Are We Better Off If Our Politicians Have More Information?

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Abstract

This paper studies a model of public policy with heterogenous citizens/voters and two public goods: one (roads) is chosen directly by an elected policymaker, and the other (pollution) depends stochastically on the amount of roads. Both a one-country and a two-country version of the model are analyzed, the latter displaying externalities across the countries which creates incentives for free riding and strategic delegation. The welfare effects of providing the policymaker with information about the relationship between roads and pollution are investigated, and it is shown that more information hurts some — sometimes even all — citizens. In particular, the absence of an institution for information gathering can serve as a commitment device for a country, helping it avoid the free-riding problem. Implications for the welfare effects of "informational lobbying" are discussed. //[Doc: Better-off-11.tex]//

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I. Introduction

Information, or lack thereof, plays an important role in many political processes. Indeed, often public policy must be decided on without full information about the consequences of different policy alternatives.¹ There are, however, several ways through which governments actively try to acquire policy-relevant information prior to their decisions. For example, most governments provide funding of applied research, and they do this regularly and on a long term basis; in many countries there exist public agencies like the US Census Bureau, which have the task of gathering statistics; in some countries, like Sweden, traditions have developed that require interest groups to be invited to provide information and give opinions on a matter before it is decided. Information is also gathered on a more day-to-day basis: governments that face an important decision often commission reports and investigations by experts and special committees.² Moreover, even when governments do not themselves actively try to acquire information, it is often provided to them by interest groups and lobbyists (so-called informational lobbying).

It may be natural to presume that as long as such public information acquisition is not requiring too much resources, it is socially desirable. After all, access to relevant information is often useful when making political decisions. In a society where citizens have conflicting preferences, however, it is not clear whether all of them are better off if public policy is made with access to more information. Likewise, when a government is making its decisions in a strategic environment, more information is not necessarily

¹For example, the information that is lacking could concern questions such as: What are the effects of a membership in the EMU on growth and unemployment? What are the effects of a new tax system on the income distribution and the incentives for becoming an industrial entrepreneur? What are the effects on the environment and economic growth of increased investment in the infrastructure?

²Larsson (1994, p. 180) writes the following about the use of royal commissions in Sweden: "Almost every government bill of any importance that is presented to Parliament has been developed in one or more royal commissions. The use of royal commissions has decreased somewhat in recent years, but nevertheless about two hundred royal commissions are at work every year. (...) Interest groups, agencies, and representatives of boards are often invited to sit on these, as are politicians from the parliamentary opposition."

beneficial. If some citizens indeed are worse off when the policymaker gets access to more information, this is important to understand for at least two reasons. First, it may help to explain why there sometimes are conflicting views on whether governments should gather more information prior to decisions on public policy.³ Second, as argued above, the information that the policymaker gets access to is in many cases generated through "informational lobbying" — that is, strategic information transmission on the part of interest groups that try to influence public policy.⁴ In order to understand the welfare effects of such lobbying, it is crucial to know how more informed decisions on public policy affect the welfare of different citizens. Will all citizens really be better off, and who are the winners and who are the losers?

This paper tries to shed light on these questions by studying a relatively simple linear-quadratic model of public policy under uncertainty. In the model there are a large number of citizens with (quadratic) preferences over the amount of roads and pollution in their country. The citizens are heterogenous with respect to how important roads are relative to pollution. A policymaker decides directly only on the amount of roads, although indirectly this decision affects (in a linear fashion) also the amount of pollution: more roads give rise to more pollution.⁵ The exact relationship between roads and pollution is unknown, though. The policymaker is elected by the citizens, where potential policymakers have preferences of the same form as the citizens and with varying relative weights on roads and pollution. After the policymaker has been elected and taken office, she first observes a noisy signal about the true state and then decides on the amount of roads.

³Another possible explanation for this phenomenon is that asking for more investigations may be a way of delaying and perhaps even stop the proposed project.

⁴Two early papers in the literature on informational lobbying are Austen-Smith and Wright (1992) and Potters and van Winden (1992); surveys can be found in Austen-Smith (1997) and Grossman and Helpman (2001). Typically, in the equilibria of the models in this literature, at least some information is transmitted to the policymaker. This is due to the assumptions that the policymaker is sufficiently sophisticated and that the interests of the lobbyist and the policymaker are not too disaligned (or, alternatively, that false reports can at least with some probability be discovered and punished).

⁵This assumption is made for simplicity. Of course, in the real world the relationship between roads and pollution might be more complex.

