TAX EVASION IN INTERRELATED TAXES

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ABSTRACT: In 1969, Shoup postulated that the presence of interrelated taxes in a tax system would reinforce the system of tax penalty ("self-reinforcing penalty system of taxes"). In this paper, we have tried to formally develop this idea. We find that in order that tax re-enforcement holds, it is necessary that the interrelated taxes are administered by a single tax administration, or in the case that they are administered by different tax administrations, the level of collaboration between them has to be high enough. If that is the case, tax evasion in interrelated taxes might be considered as an alternative explanation of the existing gap between the levels of tax evasion that can be guessed in practice and those much lower predicted by the classical theory of tax evasion (Allingham and Sandmo, 1972; Yitzhaki, 1974). Otherwise, the result expected by Shoup might even reverse. Moreover, as long as collaboration is imperfect, the classical results of the comparative statics might change, since in some cases although global tax compliance increases in front of a variation in a tax parameter, it can decrease in a tax.

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1. Introduction

Kaldor (1956) argued that a tax system in which a capital gains tax, a personal income tax, an expenditure tax, a wealth tax and an inheritance and donations tax were present, just auditing one tax return, the extent of tax evasion could be checked comprehensively. This is the so-called “self-checking system of taxes”. The reason of his argument is based on the evident relationships among the tax bases of all those five taxes. Thus, the sum of the amount of tax base declared in the expenditure tax and in the capital gains tax should be congruent with the tax base declared in the personal income tax. Otherwise, if it does not match, that might be due either because the taxpayer has consumed part of her initial stock of wealth (or she has made a donation), or because some of the tax bases have been under-declared. In the first case, that should be compatible with a decrease in the tax base of the wealth tax (or with an increase in the tax base of the recipient’s donations tax) once capital gains have also been taken into account, while in the second case, that should be an useful hint to start a process of tax auditing.

This certainly seems a powerful system to ease the tasks of the tax auditors. However, note that congruity among tax bases does not necessarily imply that tax evasion is null. That is why, Shoup (1969) suggested to rename the tax system proposed by Kaldor as a “self-reinforcing penalty system of taxes”. That is, when the taxpayer faces the decision about how much tax base to evade, she should bear in mind that as long as tax bases are crosschecked, her decision might not only have consequences on that tax, but also on other interrelated taxes. Hence, having increased the expected cost of tax evasion, a priori such type of tax systems should be useful in promoting tax compliance. In our paper, we will try to check that supposition by means of formally developing the original ideas of Kaldor (1956) and Shoup (1969), and we will do it by focusing our analysis on the interrelation between a wealth tax and a personal income tax. However, this analysis cannot only be applied to individual taxes.

For example, Das-Gupta and Gang (2001) have recently developed a similar model of tax evasion applied to the Value Added Tax (VAT) (see also the analysis of interrelated tax evasion in the VAT by Fedeli and Forte, 1999). The reason to analyze the VAT arises from the possibility of matching sales invoices against purchase invoices by part of the tax administration. These authors find that although crosschecking might distort purchase and sale decisions, a sufficiently high level of crosschecking can induce truthful reporting. To a certain extent, Engel and Hines (1999) have also applied Shoup (1969)’s idea in a dynamic setting (in fact, the seminal work by Allingham and Sandmo, 1972, section 5, also considered tax evasion within a dynamic setting). In their work, these authors find that for a rational taxpayer, current evasion is a decreasing function of prior evasion, since, if audited for evading taxes the current year, she may incur penalties for past evasions as well. Within this framework, they estimate that tax evasion is 42% lower than it would be if taxpayers were not concerned about retrospective audits. These results are certainly very interesting, and make evident the necessity to expand the classical analysis of tax evasion (Allingham and Sandmo, 1972; Yitzhaki, 1974) taking into account the interrelation between tax bases, and so the possibility of crosschecking by part of the tax administration.

In our paper, when incongruity is detected by the tax administration, the tax audit probability tends to increase above its “normal” level. This provokes that as long as both taxes are administered by a single tax administration, congruity is the optimal choice for the taxpayer. However, in some cases (typically, in federal systems) taxes are not administered by the same layer of government. Then, as long as collaboration between tax administrations is not perfect, it is possible that crosschecking is not sufficient to induce congruity between tax returns. In particular, by imperfect collaboration we refer to that situation under which when one tax administration is carrying out an audit, it does not put too much effort in detecting tax evasion on behalf of the other[1]. Actually, imperfect collaboration implies that the level of tax compliance might be even lower than the one predicted by the classical analysis! In any case, as long as collaboration is perfect, or being imperfect it is not too low, tax evasion in interrelated taxes might be considered as a partial explanation to the paradox of tax evasion. The paradox of tax evasion comes out when the observed (or guessed) levels of tax compliance and those predicted by the classical analysis are compared. In order to achieve the observed levels of tax compliance from the classical analysis, the degree of risk-aversion and/or the level of the tax enforcement parameters have to be abnormally high. In order to overcome such paradox, the literature has proposed the existence both of economic and non-economic factors (see, e.g., the clear and detailed review of this literature by Alm, 1999). In general, the conclusion of the literature is that the original model of gambling applied to tax evasion might be too simple as to take into account the numerous factors that affect the reporting decisions of individuals. In this sense, interrelated tax evasion might be considered as another factor to be taken into account, and as we will show in the numerical simulations, in some occasions that factor can by itself solve the paradox of tax evasion.

In the context of interrelated tax evasion, we have also performed a comparative static analysis. Under the classical analysis, a reinforcement of any tax parameter tends to promote tax compliance (see the review by Andreoni et al., 1998)[2]. In our analysis, as long as collaboration between tax administrations is perfect, those results remain unchanged. Nonetheless, when collaboration is imperfect, the results might change. Due to the ambiguity of the theoretical analysis in this latter situation, we have had to make use of the methodology of numerical simulations. All the results of the exercise of numerical simulation confirm those obtained by the classical analysis with respect to global tax compliance, or at least when tax compliance in each tax is weighted by the importance of their respective tax burdens. However, that result does no longer hold

[1] Niepelt (2002) analyzed tax evasion in a dynamic setting, and similarly to us considered the possibility that during an audit tax evasion is not fully discovered. However, he justified this assumption without referring to a lack of collaboration between tax administrations, but simply as a handicap of a tax administration. In any case, according to his words, that provokes an "uncorrelated detection risk", which calls for analyzing tax evasion focusing on the many sources of taxpayer's tax base, instead of on the very taxpayer. This conclusion resembles very much our differential analysis depending on whether congruity or incongruity is optimal form the taxpayer's point of view. In the former case, the unit of analysis will be the taxpayer, since there do not arise differences in the level of tax compliance between tax bases, while in the latter case, the unit of analysis is each tax base, since tax compliance is not homogenous across taxes.

[2] However, slightly modifying the original framework of the classical analysis, some authors have recently shown that the signs of the comparative statics can reverse. For instance, Boadway et al. (2002) or Brock (2002) have demonstrated that in certain circumstances an increase in the penalty per unit of tax evaded can decrease tax compliance; while Lee (2001) has shown the same, but for the case of an increase in the marginal tax rate.

when tax compliance is analyzed tax by tax (see fn. 1). This result is extremely important once we take into account that the policy decisions of one tax administration (i.e., level of government) will have consequences not only on its tax base, but also on the tax base of the other tax administration (level of government). Therefore, a tax externality stemming from the tasks of tax administration arises as long as the responsibilities on each tax hang on different layers of government. Cremer and Gahvari (2000) considered the audit rate as an additional strategic tax parameter among sub-national governments within a federal system; while Baccheta and Espinosa (1995) analyzed the incentives to share information between national governments in an open economy, although they did not included in their model the possibility of tax evasion. Hence, the identification of a potential tax externality in the context of tax administration is not totally new. Nevertheless, this confirms Andreoni et al. (1998)'s statement in the sense that in order to avoid inefficient levels of tax compliance "... how to integrate tax enforcement across different levels of government" (p. 835) may be one of the issues among tax compliance that deserve further research. In any case, this line of research is not dealt with in this paper.

In the following section, we formulate the theoretical model and our assumptions, especially those referring to the tax audit probability in the presence of interrelated tax evasion. The taxpayer's decision over tax evasion is characterized, and a comparative static analysis is performed. That analysis crucially depends on the degree of collaboration between the tax administrations responsible for each tax. In section 3, we carry out an exercise of numerical simulation, which permits us to complement the results of the theoretical model. Thus, given a simple parameterization, we can ascertain to what extent interrelated tax evasion can solve the paradox of tax evasion; under which circumstances it is more likely that the tax bases declared in each tax return are incongruent; and finally, to examine some of the ambiguities detected in the analytical comparative statics. We conclude in section 4.

2. Theoretical Model

In this section, first, we will establish how the presence of interrelated taxes changes the tax enforcement parameters, in particular, the tax audit probability. Obviously, this is the key of all the theoretical analysis carried out in the paper. Next, we will analyze the behavior of the taxpayer in this context of interrelated tax evasion, including a comparative statics analysis. This analysis will be done both in the presence of perfect and imperfect collaboration between tax administrations.

Assumptions about the tax audit probability

We suppose that the tax administration obtains valuable information from crosschecking the tax returns of the taxpayers. In particular, for each taxpayer the tax administration considers the following budget constraint:

$$Y = C + S = \beta Y + S \qquad [1]$$

where Y is income obtained by a taxpayer during the fiscal year, which can be either consumed, C, or saved, S (i.e., S is the increase in the stock of wealth obtained during that fiscal year[3]), and β is the marginal propensity to consume. Income is taxed in the personal income tax, while savings are taxed in the wealth tax. Given the level of income declared in the personal income tax, Y_D, the level of wealth declared in the wealth tax, S_D, and supposing a certain marginal propensity to consume, β[4], the tax administration can infer whether the relationship given by expression [1] holds, that is,

$$Y_D \underset{>}{\overset{\le}{=}} C' + S_D = \beta Y_D + S_D \qquad [2]$$

where C' is the level of taxpayer's consumption inferred by the tax administration, and $Y_D \le Y$ and $S_D \le S$. As long as expression [2] holds with equality, the tax administration will not appreciate any incongruity between the tax bases declared in each tax return, and will not increase the tax audit probability above the "normal" level, that is, when there is not any incongruity[5]. Otherwise, an incongruity will be a "signal of alarm" for the tax administration to audit the tax returns of the taxpayer[6]. Graphically,

[FIGURE 1]

On the left hand side, the graph shows the tax audit probability in the personal income

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[3] The budget constraint [1] could have been expressed within an inter-temporal framework, although for simplification we have left aside such possibility. For instance, *Y* could have been considered as the personal income obtained during a certain period of time, which has made possible the accumulation of a certain stock of wealth, *S*, and a certain level of inter-temporal consumption, *C*. Otherwise, given that the tax base of the wealth tax is the stock of wealth and not the increase in wealth, for our analytical purposes, expression [1] has to be considered in such a way that at the beginning of the fiscal year (i.e., at the beginning of the only period of our static analysis), the stock of wealth of the taxpayer is null. As a consequence, in our theoretical analysis there is not any difference between stock of wealth and increase in the stock of wealth.

[4] From now on, we will suppose that the marginal propensity to consume adopted by the tax administration (see expression [2] next) and the real one coincide. As we will see, this assumption will make much easier the interpretation of the results of comparative statics.

[5] Expression [2] could be modified in order to incorporate a margin of error, $|\varepsilon| > 0$, i.e., $Y_D \underset{>}{\overset{\le}{=}} \beta Y_D + S_D + \varepsilon$. For example, suppose that according to the personal income tax return Y_D= 100. Then, assuming $\beta = 0{,}8$, the stock of wealth declared in the wealth tax should be 20, but in fact, considering a certain margin of error of ±10%, S_D should be between 18 and 22. Otherwise, if S_D were above (below) 22 (18), the probability of auditing in the personal income tax (wealth tax) would increase above its "normal" level. However, the results of our marginal analysis are independent of the inclusion of a margin of error.

[6] For instance, during summer 2002, in Spain the National Tax Administration (so-called *Agencia Estatal de la Administración Tributaria*, AEAT), which is responsible for the auditing of the personal income tax while the responsibility of auditing the wealth tax is shared with the regional governments, extensively crosschecked the personal income tax and wealth tax returns. The reason of that massive crosschecking was that, according to the Director of the AEAT, the price of new houses and luxury cars purchased (in our words, an increase in the monetary value of the stock of wealth) did not match income declared by the taxpayer in the personal income tax. See, e.g., the information given by the newspaper *La Vanguardia*, 10/3/2002.

tax, p^Y. As long as $Y_D \geq S_D + C$, the tax audit probability remains at its "normal" level, $\bar{p}^Y$. Otherwise, the audit probability is increasing in the value of the incongruity, $S_D + C - Y_D > 0$. Similarly, the graph on the right hand side shows the tax audit probability in the wealth tax, p^S. In fact, summing both functions of probability of tax auditing, we obtain the following function:

[FIGURE 2]

where we have assumed that $\bar{p}^Y = \bar{p}^S$. Therefore, keeping S_D unchanged, for values of Y_D below (above) S_D+C an increase in Y_D decreases (increases) the probability of being audited in the personal income tax (wealth tax), and so $p^Y + p^S$.

Hence, in our model the tax audit probability is endogenous, since it depends on the level of tax bases declared, S_D and Y_D. In the next section, we will try to identify those situations under which the taxpayer might find it optimal to deviate from the strategy that implies the minimisation of the probability of being audited, $\bar{p}^Y + \bar{p}^S$, that is, being congruent ($S_D + C = Y_D$).

Definition: the tax audit probability, *p*, is a function $p(S_D, Y_D) = p^S + p^Y$, such that for $S_D + C > Y_D$, $\partial p^Y / \partial S_D > 0$ and $\partial p^Y / \partial Y_D < 0$, while p^S is a constant; for $S_D + C < Y_D$, $\partial p^S / \partial S_D < 0$ and $\partial p^S / \partial Y_D > 0$, being p^Y a constant; finally, for $S_D + C = Y_D$, p is a parameter, i.e., it is independent of the amount of tax bases declared.

... When collaboration between tax administrations is perfect.

In this section, we will analyse a situation under which either both tax returns (personal income tax and wealth tax) are administered by only one tax administration or they are administered by two different tax administrations (i.e., each one of them is a part of a different layer of government) but collaboration between them is perfect. By perfect collaboration we refer to that situation under which, for example, when the wealth tax return is audited, not only tax evasion in the wealth tax is fully discovered but also in the personal income tax. To the extent that there exists only one tax administration, it is perfectly understandable that tax evasion in both taxes will be fully discovered independently of which tax return is audited, since the total amount of tax revenue collected will remain in hands of that single tax administration. However, when there are two independent tax administrations, the situation is different: perfect collaboration implies that the tax administration that is carrying out an audit (e.g., in the wealth tax) will exert an additional effort in discovering tax fraud (e.g., in the personal income tax) that will only benefit the other tax administration. In any case, by now, we will suppose that even in the case that there were two (institutionally independent) tax administrations, each one of them would have an incentive to fully discover evasion in both taxes.

