A Liberal Paradox for Judgment Aggregation

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In the emerging literature on judgment (as opposed to preference) aggregation, expert rights or liberal rights have not been investigated yet. When a group forms collective beliefs, it may assign experts with special knowledge on certain propositions the right to determine the collective judgment on those propositions; and, when a group forms collective goals or desires, it may assign individuals specially affected by certain propositions similar rights on those propositions. We identify a problem similar to, but more general than, Sen's 'liberal paradox': Under plausible conditions, the assignment of such rights to two or more individuals (or subgroups) is inconsistent with the unanimity principle, whereby propositions accepted by all individuals must be collectively accepted. So a group respecting expert or liberal rights on certain propositions must sometimes overrule its unanimous judgments on others. The inconsistency does not arise if *either* different individuals' rights are 'disconnected' *or* individuals are 'agnostic/tolerant' or 'deferring/empathetic' towards other individuals' rights. Our findings have implications for the design of mechanisms by which groups (societies, committees, expert panels, organizations) can reach decisions on systems of interconnected propositions.

1 Introduction

Suppose the individual members of a group each make judgments on some interconnected propositions. For instance, three members of a committee make judgments on the following three propositions:

a : Carbon dioxide emissions are above the threshold x.

b : There will be global warming.

$a \to b$: If carbon dioxide emissions are above the threshold x, then there will be global warming.

One individual accepts a, $a \to b$ and b; another accepts a, but rejects both $a \to b$ and b; a third accepts $a \to b$, but rejects both a and b. How can the group aggregate these individual judgments into corresponding collective judgments? This problem – 'judgment aggregation' – is non-trivial, as our example illustrates. A majority accepts a, a majority accepts $a \to b$, and yet a majority rejects b, an inconsistent set of judgments. So 'propositionwise majority voting' – one possible aggregation method – does not guarantee a consistent collective set of judgments. Drawing on earlier examples of such inconsistencies in multi-member court decisions (Kornhauser and Sager 1986), Pettit (2001a) has argued that groups making collective judgments

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on multiple propositions often face a trade-off between majoritarian responsiveness to the individual judgments on every proposition and collective consistency.

Judgment aggregation allows several interpretations. On a 'belief' interpretation, as in the example above, the set of propositions accepted by an individual or a group is interpreted as the set of propositions *believed* by the individual or group to be true. Under a 'desire' interpretation, as illustrated further below, it is interpreted as the set of propositions *desired* to be true. Several recent papers have developed general models of judgment aggregation, proved some impossibility results (List and Pettit 2002; Pauly and van Hees 2003; Dietrich 2003) and possibility results (Bovens and Rabinowicz 2004; List 2003, 2004a; Dietrich 2003), and explored the parallels and disanalogies to the more familiar problem of preference aggregation in the tradition of Condorcet and Arrow (Chapman 1998, 2002; Brennan 2001; Ferejohn 2003; List and Pettit 2004).[2]

In this paper, we address an important aspect that has not been investigated yet. Sometimes a group assigns special rights to particular individuals or subgroups to be decisive on certain propositions. When a group forms collective beliefs (i.e. determines what propositions it considers true), a particular individual or subgroup may have special knowledge or expertise on some proposition, and the group may therefore assign to that individual or subgroup the right to determine the collective judgment on that proposition (an *expert right*). For instance, legislatures or committees may grant expert rights to certain subcommittees to draw on their specialization and expertise, or to achieve a division of labour. When a group forms collective goals or desires (i.e. determines what propositions it wants to make true by collective action), a particular individual or subgroup may be specially affected by the judgment on some proposition, such as when the proposition concerns an individual's private sphere or a minority group's condition. Here, too, the group may assign to the individual or subgroup the right to determine the collective judgment on that proposition (a *liberal right*).

What does the assignment of expert rights or liberal rights imply for judgment aggregation? We prove a result similar in spirit to Sen's famous 'liberal paradox' on preference aggregation, the result that certain liberal rights conflict with the Pareto principle (Sen 1970), which has recently received renewed interest among political scientists (e.g. Dowding and van Hees 2003; Pettit 2001b, 2003). Specifically, we prove that, in judgment aggregation, under plausible conditions, the assignment of expert rights or liberal rights to two or more individuals (or subgroups) (on one proposition each) is inconsistent with another plausible principle, namely that, if *all* individuals *unanimously* accept some proposition, then that proposition should also be collectively accepted. Therefore a group respecting the expert rights or liberal rights of two or more of its members (or subgroups) on certain propositions may sometimes have to overrule its *unanimous* judgments on other propositions.[3] For example, a legislature or committee that assigns expert rights to two or more subcommittees on certain issues may be unable to respect its unanimous judgments, at plenary sessions, on other

[2] The problem of judgment aggregation is also formally related to the problem of aggregating multiple truth-functions into a single one in logic, and merging multiple overlapping knowledge bases in formal epistemology and computer science.

[3] Since the rights-bearers are members of the group themselves, the unanimous judgments that may have to be overruled are shared by those rights-bearers too.

issues – although those judgments are endorsed even by the expert subcommitees themselves.

The problem identified here is distinct from Sen's original result, as it concerns the aggregation of judgments over multiple interconnected propositions rather than preferences and therefore sheds light on a different class of decision problems. In particular, our results highlight a problem that committees and other collective organizations with expert members or specialist subcommittees may face when making collective decisions on complex systems of propositions. The formal proofs of our results are stated in the appendix.

2 Two examples

We present two examples, one on the belief interpretation of judgment aggregation and the other on the desire interpretation. The first example is inspired by an example given in Pauly and van Hees (2003), the second by Sen's original 'liberal paradox' example, though both examples are somewhat different from these earlier ones. Also, neither of our examples can be formalized in a preference aggregation framework; in particular, logical connections of the form $a \to b$ or $l \to p$ require a propositional logic representation.

Example 1: expert rights. Suppose a committee has to make judgments on the factual propositions a, $a \to b$ and b, as in the global warming example above. Half of the committee members are experts on a, the other half experts on $a \to b$. So the committee assigns to the first half the right to determine the collective judgment on a and to the second a similar right on $a \to b$. The committee's constitution further stipulates that unanimous judgments within the committee must be respected. Now suppose that all the experts on a judge a to be true, and all the experts on $a \to b$ judge $a \to b$ to be true. By the expert rights, the committee accepts both a and $a \to b$. We may therefore expect that the committee also accepts b. However, when a vote is taken on b, *all* committee members reject b. How can this happen? Table 1 shows the committee members' judgments on all propositions.

	a	$a \to b$	b
Experts on a	True	False	False
Experts on $a \to b$	False	True	False

Table 1: A paradox of expert rights

The experts on a accept a, but reject $a \to b$ and b. The experts on $a \to b$ accept $a \to b$, but reject a and b. So all committee members are individually consistent. Nonetheless, respecting the rights of the a-experts on a and of the $a \to b$-experts on $a \to b$ is inconsistent with respecting the committee's unanimous judgment on b. To achieve consistency, the committee must either restrict the expert rights or overrule its unanimous judgment on b.

