Dynamic Enfranchisement

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Abstract

Why would a political elite voluntarily dilute its political power by extending the voting franchise? This paper develops a dynamic recursive framework for studying voter enfranchisement. We specify a class of dynamic games in which political rights evolve over time. Each period, private decisions of citizens co-mingle with government policies to act upon a state variable such as a capital stock, a public good, or the likelihood of an insurrection. Policies are determined by a pivotal decision maker in a potentially restricted franchise. The pivotal decision maker can also delegate decision authority to a new decision maker in the subsequent period. We describe conditions under which an equilibrium of this "dictator delegation game" corresponds to a majority vote decision by the enfranchised group to expand the set of citizens with voting rights. Under these conditions, each period's pivotal decision maker is a median voter who can designate authority to a new median of a larger voting franchise in the next period. We characterize the equilibria by their Euler equations. In certain games, the equilibria generate paths that display a gradual, sometimes uneven history of enfranchisement that is roughly consistent with observed patterns of extensions. Our main result shows that extensions of the franchise occur in a given period *if and only if* the private decisions of the citizenry have a net positive spillover to the dynamic payoff of the current median voter. The size of the extension depends on the size of the spillover. Since the class of games we study can accommodate a number of proposed explanations for franchise extension (e.g., the threat of insurrection, or ideological or class conflict within the elite, etc), the result suggests a common causal mechanism for these seemingly different explanations. We describe a number of parametric environments that correspond to the various explanations, and show how the mechanism works in each.

Keywords: Dynamic games, voter enfranchisement, franchise extension, dictator delegation game

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"There is no more invariable rule in the history of society. The further electoral rights are extended, the greater the need for extending them; for after each concession, the strength of the democracy increases, and its demands increase with its strength."

Alexis de Tocqueville[1]

1 Introduction

Voluntary expansion of political rights by a ruling elite is at first glance paradoxical. The elite, after all, dilutes its power when it extends these rights to others. Yet, significant extensions of the voting franchise took place in Europe throughout the 19th and early 20th centuries. Instances of franchise extensions date back, in fact, much further. The constitutional reforms of Cleisthenes in 508 BC in Athens was arguably an early form of franchise extension.[2] Another early instance occurred in 494 BC, when the patricians in the early Roman Republic conceded the right of the plebs (the "commoners") to participate in the election of magistrates.

This paper examines the determinants of franchise extension. We have two goals in mind. First, rather than describing a specific, stylized model to "explain" the history of voting rights, we propose a general framework in which competing explanations of franchise extention can be usefully evaluated. The few existing models tend to differentiate themselves by whether franchise extensions are modeled as being driven by external conflicts (i.e., the threat of revolution) or whether they result from internal discord (i.e., political competition between members of the elite). We seek a canonical framework that can accommodate, as special cases, the essential elements of most existing models and existing explanations of franchise extension, some of which we review later in Section 2. Second, we seek a model which produces outcomes that are consistent, broadly speaking, with the observations on franchise extensions. Though the "data" of enfranchisement are often hard to interpret, we identify certain tendencies associated with many observed extensions of rights. These observations are also discussed in Section 2.

To address these goals, we specify a class of dynamic games in which the set of eligible voters is endogenously determined each period. Political rights evolve as a fully dynamic recursive phenomenon.

Specifically, we posit a society of n infinitely lived citizens. Each period, private decisions of citizens co-mingle with a government policy to determine the value of an economic state variable in the subsequent period. This economic state, which may be a capital stock, a public good, or the likelihood of an insurrection, evolves according to a simple non-stochastic transition function of the previous period's state, private actions, and the policy decision. The government's policy decision may correspond to a tax, a public expenditure, or a public investment. Each citizen's private decision may affect others. For example, the decision may be labor effort, or savings, or voluntary contributions to a public good, or participation in a popular revolt. These decisions spill

[1] Alexis de Tocqueville, *Democracy in America*, Vol. 1, ch.4

[2] Among other things, these reforms delineated citizenship and allowed for participation in the citizen Assembly. See Fine, (1983).

over to others' payoffs either directly by entering their preferences or indirectly through changes in the state. Payoffs in a given period depend on these decisions and on the current state.

Initially, the voting franchise is restricted — a subset of the citizens has voting rights. These voting rights enable the current franchise to choose the current policy and also to possibly expand voting rights to a larger set of citizens in the subsequent period. Because natural technical issues arise in obtaining well defined majority winning outcomes under endogenous dynamic game payoffs, we formulate this decision problem as follows. Consider an alternative game in which a pivotal decision maker — a dictator — from the currently enfranchised group, makes all the public decisions in the current period. This dictator chooses a current policy and designates a dictator (possibly himself) for the subsequent period. With the possible delegation of decision authority from one individual to another, a complete description of the state in each period is given by the economic state variable and the identity of the current dictator (the "political state" variable). An equilibrium of this *dictator delegation game* is a state-contingent profile of private actions, public policies, and delegation decisions that constitutes a Markov Perfect equilibrium.

We show, under certain conditions, that the equilibrium outcomes of this game correspond to the outcome that would occur if public decisions were determined by a majority-vote. Under these conditions, the majority outcome of the restricted voting franchise corresponds to the preferred outcome of the median voter within that franchise. This median voter, in turn, designates a new median voter of a larger voting franchise in the next period. Consequently, the outcomes correspond to a new franchise decision by the current franchise each period.

This model exhibits three critical characteristics. First, *political rights are explicitly chosen to solve a strategic delegation problem.* Each period's dictator is a median voter who can effectively delegate decision authority to a new median in the next period by changing the set of eligible voters. Because the franchise option is a carefully calibrated instrument in the hands of the currently enfranchised, universal suffrage need not result. A current median may choose to extend the vote to only a subset of the remaining citizens.

Second, *this strategic delegation is recursive.* Since no commitment is attributed to a current franchise extension, an extension of rights is not a once-and-for-all decision. A franchise extension now does not preclude the future enfranchised group from extending even further later on. In particular, because future pivotal voters' extensions are beyond what would be considered ideal from the point of view of the current pivotal voter, his current choice of extension is dampened below that of a once-and-for-all decision. This is an implication of the recursive (infinite horizon) formulation that cannot be captured in, for example, a two period model. The net result is gradualism: extension may be a slow, tedious process by which elites extend to lessor elites, who proceed eventually to elites-in-waiting, and so forth.

Third, *the franchise is an instrumental rather than fundamental objective for each voter.* The model explains little if rights are extended simply because exogenous costs or benefits of the franchise are inserted directly into preferences or technology. Instead, we assume preferences for greater enfranchisement are derived from fundamental preferences about its affect on policies and private decisions of individuals.

We regard these three features as critical to the formulation of a credible theory that can account for the wide heterogeneity of historical enfranchisement (discussed in more detail in Section 2). There are other models, which we also review in Section 2, that satisfy one or, possibly, two of these features. However, we are not aware of other work that satisfies all three.

We show that equilibria in this model may exhibit partial, gradual, and possibly uneven franchise extensions. The unevenness may be due to the particular evolution of the economic state variable, or it may be due to peculiarities in the distribution of heterogeneous citizens. We provide a characterization of equilibrium in terms of the associated Euler equations, each corresponding to a participant's decision problem. These Euler equations are analogous to those in dynamic politico-economic models of policy such as the "Generalized Euler equation" approach of Klein, Krusell, and Ríos Rull (2002). However, Euler equations in the present model contain strategic interaction terms not present in politico-economic models.

In fact, these extra terms are the key to understanding franchise extension. Our main result shows that an extension of the franchise occurs in a given period *if and only if* the private decisions of the citizenry have a net positive (marginal) spillover to the dynamic payoff of the current median voter. The size of the extension depends on the size of the spillover.

Among other things, the result implies that in the absence of private decisions of the citizens, policies alone cannot cause a pivotal decision maker to relinquish power. Private decisions of the citizenry represent an implicit policy-relevant externality that the pivotal decision maker does not control. Because of the dynamic nature of the problem, current "policy-bribes" cannot induce the appropriate effort from the public since they do not guarantee favorable policies in the future.

The franchise extension, however, does offer a guarantee. A change in voting rights places decision authority in the hands of a different pivotal voter in the future. Hence, a franchise extension represents a credible commitment to future policies that are closer to those preferred by the disenfranchised citizens. If this commitment elicits a net positive effort response from other citizens, the pivotal voter is willing to sacrifice his power.

The idea that franchise is a commitment device was also explored in a seminal paper on the franchise by Acemoglu and Robinson (2000). They posit a model in which a ruling elite can choose whether in any period to make a once-and-for-all, universal extension of voting rights to the rest of the population. The motive is to pre-empt a threat of uprising or to resolve a hold-up problem. We refer to this pre-emption motive as the "external conflict" explanation. The external conflict explanation contrasts with an "internal conflict" explanation, an example of which is a recent paper by Lizzeri and Persico (2003). According to the "internal conflict" explanation, rights are extended to gain support in an environment with ideological or class conflict among the elite.

Section 2 discusses similarities and differences between our approach and these and other models of franchise extension. Section 3 describes the basic framework. We show by means of several examples that the class of dynamic enfranchisement problems posited here is broad enough to accommodate both internal and external conflict explanations. The results therefore suggest *a common causal mechanism that underlies both types of rationale.* In Section 4 we characterize equilibria that admit a *first order characterization*, i.e, that satisfy and are fully characterized by interior Euler equations. The main results are described there. Section 5 finds explicit analytical solutions in a number of parameterized environments. These cases illustrate how the model can produce franchise extension paths that are qualitatively consistent with observed political reforms. Section 6 contains concluding remarks. Proofs of the main results are relegated to the Appendix.

2 Three Observations and Two Types of Models

Many of the franchise extensions observed throughout history have common characteristics. There are three qualitative characteristics of observed franchise extensions that the present framework should confront.

(I) Most extensions are *partial* extensions. Historically, ruling elites have not had to choose exclusively between dictatorship and universal suffrage. More often, voting rights are offered to the "adjacent" group in the social hierarchy. Often the restricted franchise was defined by wealth.[3] Finer (1997, p. 336) writes of nascent democracy in the Greek city states:

> "In the earliest forms of restricted participation, that is, in the oligarchies, a property qualification constituted the basis for full citizenship. Later, in some cities, all sources of wealth were put on equality with land, and citizens' rights and duties were gradated according to one's riches."

In the 19th century, England partially expanded along lines of wealth or property ownership as well. However, in Italy, the franchise was granted to citizens who passed certain educational as well as financial criteria in 1849. 19th century Prussia presents an interesting case: in 1849, voting rights were extended to most citizens, but these rights were accorded proportionately to the percentage of taxes paid.[4] Finally, even today in most countries the franchise is usually restricted in some way.[5]

(II) Extensions are typically *gradual* processes, not one shot decisions. England's history bears this out. A brief chronology of 19th and early 20th century franchise extensions in the U.K. indicates a gradual broadening of political rights.[6]

[3] The term *timocracy* was introduced by Aristotle to characterize systems restricted in this way (see http://classics.mit.edu/Aristotle/nicomachaen.html).

[4] The electorate was divided into three groups, each group given equal weight in the voting. The wealthiest individuals who accounted for the first third of taxes paid accounted for 3.5% of the population. The next wealthiest group — the "middle class" — accounted for 10-12% of the population. The rest of the population (about 85%) accounted for the remaining third of the voting power.

[5] In the U.S., convicted felons cannot typically vote, and, until recently, "on-site" registration in some states effectively limits voting rights of the immobile and the mentally ill.

[6] Finer (1997), p. 1638.

1830	Voting franchise restricted to some 2% of population
1832	Reform Act extends franchise to 3.5% of population
1867	Second Reform Act extends to some 7.7% of population
1884	Extension to 15% of population
1918	Universal male (over 21) suffrage and female (over 30) suffrage
1928	Universal suffrage (over 21)

Franchise extension in England had, in fact, a longer history whose beginnings predated these extensions. In a number of other European countries, gradual extensions corresponded to technological innovations such as those of the industrial revolution. In ancient Rome, extensions occurred as the state's boundaries gradually expanded.

