible explanations of the puzzle, while the latter lacked the robustness against small arbitrary changes of functional forms.[1]

The second generation of tax evasion models - initiated by Reinganum and Wilde (1985) - came from game and contract theory. These papers (see e.g. Border and Sobel (1987), Mookherjee and Png (1989), Mookherjee and Png (1990), or Chander and Wilde (1998)) rather searched for an optimal, incentive compatible, environment by optimizing tax, penalty and audit schemes, than to try to solve the puzzle described above. Economic psychologists - mainly in experiments - found a variety of influence factors for tax evasion. But the resulting frameworks were mainly descriptive and did not have too much predictive power for expected behavioural reactions on changes in the environment (see Webley et al. (1991) for a good overview).

This paper tries to provide a solution of the tax evasion puzzle by stepping back to the early models, where we slightly change some assumptions. We assume risk-neutral instead of risk-averse taxpayers and argue that this might be a viable approximation for the risk preferences in the case of tax evasion. This assumption is justified by experimental evidence and psychological theories. In some models dealing with optimal taxation and tax evasion (Cremer and Gahvari, 1994), or with the black market economy (Cowell and Gordon, 1995), this assumption has been used to keep the models tractable. Furthermore, we introduce evasion costs, such as the fixed moral cost of doing something illegal or the variable costs for concealing income and creating opportunities to evade.[2]

[1] For a recent review of the relevant theoretical and empirical literature see Slemrod and Yitzhaki (2002).

[2] A recent model involving avoidance costs is Slemrod (2001).

In the following section the assumptions are justified and an "example", which has the generality level of the early models, is set up. The comparative statics in section 3 show that we can reproduce the empirically observed effects. Section 4 extends this result to a wide range of tax systems, penalty schemes, and evasion cost functions. The main conditions for our results to hold are a non-regressive tax system and what we call a "fair" penalty scheme. We conclude with some final remarks.

2 The model

Within the following sections we use a certain specification of the evasion costs, the tax system and the penalty scheme. This makes the analysis quite easy, since closed form expressions for equilibrium values and comparative static effects are obtained. In addition the formulation below gives rise to a possible comparison of our results with the results obtained in the literature. However, the derived results hold for a broad range of different cost functions, tax and penalty schemes. The treatment of a general setting can be found in section 3.

2.1 Opportunities and evasion costs

Different taxpayers have different tax-evasion opportunities. These different opportunities may stem from different sources of income. For example employees have few possibilities to evade their working income in many countries, since their taxes are directly collected and delivered to the tax authorities by the employer.[3] Opportunities to evade have to be created. Collusion with the employer and working in the shadow economy are examples for creating such opportunities. On the other hand, self-employed taxpayers have some more means of evading taxes. They can simply not report issued bills or make too high deductions by handing in bills paid for private purposes. These different opportunities also apply for other sources of income. We can think of gains from the capital markets, to. In Germany, for example, taxes on interest payments have to be collected and delivered by the banks in behalf of their customers. It is not too easy to get around this legal evasion obstacle. But, there are cases where collusion between the banks and the taxpayers took place. This opportunity had to be created. By contrast, speculative gains from trading with shares are easy to hide.[4]

This story tells us two things. Taxpayers have different opportunities to evade, and since opportunities often have to be created or at least information about opportunities has to be gathered, underreporting is costly. Obviously, the evasion opportunities a person has are closely related to the potential evasion costs it has to bear. The more opportunities a taxpayer has the easier it is for him to evade, and the lower are the evasion costs.

If we now consider a rational tax evader with an income y stemming from different sources, which is the first part of income he will underreport? Obviously, the income from the source with the lowest evasion costs. Additionally, he will use the cheapest means of underreporting first. To conceal further

[3] This is e.g. the case in Germany and Switzerland.

[4] Gains from trading with shares in Germany are considered to be speculative and regarded as taxable income, if the shares are held for less than twelve months.

income he might have to use costlier means and/or sources. In the German example for capital gains, a person with income from the capital market may first underreport his gains from share trading, which is related to low evasion costs, than bring some money abroad to create the opportunity to underreport interest payments, and than try to establish collusion with the bank, which is very costly indeed.

Thus, the additional costs for further evasion are positively related to the share of income already evaded. Furthermore, these costs decline with the individual opportunities to evade. To avoid the technical problem to deal with a discontinuous cost function we use a continuous cost function as an approximation.[5] If we consider the relations being linear at the margin, the marginal costs of evading can be written as:

$$C'(h) = \frac{h}{y\theta},$$

where h/y is the share of income not reported and θ denotes the individual evasion opportunities. The total evasion costs depend on the unreported income h and can be found by integration. This yields:

$$C(h) = \begin{cases} \kappa + \frac{h^2}{2y\theta} & if \quad h > 0 \\ 0 & if \quad h = 0 \end{cases} \tag{1}$$

The integration constant κ can be interpreted as the initial fixed costs of evading; i.e. the cost for acquisition of information about opportunities to evade and the often claimed moral costs to do something illegal. Furthermore, they can be seen as the cost of the first monetary unit evaded. This fixed cost might be individually different as evasion opportunities are. Our notion of fixed evasion costs is related to the approach in Myles and Naylor (1996). There an honest taxpayer enjoys some utility from conforming with other honest taxpayers. A motivation why there may be a utility loss simply caused by the act of evasion was first provided by Gordon (1989).

The quite arbitrary looking formulation of evasion costs is less crucial for the results to be derived later than one might suspect. The properties we need are $C_h > 0$, $C_{hh} > 0$, $C_\theta < 0$ and $C_{yh} \leq 0$, where subscripts denote partial derivatives. The explicit formulation is used for expositional reasons only.

2.2 Attitudes towards risk

The crucial assumptions driving the results in the early tax evasion models are those about the risk preferences of the taxpayers. On the first sight, it seems very reasonable to assume risk-averse actors, and consequently to use von Neumann-Morgenstern expected utility functions. But the empirical evidence about risk preferences is somewhat mixed.[6] Decision-making under risk is very sensitive against small changes in the environment. Hence, to use the same model structure for portfolio decisions with risky assets and tax evasion does not necessarily mean that it is sensible to use the same risk preferences, too. Additionally, even for portfolio decisions concave expected utility functions may not be appropriate at all.[7] In our opinion, it is possible to resolve many decision anomalies in the context of tax evasion, which

[5] This seems to be justified by the fact that there are many means and actions - with different costs associated - that can be taken to evade taxes.

[6] For a survey see Camerer (1995) or Camerer (1998). An older, but more rigorous treatment is found in Machina (1987).

[7] For obvious deficiencies of expected utility theory see the stunning calibration exercise in Rabin (2000).

are widely discussed in the economic psychology literature, by assuming - as an approximation - risk neutral taxpayers.