Example 2: liberal rights. Imagine a society of two individuals, Lewd and Prude, who each have a personal copy of the book *Lady Chatterley's Lover*.[4] Consider three propositions:

l : Lewd reads the book.

p : Prude reads the book.

$l \rightarrow p$: If Lewd reads the book, then so does Prude.

Lewd desires not only to read the book himself, but also that, if he reads it, then Prude should read it too, as he anticipates that his own pleasure of reading the book will be enhanced by the thought of Prude finding the book offensive. Prude, by contrast, desires *not* to read the book, and also that Lewd should not read it either, as he fears that the book might corrupt Lewd's moral outlook. However, in the event of Lewd reading to book, Prude desires to read it too, so as to be informed about the dangerous material Lewd is exposed to. Table 2 shows Lewd's and Prude's desires on the propositions.

	l	p	$l \rightarrow p$
Lewd	True	True	True
Prude	False	False	True

Table 2: A paradox of liberal rights

Society assigns to each individual the liberal right to determine the collective desire on those propositions that concern only the individual's private sphere. Since l and p are such propositions for Lewd and Prude, respectively, society assigns to Lewd the right to determine the collective desire on l, and to Prude a similar right on p. Further, according to society's constitution, unanimous desires of all individuals must be collectively respected. But now, by Lewd's liberal right on l, l is collectively accepted; by Prude's liberal right on p, p is collectively rejected; and yet, by unanimity, $l \rightarrow p$ is collectively accepted – an inconsistent collective set of desires. To achieve consistency, society must either restrict the liberal rights of the individuals or relax its constitutional principle of respecting unanimous desires.

3 The model

In this section we introduce the main components of our model of judgment aggregation (similar to other judgment aggregation models in the literature, e.g. List 2003, 2004a, Pauly and van Hees 2003).

The individuals. We consider a group of individuals, labelled 1, 2, ..., n ($n \geq 2$).

The propositions. The propositions on which the group makes judgments are represented in standard propositional logic, **L**, with connectives $\neg$ ('not'), $\wedge$ ('and'), $\vee$ ('or'), $\rightarrow$ ('implies'), $\leftrightarrow$ ('if and only if').[5] Propositions are generally denoted p, q,

[4] While in Sen's original example there is only one copy of the book – to be borrowed and read by at most one individual – in our example there are two copies; so the book may be read by both individuals, or just by one, or by none.

[5] Formally, the set of all propositions in **L** is defined by three rules. Let there be a set of symbols a, b, c, ... called *propositional letters.* (i) The letters a, b, c, ... are each propositions (*atomic propositions*). (ii) If p and q are propositions, then so are $\neg p$, $(p \wedge q)$, $(p \vee q)$, $(p \rightarrow q)$, $(p \leftrightarrow q)$

r, ... A *truth-value assignment* is a function assigning the value 'true' or 'false' to each proposition in $\mathbf{L}$, with standard properties.[6] A set of propositions S is *consistent* if there exists a truth-value assignment under which all propositions in S are true; S *entails* a proposition p (written $S \vDash p$) if, for every truth-value assignment under which all propositions in S are true, p is also true.

The agenda. Now the *agenda* is a non-empty subset $X \subseteq \mathbf{L}$, interpreted as the set of propositions on which judgments are to be made. For simplicity, we assume that X does not contain any double-negated propositions ($\neg\neg p$), and X consists of proposition-negation pairs in the following sense: If $p \in X$, then also $\sim p \in X$, where

$$\sim p := \begin{cases} \neg p & \text{if } p \text{ is not of the negated form } \neg q, \\ q & \text{if } p \text{ is of the negated form } \neg q. \end{cases}$$

We also assume that X contains no tautologies or contradictions (so the propositions are 'open').[7] In example 1 above, the agenda is $X = \{a, b, a \to b, \neg a, \neg b, \neg(a \to b)\}$; in example 2, it is $X = \{l, p, l \to p, \neg l, \neg p, \neg(l \to p)\}$.

Connectedness. As indicated above, judgment aggregation is made non-trivial by the interconnections between the propositions. We here assume that the propositions in the agenda are connected in the following technical sense. Two propositions $p, q \in X$ are *connected* (in X) if there exists a minimal inconsistent subset $Y \subseteq X$ that contains p or $\sim p$ and contains q or $\sim q$.[8] Now the agenda X is *connected* if any two propositions $p, q \in X$ are connected.

The agendas in examples 1 and 2 above are each connected. For example, propositions l and p in the agenda of example 2 are connected as they are contained in the minimal inconsistent subset $\{l, p, \neg(l \to p)\}$. But if the proposition $l \to p$ (and its negation) were removed from that agenda, then l and p (and hence the agenda) would no longer be connected. Several other agendas discussed in the literature on judgment aggregation are also connected, including the agendas of conjunctive or

(*compound propositions*). (iii) There are no other propositions. For notational simplicity, we always drop the external ()-brackets around a proposition.

[6]For any $p,q \in \mathbf{L}$, $\neg p$ is true if and only if p is false; $(p \wedge q)$ is true if and only if both p and q are true; $(p \vee q)$ is true if and only if at least one of p or q is true; $(p \to q)$ is true if and only if it is not the case that [p is true and q is false]; $(p \leftrightarrow q)$ is true if and only if p and q are both true or both false.

[7]In fact, our impossibility theorems below require only that the agenda contains at least one proposition that is neither a tautology nor a contradiction, and our possiblity theorems require no such assumption at all. However, in the impossibility theorems the so modified agenda assumption is only apearently weaker. These theorems also assume the agenda to be *connected* (see below), in which case the existence of a single non-tautological and non-contradictory proposition implies that X contains *no* tautology (and hence no contradiction, since X contains proposition-negation pairs). To prove this claim, assume for contradiction that $p \in X$ is a tautology. We show that X contains only tautologies or contradictions, violating the existence of a non-tautological and non-contradictory member of X. Let $q \in X$. Since X is connected, there exists a minimal inconsistent set $Y \subseteq X$ and propositions $p^* \in \{p, \sim p\}$ and $q^* \in \{q, \sim q\}$ such that $p^*, q^* \in Y$. Since p is a tautology, p^* is either a tautology or a contradiction. But p^* cannot be a tautology, since otherwise Y would not be minimal inconsistent. So p^* is a contradiction. This implies $Y = \{p^*\}$, since otherwise Y would not be minimal inconsistnet. Hence $q^* = p^*$. So q^* is a contradiction, so that q is either a tautology or a contradiction.

[8]A set S is *minimal inconsistent* if S is inconsistent and every proper subset of S is consistent.

disjunctive decision problems as in the so-called 'doctrinal paradox' or 'discursive dilemma' (Kornhauser and Sager 1986, Pettit 2001a).

Characterizing connectedness. Connectedness can be characterized in terms of logical dependence. For any propositions p, q and any set of propositions Z, we say that

- p *entails* q given Z (written $p \vDash_Z q$) if $\{p\} \cup Z \vDash q$;
- p and q are *(logically) dependent* if $p \vDash q$ or $p \vDash \neg q$ or $q \vDash p$ or $q \vDash \neg p$;
- p and q are *(logically) dependent given* Z if $p \vDash_Z q$ or $p \vDash_Z \neg q$ or $q \vDash_Z p$ or $q \vDash_Z \neg p$.