(III) Extensions are often *uneven*. In many countries, large delays, lasting decades or longer have occurred between successive extensions. Again, England's chronology is an example. In the Netherlands, voting rights were extended in 1857 from 2% to 14% of the population. The next major expansion occurred in 1894 when rights were extended to all males. In Italy, universal male suffrage in 1912 was preceded by an extension in 1882 (14%) which, in turn, was preceded by the partial extension in 1849. In the ancient Roman Republic, various extensions not associated with territorial expansion occurred in 494 BC, 336 BC, and 287 BC.[7]

Little is known about whether and what types of models can accommodate these three criteria. There is a sizable informal literature in which a number of rationales for the franchise — including the ones discussed here — have been proposed. For this we refer the reader to the useful surveys in Acemoglu and Robinson (AR) (2000) and Lizzeri and Persico (LP) (2003). We concentrate instead on the much sparser formal modeling that has been done, starting with Acemoglu and Robinson's work (2000, 2001), itself.[8] The essential claim in Acemoglu and Robinson's work is that the primary force behind, at least, the 19th century extensions was the desire by the elite to head off social unrest. AR postulate a dynamic game in which the timing of an all-or-nothing franchise extension is determined by the median voter of a ruling elite. A state variable evolves stochastically which determines the rate of success of any popular revolt. In the absence of a franchise decision, the disenfranchised mob, acting as a unitary actor, revolts in certain states of the world, and refrains in others. Redistribution to the disenfranchised is not a credible deterrent since it will only be used in threatening states of the world. By contrast, an extension of voting rights to the entire population puts the decision in the hands of the population median who chooses redistribution in all states. Extensions are then a credible way to buy-off the populace. Hence, franchise extensions pre-empt revolutions.

Similar motives for extending the franchise appear in models by Justman and Gradstein (1999) and Conley and Temimi (2001). They both examine games in which extension of voting rights occurs because of the potential for the disenfranchised group to impose costs on the elite through rioting,

[7] In 336 BC, one of the consulships became available for election by plebians. In 287 BC the Hortensian Law was introduced which gave resolutions in the plebian council the force of law. Again, see Finer (1997).

[8] We limit our attention to models in which franchise decisions are explicitly endogenous. In particular, we acknowledge but do not discuss a large literature that examines the *consequences* of the expansion of rights. To name one example, Husted and Kenny (1997) examine the effect of extensions on the size of government expenditures.

protest, or some other form of alienation if the franchise is not extended. These costs induce a trade-off similar to AR. Expansion entails a loss of decision making power, but it also pre-empts the costly social unrest.[9] In contrast to AR, the Conley and Temimi model is static and so it cannot address dynamic issues. On the other hand it can address the explicit free rider problems (unlike AR) in the decision to revolt. The Justman and Gradstein model operates in an overlapping generations environment and so it can address issues of gradual extension. However, they exogenously assume (rather than derive, as in AR) costs of disenfranchisement.

These "external conflict" models may be contrasted with an alternative "internal conflict" story in which political competition within the elite leads one or another faction to reach out to disenfranchised citizens. Lizzeri and Persico (2003) formulate a game with elements of this story. They examine a static, random voter model of Downsian competition between two candidates who vie for votes among a restricted franchise. The competition creates an inefficiency when there are relatively few eligible voters. A franchise extension, determined by referendum, is shown to lead to a more efficient electoral process in terms of the allocation of expenditure between public goods and private transfers.[10]

In contrast with the aforementioned models, Roberts (1998, 1999) and Barbera, Maschler, and Shalev (2001) examine long horizon dynamic game models with forward looking decision makers who can choose the decision maker(s) in the subsequent period. In this respect, our work is closest to theirs. Both papers examine dynamic games in which a country or an organization can "invite in" desirable outsiders to join the group from abroad. In Barbera, et al., this decision is made unilaterally by any member of society. In Roberts, the decision is made by the median voter as a way to generate endogenous hysteresis in the size of the group. Another innovation of Robert's papers is that it derives well defined majority voting outcomes each period in the dynamic game from single crossing properties on the primitive preferences.[11] The main differences between these papers and ours are, first, that these are essentially models of immigration rather than of franchise extension (since outsiders are not members of society before they enter), and, second, voters in these models have exogenous, rather than derived, preferences over the size or composition of the group.

In the subsequent section, we describe a class of dynamic games that can accommodate many of the key elements of these diverse models. Rather than focusing on one source of conflict (external threat) or another (internal political competition), or assuming exogenous preferences over the franchise, the present model derives such conflicts and preferences. At the same time, we require a rich enough class of environments that can produce dynamic paths of franchise extensions roughly consistent with the aforementioned facts.

[9] Though the rationale for extension is slightly different, Fleck and Hanssen (2003) examine a simple hold-up problem between two actors with the same type of trade-off.

[10] We discuss the relation between their work and the present paper further in Section 5.3.

[11] Because the stage game payoffs in Robert's framework depend on group membership directly, these assumptions cannot be adapted to the present paper.

3 Dynamic Enfranchisement

In this section we first describe a class of dynamic games in which policy decisions are made each period by a dictator, who may then delegate decision authority to another individual, who becomes the dictator in the subsequent period. We refer to these games as *dictator delegation games (DDGs)*. We then show that the equilibrium outcomes of these games correspond to the decisions that would be made in accordance with a simple majority rule in a restricted voting franchise. The delegation of a dictator corresponds to a decision by the currently enfranchised group to alter the franchise. We then provide several examples subsumed by the general framework.

3.1 Dictator Delegation Games

There are n citizens in a society, each labeled $i = 1, ..., n$. Citizens are assumed to differ according to a taste, productivity, or income parameter. These differences induce a natural ordering of citizens which, in turn, coincides with the ordering defined by the index i.[12] The population of all citizens is denoted N.

Time is discrete: $t = 0, 1, 2, \ldots$. At time t, each individual chooses some action e_{it} that describes a private decision taken by citizen i at date t. We let E denote the set of feasible private decisions for each citizen, and denote the vector of private decisions by

$$e_t = (e_{1t}, \ldots, e_{nt}).$$

These decisions may capture any number of activities, including labor effort, savings, or investment activities. They may also include "non-economic" activities such as religious worship. To simplify language, we refer to the decision as simply the *effort choice*.

Also at time t, a policy variable p_t, chosen from some feasible set P. For example, p may be a flat tax rate on income which generates revenue to produce a public good. We assume that policies, whatever they happen to be, are chosen at date t by a single individual, whom we refer to as a *dictator*. Let, $m_t \in N$ be the identity of the dictator at time t, with m_0 an exogenously given initial dictator. As well as choosing the policy variable, in any period the current dictator can also choose the identity of the subsequent period's dictator. That is, m_t chooses m_{t+1} (not necessarily different from m_t).

At each date t, effort and policy choices interact to influence a physical state variable denoted by $\omega_t \in \Omega$. In most of the analysis the state is one-dimensional, i.e., $\Omega \subset \mathbb{R}$. This state may represent a level of capital stock or a stock of natural resource. Alternatively, it could represent aggregate wealth or another moment of the distribution of income, or the strength of an overthrow threat. This physical state ω_t is assumed to evolve according to a transition function Q where

$$\omega_{t+1} = Q(\omega_t, e_t, p_t)$$

and ω_0 is given exogenously in order to begin the process. A complete description of the state of the game at date t is given, then, by (ω_t, m_t).

[12] For example, if citizens differ in exogenous wealth y, then we shall assume $y_i < y_j$ whenever $i > j$.

The payoff to each individual, $i \in N$, is a time separable function,

$$\sum_{t=0}^{\infty} \delta^t \; u_i(\omega_t, \; e_t, \; p_t \;)$$

where δ is a common discount factor, and the stage payoff is u_i. Note that since a citizen's private decision can affect others, his decision may be subject to a "free rider" problem in the sense that under (or over) provision of e_i, relative to some socially optimum benchmark, is likely.

A *Dictator Delegation Game (DDG)*, G, is summarized by the collection

$$G = \langle (u_i)_{i=1}^n, Q, \Omega, E, P, \omega_0, m_0; N \rangle$$

We study DDGs in which $\Omega \subset \mathbb{R}$ and both E and P are compact, convex intervals in $\mathbb{R}_+$, and in which for each i, u_i and Q are twice continuously differentiable and strictly, jointly concave in all variables.

This specification makes two key simplifying assumptions. First, we assume that all decisions are one dimensional. This is clearly made for reasons of tractability. Second, we assume that the dynamic game is deterministic. This could easily be modified to allows for shocks and other stochastic features. In general, one would ordinarily use the language of stochastic games (where Q evolves according to a Markov kernel). The deterministic assumption is made, not so much for tractability, but for ease of illustration. The basic ideas are expressed most directly in the deterministic case.

3.2 Dictator Delegation as Enfranchisement

There is a clear relationship between DDGs and the enfranchisement problem. Under limited enfranchisement, political rights are restricted to a subset of the population $M_t \subseteq N$, whose preferences are aggregated through some political process (e.g., a voting mechanism).[13] By their vote, these citizens have the right to choose current policies. However, they can also choose to *extend* these rights to others in the future. This may be done for a number of reasons, some of which were outlined in the Introduction. Each period, therefore, a currently enfranchised group can choose, along with the policy p_t, a group next period that will have the same rights in period $t+1$. Specifically, citizens in M_t can choose to enfranchise a group M_{t+1} next period.

In DDGs the preferences of a single individual, whom we call the dictator, determine policy outcomes. However, this attribute is true of any pivotal decision making process. In particular, a pivotal decision maker may be, under certain conditions, the median voter arising from a political process in which political rights are voting rights. If m_t is the dictator in a DDG, then ordering the enfranchised citizenry such that $M_t = \{1, 2, \ldots, 2m_t - 1\}$, m_t is also the median index in the enfranchised group, M_t.

If the aggregation procedure were to reflect the preferences of a median voter in a well-defined sense, then it would be natural to think of the current period's median voter as choosing both

[13] We do not model how these rights are enforced or preserved. Of course, the state variable could capture some the technology for of preserving these restricted rights.

current policy, and, by her choice of $M_{t+1} \equiv \{1, 2, \ldots, 2m_{t+1} - 1\}$, the identity of the next period's median voter, m_{t+1}.[14] Indeed, since each period's median voter (if well-defined) is pivotal, we might reasonably expect her choices of curent policy and future median to coincide with the policy and delegation choices she would have made if she had been the dictator in the corresponding DDG.

The problem that we face, of course, is to ensure that such a median voter exists in each period and is well-defined. In what follows we establish conditions under which this is the case. In the examples below, we refer to the pivotal decision maker, which covers both the dictator in a DDG, and a median voter when one exists.

3.3 Examples

In this Section, we show that the class of Dictator Delegation Games (which as discussed above, can be interpreted as franchise extension problems) is broad enough to cover a large number of interesting political/policy examples including environments in which internal or external conflicts exist.

3.3.1 Internal Conflicts over Public Goods

This example captures elements of the "internal conflict" explanation of franchise extension.[15] Tax revenue is again used to invest in an asset, but now it yields a public consumption good. Each citizen holds wealth in the form of land. The land endowment, y_i, of citizen i is exogenous, and it does not vary over time. Aggregate income is $Y = \sum_i y_i$. The policy p_t in period t is a flat tax on land, yielding revenue $p_t Y$. Aggregate individual effort, $\sum_i e_{it}$, instead of augmenting personal incomes, increase the value of the public good next period. That is, at each date t,

$$\omega_{t+1} \ = \ f(p_t Y, \sum_i e_{it}, \omega_t)$$

where ω_{t+1} is the public good produced next period. Finally, citizen i cares about after-tax wealth, about leisure, and about the public good. His payoff in period t is

$$u_i(\omega_t, e_t, p_t) \ = \ u(y_i(1 - p_t), e_{it}, \alpha_i \omega_t)$$

Here, citizens in the population could differ in at least two ways. First, they could differ according to a taste parameter $\alpha_i \in [0, 1]$. Citizens with higher values of α may place higher value of on public good. Examples of this type of conflict include views on of state-supported religion, or the support of certain social policies, such as opposition to scientific theories of evolution, the promotion of liberal attitudes towards race and sexual preference issues, and the enactment and enforcement

[14] Here we ignore integer problems by framing the problem as if $|M_t|$ were always odd. Note also that we restrict the analysis to (weak) franchise *extensions* although, in principle, franchise *contraction* could be permitted.

[15] However, the specifics here are very different from the "internal conflict" explanation in Lizzeri and Persico (LP) (2003). We discuss the differences between LP and the present work at some length in Section 5.3.

of anti-abortion laws. One would expect in this case that preferred tax rates will differ across the population. We refer to cases of taste heterogeneity such as this as cases of *ideological conflict*.

Second, citizens may differ in the amount of land wealth, y_i, they have. We refer to cases of income or wealth heterogeneity as cases of *class conflict*. Class conflict of this type is common in public economics, and can be shown to induce differences in voting behavior regarding redistribution, public goods, and tax policies generally.

