Contest with Attack and Defence: Does Negative Campaigning Increase or Decrease Voters' Turnout?*

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Abstract

We present a general model of two players contest with two types of efforts. Contrary to the classical models of contest, where each player chooses a unique effort, and where the outcome depends on the efforts of all the players, contestants are allowed to reduce the effort of the opponent. Defence increases one's chance of winning while attack annihilates the defence of the opponent. This model has many applications like political campaigning, wars, competition among lobbies, job promotion competitions, or sport contests. We study the general model of contest with attacks and defence and propose an application to negative political campaigns, where two candidates arbitrate between disparaging their opponent or enhancing their own image. We propose sufficient conditions for the existence and uniqueness of a symmetric Nash equilibrium of the contest game. In the application, we contribute to the empirically debated question dealing with the effect of attack on voters turnout, and show that the conclusion depends on the distribution of voters sensitivity to defence and attack. Furthermore, contrary to the literature, we show that an underdog candidate may be less aggressive than his opponent.

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1 Introduction

There exist two ways of winning a competition, by increasing one's chances of winning or by decreasing one's opponents chances of winning. We refer to this as the difference between positive and negative competition. There exist many real life situations in which individuals have a choice between positive or negative competition. In political campaigns, candidates can promote their image, their ideas and their program or denigrate their opponent ideas, image or program. In lobbies competitions, one lobby can try to promote his interest or to attack the interest of an other lobby. In job seeking competitions, candidates can invest in productive activities or try to discourage the firm to hire another candidate. In wars, armies can defend or attack a territory. In industrial advertizing competitions, a firm can promote the qualities of a product or can denigrate a competitor's product. There is no reason to think that positive and negative efforts have identical effects.

We propose a theoretical model of contest that allows to differentiate between positive and negative activities. Contrary to classical models of contest, where each player chooses a unique level of effort, and where the result depends on the efforts of the players, we suppose, as in the literature on sabotage in contests, that players are allowed to reduce the effective effort of their adversaries. In this effort, we do not focus attention on the dissipation of the rent but on the choice between positive and negative efforts. That is why we suppose that contestants have fixed budgets. In the first part of the paper, we study the general model of contest with attacks and defences and give sufficient conditions for the existence and uniqueness of a symmetric Nash equilibrium. In a second part, we propose an application to negative political campaigns inspired by Shachar and Nalebuff (1999), where two candidates choose between disparaging their opponent or enhancing their own image. In this application, we contribute to the hotly debated question on the effect of attacks on voters turnout, and show that the conclusion depends on voters sensitivity to defences and attacks. Furthermore, we show that an underdog candidate may attack less than his adversary.

The huge literature on contest has been mainly focused on one-dimensional efforts. In these models, each competitor chooses an effort level that increases his probability of winning a prize. Following the seminal work by Tullock (1967), this literature has considered a large number of variations on the contest model. There exist a small number of papers studying positive and negative efforts in contests, in which negative effort is called "sabotage". The first paper that has addressed this topic is the one by Lazear (1989). Chen (2003) considers a model of job promotion tournament with n players, where the effective efforts (resulting from classical rent-seeking efforts and sabotage) is additively separable in positive and negative efforts. The main result of this paper is that the contestant which is the more productive in positive lobbying is the most attacked in any equilibrium. In a different setting, Kräkel (2004) proposes a two stage model with either help or sabotage. In the first stage, contestants choose to help, to sabotage or to do nothing, and in the second stage, players choose their rent-seeking effort. The main result of this paper is that there can exist asymmetric equilibria in which one contestant helps his adversary and the second uses sabotage. The closest paper to ours (first part) is certainly the one by Konrad (2000) who proposes a model of contest with sabotage with n players and linear costs. The main result of this paper is that in a symmetric equilibrium, sabotage can be eliminated when the number of contestants is large and sabotage can lower or increase the rent dissipation. In the present paper, we consider the case of two contestants with fixed budgets. We give sufficient conditions for the existence and the uniqueness of a symmetric equilibrium.

The main contribution of the paper is the application to negative campaigning. We try to clarify the empirical debate started by the work by Ansolabehere, Iyengar, Simon and Valentino (1994) (AISV in the following). Their experiment reveals that negative advertisements lower voters turnout. They confirm the experimental result for the case of 1992 U.S. Senate election. They propose an explanation of the candidates rationality in going negative: a candidate who criticizes her opponent will reinforce his partisans' support and will give to her opponent supporters reasons not to vote for their favored candidate. This result has been challenged by Wattenberg and Brians (1999) in an empirical analysis based on NES data from 1992 and 1996 U.S. elections. On the contrary, they conclude that negative campaigning raises voters participation. This result would come from the fact that negative advertising may have a positive informative effect on voters; Ansolabehere, Iyengar and Simon (1999) respond to this "criticism" in reanalyzing NES data from 1992 and confirm their first conclusion. As for Finkel and Geer (1998), using NES survey data set of presidential campaign advertisement from 1960 to 1992, they find that attack has no negative effect on voters turnout. Delving deeper into details, Kahn and Kenney (1999) , distinguish two kinds of negative campaign advertising: useful negative advertising and mudslinging. They use 1990 U.S. Senate election data and find that relevant negative advertising was an incentive to vote whereas mudslinging disgusted voters and pushed them to choose not to go to the election booth. There has been so far no theoretical model to study the effect of negative campaigning on voters' turnout.