Proposition 1. Two propositions $p, q \in X$ are connected (in X) if and only if p, q are logically dependent given some subset $Z \subseteq X$ consistent with each of $p, q, \sim p, \sim q$.

So the agenda X is connected if and only if any two propositions $p, q \in X$ are logically dependent given some subset $Z \subseteq X$ consistent with each of $p, q, \sim p, \sim q$. In the agenda of example 2, propositions l and p are connected as they are logically dependent given $Z = \{l \rightarrow p\}$.

Individual judgment sets. Each individual i's *judgment set* is a subset $A_i \subseteq X$. On a belief interpretation, A_i is the set of propositions believed by individual i to be true; on a desire interpretation, A_i is the set of propositions desired by individual i to be true. As a shorthand, we read $p \in A_i$ as 'individual i accepts proposition p'. A judgment set A_i is *consistent* if A_i is a consistent set of propositions, as defined above; A_i is *complete* (in X) if, for every proposition $p \in X$, $p \in A_i$ or $\sim p \in A_i$. Unless otherwise stated, we require that individual judgment sets are consistent and complete. A *profile* is an n-tuple $(A_1, \ldots, A_n)$ containing each individual's judgment set.

Aggregation rules (constitutions). An *aggregation rule* (or *constitution*) is a function F that assigns to each admissible profile $(A_1, \ldots, A_n)$ a single (collective) judgment set $F(A_1, \ldots, A_n) = A \subseteq X$, where we read $p \in A$ as 'the group accepts proposition p'. The judgment set A can be interpreted as the set of propositions collectively believed to be true, or as the set collectively desired to be true. The set of admissible profiles is called the *domain* of F, denoted $Domain(F)$. In the next section we impose minimal conditions on aggregation rules.

Examples of propositions in politics. As argued in List (2004a), a model of judgment aggregation over multiple interconnected propositions allows the representation of several collective decision problems in politics, including problems of belief aggregation that cannot be represented in a standard preference aggregation model. Proposals offered for acceptance or rejection in political decisions are instances of propositions. Also, constraints between different proposals, such as budget constraints, can be represented as propositions involving logical connectives; for instance, if the implementation of any two of the proposals p, q and r is financially feasible, but the joint implementation of all three is too costly, this can be captured by the proposition

$\neg(p \wedge q \wedge r)$. To give further examples, if p is some proposal and a some amendment, then the proposal with the amendment can be represented as the proposition $p \wedge a$. The claim that some action a is a necessary condition for some political goal g can be represented as the proposition $g \rightarrow a$. Claims about how different political propositions constrain each other, which are frequently made in political debate, can also be represented as propositions involving logical connectives.

4 An impossibility result

We first state an impossibility result on the assignment of expert rights or liberal rights to individuals; we then state a similar result on the assignment of such rights to subgroups. The rights (expert or liberal) of individuals or subgroups are formalized by an appropriate concept of 'decisiveness'. Our account can be seen as the judgment aggregation counterpart of Sen's original account of rights in terms of decisive preference (Sen 1970).

Individual rights. Under an aggregation rule F, individual i is *decisive* on some proposition $p \in X$ if i determines the collective judgment on p and $\sim p$; specifically, for every profile $(A_1, \ldots, A_n) \in Domain(F)$, $[p \in F(A_1, \ldots, A_n)$ if and only if $p \in A_i]$ and $[\sim p \in F(A_1, \ldots, A_n)$ if and only if $\sim p \in A_i]$. So, on our account, an individual has a right on some proposition if the individual is decisive on that proposition. Note that (i) the second biconditional is redundant in case all individual or collective judgment sets accepted or generated by F are complete and consistent (i.e. contain exactly one member of each proposition-negation pair); (ii) decisiveness on p is equivalent to decisiveness on $\sim p$.

Suppose we want to find an aggregation rule with the following properties:

Universal Domain. The domain of F, *Domain*(F), is the set of all logically possible profiles of complete and consistent individual judgment sets.

Minimal Individual Rights. There exist (at least) two individuals who are each decisive on (at least) one proposition in X.

Unanimity Principle. For any profile $(A_1, \ldots, A_n) \in Domain(F)$ and any proposition $p \in X$, if $p \in A_i$ for all individuals i, then $p \in F(A_1, \ldots, A_n)$.

Like Sen's (1970) condition of *minimal liberalism*, minimal individual rights is a weak requirement that does not even specify which individuals have any decisiveness rights and to which propositions these rights apply. However, by using an undemanding rights requirement, our impossibility result becomes stronger. Below we introduce an explicit rights system and state a stronger rights requirement. The following theorem holds.

Theorem 1. Let the agenda X be connected. Then there exists no aggregation rule (generating consistent collective judgment sets) that has universal domain and satisfies minimal individual rights and the unanimity principle.

So a group (society, committee, legislature) whose constitution accepts all profiles of individual judgment sets in the universal domain as admissible cannot *both* assign (liberal or expert) rights to more than one individual *and* respect the unanimous judgments of its members (reached, for instance, at its plenary meetings). Let us make some remarks about theorem 1.

- This impossibility holds although we do not require collective judgment sets to be complete; the only collective rationality requirement is consistency.
- The theorem continues to hold if decisiveness in minimal individual rights is weakened to *positive* decisiveness. Individual i is *positively decisive* on $p \in X$ if, for every profile $(A_1, \ldots, A_n) \in Domain(F)$, [$p \in A_i$ implies $p \in F(A_1, \ldots, A_n)$] and [$\sim p \in A_i$ implies $\sim p \in F(A_1, \ldots, A_n)$].
- The theorem also continues to hold if F is required to generate consistent *and* complete judgment sets and decisiveness in minimal individual rights is weakened to either *negative* decisiveness or *weak* decisiveness. Individual i is *negatively decisive* (or has *veto power)* on $p \in X$ if, for every profile $(A_1, \ldots, A_n)$ $\in Domain(F)$, [$p \notin A_i$ implies $p \notin F(A_1, \ldots, A_n)$] and [$\sim p \notin A_i$ implies $\sim p \notin F(A_1, \ldots, A_n)$]. Individual i is *weakly decisive* on $p \in X$ if, for every profile $(A_1, \ldots, A_n) \in Domain(F)$, [$p \in F(A_1, \ldots, A_n)$ if and only if $p \in A_i$] (no requirement on $\sim p$).
- Without assuming a connected agenda, a modified result holds in which minimal individual rights is strenghened to the requirement that there exist (at least) two individuals who are each decisive on (at least) one proposition in X such that these two propositions are connected.