3.3.2 The Threat of Insurrection

According to the "external conflict" explanation, franchise expansion occurs to head off the threat of revolution, uprising, or insurrection. Implicitly, such threats arise from the non-satisfaction of the preferences of the disenfranchised by the policies chosen by the elite,[16] and franchise extension may be an effective means of reducing the incentives of agents to engage in uprising.

In this example, a class conflict coupled with the threat of insurrection is the driving force behind a franchise extension. This example is constructed deliberately to be close in certain respects to Acemoglu's and Robinson's model of "threat of revolt" as an explanation for the 19th century extensions.

To simplify things there are two distinct groups, referred to concretely as the nobility (Group A) and the peasantry (Group B), respectively. There are J such peasants, and $n - J$ noblemen. The franchise belongs to a subset of the nobility. A nobleman with index i has a quantity of land y_i where, as before, y_i is constant across time. Each period, a unit of land generates a unit of a consumption good, so that y_i acres generate y_i units of potential consumption. By contrast, peasants are completely disenfranchised and possess no land.

Each period t, there is a possibility that the peasants may successfully revolt and confiscate the nobility's aggregate return, $Y \equiv \sum_{i \in A} y_i$ from land. Each peasant $j = 1, \ldots, J$, contributes e_{jt} toward this effort, while each nobleman $i = J+1, \ldots, n$ contributes effort e_{it} toward suppressing the revolt. As before, effort is costly to all citizens.

Let $E_{At} = \sum_{i \in A} e_{it}$ and $E_{Bt} = \sum_{j \in B} e_{jt}$ denote the aggregate effort by nobility and peasantry, respectively, in period t. The state variable, ω_t, is the probability in period t that the confiscation by the peasants is unsuccessful. Formally,

$$\omega_{t+1} = f(E_{At}, E_{Bt}, \omega_t)$$

so that the success likelihood depends on the aggregate effort of each group, presumably increasing in E_{At} (less likely confiscation) and decreasing in E_{Bt} (more likely confiscation).

If a confiscation is successful, then the entire return Y is expropriated by the peasantry who split it evenly. On the other hand, if the revolt is unsuccessful, then peasants receive a redistributive

[16] One motivation for rebellion that we do not consider is the simple desire to be part of the decision-making process, independent of whether existing decisions are in accordance with a disenfranchised individual's preferences. That is, there is no explicit utility gained from "having the vote".

subsidy chosen by the pivotal decision maker in the restricted franchise before the state revolt's success is known. Roughly, the idea is that redistribution is used to "buy off" the peasants by inducing them to reduce their effort toward the uprising.

Each period t, the pivotal nobleman chooses a redistributive tax rate p_t which produces revenue p_tY. However, the technology for redistribution is concave — implying that some of the revenue is potentially lost in the redistributive process. Formally, revenue p_tY produces $g(p_tY)$ available to be equally distributed to all members of society if there is no confiscation, where g is a concave function.

All citizens have von Neumann Morganstern utility u defined on consumption and effort. Members of the nobility have expected utility in period t of

$$u_{it} \;=\; \omega_t \; u\left((1-p_t)y_i + g(p_tY)/n, \; e_{it}\;\right) \;+ (1-\omega_t)u(0, e_{it})$$

while members of the peasantry have utility

$$u_{jt} \;=\; \omega_t \; u\left(g(p_tY)/n, \; e_{jt}\right) \;+\; (1-\omega_t)\; u\left(Y/J, \; e_{jt}\right)$$

To summarize, individuals in the nobility differ by income, and the policy instrument is a redistributive tax. Individuals can either be supportive of the current policy or they can undermine it. Their current efforts determine the likelihood that the currently enfranchised group remains in power.

3.4 Equilibria of Dictator Delegation Games

Fix a dictator delegation game G. We assume that all citizens condition their behavior only on payoff relevant information. The payoff relevant state at time t is a pair (ω_t, m_t). Here, ω_t is interpreted as the "economic" state while m_t represents the "political" state. Strategies that condition only on the state are commonly referred to as *Markov strategies.* A Markov strategy profile is a triple $\Pi \equiv (\sigma, \psi, \mu)$ where

$$\sigma = (\sigma_1, \ldots, \sigma_n)$$

and $\sigma_i : \Omega \times N \to E$ for each i. Here, $\sigma_i(\omega_t, m_t) = e_{i\,t}$ is the action taken by citizen i when the physical state is ω_t and the current dictator is m_t in period t. Analogously, $\psi : \Omega \times N \to P$ where $\psi(\omega_t, m_t) = p_t$ is the policy chosen by the current dictator m_t when the physical state is ω_t in period t. Finally, $\mu : \Omega \times N \to N$ where $\mu(\omega_t, m_t) = m_{t+1}$ is next period's dictator chosen by the current dictator m_t when the physical state is ω_t.

To summarize, σ is a profile of individual behavioral rules of the citizenry; ψ is the policy rule; μ is the delegation rule. The last two are determined by the dictator in each period.

The payoffs to each individual i of a Markov strategy profile, $\Pi = (\sigma, \psi, \mu)$ in state (ω_t, m_t) can be expressed recursively as

$$V_i(\omega_t, m_t;\ \Pi\ ) \ \equiv\ u_i(\omega_t, \sigma(\omega_t, m_t), \psi(\omega_t, m_t)\ )\ +\ \delta\ V_i(\omega_{t+1}, m_{t+1};\ \Pi\ ) \tag{1}$$

where

$$\omega_{t+1}\ =\ Q(\omega_t, \sigma(\omega_t, m_t), \psi(\omega_t, m_t)\ ) \tag{2}$$

and

$$m_{t+1}\ =\ \mu(\omega_t, m_t) \tag{3}$$

Definition 1 An *equilibrium of a DDG* is a Markov profile, $\Pi = (\sigma, \psi, \mu)$, consisting of state contingent efforts, policies, and delegation choices such that at each date $t = 0, 1, 2, \ldots$, the following hold.

(i) *Optimal effort decisions:* For any state (ω_t, m_t), each i, and each $\hat{\sigma}_i$, and

$$V_i(\omega_t, m_t;\ \Pi\ )\ \geq\ V_i(\omega_t, m_t;\ \hat{\sigma}_i, \sigma_{-i}, \psi, \mu\ )$$

(ii) *Optimal policy and delegation decisions:* For any state (ω_t, m_t) and for any $\hat{e}_{m_t},\ \hat{p}_t$, and $\hat{m}_{t+1}$,

$$V_{m_t}(\omega_t, m_t;\ \Pi\ )\ \geq\ u_{m_t}(\omega_t, \sigma_{-m_t}(\omega_t, m_t), \hat{e}_{m_t}, \hat{p}_t)\ +\ \delta\ V_{m_t}(\hat{\omega}_{t+1}, \hat{m}_{t+1};\ \Pi\ )$$

where $\hat{\omega}_{t+1}\ =\ Q(\omega_t, \sigma_{-m_t}(\omega_t, m_t), \hat{e}_{m_t}, \hat{p})$.

It can be readily verified that an equilibrium of a DDG is a Markov Perfect equilibrium. Each citizen chooses his own effort optimally given the state and his (correct) forecast of others' effort rules and the policy/delegation rules. The dictator chooses policy, effort, and the identity of next period's dictator optimally given the state and his (correct) forecast of the effort rules of the rest of the citizenry. The question of general existence of equilibria is taken up in a companion paper (Lagunoff (2003)). In the present paper, we construct equilibria in a number of parametric examples in Section 5.

Let $\Pi = (\sigma, \psi, \mu)$ be an equilibrium of a dictator delegation game, G. Let $\{m_t^*\}$ be the identities of the dictators along the equilibrium path, and let $\{\omega_t^*\}$ be the equilibrium path of economic states. Finally, define

$$W_i(p_t, m_{t+1}; \omega_t, m_t, \Pi) = u_i(\omega_t, \sigma(\omega_t, m_t), p_t)\ +\ \delta\ V_i(\omega_{t+1}, m_{t+1};\ \Pi\ ) \tag{4}$$

where $\omega_{t+1} = Q(\omega_t, \sigma(\omega_t, m_t), p_t)$, is individual i's payoff starting at date t when all players follow the strategies defined by Π, except that the dictator in period t chooses an arbitrary policy and extension rule (p_t, m_{t+1}).

Definition 2 A strategy profile Π is *rationalized* by franchise extension at date t if there is an ordering of N (without loss of generality, $i = 1, ..., n$), such that $(p, m') \equiv (\psi(\omega_t^*, m_t^*), \mu(\omega_t^*, m_t^*))$, is a Condorcet winner within the set $M_t^* \equiv \{1, 2, ..., 2m_t^* - 1\}$, with $2m_t^* - 1 \leq n$. That is, there does not exist a $(\widehat{p}, \widehat{m}')$ that defeats (p, m') in a strict majority vote in M_t^* when voter preferences are given by $W_i(\cdot, \cdot; \omega_t^*, m_t^*, \Pi)$.

Notice that the definition requires only that states along the equilibrium path produce pivotal decision makers that arise from a majority vote. The interpretation is as follows: each period an enfranchised groups votes to alter the voting institution used in the future. One option among many is to expand the franchise to M_{t+1} in such a way as to produce a new median voter m_{t+1} next period. The strategy profile Π is rationalized precisely when this occurs. Our next task is to establish conditions under which equilibria of DDGs are rationalized by franchise extension - that is, conditions under which a median voter exists. To this end, we make the following definition.

Definition 3 *(Single Crossing)* Fix a strategy profile Π and current state (ω, m). Let $W = (W_1, ..., W_n)$ be a profile of voter preferences defined in (4), and just write $W_i(p_t, m_{t+1})$ (suppressing the notational dependence of W_i on Π and (ω, m)). A profile W satisfies the single crossing property if there exists an ordering of N (without loss of generality, $i = 1, ..., n$), such that for any two pairs (p, m') and $(\widehat{p}, \widehat{m}')$, for each i for which

$$W_i(p, m') - W_i(\widehat{p}, \widehat{m}') > 0, \tag{5}$$

it is the case that either

$$W_j(p, m') - W_j(\widehat{p}, \widehat{m}') > 0 \quad \forall \quad j > i \tag{6}$$

or

$$W_j(p, m') - W_j(\widehat{p}, \widehat{m}') > 0 \quad \forall \quad j < i. \tag{7}$$

Lemma 1 *(Median Voter Theorem) Let M be a set of voters. Suppose $(W_i)_{i \in M}$ satisfies the single crossing property and let m be the identity of the individual with the median index in M. Let (p_m, m'_m) denote individual m's most preferred voting outcome. Then (p_m, m'_m) is a Condorcet winner.*

This Lemma is an immediate consequence of a well known result by Gans and Smart (1996), in which a single crossing property on voter preferences implies existence of a Condorcet Winner that coincides with the individual with the median index, m.[17]. The lemma allows us to infer that if the single crossing property holds at each state along the equilibrium path of an equilibrium, then the equilibrium is rationalized by franchise extension.

The following result shows that when individuals have stage game payoffs that admit an Intermediate Preference representation in sense of Grandmont (1978), then DDEs are rationalized by franchise extension.

[17] See also Roberts (1977) and Rothstein (1990) for virtually equivalent conditions

Proposition 1 *Suppose that in a DDG, stage game preferences can be expressed as*

$$u_i(\omega, e, p) = h(\omega, e, p) f(i) + g(\omega, e, p) \tag{8}$$

where f is monotone. Then any equilibrium $\Pi = (\sigma, \psi, \mu)$ is rationalized by franchise extension at date t if $2m^*_{t-1} \leq n$.