The specification of risk preferences according to the Prospect Theory proves to be a good working hypothesis (Kahneman and Tversky, 1979). Psychologically, changes in the environment that lead to a reduction in (economic) freedom (e.g. higher tax rates) are very likely to lead to the so called reactance phenomenon (Brehm (1966) and Brehm and Brehm (1981)) if we consider the situation of the taxpayer's reporting decision. Reactance - in this context - means that people use an available instrument (here: tax evasion) to win back their freedom.[8] This is the basis for the assumption of the risk loving taxpayer in situations where he wants to avoid a sure loss - as predicted by prospect theory. Beyond the reference point, where the prospect is a possible gain, it is reasonable to stick to risk aversion as an assumption, since the taxpayer sees the situation as a usual gamble - again, as prospect theory predicts.[9] But just to incorporate such preferences into a usual framework of tax evasion is not viable and needs further assumptions. First of all, the individual reference point has to be determined and, secondly, we have to decide the extent of risk aversion and risk love for the different net income levels.

The assumptions about relative and absolute risk aversion (using von Neuman-Morgenstern utility functions) were crucial for the predictions of the early tax evasion models. We claim that in the tax evasion game - because of the game being played repeatedly and with high stakes - people are approximately risk neutral. For the case of losses and high stakes experimental evidence shows that people are in fact approximately risk neutral (Kachelmeier and Shehata, 1992). Furthermore, the fact that the game is played repeatedly leads to the reasoning that the variance of the average period payoff (over all periods) is much smaller than the variance of an actual period payoff. This means that the uncertainty in the repeated game is much smaller than in the one shot game. This should reduce risk aversion. Evidence for this claimed effect was also found in the mentioned experiments of Kachelmeier and Shehata where gains or losses were added to or taken from virtual lifetime accounts.

In using risk neutrality as an assumption we have a fairly good approximation for preferences in risky games with high stakes, regardless whether they are considered as possible gains or possibly avoidable losses. In the case of tax evasion this assumption might be a better approximation than the traditional von Neumann-Morgenstern approach. In addition, we do not run into the problem of finding a reference point as if we wanted to use the preferences proposed by the Prospect Theory. This makes our further analysis comparatively simple, and reproduces - as we will show - empirically observed behavioural reactions of taxpayers to changes in the environment.

[8] There are several conditions determining whether reactance occurs and what reduction instruments are used. The very interesting discussion about the consequences for situations where the Prospect Theory can be applied has still to be led.

[9] That this preference reversal phenomenon is relevant in the case of tax evasion is supported by experimental data reported in Bayer and Reichl (1997). There tax evasion behaviour is negatively correlated with the change in the degree of satisfaction with the system, which is used as an indicator for externally caused prospective utility changes.

2.3 Tax system

We use a progressive tax system to cope with reality. On the other hand, to keep things simple, , we assume in this example that the tax rate is linearly dependent on the reported income. Furthermore, to ensure that the decision function of the taxpayers is continuous and differentiable, we assume that already for the first unit of income tax has to be paid and that the maximal tax rate is reached at the highest income in the population.[10] Then (true) tax liability T for a certain income y is given by $T(y) = t(y)y$, with $t(y) = \tau y + \alpha$, where τ represents the marginal rise of the tax rate with respect to income, which is a measure for the progression of the system. The α is a constant part of the tax rate. Variations of α can be used to change the tax rate for all incomes by the same amount. We get:

$$T(y) = \tau y^2 + \alpha y. \tag{2}$$

Certainly, to obtain the tax liability with unreported income, the true gross income y has to be replaced by the declared income ($d = y - h$). The assumed linear dependency of the marginal tax rate on the declared income is not crucial for our analysis. The main results hold, as long the tax system is not regressive.

2.4 Detection, penalties and expected payoff

As in the basic models of tax evasion we assume a fixed probability p of being audited. This can be interpreted as the given strategy of the tax authority being to audit a certain amount of taxpayers randomly. We assume further that an audit reveals the true income with certainty. If an audit detects underreporting the tax cheat will have to pay his true tax liability and an additional penalty. Here, we assume that the penalty is a linear function of the amount of taxes the taxpayer tried to evade, which is $T(y) - T(y - h) = h(\alpha - h\tau + 2y\tau)$. Thus, denoting the penalty parameter by f, the payoff after an audit $D(h)$ is given by

$$D(h) = y - T(y) - f[T(y) - T(y - h)] - C(h). \tag{3}$$

We chose this specification of the penalty scheme, since this is the same as Yitzhaki (1974) used to find that for a proportional tax system ($\tau = 0$ in our setting) the relation between tax evasion and tax rate definitely has another sign than the relation between tax evasion and gross income.[11] The main results hold as well for other specifications.[12] On the other hand, if a tax cheat gets away with his underreporting his payoff $G(h)$ will be his true income minus the tax payments associated with his reported income and the evasion cost. This is expressed by the following equation:

$$G(h) = y - T(y - h) - C(h). \tag{4}$$

[10] The main findings of this chapter are not affected by these assumptions as section 4 shows.

[11] There, for a decreasing (increasing) absolute risk aversion the relationship between tax rate and evasion is negative (positive), that between gross income and evasion is positive (negative).

[12] For a general treatment see section 4, where we define the class of "fair" penalty schemes and show that this is a sufficient condition for our results to hold.

Since we assume that there is no reward for overreporting of income we can restrict h to values bigger or equal to 0 in both cases.[13] A rational risk-neutral taxpayer maximizes his expected ex post income by choosing an optimal level of non-reported income. The expected ex post income $E(h)$ is the sum of the with their probability weighted play-offs for the two states of the world; i.e. being audited or not:

$$E(h) = pD(h) + (1 - p)G(h) \tag{5}$$

2.5 Optimal underreporting

Denoting the declared income $(y - h)$ by d, the first-order condition for this maximization problem is given by:

$$\frac{dE(h)}{dh} = (1 - p)\frac{\partial T(d)}{\partial d} - pf\frac{\partial T(d)}{\partial d} + \frac{\partial C(h)}{\partial h} = 0. \tag{6}$$

Examining the first-order condition part by part we see that the first (positive) part is the expected marginal gain for a further monetary unit of unreported income, while the following (negative) parts are the expected marginal penalty from undetected evasion and the marginal evasion cost of a further unit of unreported income.

We can find combinations of the auditing parameters f and p that ensure everyone to be honest. The condition deterring tax evasion is $p + pf \geq 1$. If this condition holds the marginal gain (net of evasion costs) of not reporting a unit of income is always negative. The gamble against the tax authority is an unfair one and no risk neutral taxpayer will evade. Since in reality the parameters are such that people evade taxes, we will not look at such cases. Bernasconi (1998) reports $p + pf = .054$ for the US and similar values for other countries. So we restrict our parameters in a way that the following inequality holds:[14]

$$p + pf < 1.$$

As easily can be checked, the second derivative is negative and the second order condition for a maximum is fulfilled if we impose this restriction:[15]

$$\frac{d^2E(h)}{dh^2} = (p + pf - 1)\frac{\partial^2 T(d)}{\partial d^2} - \frac{\partial^2 C(h)}{\partial h^2} < 0.$$

Plugging the values for our specification into the first-order condition (equation 6) and solving for h yields, an interior solution assumed, a closed form solution for the optimal amount of income not reported h^*, which is:[16]

$$h^* = \frac{(1 - p - pf)(2\tau y + \alpha)}{2\tau(1 - p - pf) + 1/(\theta y)} \tag{7}$$

For an interior solution the tax gamble has to be fair and the fixed evasion costs κ have to be sufficiently small. In the following section we will assume this to be the case. For a further analysis see section 3.4.

