THE BEQUEST MOTIVE AND SINGLE PEOPLE'S DEMAND FOR LIFE ANNUITIES♦

Carlos Vidal-Meliá & Ana Lejárraga-García♣

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ABSTRACT
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The main objective of this paper is to go deeper into the "annuity puzzle" by introducing the altruistic and strategic bequest motive and determining whether this really is a relevant issue affecting the theoretical decision to purchase life annuities. With this end in view we develop an optimization model based on that first put forward by Lejárraga et al. [2002], then add to it elements from other models such as Friedman and Warshawsky's [1990] and Jousten's [1998 & 2001] which include the bequest motive. We also analyse welfare by calculating the equivalent wealth in different contexts: the possibility of having access to an actuarially fair life annuity or programmed withdrawal market, the incorporation of so-called market *imperfections,* and the inclusion of the assumption that the individuals already have part of their wealth in pre-existing life annuities.

JEL: G23, H55, J26

Key words: Capitalization, Pension Funds, Phased Withdrawal, Retirement, Utility.

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♣ Facultad de Economía. Departamento de Economía Financiera. Universidad de Valencia. Avenida de los Naranjos, s/n. 46022 Valencia (Spain). e-mail: carlos.vidal@uv.es (Corresponding author). e-mail: ana.lejarraga@gseguros.com

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1. INTRODUCTION

It is well known that there are various reasons for individuals' low level of demand for annuities. Fundamentally these are:

1) There already exists a certain level of protection against the risk of longevity in the annuities of the public pension system which provides a minimal level of resources in retirement.
2) The annuity market is not "actuarially fair". This is a consequence of the application of administrative and management costs by insurance firms.
3) The problem of *adverse selection*. This obliges insurers to use mortality and survival tables that are different from those applicable to the general population.
4) Individuals use family *self-insurance*. This allows them to take advantage of the possibilities of joint consumption, sharing out financial resources among the members of the family, thereby reducing the attractiveness of annuities.
5) The perception of the accumulated retirement fund in the form of lump sum allows the individual to face the appearance of unforeseen future expenditures such as extraordinary medical costs not covered by the usual medical insurance, the cancellation of an outstanding mortgage, or simply the possibility of going on a long-desired journey.
6) The different ways in which annuities are taxed. In some countries tax regulations penalize annuities; in Belgium, for instance, this is the main reason why they are unpopular.
7) In addition, according to Impavido et al. [2003], the low level of demand for annuities may be related to traditional products that seem to provide consumers with only a binomial allocation of risk. Either self insurance or complete insurance against the most important types of risks are available in many countries.

All these aspects have been dealt in the work of Benitez-Silva [2003], Brown [1999, 2001, 2003], Brown & Poterba [2000], Friedman & Warshawsky [1990], Kotlikoff & Spivak [1981], Lejárraga et al. [2002], Mitchell et al. [1999], Vidal et al. [2003], and Yaari [1965].

Another factor that could be decisive in the choice of the mode of withdrawing the funds available is the existence of motivations to leave a bequest to the heirs, although there are quite contradictory results in the literature concerning this aspect.

The "annuity puzzle" is that the empirical evidence shows the extreme rarity of voluntary private individual annuity contracts even though, according to Yaari [1965], individuals would be better off holding only annuitized assets in the absence of a bequest motive, or a portfolio of annuitized and traditional assets in the presence of a bequest motive. It must be emphasized that in Yaari's model the purchase of life annuities is optimal under the following assumptions:

1) Consumers maximize the expected (Neumann-Morgenstern) utility with separability and additivity.
2) The only risk faced by consumers is longevity risk.
3) There are no other assets, such as housing, that offer another source of annuity that is characteristically uncorrelated.
4) The individual is single and has no descendents.

5) The individual has no access to pensions in the form of life annuities from the first pillar of the social security system.
6) The annuity market is actuarially fair. This simultaneously implies that:
a) The mortality tables that insurance firms apply coincide with the consumer's probabilities.
b) The technical interest rate net of administrative costs coincides with the risk free rate.
7) There is only one conventional asset which pays a given interest rate
8) Consumers can borrow and lend at this same rate.

Davidoff et al. [2003] show that the conditions under which the purchase of annuities is optimal are not as demanding as those set out by Yaari [1965]. If financial markets are complete, one only requires that no bequest motive exists and that the expected rate of return on annuities is greater than that on a reference financial asset. Partial annuitization is optimal when the condition of complete insurance market is relaxed. There is a curious situation in Switzerland, Butler [2003], which contradicts the "annuity puzzle": most individuals choose to receive annuities, even though they can already count on a first pillar of support with a high degree of protection against longevity in the form of large annuities, and though benefits received in the form of capital sums receive far better tax treatment.

The aim of the present study is to go deeper into clarifying the "annuity puzzle" through the introduction of the bequest motive[1], both altruistic (the pensioner simply wants to leave a bequest to his family without expecting anything in return) and strategic (the pensioner wants to give his family an incentive to look after him in his old age by promising to leave them a bequest), and determine whether this is really a major factor influencing the theoretical decision to acquire annuities. To bring the model closer to reality, the possibility is also considered that the individual can already count on a pre-existing life annuity, and that the annuity market is not actuarially fair.

The work is structured as follows. After this brief Introduction, Section 2 presents an optimization model based on that first put forward by Lejárraga et al. [2002], supplemented by elements from other models that include the bequest motive, in particular, those of Friedman & Warshawsky [1990] and Jousten [1998 & 2001]. The following three sections analyse an individual's welfare by calculating the equivalent wealth in different contexts: with the possibility of access to an actuarially fair annuity or programmed withdrawal market (Section 3), incorporating so-called market *imperfections* (Section 4), and assuming that individuals already hold part of their wealth in pre-existing annuities (Section 5). Section 6 presents the conclusions, and Section 7 the references cited.

2. THE MODEL

One of the reasons why it has not been customary in the analysis of welfare to include the existence of the motivation to leave a bequest could well be for the sake of maintaining the analytical simplicity of the model. Also, however, Benitez-Silva [2003], it could be due to

[1] Acorrding to Impavido et al. [2003], there are three main reasons for bequests: uncertain lifetimes, altruism, and strategic behavior towards heirs. Uncertain lifetimes and incomplete insurance markets result in involuntary bequests as individuals need to save for precautionary motives. If insurance and capital markets are imperfect, uninsured risks relating to health and longevity may give rise to precautionary motives for preserving wealth in old age. We do not consider *uncertain lifetimes* in this paper.

the lack of consensus about the relevance of the bequest motive in an individual's decision with respect to contracting annuities, and about how to model this bequest factor.

Brown [1999] questions the importance of the bequest motive in influencing marginal annuity purchasing decisions – neither the presence of children nor of the bequest motive are determinants in a life annuity purchasing decision. The author states that a simple life cycle model without bequests gives predictions that are consistent with marginal annuity purchasing behaviour, and is therefore a useful first approximation to behaviour. These results coincide with those of Hurd [1987], who concludes that bequest motives have no significant effect on the marginal financial behaviour of elderly individuals. This is further supported in a later work, Hurd [1989], which finds that many bequests are apparently accidental, resulting from the uncertainty in the time of death, without there being any evidence for real bequest motives.

These findings are in contradiction with the contributions of Bernheim [1991] and Laitner & Juster [1996], who claim that bequest motives do indeed influence the decision to purchase annuities. Also, Friedman & Warshawsky [1990] conclude that the presence of a bequest motive will reduce or eliminate the demand for annuities, even when their return surpasses the real market interest rate. In a previous work, Friedman & Warshawsky [1988], these authors had indicated that the interaction of a deliberate bequest motive and the actuarial unfairness of the annuities offered by insurance firms could lead individuals to reject purchasing them.