Proof. If, for a given state (ω, m) and strategy profile Π, for each i, $W_i(p, m'; \omega, m, \Pi)$ admits the following of Intermediate Preference representation,

$$W_i(p, m'; \omega, m, \Pi) = H(p, m'; \omega, m, \Pi) F(i) + G(p, m'; \omega, m, \Pi)$$

for some monotone function F, then it can be readily verified that W satisfies the single crossing condition. Suppose then that the stage game utility function u_i satisfies (8). We now show that given a strategy profile $\Pi = (\sigma, \psi, \mu)$, i's dynamic preferences exhibits this decomposition. At time t,

$$\begin{aligned}
& W_i(p_t, m_{t+1}; \omega_t, m_t, \Pi) \\
= \;& u_i(\omega_t, \sigma(\omega_t, m_t), p_t) + \delta V_i(\omega_{t+1}, m_{t+1}; \Pi) \\
= \;& h(\omega_t, \sigma(\omega_t, m_t), p_t) f(i) + g(\omega_t, \sigma(\omega_t, m_t), p_t) \\
& + \delta \left(\sum_{\tau=t+1}^{\infty} \delta^{\tau-1} \left[h(\omega_\tau, \sigma(\omega_\tau, m_\tau), \psi(\omega_\tau, m_\tau)) f(i) + g(\omega_\tau, \sigma(\omega_\tau, m_\tau), \psi(\omega_\tau, m_\tau)) \right] \right) \\
= \;& \left[h(\omega_t, \sigma(\omega_t, m_t), p_t) + \delta \sum_{\tau=t+1}^{\infty} \delta^{\tau-1} h(\omega_\tau, \sigma(\omega_\tau, m_\tau), \psi(\omega_\tau, m_\tau)) \right] f(i) \\
& + \left[g(\omega_t, \sigma(\omega_t, m_t), p_t) + \delta \sum_{\tau=t+1}^{\infty} \delta^{\tau-1} g(\omega_\tau, \sigma(\omega_\tau, m_\tau), \psi(\omega_\tau, m_\tau)) \right] \\
\equiv \;& H(p_t, m_{t+1}; \omega_t, m_t, \Pi) F(i) + G(p_t, m_{t+1}; \omega_t, m_t, \Pi)
\end{aligned}$$

where $F(i) \equiv f(i)$. This gives the desired result. ■

The class of Intermediate preferences is restrictive, although more general conditions under which the single crossing property is satisfied are scarce. On the other hand, in all the parametric environments we study later the preferences satisfy this condition. This means that one need not check explicitly in each state whether a Condorcet winner exists.

3.5 Finite Agents versus the Continuum

We find it necessary to make two further assumptions which deserve comment. First, it will prove more tractable to treat the voter type as chosen from a continuum rather than from a discrete set M. Specifically, let $N \subset [0, 1]$. If the finite set of voters is sufficiently and uniformly dense in the continuum, then the resulting franchise choices constitute an approximation of the actual equilibrium, and outcomes of the DDG game with the continuum are only approximately rationalized by majority voting.

An alternative modeling strategy might have posited a continuum of voters from the beginning. However, the continuum presents a problem. In much of the history of voter enfranchisement, the effort choices of citizens correspond to voluntary decisions in a collective action problem such as volunteering to take part in a protest or public insurrection. But with the continuum, free rider problems in these decisions are extreme. An individual in a continuum would never choose to riot or threaten the status quo, or alternatively, to defend the status quo. The finite agent assumption is therefore critical to prevent the unreasonable boundary solution $e_i = 0$ in effort choices of citizens. Indeed, we later show that for franchise extension to exist, these boundary solutions *must not* occur. To sum up, franchise choices are characterized in the next sections as if the current median could choose the subsequent median from a continuum of types, but in the citizens' private decisions, the finite agent assumption is taken literally.

Second, though we treat the indices m as choice variables for voters, the Markov strategies are actually functions of the *types* of players, rather than their identities. For example, in the class conflict examples, individuals are ordered by wealth, $y_1 \geq y_2 \geq \cdots \geq y_n$. In that case, the strategy $\sigma_i(\omega, m)$ is just notational shorthand for $\sigma_i(\omega, y_m)$. [18]

4 First Order Characterization

We first characterize necessary conditions for an equilibrium assuming differentiability of the value function. Later we establish conditions under which differentiability holds. In all that follows, we drop the time notation, t, and adopt the usual convention in which primes, e.g., ω', are used to denote variables in the subsequent period $t+1$, and double primes, e.g., ω'', used to denote the variable two periods ahead $t+2$.

Let $\Pi = (\sigma, \psi, \mu)$ be an equilibrium of the dictator delegation game. Consider, first, a citizen's effort decision. One can write the recursive payoff evaluated at an equilibrium as the functional equation:

$$V_i(\omega, m;\ \Pi\ ) \ = \ \max_{e_i} \quad \left[u_i(\omega, e_i, \sigma_{-i}(\omega, m), \psi(\omega, m)) \ + \ \delta\ V_i(\omega', \mu(\omega, m);\ \Pi\ )\ \right] \tag{9}$$

subject to $\omega' = Q(\omega, e_i, \sigma_{-i}(\omega, m), \psi(\omega, m)\ )$. If this value function is differentiable, then the (interior) Euler equation is

$$\frac{\partial u_i}{\partial e_i} + \delta \frac{\partial V_i}{\partial \omega'} \frac{\partial Q}{\partial e_i} \ = \ 0 \tag{10}$$

As for the pivotal decision maker's problem, recall that he makes two choices. He chooses a policy in the current period given the state ω. He also chooses next period's pivotal decision maker by making a franchise decision in the current period. That is, a pivotal decision maker with index

[18] Of course, it must be assumed that citizens are sufficiently dense in the type space to justify the assumption that wealth is a continuous variable. Also, if types are not uniformly distributed, then there will be differences between the type contingent strategy, $\sigma_i(\omega, y_m)$, and the strategy $\sigma_i(\omega, m)$ that merely keeps track of player index. In order to examine these explicit distributional considerations, we will use the "type" notation explicitly when parametric examples are examined.

m chooses next period's pivotal decision maker, m'. The functional equation resulting from the dual choice of policy and franchise is

$$V_m(\omega, m;\ \Pi\ ) \ = \ \max_{e_m, p, m'} \ \left[u_m(\omega, e_m, \sigma_{-m}(\omega, m), p) \ + \ \delta\ V_m(\omega', m';\ \Pi\ ) \right]$$

subject to $\omega' = Q(\omega, e_m, \sigma_{-m}(\omega, m), p\ )$. Derived from this value function, the interior Euler equation for the policy decision, p is

$$\frac{\partial u_m}{\partial p} + \delta \frac{\partial V_i}{\partial \omega'} \frac{\partial Q}{\partial p} \ = \ 0 \tag{11}$$

and, the interior Euler equation for franchise decision, m', made by pivotal voter m is

$$\delta \frac{\partial V_m}{\partial m'} \ = \ 0 \tag{12}$$

Definition 4 We will say that an equilibrium, $\Pi = (\sigma, \psi, \mu)$, admits a *first order characterization* if for each citizen i and each voter m, in every state (ω, m), (i) the profile $\Pi = (\sigma, \psi, \mu)$ satisfies the Equations (10), (11), and (12); (ii) the expression in (10) is strictly decreasing in e_i; and (iii) if the matrix of second derivatives of the system formed by (the left-hand sides of) (10), (11), and (12) is negative semi-definite.

Any equilibrium that admits a first order characterization is fully characterized by its Euler equations. Among them, Equation (12), is the most relevant for understanding franchise expansion. Expressed in terms of a useful decomposition of marginal effects, Equation (12) is given by,

$$\begin{aligned}
\frac{\partial V_m}{\partial m'} \ &= \ \overbrace{\left[\frac{\partial u_m}{\partial p'} \ + \ \delta \frac{\partial V_m}{\partial \omega''} \frac{\partial Q}{\partial p'} \right] \frac{\partial \psi}{\partial m'}}^{\text{effect of } m' \text{ on future policy}} \ + \ \overbrace{\delta \frac{\partial V_m}{\partial m''} \frac{\partial \mu}{\partial m'}}^{\text{effect of } m' \text{ on future franchise decision}} \\
&\quad + \ \overbrace{\sum_{j \neq m} \left[\frac{\partial u_m}{\partial e'_j} \ + \ \delta \frac{\partial V_m}{\partial \omega''} \frac{\partial Q}{\partial e'_j} \right] \frac{\partial \sigma_j}{\partial m'}}^{\text{effect of } m' \text{ on future citizen behavior}} \\
&= \ 0
\end{aligned} \tag{13}$$

Clearly, a voter m chooses to expand the current franchise only if (13) is satisfied at values $\mu(\omega, m) > m$. The decomposition illustrates the various marginal effects that a change in the future pivotal voter has on the payoff of the current pivotal voter. This means that the current pivotal voter, m, rationally anticipates his choice of m' on future effort choices of the citizenry, and future policies and franchise decisions of subsequent median voters (including himself, should he choose to retain political power). Among other things, the current median realizes that his choice of franchise expansion may not be the end of the process. Since next period's pivotal voter, m', also satisfies his Euler equations, (11) and (12), if the current pivotal decision maker, m, extends the franchise to $m' > m$, then the Single Crossing Property implies

$$\overbrace{\left[\frac{\partial u_m}{\partial p'} + \delta \frac{\partial V_m}{\partial \omega''}\frac{\partial Q}{\partial p'}\right]\frac{\partial \psi}{\partial m'}}^{\text{effect of } m' \text{ on future policy}} + \overbrace{\delta \frac{\partial V_m}{\partial m''}\frac{\partial \mu}{\partial m'}}^{\text{effect on future franchise decision}} \leq 0 \tag{14}$$

A franchise extension, therefore, implies that the marginal payoff from other citizens' effort responses to the extension be nonnegative, i.e.,

$$\overbrace{\sum_{j \neq m}\left[\frac{\partial u_m}{\partial e'_j} + \delta \frac{\partial V_m}{\partial \omega''}\frac{\partial Q}{\partial e'_j}\right]\frac{\partial \sigma_j}{\partial m'}}^{\text{effect on future citizen behavior}} \geq 0 \tag{15}$$

Hence, an optimal enfranchisement for voter m balances the positive marginal effect from future effort choices (15) from the citizenry with the negative marginal effect of putting future policy and franchise decisions in the hand of other agents (14). This is illustrated by the two solid lines in Figure 1. If the current median voter is m, retaining the franchise results in no loss of control - that is, a zero marginal cost. On the other hand, extending the franchise to a median $\widehat{m}'$ generates maximal benefits associated with effort inducement, but imposes large costs in terms of future policy and franchise decisions. The index m' balances these two effects. In fact, the logic can be extended to obtain the following necessary and sufficient condition for franchise extension.

Proposition 2 *In any equilibrium that admits a first order characterization, the franchise is extended in state (ω, m), i.e., $\mu(\omega, m) = m' > m$, if and only if*

$$\sum_{j \neq m}\left[\frac{\partial u_m}{\partial e'_j} - \left(\frac{\partial u_m/\partial e'_m}{\partial Q/\partial e'_m}\right)\frac{\partial Q}{\partial e'_j}\right]\frac{\partial \sigma_j}{\partial m'} > 0 \tag{16}$$

holds at $m = m'$.

Though the result is a straightforward application of the Envelope Theorem, we include the complete proof in the Appendix. Roughly, the idea is that franchise extension requires the spillover of effort choices of ordinary citizens, without which a current policy maker would preserve his own power to make future policy decisions into perpetuity. This is true regardless of whether the effort choices are directed toward investment in public goods or whether the investment is in political upheaval.

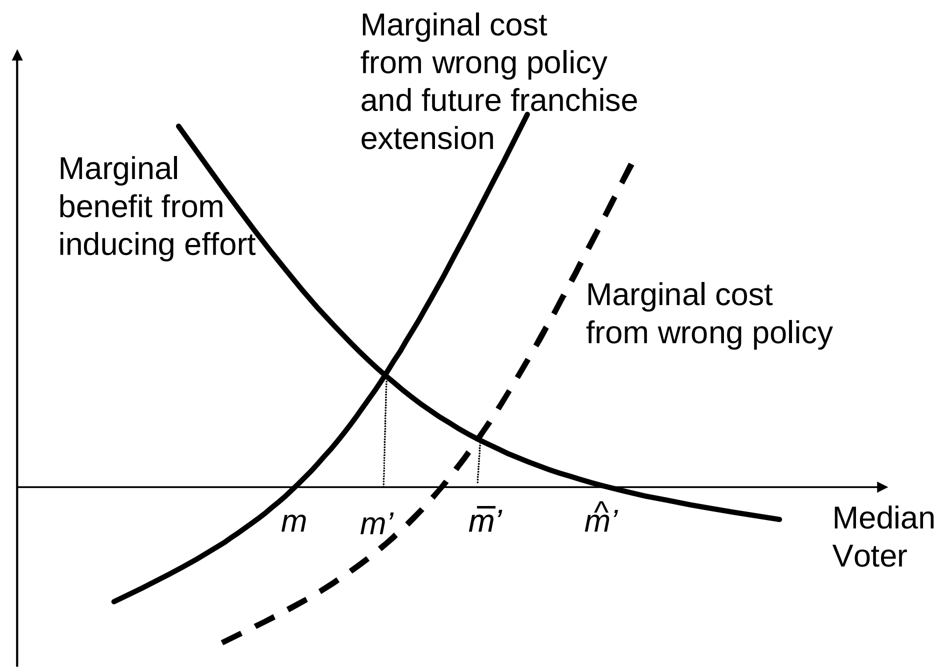

Figure 1: Optimal enfranchisement equates marginal benefits from preferable effort decisions with marginal costs of future policy and franchise distortions

This last point is worth emphasizing. Specifically, *the same causal mechanism underlies both the so-called "internal conflict" and "external conflict" explanations for franchise extension.* In the internal conflict story, disagreements within the elite over public goods create a motive by some to extend voting rights to "sympathetic outsiders." The effort choice is, for instance, a private input needed to produce the controversial good. In the external conflict story, the threat of uprising or insurrection creates a "buy off" motive for expansion of rights. The effort choice, in that case, is one's contribution either to the cause of overturning or to the cause of defending the current regime. In either case, the franchise is extended if and only if the aggregate effect of these spillovers are positive. Presumably, the larger the spillover effect, the larger is the extension.