Clearly, a citizen who could decide on the amount of roads herself (e.g., because she knew that she would be elected policymaker) would prefer to have access to a signal with a quality that is as high as possible (provided a higher quality is not costly per se, which I assume is the case here). In the political setting sketched above, however, some citizens would (from an ex ante perspective) be worse off if the policymaker had access to a signal with higher quality. In particular, this will be the case for those citizens who care the least about the environment. The reason why these citizens are worse off is that they have a concave utility function, which makes uncertainty costly. Moreover, even though the policymaker's getting more information decreases ex post uncertainty, it also *increases* ex ante uncertainty. The latter is true because at the time when the welfare evaluation is made, the signal to be received by the policymaker is not known. The ex ante uncertainty manifests itself in a greater variability in the decision on the amount of roads. Therefore, the presence of this uncertainty has an adverse effect also on the welfare of those citizens who care very little about the environment. Indeed, for those citizens whose concern about the environment is sufficiently small, the adverse welfare effect of a greater ex ante uncertainty dominates the positive welfare effect of a smaller expost uncertainty. A majority of citizens, however, including the median citizen, are always in favor of the policymaker's getting access to more information.

The model discussed above is also extended to allow for two countries, each of which elects a policymaker who is to choose the amount of roads in the own country. The total amount of roads determines, in a stochastic fashion, the amount of (global) pollution affecting both countries. Because of the externality, the electorates of the two countries elect policymakers who care less about the environment than do the median citizens. In this version of the model, if the median citizens of the countries care sufficiently much about the environment, a majority of citizens of one of the countries are worse off when the own policymaker gets access to information, given that also the other country's policymaker has access to information — it can even be the case that *all* citizens are worse off. The reason why a majority prefers the own policymaker to be uninformed is that if both policymakers were informed (and if this were common knowledge), then this would increase the amount of ex ante uncertainty, just as in the one-country model. Moreover, and in contrast to that model, the amount of ex post uncertainty would decrease only moderately or possibly even increase. This is because of the incentives to delegate: if both policymakers were informed, then the elected policymakers would care relatively little about the environment and thus make their decisions on roads, from the median citizen's point of view, too unresponsive to the information. Hence, a country's choice not to build an institution that provides its policymaker with policy-relevant information can serve as a commitment device, helping it avoid a situation where the countries delegate to policymakers who care too little about the environment.

This paper is related to a literature on the value information in economic environments. A number of authors have provided examples of non-zero sum games where a player is hurt by having more information *herself*, provided that this fact is common knowledge among the players.⁶ For examples of this phenomenon in a political framework, see Reed (1989) or, in a Cournot duopoly setting, e.g., Sakai (1985). There are also papers that study the value of information being *publicly* known among a group of individuals.⁷ The present paper, in contrast, addresses the question whether the citizens of a society are hurt if *someone else* (a policymaker) gets access to more information upon which she can act, a question which is particularly relevant for the study of the welfare effects of informational lobbying in a heterogenous society.

The remainder of the paper is organized as follows. In Section II, the one-

⁶Ponssard (1976) shows that additional information can never be detrimental to a player in a zero-sum game. It is also quite obvious that in a single-agent decision problem, more information can never hurt.

⁷For example, see Hirschleifer (1971), Gersbach (1991), and Heidhues and Lagerlöf (2001). Heidhues and Lagerlöf develop a model of electoral competition in which private information is dispersed between two political candidates. The authors show that the electorate can be worse off when the prior information that is publicly available becomes more accurate. The reason for this is that, when the prior becomes more accurate, the candidates' incentives to truthfully transmit their additional private information to the electorate are weakened.

country model is presented and analyzed. Section III considers the extension with two countries. Section IV summarizes the results and at the same time discusses their implications for the welfare effects of informational lobbying. Proofs are found in an appendix.

II. One Country

Model

Consider a society with a continuum of citizens who are each having preferences over two public goods, provided in quantities $x \in \Re$ and $y \in \Re$. Citizen *i*'s preferences are described by the von Neumann-Morgenstern utility function

$$U_{i}(x,y) = -(x-\overline{x})^{2} - 2k\lambda_{i}(x-\overline{x})(y-\overline{y}) - \lambda_{i}(y-\overline{y})^{2}, \qquad (1)$$

where $\overline{x}, \overline{y}, k$, and λ_i are fixed parameters. The citizens differ from each other only with regard to the parameter λ_i . The λ_i 's are continuously distributed on $[0, \Gamma]$ for some $\Gamma > 0,^8$ and the median λ_i is denoted λ_m . Of course, for any $\lambda_i > 0$, we want to interpret the pair of quantities $(\overline{x}, \overline{y})$ as citizen *i*'s ideal point. For this interpretation to make sense, however, we must have $U_i(\overline{x}, \overline{y}) > U_i(x, y)$ for all $(x, y) \neq (\overline{x}, \overline{y})$. This is true if and only if $\lambda_i > 0$ and $\lambda_i k^2 < 1.^9$ Motivated by this, I impose the following assumption throughout this section:

Assumption 1. $\Gamma k^2 < 1$.