In a more recent work, Brown [2001] introduces the bequest motive into the single person model, without distinguishing between altruistic and strategic motives. The results indicate that in very few cases does the obligatory purchase of annuities lead to lower welfare than when no annuities are purchased, although the welfare gains are less than those obtained in the no-inheritance life cycle model. According to Bernheim [1991], if the retiree can generate savings that are external to the life annuities and/or purchase additional, annually renewable, life insurance, no major differences result in welfare gains by incorporating a bequest motive.

The consumption optimization model with a bequest motive is based on that developed by Lejárraga et al. [2002], it being necessary to introduce a function that represents the utility of the wealth existing at each point in time that will allow the individual to bequeath an amount to the heirs on the date of death.

Let $U(C_t)$ be the welfare function of the period corresponding to age t, defined over the consumption (C_t); $V(W_t)$ the welfare function for the period corresponding to age t, defined over the wealth (W_t); δ the pure rate of time preference, i.e., the classical exponential discount factor of future utility; ω an individual's maximum lifespan; and e_r the retirement age. The expected utility (EU) is given by:

$$EU = \sum_{t=e_r}^{\omega-1} \left[\frac{U(C_t)}{(1+\delta)^{t+1-e_r}} \; {}_{t+1-e_r}P_{e_r} + h_t \frac{V(W_{t+1})}{(1+\delta)^{t+1-e_r}} \; {}_{t-e_r}P_{e_r} \cdot q_t \right] \qquad \textbf{[1]}$$

where (ages in years):

W_{t+1}: Wealth corresponding to age t+1.
${}_{t+1-er}P_{er}$: Probability that an individual of age e_r survives t+1- e_r years more.
q_t: Probability that an individual dies at age t.
h_t : Relative weight of the utility of the bequest considered by the individual at age t with respect to the expected utility corresponding to the flow of consumption at age t: C_t.
${}_{t-er}P_{er}\ q_t$: Probability that an individual of age e_r survives to age t but dies before reaching age t+1.

It has been assumed that the discount rate of the future utility of the wealth to bequeath, δ, coincides with the rate of preference of the utility of consumption, since there exists no justification for any possible alternative values. This is the assumption used by most researchers, including Brown [2001], Friedman & Warshawsky [1990], Hurd [1989], and Fischer [1973]. Jousten [1998 & 2001], however, considers that δ, coincides with the real market interest rate.

One of the determining elements in the consumption optimization model when the bequest motive is included is the parameter h_t, which indicates the individual's assessment of the amount of wealth that could be bequeathed at any given time to his or her heirs. Friedman & Warshawsky [1990] consider that this parameter does not depend on age, and that it is related to the bequest-consumption ratio corresponding to the final period: W_ω / $C_{\omega-1}$. Likewise Brown [2001] and Jousten [1998] assume it to be constant over the life cycle. Other researchers, Hurd [1989 & 1999] for instance, do not weight the utility of bequests relative to that of consumption, and apply to both the same valuation on the part of the individual.

Fischer [1973] and Yaari [1965] consider the parameter that reflects the value given to the possibility of leaving a bequest, h_t, to be a hump-shaped function, due to the greater importance that individuals give the bequest in the mid-years of life when family members have a greater dependence on them, decreasing in retirement. This is valid for the case of the individual's motivations for leaving a bequest being altruistic. If, however, the retiree's ends are strategic in the sense of seeking to encourage family members to care for him or her in old age in exchange for the promise of a bequest, it would be more appropriate to consider that the parameter h_t increases with age.

2.1. The individual does not have access to the annuity market

With the assumptions that individuals have no access to the annuity market and that they value the existence of a bequest to leave their heirs, the consumption optimization model is, following Friedman & Warshawsky [1990]:

$$\max_{C} \sum_{t=er}^{\omega-1} \left[\frac{U(C_t)}{(1+\delta)^{t+1-er}}\ {}_{t+1-er}P_{er} + h_t \frac{V(W_{t+1})}{(1+\delta)^{t+1-er}}\ {}_{t-er}P_{er} \cdot q_t \right] \quad \textbf{[2]}$$

$$\text{s.t. } C_t = W_t(1+r) - W_{t+1} \quad \textbf{[3]}$$

$$W_t \geq 0, \quad \forall\, t \quad \textbf{[4]}$$

where:

r: Nominal expected risk free rate (assumed constant over the retiree's lifetime).

Only the utility deriving from the possible bequests once death occurs is considered. No account is taken of the fact that in certain cases the individual may want to make gifts of wealth while still alive, instead of waiting for the bequest to be made once dead. Making a gift of money before death could come about because of the different tax treatment applied to the two situations, or simply because of the individual's preference for his or her relatives to enjoy as soon as possible the goods (financial wealth) that he or she is able to give them in life.

2.2. The individual can contract a single-premium life annuity

Life annuities are a type of pension marketed by insurance firms in which, in exchange for an initial premium, they commit themselves to paying certain periodic amounts until the death of the policyholder, thereby taking on the annuitant's longevity risk. In the absence of annuities, retirees could reduce their annual consumption in response to the uncertainty in the date of their death. They then, however, would risk dying before consuming all their available wealth, which represents a cost of the missed opportunity for consumption. This cost will be less if the individuals value the wealth that remains at the time of their death as a bequest.

With the assumptions that the individual has access to the annuity market and has a bequest motive, the consumption optimization model is based on maximizing the objective function given in expression [2], but subject to the following constraints:

$$\text{s.t.} \qquad C_t = W_t(1+r) - W_{t+1} + A_{e_r}(1+\alpha)^{t-e_r} \qquad \textbf{[5]}$$

$$W_t \geq 0, \quad \forall\, t \qquad \textbf{[6]}$$

Expressed in this form, constraint [5] is similar to that in Brown et al. [1999, 2000], with W_{e_r} being the initial wealth after the deduction of the wealth allocated to annuitization, and A_{e_r} the initial amount of the life annuities purchased in exchange for a single premium $W_{ANNUITIES}$, given, when there exists no reversion on any beneficiary, by the following formula:

$$A_{e_r} = \frac{W_{ANNUITIES}}{\sum_{t=e_r}^{\omega} \frac{(1+\alpha)^{t-e_r} \cdot \theta}{\left[(1+i)\right]^{t-(e_r-1)}} \; {}_{t+1-e_r}P^*_{e_r}} \qquad \textbf{[7]}$$

$$A_t = A_{e_r}(1+\alpha)^{t-e_r}, \qquad \forall t \qquad \textbf{[8]}$$

This takes into account both the administrative costs (the term θ)) and the survival and mortality probabilities that the insurer applies in pricing the annuity ($_{t+1-e_r}P^*_{e_r}$) – which may differ from the consumer's subjective probabilities – as well as the variations that may exist

between the real technical interest rate guaranteed by the insurer (i) and the nominal market interest rate.

The parameter α represents the annual accumulated growth of the annuity. Two cases are considered in the calculations that follow: α equal to zero, with the annuity constant in nominal terms; and α equal to the expected Consumer Price Index (CPI), with the annuity constant in real terms. Although the CPI is here taken as constant, the results would be similar if it were allowed to vary, assuming that the annuity is perfectly indexed to the CPI. This is possible, Mitchell & McCarthy [2002a & 2000b] and James & Song [2001], in countries such as the United States, the United Kingdom, Chile, Israel, and more recently France, in which the government issues bonds fully indexed to the CPI. This allows insurers to offer annuity policies in which the variation of the annual contribution or term depends on the variation in the CPI for each fiscal year.

2.3. The individual can purchase a phased withdrawal contract

In a programmed withdrawal contract, Devesa & Vidal [2001], the retiree maintains an individual account of annuitization capital, and can withdraw each year an amount equal to the quotient of the accumulated fund in that account and the sum needed to pay out one unit of pension, as a function of the real return on the accumulated fund, of the life expectancy in the corresponding year, and of the technical interest rate used which will vary each year according to the evolution of the markets.