In the presence of spillovers, a franchise extension can accomplish what a policy change cannot. Namely, the franchise extension is a credible commitment to future policy changes. The pivotal voter cannot credibly use current policy instruments to change future behavior except through (blunt) changes in the physical state. Since current policy changes do not imply future policy changes, citizens with preferences that differ widely from those of the pivotal voter expect the same median voter to continue to produce poor policy choices in the future from their point of view.

By contrast, an extension delegates authority to a different pivotal voter tomorrow. This guarantees that future policies in subsequent periods are closer to those that the current median voter would like to be able to commit to. Since this elicits a positive spillover in their effort choices, the pivotal voter today is willing to sacrifice his power. In this sense, the role of franchise extension is a familiar one in time-consistent models of policy. Extensions are credible since they delegate policy-making authority to a median whose tastes are closer to the large group of citizens.

Notice, however, that while the enfranchisement option may improve things, it is not generally a perfect substitute for the optimal, time inconsistent policy sequence. Given the recursive environment, the initial voter cannot limit future franchise extensions. A future median may delegate beyond the point at which the first median would choose if the first median could make a once-and-for-all franchise decision. In turn, this possibility distorts the current decision. To see this, consider an optimal once-and-for-all extension. A once-and-for-all extension trades off the marginal benefits of extra effort against the marginal costs of future policy changes (the dashed curve) as illustrated in Figure 1. Since these costs do not include the costs of future extensions, the new median is $\overline{m}' > m'$. Since $\overline{m}' > m'$ in Figure 1, the current median limits the extension of the franchise below that of a once-and-for-all decision.

An immediate corollary of the Proposition is: *absent spillovers in private decisions, the level of voter enfranchisement remains fixed.* This statement has predictive content. Consider an example of a policy that subsidizes a particular "state religion." Current subsidies determine, say, the subsequent available stock of churches. Suppose Citizen i's church attendance does not affect others' payoffs, and it does not affect the technology for building churches. In this case, the current median voter will not delegate authority to another. Though conflicts over state-funded religion may, in fact, create serious social conflict, it would not then lead to broader political rights.

Proposition 1 provides a relatively simple way to check if an expansion of the franchise occurs in equilibrium. To make full use of it, however, requires practical use of all the Euler equations, since the Inequality (16) depends on knowing both values of the equilibrium strategies, and their curvature. Consequently, the Euler equations (10)-(12) require a reformulation that depends, to the extent possible, only on the "primitives" of the problem.

Proposition 3 *Let $\Pi = (\sigma, \psi, \mu)$ denote a profile of continuously differentiable Markov strategies such that in every state (ω, m), the values $\sigma(\omega, m)$, $\psi(\omega, m)$, and $\mu(\omega, m)$ lie in the interior of their respective strategy sets. Then Π is an equilibrium that admits a first order characterization if and only if it satisfies the following.*

I. In every state (ω, m), the profile $\Pi = (\sigma, \psi, \mu)$ satisfies:
For median voter $i = m$,

$$G_m^1 \equiv \frac{\partial u_m}{\partial p} - \left(\frac{\partial u_m/\partial e_m}{\partial Q/\partial e_m}\right)\frac{\partial Q}{\partial p} = 0 \tag{E-1}$$

and

$$\begin{aligned} G_m^2 \equiv & \left[{G_m^1}'\right]\frac{\partial \psi}{\partial m'} - \delta\frac{\partial \mu/\partial m'}{\partial \mu/\partial \omega'}\left(\frac{\partial u_m/\partial e_m}{\partial Q/\partial e_m} + \Lambda_m(\omega';\Pi)\right) \\ & + \sum_{j\neq m}\left[\frac{\partial u_m}{\partial e_j'} - \left(\frac{\partial u_m/\partial e_m'}{\partial Q/\partial e_m'}\right)\frac{\partial Q}{\partial e_j'}\right]\frac{\partial \sigma_j}{\partial m'} = 0 \end{aligned} \tag{E-2}$$

where ${G^1_m}'$ *is the left side of Equation (E-1) iterated forward one period, and where*

$$\Lambda_i(\omega;\Pi) \equiv \frac{\partial u_i}{\partial \omega} + \frac{\partial u_i}{\partial p}\frac{\partial \psi}{\partial \omega} + \sum_{j\neq i}\frac{\partial u_i}{\partial e_j}\frac{\partial \sigma_j}{\partial \omega} - \frac{\partial u_i/\partial e_i}{\partial Q/\partial e_i}\left[\frac{\partial Q}{\partial \omega} + \frac{\partial Q}{\partial p}\frac{\partial \psi}{\partial \omega} + \sum_{j\neq i}\frac{\partial Q}{\partial e_j}\frac{\partial \sigma_j}{\partial \omega}\right],$$

and for each citizen i*,*

$$G^3_i \equiv \frac{\partial u_i}{\partial e_i} + \delta\ \Lambda_i(\omega';\Pi)\frac{\partial Q}{\partial e_i} + \delta^2\ \left[G_i^{2\prime}\right]\ \frac{\partial \mu}{\partial \omega'}\frac{\partial Q}{\partial e_i} = 0,\quad \forall i \in N \tag{E-3}$$

II. *The expression* G^3_i *for each* i *is strictly decreasing in* e_i*.*

III. *The matrix of second derivatives of the system formed by* (G^1_m, G^2_m, G^3_m) *is negative semi-definite.*

IV. *For all* i *,if* $G^1_i dp + G^2_i dm' > 0$ *, then either* $G^1_j dp + G^2_j dm' > 0$ *for all* $j > i$ *or* $G^1_j dp + G^2_j dm' > 0$ *either for all* $j < i$ *or for all* $j > i$*.*

Property IV above is a slightly more restrictive version of the single crossing condition adapted to the differentiable case. The Euler equations (E-1), (E-2), and (E-3) are reformulated from the original Euler equations (11), (12), and (10), respectively, in order to eliminate their functional dependence on the value functions. What remains is a collection of n partial differential equations in the strategy profile Π. Klein, Krusell, and Ríos Rull (KKR) (2002) examine properties of similar "Envelope-adjusted" Euler equations in recursively competitive equilibrium models of policy. They refer to these equations as Generalized Euler Equations. As in their reformulation, the Euler equations above differ substantially from those of single agent, dynamic programming problems. Unlike in dynamic programming problems, these Euler equations depend on one's equilibrium decision rules in the future, and on others' equilibrium decision rules in the present and in the future. Hence, they cannot be reduced to pure expressions of primitives as is typical of Euler equations in DP problems.[19] Despite their apparent complexity, the primary virtue of Properties I-IV is that they provide a computationally tractable characterization of equilibria. We make further use of these properties in examples below.

5 Parametric Environments

In this section we examine a series of parametric cases. These cases illustrate how the first order characterization may be used to understand enfranchisement. They also illustrate how enfranchisement

[19] Moreover, the politico-economic and policy models used by KKR and others in the literature are not, strictly speaking, dynamic games since individual behavior is filtered out in those models by the competitive price mechanism. Even without a franchise decision, the Euler equations in the present paper contain a number of extra terms not found in the competitive, "hybrid" models. A related, though simpler (no policy or franchise decisions), version of the Euler equations in (I) also shows up in Basar and Olsder (1995, Theorem 6.5).

may exhibit many of the qualitative features outlined in Section 2. Equilibria that are stationary in the economic state, ω, exist in each of these environments. This means $\partial\psi/\partial\omega = 0$, $\partial\sigma/\partial\omega = 0$, and $\partial\mu/\partial\omega = 0$. Consequently, instead of the Euler Equation, (E-3), in Proposition 3, the Euler Equation for franchise decision becomes:

$$\frac{\partial u_i}{\partial e_i} + \delta\left[\frac{\partial u_i}{\partial \omega'} - \left(\frac{\partial u_i/\partial e_i'}{\partial Q/\partial e_i'}\right)\frac{\partial Q}{\partial \omega'}\right]\frac{\partial Q}{\partial e_i} = 0 \tag{17}$$

Finally, all the parametric examples utilize stage game payoffs that exhibit Intermediate preference representations. Our prior results therefore imply that the equilibria are all rationalized by franchise extension.

5.1 Internal Class Conflict Generates Franchise Extension

Recall the example in Section 3.3.1 of a class conflict over a public good. y_i is each individual's exogenous land endowment (measured in terms of the flow of income). Citizens differ in their endowments. Specifically, the citizens are ordered so that $y_1 \geq y_2 \geq \cdots y_n > 0$. Citizen 1 is the wealthiest while Citizen n is the poorest.[20] The aggregate endowment is $Y = \sum_i y_i$. For simplicity, assume that the population median wealth, $\bar{y}$, is less than the wealth of the initial pivotal voter m_0, that is, $y_{m_0} > \bar{y}$. This means the initial franchise is restricted to individuals who are relatively wealthy. Notice that if $y_{m_0} < y_1$, then internal class conflict exists within the enfranchised elite. Citizens with wealth levels on the outer fringe of the elite may have more in common with their neighbors just below them in the income strata than with other members of the elite.

A flat tax rate p on land is chosen by the median enfranchised voter, the revenue from which finances investment in a public good, ω. The public good fully depreciates each period, and current tax revenues fund next period's public good. Citizens' privately chosen efforts, e_i, augment the productivity of resources devoted to provision of the public good. In particular, the transition law for the public good is

$$\omega' = (pY)^\theta E$$

where $E = \sum_i e_i$ is aggregate effort.

Payoffs each period are given by

$$u_i(\omega, e, p) = y_i(1-p) + \omega - ce_i^2$$

All citizens value the public good the same way, but income heterogeneity induces differences in the way that rich or poor citizens view a tax increase. Since preferences are separable, decision rules do not vary with the current level of the public good. The first order condition gives an individual's effort choice as a function of the tax rate p,

$$e_i = \frac{\delta(pY)^\theta}{2c}.$$

[20] We have "reversed" the ordering so that higher indices correspond to lower wealth classes. This maintains consistency with the earlier notation in which extension proceeds to citizens with higher indices.

As before, the effort and policy rules can be expressed as direct functions of an individual's type, in this case, y_i. The policy rule chosen by the current median voter m, is given by

$$\psi(y_m) \;=\; C\left(\frac{1}{y_m}\right)^{\frac{1}{1-2\theta}}$$

where $C = \left(\frac{\theta n\delta^2 Y^{2\theta}}{2c}\right)^{\frac{1}{1-2\theta}}$ is a positive constant.[21] Effort levels do not vary with one's own land wealth, but policy preferences do. If $\theta < 1/2$, then preferred tax rates decrease in one's own wealth. Wealthier individuals prefer lower taxes. Substituting the policy rule into the effort choices, we derive the effort rule for each individual as

$$\sigma(y_m) \;=\; K\left(\frac{1}{y_m}\right)^{\frac{\theta}{1-2\theta}}. \tag{18}$$

where $K = \frac{\delta Y^\theta}{2c}C^\theta$, another positive constant. Optimal efforts depend only on the identity of the median voter. According to σ, effort choices are decreasing in the wealth of the median voter — wealthier voters indirectly induce lower effort. Since one's effort contributes public capital to the creation of the public good, franchise extension is a mechanism by which a current decision maker can, by delegating his authority, change the level of public capital.