Characterization of the Taxpayer Behaviour

Following the classical analysis due to Allingham and Sandmo (1972), the taxpayer attempts to minimise the amount of taxes paid. However, such decision is not without risk, since her tax fraud might be detected by the tax administration depending on the tax auditing probability. Then, if she is audited, she will be fined proportionally to the amount of taxes evaded. We assume that the taxpayer is risk-averse, $U(Y)' > 0 > U(Y)''$, where $U(Y)$ is the utility that the taxpayer derives from income[7], Y, which is an exogenous variable in our model[8]. Analytically, the objective function of the taxpayer, W, is the following:

$$W \equiv (p^Y + p^S) U\left[Y - t_R Y_D - t_P S_D - F t_R (Y - Y_D) - F t_P (S - S_D)\right] + \\ + (1 - p^Y - p^S) U\left[Y - t_R Y_D - t_P S_D\right] \qquad [3]$$

The first summand in square brackets (from now on, denoted by A) is income at disposal of the taxpayer net of paying taxes when she is audited. In that case, the taxpayer pays taxes in the personal income tax according to the (marginal) tax rate t_R ($0 \le t_R \le 1$), but as long as she has evaded taxes ($Y > Y_D$), she will also have to pay F per each unit of tax evaded in the personal income tax, $t_R(Y-Y_D)$, where $F \ge 1$. The same reasoning applies to the case of the wealth tax, where the marginal tax rate in that case is t_P ($0 \le t_P \le 1$). The second summand in square brackets (from now on, denoted by B) is income at disposal of the taxpayer when none of the tax returns is audited. Given the presence of perfect collaboration between tax administrations, only those two states can occur: A or B. The probability of occurrence of the first one is $p^S + p^Y$, i.e., it occurs when any of the two tax administrations audits[9], while state B occurs when none of them audits, being $1 - p^S - p^Y$ the probability of occurrence of that state.

As we have said before, the objective of the taxpayer is minimising the amount of taxes paid, that is, given the existence of taxes, the taxpayer aims at maximising net income[10]. Therefore, she will choose Y_D and S_D such that expression [3] is maximised. Nevertheless, we know that as long as she is incongruent with respect to the amount of tax bases declared ($Y_D + C' \underset{>}{\le} S_D$), the tax audit probabilities of expression [3] are endogenous. Thus, before solving the maximisation problem of the taxpayer, we need to know whether (in)congruity can be an optimal strategy for her.

Is optimal to be congruent in the tax bases declared?

We are assuming that independently of which tax return is originally audited, both tax

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[7] Partial derivatives of functions of only one variable will be denoted by a prime, while for functions of more than one variable, a subscript will indicate the variable of the corresponding partial derivative.

[8] See Pencavel (1979), for a model of tax evasion in which Y (labor supply) is considered as an endogenous variable; and other references cited in Andreoni et al. (1998), p. 824.

[9] Although it does not modify the results of the present analysis, the possibility that both tax administrations simultaneously carry out a tax audit can be reasonably ruled out, i.e., $p^Y \times p^S = 0$. Therefore, those two events can be considered as mutually exclusive.

[10] This characterization of a rational taxpayer is consistent with the following description given by Cowell (1990): "(he) is "predisposed to dishonesty" because the taxpayer does not put responsibility to the State before his own interests" (p. 50).

evasion in the personal income tax and in the wealth tax are fully discovered by the tax administration. Moreover, we suppose that both functions of tax auditing are symmetric, i.e., $\left|p^Y_{S_D}\right| = \left|p^Y_{Y_D}\right| = \left|p^S_{S_D}\right| = \left|p^S_{Y_D}\right|$, and $\bar{p}^Y = \bar{p}^S$. Therefore, from now on, unless necessary, we will not distinguish between p^Y and p^S, and will simply refer to *p*, which is p^Y + p^S. Under such assumptions, we wonder whether under certain circumstances it will be optimal for the taxpayer to be incongruous.

For instance, we wonder whether it could be optimal that $S_D + C' > Y_D$. In that case, and keeping Y_D constant, the following conditions should hold:

$$\left.\frac{\partial W}{\partial S_D}\right|_{S_D+C'<Y_D} = p^1_{S_D}\left[U(A_1) - U(B_1)\right] + t_P\left[p_1(F-1)U(A_1)' - (1-p_1)U(B_1)'\right] > 0 \qquad [4]$$

where the index 1 is necessary since obviously the (marginal) utility of income is not the same for all levels of S_D and Y_D, but also the tax audit probability might vary according to those two variables. Hence, in expression [4], the index 1 is referring to *a* situation under which $S_D + C' < Y_D$. The next condition – which implies that S_D+C'=Y_D is not an optimal strategy for the taxpayer - should also hold

$$\left.\frac{\partial W}{\partial S_D}\right|_{S_D+C'=Y_D} = p_2(F-1)U(A_2)' - (1-p_2)U(B_2)' > 0 \qquad [5]$$

where $p^1_{S_D} < 0$, $p_1 > p_2$, $U(A_i) < U(B_i)$ and $U(A_i)' > U(B_i)'$ $\forall$ *i*; while at the (supposed) optimum,

$$\left.\frac{\partial W}{\partial S_D}\right|_{S_D+C'>Y_D} = p^3_{S_D}\left[U(A_3) - U(B_3)\right] + t_P\left[p_3(F-1)U(A_3)' - (1-p_3)U(B_3)'\right] = 0 \qquad [6]$$

where $p^3_{S_D} > 0$, $p_3 > p_2$, and $p_3 \underset{>}{\leq} p_1$.

Additionally, in order to guarantee an interior solution (S>S_D), we need that the following condition holds:

$$\left.\frac{\partial W}{\partial S_D}\right|_{S_D=S} < 0 \qquad [7]$$

The next Lemma states the non-optimality of incongruity from a rational taxpayer's point of view.

Lemma: *As long as* $\left|p_{S_D}\right| > 0$ *and* $\left|p_{Y_D}\right| > 0$, *it will always be optimal for the taxpayer to be congruous. Otherwise, the optimal strategy for the taxpayer is indeterminate.*

Proof: the reasoning is as follows. Expression [6] must hold both for S_D and Y_D, that is, at the optimum,

$$\left.\frac{\partial W}{\partial Y_D}\right|_{S_D+C'>Y_D} = p^3_{Y_D}\left[U(A_3)-U(B_3)\right]+t_R\left[p_3(F-1)U(A_3)'-(1-p_3)U(B_3)'\right]=0 \qquad [8]$$

Then, given that $p^3_{Y_D}<0$ and $U(A_3)<U(B_3)$, in order expression [8] holds, it is necessary that $p_3(F-1)U(A_3)'-(1-p_3)U(B_3)'<0$. However, according to expression [6], and given that $p^3_{S_D}>0$, $p_3(F-1)U(A_3)'-(1-p_3)U(B_3)'>0$. Therefore, expressions [6] and [8] cannot hold simultaneously, and incongruity cannot be an optimum. Given that the same reasoning is applicable for the case in which $S_D+C'<Y_D$, congruity is the only possible solution as long as $\left|p_{S_D}\right|>0$ and $\left|p_{Y_D}\right|>0$. In the case that the probability of auditing is independent of incongruity, i.e. $\left|p_{S_D}\right|=0$ and $\left|p_{Y_D}\right|=0$, the solution to the maximisation of expression [3] is indeterminate, being congruity one of the many solutions $\blacksquare$

We have shown that incongruity can never be an optimal strategy for a rational taxpayer in the case that the tax audit probability is conditioned on the incongruity between tax returns and collaboration between tax administrations is perfect. The reason is that, for example, in order $S_D+C'>Y_D$ to be optimal, the taxpayer's welfare reduction due to a higher probability of tax auditing with respect to its "normal" level, $p^Y>\bar{p}^Y$, must be overcome by the net expected gains of increasing tax compliance in the wealth tax keeping the tax audit probability constant. Nevertheless, given that the sign of these latter expected gains is independent of the (marginal) tax rate, there would still be gains of increasing Y_D. Moreover, in that case, increasing Y_D would also produce a decrease in the tax auditing probability (through a reduction in p^Y). Thus, the strategy under which $S_D+C'>Y_D$ can never be optimal, since in that situation it would always be welfare enhancing to increase Y_D [11]. From the point of view of the taxpayer, that means that both tax bases are perfect substitutes, and so she simply aims at minimizing the tax auditing probability, *p*.

Optimal level of tax base declared

We have previously described the taxpayer as a rational individual predisposed to dishonesty (see also fn. 10). That is, an individual who aims at maximising her own

[11] To a certain extent, the result provided by the Lemma is similar to the one obtained under a "cut-off" rule (see Border and Sobel, 1987; Reinganum and Wilde, 1985; Sánchez and Sobel, 1993). Under such a rule, the tax administration establishes a threshold below which all taxpayers are audited, while above it all taxpayers are unaudited. Then, assuming that taxpayers are risk-neutral and that the tax administration can commit to such audit rule, all those taxpayers with a tax base above the threshold declare just the amount fixed by the threshold. In our case, the threshold is endogenous. For instance, from the point of view of the tax administration responsible for the personal income tax, the relevant threshold is S_D+C', being S_D endogenous from the point of view of the taxpayer. Then, in our model, despite the assumption of risk-aversion, the taxpayer finds it optimal to declare just the amount fixed by the threshold, or in our words, finds it optimal to be congruent.

welfare by means of deciding how much tax base to declare independently of the consequences of her decision over the rest of the society[12]. The social consequences of her actions basically refer to the loss of tax revenues for the government (and so public good provision) due to the erosion of the tax base. Analytically, the taxpayer solves the following maximisation problem:

$$\max_{S_D, Y_D} W \quad s.t. \quad Y_D = S_D + C'$$

Therefore, once Y_D has been substituted into W (which has been previously defined by means of expression [3]), the only decision variable of the taxpayer is S_D. This is the decision we will deal with. Then, the FOC of the maximisation problem with respect to S_D is the following:

$$pU(A)'(F-1) = (1-p)U(B)' \qquad [9]$$

That is, at the optimum, the marginal cost of evading taxes (left hand side of expression [9]) equals the marginal benefit of evading taxes (right hand side of expression [9]). Finally, in order to guarantee that full tax compliance (S=S_D) is not an optimal strategy for the taxpayer, and making use of expression [7], we obtain the classical condition that $pF < 1$ (vid. Yitzhaki, 1974, expression [6']*)[13]. From now on, we will assume that such condition holds, and so at the optimum $S > S_D$.

Comparative statics

As we have shown above, in the case of perfect collaboration between tax administrations, congruity is the only optimal strategy. Then, in order to perform an exercise of comparative statics, we will just analyse the way in which reported wealth, S_D, depends on the parameters of the model F, t_P, t_R, p, since congruity implies that Y_D can be directly obtained from $Y_D = S_D/(1-\beta)$.

In order to obtain dS_D/dF, first, we totally differentiate expression [9], Φ,

$$\begin{aligned} d\Phi = 0 = (t_R' + t_P)\left[pU(A)' + pU(A)'R(A)(t_R(Y-Y_D) + t_P(S-S_D))(F-1)\right]dF - \\ -(t_R' + t_P)^2\left[pU(A)'R(A)(F-1)^2 + (1-p)U(B)'R(B)\right]dS_D \end{aligned} \qquad [10]$$

where we have used the Arrow-Pratt measure of absolute risk aversion, $R(A) = \left(-U''(A)/U'(A)\right) \geq 0$, and identically for state B. In expression [10], t_R'=t_R/(1-

[12] See, e.g., Bordignon (1993) for a model that takes into account moral issues when describing the taxpayer's behavior; or Cowell and Gordon (1988), and Alm et al. (1992), who consider the taxpayer's evaluation of the activity of the public sector; see also the references cited by Andreoni et al. (1998), section 8; and the complete review by Alm (1999), already cited in the introduction.

[13] Although in our case, recall that p=p^Y+p^S.

β)[14]. Then, operating over expression [10], we have

$$\frac{dS_D}{dF}=\frac{pU(A)'(1+R(A)(F-1)(Y-Y_D)(1-\beta)(t_R'+t_P))}{(t_R'+t_P)\left[pU(A)'R(A)(F-1)^2+(1-p)U(B)'R(B)\right]}\geq 0 \qquad [11]$$

Thus, an increase in the penalty per unit of tax evaded reduces the level of tax evasion in the wealth tax, and given congruity, also in the personal income tax[15]. The numerator of expression [11] can be disintegrated into an income effect and a substitution effect. On the one hand, this latter effect, $pU(A)'$, picks up the increase in the profitability of tax compliance due to the increase in the fine per unit of tax evaded; on the other hand, an income effect, $pU(A)'R(A)(F-1)(Y-Y_D)(1-\beta)(t_R'+t_P)$, is also positive, since the increase in F reduces net income of the taxpayer both in state A and B, and given the assumption of decreasing risk-aversion, this tends to increase the valuation of the marginal cost of tax evasion more than the valuation of the marginal benefit, and so to increase tax compliance.

In the case of an increase in t_P, operating as above but also making use of the FOC (expression [9]), we obtain the following reaction:

$$\frac{dS_D}{dt_P}=\frac{S_D\left[R(A)-R(B)\right]+R(A)F(S-S_D)}{(t_R'+t_P)\left[R(A)(F-1)+R(B)\right]}\geq 0 \qquad [12]$$

since according to the usual assumption about decreasing absolute risk aversion, $R(A)>R(B)$; while in the case of an increase in t_R,

$$\frac{dS_D}{dt_R}=\frac{Y_D\left[R(A)-R(B)\right]+R(A)F(Y-Y_D)}{(t_R'+t_P)\left[R(A)(F-1)+R(B)\right]}\geq 0 \qquad [13]$$

In front of an increase in any of the marginal tax rates, only an income effect is present, since a rise in the marginal tax rate simultaneously increases the penalty per unit of tax evaded, and so the substitution effect vanishes (see Yitzhaki, 1974). Note that as long as the wealth tax is assigned to one government and the personal income tax to another, considering tax evasion in interrelated taxes permits the detection of a tax externality between governments. For instance, according to expression [13], an increase in the marginal tax rate of the personal income tax will not only affect the amount of tax base declared in that tax (and so the amount of tax revenue collected), but also in the wealth tax. Finally, comparing [12] and [13], it is easily verifiable that $dS_D/dt_R > dS_D/dt_P$ as long as β>0.

[14] This alternative definition of t_R comes out from expression [2] when it holds with equality, $Y_D(1-\beta)=S_D$. Then, an increase in S_D (and consequently in Y_D by $1/(1-\beta)$) makes that the marginal tax rate of the personal income tax borne by the taxpayer is $t_R/(1-\beta)$, and not just t_R, such that $t_R'>t_R$.