Programmed withdrawal, Valdés & Edwards [1998] and Valdés-Prieto [2002], allows a bequest to be left that will be equal to the unused balance of the individual account, but the date of payment and the amount of that bequest are random. An alternative in this sense is the (fixed or variable) life insurance policy, which has the advantage that the retiree can set the amount and the payment date.

Under this assumption, the maximization problem including the bequest motive becomes

$$\max_{C} \sum_{t=e_r}^{\omega-1} \left[\frac{U(C_t)}{(1+\delta)^{t+1-e_r}} \; {}_{t+1-e_r}P_{e_r} + h_t \frac{V(W_{t+1}+W_{t+1}^{PW})}{(1+\delta)^{t+1-e_r}} \; {}_{t-e_r}P_{e_r} \cdot q_t \right] \qquad [9]$$

$$\text{s.t. } C_t = W_t(1+r) - W_{t+1} + A_t \qquad [10]$$

$$W_t \geq 0, \quad \forall\, t \qquad [11]$$

with

$$W_{e_r} = W_0 - W_{e_r}^{PW} \qquad [12]$$

$$W_t^{PW} = W_{t-1}^{PW}(1+i) - A_{t-1}, \qquad \forall t \qquad [13]$$

where W_t^{PW} is the accumulated fund in the annuitization account at the beginning of the year corresponding to age *t*, and *i* is the real expected return on the annuitization account.

At each age *t*, the maximum annuity that can be withdrawn, A_t, will be given by:

$$A_t = \frac{W_t^{PW}(1+i)}{\sum_{s=t}^{\omega} \frac{(1+\alpha)^{s-t}\cdot\theta}{[(1+i)]^{s+1-t}}\ {}_{s+1-t}P^*_t}, \qquad \forall t \qquad \textbf{[14]}$$

The individual is assumed always to withdraw the maximum permitted annuity (A_t), and to hold an initial unannuitized wealth of W_{er}. It is also assumed that if the consumption in the year corresponding to age t is less than the amount of the programmed withdrawal, A_t, the surplus will be added to the unannuitized wealth. This assumption is easily relaxed by adding a new variable to the model – the wealth withdrawn from the annuitization account under the constraint that its amount is less than the maximum permitted annuity.

2.4. The individual has a pre-existing life annuity

Considering the annuity purchasing decision of an individual who already holds part of his or her wealth in a pre-existing life annuity, has a bequest motive, and has no access to the annuity market, the model has the following constraints:

$$\text{s.t.} \quad C_t = W_t(1+r) - W_{t+1} + R_t \qquad \textbf{[15]}$$

$$W_t \geq 0, \quad \forall\, t \qquad \textbf{[16]}$$

where

$$W_{\text{er}} = W_{UW}, \qquad \textbf{[17]}$$

and

$$W_0 = W_{UW} + W_{PA}, \qquad \textbf{[18]}$$

with:

W_{UW} : Level of initial unannuitized wealth.
W_{PA} : Level of initial wealth in pre-existing annuities.
R_t is a post-payable life annuity, assumed to derive from a pre-existing public pension scheme, given by the expression:

$$R_{e_r} = \frac{W_{PA}}{\sum_{t=e_r}^{\omega} \frac{(1+\pi)^{t-e_r}}{[(1+r)]^{t+1-e_r}}\ {}_{t+1-e_r}p^g_{e_r}} \qquad \textbf{[19]}$$

$$R_t = R_{e_r}(1+\pi)^{t-e_r}, \quad \forall\, t \qquad \textbf{[20]}$$

in which the wealth providing this pre-existing annuity is evaluated according to the general mortality probabilities (${}_{t+1-er}p^g_{er}$) and the real market interest rate, and the annuity is taken to be CPI-indexed (π).

Similarly, in the case that the individual can voluntarily contract a life annuity, the model involves maximizing the same objective function [2], but subject to the following constraints:

$$C_t = W_t(1+r) - W_{t+1} + R_t + A_t \qquad \textbf{[21]}$$

$$W_t \geq 0, \quad \forall\, t \qquad \textbf{[22]}$$

where W_{er} is the initial unannuitized wealth:

$$W_{\text{er}} = W_{UW} \qquad \textbf{[23]}$$

and

$$W_0 = W_{UW} + W_{PA} + W_{ANNUITIES} \qquad \textbf{[24]}$$

and A_t is calculated from expressions [7] and [8].

2.5. The utility function and the solution of the model

Under the assumption of constant relative risk aversion (which implies decreasing absolute risk aversion), the utility function corresponding to the individual's consumption is given by the expression:

$$U(C_t) = \begin{cases} \dfrac{\left[\dfrac{C_t}{(1+\pi)^{t+1-er}}\right]^{1-\beta} - 1}{1-\beta}, & \text{if } \beta \neq 1 \\ Ln\left[\dfrac{C_t}{(1+\pi)^{t+1-er}}\right], & \text{if } \beta = 1 \end{cases} \qquad \textbf{[25]}$$

where:

$\beta > 0$, the Pratt-Arrow coefficient, representing both risk aversion and the inverse of the elasticity of intertemporal consumption substitution. In practice, the aversion coefficient and the consumption substitution elasticity do not have to be inversely related, nor even necessarily linked. While substitution elasticity reflects consumer preferences for different periods, the risk aversion coefficient indicates how individuals wish to move their consumption between alternative states of nature. Despite these limitations, the potential utility function with decreasing absolute risk aversion and constant relative risk aversion has become the most widely used assumption in the financial (and even the macroeconomic) literature in the intertemporal context. There is currently a trend towards the revision of this concept. Davidoff et al. [2003] explore the possibility that the utility depends on a standard of living, i.e., that the period's utility is a function of present and future consumption. Ponzetto [2003], using the recursive utility function of Epstein & Zin, attempts to separate risk aversion from substitution elasticity.

The utility deriving from the bequest that remains at each time t is given by the same isoelastic function, in which death has been assumed to occur half way through the corresponding year (uniform distribution of deaths), so that the accumulated wealth on the date of death will be equal to that at the beginning of the period t+1, discounted for one half year: $W_{t+1}\ [(1+r)]^{-1/2}$. Hence, the function that gives the value of the utility of the bequest is given by the expression:

$$V(\mathrm{W}_{t+1}) = \begin{cases} \dfrac{\left[\dfrac{W_{t+1}\cdot\left[(1+r)\right]^{-1/2}}{(1+\pi)^{t+1-er}}\right]^{1-\beta} - 1}{1-\beta}, & \text{if } \beta \neq 1 \\ Ln\left[\dfrac{W_{t+1}\cdot\left[(1+r)\right]^{-1/2}}{(1+\pi)^{t+1-er}}\right], & \text{if } \beta = 1 \end{cases} \qquad \textbf{[26]}$$

The level of risk aversion that is used in the bequest utility function is the same as that applied to consumption. This is the criterion adopted by most researchers, Brown, Fischer [1973], and Friedman & Warshawsky [1990] for instance, although for Fischer [1973], the level of risk aversion might be expected to be higher in the bequest than in the consumption utility function, since individuals may be less willing to accept risk with respect to the welfare of their heirs than with respect to their own welfare. Jousten [1998] takes the completely contrary standpoint, considering that individuals are less averse to risk with respect to gifts and bequests than in their personal consumption, justifying the use of zero-valued risk aversion parameter reflected in the choice of a quasi-linear bequest utility function. Hurd [1989], too, considers that the level of risk aversion associated with the bequest motive is equal to zero.

The mathematical model was implemented in the language of the LINGO® suite of optimization programs. The results of the computations are presented in the various figures and tables.