An other question addressed in the application is whether or not an underdog candidate is more or less aggressive than his opponent. Skaperdas and Grofman (1995) have studied a model of negative campaigning in which defence efforts make voters change their votes and attack efforts lead initial candidates supporters to abstain. They define the underdog candidate as the one with the smallest initial support. The model is specified such that, with the same negative advertising effort, the number of voters that will abstain is proportional to the initial support. Skaperdas and Grofman [15], as Harrington and Hess (1996), show that the underdog is more aggressive. In our model, an underdog candidate is the one with the smallest financial support. We show that the underdog candidate may be less aggressive than his adversary (in absolute as well as in relative terms).

The paper is organized as follows. In section 2 we present the general model of attack-defence contest, in section 3 we analyze the equilibrium properties, in section 4 we examine the application to negative political campaigns, in section 5, we discuss the case of heterogeneous candidates and the case of proportional election with N candidates, and we conclude in section 6.

2 The Model

Two players, L and R compete in a contest and choose two types of actions, a defence level d and an attack level a. The probability of victory is given by the comparison of the effective efforts resulting of attacks and defences. Let ψ be the synergy function of the contest. Each player is associated with a value of the synergy function that represents his effective effort in the competition. Let ψ^R be the effective effort of player R and ψ^L the effective effort of player L. Formally, as in classical models of contest, the probability of victory π^R of player R is given by the following logit-form:

$$\pi^R = \frac{\psi^R}{\psi^R + \psi^L},$$

We suppose that the effective effort of player R depends on his defence and the attack of the adversary. The function ψ (twice continuously differentiable on $\Re_+ \times \Re_+$ and three times differentiable) increases with the defence of the player and decreases with the attack of his adversary. Formally,

$$\psi^R = \psi\left(d_R, a_L\right),$$

and,

$$\begin{aligned}
\psi_1^R &= \frac{\partial \psi}{\partial d_R}\left(d_R, a_L\right) > 0, \\
\psi_2^R &= \frac{\partial \psi}{\partial a_L}\left(d_R, a_L\right) < 0
\end{aligned}$$

The two types of effort can have different interpretation in real world, depending on the context. In electoral campaigns, d is a positive campaigning effort and a is a negative advertisement effort. In a war, d can be interpreted as the spending for weapons and a as the spending for anti-weapons forces. In a job promotion competition, d is the productive activity and a is a sabotage effort (see Chen (2003)).

We suppose that defence and attack have decreasing marginal effects on $\varphi = \ln \psi$. Furthermore, we consider that ψ is (strictly) log-concave in d and (strictly) log-convex in a. Here, the log-convexity in a is *not a strong assumption*, this is simply the symmetric hypothesis with the log-concavity in d, because φ increases with d and decreases with a.

$$\varphi_{11}^{R} = \frac{\partial^2 \varphi}{\partial d_R^2} (d_R, a_L) < 0,$$

and,

$$\varphi_{22}^{R} = \frac{\partial^2 \varphi}{\partial a_L^2} (d_R, a_L) > 0.$$

This assumption signifies that the marginal effect of attack on the adversary's effective effort is decreasing. In other words, the more a player attacks his opponent, the less the decrease of the adversary effective effort is important.

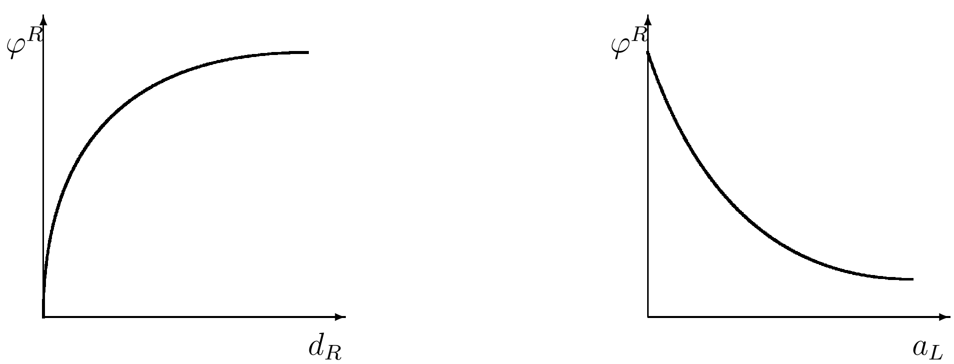

Figure 1: Synergy function and efforts

We suppose that, players have an incentive to defend and attack, that is $\lim_{d\to 0}\varphi_1(d,a) = +\infty$ and $\lim_{a\to 0}\varphi_2(d,a) = -\infty$. As in classical contest models, we suppose that players incur a cost of effort. In the present model, the cost depends on the attack and the defence levels. When player R chooses a defence level d_R and an attack level a_R, he pays the cost $C(d_R + a_R)$. This functional form implicitly assumes that positive and negative campaigning have similar costs. Indeed, the cost of an advertising campaign is independent of its contents. We suppose that C is twice continuously differentiable, strictly increasing ($C' > 0$) and convex ($C'' \geq 0$).

We are interested in the trade-off between attack and defence and we do not study total spending choices. We suppose that players have (identical) fixed budgets.[1] Let B be the budget of contestants R and L. Player R faces the following budget constraint:

$$C(d_R + a_R) \leq B.$$

Player R has to choose the levels of attack and defence which maximize his probability of victory subject to his budget constraint. Hence, the optimization program of player R is (the value of the rent is normalized to 1):

$$\underset{(d_R,a_R)}{Max}\left[\pi^R = \frac{\psi(d_R,a_L)}{\psi(d_R,a_L)+\psi(d_L,a_R)}\right],$$
$$s.t.:\ C(d_R + a_R) \leq B$$

At this point, it is important to note that attacking and defending have different effects on the probability that a player wins the tournament. Consider an infinitesimal increase of ψ^R and an infinitesimal decrease of ψ^L. The relative effect on the probability that R wins the contest is:

$$\left|\frac{\frac{\partial\pi^R}{\partial\psi^L}}{\frac{\partial\pi^R}{\partial\psi^R}}\right| = \frac{\psi^R}{\psi^L},$$

Hence, the effect of an increase in one candidate's effective effort will be greater than a decrease in the opponent's one if the opponent has a higher

[1]We provide an example where this assumption is relaxed in the final discussions.

effective effort. This remark underlines an incentive, for a strong player to attack a weakest one, and an incentive, for a weak player, to defend.