Notice that citizen *i*'s preferences are separable across the two public goods only if k = 0. If k is positive, then there is a negative complementarity between the two goods; that is, if k > 0 and if y is greater than \overline{y} , then a citizen's preferred level of x is smaller than \overline{x} . Similarly, if k is negative there is a positive complementarity: if y is greater than \overline{y} , then a citizen's preferred level of x is greater than \overline{x} .

⁸We can have $\Gamma = \infty$ if we also set k = 0; cf. Assumption 1 below.

⁹See e.g. Sydsæter and Hammond (1995, Ch. 15.8).

Public policy is decided on by a representative, who is elected by the citizens. The representative can control only x. There is, however, a stochastic relationship between x and y, given by

$$y = \beta x - \varepsilon. \tag{2}$$

Here $\beta > 0$ is a fixed parameter and ε is a stochastic variable with zero mean. We may think of x as the amount of roads in the country, and y as the amount of pollution caused by the traffic on these roads (or perhaps rather the adverse environmental effects of the pollution). Everybody has some ideal amount of roads, \overline{x} , and some ideal amount of pollution, \overline{y} . The uncertainty as to the exact relationship between the amount of roads and pollution may be due to the fact that the technology giving rise to the relationship is not perfectly known, or to the fact that the amount of pollution also depends on weather conditions which are not known at the time when the decision on x must be made.

Substituting (2) into (1) yields citizen i's induced preferences over x only:

$$u_i(x,\varepsilon) = -(x-\overline{x})^2 - 2k\lambda_i(x-\overline{x})\left(\beta x - \varepsilon - \overline{y}\right) - \lambda_i\left(\beta x - \varepsilon - \overline{y}\right)^2.$$
 (3)

This means that if ε were known, citizen *i* would like the representative to set *x* equal to $\hat{x} = \psi(\lambda_i) + \varphi(\lambda_i) \varepsilon$, where

$$\psi(\lambda_i) \equiv \frac{\overline{x}\left(1 + k\beta\lambda_i\right) + \overline{y}\lambda_i\left(k + \beta\right)}{1 + 2k\beta\lambda_i + \lambda_i\beta^2},\tag{4}$$

$$\varphi(\lambda_i) \equiv \frac{\lambda_i \left(k + \beta\right)}{1 + 2k\beta\lambda_i + \lambda_i\beta^2}.$$
(5)

Throughout this section I make the following assumption:

Assumption 2. $k > -\frac{\beta}{2} - \frac{1}{2\beta\Gamma}$.

Assumption 2 guarantees that the above expressions are well defined and the second-order condition to the problem of maximizing $u_i(x,\varepsilon)$ with respect to x is satisfied. Since $\varphi(0) = 0$ and $\varphi' > 0$, the parameter λ_i measures how responsive a citizen is to changes in ε . Someone who has a low λ_i (i.e., someone who cares relatively little about pollution) would like the representative to make x contingent on ε to a lesser degree than someone for whom λ_i is large. In the following, the parameter λ_i will often be called citizen *i*'s responsiveness parameter.¹⁰

The sequence of events is as follows. (i) The representative is elected. (ii) The representative observes a signal s, which is correlated with ε , and then chooses x. (iii) The shock ε is realized. The election at stage (i) works as follows. The elected representative is assumed to be a citizen having a responsiveness parameter λ_i such that she cannot be beaten in a pair-wise comparison when each citizen votes for the one of the two candidates who gives her the highest expected utility. In other words, the representative is a Condorcet winner among the citizens.

It is assumed that the signal s and the shock ε are jointly distributed according to the density function $f(\varepsilon, s)$. Moreover, the expected value of ε equals zero and the expected value of s is denoted μ_s ; the variances of ε and s are denoted σ^2 and σ_s^2 , respectively. The correlation coefficient between sand ε is defined by $\rho = Cov(\varepsilon, s) / (\sigma \sigma_s)$, where ρ is assumed to be distinct from zero: $\rho \in [-1, 0) \cup (0, 1]$.