2.6. Optimal consumption path with a bequest, for each type of pension

The solution of the model yields the optimal consumption path that maximizes the individual's expected utility including a possible bequest. The following assumptions and parameter values were used in the calculations:

1. The GRMF-95 survival and mortality tables.
2. The insurance firm applies no type of costs in the single-premium life annuity contract (θ=1).
3. The survival probabilities that the insurer uses in setting the premium coincide with the consumer's probabilities (${}_{t+1-er}\mathrm{P}^{*}_{er} = {}_{t+1-er}\mathrm{P}_{er}$).
4. The nominal market interest rate, r, coincides with the annuity's technical interest rate, and is equal to 4.545%.

5. The level of risk aversion β takes the three values (0.7, 2.9, and 4.4). While there is no consensus in the literature on which values should be used for the degree of aversion to risk, the present study uses values close to those employed in Brown [1999].
6. Retirement age for both men and women is 65.
7. Expected inflation rate equal to 1.5%.
8. Rate of preference according to the individual's level of impatience given by the expression $\delta = \lambda[(1+r)-1]$, where the values of the parameter λ (2, 1, and 0.25) classify the individuals as, respectively: (A) very impatient, (C) indifferent to impatience, and (E) very patient. In most of the papers cited the level of impatience is not usually emphasized. Seldom are impatient or very impatient individuals considered which could be due to the fact that they present greater difficulties to calculate.

Two alternatives were also taken into account for the function h_t. On the one hand, when there exist altruistic motives for the bequest, the function was taken to decrease after the age of 65, since this is the individual's inactive phase. On the other hand, if there exists a strategic interest in bequeathing wealth in exchange for possible assistance from the family in old age, the function h_t was taken as increasing with age. In particular, the values of h_t are the following:

a. Altruistic bequest motives:

$$h_t = h_{t-1}/1.02 \quad ; \quad h_\omega = 2 \qquad , \forall\ t < \omega \qquad \textbf{[27]}$$

b. Strategic bequest motives:

$$h_t = h_{t-1} \cdot 1.02 \quad ; \quad h_{e_r} = 2 \qquad , \forall\ t > e_r \qquad \textbf{[28]}$$

As was noted above, there is no consensus in the literature on how to model bequest motives, and the values considered for the parameter weighting the utility of bequest relative to the utility of consumption are quite disparate. Thus, Brown [2001] uses two different hypotheses 0.5 and 1. Fischer [1973] considers values in a range of approximately 4.5-9.8 – or 28.2-120.8 with the rate of consumption preference hypothesis[2] – starting from the age of 65. The bequest parameter[3] applied in the model of Jousten [1998] is equal to $5.5*10^{-5}$. Finally, Friedman & Warshawsky [1990] determine the optimal percentage of wealth to annuitize assuming that the bequest parameter can vary between 0 and 100.

Figure 1 shows the optimal consumption path that maximizes the expected utility for each of the types of pension studied, for a man who wants to leave a bequest to his heirs, considering the motives to be altruistic in the first case and strategic in the second[4]. During

[2] To determine the range of values used by Fischer [1973], account was taken of the effect of the utility function discount rate for each of the two hypotheses considered (approximately 4.17% and 8.70%, respectively) which is incorporated into the weight of the bequest utility function.

[3] Bequest motive is in the form of a linear bequest utility term. The parameter on the linear bequest utility term is $5.5*10^{-5}$.

[4] Due to the large number of graphs that would be needed to represent the consumption flow associated with each of the results shown in the tables, we decided to select those we considered the most representative

the first years, the level of consumption is, in all the assumptions, less than the corresponding level in the individual case[5], since in this model the amount of wealth remaining in each period contributes positively to the expected utility that is to be maximized, so that the individual prefers to give up a part of his consumption in exchange for more wealth to bequeath.

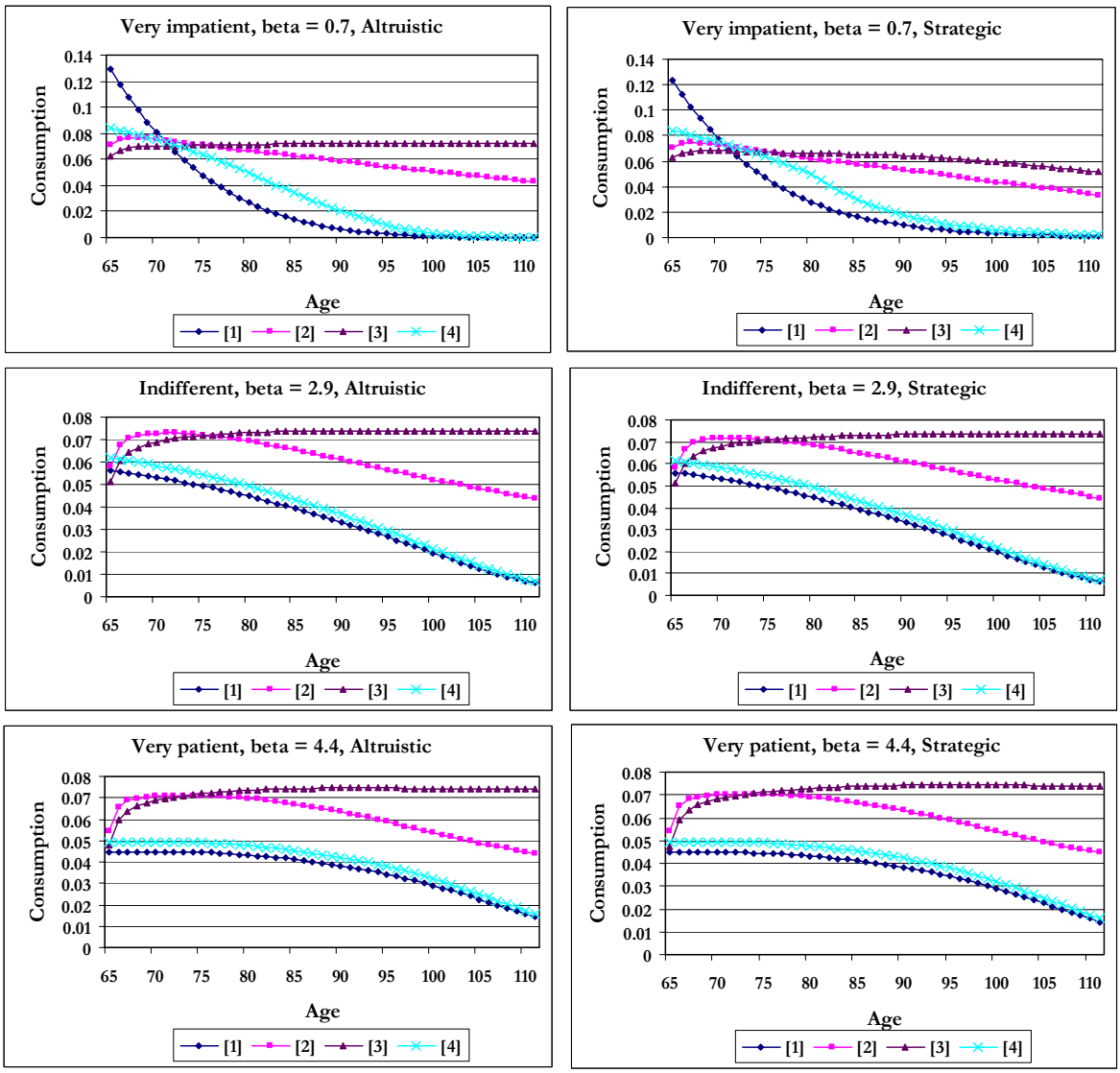

Figure 1. Optimal consumption path for men with bequest motive.
[1]No annuities, [2]Unindexed annuity, [3]Indexed annuity, [4]Programmed withdrawal.