[13] To assure this, we furthermore have to assume that there is no reward to a negative income such as a negative income tax.

[14] For other specifications this "more than a fair gamble" condition is slightly more complicated. E.g. in the Allingham/Sandmo setting for fines we would get $ps < 2y\tau(1 - p)$.

[15] This is the case since we assumed an indirectly progressive tax system ($T'' > 0$) and increasing marginal evasion costs ($C'' > 0$).

[16] Recall that θ was the parameter for the evasion opportunity.

3 Comparative statics

The standard models of tax evasion with taxpayers being risk-averse and having risk preferences, that are not affected by changes in parameters, are not capable of simultaneously explaining the empirically observed positive relations between unreported income and tax rates and between true and non-reported income.[17] So it is useful to have a look on the relationships our model predicts. In this and the next two sections we will restrict ourselves to parameter constellations with interior solutions. The conditions for corner solutions are examined in section 3.4.

3.1 Changes in the tax system

In the case of tax rate variations there are two different sub cases of interest. What happens,

1. if the tax rate rises due to a ceteris paribus increase in the progression τ,
2. if the tax rate rises for everyone by the same amount (increase in α)?

The marginal changes in non-reported income in equilibrium is given by implicit differentiation of the first-order condition. For case one, where the marginal tax rate rises, while the income independent component of the tax rate stays the same, the change in optimal underreporting is given by:[18]

$$\left.\frac{\partial h}{\partial \tau}\right|_{h=h^*} = -\frac{E_{h\tau}}{E_{hh}} = \frac{(1-p-pf)(y-h)^*}{1/(2y\theta)+\tau(1-p-pf)} > 0. \tag{8}$$

We already know that the second derivative of the expected ex-post income with respect to the unreported income is negative. Thus the sign of equation 8 is given by the sign of the numerator. It is obvious that for an interior solution ($0 < h^* < y$) ceteris paribus the influence of the marginal tax rate on the unreported income is positive. This result holds for all non-regressive tax systems since the conditions for the positive sign are $T_{y\tau} > 0$, $T_{yy} \geq 0$ and $C_{hh} > 0$. The first condition can be interpreted as the fact that a ceteris paribus rise in the progression, holding the tax rate for the poorest taxpayer constant, leads to a higher marginal tax rate. The second condition assures that the tax system is not regressive; the third that the marginal evasion costs are rising with unreported income.

The second case, where the tax rate rises for everybody by the same amount, is represented in our model by a rise in α. Again, implicit differentiation of the first-order condition leads to the equilibrium change of the unreported income:

$$\left.\frac{\partial h}{\partial \alpha}\right|_{h=h^*} = \frac{1-p-pf}{1/(y\theta)+2\tau(1-p-pf)} > 0.$$

[17] For the early basic models see Allingham and Sandmo (1972) and Yitzhaki (1974), who assume constant tax rates, Christiansen (1980) for progressive tax systems, and Clotfelter (1983) who estimates the effects of changing tax rates with TCMP data.

[18] Subscripts denote partial derivatives. Furthermore, we use implicit instead of explicit differentiation, since this makes the analysis much simpler.

We see that the influence of an increased constant part of the tax rate on unreported income is positive as well.[19] To see how tax evasion is influenced by changes in the tax rate we have to examine the relation between evaded tax (denoted by F) and unreported income (h). This relationship is purely technical, and is determined by the tax system as the difference between the true tax burden and the tax burden with cheating:

$$F(h) = T(y) - T(y-h) = h(\alpha - \tau h + 2\tau y) \tag{9}$$

To find the change in the amount of tax evaded if one of the tax rate parameters rises, we have to examine the sign of the derivatives of $F(h)$ in equilibrium with respect to the interesting parameters.

$$\frac{\partial F(h^*, \alpha, \cdot)}{\partial \alpha} = \alpha \frac{\partial h^*}{\partial \alpha} + h^* + 2\tau \frac{\partial h^*}{\partial \alpha}(y - h^*) > 0 \tag{10}$$

$$\frac{\partial F(h^*, \tau; \cdot)}{\partial \tau} = \alpha \frac{\partial h^*}{\partial \tau} + (2\tau \frac{\partial h^*}{\partial \tau} + h^*)(y - h^*) > 0 \tag{11}$$

We see that both derivatives are positive. Considering the assumed inner solution for h^* we can state that raising the tax parameters leads to more income unreported and through that channel to more taxes evaded. If the taxpayer reported no income before, he will report no income after the change of the tax rate again. The relations shown above lead to the following Proposition.

Proposition 1 *In our example an increase in the tax rate, interpreted as an increase in τ or in α, leads to*

1. *more income underreported,*
2. *to more tax evasion if an interior solution is realized before the tax rate change,*
3. *to a taxpayer that has hidden his entire income before the change, to do so afterwards.*

3.2 Changes in the individual parameters

In our model, two individual characteristics, the gross income y and the evasion opportunity θ, are exogenously given parameters. Let us now consider the changes of the taxpayers' behaviour due to exogenous changes in these parameters. It is quite obvious that a greater opportunity for evasion - exogenously determined by sources of income, knowledge of evasion possibilities, etc. - should lead to higher tax evasion. And indeed, a higher θ induces more non-reported income, and ceteris paribus more tax evasion. This is shown by the following equation:

$$\left.\frac{\partial h}{\partial \theta}\right|_{h=h^*} = -\frac{E_{h\theta}}{E_{hh}} = \frac{h^*}{\theta + 2\theta^2(1-p-pf)/y} \geq 0 \tag{12}$$

As intuition and real world data suggest, the non-reported income should increase with the gross income. The following implicit derivative shows that this is in fact true for our example, since the right hand side is positive, whenever an interior solution is achieved ($1-p-pf > 0$):

$$\left.\frac{\partial h}{\partial y}\right|_{h=h^*} = -\frac{E_{hy}}{E_{hh}} = \frac{h^*/(y^2\theta) + 2\tau(1-p-pf)}{1/(y\theta) + 2\tau(1-p-pf)} \geq 0 \tag{13}$$

[19] The effect for a proportional tax system, which makes our result comparable to the Yitzhaki result, is obtained by setting τ equal to 0. Certainly, the result still holds.