3 Equilibrium

In this section, we study the equilibrium properties (existence and unicity) of the general model with two players presented above, when the budgets are equal. We note f^R the value of function f in (d_R, a_L), f^L the value of function f in (d_L, a_R), and f_k the partial derivative of the function f with respect to its k^{th} argument.

Straightforwardly, with our assumptions, the budget constraints will be satisfied with equality. Then, d_R can be defined as a function of a_R, noted δ, such that:

$$\delta\left(a_R\right) = C^{-1}\left(B\right) - a_R, \tag{1}$$

Hence, we can focus on the choice of a_R, with equation 1 determining the corresponding unique value of d_R. The first order condition for candidate R is given by:

$$\frac{\varphi_2^L}{\varphi_1^R} = -1, \tag{2}$$

This condition says that in an interior equilibrium, the rate of marginal effects of attack and promotion must be equal to the rate of the marginal costs . This implicitly define the reaction correspondence of candidate R to the attack of candidate L. Let us denote by $\Gamma\left(a_L\right)$ candidate's R best reply, defined by:

$$\varphi_1\left(\delta\left(\Gamma\left(a_L\right)\right), a_L\right) = -\varphi_2\left(\delta\left(a_L\right), \Gamma\left(a_L\right)\right), \tag{3}$$

Proposition 1 *There exists a unique symmetric equilibrium of the negative campaigning game.*

The strategic effects are driven by the marginal cross-effect of attack and defence, $\frac{\partial^2\varphi}{\partial a \partial d}$. This represents the effect of simultaneous attack and defence on a player's effective effort. Differentiating equation 3 leads to the following expression of the slope of candidate's R reaction function:

$$\Gamma'(a_L) = \frac{\varphi_{12}^L - \varphi_{12}^R}{\varphi_{22}^L - \varphi_{11}^R},$$

The denominator is equal to the second order derivative of the payoff and is positive because $\varphi_{22}^L > 0$ and $\varphi_{11}^R < 0$ (for second order conditions, see the proof of proposition 1). Finally, the sign of the slope of candidate's R best-reply function is given by:

$$\Gamma'(a_L) \propto \varphi_{12}(\delta(a_L), \Gamma(a_L)) - \varphi_{12}(\delta(\Gamma(a_L)), a_L), \tag{4}$$

Since the sign of the right-hand side may change, the attacks are not always strategic substitutes or always strategic complements. Let ε_a be the elasticity of effective effort with respect to attack and ε_d the elasticity of effective effort with respect to defence:

$$\varepsilon_d(d,a) = \frac{\psi_1(d,a)}{d\psi(d,a)} \text{ and } \varepsilon_a(d,a) = \frac{\psi_2(d,a)}{a\psi(d,a)},$$

Hence, we obtain the following result:

Proposition 2 *(i) If* $\frac{\partial^2 \varepsilon_d}{\partial a^2}, \frac{\partial^2 \varepsilon_a}{\partial d^2} < 0$ *the equilibrium is unique.*
(ii) If $\frac{\partial^2 \varepsilon_d}{\partial a^2}, \frac{\partial^2 \varepsilon_a}{\partial d^2} > 0$ *the equilibrium is unique.*

The proof uses the result of proposition 1. Since there exists a unique symmetric equilibrium, there exists a unique value $a^* = \Gamma(a_L^*) = \Gamma(a_R^*)$ such that $a_L^* = a_R^*$. In both cases (i) and (ii), when the levels of attack are different, the attack of a player is a strategic complement of the opponent's one, and the attack of the opponent is a strategic substitute of the player's attack. Since the symmetric equilibrium is unique, the reaction functions can not cross in any other point. The following graphs illustrate this remark:

In the case where $\frac{\partial^2 \varepsilon_d}{\partial a^2}, \frac{\partial^2 \varepsilon_a}{\partial d^2} < 0$ (i), the reaction functions are quasi-convex:

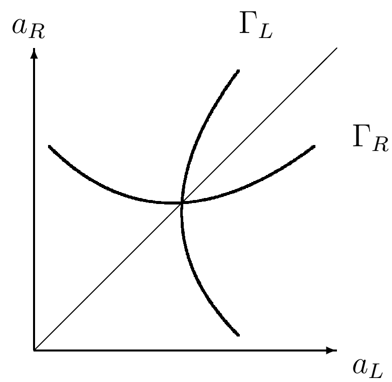

Figure 2: Reaction curves (case (i))

In the case $\frac{\partial^2 \varepsilon_d}{\partial a^2}, \frac{\partial^2 \varepsilon_a}{\partial d^2} > 0$ (ii), the reaction functions are quasi-concave:

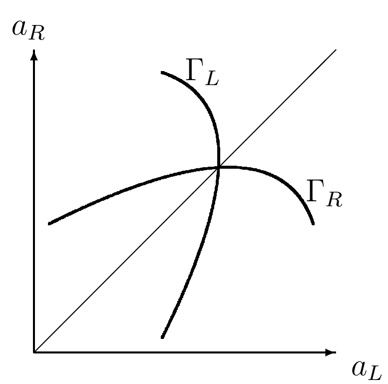

Figure 3: Reaction curves (case (ii))