It is further assumed that the distribution of s and ε is such that ε has *linear regression* with regard to s; that is, $E(\varepsilon | s)$, where $E(\varepsilon | s) \equiv \int \varepsilon f(\varepsilon | s) d\varepsilon$ is the conditional expectation function, is a linear (affine) function of s.¹¹ It is well known that if ε has linear regression with regard to s(and if $E(\varepsilon) = 0$), then

$$E\left(\varepsilon \mid s\right) = \rho \frac{\sigma}{\sigma_s} \left(s - \mu_s\right). \tag{6}$$

¹⁰This kind of heterogeneity is also found in the models of, for example, Melumad and Shibano (1991), Lagerlöf (1997), and Schultz (2002). In those papers, however, the heterogeneity is simply postulated when specifying the functional form. Here, in contrast, the heterogeneity is derived from differences in the relative weights on two policy issues and the stochastic relationship between them. Moreover, the relationship between the representative's and the median citizen's responsiveness is endogenous to the model.

¹¹For instance, a bivariate normal distribution has this property.

Analysis

Let us denote the representative's responsiveness parameter by λ_r . At stage (ii), conditional on having observed the signal *s*, the representative will implement the policy *x* that maximizes her expected utility:

$$\max_{x \in \Re} \int u_r(x,\varepsilon) f(\varepsilon \mid s) d\varepsilon.$$
(7)

The unique solution to this problem is given by

$$x_{r}^{*} = \psi(\lambda_{r}) + \varphi(\lambda_{r}) E(\varepsilon \mid s).$$
(8)

Now consider a citizen/voter. At the time of the election, this person only knows the prior distribution of s and ε . She anticipates, however, that a representative with responsiveness parameter λ_r will set x equal to x_r^* . Hence, citizen *i*'s expected utility at the time of the election, denoted by Eu_i , may be written as

$$Eu_{i} = \int \int u_{i} (x_{r}^{*}, \varepsilon) f(\varepsilon, s) d\varepsilon ds$$

$$= -(1 + 2k\beta\lambda_{i} + \lambda_{i}\beta^{2}) [\psi^{2} (\lambda_{r}) - 2\psi (\lambda_{r}) \psi (\lambda_{i})]$$

$$-(1 + 2k\beta\lambda_{i} + \lambda_{i}\beta^{2}) \rho^{2}\sigma^{2} [\varphi^{2} (\lambda_{r}) - 2\varphi (\lambda_{r}) \varphi (\lambda_{i})]$$

$$-\overline{x}^{2} - 2k\lambda_{i}\overline{xy} - \lambda_{i}\sigma^{2} - \lambda_{i}\overline{y}^{2}$$
(9)

The expression after the second equality sign in equation (9) was obtained by using equations (3), (6), (8), and by carrying out some algebra.

The function Eu_i represents citizen *i*'s preferences over a potential representative. The potential representatives differ from each other along only one dimension, $\lambda_r \in [0, \Gamma]$. Moreover, in the proof of Lemma 1 below it is shown that Eu_i is single peaked in λ_r . Hence, we can invoke the median voter theorem (see, e.g., Persson and Tabellini, 2000), which states that if those two conditions (i.e., one dimension and single-peakedness) are met then the median voter's favorite representative cannot lose under majority rule. This means that, in a political equilibrium, the representative will be the favorite of the median citizen/voter. Unsurprisingly, the responsiveness parameter of this favorite representative equals the median voter's, $\lambda_r = \lambda_m$;

there is no reason for any member of the electorate to delegate the task of deciding on public policy to someone with other preferences than herself.¹²

Lemma 1. The representative's responsiveness parameter is the same as the median citizen's, $\lambda_r = \lambda_m$.

Welfare Effects of More Information

Let us now investigate whether members of the society would be better off if the representative got access to a signal with higher quality. The welfare evaluation will be made ex ante; that is, I will consider citizen *i*'s expected utility, as measured by Eu_i in equation (9) (with $\lambda_r = \lambda_m$). The expression "higher quality" will be understood as an increase in ρ^2 .

Let λ be defined by

$$\widetilde{\lambda} = \frac{\lambda_m}{2 + 2k\beta\lambda_m + \lambda_m\beta^2}.$$
(10)

Proposition 1. An increase in ρ^2 benefits those with $\lambda_i > \widetilde{\lambda}$ and makes those with $\lambda_i < \widetilde{\lambda}$ worse off (i.e., $\frac{\partial E u_i}{\partial \rho^2} \mid_{\lambda_r = \lambda_m} \gtrsim 0$ as $\lambda_i \gtrsim \widetilde{\lambda}$).