[15] Note that, on the one hand, $dY_D/dF=(1/(1-\beta))(dS_D/dF)$, and so the total effect of increasing F on the amount of tax bases declared is $dS_D/dF(1+(1/(1-\beta)))$. On the other hand, in terms of elasticity, ε, there is not any difference between the variation in S_D and the variation in Y_D, i.e., $\varepsilon_{S_D,F}=\varepsilon_{Y_D,F}$.

Finally, in front of an increase in $p_{,}$

$$\frac{dS_D}{dp} = \frac{U(A)'(F-1)+U(B)'}{(t_R'+t_P)(1-p)U(B)'\left[R(A)(F-1)+R(B)\right]} \geq 0 \qquad [14]$$

where only a substitution effect is at work.

From the results of this section, we can conclude that when collaboration between tax administrations is perfect, the results of the comparative statics do not differ from the original results due to Allingham and Sandmo (1972) and Yitzhaki (1974). However, as we suggested above, it is important to note that as long as we consider tax evasion in interrelated taxes, the statutory tax parameters of any of both taxes (t_R or t_P) or those instruments set by a tax administration (F, p^Y or p^S)[16] simultaneously affect the behaviour of the taxpayer in both taxes. That is, we have been able to identify a tax externality. Therefore, from a social point of view, it seems necessary that those parameters are decided taking into account their effects on both taxes, otherwise their level will not be optimal with respect to that situation under which the tax administration is fully integrated and the power to change the statutory tax parameters is at hands of just one government. In Shoup (1969)'s terminology, the interrelation between tax bases creates a "self-reinforcing penalty system of taxes"[17]. In the numerical simulations of Section 3, we will analyse these issues in more detail in the sense of checking how this system of taxes raises the level of tax compliance.

... When collaboration between tax administrations is imperfect.

In the case collaboration between tax administrations is imperfect, when the taxpayer is caught evading taxes only a share of the tax revenue due to the other tax administration is discovered. That might be understood as a low powered incentive of the tax administration that has audited to collect tax revenue on behalf of the other tax administration[18].

Then, in the case of imperfect collaboration, net income at disposal of the taxpayer when the tax administration responsible for the personal income tax audits is

[16] Certainly, the value of F is legally set by the political power. However, a tax inspector might discretionally vary its value depending on the development of the tax auditing process. In this sense, see OECD (1990) for a comparison among OECD's countries about the divergence between the legal value of F and the real one set by tax auditors.

[17] Note that, as suggested in the introduction, this is quite different from a "self-checking system of taxes" (Kaldor, 1956), since from the level of income (wealth) declared, it is not possible to be fully certain that the inferred level of wealth (income) is such that tax evasion is null.

[18] Obviously, if there exists only one tax administration, there might also exist internal inefficiencies within that tax administration. For instance, it could be the case that different departments within the same tax administration - each one of them in charge of a tax or of a group of taxes - might not fully cooperate between them. Then, it would not be necessary to consider the possibility that there were two imperfectly coordinated tax administrations in order that in some occasions the percentage of tax evasion discovered is less than 100%. In any case, although it is not relevant for our analysis, the non-cooperative possibility seems less likely within a tax administration than between two institutionally independent tax administrations.

$$A \equiv Y - Y_D t_R - S_D t_P - F t_R (Y - Y_D) - F t_P \alpha_Y (S - S_D) \quad [15]$$

where α_Y is the percentage of tax evasion discovered in the wealth tax, such that $0 \leq \alpha_Y < 1$. In the case of perfect collaboration between tax administrations, $\alpha_Y = 1$[19]. Similarly, when the tax administration responsible for the wealth tax audits, net income is

$$D \equiv Y - Y_D t_R - S_D t_P - F t_R \alpha_S (Y - Y_D) - F t_P (S - S_D) \quad [16]$$

where again $0 \leq \alpha_S < 1$. Finally, when none of both tax administrations carries out a tax audit, net income is

$$E \equiv Y - Y_D t_R - S_D t_P \quad [17]$$

Hence, for instance, from [16], as long as $\alpha_S < 1$, it is not clear whether an increase in the amount of tax base declared in the personal income tax, Y_D, increases the amount of net income at disposal of the taxpayer, since

$$D_{S_D} = t_R'(F\alpha_S - 1) + t_P(F - 1) \underset{\leq}{\overset{>}{}} 0 \quad [18]$$

where recall that $t_R' = t_R/(1-\beta)$ (vid. footnote 14). Only as long as $F > [(t_R' + t_P)/(t_R'\alpha_S + t_P)] > 1$, expression [18] will be positive as in the case in which collaboration is perfect and the tax administration responsible for the wealth tax is auditing. Thus, contrary to that situation, in order marginal net income increases as a consequence of having reduced the level of tax evasion, it is no longer sufficient that $F > 1$ if $\alpha_S < 1$. Otherwise, as long as $F < [(t_R' + t_P)/(t_R'\alpha_S + t_P)]$, although one of the two tax administrations were auditing, the taxpayer would still obtain marginal increases in net income evading taxes[20]. As we will check, in part that possibility will make ambiguous the results of the comparative statics when collaboration between tax administrations is imperfect. However, before performing the exercise of comparative statics, again we previously need to know whether incongruity or just congruity as before is an optimal strategy for the taxpayer.

Is optimal to be congruent in the tax bases declared?

In the case of imperfect collaboration between tax administrations incongruity might be an optimal strategy for the taxpayer. In order to show such result, let analyse the possibility under which $S_D + C' < Y_D$. Then, the following FOC's should hold:

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[19] Given that the percentage of tax fraud discovered depends on the effort carried out by the tax administration (that is, the level of collaboration between tax administrations), α_Y could also be interpreted as the effort of the tax administration in discovering tax evasion on behalf of the other tax administration. Thus, for example, for $\alpha_Y = 1$, that level of effort is maximum.

[20] An alternative explanation for the negative sign of expression [18] is that the level of t_R' (with respect to t_P) is relatively high, while the level of α_S is low enough and in any case it is not compensated by a big value of F.

$$\left.\frac{\partial W}{\partial Y_D}\right|_{S_D+C'>Y_D} = p^1_{Y_D}\left[U(A_1)-U(E_1)\right]+t_R\left[p^Y_1(F-1)U(A_1)'+p^S_1(F\alpha_S-1)U(D_1)'-(1-p^Y_1-p^S_1)U(E_1)'\right]>0$$
[19]

such that $U(A_i)<U(E_i)$, and $U(A_i)'>U(E_i)'\ \forall i$, and $p^1_{Y_D}<0$; but also

$$\left.\frac{\partial W}{\partial Y_D}\right|_{S_D+C'=Y_D} = t_R\left[p^Y_2 U(A_2)'(F-1)+p^S_2 U(D_2)'(F\alpha_S-1)-(1-p^Y_2-p^S_2)U(E_2)'\right]>0 \qquad [20]$$

while at the (supposed) optimum, the following two conditions should hold:

$$\left.\frac{\partial W}{\partial Y_D}\right|_{S_D+C'<Y_D} = p^3_{Y_D}\left[U(D_3)-U(E_3)\right]+t_R\left[p^Y_3 U(A_3)'(F-1)+p^S_3(F\alpha_S-1)U(D_3)'-(1-p^Y_3-p^S_3)U(E_3)'\right]=0$$
[21]

such that $U(E_i)>U(D_i)$, and $U(D_i)'>U(E_i)'\ \forall i$, and $p^3_{Y_D}>0$; and

$$\left.\frac{\partial W}{\partial S_D}\right|_{S_D+C'<Y_D} = p^3_{S_D}\left[U(D_3)-U(E_3)\right]+t_P\left[p^Y_3 U(A_3)'(F\alpha_Y-1)+p^S_3 U(D_3)'(F-1)-(1-p^Y_3-p^S_3)U(E_3)'\right]=0$$
[22]

where $p^3_{S_D}<0$. According to expression [21], at equilibrium, the welfare cost of marginally increasing Y_D caused by a higher level of p, $p^3_{Y_D}\left[U(D_3)-U(E_3)\right]<0$, is exactly compensated by the welfare benefit of increasing tax compliance keeping the tax audit probability constant. Instead, according to expression [22], the welfare benefit that produces an increase in S_D due to a lower level of p is compensated by the welfare cost when the tax audit probability remains constant. In any case, note that as long as collaboration between tax administrations is imperfect (see again fn. 18), nothing impedes incongruity to be an optimal strategy for the taxpayer.

In order to ascertain under what circumstances it is more likely that from the taxpayer's point of view $S_D+C'<Y_D$ is an optimal strategy, using expressions [21] and [22] we can obtain the following necessary condition:

$$p^Y_3 U(A_3)'(F\alpha_Y-1)+p^S_3 U(D_3)'(F-1)<(1-p^Y_3-p^S_3)U(E_3)'<p^Y_3 U(A_3)'(F-1)+p^S_3 U(D_3)'(F\alpha_S-1)$$
[23]

so,

$$p^S_3 U(D_3)'(1-\alpha_S)<p^Y_3 U(A_3)'(1-\alpha_Y) \qquad [24]$$

Note that if the tax audit probability functions are symmetric, for $S_D+C'<Y_D$, $p^S_3>p^Y_3$. In general, expression [24] implies that all the tax parameters referred to the personal income tax have to be more stringent than those referring to the wealth tax, that is, $\alpha_S>\alpha_Y$ and $t_R>t_P$ (and, leaving aside the assumption of symmetry, also $p^Y>p^S$). For instance, if we suppose that $\alpha_Y, \alpha_S < 1$, but $\alpha_Y = \alpha_S$, it can be shown that expression [24] necessarily implies $t_R' > t_P$, since only then $U(A_3)' > U(D_3)'$. Thus, given $p_3^S > p_3^Y$ and supposing $\alpha_Y = \alpha_S$, a necessary condition for being optimal that a taxpayer evades less taxes in the personal income tax than in the wealth tax is simply that the tax rate of the former tax is lower than the tax rate of the wealth tax weighted by the marginal propensity to save. That is, the reduction in disposable income has to be relatively greater when the tax administration responsible for the personal income tax audits than when the other tax administration carries out an audit[21]. On the whole, contrary to the case of perfect collaboration, S_D and Y_D are no longer perfect substitutes with respect to the optimal decision over tax evasion.

a) … *when it is optimal to be congruent*

Optimal level of tax base declared

In the case of congruity and imperfect collaboration, the FOC is obtained from the following maximisation problem:

$$\underset{S_D, Y_D}{Max} \quad W' \quad s.t. \quad Y_D = S_D + C'$$

where $W' = p^Y A + p^S D + (1 - p^Y - p^S)E$, and *A*, *D* and *E* have been previously defined by expressions [15], [16] and [17], respectively. Now, in contrast to the case of perfect collaboration, net income is not the same after having audited each tax administration, i.e., $A \neq D$. Once we have substituted Y_D into *W'*, we obtain the following FOC with respect to S_D:

$$\begin{aligned} p^Y U(A)' \{t_R'(F-1) + t_P(F\alpha_Y - 1)\} + p^S U(D)' \{t_R'(F\alpha_S - 1) + t_P(F-1)\} = \\ = (1 - p^Y - p^S) U(E)' (t_R' + t_P) \end{aligned} \qquad [25]$$

As long as marginal income is positive under states *A* and *D*, the left-hand side of the equation (first row) can be defined as the marginal cost of evading taxes (or marginal benefit of tax compliance), while the right-hand side (second row) is the marginal benefit of evading taxes (or marginal cost of tax compliance). Nevertheless, as we already know (see, e.g., expression [18]), marginal income is not always positive (i.e., $A_{S_D}, D_{S_D} \lesseqgtr 0$). Thus, as long as one of the summands of the first row had a negative sign, it should be considered as a marginal benefit of tax evasion and not as a marginal cost

[21] In fact, in our static model where the initial stock of wealth is null (see fn. 3), this seems the most plausible assumption, since the marginal tax rates of the wealth tax tend to be much lower than those of the personal income tax, and additionally the tax base of the former tax is just a percentage (1-β) of the tax base of the latter tax. However, as long as we were dealing with a dynamic model which had made possible the accumulation of a stock of wealth, although the tax rate of the wealth tax were lower than the tax rate of the personal income tax, the tax base of the wealth tax (now, a real stock of wealth) could be big enough in terms of current personal income as to make more burdensome the wealth tax than the personal income tax, and so $S_D + C' > Y_D$ could equally be a plausible optimal strategy for the taxpayer.

of tax evasion[22].

From now on, we will assume that $S_D<S$, which requires that expression [7] holds, in this case applied to the case of imperfect collaboration between tax administrations,

$$t_R'\left\{F(p^Y+p^S\alpha_S)\right\}+t_P\left\{F(p^S+p^Y\alpha_Y)\right\}<t_R'+t_P \qquad [26]$$

Therefore, expression [26] has the same interpretation than the classical condition that guarantees an inner solution, that is, the summation of expected penalties when the taxpayer decreases tax compliance and is caught evading taxes (left-hand side of the inequality) is smaller than the certain amount of taxes due when the taxpayer increases tax compliance (right-hand side of the inequality)[23].

Comparative statics

First, we will analyze how the tax base declared, S_D, varies in front of an increase in F. Nevertheless, given that in this case the exercise of comparative statics is much more cumbersome than when collaboration between tax administrations is perfect, and given that we are only interested in the sign of each reaction, we will make use of the fact that

$$\frac{dS_D}{dF}=\frac{\Phi_F}{-\Phi_{S_D}} \qquad [27]$$

where Φ is the FOC of the taxpayer's maximization problem (expression [25]). Thus, since the SOC of the maximisation problem really holds[24], $\Phi_{S_D}<0$, $sign\{dS_D/dF\}=sign\{\partial\Phi/\partial F\}$, we will simply have to calculate the partial derivative $\partial\Phi/\partial F$, and equally for the rest of parameters of the model. Hence,

$$\begin{aligned}\Phi_F = {} & p^S U(D)'\left[t_R'(F\alpha_S-1)+t_P(F-1)\right](Y-Y_D)(1-\beta)\left[R(D)\{t_R'\alpha_S+t_P\}-R(A)\{t_R'+t_P\alpha_Y\}\right]+\\ & +(1-p^Y-p^S)U(E)'(t_R'+t_P)(Y-Y_D)(1-\beta)R(A)\{(t_R'+t_P\alpha_Y\}+\\ & +\left\{\frac{(1-p^Y-p^S)U(E)'(t_R'+t_P)-p^S U(D)'\left[t_R'(F\alpha_S-1)+t_P(F-1)\right]}{t_R'(F-1)+t_P(F\alpha_Y-1)}\right\}(t_R'+t_P\alpha_Y)+\\ & +p^S U(D)'(t_P+t_R'\alpha_S)\geq 0\end{aligned}$$

[28]

[22] Obviously, it cannot be the case that those two summands are negative at the same time, since then there would not be a solution to the maximization problem.