What perhaps attracts the attention most in Figure 1 is the optimal consumption profile in the case where an individual does not have access to the annuities market **[1]**, changing from concave to convex because it deals with extreme cases. The convex profile represents an individual who is very impatient to consume, who places much more value on present rather than future consumption and, in addition, does not have much aversion towards

to reflect the variations in this consumption flow, when the main parameters that are used to determine them are modified.

[5] Throughout the study, *individual* case is used to refer to the situation of a single individual with no bequest motive.

outliving his resources. Because of this and his probabilities of survival, which are much higher in the years immediately following retirement, his consumption is concentrated in the initial years. An individual with little impatience for consumption, on the other hand, places more value on future rather than present consumption. With a high level of aversion to the risk of longevity, this individual tends to hold back on consumption to cover the number of years he may possibly live.

Table 1 shows the differences between optimal consumption for each type of pension, with special emphasis on the bequest motive. Although Figure 1 gives an idea of the path of optimal consumption, due to there being only small differences it is difficult in some cases to distinguish which individual consumes more, the one with an altruistic motive or the one with a strategic motive.

Table 1: Comparison between optimal consumptions with bequest motives

Type of pension	Very impatient beta = 0.7	Indifferent beta = 2.9	Very patient beta = 4.4
No annuities	Al > St; 65-77 years Al < St; 78-110 years	Al >St; 65-96 years Al < St; 97-110 years	Al > St; 65-109 years Al < St; 110 years
Unindexed annuity	Al > St; 65-110 years	Al > St; 65-91 years Al < St; 92-110 years	Al > St; 65-92 years Al < St; 93-110 years
Indexed annuity	Al > St; 65-110 years	Al > St; 65-110 years	Al > St; 65-104 years Al < St; 105-110 years
Programmed withdrawal	Al = St; 65-80 years Al >St; 81-93 years Al < St; 94-110 years	Al >St; 65-96 years Al < St; 97-110 years	Al > St; 65-109 years Al < St; 110 years

As Table 1 shows, it can be said that in general terms the individual with an altruistic motive will always consume more than the individual with a strategic motive, although some exceptions can be found which will depend on the type of pension and the combination of impatience for consumption and the degree of risk aversion. In those cases where consumption is higher for an individual with strategic motives, it always begins at a more advanced age - from ages 78, 92, 105 and 81 years respectively for No annuities, Unindexed annuity, Indexed annuity and Programmed withdrawal. This consumption behavior is perfectly understandable since the pensioner with a strategic bequest motive wants to build up savings during the initial years and thereby give his family an incentive for looking after him.

3. EQUIVALENT WEALTH

In this paper the gain in welfare is computed by means of the equivalent wealth, a measure of what would be the level of wealth required to be on the same expected utility curve in whichever of the cases analysed. This measure is aimed at determining how much the individual – averse to risk and with a bequest motive – would value the possibility of purchasing either a life annuity or a programmed withdrawal contract and thus being able to protect him or herself against the risk of excessive longevity, including both financial and psychological parameters.

The *equivalent wealth* is given by μW_0, where the coefficient μ is defined as:

$$\mu = \frac{W_0 + \Delta W}{W_0} = 1 + \frac{\Delta W}{W_0} \qquad \textbf{[29]}$$

and ΔW is the amount of additional wealth the consumer would need to have access to so that, following his or her optimal consumption rate in any one of the cases considered, the same level of utility would be attained as when the consumption rate is optimized in any other of the cases. The gain in welfare, expressed as a percentage of the initial wealth W_0, is given by the ratio $\Delta W / W_0$.

More precisely, the equivalent wealth will be determined by the percentage μ_{ij} which satisfies:

$$UE_i(\mu_{ij} W_0) = UE_j(W_0), \qquad i, j = 1,2,3,4 \qquad \textbf{[30]}$$

where:

$UE_1(W)$: Expected utility derived from the optimal consumption rate that the individual would choose in Case 1 (no access to the annuity market), for an initial wealth at the age of retirement equal to W.
$UE_2(W)$: Expected utility derived from the optimal consumption rate that the individual would choose in Case 2 (purchasing an unindexed life annuity with a single premium equal to the total wealth at retirement W).
$UE_3(W)$: Expected utility derived from the optimal consumption rate that the individual would choose in Case 3 (purchasing an indexed life annuity with a single premium equal to the total wealth at retirement W)
$UE_4(W)$: Expected utility derived from the optimal consumption rate that the individual would choose in Case 4 (purchasing a programmed withdrawal contract with the wealth at retirement).

The parameter μ_{ij} can take three values: if it is greater than one, then alternative j is preferred over i; if it is less than one, the contrary is the case; and if it is equal to one, there is indifference to which of the two alternatives is chosen.

3.1. Equivalent wealth with the bequest motive and life annuities

In this subsection, the same parameter values and assumptions that were applied in Section 2.6 are used to compute the value of the equivalent wealth for men and women with altruistic and with strategic bequest motives.

Tables 2-3 list the results[6] comparing the purchase of an indexed or unindexed life annuity with the situation in which the individual has no access to the annuity market. The equivalent wealth is also compared with the same situations when the model does not include the bequest motive (listed under 'individual').

[6] Unless otherwise stated in the source, all the tables in this paper were drawn up by the authors.

Table 2: Equivalent wealth for men.										
δ ↓	β→	Altruistic			Individual			Strategic		
		0.7	2.9	4.4	0.7	2.9	4.4	0.7	2.9	4.4
A) Unindexed annuity										
A		**0.801**	1.222	1.314	1.092	1.448	1.546	**0.938**	1.289	1.377
C		**0.933**	1.362	1.442	1.292	1.545	1.609	1.082	1.415	1.491
E		1.043	1.460	1.529	1.433	1.596	1.641	1.184	1.497	1.562
B) Indexed annuity										
A		**0.764**	1.146	1.211	1.049	1.406	1.515	**0.901**	1.222	1.286
C		**0.910**	1.314	1.367	1.281	1.555	1.638	1.068	1.382	1.436
E		1.037	1.449	1.495	1.467	1.662	1.724	1.196	1.506	1.554

Table 3: Equivalent wealth for women.										
δ ↓	β→	Altruistic			Individual			Strategic		
		0.7	2.9	4.4	0.7	2.9	4.4	0.7	2.9	4.4
A) Unindexed annuity										
A		**0.832**	1.140	1.207	**0.994**	1.266	1.333	**0.915**	1.175	1.241
C		**0.962**	1.246	1.298	1.176	1.336	1.373	1.051	1.268	1.319
E		1.047	1.307	1.348	1.273	1.367	1.392	1.123	1.317	1.358
B) Indexed annuity										
A		**0.787**	1.065	1.110	**0.943**	1.218	1.297	**0.870**	1.108	1.156
C		**0.939**	1.210	1.244	1.162	1.346	1.400	1.037	1.244	1.283
E		1.047	1.315	1.345	1.310	1.426	1.464	1.137	1.338	1.373

The bequest motive always noticeably reduces the equivalent wealth with respect to the individual case model, i.e., it makes annuities less attractive, increasing the number of individual profiles who will prefer not to purchase an annuity. This extends even to those who, showing little risk aversion, feel little impatience for consumption – as in the case of men who purchase fully CPI-indexed annuity contracts. As also is the case for individuals with no bequest motive, the welfare attained with annuities increases with increasing risk aversion and decreasing impatience for consumption.

Also, unlike the case in which the individual has no bequest motive, the results indicate that it is preferable to purchase an unindexed than an indexed annuity in practically all the profiles. The reason is that, in the initial years, the indexed annuity provides a smaller income flow, so that the individual does not have the desired level of wealth in order to leave a bequest to the heirs. Only in the case of strategic bequest motives can greater welfare be attained with the purchase of an indexed annuity, as long as the level of impatience for consumption is very low. This is because, in the case of strategic bequest motives, future bequeathable wealth is valued more than current wealth, and the indexed annuity allows consumption to be rationalized in the initial years in order to provide greater income in the future, thereby increasing the amount of bequeathable wealth.