Now observe that $\frac{\partial \sigma_j}{\partial y_m} < 0$ whenever $\frac{\partial \sigma_j}{\partial m} > 0$ since y_m is ordered from highest wealth type to lowest. A franchise extension therefore requires a movement of the median evaluation toward lower, rather than higher, land endowment, y_m. This means that the inequality in Proposition 2 is reversed, using y_i as an individual's type. Hence, from Proposition 2, the equilibrium admits a franchise extension if and only if

$$\sum_{j\neq m}\frac{\partial \sigma_j}{\partial y_m} \;<\; 0 \tag{19}$$

Therefore, differentiating the behavioral rule σ with respect to the state y_m, we see that (19) holds iff $\theta < 1/2$. Hence, if $\theta < 1/2$, then $\mu(y) < y$, for all y, meaning that the franchise is extended to successively lower classes in the income strata. Each extension elicits a higher effort from the citizens. Extension also produces a higher tax rate since taxes and effort are complementary inputs in the production of public goods.

We now turn to the issue of derivation of an equilibrium franchise rule. Formally, the current median delegates to a voter with the land endowment $y_{m'}$, which is chosen to satisfy the Euler equation $\frac{\partial V_m}{\partial y_{m'}} = 0$. Expanding this equation gives

[21] We write $\psi(y_m)$ instead of $\psi(\omega, m)$ as in earlier sections of the paper because (i) the policy choice is independent of the economic state ω, and (ii) the optimal policy depends on the median's income, not his/her index. We could, of course, have written $\psi(\omega, m) \equiv \widetilde{\psi}(y(m))$, where $y(m) = y_m$, but we hope the convenience of the notation we adopt outweighs any small chance of confusion.

$$
\begin{aligned}
\frac{\partial V_m}{\partial y_{m'}} &= \left[\frac{\partial u_m}{\partial p'} - \left(\frac{\partial u_m/\partial e'_m}{\partial Q/\partial e'_m}\right)\frac{\partial Q}{\partial p'}\right]\frac{\partial \psi}{\partial y_{m'}} + \sum_{j\neq m}\left[\frac{\partial u_m}{\partial e'_j} - \left(\frac{\partial u_m/\partial e'_m}{\partial Q/\partial e'_m}\right)\frac{\partial Q}{\partial e'_j}\right]\frac{\partial \sigma_j}{\partial y_{m'}} + \delta\frac{\partial V_m}{\partial y_{m''}}\frac{\partial \mu}{\partial y_{m'}} \\
&= \left[-y_m + \frac{2cK\left(\frac{1}{y'_m}\right)^{\frac{\theta}{1-2\theta}}}{C\left(\frac{1}{y'_m}\right)^{\frac{1}{1-2\theta}}}\left(nK\left(\frac{1}{y'_m}\right)^{\frac{\theta}{1-2\theta}}\right)\theta\right] C\frac{-1}{1-2\theta}\left(\frac{1}{y_{m'}}\right)^{\frac{1}{1-2\theta}+1} \\
&\quad + \sum_{i\neq m} 2c\left[K\left(\frac{1}{y_{m'}}\right)^{\frac{\theta}{1-2\theta}}\right]\left[K\frac{-\theta}{1-2\theta}\left(\frac{1}{y_{m'}}\right)^{\frac{\theta}{1-2\theta}+1}\right] + \delta\frac{\partial V_m}{\partial y_{m''}}\frac{\partial \mu}{\partial y_{m'}} \\
&= \left[-y_m + \theta y_{m'}\right] C\frac{-1}{1-2\theta}\left(\frac{1}{y_{m'}}\right)^{\frac{1}{1-2\theta}+1} \\
&\quad + \sum_{i\neq m} 2c\left[K\left(\frac{1}{y_{m'}}\right)^{\frac{\theta}{1-2\theta}}\right]\left[K\frac{-\theta}{1-2\theta}\left(\frac{1}{y_{m'}}\right)^{\frac{\theta}{1-2\theta}+1}\right] + \delta\frac{\partial V_m}{\partial y_{m''}}\frac{\partial \mu}{\partial y_{m'}} \\
&= C\left[-y_m + \theta y_{m'}\right]\frac{-1}{1-2\theta}\left(\frac{1}{y_{m'}}\right)^{\frac{1}{1-2\theta}+1} + 2c(n-1)K^2\frac{-\theta}{1-2\theta}\left(\frac{1}{y_{m'}}\right)^{\frac{1}{1-2\theta}} + \delta\frac{\partial V_m}{\partial y_{m''}}\frac{\partial \mu}{\partial y_{m'}} \\
&= 0
\end{aligned} \tag{20}
$$

The first equality in (20) follows by definition of the Euler equation (13). The second and third equalities are just algebra. The final equality is the first order condition. By iterating forward (20), we obtain the Euler equation for an initial median voter $m = m_0$:

$$
\begin{aligned}
\frac{\partial V_{m_0}}{\partial y_{m_1}} &= \sum_{t=1}^{\infty}\delta^{t-1}\left\{\left[-y_{m_0} + \theta y_{m_t}\right]\frac{-C}{1-2\theta}\left(\frac{1}{y_{m_t}}\right)^{\frac{1}{1-2\theta}+1} - \frac{2c(n-1)K^2\theta}{1-2\theta}\left(\frac{1}{y_{m_t}}\right)^{\frac{1}{1-2\theta}}\right\}\prod_{\tau=0}^{t-1}\frac{\partial \mu}{\partial m_\tau} \\
&= 0
\end{aligned} \tag{21}
$$

We now verify a "guess" that there exists an equilibrium franchise rule which is linear. Specifically, verify $y_{m'} = \mu y_m$ with $0 < \mu < 1$. Given this form, the Euler equation, (21) can be expressed as

$$
\begin{aligned}
\frac{\partial V_m}{\partial y_{m'}} &= \sum_{t=1}^{\infty}(\delta\mu)^{t-1}\left\{C\left[\theta\mu^t - 1\right]\left(\frac{1}{\mu^t}\right)^{\frac{1}{1-2\theta}+1} + 2c(n-1)K^2\theta\left(\frac{1}{\mu^t}\right)^{\frac{1}{1-2\theta}}\right\} \\
&= 0
\end{aligned} \tag{22}
$$

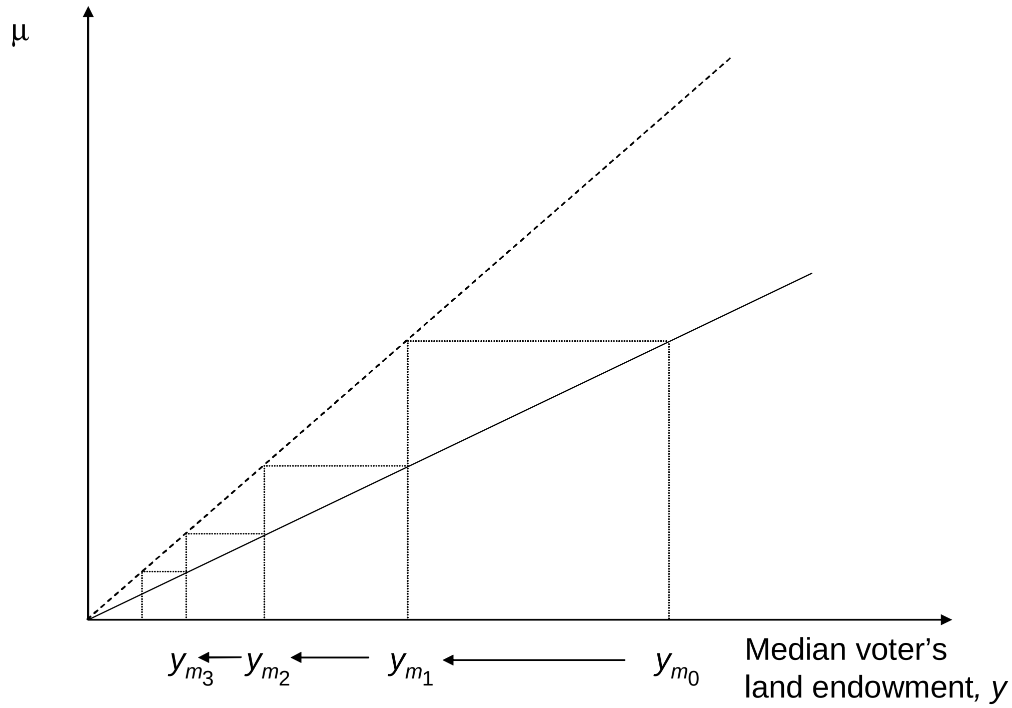

Figure 2: Linear franchise extension rule

Note that Equation (22) no longer depends on the wealth, y_m, of the decision maker. Hence, Equation (22) is an equation in one unknown, namely μ, the proposed coefficient of a linear franchise rule. Using the definitions of C and K, Equation (22) reduces to

$$\sum_{t=1}^{\infty}\left(\frac{\delta}{\mu^{\frac{1}{1-2\theta}}}\right)^t\left\{\left(\theta+\frac{n-1}{n}\right)\mu^t-1\right\} \;=\; 0.$$

As long as $\delta < \mu^{\frac{2\theta}{1-2\theta}}$ (and therefore, $\delta < \mu^{\frac{1}{1-2\theta}}$), this reduces to

$$\left[\mu^{\frac{2\theta}{1-2\theta}}-\delta\right] \;=\; \left[\mu^{\frac{1}{1-2\theta}}-\delta\right]\left(\theta+\frac{n-1}{n}\right) \tag{23}$$

Hence, any $\mu \in (0,1)$ which solves (23) and satisfies $\delta < \mu^{\frac{2\theta}{1-2\theta}}$, is the coefficient in a linear, equilibrium franchise rule. That is, if μ solves (23), then an equilibrium enfranchisement rule is given by $y_{m'} = \mu y_m$. Since the rule is linear, extensions occur until universal suffrage is attained. The linear rule is illustrated in Figure 2.

5.2 Internal Ideological Conflict Generates Partial Franchise Extension

This environment is similar to that of the previous example, except that instead of wealth heterogeneity, citizens differ in ideological views toward a public good. Land holdings are now identical for all citizens, but valuations of the public good differ. Indeed, some citizens may place a negative value on the public good. Thus, there are two groups who have fundamentally conflicting views toward the good, with differing preference intensities within each group. We use this case to compare the "internal" mechanism for extension with that of Lizzeri and Persico (2003), who also model this type of situation.

Tax revenue is used to provide the public good, but the production is augmented (reduced) by individual contributions of positive (negative) effort. Worldly examples of this type of conflict are given in Section 3.

Formally, preferences are given by

$$u_i(\omega, e, p) = y(1-p) - ce_i^2 + \alpha_i \omega$$

where land holdings, y, are the same across all citizens. However α_i, which is a citizen's utility weight on the public good, can vary across individuals. We order the weights so that $\alpha_1 \geq \alpha_2 \geq \cdots \alpha_J > 0 > \alpha_{J+1} \geq \cdots \geq \alpha_n$. Citizens for whom $\alpha_i > 0$ support the provision of the public good, while others suffer a utility loss from it. Assume that $\sum_i \alpha_i > 0$ so that, on balance, positive feeling toward the good is more intense than negative feeling toward it. Assume also that $\alpha_{m_0} > 0$ so that political power initially rests with the "positives." For simplicity, assume that the population median is 0 so that $\alpha_{m_0} > 0$ represents a restricted franchise. Since these individuals may disagree about intensity of preference even if they agree that the public good is a "good," internal conflicts that lead to franchise extension are possible.

Each individual can choose costly effort to either increase or decrease the public good in the subsequent period. Let $e_i \in [-b, b]$ where b is a large enough bound so that interior solutions always exist. Hence if $e_i > 0$ is chosen by i then this individual augments the investment in the public good, whereas if $e_i < 0$, then he exerts effort to resist such investments.

Aggregate effort provision is $E = \sum_i e_i$, and the size of the public good is multiplicative in this effort and tax revenue. Benefits from the public good accrue in the following period, so the law of motion is, as in the previous example,

$$\omega' = (pY)^\theta E$$

where $Y = \sum_i y = ny$ is the aggregate land in this society. The public good is assumed to fully depreciate each period. For reasons that will be clear later on, we assume $1/2 > \theta > 1/4$. If $E > 0$, then tax revenue is used to produce positive amounts of the public good. If, however, $E < 0$, then revenue is used to produce negative amounts of the good. In such a case, a voter with a positive marginal evaluation, $\alpha_m > 0$, would prefer a tax rate of 0.

Once again, franchise extension may represent a mechanism by which a decision maker can commit to a certain tax rate, and the relative strength of preferences now determines whether this is desirable. For example, if aggregate effort is decreasing in the tax rate (opponents of the public good dissent more strongly than supporters), then it may be possible to commit to a lower tax rate and thus increase net effort, by delegating the choice of tax policy to an agent with a smaller value of α.