Our assumptions are verified for a natural example, when candidate image is the outcome of a contest between attack and defence:

Corollary 3 *If* $\psi(d, a) = \frac{d^\alpha}{d^\alpha + a^\beta}$ *with* $\alpha, \beta \in]0, 1[$, *then the equilibrium is unique and symmetric.*

This example will illustrate the debate on the link between participation and the tone of the campaign:

4 Application: does negative campaigning increase or reduce turnout?

In this section, we analyze the important application to the political campaigns. Political advertisements can be of different natures, politicians can choose to defend their ideas, their image, their morality... They can also choose to attack their opponent's program, image or morality... How do these two kinds of advertisement influence voters' choice? Will they be more or less likely to vote? Will an underdog candidate be more or less aggressive? In this section, we try to clarify these questions. The model is inspired on Shachar and Nalebuff (1999), who state that voters do not choose whether or not to vote strategically. We consider that the population is split into two types of agents. On the one hand, we consider the leaders (lobbies, medias, candidates...), the agents who spend resources to support the campaign of one candidate. These agents strategically (and cooperatively) choose whether to invest or not for their preferred candidate. On the other hand, we consider the followers, the voters, who choose whether or not to vote for a candidate non strategically. We suppose that the followers are influenced by campaign spending. Abstention is due to the existence of a positive cost of voting[2]. Candidates' payoffs depend on candidate images and on the cost of voting which is the dominant factor for explaining voters turnout and abstention (see Xu, 2002; Börgers, 2001; Ledyard, 1981; and Palfrey and Rosenthal, 1983, 1985). To study the effect of negative advertising, we introduce a campaign game in which leaders have fixed budgets and have to choose between positive and negative advertising. In other words, leaders decide whether they denigrate their opponent or promote their favorite candidate. The can-

[2](for a model of abstention in a spatial competition setting, see Llavador, 2000)

didate's image is positively related to the candidate's amount of positive campaigning activities and negatively related to the other candidate's negative campaigning activities. In the spirit of Shachar and Nalebuff [14], we suppose that a candidate image is not affected by the candidate's attack and by the opponent's positive campaigning. This assumption is justified by the fact that these effects are weaker than the ones we consider. Making a voter change his vote is harder than making him not to vote for his favored candidate. Indeed to make a citizen change his vote, he would have first to be convinced not to vote for his favored candidate, and, secondly, to be convinced to vote for the adversary. We now explain how the attack-defence contest can be applied to this campaigning game.

4.1 Negative Campaigning: a follow the leader approach

We introduce attack and defence in the model by Shachar and Nalebuff [14]. We suppose that two candidates, R and L compete in a winner-take-all election. The population is divided into two types of agents. The leaders engage resources in the campaign, and voters choose whether or not to vote for their favored candidate.

The followers*:* the population of voters, with mass 1, is divided into two types. Let r be the share of citizens preferring candidate R to candidate L with the cumulative H and the density h with support $[0,1]$ and h has strictly positive values. H is an increasing and continuous function. When this citizen chooses to vote for his preferred candidate, he gets a benefit ψ^R and he faces a cost of voting μ, where μ is an idiosyncratic component drawn from a uniform distribution over $[0,1]$. Then she chooses to vote for R if and only if:

$$\mu \leq \psi^R,$$

The leaders*:* in Shachar and Nalebuff [14], ψ^R depends on E_R, that is the leaders spending in favor of candidate R. Since we want to study attack advertising, we will modify this assumption by assuming that the leaders' spending in favor of candidate R is a vector with two components a_R and d_R, where a_R is the negative advertising effort of leaders supporting R to attack his opponent, and d_R represents their promotion effort in favor of candidate R. The benefit of voting for candidate R is an increasing function of his promotion effort and a decreasing function of his opponent attack effort. Formally:

$$\psi^R \equiv \psi\left(d_R, a_L\right),$$

The probability that R wins the election is equal to the probability that he gets more votes than L, i.e. the probability that $r\psi^R \geq (1-r)\,\psi^L$, or the probability that $r \geq \frac{\psi^L}{\psi^R+\psi^L}$. Then, the probability that R wins the election, noticed Π^R is:

$$\Pi^R = 1 - H\left(\frac{\psi^L}{\psi^R+\psi^L}\right),$$

The participation is the expected sum of the votes of both sides, formally,

$$P = \int_0^1 r\psi^R + (1-r)\,\psi^L dH\left(r\right),$$

Furthermore, we keep the same assumptions on function ψ as in section 2. We now present our main example and draw conclusions on the (de)mobilizing effect of negative campaigning.

4.2 Main example

Suppose that a candidate's image results from a contest between her promotion and her adversary's attack. If the electorate's sensitivity to promotion is α and the sensitivity to attack is β, then candidate's R image function can be written as:

$$\psi^R\left(d_R, a_R\right) = \frac{\left(d_R\right)^\alpha}{\left(d_R\right)^\alpha + \left(a_L\right)^\beta},$$

with $\alpha, \beta \in]0,1[$. Furthermore, suppose that the cost function is linear:

$$C\left(d_R + a_R\right) = d_R + a_R.$$

Let B_R and B_L be the respective budgets of leader R and leader L. With these specifications, Candidate R's program is:

$$\underset{d_R, a_R \geq 0}{Max} \left[\Pi^R = 1 - H \left(\frac{\frac{(d_L)^\alpha}{(d_L)^\alpha + (a_R)^\beta}}{\frac{(d_R)^\alpha}{(d_R)^\alpha + (a_L)^\beta} + \frac{(d_L)^\alpha}{(d_L)^\alpha + (a_R)^\beta}} \right) \right],$$

such that the budget constraint is not violated.