When the level of risk aversion is low and the bequest motive is altruistic, women achieve greater welfare than men by purchasing life annuities. People with altruistic bequest motives attain less welfare with the purchase of an annuity than those with strategic motives. This is logical since the altruistic individual values the bequest more in the initial years, and thus consumes less in this period. The consequence is a decrease in the utility deriving from that consumption, which is that with greatest weight in the expected utility.

As was noted above, an increase in the parameter b_t reduces the welfare obtained by purchasing life annuities when the bequest motive is included. Greater values of this parameter lead to notably smaller values of the equivalent wealth. As an example, Table 2, for a man with a level of risk aversion of 0.7, who is very impatient to consume (case A), and who has an altruistic bequest motive, the equivalent wealth decreases by 11% from 0.801 to 0.711 when the value of the weight of the bequest in the expected utility is taken to be constant and equal to 20. If the level of risk aversion is equal to 2.9 and the attitude towards consumption is neutral (case C), the decrease is somewhat less – 8% – from an equivalent wealth of 1.362 with the value of the parameter used in the present paper to 1.257 with a bequest parameter equal to 20.

In all the tables the values for the equivalent wealth of the non-indexed (indexed) annuities that are greater than those for indexed (non-indexed) annuities have been underlined. Values less than 1, which mean that wealth without annuities is preferable, are shown in bold.

Another way to measure the preference that individuals show in accessing the annuity market is to calculate the maximum percentage of wealth accumulated at retirement that they are willing to annuitize. To determine this maximum percentage it has to be established that the wealth allocated to annuities is a control variable of the problem set. It should not be considered as taking on a fixed value equal to the initial wealth the individual has available at the time of retirement; this assumption is made to obtain the value of equivalent wealth. As was noted before, the mathematical models in this paper have been translated into LINGO® software programming language, and this program was used to obtain the numerical results shown in the various tables. More details can be found in Lejárraga [2003].

As can be seen in Table 4, the greater their impatience to consume, the smaller the annuitization percentage. Retirees, both men and women, with a very low level of risk aversion and a high level of impatience to consume ($\beta = 0.7$ and case A) would prefer to consume their accumulated wealth directly and would allocate no amount to a life annuity.

Table 4: Optimal percentage of wealth allocated to purchasing an indexed annuity.

δ ↓ / β →	Men (%)						Women (%)					
	Altruistic			Strategic			Altruistic			Strategic		
	0.7	**2.9**	**4.4**	**0.7**	**2.9**	**4.4**	**0.7**	**2.9**	**4.4**	**0.7**	**2.9**	**4.4**
A	0.00	74.61	81.52	31.41	79.14	84.05	0.00	72.18	80.07	22.23	75.42	81.90
C	24.64	84.83	88.04	74.39	89.09	90.47	29.80	85.97	89.18	71.71	89.37	91.11
E	69.64	91.40	92.10	100.0	94.84	94.21	80.80	94.14	94.29	100.0	96.71	95.91

As indicated by Brown [2001], in the presence of a bequest motive, greater welfare gains result when only a certain percentage of the wealth available at retirement is annuitized instead of the whole of that wealth. Indeed, the only case in which the optimal choice is 100% annuitization is that of an individual with a strategic bequest motive, a very low level of risk aversion, and who is very patient with respect to consumption.

Greater risk aversion implies a greater annuitization percentage, except in the case that the retiree has a strategic bequest motive and is very patient with respect to consumption, i.e., in the case where the level of risk aversion has greater weight in the bequest utility function

than in the consumption utility function. Also, individuals with an altruistic bequest motive will always annuitize a smaller percentage than those with a strategic motive.

3.2. Equivalent wealth with the bequest motive and programmed withdrawal

In a programmed withdrawal contract, when the real return obtained on the annuitization account is equal to the real market interest rate, one observes in Tables 5-6 that from a certain level of risk aversion and impatience for consumption the individual obtains the same utility by purchasing this type of pension as when no annuity is purchased. This is because the optimal consumption rate in these cases is less than the amount of the available annuity, so that the two situations are valued the same. When, however, the individual shows little risk aversion and is also very impatient to consume (case A), the utility of not purchasing annuities is greater than that with a programmed withdrawal contract.

Table 5: Equivalent wealth for men.

δ↓	β →	Altruistic				Individual				Strategic			
		i=3%		i=4%		i=3%		i=4%		i=3%		i=4%	
		0.7	2.9	0.7	2.9	0.7	2.9	0.7	2.9	0.7	2.9	0.7	2.9
A) Unindexed programmed withdrawal													
A		**0.997**	1.000	1.074	1.092	**0.927**	1.000	1.018	1.092	**0.978**	1.000	1.064	1.092
C		1,000	1.000	1.080	1.092	**0.995**	1.000	1.089	1.092	1.000	1.000	1.086	1.092
E		1,000	1.000	1.083	1.092	1.000	1.000	1.092	1.092	1.000	0.999	1.087	1.092
B) Indexed programmed withdrawal													
A		**0.988**	1.000	1.072	1.101	**0.888**	1.000	0.985	1.102	**0.957**	1.000	1.049	1.102
C		1.000	1.000	1.086	1.101	**0.981**	1.000	1.084	1.102	1.000	1.000	1.093	1.102
E		1.000	1.000	1.090	1.101	1.000	1.000	1.101	1.102	1.000	0.999	1.095	1.102

Unlike the life annuities case, programmed withdrawal is not always more attractive when the bequest motives are strategic. Instead, one observes that individuals who are very impatient for consumption and, in general, with little risk aversion obtain greater welfare with this type of pension when their bequest motive is altruistic than when it is strategic.

Likewise, only individuals with little risk aversion and great impatience for consumption obtain greater welfare through an unindexed programmed withdrawal contract than through an indexed contract. This is different from the life annuities case, in which an unindexed annuity was preferable for practically all the individual profiles.

The level of risk aversion is the determining factor in the level of welfare that is attained, since, when it is high, the same equivalent wealth is obtained independently of the individual's profile of impatience for consumption.

In practically all cases for both men and women, when the level of risk aversion is low, the equivalent wealth is greater for an indexed programmed withdrawal contract when a bequest motive is included. This is because in the individual case no valuation is made of the funds that remain in the individual annuitization account, and hence they do not contribute to the expected utility. When the level of risk aversion is high, the welfare attained is independent of the consumption impatience profile, in all three bequest motive contexts – altruistic, strategic, and no motive to leave a bequest.

Table 6: Equivalent wealth for women.													
δ↓	β →	Altruistic				Individual				Strategic			
		i=4.55%		i=5.56%		i=4.55%		i=5.56%		i=4.55%		i=5.56%	
		0,7	2,9	0,7	2,9	0,7	2,9	0,7	2,9	0,7	2,9	0,7	2.9
A) Unindexed programmed withdrawal													
A		**0.973**	1,000	1,073	1,110	**0,913**	1,000	**1,019**	1,110	**0,954**	1,000	1,059	1,110
C		1.000	1,000	1,102	1,110	**0,997**	1,000	1,108	1,110	1,000	1,000	1,106	1,110
E		1.000	1,000	1,104	1,110	1,000	1,000	1,110	1,110	1,000	1,000	1,107	1,110
B) Indexed programmed withdrawal													
A		**0.945**	1.000	1.054	1.123	**0.865**	1.000	**0.977**	1.123	**0.919**	1.000	1.031	1.123
C		1.000	1.000	1.112	1.123	**0.981**	1.000	1.104	1.123	**0.999**	1.000	1.116	1.123
E		1.000	1.000	1.115	1.123	1.000	1.000	1.123	1.123	1.000	1.000	1.118	1.123

In the individual case, greater welfare always results from purchasing a life annuity than a programmed withdrawal contract. In the bequest motive contexts, however, the contrary is the case for retirees showing a low level of risk aversion and great impatience to consume. This preference is more marked in the altruistic case when they value the bequest more in the first years of retirement than in the strategic case. This is so even when the expected return on the individual annuitization account coincides with the real market interest rate.