[23] In any case, as expected, the condition given by expression [26] is less stringent than the classical one, $pF<1$. This can be easily shown once expression [26] is re-written as follows:

$$pF-\frac{1}{t_R'+t_P}\left\{t_R'Fp^S(1-\alpha_S)+t_PFp^Y(1-\alpha_Y)\right\}<1 \qquad [26']$$

[24] $$\begin{aligned}\Phi_{S_D} = {} & -p^Y U(A)'R(A)\{t_R'(F-1)+t_P(F\alpha_Y-1)\}^2-\\ & -p^S U(D)'R(D)\{t_R'(F\alpha_S-1)+t_P(F-1)\}^2-(1-p^S-p^Y)U(E)'R(E)(t_R'+t_P)^2<0\end{aligned}$$

Both a substitution and an income effect provoke that the optimal reaction of the taxpayer is unambiguously positive. The first two rows show the latter effect, which is always positive, and so points out in the direction of increasing S_D. This is so since, on the one hand, in the case in which $t_R'(F\alpha_S-1)+t_P(F-1)<0$, $R(A)>R(D)$ and $t_R'+t_P\alpha_Y>t_P+t_R'\alpha_S$, while the reverse occurs when $t_R'(F-1)+t_P(F\alpha_Y-1)<0<t_R'(F\alpha_S-1)+t_P(F-1)$, which ensures the positive sign of the first row. On the other hand, when $t_R'(F\alpha_S-1)+t_P(F-1)>0$ and $t_R'(F-1)+t_P(F\alpha_Y-1)>0$, the income effect is also positive, which can be easily checked if $\partial\Phi/\partial F$ is analyzed without making use of the FOC. A substitution effect in favour of increasing S_D is shown in the last two rows of expression [28][25].

In the case of an increase in t_R,

$$\begin{aligned}\ddot{O}_{t_R}&=-p^SU(D)'[t_R'(F\alpha_S-1)+t_P(F-1)]\{[R(A)-R(D)]Y_D+[R(A)-R(D)\alpha_S]F(Y-Y_D)\}+\\&+(1-p^Y-p^S)U(E)'(t_R'+t_P)\{[R(A)-R(E)]Y_D+R(A)F(Y-Y_D)\}+\\&+\frac{Ft_P}{t_R'(F-1)+t_P(F\alpha_Y-1)}\{(1-p^Y-p^S)U(E)'(1-\alpha_Y)+p^SU(D)'[2-(\alpha_Y+\alpha_S)-F(1-\alpha_Y\alpha_S)]\}\overset{<}{\geq}0\end{aligned}$$

[29]

In the first two rows, there appears an income effect, while in the third row there appears a substitution effect. On the one hand, as long as $\alpha_Y=1$, a substitution effect always points out in the direction of decreasing S_D, while when $\alpha_Y<1$, the sign of that effect is ambiguous. The reason is as follows: if $\alpha_Y=1$, $\alpha_Y>\alpha_S$ (given the hypothesis of imperfect collaboration), and then in front of an increase in t_R the relative benefit of evading taxes when collaboration is imperfect increases, since in that case tax evasion in the personal income tax is not fully discovered is state *D*, while it is in the wealth tax. Instead, if $\alpha_Y<1$, it is not possible to ascertain the sign of the substitution effect, since $\alpha_Y \overset{\leq}{>} \alpha_S$, and the final net effect will also depend on the marginal utility of income in each one of the three possible states (*A*, *D* and *E*)[26]. On the whole, we can conclude that

[25] In the third row of expression [28], note that the fraction that appears in brackets is simply $p^YU(A)'$, which confirms the positive sign of a substitution effect independently of the sign of marginal income in state *A* and state *D*.

[26] In fact, the profitability of diminishing the amount of tax base declared, $r(-S_D)$, can de defined as $r(-S_D)\equiv 1-F\left\{\frac{t_R'(p^Y+\alpha_Sp^S)+t_P(\alpha_Yp^Y+p^S)}{t_R'+t_P}\right\}$, i.e., supposing a risk-neutral taxpayer, it is calculated as the marginal income that would be obtained increasing tax evasion with respect to that situation under which tax evasion is null. Then, $r_{t_R}=\frac{Ft_Pp}{(1-\beta)(t_R'+t_P)^2}(\alpha_Y-\alpha_S)$, where in order to simplify we have supposed that $p=p^Y=p^S$. Then, it is clear that as long as $\alpha_S<\alpha_Y$, in front of an increase in t_R, the profitability of increasing tax evasion (i.e., reducing S_D) has increased, $r_{t_R}>0$, while the reverse happens when $\alpha_S>\alpha_Y$. As in the classical analysis, if $\alpha_S=\alpha_Y$, the substitution effect vanishes independently of whether the degree of collaboration between tax administrations is perfect or not. Thus, in front of an increase in the

the lower (higher) the level of collaboration of the tax administration responsible for the wealth tax with respect to the level collaboration of the other tax administration, the more likely that the increase in t_R tends to promote tax evasion (compliance). Therefore, the analysis of tax evasion in interrelated taxes has permitted to identify a situation (imperfect collaboration between tax administrations) under which the theoretical classical results on tax evasion might fail, that is, an increase in the (marginal) tax rate might not produce an increase in tax compliance.

On the other hand, the net impact of the income effect is more difficult to ascertain due to the ambiguity of the sign of the first row of expression [29] when $t_R'(F-1)+t_P(F\alpha_Y-1)<0$, while the sign of the second row is clearly positive, that is, in favor of increasing S_D. The reason of that ambiguity is the following: an increase in t_R will certainly diminish net income both in state A and in state D, and so the valuation of marginal income will have increased. However, as long as $t_R'(F-1)+t_P(F\alpha_Y-1)<0$, marginal increases in S_D under state A have to be considered as a marginal cost of tax compliance. Then, contrary to the traditional case, given that the marginal impact of the increase in t_R is greater under state A than under state D, i.e., $|A_{t_R}|>|D_{t_R}|$, the valuation of the marginal cost of tax compliance has increased more than the valuation of the marginal benefit of tax compliance (i.e., net income in state D). Under these circumstances, for $t_R'(F-1)+t_P(F\alpha_Y-1)<0$, a sufficient condition to avoid such ambiguity is that the valuation of the marginal benefit of tax compliance is big enough with respect to the valuation of the marginal cost of tax compliance such that $\alpha_S R(D)>R(A)$.

On the whole, the sign of expression [29] is not clear-cut, since a substitution and an income effect might have contradictory signs. Hence, we are back to the ambiguity originally detected by Allingham and Sandmo (1972). For instance, note that if $\alpha_Y=1$, a substitution effect incentives a decrease in S_D, while an income effect points out right in the contrary direction, since for $\alpha_Y=1$, there is not ambiguity with respect to the income effect.

In the case of an increase in t_P,

$$\begin{aligned}\Phi_{t_P} = & -p^S U(D)'\left[t_R'(F\alpha_S-1)+t_P(F-1)\right]\left\{\left[R(A)-R(D)\right]S_D+\left[R(A)\alpha_Y-R(D)\right](F(S-S_D))\right\}+\\ & +(1-p^Y-p^S)U(E)'(t_R'+t_P)\left\{\left[R(A)-R(E)\right]S_D+R(A)F\alpha_Y(S-S_D)\right\}-\\ & -\frac{Ft_R'}{t_R'(F-1)+t_P(F\alpha_Y-1)}\left\{(1-p^Y-p^S)U(E)'(1-\alpha_Y)+p^S U(D)'\left[2-(\alpha_Y+\alpha_S)-F(1-\alpha_Y\alpha_S)\right]\right\}\lesseqgtr 0\end{aligned}$$

[30]

As long as $\alpha_Y=1$, a substitution effect always points out in the direction of increasing S_D, while when $\alpha_Y<1$, the sign of the substitution effect is ambiguous. The reasoning is identical to the one given above with respect to Φ_{t_R}, although the signs are obviously

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marginal tax rate, imperfect collaboration only modifies the profitability of tax evasion as long as both tax administrations do not exert the same level of effort in auditing on behalf of the other tax administration.

reversed[27]. In the first two rows, there appears an income effect, which sign is again ambiguous. In this case, the ambiguity comes from those situations under which $t_P(F-1)+t_R'(F\alpha_S-1)<0$, being $\alpha_Y R(A)>R(D)$ a sufficient condition to avoid it.

In the case of an increase in p^Y,

$$\Phi_{p^Y}=U(A)'\{t_R'(F-1)+t_P(F\alpha_Y-1)\}+U(E)'(t_R+t_P)\underset{>}{\overset{\leq}{}}0 \qquad [31]$$

Only a substitution effect is at work. As long as $t_R'(F-1)+t_P(F\alpha_Y-1)>0$, an increase in p^Y always provokes an increase in S_D. Otherwise, the sign is ambiguous. Paradoxically, an increase in p^Y might be welcome by the taxpayer as long as the tax administration on behalf of the personal income tax does not collaborate very much with the other tax administration, and then given the value of the rest of relevant parameters, $t_R'(F-1)+t_P(F\alpha_Y-1)<0$. In that case, the expected profitability of evading taxes will have increased, since the rise in p^Y has made more likely a state under which, even though one tax administration is auditing, the taxpayer can still obtain increases in net income evading taxes. This is certainly a curious result that directly stems from the absence of perfect collaboration between tax administrations.

Similarly in the case of an increase in p^S,

$$\Phi_{p^S}=U(D)'\{t_R'(F\alpha_S-1)+t_P(F-1)\}+U(E)'(t_R+t_P)\underset{>}{\overset{\leq}{}}0 \qquad [32]$$

Again, as long as $t_R'(F\alpha_S-1)+t_P(F\alpha-1)>0$, the sign is unambiguously positive. Otherwise, being only a substitution effect at work, the reason of the ambiguity is the same than the one given above with respect to expression [31].

Finally, we are interested in showing how a reinforcement of the collaboration between tax administrations varies the level of S_D. First, when the tax administration responsible for the personal income tax increases its auditing effort with respect to the wealth tax:

$$\Phi_{\alpha_Y}=p^Y U(A)'Ft_P\{1+R(A)(S-S_D)[t_R'(F-1)+t_P(F\alpha_Y-1)]\}\underset{>}{\overset{\leq}{}}0 \qquad [33]$$

As long as $t_R'(F-1)+t_P(F\alpha_Y-1)>0$, $\Phi_{\alpha_Y}>0$, otherwise, the sign is ambiguous. A substitution effect always points out in the direction of increasing tax compliance, $p^Y U(A)'Ft_P>0$, while the sign of an income effect can go either way, depending on the sign of marginal income in state *A*. In the case that $t_R'(F-1)+t_P(F\alpha_Y-1)<0$, a reinforcement of the collaboration by part of the tax administration responsible for the personal income tax certainly reduces net income in state *A*, which increases the valuation of a marginal cost of tax compliance. As a consequence of the increase in that

[27] Following the methodology used in the previous footnote, $r_{t_P}=\dfrac{Ft_R'p}{(t_R'+t_P)^2}(\alpha_S-\alpha_Y)\underset{>}{\overset{\leq}{}}0$.

marginal valuation, there is an incentive to decrease the level of tax compliance.

Second, we analyse the variation in S_D when the tax administration responsible for the wealth tax increases its auditing effort with respect to the personal income tax:

$$\Phi_{\alpha_S} = p^S U(D)' F t_R' \{1 + R(D)(S - S_D)[t_R'(F\alpha_S - 1) + t_P(F - 1)]\} \substack{\leq \\ >} 0 \qquad [34]$$

If $t_R'(F\alpha_S - 1) + t_P(F - 1) > 0$, $\Phi_{\alpha_S} > 0$, otherwise, the sign is ambiguous. The reasoning of this ambiguity is identical to the one given above with respect to expression [33].

Undoubtedly, the results of the comparative statics concerning collaboration between tax administrations are quite interesting. An increase in collaboration between tax administrations is always a good thing in the sense of promoting higher levels of tax compliance only as long as marginal net income is positive in all those states where one tax administration is auditing (so, note that it is not strictly necessary that α_i=1). Otherwise, paradoxically, an increase in collaboration between tax administrations might produce a lower level of tax compliance! In Figure 3, on the left-hand side, there appears the level of α_S from which an increase in α_S creates a substitution and an income effect that unambiguously promote tax compliance. Similarly, on the right-hand side, there appears the threshold with respect to the level of collaboration of the tax administration responsible for the personal income tax, α_Y.

[FIGURE 3]

b) … *when it is optimal to be incongruent*

When collaboration between tax administrations is not perfect and incongruity between tax returns conditions the tax audit probability, the taxpayer might find it optimal not to be congruent. In this section, we will simply try to sketch how this strategy affects the results of the comparative statics, while the methodology of numerical simulation will complement this initial analysis.

Optimal level of tax base declared

We will analyze the case in which Y_D>S_D+C. The objective function of the taxpayer does not vary with respect to the previous case. Thus, there are still three possible states: A, D and E (expressions [15], [16] and [17], respectively), but now the tax audit probability for each one of those three states is endogenous. Moreover, there are two decision variables: S_D and Y_D. Taking all this into account, the FOC's of the maximization problem are the following:

$$Y_D:\ p_{Y_D}[U(D) - U(E)] + t_R\{p^Y U(A)'(F-1) + p^S U(D)'(F\alpha_S - 1) - (1 - p^Y - p^S)U(E)'\} = 0 \qquad [35]$$

$$S_D:\ -p_{Y_D}[U(D) - U(E)] + t_P\{p^Y U(A)'(F\alpha_Y - 1) + p^S U(D)'(F-1) - (1 - p^Y - p^S)U(E)'\} = 0 \qquad [36]$$

where $p_{Y_D} > 0$. In fact, given that Y_D>S_D+C, an increase in Y_D provokes a greater tax audit probability in the wealth tax, $p^S_{Y_D} > 0$; while from the same situation, an increase in S_D brings about a smaller tax audit probability in that tax, $p^S_{S_D} < 0$. However, given the assumption of symmetry, $\left|p^S_{Y_D}\right| = \left|p^S_{S_D}\right|$. That is why, in expression [36], we have used $-p_{Y_D}(< 0)$ instead of p_{S_D}, while the super-index *s* has been suppressed for clarity of exposition.

Expression [35] can be rewritten as follows:

$$t_R\left\{p^Y U(A)'(F-1) + p^S U(D)'(F\alpha_S - 1)\right\} = -p_{Y_D}\left[U(D) - U(E)\right] + t_R(1 - p^Y - p^S)U(E)' \qquad [35']$$

On the left-hand side, there appears the marginal benefit of tax compliance, and on the right-hand side, the marginal cost of tax compliance. Precisely, the novelty with respect to the case in which congruity is optimal is the additional marginal cost in which incurs the taxpayer when increases tax compliance, $-p_{Y_D}\left[U(D) - U(E)\right] > 0$. Incongruity implies that an increase in Y_D causes a higher level of *p*, and so a loss of welfare since *U(E)*>*U(D)*. Instead, in expression [36], an increase in S_D brings about a higher level of welfare due to the decrease in *p*. Finally, note that as long as $\alpha_S < (1/F)$, the second summand of the left-hand side in expression [35'] has to be considered as a marginal cost of tax compliance, and equally in expression [36] for $\alpha_Y < (1/F)$.