This example would be complicated to solve directly because of the embedded logit-form functions, but the general results of section 2 enable us to compute it easily.

4.3 Negative campaigning: Increasing or decreasing turnout?

In this section we analyze the main example when the budgets are identical, $B_R = B_L = 1$.

Proposition 4 *There exists a unique equilibrium, and the equilibrium levels of attack and promotion are given by:*

$$a_R^* = a_L^* = \frac{\beta}{\beta + \alpha},$$

and,

$$d_R^* = d_L^* = \frac{\alpha}{\beta + \alpha}.$$

Not surprisingly, the more voters are sensitive to attack, the higher the level of equilibrium attacks, and the more voters are sensitive to promotion, the higher the equilibrium promotion levels.

The equilibrium participation rate is:

$$P^* = \psi^*,$$

Then,

$$P^* = \frac{\left(\frac{\alpha}{\beta+\alpha}\right)^\alpha}{\left(\frac{\alpha}{\beta+\alpha}\right)^\alpha + \left(\frac{\beta}{\beta+\alpha}\right)^\beta},$$

Now, we can analyze the sign of the correlation between attack and participation. Suppose to simplify that $\beta = 1 - \alpha$. Then β measures the voters' relative sensitivity to attacks. Comparing the outcomes of an election in different States in U.S., or different national elections, there is no reason to think that β will be equal in each State or at each election. The empirical result can be summarized with a graph. Each point of the graph represents the participation rate and the corresponding attack equilibrium level in the State. Here, we suppose that β varies across States or national elections, and then look at the variations of participation and the variations of attack levels in the different equilibria. The equilibrium turnout rate is:

$$P^*(\alpha) = \frac{(1-\beta)^{1-\beta}}{(1-\beta)^{1-\beta} + \beta^{\beta}},$$

and, the equilibrium attack level is also a function of β, denoted $a(\beta) = \beta$.

The following proposition states that the participation can be high in one election when the campaign is negative and the participation can be low when the campaign tone is positive.

Proposition 5 $a'(\beta) P^{*\prime}(\beta) \leq 0$ *if and only if* $\beta \in \left[\frac{1-\sqrt{1-\frac{4}{e^{-2}}}}{2}, \frac{1+\sqrt{1-\frac{4}{e^{-2}}}}{2}\right]$.

Hence, when β is small enough or large enough, when the equilibrium attack level increases, the equilibrium turnout rate increases. That is states where leaders are more aggressive can present higher participation rates. The following graph illustrates the proposition, it represents the variations of the equilibrium attack and the participation when β increases:

[Insert Figure 4 about here]

To understand Proposition 5, notice that there are two competing effects. The first effect is a direct effect on equilibrium attack and promotion levels. When the sensitivity to attack increases, then the equilibrium attack level increases and the equilibrium promotion decreases. This effect makes participation fall. A second effect is the "impact effect". When the sensitivity to attack increases, the relative effect of attack decreases and participation rises. The first effect is constant while the second effect changes when the sensitivity to attack increases. Since the marginal effects of attack and promotion on a candidate's effective effort are decreasing (because $\beta < 1$), the "impact effect" is high for heterogeneous values of attack and promotion and is small for homogeneous values of attack and promotion levels. Then, when comparing different States elections or National elections, one compares heterogeneous populations in term of sensitivities to attack and promotion, and then, one can observe a positive correlation between attack and participation (when the populations are almost equally sensitive to both tones) like in Wattenberg and Brians (1999), or a negative correlation (when the populations are very sensitive to one of the tone) like in AISV (1994) , or one can observe no correlation (when the range of sensitivities is large) like in Finkel and Geer (1998).

5 Discussions

In this section, through two different examples, we relax two assumptions of the model. In a first sub-section, we relax the equal budget hypothesis and derive a relation between the budget and the level of aggressiveness of a candidate. In the second sub-section, we compare the case of a proportional election with N players to the case of majority election with two candidates.

5.1 Is an underdog candidate more aggressive?

In our context, we consider an underdog candidate who has less financial support than her adversary. Let R be the underdog candidate and L the advantaged candidate, with $B_R < B_L$. Unfortunately, it seems difficult to obtain general results with this assumption. In different models, Skaperdas and Grofman (1995) and Harrigton and Hess (1996) show that the underdog candidate, defined as the candidate with the smaller initial popular support, is more aggressive than his adversary. We provide an example in which the underdog candidate is, in equilibrium, less aggressive than the advantaged candidate. Consider the main example with $\alpha = \beta$. Candidate R's optimization program is equivalent to:

$$\underset{d_R, a_R > 0}{Max} \left[\Pi^R = 1 - H \left(\frac{\frac{d_L^\alpha}{d_L^\alpha + a_R^\alpha}}{\frac{d_R^\alpha}{d_R^\alpha + a_L^\alpha} + \frac{d_L^\alpha}{d_L^\alpha + a_R^\alpha}} \right) \right],$$

s.t.:

$$B_R = d_R + a_R.$$

The equilibrium of this campaign game is unique and the candidates efforts in negative and positive advertisement are given in the following proposition:

Proposition 6 *There exists a unique equilibrium. The underdog candidate levels of promotion and attack are:*

$$\begin{aligned} d_R^* &= \frac{(B_L)^\alpha}{(B_R)^\alpha + (B_L)^\alpha} B_R, \\ a_R^* &= \frac{(B_R)^\alpha}{(B_R)^\alpha + (B_L)^\alpha} B_R, \end{aligned}$$

the advantaged candidate levels of promotion and attack are:

$$\begin{aligned} d_L^* &= \frac{(B_R)^\alpha}{(B_R)^\alpha + (B_L)^\alpha} B_L, \\ a_L^* &= \frac{(B_L)^\alpha}{(B_R)^\alpha + (B_L)^\alpha} B_L, \end{aligned}$$

And the participation rate is:

$$P^* = \frac{(B_L)^\alpha + \left[(B_R)^\alpha - (B_L)^\alpha \right] E(r)}{(B_L)^\alpha + (B_R)^\alpha},$$

with $E(r) = \int_0^1 r dH(r)$.