4. EQUIVALENT WEALTH AND MARKET *IMPERFECTIONS*[7]

Annuity markets, Blake [1999], are insufficiently developed even in many of the financially most advanced countries. There is enough empirical evidence in the economics literature to show that people who voluntarily decide to purchase annuities live longer than the average general population. In their international study, Mitchell & McCarthy [2002b] show that the adverse selection associated with voluntary purchasers of life annuities reduces the expected mortality rates by at least 25%. Further evidence relating to adverse selection, can be found in papers by Mung [2002], Poterba [2001] and Villeneuve [2003].

Friedman & Warshawsky [1990] and more recently Mitchell et al. [1999] have shown that annuities in the United States tend to be more expensive than would be expected from the life expectancy of the general population. In particular, Mitchell et al. [1999] calculate that, for a 65-year-old male, the value of the annuity is 15-25% less than would correspond to the general mortality tables. They conclude that about half of this reduction is because the annuitants live longer than the US mean, and that the rest is imputable to administrative costs. Finkelstein & Poterba [2000] find that the value of annuities contracted voluntarily by 65-year-old men in the United Kingdom is 10-15% less than that deriving from the use of the mortality tables of the overall population. More than 60% of this reduction is due to the longer life expectancy of the annuitants compared with the population mean. There exists international evidence, Devesa et al. [2002] and [2003], that the transformation of the accumulated savings of an individual annuitization account into a life annuity may involve high costs for the participant.

Together with the mortality[8] and the intermediation costs, another major factor in determining life annuities is the technical interest rate that is applied. While individuals

[7] Some authors show their complete disagreement with the term *imperfection*, and simply attribute the decrease in the amount of the annuities on the market over the actuarially fair amount to the price that has to be paid to the company for assuming the financial risk and the risk of longevity.

might not understand how the use of a mortality rate or the administrative costs applied by the insurance company influence the price that they pay for the annuities, they may be very sensitive with respect to guaranteed interest rates.

One final aspect that could be considered an imperfection of the annuities market is the fact that in some countries - Belgium, for instance, as already mentioned in the introduction - annuities are taxed at a higher rate than other types of pension.

The inclusion in the model of the existence of so-called market *imperfections*, Tables 7-8, leads to lower values of the equivalent wealth than in the context of actuarially fair markets. In particular, it was considered that the combined effect of the different mortality and survival probabilities used by the insurance firm in marketing life annuities, the application of administrative costs, and the possible differences between the interest rate guaranteed in the annuity and the market rate (the term θ in expression [7]), reduces the annuities by 15% with respect to the actuarially fair value.

Table 7: Equivalent wealth for men with 15% reduction factor.

δ↓	β→	Altruistic			Individual			Strategic		
		0.7	**2.9**	**4.4**	**0.7**	**2.9**	**4.4**	**0.7**	**2.9**	**4.4**
A) Unindexed annuity										
A		**0.681**	**1.039**	**1.117**	**0.928**	**1.231**	**1.314**	**0.798**	**1.096**	**1.171**
C		**0.793**	1.158	1.226	1.098	1.313	1.368	**0.919**	1.203	1.267
E		**0.886**	1.241	1.300	1.218	1.357	1.395	1.007	1.272	1.328
B) Indexed annuity										
A		**0.649**	**0.974**	1.029	**0.892**	1.195	1.288	**0.766**	1.039	1.093
C		**0.774**	1.117	1.162	1.089	1.322	1.392	**0.908**	1.175	1.221
E		**0.881**	1.232	1.271	1.247	1.413	1.466	1.016	1.280	1.321

Table 8: Equivalent wealth for women with 15% reduction factor.

δ ↓	β→	Altruistic			Individual			Strategic		
		0.7	**2.9**	**4.4**	**0.7**	**2.9**	**4.4**	**0.7**	**2.9**	**4.4**
A) Unindexed annuity										
A		**0.707**	**0.969**	1.026	**0.845**	1.076	1.133	**0.778**	**0.999**	1.054
C		**0.817**	1.059	1.104	**0.999**	1.135	1.167	**0.893**	1.077	1.121
E		**0.890**	1.111	1.146	1.082	1.162	1.183	**0.955**	1.119	1.155
B) Indexed annuity										
A		**0.669**	**0.906**	**0.944**	**0.801**	1.036	1.103	**0.739**	**0.942**	**0.983**
C		**0.798**	1.028	1.058	**0.988**	1.144	1.190	**0.882**	1.057	1.090
E		**0.890**	1.118	1.143	1.113	1.212	1.244	**0.966**	1.137	1.167

In a similar way as occurred in the case of an individual who has no bequest motive, the reduction in the value of the equivalent wealth caused by introducing market *imperfections* coincides with the percentage decrease considered for the annuity with respect to the "actuarially fair" value, i.e., 15% for all the individual profiles that were analyzed.

[8] As Pierre Devolder has pointed out, the use of different mortality tables is not really the imperfection. If the tables used by the insurer give a fair estimate of the mortality rate of people choosing annuities (which is lower), there is no imperfection even though it means that tables other than those covering the general population are used. The imperfection arises from the "extra" safety margin the insurer can include, which is not justified judging by the experience of annuitants.

It is important to note that, in almost all cases, individuals with little risk aversion would prefer not to purchase a life annuity.

The trend of the preferences between indexed and unindexed annuities is the same for the different individual profiles as in the "actuarially fair" case. As also in this latter case, when market *imperfections* are included, women with a low level of risk aversion attain greater welfare with life annuities, whether indexed or not, than men do. In no case, however, do they prefer the purchase of annuities over the consumption of current wealth.

5. EQUIVALENT WEALTH AND PRE-EXISTING LIFE ANNUITIES

In some countries there is a certain freedom of choice, and there can be pre-existing annuities from the first pillar of the public pension system. This is the case in Argentina, Costa Rica, and the United Kingdom. One must also include the case of countries, Spain for example, which have a voluntary system that complements the public pension system, in which the benefits are exclusively in the form of joint annuity with contingent survivor benefit, with the participant having the possibility of transforming the accumulated savings at retirement into an additional life annuity.

5.1. - Without market imperfection

The results listed in Tables 9-10 indicate that in the presence of pre-existing annuities there is a reduction in the welfare attained with the purchase of an actuarially fair life annuity. The reduction is up to 22% and 15% for men and women, respectively, with respect to the value attained when no bequest motive is considered.

Table 9: Equivalent wealth with 50% of the wealth in pre-existing annuities for men.

δ ↓	β→	Altruistic			Individual			Strategic		
		0.7	**2.9**	**4.4**	**0.7**	**2.9**	**4.4**	**0.7**	**2.9**	**4.4**
A) Unindexed annuity										
A		**0.852**	**0.928**	**0.933**	**0.959**	1.102	1.155	**0.910**	**0.985**	**0.995**
C		**0.926**	1.028	1.036	1.074	1.178	1.222	**0.997**	1.081	1.092
E		**0.989**	1.111	1.120	1.168	1.237	1.261	1.063	1.154	1.166
B) Indexed annuity										
A		**0.834**	**0.892**	**0.882**	**0.939**	1.078	1.131	**0.892**	**0.952**	**0.948**
C		**0.914**	1.001	**0.993**	1.067	1.182	1.223	**0.995**	1.060	1.057
E		**0.989**	1.099	1.092	1.183	1.260	1.289	1.068	1.151	1.150

In general, consumers with little risk aversion independently of their attitude towards consumption, or who are very impatient with respect to consumption independently of their attitude towards risk, will lose welfare by acquiring additional life annuities. It is notable that, considering indexed annuities, when an individual is already receiving part of his or her wealth in the form of life annuities, the welfare attained with the purchase of an additional annuity decreases after a certain level of risk aversion except for women with a strategic bequest motive, i.e., in these cases the level of risk aversion has more weight in the bequest utility function than in the consumption utility function.