Accordingly, those members of the electorate who have a sufficiently low responsiveness parameter λ_i are *worse* off if the representative gets access to better information about the relationship between the amount of roads and the amount of pollution, in the sense that ρ^2 increases. Before looking at the intuition for this result, let us consider the question whether the *majority* of citizens may be worse off from an increase in ρ^2 . Since $\tilde{\lambda} < \lambda_m/2$ (this follows from equation (10) and Assumption 2), it follows immediately that the answer to this question is no: everyone with a responsiveness parameter $\lambda_i \in [\lambda_m/2, \Gamma]$ is strictly better off from a larger ρ^2 , and this group of citizens form a majority.

In order to understand the intuition behind the result that those citizens who have a low responsiveness parameter are worse off if the signal quality

¹²In Lagerlöf (2001), a model that is similar to the present one is used to study monetary policy and the optimal choice of a central banker. In that model, however, the signal quality ρ^2 is endogenous: the appointed banker can, by incurring a private cost, choose a higher signal quality. This possibility creates an incentive to delegate to a relatively "liberal" banker.

 ρ^2 becomes larger, it is useful to look at how the different terms in the expression for citizen *i*'s expected utility are affected by an increase in ρ^2 . Of course, if $\lambda_i = 0$, then the expected utility is independent of ρ^2 . Let us suppose that $\lambda_i > 0$. Then it turns out that an increase in ρ^2 has the effects that are indicated in the following equation:

$$Eu_{i}(x_{r}^{*},\varepsilon) \mid \lambda_{r}=\lambda_{m} = \underbrace{-\int \int (x_{m}^{*}-\overline{x})^{2} f(\varepsilon,s) d\varepsilon ds}_{\text{decreasing in }\rho^{2}}$$

$$\underbrace{-2\lambda_{i}k \int \int (x_{m}^{*}-\overline{x}) \left(\beta x_{m}^{*}-\varepsilon-\overline{y}\right) f(\varepsilon,s) d\varepsilon ds}_{\text{decreasing in }\rho^{2} \text{ iff } k\in(-1/\beta\lambda_{m},0)}$$

$$\underbrace{-\lambda_{i} \int \int (\beta x_{m}^{*}-\varepsilon-\overline{y})^{2} f(\varepsilon,s) d\varepsilon ds.}_{\text{increasing in }\rho^{2}}$$

$$(11)$$

That is, whereas the third term of citizen *i*'s expected utility is increasing in ρ^2 , the first term is decreasing in ρ^2 ; the second term is increasing in ρ^2 if there is a negative complementarity or a strong positive complementarity. Hence, it is from the first (and sometimes also the second) term that the cost of having a more informed representative comes. The reason why the first term is decreasing in ρ^2 is that it gets smaller (which means lower utility) when the representative's decision x_m^* varies more, and x_m^* will indeed vary more when the signal quality ρ^2 increases. In other words, a better informed representative will make her decision contingent on ε to a greater extent, which is bad for the citizen since her preferences are concave.

Stated differently, whereas the ex post uncertainty decreases when the representative gets access to a signal with higher quality, the ex ante uncertainty *increases*. The decrease in ex post uncertainty is of course beneficial for all citizens for whom $\lambda_i > 0$; this is reflected in the fact that the third term in citizen *i*'s expected utility is increasing in ρ^2 . For those citizens for whom λ_i is small enough (smaller than $\tilde{\lambda}$ according to the algebra), the positive welfare effect of a lower ex post uncertainty is dominated by the adverse welfare effect of a larger ex ante uncertainty.¹³

¹³The intuition for the result in Proposition 1 is related to the intuition for a result

III. Two Countries

Model

Let us now assume that there are two countries, indexed by $j \in \{1, 2\}$, each with a continuum of citizens. The preferences of citizen *i* of country *j* are described by the von Neumann-Morgenstern utility function $U_{ij}(x_j, y) =$ $-x_j^2 - \lambda_{ij}y^2$, where $x_j \in \Re$ is the quantity of a public good specific for country *j* (e.g., the amount of roads in that country) and $y \in \Re$ is the quantity of a public good common for the two countries (e.g., the amount of global pollution). The parameter $\lambda_{ij} \in [0, \infty)$ is a weight for citizen *i* in country *j*; the median λ_{ij} is the same in both countries and denoted λ_m . (Relative to the model in the previous section, I thus simplify by setting $k = \overline{x} = \overline{y} = 0$ and $\Gamma = \infty$.) The amount of pollution, *y*, is determined by the following relationship:

$$y = \beta \left(x_1 + x_2 \right) - \varepsilon,$$

where, as before, β is a fixed parameter and ε is a stochastic variable with zero mean. The induced preferences of citizen *i* of country *j* are thus given by

$$u_{ij}(x_1, x_2, \varepsilon) = -x_j^2 - \lambda_{ij} \left[\beta \left(x_1 + x_2\right) - \varepsilon\right]^2.$$
 (12)