Contrary to the case where candidates have equal budgets, the equilibrium participation depends on the expected value of candidate R support share, $E(r)$. Since $B_L > B_R$, then the more candidate L expected support $(1 - E(r))$ is large, the higher the participation rate. Indeed, the advantaged candidate can generate more participation ($\psi^{L*} > \psi^{R*}$), but he is more aggressive than the underdog, in relative and absolute terms:

Corollary 7 *The underdog candidate is less aggressive than the advantaged candidate:*

$$a_R^* < a_L^*,$$

And he is relatively less aggressive than the advantaged candidate:

$$\frac{a_R^*}{d_R^*} < \frac{a_L^*}{d_L^*}.$$

This result directly follows from proposition 6. The underdog candidate is less aggressive than the advantaged one and he is relatively less aggressive. The intuition of this result is linked to the remark made in section 2. A strong candidate has an incentive to be more aggressive, and a weak candidate has an incentive to be more defensive. This result can be understood in the light of the remark made in section 2, that is a candidate with a better image increases his level of attack. When a candidate has a greater budget, he can easily have a better image than her adversary, and then is more aggressive. Indeed, when a candidate's image is high, the marginal effect of promotion becomes small compared to the marginal effect of aggressiveness. Concerning contests in general, this result seems to be realistic, in a conflict, the more aggressive being generally the strongest contestant. In the context of elections, this is certainly not always the case, but we think that other important effects would have to be considered, as incumbency. Indeed, the effect of attacking a party which have never been in power is certainly smaller than attacking a governing party with verifiable arguments.

5.2 Majority VS Proportionality

We now discuss the question addressed by Konrad [10]. The question is whether or not an increase in the number of candidates leads to an increase of aggressiveness. Konrad [10] shows that in a symmetric equilibrium, when budgets are not fixed, sabotage can be eliminated if the number of players is large enough. Through an example, we conclude that, in equilibrium, candidates attacks decrease with the number of candidates. We suppose that candidates maximize their share of votes. Consider N candidates competing in the proportional election. The share of votes of candidate i is given by the following expression:

$$\pi^i = \frac{\psi^i}{\sum\limits_{j=1,\dots,N} \psi^j},$$

We specify the model such that:

$$\psi\left(d_i, (a)_{j\neq i}\right) = e^{\sqrt{d_i} - \sum\limits_{j\neq i} \sqrt{a_{ji}}},$$

where a_{ij} is the level of attack from i targeted on candidate j, d_i is the level of defence of player i, $(a)_{j\neq i} = (a_1, ..., a_{i-1}, a_{i+1}, ..., a_N)$ is the vector of attacks targeted on i. The cost function is linear and the budget fixed to 1, so that the budget constraint of candidate i can be written:

$$d_i + \sum_{j\neq i} a_{ij} = 1,$$

The main difference with the two candidates case is the effect of a candidate's attack on the payoff of the candidates that are not targeted. The derivative of candidate's k vote share when i increases his attack against j is:

$$\frac{\partial \pi^k}{\partial a_{ij}} = -\frac{\partial \psi^j}{\partial a_{ij}} \frac{\psi^i}{\sum\limits_{j=1,\dots,N} \psi^j} > 0,$$

Thus, an attack from i to j generates positive externalities on the other candidates. Solving this example leads to the following result:

Proposition 8 *In the proportional election with N candidates there is a unique equilibrium and the attack levels decrease with the number of candidates:*

$$\frac{\partial a_i^*}{\partial N} < 0.$$

Finally, the more candidates in the competition, the less they are aggressive. The intuition underlying this result stems from the positive externalities of attacks on the other candidates. This externality leads candidates to reduce their attack level, and this reduction is even greater the larger the number of candidates.

6 Conclusion

We have presented a model of contest with two players choosing between positive or negative campaigning and given sufficient conditions for the existence and the uniqueness of a symmetric equilibrium. We have proposed an application to negative political campaigns. Through an example, our results suggest that the relation between attack and participation can be positive or negative, depending on the distribution of the sensitivities to positive and negative advertisements in the electorate. Furthermore, we have shown that a candidate with a smaller financial support may be less aggressive than his adversary.

7 Appendix

Proof of Proposition 1: A symmetric equilibrium exists only if the following equation has a solution a^* in $[0, C^{-1}(B)]$

$$\varphi_1(\delta(a), a) + \varphi_2(\delta(a), a) = 0, \tag{5}$$

Let $f(a) = \varphi_1(\delta(\Gamma(a)), a) + \varphi_2(\delta(a), \Gamma(a))$, its derivative is given by:

$$f'(a) = [\varphi_{22}(\delta(a), a) - \varphi_{11}(\delta(a), a)] + [\varphi_{12}(\delta(a), a) - \varphi_{21}(\delta(a), a)],$$

Since φ is twice continuously differentiable, the second term in brackets is null, then $f'(a) > 0$. Since $\lim\limits_{a\to 0^+} f(a) = -\infty$ and $\lim\limits_{a\to C^{-1}(B)^-} f(a) = +\infty$. Hence, there exists at most one a^* such that (a^*, a^*) is a symmetric equilibrium. The second order conditions are verified:

$$\frac{d^2\pi^R}{da_R^2} = \pi^R\left(1-\pi^R\right)\left[\left(-\varphi_1^R - \varphi_2^R\right)\left(1-2\pi^R\right) + \left(\varphi_{11}^R - \varphi_{22}^L\right)\right],$$

Then, in the symmetric equilibrium,

$$\frac{d^2\pi^{R*}}{da_R^2} = \left(\varphi_{11}^R - \varphi_{22}^L\right) < 0.$$

Then there exists a unique symmetric equilibrium.