This rather trivial result is quite important. Combining this result with the result of proposition 1 we get the empirically observed result that both relations - tax rate and income to evasion - point in the same positive direction. The existing theoretical literature could not unambiguously reproduce these empirical findings [20] A further result we get is that a taxpayer that reported at least a certain amount of income ($h^* < y$) will at least report a fraction of his additional income. This is true since the implicit derivative in equation 13 is smaller than one in that case. Using equations 12 and 13 we can state the following proposition.

Proposition 2 *Ceteris paribus in our example,*

1. *a taxpayer will evade more (less) taxes the higher (lower) his evasion opportunity is,*
2. *an interior solution assumed, rising income leads to more tax evasion, and*
3. *an additional unit of income is reported at least partly, if the taxpayer reported some income before.*

3.3 Changes in the enforcement parameters

The most obvious results we get for the comparative statics of changes in the enforcement parameters, i.e. the exogenously given auditing probability and the penalty scheme. As in the classic tax evasion models a higher audit probability p leads to lower underreporting. The same is true for higher penalties which is indicated by a higher penalty parameter f. Applying the same procedure as above to determine the sign of equilibrium changes in the non-reported income due to a variation of the enforcement parameters we get:

$$\left.\frac{\partial h}{\partial p}\right|_{h=h^*} = -\frac{E_{hp}}{E_{hh}} = -\frac{(1+f)(\alpha + 2\tau(y-h^*)}{1/(y\theta) + 2\tau(1-p-pf)} < 0 \quad \text{and} \tag{14}$$

$$\left.\frac{\partial h}{\partial f}\right|_{h=h^*} = -\frac{E_{hf}}{E_{hh}} = -\frac{p(\alpha + 2\tau(y-h^*)}{1/(y\theta) + 2\tau(1-p-pf)} < 0 \tag{15}$$

The two equations above show that audit probability and fines are both appropriate instruments to lower underreporting, since both implicit derivatives are negative.[21] As easily can be checked, the effect on the taxes evaded points in the same direction.[22] This is the standard finding in the tax evasion literature. More interesting than this rather trivial statement is the question, which instrument is the more effective in reducing the amount of unreported income. To make the effects on underreporting comparable we derive the elasticities that show the percentage of reduced underreported income as consequence of a percentage rise in the enforcement parameter. The two elasticities are:

$$\eta_{h^*,p} = \frac{p}{h^*}\left.\frac{\partial h}{\partial p}\right|_{h=h^*} = -\frac{(p+pf)(\alpha + 2\tau(y-h^*)}{h^*\left[1/(y\theta) + 2\tau(1-p-pf)\right]} \tag{16}$$

$$\eta_{h^*,f} = \frac{f}{h^*}\left.\frac{\partial h}{\partial f}\right|_{h=h^*} = -\frac{pf(\alpha + 2\tau(y-h^*)}{h^*\left[1/(y\theta) + 2\tau(1-p-pf)\right]} \tag{17}$$

[20] In the Yitzhaki model specification this is not possible. In the original Allingham/ Sandmo specification it is possible. But there are strong conditions the expected utility functions and the system parameters have to fulfil. Further unrealistic is that there the higher the tax rate is, the less likely is the positive sign for both relations.

[21] That is true for an interior solution ($y > h > 0$), which implies a gamble with positive expected value ($1 - p - ps > 0$). Note, that we assumed an interior solution to exist.

[22] This is due to the implicit derivatives of th^* having the same sign as equations 14 and 15.

We immediately find that the absolute value of the audit probability elasticity $\eta_{h^*,p}$ is higher than that of the penalty parameter $\eta_{h^*,f}$. This means that - as empirical evidence suggests - raising the audit probability is more effective than imposing more severe penalties.[23] However, the desirability of using the instruments depends heavily on the associated costs.

Proposition 3 *Raising the audit probability and imposing higher fines are means of achieving lower underreporting, but audit probabilities are more effective than fines.*

3.4 Honest taxpayers, evaders and ghosts

Since we assumed that there is an initial fixed cost κ of behaving as a cheat, for some taxpayers - in expected terms - it will not be profitable to evade taxes, even if the game is a gamble that is better than fair. The individual will compare the expected payoff she yields if she decides to bear the initial fixed evasion costs and chooses the optimal amount of underreporting with the certain payoff she yields if she does not evade. She will evade if and only if $E(h^*) > y - T(y)$. Dividing the net expected value in a gross component $\hat{E}(h^*)$, which depends on the non-reported income, and in the fixed cost κ we get the following condition for evasion $\hat{E}(h^*) - \kappa > y - T(y)$. Solving for κ we obtain the minimum fixed evasion cost κ_l to force an individual taxpayer to be honest:

$$\kappa_l = \hat{E}(h^*) - (y - T(y)) \tag{18}$$

To study the change of the behaviour of former honest taxpayers due to changed personal or tax system parameters, we have to examine the change of this minimum fixed cost necessary to prevent cheating. If the reaction of κ_l is positive and a continuous distribution of κ exists, which assigns positive frequencies to the whole range of $[0, \kappa_h]$ with $\kappa_h > \kappa_l$, then at least one formerly honest taxpayer is becoming a cheat.

Since the fixed cost is an additive constant within the net expected payoff function, the implicit derivative of the net payoff with respect to the interesting parameters is equal to that of the gross payoff; i.e. $\partial\hat{E}(h^*)/\partial(\cdot) = \partial E(h^*)/\partial(\cdot)$. This gives the following equation that decides the change in κ_l for an individual taxpayer:

$$\frac{\partial\kappa_l}{\partial(\cdot)} = \frac{\partial E(h^*)}{\partial(\cdot)} - \frac{\partial[y - T(y)]}{\partial(\cdot)} \tag{19}$$

We immediately see that for changes in the parameters that have no influence on the net income after being honest; i.e. p, f and θ; we get the same sign for our change in the minimum fixed evasion cost that deters from evasion as for the comparative static analysis above. In conclusion we can say - as intuition suggests - that lower audit probabilities and fines, as well as greater opportunities to evade, lead to more formerly honest taxpayers becoming tax cheats.

In the case of a higher tax rate we have to calculate $\partial\kappa_l/\partial\tau$ and $\partial\kappa_l/\partial\alpha$. Both derivatives are

[23] This finding is robust to the changes in the penalty scheme. E.g. the Allingham and Sandmo specification leads to the same result.

positive:[24]

$$\frac{\partial \kappa_l}{\partial \tau} = (1-p-pf)(2y-h^*)h^* > 0 \quad (20)$$

$$\frac{\partial \kappa_l}{\partial \alpha} = (1-p-pf)h^* > 0 \quad (21)$$

Thus, rising tax rates - from an increase either in the income-dependent (τ) or the independent (α) component - induce formerly honest taxpayers to underreport their income.