The sequence of events is as follows. (i) Each country elects a representative. (ii) The representative of each country j first learns about the outcome of the election in the other country. She then observes a signal s, which is correlated with ε . (The representatives are thus assumed to observe the same signal s, which means that they have access to exactly the same information.¹⁴) Thereafter, simultaneously with the other representative,

in Freixas and Kihlstrom (1984). These authors consider a situation in which a patient must choose a doctor in the face of imperfect information about the distribution of service quality across doctors. In particular they study the effect of risk aversion on demand for information about this distribution. They find that, in their model, an increase in the degree of risk aversion unambiguously reduces the demand for information.

¹⁴An alternative assumption would be to say that the representatives observe signals that are not perfectly correlated (for example, independent conditionally on ε). The assumption of perfect correlation simplifies the analysis and the exposition considerably, however. I would expect qualitatively unaltered results in a model with conditionally independent or otherwise imperfectly correlated signals.

she chooses x_j . (iii) The shock ε is realized.

The way in which the election of a representative in country j works is similar to before. In particular, a *political equilibrium* at stage (i) is defined as a pair $(\lambda_{r1}^*, \lambda_{r2}^*)$ such that each λ_{rj}^* is a Condorcet winner in country j, given that the other representative's responsiveness parameter equals λ_{rl}^* . Formally, denoting a Condorcet winner in country j given some λ_{rl} (for $l \neq j$) by $C_j(\lambda_{rl})$, a political equilibrium is a pair $(\lambda_{r1}^*, \lambda_{r2}^*)$ such that $(\lambda_{r1}^*, \lambda_{r2}^*) = (C_1(\lambda_{r2}^*), C_2(\lambda_{r1}^*)).$

The rest of the model and the notation is the same as in the previous section. In particular, the signal technology is identical to the previous one (recall that the representatives observe the same signal s), and the relationship (6) will be used extensively here, too.

Analysis

Let us begin the analysis by solving the game between the two representatives at stage (ii). Given that she has observed the signal s, the representative of country j maximizes $E\{u_{rj}(x_1, x_2, \varepsilon) \mid s\}$ with respect to x_j , while taking the other representative's strategy as given. The first-order condition for this problem is

$$-x_j - \lambda_{rj}\beta \left[\beta \left(x_1 + x_2\right) - E\left(\varepsilon \mid s\right)\right] = 0,$$

which means that the two representatives' first-order conditions form an equation system that is linear in x_1 and x_2 . Solving for this yields the unique solution $x_j^* = B_j^*(\lambda_{r1}, \lambda_{r2}) E(\varepsilon \mid s)$, where

$$B_j^*(\lambda_{r1}, \lambda_{r2}) = \frac{\beta \lambda_{rj}}{1 + \beta^2 (\lambda_{r1} + \lambda_{r2})}.$$
(13)

Let us now turn to stage (i). Given the equilibrium behavior at stage (ii) and some pair of responsiveness parameters $(\lambda_{r1}, \lambda_{r2})$, the expected utility of citizen/voter *i* in country *j* can be written as

$$E u_{ij} \left(\lambda_{r1}, \lambda_{r2}\right) \equiv \int \int u_{ij} \left(x_1^*, x_2^*, \varepsilon\right) d\varepsilon ds$$

= $-\sigma^2 \rho^2 \left(B_j^*\right)^2 + \lambda_{ij} \left[\sigma^2 \rho^2 \beta \left(\sum_{l=1}^2 B_l^*\right) \left(2 - \beta \sum_{l=1}^2 B_l^*\right) - \sigma^2\right],$
(14)

where the arguments of B_1^* and B_2^* have been suppressed. The second line of this expression was obtained by using (6), (12), and $x_j^* = B_j^*(\lambda_{r1}, \lambda_{r2}) E(\varepsilon \mid s)$, and then performing some straightforward algebra.

Similarly to the previous section, $Eu_{ij}(\lambda_{r1}, \lambda_{r2})$ can be shown to be single peaked in λ_{rj} (see the proof of Lemma 2). Hence, a Condorcet winner in country j exists and coincides with the favorite of a citizen/voter with $\lambda_{ij} = \lambda_m$. This, in turn, means that the set of political equilibria at stage (i) coincides with the set of Nash equilibria of a game where the two median voters simultaneously choose the own representative's responsiveness parameter and get the payoffs $Eu_{m1}(\lambda_{r1}, \lambda_{r2})$ and $Eu_{m2}(\lambda_{r1}, \lambda_{r2})$, respectively. Solving for this set of Nash equilibria yields the following result.