Proof of Proposition 2: The condition $\frac{\partial^2 \varepsilon_d}{\partial a^2}, \frac{\partial^2 \varepsilon_a}{\partial d^2} > 0$ is equivalent to $\varphi_{112}, \varphi_{221} > 0$. We first show that if $a_L \neq a_R$, then $\varphi_{112}, \varphi_{221} > 0$ or $\varphi_{112}, \varphi_{221} < 0 \Rightarrow \frac{d\Gamma^R}{da_L}(a_L)\frac{d\Gamma^L}{da_R}(a_R) < 0$. Indeed:
Suppose $\varphi_{112}, \varphi_{221} > 0$. Consider the case $a_L > a_R$, then $\delta(a_L) < \delta(a_R)$. These two inequalities implies that $\varphi_{12}(\delta(a_R), a_L) > \varphi_{12}(\delta(a_L), a_R)$, hence, $\frac{d\Gamma^R}{da_L}(a_L) < 0$ and $\frac{d\Gamma^L}{da_R}(a_R) > 0$. Now consider $a_R > a_L$, with the same reasoning, we obtain: $\frac{d\Gamma^R}{da_L}(a_L) > 0$ and $\frac{d\Gamma^L}{da_R}(a_R) < 0$.
Suppose $\varphi_{112}, \varphi_{221} < 0$. if $a_L > a_R$, then $\frac{d\Gamma^R}{da_L}(a_L) > 0$ and $\frac{d\Gamma^L}{da_R}(a_R) < 0$. If $a_R > a_L$, then $\frac{d\Gamma^R}{da_L}(a_L) < 0$ and $\frac{d\Gamma^L}{da_R}(a_R) > 0$.

Since in the unique symmetric equilibrium the two reaction functions derivatives are null, $\varphi_{112}, \varphi_{221} > 0$ implies that if $a_L \neq a_R$, then $\Gamma^R(a_L) \neq \Gamma^L(a_R)$. Then there does not exist any asymmetric equilibrium.

Proof of Corollary 3: The example verifies the assumptions of the previous propositions. Indeed, when $a, d \neq 0$, $\varphi(d,a) = \alpha \ln d - \ln\left((d)^\alpha + (a)^\beta\right)$, then

$$\begin{aligned}\psi_1(d,a) &= \frac{\alpha (d)^{\alpha-1}(a)^\beta}{\left((d)^\alpha + (a)^\beta\right)^2} > 0,\\ \psi_2(d,a) &= -\frac{\beta (d)^\alpha (a)^{\beta-1}}{\left((d)^\alpha + (a)^\beta\right)^2} < 0.\end{aligned}$$

and, second order derivatives,

$$\psi_{11}(d,a) = \alpha (a)^\beta \frac{(\alpha-1)(d)^{\alpha-2}\left((d)^\alpha + (a)^\beta\right)^2 - 2\alpha\left((d)^\alpha + (a)^\beta\right)(d)^{2\alpha-2}}{\left((d)^\alpha + (a)^\beta\right)^4} < 0,$$

Then φ is concave in d.

$$\varphi_{22}(d,a) = -\beta \frac{(\beta-1)(a)^{\beta-2}\left((d)^\alpha + (a)^\beta\right) - \beta (a)^{2\beta-2}}{\left((d)^\alpha + (a)^\beta\right)^2} > 0,$$

Then φ is convex in a. Furthermore,

$$\varphi_{12}(d,a) = \alpha\beta \frac{(a)^{\beta-1}(d)^{\alpha-1}}{\left((d)^\alpha + (a)^\beta\right)^2},$$

Hence, with simple computations, we obtain:

$$\varphi_{221}(d,a) \propto \alpha\beta\left((\beta-1) d^\alpha - (\beta+1) a^\beta\right) < 0,$$

and,

$$\varphi_{112}(d,a) \propto \alpha\beta\left((\alpha-1) a^\beta - (\alpha+1) d^\alpha\right) < 0,$$

This example also verify the Inada conditions: $\lim\limits_{d\to 0}\varphi_1(d,a)=+\infty$, and $\lim\limits_{a\to 0}\varphi_2(d,a)=-\infty$. Finally, with proposition 2, there exists a unique equilibrium.

Proof of Proposition 4: With proposition 2 and corollary 3, since H is a strictly increasing function, the example admits a unique equilibrium and it is symmetric. The equilibrium attack level is given by the following equation:

$$\frac{\beta\,(a^*)^{\beta-1}}{(B-a^*)^{\alpha}+(a^*)^{\beta}}=\frac{\alpha}{B-a^*}-\frac{\alpha\,(B-a^*)^{\alpha-1}}{(B-a^*)^{\alpha}+(a^*)^{\beta}},$$

Then,

$$\beta\,(B-a^*)=\alpha a^*,$$

Thus,

$$\begin{aligned} a^* &= \frac{\beta}{\beta+\alpha}B, \\ d^* &= \frac{\alpha}{\beta+\alpha}B. \end{aligned}$$

Proof of Proposition 5:
Simple computations lead to: $a'(\beta)\,P^{*\prime}(\beta)\ \propto\ -2-\ln(\beta(1-\beta))$, then $a'(\beta)\,P^{*\prime}(\beta)\propto e^{-2}+\beta^2-\beta$. Furthermore $\frac{1-\sqrt{1-\frac{4}{e^{-2}}}}{2}$ and $\frac{1+\sqrt{1-\frac{4}{e^{-2}}}}{2}$ are the roots of $e^{-2}+\beta^2-\beta=0$. Hence, the result holds.