The question whether a rise in personal income promotes honest taxpayers to become evaders depends on the sign of the following derivative:

$$\frac{\partial \kappa_l}{\partial y} = h^* \left(2(1-p-pf) + \frac{h^*}{2\theta y^2} \right) > 0 \quad (22)$$

The effect of a rise in gross income on a marginally honest taxpayer is positive as well. Summing up our findings about formerly honest taxpayer's reaction to changes in the model parameters (from equations 20 to 22) gives us the following proposition.

Proposition 4 *Under the assumption that the fixed evasion cost distribution provides positive frequencies for all $\kappa \in [0, \bar{\kappa}]$ such that there always exists at least one taxpayer that is indifferent between reporting his entire income or not reporting h^* we can say that there is at least one formerly honest taxpayer that starts cheating*

1. *if the tax rate (τ and/or α), the income (y), or the evasion opportunity (θ) increases, and*

2. *if the audit probability (p), or the fine rate (f) decreases.*

Another interesting question concerns the condition under which a taxpayer prefers to declare no income at all and becomes what in the literature is called a "ghost".[25] To find the necessary (and for small κ sufficient) condition for a taxpayer to prefer to be a ghost, we take equation 7, which determines the optimal non-reported income h^* and set h^* equal to y. Solving for θ and recalling that h^* increases with θ, leads to the following handy inequality:

$$\theta \geq \frac{1}{\alpha(1-p-pf)} \quad (23)$$

Proposition 5 *For sufficiently large opportunities for evasion, a high income independent part of the tax rate and sufficient low audit probabilities and fines an individual taxpayer will become a ghost and report no income at all.*

4 The general case: income and policy effects

We have already argued that our findings about the taxpayers' reactions on changes in parameters are quite general. In this section we show the necessary conditions for our main results to hold. For this

[24] Note that here h^* is the hypothetical optimal amount of not declared income if honesty was not possible.

[25] More precisely, a ghost is someone who does not make a tax declaration. However, it is not possible in our model to distinguish between zero declarations and no declarations at all.

purpose, we set up a general framework and formulate requirements for the functions that ensure that a taxpayer ceteris paribus reacts with more tax evasion on higher taxes and on more personal gross income. More generally, we introduce a policy parameter β that the government can influence. This gives us the possibility of analyzing the reactions of the taxpayer to certain policies.

To make the general analysis as easy as possible we define the decision problem in terms of undeclared income h and use functional forms for the evasion cost $C(h, y, \theta, \beta)$, the tax system $T(y, \beta)$ and the fine $F(h, y, \beta)$, where y denotes the true income. Sometimes, it might be convenient to replace the declared income $y - h$ by the variable d. Later on, we impose reasonable restrictions on the functional forms, which stem from real world observations and allow us to get clear comparative static results.

4.1 Tax system, penalty scheme, and evasion cost

4.1.1 Tax system

The tax system assigns a tax liability to every (declared or true) income. Without much loss of generality we can specify the tax systems in terms of parameters:

$$T(y, \beta) = t[y, \beta]y = [r(y, \beta) - z(y, \beta)]y \tag{24}$$

The tax system $T(y, \beta)$ is assumed to be continuous and differentiable with respects to its arguments. With this specification we can generate nearly every possible continuous and differential tax system. To exclude non-differentiable (with respect to the average tax rate) tax systems seems to be a severe loss of generality, since most real systems are not. But if the systems are at least continuous and monotonous in the average tax rate our results still hold with weak inequality. For other systems a reasonable continuous approximation might lead to reliable results as section **??** shows. The intuition for our results going through with weak inequalities in a monotonous tax system, which is continuous but not globally differentiable, is the following. Changes in parameters will cause changes in the directions we predict, unless the taxpayers' optimal declaration is at a kink before the parameter change takes place. Then it is possible that the optimal declaration after the parameter change will still be at the kink. But the reaction will never be in the direction opposite to the predictions our differentiable model provides.

In our differentiable model $t(\cdot)$ denotes the average net tax rate, which is composed of the tax rate $r(y)$, and an income-dependent transfer rate $z(y)$.[26] It is reasonable to use an additive formulation with income and the policy parameter β as the common independent arguments. It may seem a bit unfamiliar to formulate transfers in the way we did, but to express transfers as negative taxes with a certain tax rate $z(y, \beta)$ will soon prove to be very convenient. Furthermore, a negative income tax fits into this specification, as a constant subsidy does.[27] The average tax rate rises when its tax component increases. It falls with the subsidy component. But the only important decision criteria for the taxpayer is the

[26] Note, that here, in contrast to the example used above t is a function of y to allow for other than linearly progressive tax systems.

[27] Tax allowances are not covered by our specification, since the tax liability function in that case is neither continuous, nor differentiable. But our specification covers lump sum payments as well as a negative income tax.

aggregate average net tax rate $t(y,\beta)$. We may conveniently concentrate on this function, without loss of generality.

Let us establish the conditions for a tax system to be non-regressive. A tax system is called globally non-regressive if the average tax rate is monotonously increasing in income over the whole domain. The condition is:

$$\frac{\partial r(y,\beta)}{\partial y} - \frac{\partial z(y,\beta)}{y} \geq 0 \quad \forall y$$

Without any loss of generality we can state the following definition:

Definition 1 *A tax system is called globally non-regressive if and only if the following condition holds:*

$$\frac{\partial t(y,\beta)}{\partial y} \geq 0 \quad \forall y \tag{25}$$

4.1.2 Penalty schemes

Following our general approach, we allow the penalty scheme $F(h,y,\beta)$ to be dependent on undeclared income h, true income y and the policy parameter β. Thus the specific penalty described by our general penalty scheme can depend on the amount of non-reported income and on the evaded tax as well, which were the specifications of Allingham and Sandmo (1972) and Yitzhaki (1974), that proved to be crucial for the results. Furthermore, this general representation allows for an income dependent penalty like it is the case for many real tax systems. Examining existing penalty schemes more closely we see that usually two components determine penalties for tax evasion: income and a measure for the severity of the offence. The amount of tax the taxpayer tries to evade (denoted by T_e) or the amount of concealed income h are the two natural possibilities for the latter. Following the observation that the income and severity parts are multiplicatively combined in real world tax laws, we can write:

$$\begin{aligned} F(h,y,\beta) &= g(y,\beta)\cdot f(T_e,\beta) \text{ or} \\ &= g(y,\beta)\cdot f(h,\beta) \end{aligned} \tag{26}$$

The German law, for example, uses an income component, which is the income per day ($y/365$), not only for tax fraud but for many different kinds of crime. To include the specifications of earlier models, we also allow for a penalty scheme that has no income component (i.e. $g(y,\beta)=1\ \forall y$). We impose further restrictions on the parts of the penalty scheme, to obtain a class of penalty schemes, which we will call "fair". If a fair penalty scheme has an income component the fine should be proportional to the income. The severity component has to be proportional to the severity measure used.[28] If we denote the product of the constant proportionality factors with $f(\beta)$, we can sum up potentially fair tax systems in table 1.