Incongruity implicitly impedes S_D=S and Y_D=Y from being an optimal strategy for the taxpayer. Thus, in order to obtain an inner solution, the following conditions should hold:

$$\left.\frac{\partial W}{\partial S_D}\right|_{S_D=S,Y_D<Y} < 0; \qquad \left.\frac{\partial W}{\partial Y_D}\right|_{S_D=S,Y_D<Y} < 0 \qquad [37a]$$

$$\left.\frac{\partial W}{\partial S_D}\right|_{S_D<S,Y_D=Y} < 0; \qquad \left.\frac{\partial W}{\partial Y_D}\right|_{S_D<S,Y_D=Y} < 0 \qquad [37b]$$

which from now on, we assume that hold, and so an interior solution is obtained from the taxpayer's maximization problem.

Comparative statics

We will skip the analytical comparative statics corresponding to the situation of incongruity due to its difficulty, which is mainly caused by the cross-effects between declared tax bases (Y_D and S_D). Instead, we will carry it out by means of an exercise of numerical simulations. However, before that, it might be useful to briefly analyze the main difference with respect to that situation in which congruity is optimal. Thus, from expression [35], we define the cost of incongruity, *K*, as

$$K \equiv p_{Y_D}\left[U(E) - U(D)\right] > 0 \qquad [38]$$

From this definition, it is easily verifiable that an increase either in α_S , t_R, t_P, F, p_S or in the sensitiveness of this latter variable with respect to incongruity[28] will produce an increase in the cost of incongruity. Hence, leaving aside the corresponding income and substitution effects, and being the initial situation one under which $Y_D > S_D + C$, the rise in K should produce a reduction in Y_D and/or an increase in S_D, that is, a decrease in the level of incongruity. Another new effect at work is a substitution effect between tax bases declared. Given that they are independently decided, as long as one parameter exclusively affects one tax base (e.g., the statutory tax rate), it will tend to promote an increase/decrease in tax compliance of that tax base with respect to the other one. Therefore, when interpreting the results of the numerical simulations, these two new effects will have to be taken into account join with the income and substitution effects already identified in the theoretical analysis.

3. Numerical simulations

The methodology of numerical simulations must be helpful in addressing some key issues that were not totally solved by means of the theoretical analysis. Among those issues are the following:

- Does the approach of considering tax evasion in interrelated taxes overcome, at least partially, the paradox of tax evasion?

- Given this theoretical approach and considering the possibility of imperfect collaboration between tax administrations, under which circumstances is incongruity between tax returns an optimal choice for the taxpayer?; and finally,

- The numerical simulations should be helpful in solving the inconclusive results of the analytical comparative statics.

In order to carry out the numerical simulations, we will employ the following well-known iso-elastic utility function:

$$U(Y_N) = \frac{Y_N^{\,1-\sigma}}{1-\sigma}, \sigma \neq 1 \qquad [39]$$

where $\sigma (>0)$ is the coefficient of absolute risk-aversion, and Y_N is net income after paying taxes, and in the presence of tax evasion also the corresponding fine per unit of tax evaded. The greater the value of σ, the greater the degree of risk-aversion. According to the economic literature, a reasonable value of this parameter is 1,8 (see Karni and Schmeidler, 1990; Epstein, 1992).

The rest of values given to the basic parameters of the model are the following:

$$Y = 1; \beta = 0{,}8; S = 0{,}2; t_R = 0{,}5; \; t_P = 0{,}005; F = 2$$

The numerical simulations do not pretend to replicate any real situation. That is why, the value of the above parameters do not necessarily reflect those of any potentially average taxpayer. However, the value of the tax audit probability will be obtained from

[28] Later on, in the exercises of numerical simulation, such sensitiveness will be denoted by h.

the model in such a way that the equilibrium values of tax evasion (Y_D/Y and S_D/S) range within a reasonable interval, as we will check below.

In the presence of incongruity, e.g. $Y_D(1-\beta) > S_D$, the tax audit probability of the wealth tax will adopt the following function:

$$p^S = \bar{p}^S \times exp\left[h \times (Y_D(1-\beta) - S_D)\right] \qquad [40]$$

where h>0. In the presence of incongruity, the greater the value of h, the greater the value of p^S above its normal level ($\bar{p}^S$). Similarly, in the case that $Y_D(1-\beta) < S_D$,

$$p^Y = \bar{p}^Y \times exp\left[h \times (S_D - Y_D(1-\beta))\right] \qquad [40']$$

In order to make significant in money terms the impact on net income of the wealth tax, apart from the increase in wealth due to annual savings (S), we have assumed that at the beginning of the fiscal year the taxpayer owned an initial amount of wealth, S_0 (>0) (see fn. 3). Therefore, leaving aside tax evasion, her budget constraint becomes as follows

$$Y - Y_D t_R - (S + S_0) t_P \qquad [41]$$

Along all the numerical simulations, we will suppose that S_0=2[29]. Moreover, in order to keep things as simple as possible and so focusing exclusively on the relationship between S and Y, we will assume that the taxpayer always declares the whole amount of her initial stock of wealth[30].

Paradox of tax evasion

The traditional analysis of tax evasion predicts very low levels of tax compliance, a situation that does not seem to hold in practice. As we already said in the introduction, in order to try to overcome such paradox, the literature on tax evasion has proposed several alternative explanations. It is in this context that we propose a new one. Thus, we postulate that considering tax evasion in interrelated taxes can, at least partially, help to solve such paradox. In fact, intuitively it seems that as long as the tax instruments of the interrelated taxes were (relatively) coordinated, they should positively interact with each other making tax evasion less attractive (as we know, this is the idea which bases the so-called "self-reinforcing penalty system of taxes" due to Shoup (1969)).

[TABLE 1a]

In Table 1a, we show the first results of this exercise of numerical simulation. We have characterized a situation with tax evasion both in the personal income tax, Y_D=0,8 given

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[29] This implies that the initial stock of wealth subject to taxation is twofold current income. That seems a reasonable assumption, once we take into account that the tax law usually permits to deduct a certain amount of money in the calculus of the tax base. Thus, S_0 must be considered as the initial stock of wealth once such amount of money has already beed deducted.

[30] This assumption will prove extremely useful in the numerical simulations in order to isolate an income effect.

Y=1, and in the wealth tax, S_D=0,16 given S=0,2, and in order to ease the comparison with previous results of the literature, declared tax bases are congruous for $\beta = 0{,}8$. Next, we have obtained the value of the audit probability compatible with that level of tax evasion, supposing a taxpayer that aims at maximizing the utility function [39]. However, the results of the numerical simulation certainly depend on the assumptions regarding the context of tax evasion. Under the label *Classical analysis*, the model employed of tax evasion coincides with the original model due to Allingham and Sandmo (1972), and so each decision of tax evasion is considered separately. Hence, the maximization problem is solved for each tax, such that p^Y and p^S are obtained given the values of the basic parameters of the model[31]. Since each decision is considered separately, there is no reason to treat both events (auditing of the personal income tax return and auditing of the wealth tax return) as mutually exclusive. That is why, the probability of occurrence of any of both events is calculated as $(p^Y+p^S)-(p^Y\times p^S)$. In the rest of simulated situations, tax evasion in interrelated taxes is the behavior under analysis (*Interrelated Evasion*), having analyzed, first, that situation under which collaboration between tax administrations is perfect; and second, those situations under which collaboration is imperfect.

In the *Classical analysis*, in order to ensure those cited levels of tax compliance, the sum of tax audit probabilities has to be as high as 0,6625, while in the case of *Interrelated Evasion* and perfect collaboration, that level is "only" 0,3212[32]. Nonetheless, as long as collaboration is imperfect, the tax audit probability might be higher or lower than the value obtained in the *Classical analysis* (see also fn. 9). Hence, taxes might interact negatively with each other producing a decrease in the level of tax compliance as long as collaboration is imperfect. That impedes Shoup (1969)'s idea of the "self-reinforcing penalty system of taxes" to be universal, since it depends on the degree of collaboration between tax administrations. Thus, such negative possibility does not come out when anyone of both tax administrations is carrying out the maximum level of collaboration ($\alpha_i = 1$), becoming then that case identical to the one of perfect collaboration. Table 2a illustrates the same cases than Table 1a, but for a situation of full tax compliance (i.e., $Y_D/Y = 1$ and $S_D/S = 1$).

[TABLE 2a]

The values obtained of the tax audit probabilities that are shown in both tables are certainly very high, obviously being highest in Table 2a. For example, Bernasconi (1998), pp. 127-6, argues that in order to be accord with those in force in many countries, the individual tax audit probabilities should range from 0,01 to 0,03, whereas 0,09 might be the average for USA taxpayers (Harris, 1987). Therefore, from the results of our numerical simulations, we should conclude that *Interrelated Evasion* does not solve the paradox of tax evasion, since in order to ensure full tax compliance, p (defined as p^Y+p^S) has to be as high as 0,5 when collaboration is perfect (Table 2a), and 0,3212 to guarantee a level of tax compliance of 80% (Table 1a). Nevertheless, there is another

[31] The method used to solve the system of non-linear equations is the so-called "Gauss-Newton".

[32] As we know from the theoretical analysis, when collaboration between tax administrations is perfect, we just have p, and so from the numerical simulations it is not possible to ascertain the value of p^Y and p^S, but just p^Y+p^S.

way to read our results, that is, comparing those absolute values with those obtained in the *Classical analysis*, 0,75 and 0,66, respectively. Thus, our approach might be considered as a partial explanation to the paradox of tax evasion, since our tax audit probabilities are around half those predicted by the *Classical analysis*.

[TABLE 1b]

[TABLE 2b]

Bernasconi (1998) also carried out an exercise of numerical simulation in order to contrast whether his theory of over-weighted tax audit probabilities by part of the taxpayers - which can be justified once different orders of risk aversion are distinguished - was able to overcome the paradox of tax evasion. In order to compare his results with ours, in Table 1b and Table 2b, we have modified the value of the basic parameters of the previous numerical simulation. Now, $t_R = 0{,}3$; $t_P = 0{,}002$ (which might be considered as a low bound of the range of reasonable values of t_P), and F=4, which are the same values than those used by Bernasconi (1998) with the obvious exception of t_P[33]. In this case, the equilibrium values of the tax auditing probabilities are much lower. For instance, if we just pay attention to the value of p^Y, for levels of tax compliance of 80%, when collaboration (between taxes or tax administrations) is symmetric and above 0,5, we can see that it lies within a relatively reasonable interval (0,096 to 0,0755), and in any case much lower than in the *Classical analysis* (0,1441)[34]. Moreover, in this latter analysis, p^S should be as high as 0,2499, while in the former it should be between 0,1398 and 0,0820. In fact, although this result does not appear in Table 1b, in the case of *Interrelated Evasion*, a level of tax compliance of 60% is compatible with auditing probabilities in each tax as low as 0,03. Additionally, the necessary values of tax auditing probabilities is decreasing in S_0 (in our case, exclusively due to an income effect), so for big fortunes (i.e., those taxpayers with a high ratio S_0/Y), the tax auditing probabilities should be even lower than the values shown in tables[35].

[33] In fact, Bernasconi (1998) set F=3. However, he expressed net income when the taxpayer is audited as

$$Y - Yt_R - F' t_R(Y - Y_D)$$

Therefore, given our different way of expressing net income, it is obvious that in our case $F=F'+1$.That is why, using F=4, and given the rest of values of the basic parameters, we are exactly replicating Bernasconi (1998)'s simulations.

[34] Note that the tax audit probabilities qualified by Bernasconi (1998) as reasonable, which range from 0,01 to 0,03, are an average for the whole set of taxpayers. Hence, these average values should be perfectly compatible with point values much higher (and lower). In this sense, it could be the case that those taxpayers that submit a wealth tax return were audited in the personal income tax more often than any other taxpayer, that is, it could be the case that their "normal" tax audit probability (before considering the possibility of incongruity between tax bases) were above those average values. Once we take into account this possibility, a tax auditing probability around 0,07 or even slightly above might not be too far from reality.

[35] For instance, for a taxpayer which initial stock of wealth (S_0) is 100, maintaining the rest of values equal to those in Table 1b, p^Y+p^S=0,1083. However, given that the wealth tax is a

On the whole, from the results of our numerical simulations, we should conclude that considering tax evasion in interrelated taxes permits to partially overcome the paradox of tax evasion, since reasonable levels of tax evasion are compatible with relatively low values of the tax auditing probabilities. However, it is very important to note that such result is only valid as long as there is an important degree of collaboration between tax administrations.

Incongruity

The consideration of tax evasion in interrelated taxes can produce an interesting result. As long as collaboration between tax auditors responsible for each tax is not perfect, the tax bases declared in each tax return might not be congruous. This result has already been shown in the theoretical part of the paper. However, the numerical simulations should still provide us more information. In particular, first, they should be useful in indicating which situation is more likely, $Y_D(1-\beta) \stackrel{<}{>} S_D$, and second, which combination of values of the basic parameters of the model can produce incongruity.

The benchmark case will be that of Table 1a in which for $\alpha_Y = \alpha_S = 0{,}75$, $p^Y = 0{,}1848$ and $p^S = 0{,}2296$ (i.e., p^Y/p^S=0,8049)[36], and tax bases are congruous for β=0,8. From these initial values, we ask which new combination of tax audit probabilities should hold in order incongruity becomes an optimal choice for the taxpayer. In Table 3, first, we detect those situations under which $Y_D(1-\beta)>S_D$. As expected, maintaining constant the rest of parameters, that type of situation is only compatible with a relatively much greater level of tax enforcement of the tax administration responsible for the personal income tax (recall that in this situation p^S has to be calculated by means of expression [40]). Moreover, this difference in the relative degree of tax enforcement has to be greater, the greater the level of h. Second, as Table 3 also shows, under the reverse situation, $Y_D(1-\beta)<S_D$, those differences – now, in favor of p^S, while p^Y has to be calculated by means of expression [40'] - have to be even much more acute. Therefore, in order the taxpayer finds it optimal to be incongruent, keeping the rest of tax parameters unchanged, there must be great differences in the relative level of tax enforcement, especially in the situation under which $Y_D(1-\beta)<S_D$.

[TABLE 3]

Next, in Table 4, we have carried out the same exercise, but now in order to detect likely differences in the degree of collaboration. Again, in order $Y_D(1-\beta)>S_D$ becomes an optimal strategy for the taxpayer, the endogenous parameter concerning the personal income tax, α_S, has to be greater than the one concerning the wealth tax, α_Y. This

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progressive tax, we would expect t_P to be higher than 0,002. Then, for example, for t_P=0,005, p^Y+p^S=0,0028!