Subgroup rights. A subgroup is a non-empty set of individuals $M \subseteq \{1, \ldots, n\}$. Under an aggregation rule F, a subgroup M is *decisive* on a proposition $p \in X$ if, for every profile $(A_1, \ldots, A_n) \in Domain(F)$, [$p \in A_i$ for all $i \in M$ implies $p \in F(A_1, \ldots, A_n)$] and [$p \notin A_i$ for all $i \in M$ implies $p \notin F(A_1, \ldots, A_n)$] and the two analogous implications with p replaced by $\sim p$ hold. Note that (i) M is decisive on p if and only if M is decisive on $\sim p$, and (ii) if M contains only a single individual i, then the definition of decisiveness of $M = \{i\}$ on p is equivalent to our earlier definition of decisiveness of individual i on p.

In the interest of strength of the next impossibility theorem, we have defined a weak form of subgroup decisiveness: a subgroup determines the collective judgment on a proposition p only when the subgroup *unanimously* agrees on p. Stronger forms of subgroup rights on p are of course imaginable. A subgroup's right on p, properly understood, might also involve the right to determine the collective judgment on p by a majority vote within the subgroup.

However, are there any aggregation rules under which two or more subgroups are decisive at least in the present undemanding sense? Suppose we want to find an aggregation rule that satisfies the following condition:

Minimal Subgroup Rights. There exist (at least) two disjoint subgroups that are each decisive on (at least) one proposition in X.

Here two subgroups are *disjoint* if they have no members in common. Note also that minimal subgroup rights is logically less demanding than minimal individual rights, as minimal individual rights implies minimal subgroup rights (take subgroups containing a single individual) but not vice-versa (unless $n = 2$). The following theorem holds.

Theorem 2. Let the agenda X be connected. Then there exists no aggregation rule (generating consistent collective judgment sets) that has universal domain and satisfies minimal subgroup rights and the unanimity principle.

So, further to our earlier result, a group (society, committee, legislature) whose constitution accepts all profiles in the universal domain as admissible cannot *both* assign (liberal or expert) rights to more than one *subgroup* (minority, subcommittee, expert panel) *and* respect the unanimous judgments of its members. For theorem 2, analogous remarks to the ones given after theorem 1 apply. This includes the possible relaxation of minimal subgroup rights by only requiring *positive* decisiveness of subgroups, or (if collective judgment sets are required to be complete) by only requiring *negative* or *weak* decisiveness of subgroups.[9]

5 Possibility results

We now consider conditions under which the conflict between (expert or liberal) rights and the unanimity principle does not arise. For simplicity, we focus on individual rights, but our results can be generalized to subgroup rights too. To state our possibility results, we first refine our account of rights. The condition of minimal individual rights above does not specify which individuals have rights on which propositions. We now make (liberal or expert) rights explicit by introducing, for each individual i, a set $R_i \subseteq X$, called individual i's *right set*, where R_i contains proposition-negation pairs, i.e. if $p \in X$, then also $\sim p \in X$.[10] The elements of R_i are interpreted as the propositions that either belong to individual i's private sphere (liberal rights interpretation) or for which individual i is the expert (expert rights interpretation). The vector $(R_1, \ldots, R_n)$ is called a *rights system*. An aggregation rule respects a rights system if it satisfies the following condition.

Respectance of Rights. Every individual i is decisive on every proposition in R_i.

[9] The definitions of positive, negative, and weak decisiveness of subgroup M on proposition p are obtained by dropping from the definition of decisiveness, respectively, the two implications containing '$\notin$', the two implications containing '$\in$', and the two implications containing '$\sim p$'.

[10] It is no restriction of generality to assume that the right sets R_i contain proposition-negation pairs. If R_i did *not* contain proposition-negation pairs then individual i's decisiveness on each proposition in R_i (see *respectance of rights* below) would still guarantee i's decisiveness on every proposition in the enlarged right set $R_i^* := R_i \cup \{\sim p : p \in R_i\}$ (because decisiveness on p is equivalent to decisiveness on $\sim p$). So the right set R_i is equivalent to the right set R_i^* containing proposition-negation pairs. However, if respectance of rights is weakened to the requirement that each individual i be *positively/negatively/weakly* decisive on each proposition in R_i, then it may become interesting to consider right sets R_i not containing proposition-negation pairs.

Under what conditions can this condition be met? We call the rights system $(R_1, \ldots, R_n) \subseteq X$ *consistent* if the sets $R_1, \ldots, R_n$ are *(logically) independent.*[11] Informally, consistency of a rights system means that different individuals' rights never conflict with each other. Without a consistent rights system, respectance of rights becomes internally inconsistent, i.e. inconsistent even without the unanimity principle. More precisely:

Proposition 2. There exists an aggregation rule F (generating consistent and complete judgment sets) that has universal domain and satisfies respectance of rights *if and only if* the rights system $(R_1, \ldots, R_n)$ is consistent.

We are now able to state our possibility results. We address, firstly, special domains and, secondly, special agendas and rights systems. (For an overview of domain restriction approaches in response to the original liberal paradox in preference aggregation, including preference-based definitions of 'empathy' and 'tolerance', see Sen 1983; see also Craven 1982, Gigliotti 1986.)

Special domains 1: deferring/empathetic judgments. When one individual accepts (or takes on) the judgments of another, where those judgments concern the second individual's right set, we say that the first individual *defers* to the judgments of the second (if the rights in question are expert rights) or that he or she *shows empathy* for those judgments (if the rights in question are liberal rights). Formally:

- An individual i is *deferring/empathetic* in profile $(A_1, \ldots, A_n)$ if, for every proposition p in the right set R_j of any other individual $j \neq i$, the judgment set A_i satisfies [$p \in A_i$ if and only if $p \in A_j$].[12]
- A profile $(A_1, \ldots, A_n)$ is *deferring/empathetic* if *every* individual i is deferring/empathetic in it.
- A profile $(A_1, \ldots, A_n)$ is *minimally deferring/empathetic* if *some* individual i is deferring/empathetic in it.

Deferring/empathetic profiles require a unanimous agreement on every proposition in some individual's right set, a strong restriction on a profile. Our possibility theorem, however, is based on the less demanding restriction of *minimally* deferring/empathetic profiles.

Minimally Deferring/Empathetic Domain. The domain of F, $Domain(F)$, is the set of all minimally deferring/empathetic profiles of complete and consistent individual judgment sets.

If more than one individual has a non-empty right set, the minimally deferring/empathetic domain is a proper subset of the universal domain.[13]

[11] Formally, $R_1, \ldots, R_n$ are *independent* here if $B_1 \cup \cdots \cup B_n$ is consistent whenever $B_1, \ldots, B_n$ are consistent subset of $R_1, \ldots, R_n$, respectively.

[12] Equivalently, $A_i \cap R_j = A_j \cap R_j$ for all $j \neq i$.

[13] If there exists only a single individual i with $R_i \neq \emptyset$ (i.e. only one individual has a non-empty right set), then i is trivially deferring/empathetic in every profile; and if there exists no individual i with $R_i \neq \emptyset$ for *no* individual i (i.e. no individual has a non-empty right set), then every individual is trivially deferring/empathetic in every profile. So, if $R_i \neq \emptyset$ for at most one individual i, then the minimally deferring/empathetic domain coincides with the universal domain.

Theorem 3. There exists an aggregation rule (generating consistent and complete collective judgment sets) that has the minimally deferring/empathetic domain and satisfies respectance of rights and the unanimity principle.