Lemma 2. There exists a unique political equilibrium at stage (i). This is symmetric and has

$$\lambda_{r1}^{*} = \lambda_{r2}^{*} = \lambda_{r}^{*} = \frac{1}{2\beta^{2}} \left[\sqrt{4\beta^{2}\lambda_{m} + 1} - 1 \right].$$
 (15)

As one would expect, λ_r^* is increasing in λ_m . Moreover, we have $\partial \lambda_r^* / \partial \beta < 0$ with $\lim_{\beta \to 0} \lambda_r^* = \lambda_m$ and $\lim_{\beta \to \infty} \lambda_r^* = 0.15$ That is, the two electorates delegate the task of deciding on the amount of roads to representatives who care less than the median citizens about the environment, and the incentives to do this are stronger the larger is the parameter β .

The fundamental reason why $\lambda_r^* < \lambda_m$ is that the representatives' strategic variables at stage (ii), x_1 and x_2 , are strategic substitutes (see Bulow, Geanakoplos, and Klemperer, 1985): an increase in x_l (for $l \neq j$) decreases representative j's marginal utility. Intuitively, if a representative expects the other representative to contribute only a little or not at all to the public good of making x_j sufficiently responsive to the signal, her best response is to contribute a relatively large amount herself (and vice versa). Because of this, the median citizen of a country can gain by, at stage (i), choose a representative who is relatively unwilling to contribute to the public good. Both

¹⁵The simplest way to see this is to multiply the numerator and the denominator of λ_r^* by $\sqrt{4\beta^2\lambda_m+1}+1$, thus obtaining $\lambda_r^* = 2\lambda_m / \left[\sqrt{4\beta^2\lambda_m+1}+1\right]$. From this expression it is easily seen that the claims are true.

electorates have this incentive, and as the parameter β — which measures to what degree an increase in the amount of roads leads to an increase in the amount of pollution — becomes very large, cutthroat competition between the countries leads to an equilibrium λ_r^* that is close to zero.¹⁶

Welfare Effects of More Information

Let us now ask how the welfare of the citizens is affected when their representative gets access to more information. In contrast to the previous section, here I will not do comparative statics on the signal quality ρ^2 . Instead I will simply compare the ex ante welfare level in the equilibrium of the model above with the ex ante welfare levels in the following three benchmarks: (1) the other country but not the own having access to a signal; (2) the own country but not the other having access to a signal; and (3) none of the countries having access to a signal. Moreover, rather than considering the expected welfare of any arbitrary citizen, I will, initially, look only at the expected welfare of the median citizen.

Suppose that both representatives have access to the signal, so that in equilibrium $x_j = B_j^*(\lambda_{r1}, \lambda_{r2}) E(\varepsilon \mid s)$ and $(\lambda_{r1}, \lambda_{r2}) = (\lambda_r^*, \lambda_r^*)$. Then the median voter's expected utility can be written as

$$Eu_{mj}\left(\lambda_r^*,\lambda_r^*\right) = \frac{\sigma^2 \rho^2 \left[\left(4\beta^2 \lambda_m\right)^2 - \left(\sqrt{4\beta^2 \lambda_m + 1} - 1\right)^2\right]}{4\beta^2 \left(4\beta^2 \lambda_m + 1\right)} - \lambda_m \sigma^2.$$
(16)

If instead none of the representatives had access to a signal, then they would set $x_j = 0$ (i.e., they would behave as if $\lambda_{rj} = 0$) and the median citizen's expected utility would be given by $Eu_{mj}(0,0) = -\lambda_m \sigma^2$. Similarly, if only

¹⁶It may be illuminating to compare this result with the one of Persson and Tabellini (1992; 2000, Ch. 12.4.4). They develop a model of tax competition between two countries, where the chosen tax rates on capital (the mobile tax base) are inefficiently low. At a prior stage, however, the electorates of the two countries delegate to representatives who have a relatively strong incentive to tax capital, which mitigates the inefficiency. The reason why strategic delegation has this positive effect on the outcome in their model is that the strategic variables of the representatives in the tax competition game are strategic complements: if a representative expects the other representative to set a high tax, her best response is to set a relatively high tax, too.