[36] As we already know, these values seem quite high in comparison with those in force in many countries. However, for the purposes of this section, this is not an important issue, since what we are really interested in is in the relative differences of tax enforcement necessary in order incongruity becomes an optimal strategy for the taxpayer. In any case, note that as long as we set F=4 as Bernasconi (1998) did (see also fn. 33), the tax audit probabilities would be much lower, p^Y=0,0821 and p^S=0,1001 (see Table 1b).

difference has also to be greater, the greater the value of *h*. Curiously, the smaller the value of α_Y, the greater the value of α_S. This result, which might seem counterintuitive, can be easily understood from expression [24]. In this expression, keeping the rest of parameters unchanged, a decrease in α_Y should certainly permit a smaller value of α_S such that the sign of the inequality could still hold. However, note that as long as α_Y decreases, marginal utility in state *A* decreases as well, while marginal utility in state *D* remains unchanged. Thus, the combination of those facts might make necessary an even greater value of α_S. Finally, in Table 4, we can check that the situation $Y_D(1-\beta)<S_D$ is not compatible with differences in the degree of tax enforcement that remain within the boundaries of α_Y, since both for α_S=0,75 and α_S=0,25, the value of α_Y in those cases should be above 1.

[TABLE 4]

In conclusion, the results of these numerical simulations confirm those already obtained in the theoretical analysis. That is, incongruity as an optimal strategy for the taxpayer is only possible as long as the level of tax parameters of each tax is sufficiently different. Moreover, these differences have to be more acute, the more sensitive the tax audit probabilities to the degree of incongruity. From the analysis of the numerical simulations, it is possible to infer that incongruity is much more likely as long as there are differences in the degree of collaboration (which not only produces a substitution effect between states, but also an income effect), since otherwise the differences in the tax audit probabilities probably have to be too sharp in order they can hold in practice. In any case, note that if we focus either on the ratio p^Y/p^S or on the ratio α^S/α^Y, the more likely situation is that one under which $Y_D(1-\beta)>S_D$. Precisely, this is the situation that will be analyzed in the following exercises of comparative statics.

Comparative statics

The exercise of numerical simulation will prove extremely useful in order to analyze the results of the comparative statics in the case of incongruity. However, before that exercise, and although all the signs of the comparative statics are perfectly clear from the theoretical analysis, firstly, in Table 5, we show the results in the case of *Interrelated Evasion* and perfect collaboration such that $Y_D(1-\beta)=S_D$. In the first column, there appear the values of the variable on which the exercise of comparative statics is based; in the second and third column, the equilibrium values of the tax bases declared; in the fourth one, the percentage of tax compliance, while in the fifth column that percentage is expressed in money terms with respect to the amount of money that would be collected in the presence of full tax compliance[37]; finally, in the sixth column, we have calculated expected net income, after paying taxes and, in the presence of tax evasion, the corresponding fine per unit of tax evaded[38].

[TABLE 5]

In fact, the only new results that appear in Table 5 are the comparative statics with

[37] That is, $(((S_D+S_0)\times t_P)+(Y_D\times t_R))/(((S+S_0)\times t_P)+(Y\times t_R))$.

[38] Note that due to $S_0>0$, nothing impedes in some cases net income to be negative.

respect to σ and S_0. In both cases, the sign is also positive. An increase in S_0 produces a reduction in net income in all the states, which given the assumption of decreasing risk aversion forces the taxpayer to increase tax compliance. Obviously, an increase in σ automatically generates the same reaction by the taxpayer.

In the Appendix, we have included the whole set of numerical simulations carried out for a situation under which $Y_D(1-\beta)>S_D$. The structure of the tables is the same than in Table 5, with the exception of two new definitions. First, the ratio between p_S and the "normal" level of p^S, $\bar{p}^S$; and second, the level of incongruity, $((1-\beta)-(S_D/Y_D))/(1-\beta)$.

The values of the basic parameters used in the numerical simulations make that marginal income in states *A* and *D* are always positive. Thus, on the one hand, the income effect detected in the theoretical analysis will always point out in favor of an increase in tax compliance. On the other hand, a substitution effect always had a clear impact in favor of increasing tax compliance with the exception of those cases in which t_R or t_P varied. This latter ambiguity was caused by potential discrepancies in the level of collaboration between tax administrations (see, e.g., fn. 26). In the numerical simulations, we will check to what extent that potential situation can produce a decrease in tax compliance. These basic results apply in the case of congruity. Nonetheless, in the case of incongruity, due to cross-effects between S_D and Y_D, it might be the case that although global tax compliance is increasing, one of both declared tax bases is decreasing. In any case, recall that it also has to be taken into account the effect that occurs through variations in the cost of incongruity, *K*, as long as $h>0$.

As can be checked from tables A.5 and A.4, an increase in *h* or in p^S, respectively, provokes a higher level of unweighted tax compliance through a small reduction in Y_D and an important increase in S_D. In both cases, the cost of incongruity has augmented, calling for an increase in the ratio S_D/Y_D. Moreover, the increase in that cost has occurred through an increase in the effective tax audit probability of the tax administration responsible for the wealth tax, making tax compliance in that tax relatively more attractive. Therefore, both effects point out in the same direction of increasing S_D over Y_D. Obviously, that increase is greater, the greater the value of α_Y.

In Table A.10, it is interesting to analyze the consequences of an increase in S_0. Given that S_0 is fully declared, only an income effect is at work (see fn. 30). Such an income effect causes an increase in the level of tax compliance weighted by the relative importance of the tax burden of each tax (in the table, denoted by $(S_D+Y_D)r$). This increase in the level of tax compliance is achieved by means of a decrease in the ratio S_D/Y_D. Hence, this result will be useful in those comparative static analysis where an income effect arises.

An increase in α_Y (Table A.8) or in α_S (Table A.7) provokes the same effect both on total tax compliance and on each one of the tax bases declared than in the case of an increase in *h* or in p^S. However, the reasoning is not exactly the same than the one given above. On the one hand, an increase in α_Y does not modify the cost of incongruity, since it does not change the value of marginal income neither in state *D* nor in state *E*. However, it certainly makes more attractive increasing S_D with respect to Y_D, while at the same time generates an income effect in favor of increasing total tax compliance (in particular, as we already know that occurs through increases in Y_D). Thus, in this case, a substitution effect between tax bases prevails over an income effect. On the other hand, an increase in α_S makes incongruity more costly, so pointing out in the direction of increasing the ratio S_D/Y_D. But, at the same time, both an income effect and a substitution effect between tax bases point out in the contrary direction. Hence, in this case the increase in the cost of incongruity overcomes the impact of the two latter effects.

In the rest of cases, the comparative statics points out in the direction of increasing Y_D and decreasing S_D. In Table A.3, we can check how an increase in p^Y causes a substitution effect in favor of increasing tax compliance, and in particular decreasing the ratio S_D/Y_D, while the cost of incongruity remains unchanged. The results concerning F are shown in Table A.9. An increase in F intensifies the cost of incongruity, so promoting a rise in the ratio S_D/Y_D. However, a substitution and an income effect in favor of increasing tax compliance dominate in the sense that this increase in tax compliance is achieved through a reduction in the ratio S_D/Y_D.

Finally, an increase in any one of the statutory tax rates generates an income effect that as we know favors a decrease in S_D/Y_D, although total weighted tax compliance increases (tables A.1 and A.2). Moreover, in both cases, the cost of incongruity raises, so promoting an increase in S_D/Y_D. Given that the degree of collaboration between tax administrations is quite similar, we do not expect a substitution effect in favor of decreasing total tax compliance (see expressions [29] and [30]), but only a relative increase in the level of tax compliance of the tax base that has suffered an increase in its tax burden (i.e., a substitution effect between tax bases declared). Therefore, in the particular case of t_R, the decrease in S_D/Y_D is consequence of the domination of an income effect and a substitution effect between tax bases over the increase in the cost of incongruity; while in the case of t_P an income effect prevails over the other two effects.

4. Conclusions

The objective of this paper has been analyzing the consequences of considering the decision over tax evasion as a decision in which interrelated taxes (e.g., at the individual level, personal income tax and wealth tax, but a similar analysis could also be applied to the case of corporations, given the evident relationship between the VAT and the corporate tax) interact with each other, and so the optimal level of tax compliance from the point of view of the taxpayer might differ with respect to those analysis in which tax evasion is considered tax by tax. In particular, the source of interaction comes out from the information available to the tax administration as long as it compares the tax bases declared in each tax return. Given the evident relationship between tax bases (e.g., the increase in the wealth tax base with respect to the previous fiscal year should be a reasonable proportion of current income), that comparison should make evident any incongruity, and so it could be a hint to start a process of auditing. In the paper, such incongruity produces an increase in the auditing probability of that tax which tax base has been supposedly under-declared. However, given the possibility that those interrelated taxes were audited by different tax administrations or within a tax administration by different departments, we have also analyzed those situations under which collaboration between different tax administrations or between different departments of the same tax administration is imperfect. In the extreme case in which collaboration is null, that means that each tax administration might certainly obtain valuable information from comparing tax returns, but it does not make any effort in enforcing tax obligations on behalf of the other tax administration/department when carrying out its own tax audits. On the contrary, perfect collaboration exactly replicates that situation in which there exists just one tax administration or just one department responsible for both taxes.

The idea of interaction between taxes as a means of reinforcing tax compliance is due to Shoup (1969), but it had never been formally developed. Moreover, taking into account the possibility of imperfect collaboration can certainly affect the originally expected results. In the case of perfect collaboration between tax administrations, our theoretical analysis shows that congruity between tax bases is the only optimal decision for the taxpayer, while the results of the comparative statics do not vary from the classical analysis (Allingham and Sandmo, 1972; Yitzhaki, 1974). In that context, as expected, interrelated taxes slightly reinforce with each other making possible to achieve higher levels of tax compliance maintaining the level of tax enforcement constant[39]. However, that result –which has been obtained from an exercise of numerical simulation - might reverse for low levels of collaboration. This type of analysis has also permitted us to shed some new light on the paradox of tax evasion. Also employing the methodology of numerical simulations, we have shown that using values of the penalty per unit of tax evaded equal to those previously used by the literature (Bernasconi, 1998), it is possible to obtain relatively reasonable values of the levels of tax enforcement compatible with reasonable values of tax compliance. Nonetheless, again that result is crucially dependent on the existence of high levels of collaboration between tax administrations. At least, interrelated tax evasion might be considered as a partial explanation to the paradox of tax evasion.

Certainly, the degree of collaboration between tax administrations becomes crucial in the results concerning tax evasion in interrelated taxes. In fact, when collaboration is imperfect, our theoretical analysis has shown that incongruity might be an optimal choice for a rational taxpayer. The direction of the incongruity depends on the relative importance of the tax parameters of each tax. Thus, we should expect relatively lower levels of tax compliance in those taxes which tax parameters (including the tax auditing probability and the level of collaboration of the other tax administration) are relatively less important. For example, in the case of the personal income tax and the wealth tax, we expect the level of tax compliance to be more important in the former tax than in the latter[40].

The results of the comparative statics are also crucially affected by the degree of collaboration. In the case of imperfect collaboration, the theoretical analysis does not provide clear-cut results, and the methodology of numerical simulations becomes fundamental to ascertain its effects. Moreover, this analysis depends on whether congruity or incongruity is the optimal decision of the taxpayer. Our numerical simulations have been done for the probably most interesting case, that is, that one in which incongruity is optimal. As a general result, it is worth mentioning that there is not

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[39] For example, observe in Table 1a, 1b, 2a and 2b that in the presence of perfect collaboration, lower levels of tax enforcement are compatible with equal levels of tax compliance in the situation of interrelated tax evasion and under the classical analysis. Thus, keeping the same level of tax enforcement, in the former situation tax compliance will be greater than in the latter one.

[40] For example, although it does not appear in Table 4, the impossibility of $Y_D(1-\beta)<S_D$ is independent of the level of S_0. That is, even for very big values of S_0, α_Y is still above 1.

any tax policy that promotes tax compliance in both taxes at the same time. Additionally, although as we said in the introduction our aim is not characterizing the optimal policies of a tax administration, it is also interesting to stress that in some occasions the incentives to carry out certain policies by a tax administration are null. For instance, we have obtained that as long as the tax administration responsible for the personal income tax strengthens its collaboration with the other tax administration, the level of tax compliance in that tax decreases, while it increases in the wealth tax (see Table A.8). Thus, we should observe very low levels of collaboration on that tax administration's side. Just the reverse incentives hold for the other tax administration (see Table A.7). Hence, this analysis predicts an asymmetric degree of collaboration, null by part of the tax administration responsible for the personal income tax and maximum by part of the tax administration responsible for the wealth tax. Finally, the presence of imperfect collaboration, contrary to the case of perfect collaboration, might produce negative externalities between tax administrations. For instance, an increase in the statutory tax rate of the personal income tax promotes a higher level of tax compliance in that tax, but at the same time a decrease in the level of tax compliance in the wealth tax. On the whole, all these results call for an integration of all the processes of tax auditing, or as long as the responsibilities of auditing for different taxes are assigned to different layers of government, they call for a high level of mutual collaboration between tax administrations.