Our second condition for a fair penalty scheme is that the proportionality factor $f(\beta)$, which is the same for all taxpayers should be chosen the way that detected tax evasion does not lead under any circumstances to a negative ex post net income. That means that $y - t(y)y - F(h,\cdot) \geq 0$ has to hold for all income levels and all evasion levels.

[28] These properties are widely observed in real world tax systems as far as monetary penalties are concerned. The latter restriction may not hold for the degree of severity, when the penalty becomes imprisonment. For simplicity reasons we do not consider those discontinuities of penalty schemes.

component	*with income*	*without income*
evaded tax	$f(\beta) \cdot y \cdot T_e$	$f(\beta) \cdot T_e$
concealed income	$f(\beta) \cdot y \cdot h$	$f(\beta) \cdot h$

Table 1: Potentially fair penalty schemes

component	*with income*	*without income*
evaded tax	$f(\beta) \leq (1-\bar{t})/(\overline{ty})$	$f(\beta) \leq (1-\bar{t})/\bar{t}$
concealed income	$f(\beta) \leq (1-\bar{t})/\bar{y}$	$f(\beta) \leq 1-\bar{t}$

Table 2: Fair penalty factors

Table 2 reports the maximal proportionality factors to assure that the condition above is fulfilled. Upper bars denote maximum values in the population.[29] Using the conditions imposed above we can state the following definition for a "fair" penalty scheme.

Definition 2 *A penalty scheme is called fair if it has the following properties:*

1. *It never leaves the taxpayer with negative ex post period income.*
2. *Its severity of offence component is proportional to the evaded tax or to the concealed income.*
3. *If it has an income component, the income component is proportional to the gross income.*
4. *If it has an income component the income component is multiplicatively combined with the severity of offence component.*

4.1.3 Evasion costs

In specifying the evasion cost we stick to the definition we made above. In addition, we add the policy parameter to its arguments, since some governmental action may have an influence on the evasion cost. We assume the evasion cost to be growing with non-reported income. The marginal evasion cost is non-decreasing in the income non-reported. The costs further depend on the gross income and on the evasion opportunity. By definition evasion costs fall with an increasing evasion opportunity. We can write the evasion cost as $C(y, h, \theta, \beta)$ according to our assumptions:

$$\frac{\partial C}{\partial h} > 0, \quad \frac{\partial^2 C}{\partial h^2} > 0 \quad \forall h \in [0, y] \tag{27}$$

$$\frac{\partial C}{\partial \theta} < 0 \quad \forall \theta \tag{28}$$

A crucial question is the change in marginal evasion cost when income rises. If we think of a rise in all different income sources, then the marginal evasion costs should decrease in our continuous approximation with a rising income for all levels of evasion, because the cheapest means of evading can be used to evade

[29] To see, that the condition holds for all incomes and tax rates lower than the maximal values, use the general forms ($t(y)$ and y) and calculate the derivative with respect to y. Since the derivative is negative, and for non-regressive tax systems t is non-decreasing in y, we find that the maximal values are the crucial ones.

a bigger amount. The other extreme is that the new source of income has a higher marginal evading cost than all old sources of income. Then there is no change in the marginal cost of evading up to the old income. If the new (single) source of income lies with its marginal evasion cost somewhere in between the lowest and highest sources then the marginal evasion cost sinks for all evasion levels that cannot be achieved with this source. Expressed in technical terms with respect to the undeclared income we get:

$$\frac{\partial^2 C}{\partial h \partial \theta} \leq 0 \quad \forall h, \theta \tag{29}$$

For convenience we define a typical average evasion cost function. The assumption that a higher income does not systematically more likely stem from a certain source leads to the conclusion that on average the new income rises equally for the different income sources. The counterpart - in terms of an evasion cost function - is a marginal concealment cost, that depends somehow on the share of non-reported income to gross income. So our evasion cost function of section 2 could be a typical average evasion cost function. To allow for cost functions of different steepness we use a parameter γ as exponent which has to be larger than one. Policy influences are modelled as changes of the evasion-opportunity level. Then a typical average evasion cost can be defined as follows:

Definition 3 *A typical average evasion cost function has the form*

$$C(h, y, \beta) = \frac{h^\gamma}{\theta(\beta) \cdot y} \textit{ with } \gamma > 1. \tag{30}$$

4.2 Optimal reported income

Putting all the parts together we can write down the expected income, conditional on the amount of undeclared income:

$$E(h, \cdot) = (1 - p)[y - (y - h) \cdot t(y - h, \cdot)] - pF(h, \cdot) - C(h, \cdot) \tag{31}$$

Having set up the general model, we can state the first-order condition for individually optimal non-reported income:[30]

$$E_h = (1 - p)T_d - pF_h - C_h = 0 \tag{32}$$

We assume that this condition can be met: this means that we have a local extremum for at least one taxpayer. To check whether we get an interior solution, that maximizes the expected value of the taxpayer, we have to look on the second order condition. We obtain always an interior solution and a maximum, whenever the second derivative is globally negative.[31] The second order condition is:

$$E_{hh} = -(1 - p)T_{dd} - pF_{hh} - C_{hh} < 0 \tag{33}$$

T_{dd} is non negative if we assume a globally non-regressive tax system, since it is just the second derivative of the tax liability. It could only be negative if the marginal tax liability were falling. But this would violate the condition for a globally non-regressive tax system. In the following we will restrict ourselves to

[30] Subscripts denote partial derivatives. Using the Tax liability T is convenient to abbreviate the notation.

[31] A further necessary condition is that there exists a taxpayer with sufficiently low fixed evasion costs κ. This is assumed in the following analysis.

globally non-regressive tax systems, and hence the first term has to be non-positive. The second term is non-positive if the penalty rises proportionally or more than proportionally with the not declared income, i.e. the marginal penalty is not decreasing with concealed income. The third term is negative, since the evasion costs rise more than proportionally with the not declared income. In conclusion we can say that for a penalty scheme, where the penalty is at least proportional to the unreported income, the extremum found by the first-order condition is a maximum. A unique solution is obtained, since the optimization problem is globally convex in that case. Even for a penalty system that has falling marginal penalties the optimization problem is well behaved if the curvature of the evasion cost dominates the curvature of the penalty function. In the following we assume an interior solution.