Table 10: Equivalent wealth with 50% of the wealth in pre-existing annuities for women.										
δ ↓	β→	Altruistic			Individual			Strategic		
		0.7	2.9	4.4	0.7	2.9	4.4	0.7	2.9	4.4
A) Unindexed annuity										
A		**0.867**	**0.942**	**0.951**	**0.924**	1.044	1.084	**0.902**	**0.976**	**0.989**
C		**0.948**	1.030	1.039	1.038	1.112	1.135	**0.995**	1.057	1.069
E		1.004	1.091	1.100	1.110	1.147	1.160	1.047	1.108	1.119
B) Indexed annuity										
A		**0.845**	**0.904**	**0.900**	**0.899**	1.015	1.056	**0.880**	**0.941**	**0.942**
C		**0.936**	1.007	1.003	1.030	1.109	1.136	**0.988**	1.039	1.040
E		1.003	1.088	1.085	1.126	1.169	1.187	1.053	1.112	1.115

5.2. Pre-existing life annuities and market *imperfections*

It was considered that the retiree already receives a part of his or her wealth – in particular 50% – in the form of a life annuity linked to the expected CPI, and also purchases a life annuity that includes market *imperfections*. The results – Tables 11-12 – show a reduction in the welfare attained with respect to the same situation except that the life annuity the retiree can purchase is "actuarially fair".

Table 11: Equivalent wealth with 50% of the wealth in pre-existing annuities for men with 15% reduction factor.										
δ ↓	β→	Altruistic			Individual			Strategic		
		0.7	2.9	4.4	0.7	2.9	4.4	0.7	2.9	4.4
A) Unindexed annuity										
A		**0.796**	**0.848**	**0.845**	**0.885**	1.002	1.046	**0.844**	**0.899**	**0.901**
C		**0.859**	**0.934**	**0.935**	**0.983**	1.075	1.106	**0.919**	**0.982**	**0.987**
E		**0.913**	1.007	1.011	1.065	1.122	1.142	**0.977**	1.047	1.054
B) Indexed annuity										
A		**0.781**	**0.818**	**0.804**	**0.868**	**0.981**	1.026	**0.829**	**0.871**	**0.862**
C		**0.849**	**0.911**	**0.899**	**0.977**	1.072	1.106	**0.913**	**0.964**	**0.957**
E		**0,910**	**0.997**	**0.987**	1.077	1.141	1.165	**0.981**	1.044	1.039

Table 12: Equivalent wealth with 50% of the wealth in pre-existing annuities for women with 15% reduction factor.										
δ ↓	β→	Altruistic			Individual			Strategic		
		0.7	2.9	4.4	0.7	2.9	4.4	0.7	2.9	4.4
A) Unindexed annuity										
A		**0.808**	**0.864**	**0.867**	**0.855**	**0.955**	**0.988**	**0.838**	**0.894**	**0.902**
C		**0.878**	**0.941**	**0.946**	**0.954**	1.016	1.035	**0.918**	**0.966**	**0.974**
E		**0.926**	**0.996**	1.001	1.017	1.047	1.058	**0.964**	1.012	1.020
B) Indexed annuity										
A		**0.789**	**0.831**	**0.824**	**0.834**	**0.930**	**0.965**	**0.819**	**0.864**	**0.863**
C		**0.868**	**0.921**	**0.915**	**0.947**	1.012	1.035	**0.912**	**0.951**	**0.949**
E		**0.926**	**0.993**	**0.988**	1.030	1.066	1.080	**0.969**	1.015	1.015

As was found when the values of the equivalent wealth with no bequest motive were computed, there was an even more spectacular increase in individuals who theoretically would prefer not to purchase a life annuity since that would produce a sharp decline in the resulting welfare. The individuals with greatest risk aversion and with very little impatience for consumption, for whom the purchase of a life annuity would still be preferable, would achieve negligible welfare gains, even though these would always be greater in the strategic bequest motive case than in the altruistic case.

The foregoing results greatly clarify the annuity puzzle for single individuals. The successive incorporation of market elements together with the consideration of the bequest motive practically resolve the puzzle, since under these conditions very few individuals would be willing to purchase life annuities, as is indeed the case in reality when people are allowed freedom of choice.

6. CONCLUSIONS, POLICY RECOMMENDATIONS, AND FUTURE RESEARCH

The present paper has contributed to clarifying the "annuity puzzle" by including the bequest motive. To this end, the evaluation was presented of an ample set of assumptions that have received little attention in the economics literature. Also, some light was shed on whether the bequest motive in itself is really a major factor influencing the theoretical decision to purchase annuities.

In all the contexts and individual profiles considered, people with an altruistic motive to bequeath part of their wealth obtain less welfare by purchasing a life annuity than those who have a strategic motive, although there exist exceptions to this rule in the programmed withdrawal type of contract.

The basic model, which does not include the extra characteristics of market imperfections and pre-existing annuities, gives the best measure of the true impact of the bequest motive on the decision to purchase annuities. In the absence of interference from these other characteristics, it seems to indicate that the bequest motive in isolation is not really a major factor, although it has to be noted that:

1) It reduces the attractiveness of annuities, increasing the number of individual profiles that will prefer not to purchase an annuity, extending to those who, showing little risk aversion, even feel little impatience for consumption.

2) Unlike the case in which the individual has no interest in leaving a bequest, the results indicate that for practically all the profiles the purchase of an unindexed annuity has more utility than that of an indexed annuity. Only in the case when the motives for the bequest are strategic, can greater welfare be attained with the purchase of an indexed annuity, and then only as long as there is very little impatience for consumption.

3) With respect to the programmed withdrawal type of pension, only individuals with little risk aversion and who are very impatient for consumption would prefer an indexed programmed withdrawal contract over an unindexed contract. The level of risk aversion is the determining factor, since for high levels of this parameter the resulting equivalent wealth is the same, independently of the individual's profile of impatience for consumption.

4) Retirees who show a low level of risk aversion and a great impatience for consumption would prefer the programmed withdrawal contract over life annuities. This is so even when the return on the individual annuitization account coincides with the real market interest rate.

The generalized model, with all the characteristics included – market *imperfections* and pre-existing annuities – practically resolves the so-called "annuity puzzle", since under these conditions very few individuals would be willing to purchase life annuities, as is usually the case in reality when they have freedom of choice.

The results lend support to the conclusions drawn from previous models in the sense that: (i) a certain flexibility seems advisable in regulating the types of pension schemes so as to accommodate individual circumstances and public policy objectives; and (ii) in defined contribution schemes that complement defined benefit pension systems, in which individuals already hold a major portion of their wealth in the form of life annuities, freedom should be total, with no obligation to allocate any amount whatsoever to purchasing additional life annuities.

Finally, further research could focus on:

a) Developing a model to include the bequest motive from the standpoint of couples. The difficulty in this case would be to determine the level of welfare provided by life annuities, as this would require more assumptions to be made regarding the structure of both the utility function and the beneficiaries' pensions.

b) Modeling the fact that life annuities are virtually the only product that can guarantee an interest rate in the long-term. However, in the model followed in this paper all the other types of pension have been assigned a fixed return, which invalidates the supposed superiority of life annuities where this aspect is concerned. This could be very important when financial markets are undergoing periods of instability (which is almost always) and for those pensioners who are more averse to risk.

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