Proof of Proposition 6: Candidate R's first order condition is:

$$\left(d_R^{\alpha}+a_L^{\alpha}\right)\left(d_L^{\alpha}+a_R^{\alpha}\right)=d_R^{\alpha}\left(d_L^{\alpha}+a_R^{\alpha}\right)+\left(d_R^{\alpha}+a_L^{\alpha}\right)a_R^{\alpha-1}d_R,$$

Then,

$$\frac{a_L^{\alpha}}{a_R^{\alpha-1}d_R}=\frac{d_R^{\alpha}+a_L^{\alpha}}{d_L^{\alpha}+a_R^{\alpha}}, \tag{6}$$

Symmetrically, candidate L's first order condition is:

$$\frac{a_L^{\alpha-1}d_L}{a_R^{\alpha}}=\frac{d_R^{\alpha}+a_L^{\alpha}}{d_L^{\alpha}+a_R^{\alpha}},$$

Then,

$$d_L d_R = (B_L - d_L)(B_R - d_R),$$

Finally,

$$d_L = \frac{B_L}{B_R}(B_R - d_R),$$

With 6, we obtain:

$$\left(\frac{B_L}{B_R}\right)^{\alpha}\left(\left(\frac{B_L}{B_R}\right)^{\alpha}+1\right) d_R^{\alpha}(B_R - d_R)^{\alpha} = (B_R - d_R)^{\alpha-1} d_R^{\alpha+1}\left(\left(\frac{B_L}{B_R}\right)^{\alpha}+1\right),$$

Finally,

$$d_R^* = \frac{(B_L)^{\alpha}}{(B_R)^{\alpha}+(B_L)^{\alpha}} B_R,$$

And,

$$d_L^* = \frac{(B_R)^{\alpha}}{(B_R)^{\alpha}+(B_L)^{\alpha}} B_L.$$

Hence, the equilibrium effective efforts are:

$$\psi^R = \frac{(B_R)^{\alpha}}{(B_R)^{\alpha}+(B_L)^{\alpha}},$$

and,

$$\psi^L = \frac{(B_L)^{\alpha}}{(B_R)^{\alpha}+(B_L)^{\alpha}},$$

And the equilibrium participation is:

$$P^* = \int_0^1 r\psi^R + (1-r)\psi^L dH(r) = \psi^L + \left[\psi^R - \psi^L\right] E(r),$$

with $E(r) = \int_0^1 r dH(r)$.

Proof of Proposition 8:

$$\psi(d_i, a_{-i}) = e^{\sqrt{1-\sum_{j\neq i} a_{ij} - \sum_{j\neq i}\sqrt{a_{ji}}}},$$

Then, the first order condition of candidate's i maximization program is given by the following $N-1$ equations: for all j,

$$\pi^j = \frac{\sqrt{a_{ij}}}{\sqrt{1 - \sum\limits_{k \neq i} a_{ik}}},$$

And the same is true for each candidate i. Then, a few computation leads to, for all i:

$$a_i = a = \frac{\sum \left(\pi^j\right)^2}{1 + \sum \left(\pi^j\right)^2},$$

Then,

$$\sqrt{1-a} = \sum_{j \neq i} \sqrt{a_{ji}},$$

And, for all i,

$$\pi^i = \frac{1}{N},$$

Finally,

$$a = \frac{1}{1+N},$$

Furthermore, the Hessian matrix of candidate's i payoff is:

$$Hess^i = -\frac{1}{4} \frac{\psi^i}{1 - \sum\limits_{j \neq i} a_{ij}} \begin{pmatrix} 1 & ... & 1 \\ ... & ... & ... \\ 1 & ... & 1 \end{pmatrix},$$

Then the second order conditions are verified.

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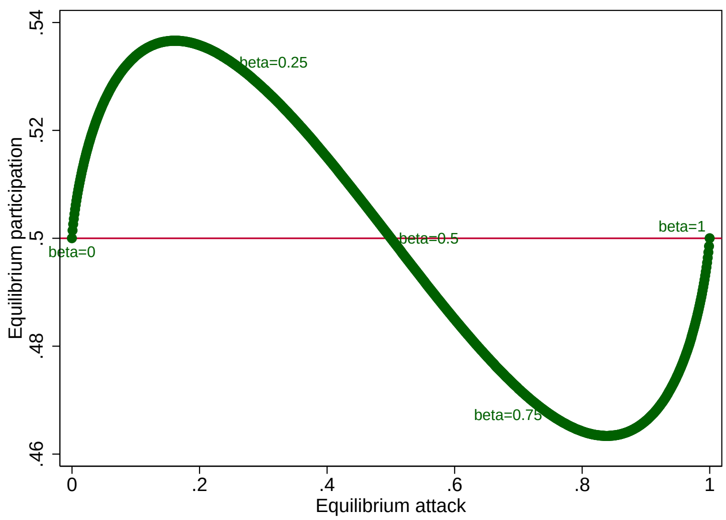

Figure 4: Equilibrium Participation and Attack when β increases