Suppose, initially that $E > 0$ is forecast. Then an individual's effort choice, as a function of tax p, is given by $e_i = \frac{\delta\alpha_i(pY)^\theta}{2c}$. As expected, this effort is positive iff $\alpha_i > 0$.

As before, all decision rules can be expressed as direct functions of type, α_i. It is not hard to show that the optimal policy rule for positive intensity types is given by

$$\psi(\alpha_m) \;=\; \bar{C}\alpha_m^{\frac{1}{1-2\theta}}$$

where $\bar{C} = \left(\frac{\theta\delta^2 Y^{2\theta}\sum_i \alpha_i}{2cy}\right)^{\frac{1}{1-2\theta}}$ is a positive constant. Substituting the policy rule into the effort choices, we derive the behavioral rule for each individual as

$$\sigma_i(\alpha_m) \;=\; \alpha_i \bar{K}\alpha_m^{\frac{\theta}{1-2\theta}} \tag{24}$$

where $\bar{K} = \frac{\delta Y^\theta}{2c}\bar{C}^\theta$, another positive constant.

Just as in the previous example, types are ordered so that $\frac{\partial\sigma_j}{\partial\alpha_m} < 0$ whenever $\frac{\partial\sigma_j}{\partial m} > 0$. A franchise extension therefore requires a movement of the median evaluation toward lower, rather than higher, weights, α_m. This means that inequality in Proposition 2 is reversed, using α_i as an individual's type. Hence, from Proposition 2, the equilibrium admits a franchise extension if and only if

$$\sum_{j\neq m}\frac{\partial\sigma_j}{\partial\alpha_m} \;<\; 0 \tag{25}$$

Next, observe that $\frac{\partial Q}{\partial e_i} > (<)0$ iff $\alpha_i > 0$ ($\alpha_i < 0$). Therefore, differentiating the behavioral rule (24) with respect to the state α_m, we see that (25) holds iff $\sum_{j\neq m}\alpha_j \;< 0$, or, in other words, $\sum_j \alpha_j \;< \alpha_m$. This implies that extension occurs only if the median lies above the aggregate welfare weight for the public good. But since $\sum_j \alpha_j > 0$ then this means, among other things, that universal suffrage is not achieved — extension stops when $\alpha_m = \sum_j \alpha_j > 0$.

A franchise expansion occurs then if the relative transfer from the "positive" group (less that of the pivotal voter) is outweighed by the relative gain to the "negative" group when taxes are lowered due to a smaller median weight on the public good. Roughly, the idea is that if the dominant group extends the franchise to at least some of the outsiders, then the tax burden is lower. Consequently, the outsiders do not fight as hard to resist taxation. If $\sum_j \alpha_j \;< \alpha_m$ holds, then, evidently, the drop in outsider effort outweighs the drop in insider effort.

This "internally driven" explanation appears to be different from the "internally driven" explanation underlying the model by Lizzeri and Persico (LP) (2003). LP consider a random voting model with two groups of citizens. Preferences over policy are uniform within each group, but individuals differ along an ideological dimension (which correlates with inherent policy-independent support for one of two political parties). In their model, the equilibrium policy choice reflects the relative electoral strengths of ideologically neutral voters in both groups, and is a kind of weighted average of the most preferred policy of each group. Extending the franchise to members of one of the groups (but not the other) can have the effect of shifting the equilibrium in the direction of that group's most preferred policy.[22]

[22] Note that the optimality of such an extension is not automatic, since by expanding the size of the elite, per capita

There are clear differences between LP's set up and ours. Initial members of the elite are, on average, no different to the disenfranchised; policy is determined by party-political competition rather than median voter preferences; and policy choices and franchise extension are determined by different political processes (while in our model they are both determined by the median voter). However, there are some more fundamental similarities: in particular, in both models, the economic outcome of the political process is in general not that which is most preferred by the individual who has the option of extending the franchise (in both models this is the median voter), and by doing so this individual can move the outcome in a desirable direction.

In LP, extension effectively strengthens the current median's voice in the policy decision, but it comes at a cost of diluting his share of redistributive transfers within the elite. In our model, extension has beneficial static efficiency effects, but these are traded off against the costs of loss of control over future decisions.

Unfortunately, an analytical solution to the equilibrium franchise rule in the present model is not tractable. However, if the discount factor δ is low, then an approximate solution is given by the one period Euler equation. Replacing ∞ with $T = 1$ in the infinite horizon Euler equation (see Appendix B), the one period or "terminal" solution is given by μ^* where

$$\alpha_{m'} = \mu^*(\alpha_m) = \alpha_m \left(2 - \frac{\alpha_m}{\sum_j \alpha_j}\right)$$

This solution is consistent with the requirement that $\sum_j \alpha_j \leq \alpha_m$.

5.3 External Conflict Generates Franchise Extension

Here we recall the "external conflict" example in which franchise expansion occurs to head off the threat insurrection. Recall that there is a set A of $n - J$ noblemen and a set B of J peasants. A nobleman i has y_i land which generates y_i units of potential consumption each period. Only noblemen can become voters, while peasants are disenfranchised and possess no land.

Each peasant chooses a contribution e_j to an uprising which, if successful, leads to the confiscation of the nobility's aggregate land rents, $Y \equiv \sum_{i \in A} y_i$. A nobleman i chooses effort e_i to suppress the revolt. As before, effort is costly to all citizens, and $E_A = \sum_{i \in A} e_i$ and $E_B = \sum_{j \in B} e_j$. Finally, the state variable, ω, is the probability in the current period that the confiscation by the peasants is unsuccessful. The initial state is $\omega_0 \in (0, 1)$. Next period's confiscation likelihood depends on current effort and the current state according to

$$\omega' = \begin{cases} 1 - (\frac{E_B - E_A}{n}) - \omega & if \quad 0 < (\frac{E_B - E_A}{n}) + \omega < 1 \\ 0 & if \quad (\frac{E_B - E_A}{n}) + \omega > 1 \\ 1 & if \quad otherwise \end{cases}$$

resources are reduced. However, if initially members of the expanding group were receiving no private transfers, this dilution effect is absent, and they can be made better off by the expansion.

The negative serial dependence means that a promising but in the end unsuccessful uprising in the current period reduces the likelihood of successful insurrection in the subsequent period.[23] A successful confiscation splits the return Y evenly among the peasants.

Each period, the median enfranchised nobleman chooses a tax rate p which produces revenue pY with which the peasants can be "bought off". Redistribution of this tax revenue is both untargeted and inefficient, in the sense that all citizens (noblemen and peasants) receive the same transfer (as long as there is no successful insurrection), and the revenue available is $(pY)^\theta$, with $\theta \in (0, 1)$.

Members of the nobility have stage game utility functions

$$u_i(e_i, p, \omega) \;=\; \omega[(1-p)y_i + \frac{(pY)^\theta}{n}] - ce_i^2,\; i \in A$$

while members of the peasantry have utility

$$u_j \;=\; \omega\frac{(pY)^\theta}{n} \;+\; (1-\omega)\frac{Y}{J} \;-\; ce_j^2,\; j \in B$$

Using the Euler equations in a first order characterization, it can be shown, once again, that behavior and policy rules are invariant to the physical state, ω. The rules are given by

$$\psi(y_m) = \left(\frac{\theta Y^\theta}{n y_m}\right)^{\frac{1}{1-\theta}}$$

for policy, and

$$\sigma_i(y_m) = \frac{1}{2cn}(\frac{\delta}{1+\delta})\left[y_i + (\frac{1}{\theta}y_m - y_i)\left(\frac{\theta Y^\theta}{n y_m}\right)^{\frac{1}{1-\theta}}\right],\; i \in A$$

and

$$\sigma_j(y_m) = \frac{1}{2cn}(\frac{\delta}{1+\delta})\left[\frac{Y}{J} - \frac{1}{\theta}y_m\left(\frac{\theta Y^\theta}{n y_m}\right)^{\frac{1}{1-\theta}}\right],\; j \in B$$

for the efforts of noblemen and peasants, respectively.

Using the same techniques as before, it is not hard to show that the franchise is extended by median income voter y_m iff

$$\sum_{i \in A,\, i \neq m} \frac{\partial \sigma_i}{\partial y_m} \;<\; \sum_{j \in B} \frac{\partial \sigma_j}{\partial y_m} \tag{26}$$

Using the expressions for effort rules above, it is not hard to show that (26) is equivalent to $y_m > Y/n$. Hence, a landowner with endowment y_m extends the franchise iff his land value is larger than average.

Despite the stationarity of the equilibrium, its analytical solution is again not tractable. However, it is instructive to see how the franchise decision affects the trajectory of the state — the likelihood

[23] The idea is that "close calls" today lead to better deterrence tomorrow. Conversely, confiscation is more likely when one's guard is down.

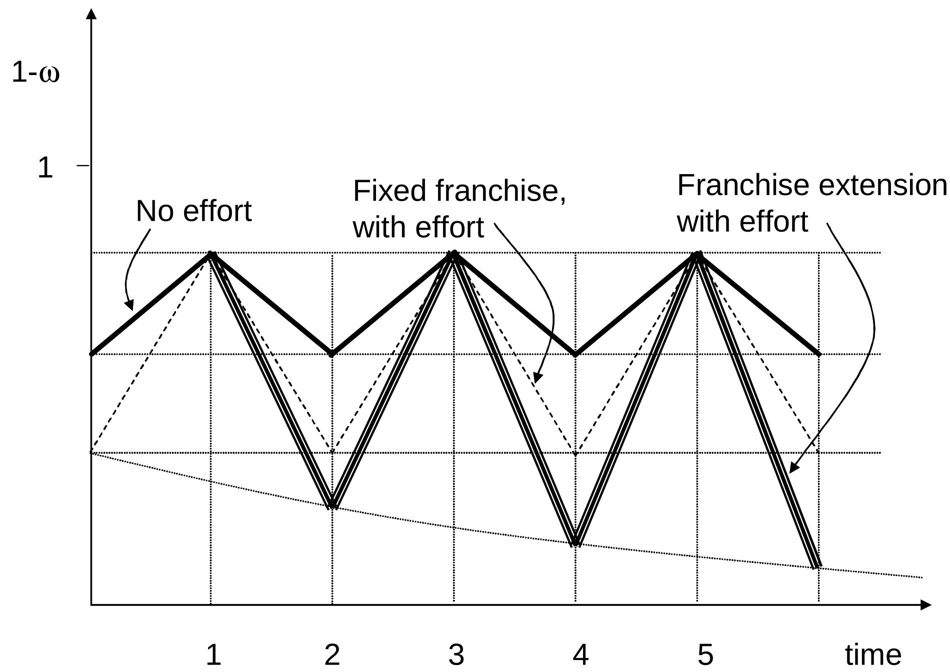

Figure 3: Path of insurrection likelihoods with and without a one-time franchise extension

of insurrection — by changing the private effort of citizens. Figure 3 illustrates the effect of repeated extensions. The extension effectively lowers the success rate of an insurrection. When authority is given to a lower income nobleman, the low income nobleman chooses a larger redistributive tax. This induces a relatively greater effort toward the defense of the status quo compared to the effort directed toward its demise. The "buy-off" is therefore successful.

6 Summary

This paper introduces a class of games in which the voter franchise is an explicit voting decision of the currently enfranchised group. This decision is formulated as a fully recursive delegation decision, and preferences for enfranchisement options are derived rather than assumed. We know of no other model with these features.

We characterize equilibria of a related game in which a current dictator may designate a new dictator whose policy decisions are absolute. We show that under certain conditions, the outcome path produced by this game may be rationalized by a well defined majority voting rule operating on a limited voting franchise. This enfranchised group votes for a possibly larger voting franchise in the next period. The outcome of a vote in any period is shown to coincide with the preferred choice of a median voter from that group.

The current median voter is motivated by a desire to change the policy-relevant private decisions of ordinary citizens. The franchise extension is therefore used as a commitment device to change private behavior through irreversible expansions of the policy-making elite, which induce credible changes in future policy choices. This underlying causal mechanism is at the heart of both "internal" and "external" explanations of observed franchise extensions.

The assumption that political aggregation occurs via a simple majority vote clearly omits some important subtleties of actual political processes. It also requires restrictive conditions in the present multi-dimensional policy space. Nevertheless, its use in this context is as good, in our view, as any alternative. Consider, for example, the influential citizen-candidate model (see Besley and Coate (1997) or Osborne and Slivinsky (1996)). While that model can be applied to multi-dimensional policy spaces, it typically requires burdensome mixed strategies in precisely those cases where majority voting is problematic. In either model, the fundamental mechanism for institutional change is the same.