Let us make some remarks about this possibility result.

- The result holds for any agenda, however rich or sparse the interconnections between propositions are.
- As shown in our proof, the possibility is established, for instance, by defining the collective judgment set as the judgment set of some deferring/empathetic individual. Note that this is never a dictatorial rule since different profiles (in the minimally deferring/empathetic domain) have different deferring/empathetic individuals.
- It may seem surprising that theorem 3 does not require the assumption of a consistent rights system $(R_1, \ldots, R_n)$. If $(R_1, \ldots, R_n)$ is inconsistent, how could a single deferring/empathetic individual prevent the other individuals from exercising their rights in a mutually inconsistent way, leading to an inconsistent collective judgment set by respectance of rights? The simple answer is that individual i's deferral/empathy *does* prevent such inconsistencies, albeit in a rather technical sense. Inconsistencies in the exercise of the other individuals' rights would (by the definition of deferral/empathy) lead individual i to have an inconsistent judgment set A_i, something excluded by the minimally deferring/empathetic domain. But this is puzzling, since it could be interpreted as meaning that the individuals $j \neq i$ are restricted in the exercise of their rights so as to allow individual i to be *both* deferring/empathetic *and* consistent. One might, therefore, want to redefine a deferring/empathetic individual as one who adopts the other individuals' judgments (where they have rights) *unless* these judgments are mutually inconsistent.[14] Under this modified definition, theorem 3 continues to hold provided that the rights system $(R_1, \ldots, R_n)$ is consistent.

Special domains 2: agnostic/tolerant judgments. When one individual makes no judgment on those propositions that lie within another individual's right set, we say that the first individual is *agnostic* about the judgments of the second (if the rights in question are expert rights) or that he or she is *tolerant* towards those judgments (if the rights in question are liberal rights). Specifically, an agnostic/tolerant individual i makes no judgment about a proposition in the right set R_j of any other individual $j \neq i$, but moreover i's judgment set A_i does not *entail* any judgment about any $p \in R_j$, i.e. A_i is consistent with p and with $\sim p$. However, even consistency of A_i with every *particular* possible judgment of another individual $j \neq i$ over a proposition in R_j is not all we require from an agnostic/tolerant individual i. We require A_i to be consistent with every *combination* of possible judgments by individuals $j \neq i$ on propositions in R_j. In short, A_i must be consistent with any possible (consistent) exercise of rights by other individuals. Formally:

[14] Formally, individual i is *deferring/empathetic* in profile $(A_1, \ldots, A_n)$ if $[A_i \cap R_j = A_j \cap R_j$ for all $j \neq i]$ whenever $\cup_{j \neq i}[A_j \cap R_j]$ is consistent.

- An individual i with judgment set A_i is *agnostic/tolerant* if A_i is consistent with every consistent set of the form $B_1 \cup \cdots \cup B_{i-1} \cup B_{i+1} \cup \cdots \cup B_n$,[15] where, for each individual $j \neq i$, $B_j \subseteq R_j$.

- A profile $(A_1, \ldots, A_n)$ is *agnostic/tolerant* if *every* individual i is agnostic / tolerant in it.

- A profile $(A_1, \ldots, A_n)$ is *minimally agnostic/tolerant* if *some* individual i is agnostic/tolerant in it.

Our possibility theorem requires only *minimally* agnostic/tolerant profiles.

Minimally Agnostic/Tolerant Domain. The domain of F, $Domain(F)$, is the set of all minimally agnostic/tolerant profiles of consistent individual judgment sets.

The minimally agnostic/tolerant domain does not require complete judgment sets, and hence is not a subset of the universal domain. In fact, an agnostic/tolerant individual i *cannot* have a complete judgment set (unless all other individuals $j \neq i$ have an empty right set), since agnosticism/tolerance forces individual i to make no judgment on (at least) those propositions in other individuals' right sets. If *at least two* individuals have a non-empty right set, then the universal domain neither contains, nor is contained by, the minimally agnostic/tolerant domain.[16]

Theorem 4. Let the rights system $(R_1, \ldots, R_n)$ be consistent. Then there exists an aggregation rule (generating consistent judgment sets) that has the minimally agnostic/tolerant domain and satisfies respectance of rights and the unanimity principle.

Comparing this result with our earlier result about the minimally deferring / empathetic domain, note the following.

- The present result holds for any agenda, but requires a consistent rights system.

- As shown in our proof, the possibility is established, for instance, by defining the collective judgment set as the union of the judgment set of some agnostic/tolerant individual *and* the set containing every proposition accepted by some individual whose right set contains that proposition.

- The aggregation rule may generate incomplete collective judgment sets, since an individual's possible incompleteness within his or her right set has to be collectively reflected by respectance of rights. However, in theorem 4 we may additionally require complete collective judgment sets if we *either* weaken respectance of rights to the requirement that every person i be *positively* decisive over each

[15] If the rights system $(R_1, \ldots, R_n)$ is consistent, the set $\cup_{j \neq i} B_j$ is *always* consistent provided that each B_j is consistent.

[16] As in the case of deferral/empathy, if $R_i \neq \emptyset$ for only one individual i, then i is trivially agnostic/tolerant in every profile; and if $R_i \neq \emptyset$ for *no* individual i, then every individual is trivially agnostic/tolerant in every profile. Therefore, if $R_i \neq \emptyset$ for *at most* one individual i, then the minimally agnostic/tolerant domain contains the universal domain.

$p \in R_i$, *or* restrict the domain to the set of all minimally agnostic/tolerant profiles $(A_1, \ldots, A_n)$ such that each A_i is consistent *and* complete within R_i (i.e. each A_i contains a member of every proposition-negation pair in R_i). In such a restricted domain, each individual may refrain from making judgments only outside his or her right set.

Special agendas and rights systems. Instead of restricting the domain, we now consider special rights systems, namely ones we call *disconnected*. We have seen in proposition 2 that consistency of a rights system is sufficient for the existence of aggregation rules that have universal domain and satisfy respectance of rights, yet the unanimity principle may be violated. We now strenghen the consistency requirement on the rights system so as to become sufficient for the existence of aggregation rules that have universal domain and satisfy *both* respectance of rights *and* the unanimity principle. The rights system $(R_1, \ldots, R_n)$ is *disconnected* (in X) if the sets $R_1, \ldots, R_n, X \backslash (R_1 \cup \cdots \cup R_n)$ are *(logically) independent*.[17] This implies in particular that the rights system is consistent, i.e. that $R_1, \ldots, R_n$ are independent.

Informally, disconnectedness requires not only that the rights of different individuals never conflict with each other, but also that they never conflict with any (collective) judgments on propositions outside any individual's right sphere. So the rights of different individuals must not be 'entangled' with each other or with any other propositions that fall into the agenda for public consideration. If each individual lives on a separate island, where the propositions relevant to each island do not constrain those relevant to the other islands, then it is possible to construct a disconnected rights system, where each individual's right set includes only propositions relevant to his or her island.