If we examine the first-order conditions for the different fair penalty schemes, we see that the necessary conditions for an interior solution are that, on the one hand, the tax system is such that the taxpayer faces a better than fair gamble, and on the other hand, the evasion costs are growing fast enough to prevent the taxpayer from becoming a ghost. The latter condition is assumed to be fulfilled in general. The (global) fair gamble conditions for the different penalty schedules are shown in table 3. The dependence of f on β is omitted.

component	*with income*	*without income*
evaded tax	$1 - p - f \cdot p \cdot y > 0$	$1 - p - f \cdot p > 0$
concealed income	$(1 - p)T_d - f \cdot p \cdot y > 0$	$(1 - p)T_d - f \cdot p > 0$

Table 3: Global fair gamble conditions for fair penalties

The fair gamble conditions have to be satisfied. Otherwise nobody will evade anything. Now it is easy to check the second order conditions for the different fair penalty schemes. They are shown in table 4. Since C_{hh} is positive and T_{dd} is non-negative (under the assumption of a non-regressive tax system), the second order conditions for a maximum are always met if the fair gamble condition holds.

component	*with income*	*without income*
evaded tax	$-C_{hh} - T_{dd}(1 - p - f \cdot p \cdot y)$	$-C_{hh} - T_{dd}(1 - p - f \cdot p)$
concealed income	$-C_{hh} - (1 - p)T_{dd}$	$-C_{hh} - (1 - p)T_{dd}$

Table 4: Second order conditions

This leads us to the following proposition.

Proposition 6 *Under a non-regressive tax system, a fair penalty scheme, sufficiently low fixed evasion costs, and sufficiently fast growing typical average evasion costs we obtain a unique, interior solution for the maximization problem.*

4.3 Changes in gross income

Assuming that the conditions for an interior solution above hold we are able to examine the effects of policy changes and changes in income. The evaded tax is given by the equation:

$$T_e(h, y) = T(y) - T(y - h)$$

Differentiation with respect to y leads to

$$\frac{\partial T_e(h, y)}{\partial y} = \frac{\partial T(y)}{\partial y} - (1 + \frac{\partial h^*}{\partial y})\frac{\partial T(d)}{\partial d}, \tag{34}$$

where $\partial h^*/\partial y$ represents the change of the optimal unreported income due to a change in gross income. We see that $\partial h^*/\partial y > 0$ is a sufficient condition for a higher gross income to lead to a higher amount of taxes evaded. The reason for this fact is that a non-regressive tax system implies $\partial T(y)/\partial y \geq \partial T(d)/\partial d$. We have to determine the sign of $\partial h^*/\partial y$, which is given by:

$$\left.\frac{\partial h}{\partial y}\right|_{h=h^*} = -\frac{E_{hy}}{E_{hh}} = \frac{(1-p)T_{dd} - C_{yh} - pF_{yh}}{-\Delta} \tag{35}$$

The second derivative of the objective function (i.e. E_{hh}) is denoted by Δ. Since $-\Delta$ is positive, we only have to look for the sign of E_{yh}. We know that $(1-p)T_{dd}$, which is the incentive to evade in order to get a lower tax bracket for rising income, is non negative. The mixed derivative of the typical cost function used here is negative - and so $-C_{hy}$ is positive. The value of the mixed derivative of the penalty scheme F_{yh}, can be interpreted as the change in the marginal penalty due to an increasing income, and depends on the specification. A closer examination leads to the following proposition.

Proposition 7 *Under a non-regressive tax system a taxpayer with a typical average evasion cost function will evade more taxes when his gross income rises, if the fair penalty scheme has no income component. If the fair tax system has an income component, a taxpayer facing a non-regressive tax system and a typical average evasion cost function for realistic detection probabilities and tax rates will evade more taxes when her gross income rises even if the system is least favourable for evasion.*

Proof. See appendix. ■

4.4 Policy effects

After checking the conditions for a higher gross income leading to more tax evasion we now examine the effects of policy changes on tax evasion. Since the effects of higher audit probabilities p and higher fines - i.e. an increased f - are not very interesting and lead to the intuitive result of less tax evasion, we concentrate on changes in the tax system and the evasion opportunity. We again restrict our attention to non-regressive tax systems, fair penalties and typical average evasion costs.

4.4.1 Changes in the tax system

A policy change affecting the tax laws is expressed by a change in the policy parameter β. Here we assume that such a policy change does not influence the evasion opportunities and the evasion costs, i.e. $C_{h\beta} = 0$. But since the penalty for detected tax evasion may depend on the tax rates, the effective penalty can be changed by a change in the tax law.

The effect of a policy change is given by:

$$\left.\frac{\partial h}{\partial \beta}\right|_{h=h^*} = -\frac{E_{h\beta}}{E_{hh}} \tag{36}$$

Since we know that the second derivative of the objective function is negative, it is sufficient for determining the sign of equation 36 to find the sign of $E_{h\beta}$. If we restrict ourselves to the fair penalty schemes defined above, we can report (in table 5) the mixed derivatives for the schemes based on evaded tax T_e or concealed income h, respectively with or without income components.

component	*with income*	*without income*
evaded tax	$(1-p-fpy)\,[t_\beta(d^*) + d^* t_{d\beta}(d^*)]$	$(1-p-fp)\,[t_\beta(d^*) + d^* t_{d\beta}(d^*)]$
concealed income	$(1-p)\,[t_\beta(d^*) + d^* t_{d\beta}(d^*)]$	$(1-p)\,[t_\beta(d^*) + d^* t_{d\beta}(d^*)]$

Table 5: Effects of policy changes

In the case where the evaded tax T_e is the measure for the severity of the offence the first term in brackets (for systems with or without income component) is positive because it is just the fair gamble condition from table 3.

For the case where the concealed income is the measure the term $(1-p)$ is obviously positive as well. We see that the sign of equation 36 for fair penalty schemes only depends on the sign of $[t_\beta(d^*) + d^* t_{d\beta}(d^*)]$, which is just the cross derivative of the tax liability for the formerly optimal income declared (denoted $T_{d\beta}(d^*)$).[32] If we postulate that the change does not lead to a regressive tax system we know that for an increase of the average tax rate (i.e. $t_\beta(d) > 0$) the change in the marginal tax liability ($T_{d\beta}(d)$) has to be positive as well. The interpretation of this finding is given in the following proposition.

Proposition 8 *A change in a non-regressive tax system with a fair penalty scheme, which leaves evasion opportunities unchanged and results in another non-regressive tax system, leads to a taxpayer concealing more income if the average tax rate at his formerly optimal declared income rises.*

Tax evasion due to changes in the tax system is promoted by two things - average tax rates and progression. The former influence - if average tax rates rise - dominates the latter in non-regressive tax systems. That means that for example a tax reform which results in higher average tax rates induces more tax evasion, even when the progression is lowered. The opposite conclusion is not necessarily true. A reform leading to a lower average tax rate for a taxpayer at his formerly declared income, but to a higher progression as well does not necessarily induce less concealed income.

4.4.2 Effects of changes in the evasion opportunities

The effects of policies that affect the evasion opportunities such as the introduction of tax collection at source are straight forward. Such policies have influence on the marginal evasion costs. Lower evasion

[32] Recall, that $t(d)$ denotes the net average tax rate for the declared income d, and $T(d)$ represents the tax liability for reported income d, which is $d \cdot t(d)$.

opportunities ceteris paribus lead to higher marginal evasion costs and consequently to less tax evasion and vice versa. In technical terms, the sign of the change in non-reported income depends only (if the policy does not change tax rates and penalties) on the negative cross derivative of the evasion costs $-C_{h\beta}$.