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FIGURE 2

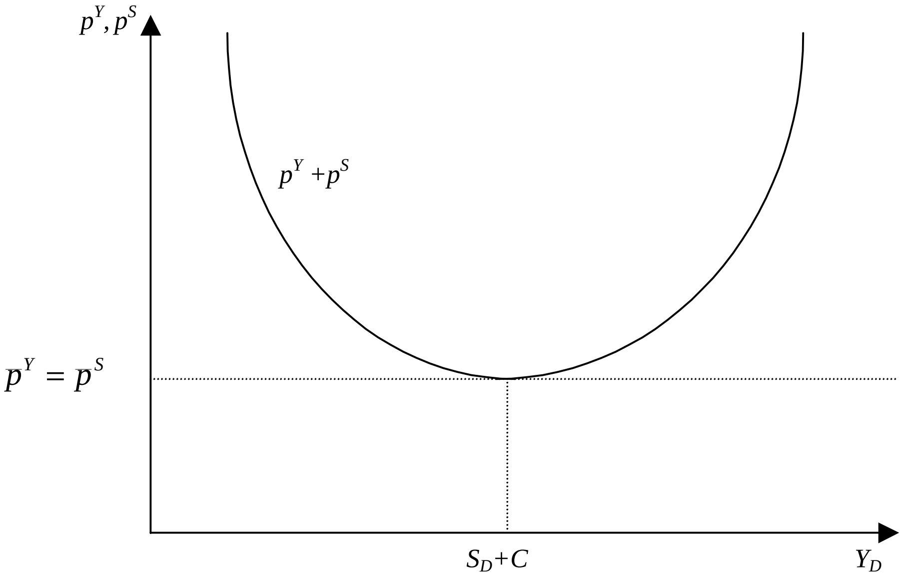

FIGURE 1

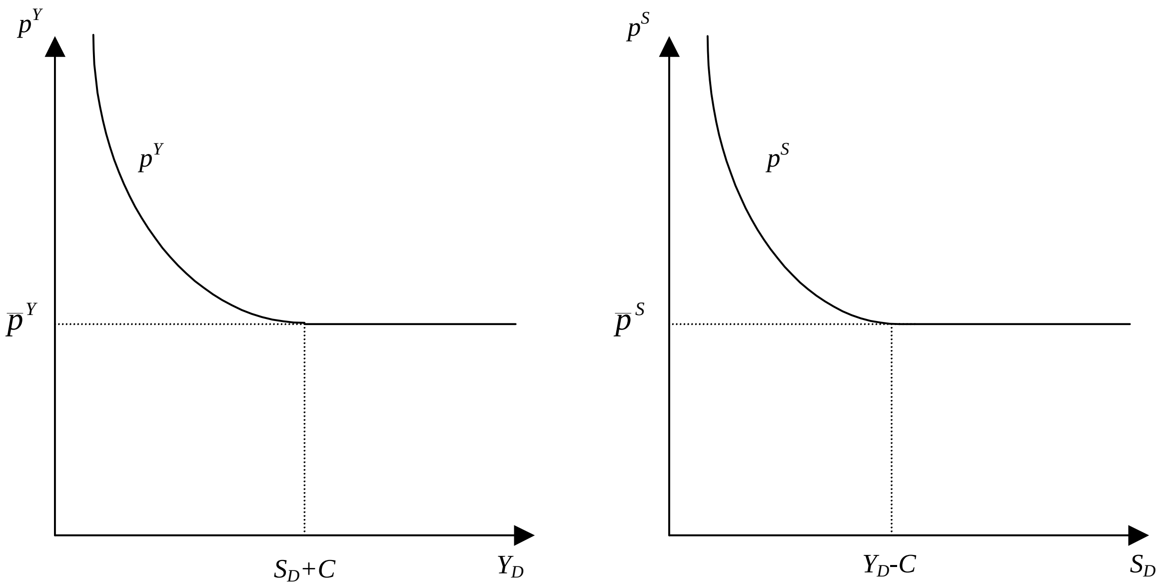

FIGURE 3

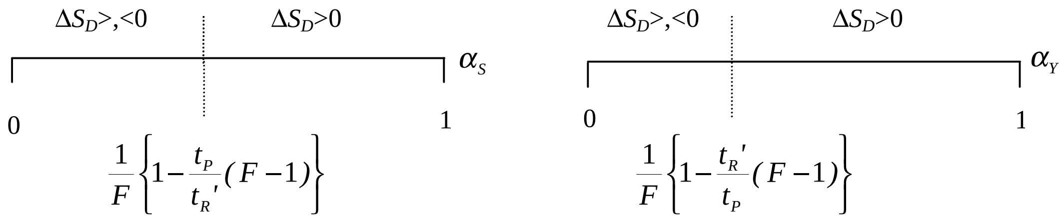
ΔS_D>,<0
ΔS_D>0
α_S
0
1
$\frac{1}{F}\left\{1-\frac{t_P}{t_R'}(F-1)\right\}$
ΔS_D>,<0
ΔS_D>0
α_Y
0
1
$\frac{1}{F}\left\{1-\frac{t_R'}{t_P}(F-1)\right\}$

Table 1a

	Y_D=0,8; S_D=0,16		
	p^Y	p^S	$p^Y \cup p^S$
Classical analysis	0,3252	0,4998	0,6625
Interrelated Evasion			
	p^Y	p^S	$p^Y \cup p^S$
Perfect collaboration	n.a.	n.a.	0,3212
Imperfect collaboration			
	Symmetric Collaboration		
$\alpha_Y=\alpha_S=0$	0,3219	0,6781	1
$\alpha_Y=\alpha_S=0{,}25$	0,2596	0,4664	0,7260
$\alpha_Y=\alpha_S=0{,}5$	0,2163	0,3261	0,5424
$\alpha_Y=\alpha_S=0{,}75$	0,1848	0,2296	0,4144
$\alpha_Y=\alpha_S=0{,}9$	0,1696	0,1855	0,3551
	Asymmetric Collaboration		
$\alpha_Y=1$; $\alpha_S<1$	0,3212	0	0,3212
$\alpha_S=1$; $\alpha_Y<1$	0	0,3212	0,3212

n.a.: non-available

Table 2a

	Y_D=1; S_D=0,2		
	p^Y	p^S	$p^Y \cup p^S$
Classical analysis	0,5	0,5	0,75
Interrelated Evasion			
	p^Y	p^S	$p^Y \cup p^S$
Perfect collaboration	n.a.	n.a.	0,5
Imperfect collaboration			
	Symmetric Collaboration		
$\alpha_Y=\alpha_S=0$	0,5	0,5	1
$\alpha_Y=\alpha_S=0{,}25$	0,4	0,4	0,8
$\alpha_Y=\alpha_S=0{,}5$	0,3331	0,3331	0,6662
$\alpha_Y=\alpha_S=0{,}75$	0,2857	0,2857	0,5714
$\alpha_Y=\alpha_S=0{,}9$	0,2632	0,2632	0,5264
	Asymmetric Collaboration		
$\alpha_Y=1$; $\alpha_S<1$	0,5	0	0,5
$\alpha_S=1$; $\alpha_Y<1$	0	0,5	0,5

n.a.: non-available

PARADOX OF TAX EVASION

(Replica of Bernasconi's analysis: F=4; t_R=0,3; t_P=0,002)

Table 1b

Y_D=0,8; S_D=0,16			
	p^Y	p^S	$p^Y \cup p^S$
Classical analysis	0,1441	0,2499	0,3580
Interrelated Evasion			
	p^Y	p^S	$p^Y \cup p^S$
Perfect collaboration	n.a.	n.a.	0,1434
Imperfect collaboration			
Symmetric Collaboration			
α_Y=α_S=0	0,1435	0,2853	0,4288
α_Y=α_S=0,25	0,1152	0,1973	0,3125
α_Y=α_S=0,5	0,0960	0,1398	0,2358
α_Y=α_S=0,75	0,0821	0,1001	0,1822
α_Y=α_S=0,9	0,0755	0,0820	0,1575
Asymmetric Collaboration			
α_Y=1; α_S<1	0,1434	0	0,1434
α_S=1; α_Y<1	0	0,1434	0,1434

n.a.: non-available

Table 2b

Y_D=1; S_D=0,2			
	p^Y	p^S	$p^Y \cup p^S$
Classical analysis	0,2500	0,2500	0,4375
Interrelated Evasion			
	p^Y	p^S	$p^Y \cup p^S$
Perfect collaboration	n.a.	n.a.	0,25
Imperfect collaboration			
Symmetric Collaboration			
α_Y=α_S=0	0,2500	0,2500	0,5000
α_Y=α_S=0,25	0,2000	0,2000	0,4000
α_Y=α_S=0,5	0,1667	0,1667	0,3334
α_Y=α_S=0,75	0,1429	0,1429	0,2858
α_Y=α_S=0,9	0,1316	0,1316	0,2632
Asymmetric Collaboration			
α_Y=1; α_S<1	0,2500	0	0,2500
α_S=1; α_Y<1	0	0,2500	0,2500

n.a.: non-available

Table 3
(*Benchmark*: p^Y= 0,1848; p^S=0,2296)

$Y_D(1-\beta) > S_D$				
Y_D=0,8; S_D=0,14; α_Y=0,75; α_S=0,75				
	p^Y	p^S	p^{SF}	p^Y/p^{SF}
h=0,5	0,3118	0,0159	0,0160	19,422
h=1,5	0,3181	0,0054	0,0056	56,604
h=10	0,3209	0,0007	0,0008	372,824
Y_D=0,8; S_D=0,12; α_Y=0,75; α_S=0,75				
	p^Y	p^S	p^{SF}	p^Y/p^{SF}
h=0,5	0,3117	0,0157	0,0160	19,426
h=1,5	0,3179	0,0053	0,0056	56,668
h=10	0,3208	0,0006	0,0009	373,249
Y_D=0,8; S_D=0,10; α_Y=0,75; α_S=0,75				
	p^Y	p^S	p^{SF}	p^Y/p^{SF}
h=0,5	0,3116	0,0155	0,0160	19,448
h=1,5	0,3178	0,0051	0,0056	56,732
h=10	0,3207	0,0005	0,0009	373,730
$Y_D(1-\beta) < S_D$				
Y_D=0,7; S_D=0,16; α_Y=0,75; α_S=0,75				
	p^Y	p^{YF}	p^S	p^S/p^{YF}
h=0,5	0,0091	0,0092	0,4590	49,842
h=1,5	0,0031	0,0032	0,4708	146,661
h=10	0,0004	0,0005	0,4762	968,198
Y_D=0,6; S_D=0,16; α_Y=0,75; α_S=0,75				
	p_Y	p_{YF}	p_S	p_S/p_{YF}
h=0,5	0,0062	0,0063	0,4008	63,613
h=1,5	0,0021	0,0022	0,4107	187,425
h=10	0,0002	0,0003	0,4152	1239,826
Y_D=0,5; S_D=0,16; α_Y=0,75; α_S=0,75				
	p^Y	p^{YF}	p^S	p^S/p^{YF}
h=0,5	0,0043	0,0044	0,3442	78,031
h=1,5	0,0014	0,0015	0,3530	229,826
h=10	0,0001	0,0002	0,3571	1520,698

p^{YF} and p^{SF} have been calculated using expressions [40'] and [40], respectively.

Table 4
(*Benchmark*: α_Y=0,75; α_S =0,75)

$Y_D(1-\beta) > S_D$				
	Y_D=0,8; S_D=0,14; p^Y= 0,1848; p^S=0,2296		Y_D=0,8; S_D=0,12; p^Y= 0,1848; p^S=0,2296	
	α_Y=0,75	α_Y=0,25	α_Y=0,75	α_Y=0,25
	α_S	α_S	α_S	α_S
h=0,5	0,7549	0,7555	0,7500	0,7508
h=1,5	0,7657	0,7663	0,7520	0,7527
h=10	0,8919	0,8925	0,8098	0,8105
$Y_D(1-\beta) < S_D$				
	Y_D=0,7; S_D=0,16; p^Y= 0,1848; p^S=0,2296		Y_D=0,6; S_D=0,16; p^Y= 0,1848; p^S=0,2296	
	α_S=0,75	α_S=0,25	α_S=0,75	α_S=0,25
	α_Y	α_Y	α_Y	α_Y
h=0,5	>1	>1	>1	>1
h=1,5	>1	>1	>1	>1
h=10	>1	>1	>1	>1

COMPARATIVE STATICS

Table 5
(*perfect collaboration*)

	Y_D	S_D	S_D+Y_D	$(S_D+Y_D)r$	$E(Y_N)$
			t_R		
0,1675	0,0010	0,0002	0,0010	0,0570	0,8820
0,3000	0,5310	0,1062	0,5309	0,5460	0,8415
0,5000	**0,8000**	**0,1600**	**0,8000**	**0,8039**	**0,5757**
0,6000	0,8674	0,1735	0,8673	0,8695	0,4169
0,7000	0,9155	0,1831	0,9155	0,9167	0,2508
0,9890	1,0000	0,2000	1,0000	1,0000	-0,2505
			t_P		
0,0001	0,7952	0,1590	0,7952	0,7953	0,5875
0,0050	**0,8000**	**0,1600**	**0,8000**	**0,8039**	**0,5757**
0,0100	0,8049	0,1610	0,8049	0,8124	0,5637
0,0500	0,8433	0,1687	0,8433	0,8690	0,4673
0,1500	0,9343	0,1869	0,9343	0,9580	0,2245
0,2273	1,0000	0,2000	1,0000	1,0000	0,0350
			p		
0,0010	0,0651	0,0130	0,0652	0,0834	0,9564
0,0100	0,1650	0,0330	0,1650	0,1813	0,8993
0,0400	0,3091	0,0618	0,3092	0,3227	0,8099
0,0700	0,3988	0,0798	0,3988	0,4105	0,7536
0,1300	0,5277	0,1055	0,5277	0,5369	0,6777
0,5000	1,0000	0,2000	1,0000	1,0000	0,5880
			F		
1,310	0,0217	0,0043	0,0218	0,0409	0,7767
1,500	0,4333	0,0867	0,4333	0,4444	0,6986
2,000	**0,8000**	**0,1600**	**0,8000**	**0,8039**	**0,5757**
2,250	0,8763	0,1753	0,8763	0,8787	0,5267
2,500	0,9273	0,1855	0,9273	0,9287	0,4805
3,113	1,0000	0,2000	1,0000	1,0000	0,3734
			σ		
0,001	0,0240	0,0048	0,0239	0,0278	0,6654
0,751	0,5504	0,1101	0,5503	0,8642	0,6046
1,800	**0,8000**	**0,1600**	**0,8000**	**0,9614**	**0,5757**
3,000	0,8789	0,1758	0,8789	0,9812	0,5666
4,500	0,9190	0,1838	0,9190	0,9891	0,5620
12,000	0,9696	0,1939	0,9696	0,9967	0,5561
			S_0		
0	0,7959	0,1592	0,7959	0,7959	0,5862
5	0,8061	0,1612	0,8062	0,8153	0,5600
10	0,8164	0,1633	0,8163	0,8330	0,5338
20	0,8368	0,1674	0,8368	0,8640	0,4815
60	0,9186	0,1837	0,9186	0,9491	0,2720
99,8	1,0000	0,2000	1,0000	1,0000	0,0636

COMPARATIVE STATICS
(*imperfect collaboration*)

Table A.1.

t_R	S_D	Y_D	S_D+Y_D	$(S_D+Y_D)r$	$p_S\ /\ \bar{p}_S$	*Incongruity*	$E(Y_N)$
				$\alpha_Y = 0{,}73$			
0,41	0,1415	0,7101	0,7097	0,7169	1,0007	0,0035	0,6316
0,46	0,0936	0,7647	0,7153	0,7691	1,0931	0,3881	0,5778
0,50	0,0499	0,8006	0,7088	0,8034	1,1799	0,6887	0,5348
0,54	0,0011	0,8312	0,6935	0,8328	1,2812	0,9937	0,4918
0,57	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0,61	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
				$\alpha_Y = 0{,}75$			
0,41	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0,46	0,1438	0,7641	0,7566	0,7690	1,0135	0,0587	0,5778
0,50	**0,1000**	**0,8000**	**0,7500**	**0,8033**	**1,0942**	**0,3749**	**0,5349**
0,54	0,0511	0,8306	0,7348	0,8327	1,1884	0,6925	0,4919
0,57	0,0037	0,8540	0,7148	0,8550	1,2848	0,9782	0,4543
0,61	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
				$\alpha_Y = 0{,}77$			
0,41	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0,46	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0,50	0,1544	0,7993	0,7948	0,8032	1,0082	0,0340	0,5349
0,54	0,1054	0,8300	0,7795	0,8326	1,0952	0,3653	0,4919
0,57	0,0579	0,8534	0,7594	0,8550	1,1844	0,6609	0,4543
0,61	0,0056	0,8741	0,7332	0,8748	1,2890	0,9679	0,4167

n.s.: no-solution

Table A.2.