Proposition 3. The rights system $(R_1, \ldots, R_n)$ is disconnected (in X) if and only if the right sets $R_1, \ldots, R_n$ are pairwise disjoint and, for every individual i, no proposition in R_i is connected with any proposition in $X \backslash R_i$.

Note that a disconnected rights system in which more than one individual has a non-empty right set can exist only if the agenda is *not* connected. The following theorem holds.

Theorem 5. If the rights system $(R_1, \ldots, R_n)$ is disconnected, then there exists an aggregation rule (generating consistent and complete collective judgment sets) that has universal domain and satisfies respectance of rights and the unanimity principle.

However, while the domain is not restricted – there need not be any deferring/empathetic or agnostic/tolerant individuals – disconnectedness is a severe constraint on a rights system, and satisfiable (if more than one individual is to have a non-empty right set) only for special agendas.

[17] Formally, the sets $R_1, \ldots, R_n, X \backslash (R_1 \cup \cdots \cup R_n)$ are *(logically) independent* here if $B_1 \cup \cdots \cup B_n \cup B$ is consistent whenever $B_1, \ldots, B_n, B$ are consistent subsets of $R_1, \ldots, R_n, X \backslash (R_1 \cup \cdots \cup R_n)$, respectively.

6 Conclusion

We have identified a paradox in judgment aggregation similar to the famous liberal paradox in preference aggregation, yet more general since it applies to various problems of belief or desire aggregation that cannot be represented in a classical preference aggregation framework. If the propositions on which judgments are to be made are connected, then, under universal domain, the assignment of (expert or liberal) rights to two or more individuals (or subgroups) is logically inconsistent with the unanimity principle whereby a proposition accepted by *all* individuals should also be accepted by the group. This is a potential inconsistency between rights and a seemingly undemanding democratic principle. For some profiles of judgments across the group, the collective judgment will, of necessity, *either* violate the (expert or liberal) rights of some members or subgroups, *or* fail to respect a unanimous judgment by all group members. This conflict occurs because propositions on which unanimous judgments are reached are sometimes logically constrained by other propositions that lie in some individual's or subgroup's sphere of rights.

We have also shown that, for the restricted domains of deferring/empathetic judgments or agnostic/tolerant judgments, the inconsistency between rights and the unanimity principle does not arise. Likewise, for a disconnected rights system, which requires an agenda that is not connected (if there is to be more than one rights-bearer), the inconsistency does not arise either. So if different individuals (or subgroups) each live on their own Robinson Crusoe island, where the propositions relevant to different islands are not connected with each other, then rights can be assigned to these individuals (or subgroups) without conflicting with the unanimity principle. But scenarios of this kind are rare, and one may question whether such islanders would ever face a collective decision problem anyway. Almost all realistic collective decision problems presuppose some degree of interaction between different agents, which makes it plausible to expect connections between different individuals' right sets.

Our results have implications for the design of mechanisms that groups (societies, committees, expert panels, organizations) can use for making decisions on complex systems of propositions. For certain groups or decision problems, the existence of agnostic/tolerant or even deferring/empathetic group members (in our technical sense) may avoid the paradox. But there is usually no guarantee that such attitudes will exist, and so constitutional provisions may be needed to deal with the possible occurrence of the paradox. Ultimately, the group faces the constitutional choice between either relaxing the (democratic) unanimity principle or relaxing (expert or liberal) rights of individuals or subgroups. Let us briefly discuss each option.

If it is deemed unacceptable to weaken any (expert or liberal) rights of individuals (or subgroups), violations of the unanimity principle will have to be allowed in collective decision making – an option advocated, among others, by Sen (1976) in the context of preference aggregation. The overruling of unanimous judgments might be defended on the grounds of 'unacceptable' individual *motivations* behind such judgments: motivations that disregard the rights of other individuals. Individual judgments driven by such 'unacceptable' motivations may be seen as the counterpart in judgment aggregation of the so-called *meddlesome* preferences in preference aggregation (Blau 1975).

On the other hand, if the unanimity principle is deemed indispensible, then some relaxation of (expert or liberal) rights is necessary. One possibility is to grant such rights in a suitably disconnected way, so that different rights never conflict with each other or with unanimous judgments on other propositions. (Here, weaker forms of disconnectedness than that assumed in theorem 5 might be sufficient.) Alternatively, rights can be made *alienable*, that is, conditional on not being in conflict with any other rights or unanimous judgments. Dowding and van Hees (2003), for instance, have suggested that rights may sometimes be overruled by other considerations; in particular, different rights may carry a different characteristic threshold of being respected, where that threshold may vary from right to right and from context to context.

The choice of whether or not to give rights priority over the unanimity principle may also depend on whether these rights are expert rights or liberal rights, i.e. whether we interpret judgments in terms of beliefs or in terms of desires. Under a desire interpretation, the choice is ultimately a normative one, which depends on how much weight we give to individual liberty as a political value relative to other political values such as certain democratic decision principles. Under a belief interpretation, by contrast, the choice is not just normative. If there exists a truth of the matter on the propositions (i.e. they are factually either true or false), then it becomes a technical epistemological question which aggregation rule is better at tracking that truth, one that respects expert rights, or one that satisfies the unanimity principle. The answer to this question – which we cannot provide here – depends on several factors: how competent the experts are, both on propositions within their area of expertise and on other propositions, how competent the non-experts are on all these propositions, and how the judgments of different individuals are related to each other (dependent or independent). The literature on the Condorcet jury theorem can in principle be modified so as to address this question (Bovens and Rabinowicz 2004, List 2004b).

As the liberal paradox continues to be much discussed in the area of preference aggregation as well as political science more generally, we hope that our findings will help to extend this discussion to the emerging research area of judgment aggregation and to inspire further work not only on liberal rights, but also on expert rights, and perhaps on combinations between the two.

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A Appendix: Proofs

Proof of proposition 1. Let $p, q \in X$. We show both implications.

(i) First, assume p, q are logically dependent given some subset of X consistent with each of $p, q, \sim p, \sim q$. Let $Z \subseteq X$ be a *minimal* such subset. By assumption, there exist $p^* \in \{p, \sim p\}$ and $q^* \in \{q, \sim q\}$ such that $p^* \vDash_Z q^*$. So, the set $Y := \{p^*, \sim q^*\} \cup Z$ is inconsistent. In fact, Y is even minimal inconsistent (which proves that p, q are connected):

- $\{p^*\} \cup Z$ and $\{\sim q^*\} \cup Z$ are consistent since Z is consistent with each of $p, q, \sim p, \sim q$;

- Assume for contradiction that $\{p^*, \sim q^*\} \cup Z'$ is inconsistent for a proper subset $Z' \subsetneq Z$. Then $p^* \vDash_{Z'} q^*$, and so p, q are logically dependent given Z'. Also, since Z is consistent with each of $p, q, \sim p, \sim q$, so is Z'. This contradicts the minimallity assumption on Z.