Our framework is shown to cover a variety of policy environments. However, some caveats apply. The present environment is deterministic and assumes simple, single dimensional policies and private decisions. Naturally, the framework can be extended to include environments with higher dimensional policies and private decisions.

The framework can also be extended to stochastic games. The extension to stochastic environments is important because, it turns out, most existence results either do not apply to deterministic environments, or apply only when all sets of states, policies, and actions are finite.[24] These issues and others are taken up in a companion paper, Lagunoff (2003).

Future research might be directed toward computational methods for generating equilibria with franchise extension. It is hoped that a broader comprehension of the dynamic game model of enfranchisement leads to a deeper understanding of the mechanisms that sustain and extend democracy.

7 Appendix A: Proofs of the Propositions

Proof of Proposition 1 Let Π admit a first order characterization. If the current median voter, m, chooses to keep the current franchise, i.e., if $\mu(\omega, m) = m' = m$, then the Envelope Theorem implies:

$$\delta \frac{\partial V_m}{\partial m''} = 0 \text{ and } \frac{\partial u_m}{\partial p'} + \delta \frac{\partial V_m}{\partial \omega''_m} \frac{\partial Q}{\partial p'} = 0$$

Next, since the citizen's Euler equation, (10), must hold each period and in each state, we obtain

$$\frac{\partial V_i}{\partial \omega''} = -\frac{\partial u_i / \partial e'_i}{\delta \; \partial Q / \partial e'_i} \tag{27}$$

[24] In which case existence of equilibrium is in mixed strategies.

Substituting these three equations in the franchise Euler equation, (13), if Π admits a franchise extension then (16) must hold at $m' = m$. To obtain the converse, observe that since Π admits a first order characterization then equation (13) is decreasing, and so if (16) holds at $m' = m$, then by the Envelope Theorem, the solution to (13) entails a choice $m' > m$. ∎

Proof of Proposition 2 First we show (E-1)-(E-3) are equivalent to the original Euler equations, (10)-(12). The techniques for showing this are fairly standard. We differentiate the value function in Equation (9) with respect to the state using the Envelope Theorem wherever possible. This gives

$$\frac{\partial V_i}{\partial \omega} = \frac{\partial u_i}{\partial \omega} + \frac{\partial u_i}{\partial p}\frac{\partial \psi}{\partial \omega} + \sum_{j \neq i} \frac{\partial u_i}{\partial e_j}\frac{\partial \sigma_j}{\partial \omega} + \delta \frac{\partial V_i}{\partial \omega'}\left[\frac{\partial Q}{\partial \omega} + \frac{\partial Q}{\partial p}\frac{\partial \psi}{\partial \omega} + \sum_{j \neq i} \frac{\partial Q}{\partial e_j}\frac{\partial \sigma_j}{\partial \omega}\right] + \delta \frac{\partial V_i}{\partial m'}\frac{\partial \mu}{\partial \omega} \quad (28)$$

From the citizen's Euler equation, (10), we obtain

$$\frac{\partial V_i}{\partial \omega'} = -\frac{\partial u_i / \partial e_i}{\delta\ \partial Q / \partial e_i} \quad (29)$$

Then, substitute (29) for $\frac{\partial V_i}{\partial \omega'}$ in the expression (28) and iterate $\frac{\partial V_i}{\partial \omega}$ one period forward to obtain

$$\begin{aligned} \frac{\partial V_i}{\partial \omega'} &= \frac{\partial u_i}{\partial \omega'} + \frac{\partial u_i}{\partial p'}\frac{\partial \psi}{\partial \omega'} + \sum_{j \neq i} \frac{\partial u_i}{\partial e'_j}\frac{\partial \sigma_j}{\partial \omega'} - \frac{\partial u_i / \partial e'_i}{\partial Q / \partial e'_i}\left[\frac{\partial Q}{\partial \omega'} + \frac{\partial Q}{\partial p'}\frac{\partial \psi}{\partial \omega'} + \sum_{j \neq i} \frac{\partial Q}{\partial e'_j}\frac{\partial \sigma_j}{\partial \omega'}\right] + \delta \frac{\partial V_i}{\partial m''}\frac{\partial \mu}{\partial \omega'} \\ &\equiv \Lambda_i(\omega'; \Pi) + \delta \frac{\partial V_i}{\partial m''}\frac{\partial \mu}{\partial \omega'} \end{aligned} \quad (30)$$

where Λ_i is defined in the statement of the Proposition.

Now substitute (30) into the original first order condition (10) to obtain

$$\frac{\partial u_i}{\partial e_i} + \delta \left[\Lambda_i(\omega'; \Pi) + \delta \frac{\partial V_i}{\partial m''}\frac{\partial \mu}{\partial \omega'}\right]\frac{\partial Q}{\partial e_i} = 0 \quad (31)$$

Using (31), we solve for $\delta \frac{\partial V_m}{\partial m''}$ to obtain

$$\delta \frac{\partial V_m}{\partial m''} = -\frac{1}{\partial \mu / \partial \omega'}\left(\frac{1}{\delta}\frac{\partial u_m / \partial e_m}{\partial Q / \partial e_m} + \Lambda_m(\omega'; \Pi)\right) \quad (32)$$

Equations (30)-(32) can now be used to obtain the adjusted Euler equations for the pivotal voter's policy and franchise decision, and all citizens' effort decisions, resp. To this end, write the pivotal voter m's Euler equation for the optimal policy choice p as

$$\frac{\partial u_m}{\partial p} + \delta \left[\Lambda_m(\omega'; \Pi) + \delta \frac{\partial V_m}{\partial m''}\frac{\partial \mu}{\partial \omega'}\right]\frac{\partial Q}{\partial p} = 0 \quad (33)$$

Then substitute (32) in place of $\frac{\partial V_m}{\partial m''}$ in (33) to obtain (E-1), the Euler equation for the pivotal voter's policy. Next, recall the Euler equation for the franchise decision expressed in terms of its explicit decomposition of effects, Equation (13). Using Equation (29) to substitute for $\frac{\partial V_m}{\partial \omega'}$ and Equation (32) to substitute for $\frac{\partial V_m}{\partial m''}$ we rewrite Equation (13) to obtain (E-2), the Euler equation for the franchise decision. Finally, for any ordinary citizen i, we iterate the left side of (E-2) one period forward. This term should then be substituted in place of $\frac{\partial V_i}{\partial m''}$ in equation (31) to obtain (E-3).

Consequently, a continuously differentiable, interior profile $\Pi = (\sigma, \mu, \psi)$ satisfies properties I-III if and only iff these same properties apply to the original Euler equations, (10)-(12). But these are the conditions for which there exists an equilibrium that admits a first order characterization, save for the single crossing property, (ii). As stated earlier, Property IV is equivalent to the single crossing property when profiles are differentiable. ∎

8 Appendix B: Equilibrium with Internal Ideological Conflict

As in the case of class conflict, we derive the an Euler equation for franchise extension, which is

$$\begin{aligned}
\frac{\partial V_m}{\partial \alpha_{m'}} &= \left[\frac{\partial u_m}{\partial p'} - \left(\frac{\partial u_m/\partial e'_m}{\partial Q/\partial e'_m}\right)\frac{\partial Q}{\partial p'}\right]\frac{\partial \psi}{\partial m'} + \sum_{j\neq m}\left[\frac{\partial u_m}{\partial e'_j} - \left(\frac{\partial u_m/\partial e'_m}{\partial Q/\partial e'_m}\right)\frac{\partial Q}{\partial e'_j}\right]\frac{\partial \sigma_j}{\partial m'} + \delta\frac{\partial V_m}{\partial \alpha_{m''}}\frac{\partial \mu}{\partial \alpha_{m'}} \\
&= \left[-y + y\frac{\alpha_m}{\alpha_{m'}}\right]\frac{\bar{C}}{1-2\theta}\alpha_{m'}^{\frac{2\theta}{1-2\theta}} + \alpha_{m'}^{\frac{\theta}{1-2\theta}}\left[\sum_{i\neq m}\alpha_i\bar{K}\frac{\theta 2c\alpha_m\bar{K}}{1-2\theta}\alpha_{m'}^{\frac{\theta}{1-2\theta}-1}\right] + \delta\frac{\partial V_m}{\partial \alpha_{m''}}\frac{\partial \mu}{\partial \alpha_{m'}} \\
&= \left[\frac{\alpha_m}{\alpha_{m'}} - 1\right]\frac{y\bar{C}}{1-2\theta}\alpha_{m'}^{\frac{2\theta}{1-2\theta}} + \alpha_m\left[\frac{\theta 2c\bar{K}^2}{1-2\theta}\sum_{i\neq m}\alpha_i\right]\alpha_{m'}^{\frac{2\theta}{1-2\theta}-1} + \delta\frac{\partial V_m}{\partial \alpha_{m''}}\frac{\partial \mu}{\partial \alpha_{m'}} = 0
\end{aligned} \tag{34}$$

Recall that since $\theta > 1/4$ and $\sum_{i\neq m}\alpha_i < 0$, then $\frac{\partial V_m}{\partial \alpha_{m'}}$ is decreasing in $\alpha_{m'}$, and so the solution to (34) is a maximizer. The optimal franchise extension is, therefore, the median voter, m', that solves (34). By iterating forward (34), the infinite horizon Euler equation for an initial median voter $m = m_0$ is

$$\begin{aligned}\frac{\partial V_{m_0}}{\partial \alpha_{m_1}} &= \sum_{t=1}^{\infty} \delta^{t-1} \left\{ \left[\frac{\alpha_{m_0}}{\alpha_{m_t}} - 1 \right] \frac{y\bar{C}}{1-2\theta} \alpha_{m_t}^{\frac{2\theta}{1-2\theta}} + \alpha_{m_0} \left[\frac{2c\bar{K}^2\theta}{1-2\theta} \sum_{i \neq m_0} \alpha_i \right] \alpha_{m_t}^{\frac{2\theta}{1-2\theta}-1} \right\} \prod_{\tau=0}^{t-1} \frac{\partial \mu}{\partial \alpha_{m_\tau}} \\ &= \sum_{t=1}^{\infty} \delta^{t-1} \left\{ \left[\frac{\alpha_{m_0}}{\alpha_{m_t}} - 1 \right] \alpha_{m_t}^{\frac{2\theta}{1-2\theta}} + \frac{\alpha_{m_0}}{\alpha_{m_t}} \left[\theta(\sum_{i \neq m_0} \alpha_i / \sum_j \alpha_j \right] \alpha_{m_t}^{\frac{2\theta}{1-2\theta}} \right\} \prod_{\tau=0}^{t-1} \frac{\partial \mu}{\partial \alpha_{m_\tau}} \\ &= \sum_{t=1}^{\infty} \delta^{t-1} \left\{ \frac{\alpha_{m_0}}{\alpha_{m_t}} \alpha_{m_t}^{\frac{2\theta}{1-2\theta}} \left(1 + \theta \frac{\Sigma - \alpha_{m_0}}{\Sigma} \right) - \alpha_{m_t}^{\frac{2\theta}{1-2\theta}} \right\} \prod_{\tau=0}^{t-1} \frac{\partial \mu}{\partial \alpha_{m_\tau}} \\ &= 0 \end{aligned} \tag{35}$$

where the third equality comes from the fact that $\sum_{i \neq m_0} \alpha_i = \Sigma - \alpha_{m_0}$ where we define $\Sigma \equiv \sum_j \alpha_j$. While this expression does not give tractable analytical solution, we do find an approximate solution when δ is small by finding the solution to the truncated game when $T = 1$.[25]

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[25] Extending this one period problem, we tried to find the infinite horizon rule by taking limits of the time-dependent rules derived in a T period, truncated game. While this approach probably feasible computationally, it is not feasible analytically. For instance, even in a *two* period game, the initial period's decision rule is the solution to

$$\alpha_{m'}^{\frac{2\theta}{1-2\theta}} \left[\frac{\alpha_m(1+\theta) - \frac{\theta}{\Sigma}\alpha_m^2}{\alpha_{m'}} - 1 \right] = \delta \left(\alpha_{m'}(1+\theta) - \frac{\theta}{\Sigma}\alpha_{m'}^2 \right)^{\frac{2\theta}{1-2\theta}} \left[1 - \frac{\alpha_m(1+\theta) - \frac{\theta}{\Sigma}\alpha_m^2}{\alpha_{m'}(1+\theta) - \frac{\theta}{\Sigma}\alpha_{m'}^2} \right] \left[(1+\theta) - 2\frac{\theta}{\Sigma}\alpha_{m'} \right]$$

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