t_P	S_D	Y_D	S_D+Y_D	$(S_D+Y_D)r$	$p_S\ /\ \bar{p}_S$	*Incongruity*	$E(Y_N)$
				$\alpha_Y = 0{,}73$			
0,0043	0,1485	0,7987	0,7893	0,8020	1,0170	0,0703	0,5351
0,0046	0,1043	0,7995	0,7532	0,8026	1,0870	0,3476	0,5350
0,0050	0,0512	0,8006	0,7098	0,8034	1,1776	0,6805	0,5348
0,0053	0,0119	0,8014	0,6778	0,8040	1,2492	0,9256	0,5347
0,0058	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0,0063	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
				$\alpha_Y = 0{,}75$			
0,0043	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0,0046	0,1546	0,7989	0,7946	0,8025	1,0078	0,0324	0,5350
0,0050	**0,1000**	**0,8000**	**0,7500**	**0,8033**	**1,0942**	**0,3749**	**0,5349**
0,0053	0,1013	0,8000	0,7511	0,8033	1,0920	0,3667	0,5349
0,0058	0,0620	0,8008	0,7190	0,8039	1,1587	0,6130	0,5347
0,0063	0,0034	0,8022	0,6713	0,8049	1,2657	0,9789	0,5345
				$\alpha_Y = 0{,}77$			
0,0043	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0,0046	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0,0050	0,1557	0,7993	0,7958	0,8032	1,0062	0,0258	0,5349
0,0053	0,1163	0,8001	0,7637	0,8038	1,0678	0,2733	0,5348
0,0058	0,0575	0,8015	0,7158	0,8048	1,1667	0,6413	0,5345
0,0063	0,0038	0,8028	0,6723	0,8058	1,2650	0,9760	0,5343

n.s.: no-solution

Table A.3.

p_Y	S_D	Y_D	S_D+Y_D	$(S_D+Y_D)r$	$p_S\ /\ \bar{p}_S$	*Incongruity*	$E(Y_N)$
				$\alpha_Y = 0{,}73$			
0,288	0,1524	0,7628	0,7627	0,7674	1,0003	0,0011	0,5485
0,302	0,1058	0,7807	0,7387	0,7845	1,0785	0,3226	0,5418
0,318	0,0492	0,8008	0,7083	0,8036	1,1810	0,6926	0,5347
0,330	0,0040	0,8158	0,6832	0,8178	1,2696	0,9754	0,5299
0,343	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0,355	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
				$\alpha_Y = 0{,}75$			
0,288	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0,302	0,1559	0,7801	0,7800	0,7844	1,0001	0,0005	0,5418
0,318	0,0994	0,8002	0,7497	0,8035	1,0953	0,3790	0,5348
0,330	0,0542	0,8151	0,7244	0,8177	1,1774	0,6677	0,5299
0,343	0,0019	0,8312	0,6943	0,8329	1,2796	0,9886	0,5251
0,355	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
				$\alpha_Y = 0{,}77$			
0,288	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0,302	0,1538	0,7995	0,7944	0,8034	1,0092	0,0382	0,5348
0,318	0,1086	0,8145	0,7692	0,8176	1,0849	0,3334	0,5300
0,330	0,0563	0,8305	0,7390	0,8327	1,1790	0,6609	0,5252
0,343	0,0044	0,8452	0,7080	0,8466	1,2802	0,9741	0,5211
0,355	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.

n.s.: no-solution

Table A.4.

p_S	S_D	Y_D	S_D+Y_D	$(S_D+Y_D)r$	$p_S\ /\ \bar{p}_S$	*Incongruity*	$E(Y_N)$
				$\alpha_Y = 0{,}73$			
0,0041	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0,0044	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0,0048	0,0080	0,8009	0,6741	0,8033	1,2563	0,9498	0,5349
0,0051	0,0473	0,8006	0,7066	0,8034	1,1844	0,7046	0,5348
0,0056	0,1079	0,8002	0,7568	0,8036	1,0814	0,3259	0,5347
0,0059	0,1417	0,8000	0,7848	0,8037	1,0279	0,1145	0,5347
				$\alpha_Y = 0{,}75$			
0,0041	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0,0044	0,0017	0,8007	0,6687	0,8030	1,2682	0,9891	0,5350
0,0048	0,0582	0,8003	0,7153	0,8032	1,1652	0,6367	0,5349
0,0051	0,0975	0,8000	0,7479	0,8033	1,0984	0,3909	0,5349
0,0056	0,1581	0,7996	0,7981	0,8035	1,0027	0,0114	0,5348
0,0059	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
				$\alpha_Y = 0{,}77$			
0,0041	0,0102	0,8004	0,6755	0,8028	1,2520	0,9361	0,5351
0,0044	0,0561	0,8000	0,7134	0,8029	1,1687	0,6496	0,5350
0,0048	0,1125	0,7996	0,7602	0,8031	1,0737	0,2964	0,5349
0,0051	0,1519	0,7993	0,7927	0,8032	1,0121	0,0500	0,5349
0,0056	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0,0059	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.

n.s.: no-solution

Table A.5.

h	S_D	Y_D	S_D+Y_D	$(S_D+Y_D)r$	$p_S \,/\, \bar{p}_S$	*Incongruity*	$E(Y_N)$
				$\alpha_Y = 0{,}73$			
1,24	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
1,33	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
1,41	0,0007	0,8012	0,6683	0,8035	1,2523	0,9956	0,5348
1,51	0,0548	0,8005	0,7128	0,8034	1,1723	0,6576	0,5348
1,64	0,1117	0,7998	0,7597	0,8033	1,0823	0,3016	0,5349
1,77	0,1570	0,7992	0,7969	0,8031	1,0050	0,0177	0,5349
				$\alpha_Y = 0{,}75$			
1,24	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
1,33	0,0055	0,8012	0,6722	0,8035	1,2285	0,9659	0,5348
1,41	0,0540	0,8006	0,7121	0,8034	1,1615	0,6630	0,5348
1,51	0,1046	0,7999	0,7538	0,8033	1,0872	0,3459	0,5349
1,64	0,1577	0,7993	0,7975	0,8032	1,0035	0,0134	0,5349
1,77	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
				$\alpha_Y = 0{,}77$			
1,24	0,0058	0,8012	0,6724	0,8035	1,2111	0,9641	0,5348
1,33	0,0666	0,8004	0,7225	0,8034	1,1324	0,5839	0,5348
1,41	0,1117	0,7999	0,7597	0,8033	1,0704	0,3016	0,5349
1,51	0,1587	0,7993	0,7983	0,8032	1,0017	0,0071	0,5349
1,64	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
1,77	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.

n.s.: no-solution

Table A.6.

σ	S_D	Y_D	S_D+Y_D	$(S_D+Y_D)r$	$p_S \,/\, \bar{p}_S$	*Incongruity*	$E(Y_N)$
				$\alpha_Y = 0{,}73$			
1,50	0,1518	0,7605	0,7603	0,7652	1,0005	0,0020	0,5418
1,64	0,1022	0,7809	0,7359	0,7847	1,0843	0,3456	0,5383
1,79	0,0530	0,7995	0,7104	0,8024	1,1739	0,6685	0,5350
1,96	0,0015	0,8172	0,6823	0,8192	1,2749	0,9907	0,5319
2,14	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
2,35	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
				$\alpha_Y = 0{,}75$			
1,50	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
1,64	0,1525	0,7803	0,7773	0,7846	1,0054	0,0230	0,5383
1,79	0,1032	0,7989	0,7517	0,8022	1,0886	0,3543	0,5350
1,96	0,0516	0,8166	0,7235	0,8191	1,1825	0,6843	0,5320
2,14	0,0012	0,8325	0,6948	0,8341	1,2813	0,9927	0,5292
2,35	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
				$\alpha_Y = 0{,}77$			
1,50	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
1,64	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
1,79	0,1576	0,7982	0,7965	0,8021	1,0031	0,0129	0,5351
1,96	0,1059	0,8160	0,7682	0,8190	1,0898	0,3514	0,5320
2,14	0,0554	0,8318	0,7393	0,8340	1,1811	0,6671	0,5292
2,35	0,0013	0,8474	0,7073	0,8487	1,2869	0,9922	0,5265

n.s.: no-solution

Table A.7.

α_S	S_D	Y_D	S_D+Y_D	(S_D+Y_D)r	$p_S\ /\ \bar{p}_S$	*Incongruity*	$E(Y_N)$
				$\alpha_Y = 0{,}73$			
0,62	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0,67	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0,71	0,0064	0,8008	0,6727	0,8032	1,2594	0,9599	0,5349
0,75	0,0499	0,8006	0,7088	0,8034	1,1799	0,6887	0,5348
0,80	0,1024	0,8004	0,7523	0,8037	1,0904	0,3606	0,5347
0,85	0,1531	0,8002	0,7944	0,8040	1,0105	0,0434	0,5346
				$\alpha_Y = 0{,}75$			
0,62	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0,67	0,0113	0,8004	0,6764	0,8029	1,2500	0,9294	0,5350
0,71	0,0564	0,8002	0,7138	0,8031	1,1682	0,6477	0,5349
0,75	**0,1000**	**0,8000**	**0,7500**	**0,8033**	**1,0942**	**0,3749**	**0,5349**
0,80	0,1527	0,7997	0,7938	0,8036	1,0109	0,0450	0,5348
0,85	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
				$\alpha_Y = 0{,}77$			
0,62	0,0064	0,8001	0,6721	0,8025	1,2592	0,9602	0,5351
0,67	0,0653	0,7998	0,7209	0,8028	1,1526	0,5919	0,5350
0,71	0,1106	0,7996	0,7584	0,8030	1,0768	0,3085	0,5350
0,75	0,1544	0,7993	0,7948	0,8032	1,0082	0,0340	0,5349
0,80	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0,85	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.

n.s.: no-solution

Table A.8.

α_Y	S_D	Y_D	S_D+Y_D	(S_D+Y_D)r	$p_S\ /\ \bar{p}_S$	*Incongruity*	$E(Y_N)$
				$\alpha_S = 0{,}73$			
0,700	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0,710	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0,720	0,0047	0,8010	0,6714	0,8034	1,2627	0,9708	0,5348
0,764	0,1159	0,7996	0,7629	0,8031	1,0683	0,2754	0,5349
0,770	0,1327	0,7994	0,7768	0,8031	1,0417	0,1702	0,5349
0,777	0,1528	0,7992	0,7933	0,8031	1,0106	0,0439	0,5349
				$\alpha_S = 0{,}75$			
0,700	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0,710	0,0034	0,8012	0,6704	0,8035	1,2653	0,9791	0,5348
0,720	0,0262	0,8009	0,6893	0,8035	1,2226	0,8366	0,5348
0,764	0,1376	0,7995	0,7809	0,8032	1,0340	0,1394	0,5349
0,770	0,1544	0,7993	0,7948	0,8032	1,0082	0,0340	0,5349
0,777	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
				$\alpha_S = 0{,}77$			
0,700	0,0024	0,8014	0,6698	0,8037	1,2672	0,9850	0,5347
0,710	0,0245	0,8011	0,6880	0,8036	1,2258	0,8472	0,5347
0,720	0,0474	0,8008	0,7068	0,8036	1,1844	0,7043	0,5348
0,764	0,1590	0,7994	0,7987	0,8033	1,0013	0,0055	0,5348
0,770	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0,777	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.

n.s.: no-solution

Table A.9.

F	S_D	Y_D	S_D+Y_D	(S_D+Y_D)r	$p_S\ /\ \bar{p}_S$	*Incongruity*	$E(Y_N)$
				$\alpha_S = 0{,}73$			
1,86	0,1444	0,7373	0,7347	0,7424	1,0046	0,0210	0,5519
1,93	0,0984	0,7712	0,7246	0,7751	1,0874	0,3622	0,5426
2,00	0,0499	0,8006	0,7088	0,8034	1,1799	0,6887	0,5348
2,06	0,0061	0,8229	0,6909	0,8248	1,2683	0,9627	0,5292
2,13	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
2,19	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
				$\alpha_S = 0{,}75$			
1,86	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
1,93	0,1486	0,7706	0,7660	0,7750	1,0083	0,0359	0,5426
2,00	**0,1000**	**0,8000**	**0,7500**	**0,8033**	**1,0942**	**0,3749**	**0,5349**
2,06	0,0563	0,8223	0,7322	0,8247	1,1762	0,6580	0,5292
2,13	0,0025	0,8455	0,7067	0,8469	1,2839	0,9853	0,5236
2,19	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
				$\alpha_S = 0{,}77$			
1,86	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
1,93	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
2,00	0,1544	0,7993	0,7948	0,8032	1,0082	0,0340	0,5349
2,06	0,1106	0,8216	0,7769	0,8246	1,0839	0,3268	0,5292
2,13	0,0568	0,8448	0,7514	0,8468	1,1832	0,6637	0,5236
2,19	0,0081	0,8627	0,7257	0,8638	1,2797	0,9529	0,5196

n.s.: no-solution

Table A.10.

$\boldsymbol{S_0}$	S_D	Y_D	S_D+Y_D	(S_D+Y_D)r	$p_S\ /\ \bar{p}_S$	*Incongruity*	$E(Y_N)$
				$\alpha_S = 0{,}73$			
0	0,0905	0,7958	0,7386	0,7951	1,1085	0,4314	0,5643
5	0,0597	0,8065	0,7218	0,8065	1,1647	0,6299	0,5500
8	0,0404	0,8129	0,7111	0,8135	1,2012	0,7515	0,5402
13	0,0068	0,8236	0,6920	0,8246	1,2673	0,9587	0,5250
16	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
19	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
				$\alpha_S = 0{,}75$			
0	0,1122	0,7957	0,7566	0,7952	1,0729	0,2950	0,5712
5	0,0812	0,8064	0,7397	0,8066	1,1276	0,4965	0,5569
8	0,0618	0,8128	0,7288	0,8137	1,1631	0,6198	0,5470
13	0,0281	0,8235	0,7097	0,8248	1,2274	0,8294	0,5318
16	0,0069	0,8299	0,6973	0,8317	1,2695	0,9584	0,5214
19	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
				$\alpha_S = 0{,}77$			
0	0,1336	0,7956	0,7743	0,7953	1,0391	0,1604	0,5781
5	0,1025	0,8063	0,7573	0,8067	1,0922	0,3644	0,5637
8	0,0830	0,8127	0,7464	0,8138	1,1267	0,4894	0,5538
13	0,0697	0,8170	0,7389	0,8188	1,1509	0,5734	0,5463
16	0,0278	0,8298	0,7147	0,8318	1,2302	0,8325	0,5281
19	0,0058	0,8363	0,7018	0,8387	1,2741	0,9653	0,5174

n.s.: no-solution