(ii) Now assume that p, q are connected. If $q \in \{p, \sim p\}$, then p and q are dependent conditional on the empty set $Z = \emptyset$ (which is consistent with each of $p, q, \sim p, \sim q$) because p entails q (if $q = p$) or entails $\sim q$ (if $q = \sim p$). Now assume $q \notin \{p, \sim p\}$. Since p, q are connected, there exist $p^* \in \{p, \sim p\}$ and $q^* \in \{q, \sim q\}$ and minimal inconsistent set $Y \subseteq X$ such that $p^*, q^* \in Y$. Define Z as $Y \backslash \{p^*, q^*\}$. Then p, q are logically dependent given Z, since $\{p^*, q^*\} \cup Z$ is inconsistent and hence $p^* \vDash_Z \sim q^*$. Moreover, Z is consistent with each of $p, q, \sim p, \sim q$. To show the latter, we prove that Z is consistent with each of $p^*, q^*, \sim p^*, \sim q^*$:

- $Z \cup \{p^*\}$ is consistent since it is (by $p^* \neq q^*$) a proper subset of the minimal inconsistent set $Y = Z \cup \{p^*, q^*\}$; analogously, $Z \cup \{q^*\}$ is consistent;

- $Z \cup \{\sim q^*\}$ is consistent, since otherwise $Z \vDash q^*$, so that the (already shown) consistency of $Z \cup \{p^*\}$ would remain after adding q^*, i.e. the set $Z \cup \{p^*, q^*\}$ would be consistent; analogously, $Z \cup \{\sim p^*\}$ is consistent. ■

Proof of theorem 1. Assume that X is connected. Suppose that the aggregation rule F satisfies minimal individual rights, the unanimity principle and universal domain. We claim that F generates an inconsistent collective judgment set on some profile. By minimal individual rights there exist two distinct individuals i, j and two propositions $p, q \in X$ such that i is decisive on p and j is decisive on q.

Case $q \in \{p, \sim p\}$. Let $(A_1, \ldots, A_n) \in \mathit{Domain}(F)$ be any profile with $p \in A_i$ and $\sim p \in A_j$; such a profile exists by universal domain (and since p and $\sim p$ are not contradictions). Since i and j are each decisive on p, we have $p, \sim p \in F(A_1, \ldots, A_n)$, and hence $F(A_1, \ldots, A_n)$ is inconsistent.

Case $q \notin \{p, \sim p\}$. Since X is connected, there exist propositions $p^* \in \{p, \sim p\}$ and $q^* \in \{q, \sim q\}$ and a minimal inconsistent set $Y \subseteq X$ that contains p^* and q^*. By the minimal inconsistency of Y, the sets $Y \backslash \{p^*\}$ and $Y \backslash \{q^*\}$ are each consistent. So $Y \backslash \{p^*\}$ and $Y \backslash \{q^*\}$ can each be extended to a complete and consistent judgment set. Let $(A_1, \ldots, A_n)$ be any profile for which A_i is an extension of $Y \backslash \{q^*\}$, A_j is an extension of $Y \backslash \{p^*\}$, and, for all other individuals $k \notin \{i, j\}$, A_k is an extension either of $Y \backslash \{p^*\}$ or of $Y \backslash \{q^*\}$, where all extensions are complete and consistent. By universal domain, $(A_1, \ldots, A_n) \in \mathit{Domain}(F)$. We show that $Y \subseteq F(A_1, \ldots, A_n)$, which completes the proof since $F(A_1, \ldots, A_n)$ is then inconsistent. First, by the unanimity principle we have $r \in F(A_1, \ldots, A_n)$ for every proposition $r \in Y \backslash \{p^*, q^*\}$. Next, note that by $q \notin \{p, \sim p\}$ we have $p^* \neq q^*$. So $p^* \in Y \backslash \{q^*\}$ and $q^* \in Y \backslash \{p^*\}$, and hence $p^* \in A_i$ and $q^* \in A_j$. Consequently, since i is decisive on p and j is decisive on q, we have $p^*, q^* \in F(A_1, \ldots, A_n)$. ■

Proof of theorem 2. The theorem can be proved by a straightforward adaptation of the proof of theorem 1. Instead of having two distinct individuals $i, j \in \{1, \ldots, n\}$ each decisive on some proposition, we now have two disjoint subgroups $M, N \subseteq \{1, \ldots, n\}$ each decisive on some proposition. The only adaptation is that, when constructing profiles, the judgment set of individual i in the old proof becomes the judgment set of *each* individual in M in the new proof, and the judgment set of individual j in the old proof becomes the judgment set of *each* individual in N in the new proof. ■

Proof of proposition 2. (i) First, assume the rights system $(R_i)_{i=1,\ldots,n}$ is consistent. For every profile $(A_1, \ldots, A_n)$ of complete and consistent judgment sets, let

$$B := B_1 \cup \cdots \cup B_n, \text{ where } B_i := A_i \cap R_i.$$

Since each A_i is consistent, so is each B_i. So, as the rights system $(R_1, \ldots, R_n)$ is consistent, B is consistent, and hence B can be extended to a complete and consistent judgment set. Let $F(A_1, \ldots, A_n)$ be any such extension. The so-defined aggregation rule F satisfies all properties.

(ii) Now assume the aggregation rule F has all properties. To show that the rights system $(R_1, \ldots, R_n)$ is consistent, let $B_1, \ldots, B_n$ be consistent subsets of, respectively, $R_1, \ldots, R_n$. For each i, as B_i is consistent it may be extended to a complete and consistent judgment set A_i. The so-defined profile $(A_1, \ldots, A_n)$ belongs to the (universal) domain of F. By acceptance of rights, $B_i \cap F(A_1, \ldots, A_n) = B_i$ for all individuals i, and so

$$\begin{aligned} B_1 \cup \cdots \cup B_n &= [B_1 \cap F(A_1, \ldots, A_n)] \cup \cdots \cup [B_n \cap F(A_1, \ldots, A_n)] \\ &= [B_1 \cup \cdots \cup B_n] \cap F(A_1, \ldots, A_n). \end{aligned}$$

So $B_1 \cup \cdots \cup B_n$ is a subset of the consistent set $F(A_1, \ldots, A_n)$, hence is itself consistent. ■

Proof of theorem 3. For each minimally deferring/empathetic profile $(A_1, \ldots, A_n)$, define $F(A_1, \ldots, A_n)$ as the judgment set A_i of some deferring/empathetic individual i (if there are several such individuals, choose one of them). The so-defined aggregation rule satisfies all conditions, because the collective judgment set, by being the judment set of a deferring/empathetic individual, is consistent and complete, reflects the judgments of any individual over propositions in the individual's right set (which guarantees respectance of rights), and contains each proposition that every individual accepts (which guarantees the unanimity principle). ■

Proof of theorem 4. For every minimally agnostic/tolerant profile $(A_1, \ldots, A_n)$, consider the set

$$B := B_1 \cup \cdots \cup B_n, \text{ where } B_i := A_i \cap R_i.$$

Since each A_i is consistent, so is each B_i. Hence, by the consistency of the rights system $(R_1, \ldots, R_n)$, the set B is consistent. So, as $(A_1, \ldots, A_n)$ is minimally agnostic/tolerant, there exists an (agnostic/tolerant) individual i such that A_i is consistent with $\cup_{j \neq i} B_j$, i.e. such that the set $C := A_i \cup [\cup_{j \neq i} B_j]$ is consistent. Note that $C = A_i \cup B$. Let $F(A_1, \ldots, A_n) := C = A_i \cup B$. The so-definied aggregation rule F satisfies all properties: F satisfies