With the typical average evasion cost function we get

$$-C_{h\beta} = \frac{\theta_\beta h^{*(\gamma-1)}}{\theta^2 y} \tag{37}$$

If the policy reduces the evasion opportunity ($\theta_\beta < 0$) the expression above is negative. The taxpayer will report more income. For a policy that makes evasion easier ($\theta_\beta > 0$) we get a positive sign, and consequentially more tax evasion.

Proposition 9 *For a typical average evasion cost function, a policy that reduces (improves) evasion opportunities leads to less (more) tax evasion.*

5 Conclusion

In the previous sections we showed that it is possible to obtain the empirically observed reactions of taxpayers to changes of tax rates and gross income for a broad range of different tax systems by using an easy portfolio choice approach, as the early tax evasion models did. In order to obtain such results a slight change of the assumptions was necessary. We claim that risk neutrality is a fair approximation of the taxpayer's risk preferences if some psychological effects as reactance are considered. Reactance (Brehm, 1966; Brehm and Brehm, 1981) is the phenomenon that people who have lost some (economic) freedom due to exogenous changes immediately try to regain their freedom without taking into account the potentially negative consequences of their actions. As the factor limiting evasion - instead of risk aversion - we introduce evasion costs. These are costs such as moral costs to do something illegal, expenses arising with tax evasion, and costs for the creation of evasion opportunities. It might be worth investigating the predictions of an extended model with our assumptions that e.g. has features like endogenous working time or public good provision. Furthermore, it might be interesting to have a closer look on the notion of evasion costs. They can be seen as some kind of investment in evasion possibilities and/or lower detection probabilities. Then, if it is assumed that tax authorities have some (at least imperfect) information or conjectures about those investments, a rich hidden action and signaling environment is created.

In general, this paper showed that in the case of tax evasion, where complex psychological influences are going along with basic optimization reasoning, economic models do not necessarily fail to explain individual behaviour. In order to obtain a viable model important psychological influences have to be considered and simplified in a way that they can be implemented in standard economic models. This might have been relatively easy in the case of the reactance phenomenon, where a reasonable translation into risk preferences was possible. To include other phenomena such as influences of attitudes on behaviour or social comparison effects (equity considerations and fairness) - without losing the robustness against small changes in the specification - remains difficult. Maybe some newer developments dealing with social preferences might be of value to further understand tax evasion.[33]

[33] For such approaches to incorporate social aspects into preferences see for example Fehr and Schmidt (1999), Bolton

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A Proof of proposition 7

Proof. To prove proposition 7 it is sufficient to evaluate $\partial h^*/\partial y$. If this derivative is positive, $\partial T_e(h,y)/\partial y$ for a non-regressive tax system is positive as well (see equation 34). We have to check our four different cases for a fair penalty scheme.

Penalties without income component

The two possible fair penalty functions without an income component are:
i) $F(h) = f \cdot h$ and *ii)* $F(h,y) = f[T(y) - T(y-h)]$.

For *i)* the derivative is given by

$$\left.\frac{\partial h}{\partial y}\right|_{h=h^*} = -\frac{E_{yh}}{E_{hh}} = \frac{(1-p)T_{dd} - C_{yh}}{-\Delta} > 0$$

This derivative is unambiguously positive, since $(1-p)T_{dd} \geq 0$, $-C_{yh} > 0$, and $-\Delta > 0$.

For the second case *ii)* we get

$$\left.\frac{\partial h}{\partial y}\right|_{h=h^*} = -\frac{E_{yh}}{E_{hh}} = \frac{(1-p-fp)T_{dd} - C_{yh}}{-\Delta} > 0$$

As above $T_{dd} \geq 0$, $-C_{yh} > 0$, and $-\Delta > 0$. The crucial term in the brackets $(1-p-fp)$ is positive, as well, because it is just the corresponding fair gamble condition (see table 3). Hence $\partial h^*/\partial y$ is unambiguously positive in this case.

Penalties with income component

For the cases with an income component the fair penalty schemes are
iii) $F(h,y) = f[T(y) - T(y-h)]$and *iv)* $F(h,y) = fyh$.

The change in concealed income due to a higher income for case iii) is

$$\left.\frac{\partial h}{\partial y}\right|_{h=h^*} = -\frac{E_{yh}}{E_{hh}} = \frac{(1-p-fpy)T_{dd} - fpT_d - C_{yh}}{-\Delta}$$

Since $-\Delta$ is positive, we can concentrate on the numerator. $1-p-fpy$ is positive (fair gamble condition from table 3). To examine the least favourable environment we consider a linear tax system where $T_{dd} = 0$. Recall, that progression was favourable for evasion. Now $\partial h^*/\partial y$ is positive whenever

$$-Cyh > fpT_d$$

Solving the first-order condition $(1-p-fpy)T_d - C_h = 0$ for T_d and substituting in the inequality above leads to

$$-C_{yh} > \frac{fpC_h}{1-p-pfy}$$

Since $Ch = (\gamma h^{\gamma-1})/(y\theta) = y(-C_{hy}) = (y\gamma h^{\gamma-1})(y^2\theta)$ we can rewrite the inequality as

$$1 > \frac{pfy}{1-p-pfy}$$

To consider the least favourable fair penalty scheme for this case we consider f to be at its upper bound (where a detected tax evader concealing his entire income will be left without any ex post period income). Plugging in the upper bound $f = (1-t)/(ty)$ from table 2 and solving for t we get:

$$t > \frac{2p}{1+p}$$

The tax rate t necessary for $\partial h^*/\partial y > 0$ rises with $p \in [0,1]$. For realistic values for the detection probability, such as $p = .05$ (or $p = .1$) t has to be larger than .095 (or .181), which is realistic.

For case *iv)* $\partial h^*/\partial y$ is given by:

$$\left.\frac{\partial h}{\partial y}\right|_{h=h^*} = -\frac{E_{yh}}{E_{hh}} = \frac{(1-p)T_{dd} - fp - C_{yh}}{-\Delta}$$

Again, it is sufficient to examine the numerator (because $-\Delta > 0$). The incentive to obtain a lower tax rate by evasion $(1- \ p)T_{dd}$ is non negative for a non-regressive tax system. The least favourable case for evasion is given again by a linear tax rate system, where $T_{dd} = 0$. Using this condition and, as above, substituting C_h/y for $-C_{yh}$ we obtain the following condition for $\partial h^*/\partial y > 0$:

$$\frac{C_h}{y} - fp > 0$$

Substituting C_h from the corresponding first-order condition (i.e. $(1-p)t - fpy - C_h = 0$) into the inequality yields

$$\frac{(1-p)t}{y} - 2pf > 0$$

Again, using the least favourable (and confiscating) penalty factor (here: $f = (1-t)/y$ from table 2) and solving for t leads to the condition

$$t > \frac{2p}{1+p},$$

which is the same as in case iii). ∎