- minimally agnostic/tolerant domain, and consistent collective judgment sets;

- respectance of rights since, for all minimally agnostic/tolerant profiles $(A_1, \ldots, A_n)$, $F(A_1, \ldots, A_n) \cap R_i = A_i \cap R_i$ for all individuals i;

- the unanimity principle since, for all minimally agnostic/tolerant profiles $(A_1, \ldots, A_n)$ and each unanimously accepted proposition $p \in A_1 \cap \cdots \cap A_n$, this proposition belongs in particular to A_i for each agnostic/tolerant individual i, hence to $F(A_1, \ldots, A_n)$ by construction of $F(A_1, \ldots, A_n)$. ■

Proof of proposition 3. (i) Assume the rights system $(R_1, \ldots, R_n)$ is disconnected.

First, assume for contradiction that the right sets $R_1, \ldots, R_n$ are not pairwise disjoint. Then there exist distinct individuals i, j such that $R_i \cap R_j$ is non-empty, hence contains a proposition-negation pair $p, \sim p$. Take any consistent subsets $B_1, \ldots, B_n, B$ of, respectively, $R_1, \ldots, R_n, X \backslash (R_1 \cup \cdots \cup R_n)$, such that $p \in R_i$ and $\sim p \in R_j$. Then $B_1 \cup \cdots \cup B_n \cup B$ contains both p and $\sim p$, hence is inconsistent. So $R_1, \ldots, R_n, X \backslash (R_1 \cup \cdots \cup R_n)$ are not independent, in violation of the disconnectedness of the rights system $(R_1, \ldots, R_n)$. It follows that $R_1, \ldots, R_n$ are pairwise disjoint.

Next, assume for contradiction that there exist an individual i and a proposition $p \in R_i$ that is connected to a proposition $q \in X \backslash R_i$. Then there exist propositions $p^* \in \{p, \sim p\}$ and $q^* \in \{q, \sim q\}$, and a minimal inconsistent set $Y \subseteq X$, such that $p^*, q^* \in Y$. Since $R_1, \ldots, R_n$ are pairwise disjoint, the sets $B_1 := Y \cap R_1, \ldots, B_n := Y \cap R_n, B := Y \backslash (R_1 \cup \cdots \cup R_n)$ are also pairwise disjoint. So, by $p^* \in B_i$ we have $p^* \notin B_1, \ldots, B_{i-1}, B_{i+1}, \ldots, B_n, B$. Note also that by $q^* \in X \backslash R_i$ we have $q^* \notin B_i$. As each of the sets $B_1, \ldots, B_n, B$ either does not contain p^* or does not contain q^*, these sets are *proper* subsets of Y. So, as Y is minimal inconsistent, each of $B_1, \ldots, B_n, B$ is consistent. Note that $B_1, \ldots, B_n, B$ are consistent subsets of $R_1, \ldots, R_n$, $X \backslash (R_1 \cup \cdots \cup R_n)$, respectively. But their union $Y = B_1 \cup \cdots \cup B_n \cup B$ is inconsistent, so that $R_1, \ldots, R_n$, $X \backslash (R_1 \cup \cdots \cup R_n)$ are not independent, violating the disconnectedness of the rights system $(R_1, \ldots, R_n)$.

(ii) Now assume $(R_1, \ldots, R_n)$ is not disconnected. We have to show that (a) the right sets $R_1, \ldots, R_n$ are not pairwise disjoint, *or* (b) for some individual i a proposition $q \in R_i$ is connected with a proposition in $X \backslash R_i$. Showing that (a) or (b) holds is equivalent with showing that (b) holds in case (a) does not hold. So suppose that (a) does not hold, i.e. that $R_1, \ldots, R_n$ *are* pairwise disjoint. Since $(R_1, \ldots, R_n)$ is not disconnected, there exist consistent subsets $B_1, \ldots, B_n, B$ of, respectively, $R_1, \ldots, R_n, X \backslash (R_1 \cup \cdots \cup R_n)$ such that $B_1 \cup \cdots \cup B_n \cup B$ is inconsistent. Let Y be a minimal inconsistent subset of $B_1 \cup \cdots \cup B_n \cup B$. Since each of $B_1, \ldots, B_n, B$ is consistent, Y is not a subset of any of these sets. So there exist $p, q \in Y$ such that $p \in B_i$ and $[q \in B_j$ or $q \in B]$ (for some $j \neq i$). (By $p, q \in Y$, which is minimal inconsistent, p and q are connected.) Hence $p \in R_i$ and $[q \in R_j$ or $q \in X \backslash (R_1 \cup \cdots \cup R_n)]$. If $q \in R_j$ then $q \notin R_i$ since $R_1, \ldots, R_n$ are pairwise disjoint; and if $q \in X \backslash (R_1 \cup \cdots \cup R_n)$ then also $q \notin R_i$. So $p \notin R_i$, hence $q \in X \backslash R_i$. This completes the proof, since a proposition in R_i (p) is connected with a proposition in $X \backslash R_i$ (q). ■

Proof of theorem 5. Let the rights system $(R_1, \ldots, R_n)$ be disconnected. Let $(A_1, \ldots, A_n)$ be any profile of complete and consistent judgment sets. By universal domain, each of the sets

$$B_1 := A_1 \cap R_1, \ \ldots, \ B_n := A_n \cap R_n, \ B := (A_1 \cap \cdots \cap A_n) \backslash (R_1 \cup \cdots \cup R_n)$$

is consistent. Since $B_1, \ldots, B_n, B$ are consistent subsets of, respectively, $R_1, \ldots, R_n$, $X \backslash (R_1 \cup \cdots \cup R_n)$, the disconnectednesss of the rights system $(R_1, \ldots, R_n)$ implies that the set $C := B_1 \cup \cdots \cup B_n \cup B$ is also consistent. So C may be extended to a complete and consistent judgment set. Let $F(A_1, \ldots, A_n)$ be such an extension. The so-defined aggregation rule F has all relevant properties: F satisfies

- universal domain, and consistent and complete collective judgment sets;

- respectance of rights since $F(A_1, \ldots, A_n) \cap R_i = A_i \cap R_i$ for all individuals i and profiles $(A_1, \ldots, A_n) \subseteq Domain(F)$;

- the unanimity principle since, for all profiles $(A_1, \ldots, A_n) \subseteq Domain(F)$, $F(A_1, \ldots, A_n)$ contains each proposition $p \in A_1 \cap \cdots \cap A_n$, as is seen by distinguishing between two cases: $p \in R_i$ for some individual i (hence $p \in A_i \cap R_i$), and $p \notin R_i$ for all individuals i (hence $p \in (A_1 \cap \cdots \cap A_n) \setminus (R_1 \cup \cdots \cup R_n)$). ■