For references to other related work on strategic delegation, see Persson and Tabellini (2000). See also Putnam (1988) for an influential paper in the political science literature on the interaction between domestic politics and international relations.

the representative of country 1 (respectively, country 2) had access to a signal, then the expected utility of the median citizen of country 1 would be given by $Eu_{m1}(\lambda_m, 0)$ (respectively, $Eu_{m1}(0, \lambda_m)$),¹⁷ where

$$Eu_{m1}(\lambda_m, 0) = \frac{\sigma^2 \rho^2 \beta^2 \lambda_m^2}{\beta^2 \lambda_m + 1} - \lambda_m \sigma^2, \qquad (17)$$

$$Eu_{m1}(0,\lambda_m) = \frac{\sigma^2 \rho^2 \beta^2 \lambda_m^2 \left(\beta^2 \lambda_m + 2\right)}{\left(\beta^2 \lambda_m + 1\right)^2} - \lambda_m \sigma^2.$$
(18)

(Eqs. (16)-(18) were obtained by using (13)-(15) and then carrying out some algebra.)

It is easy to see that $Eu_{m1}(0,0) < Eu_{m1}(\lambda_m,0)$ and $Eu_{m1}(\lambda_m,0) < Eu_{m1}(0,\lambda_m)$. That is, the median citizen of a country prefers the own country's representative to have access to a signal rather than none of the representatives' having access to a signal. And, given that only one of the representatives has access to a signal, the median citizen prefers this to be the representative of the other country. The following result tells us how $Eu_{m1}(\lambda_r^*, \lambda_r^*)$ relates to the three other expected welfare levels.

Lemma 3. We have $Eu_{m1}(\lambda_r^*, \lambda_r^*) > Eu_{m1}(\lambda_m, 0)$ for all β , λ_m , σ^2 , and ρ^2 . Moreover, $Eu_{m1}(\lambda_r^*, \lambda_r^*) > Eu_{m1}(0, \lambda_m) \Leftrightarrow \beta^2 \lambda_m < c \approx .68$ (where c is the unique root of $c(\sqrt{4c+1}+1)=2$).

This means that, whenever $\lambda_m > c/\beta^2$, we have the following relationship:

$$Eu_{m1}\left(\lambda_r^*, \lambda_r^*\right) < Eu_{m1}\left(0, \lambda_m\right).$$
(19)

In words, for λ_m and/or β sufficiently large, the median voter of one of the countries, say country 1, is better off if the own representative does *not* have access to a signal, given that the other country's representative does. As we will see, the same is true for those citizens of country 1 with $\lambda_{i1} < \lambda_m$.

¹⁷Recall from the previous section that if there is only one country (or, which here amounts to the same, if the other country's representative does not have access to a signal), then there is no reason to delegate strategically: $\lambda_r = \lambda_m$.

To understand why we get this result, consider again the expression for $Eu_{ij}(\lambda_{r1}, \lambda_{r2})$ in (14), and suppose for concreteness that j = 1. We see that the first term of $Eu_{i1}(\lambda_{r1}, \lambda_{r2})$ is a function of B_1^* only, and it is maximized at $B_1^* = 0$. Hence, this term is obviously larger with $(\lambda_{r1}, \lambda_{r2}) = (0, \lambda_m)$ than with $(\lambda_{r1}, \lambda_{r2}) = (\lambda_r^*, \lambda_r^*)$. The possible benefit with $(\lambda_r^*, \lambda_r^*)$ comes from the second term, which is a function of the sum $(B_1^* + B_2^*)$; in particular, this term is single peaked in $(B_1^* + B_2^*)$, with the peak at $1/\beta$. Straightforward calculations show that

$$B_{1}^{*}(\lambda_{r}^{*},\lambda_{r}^{*}) + B_{2}^{*}(\lambda_{r}^{*},\lambda_{r}^{*}) = \frac{1}{\beta} \left[1 - \left(4\beta^{2}\lambda_{m} + 1\right)^{-\frac{1}{2}} \right],$$
(20)

whereas

$$B_1^*(0,\lambda_m) + B_2^*(0,\lambda_m) = \frac{\beta\lambda_m}{1+\beta^2\lambda_m}.$$
(21)

Both these expressions are strictly smaller than $1/\beta$. Hence, if (20) is smaller (and thus farther away from the peak $1/\beta$) than (21), there is in fact no benefit at all with $(\lambda_r^*, \lambda_r^*)$: both terms of $Eu_{i1}(\lambda_{r1}, \lambda_{r2})$ are then larger with $(0, \lambda_m)$. Indeed, one can verify that (20) is smaller than or equal to (21) if and only if $\beta^2 \lambda_m \geq 2$, in which case *all* citizens of country 1 prefer $(0, \lambda_m)$ to $(\lambda_r^*, \lambda_r^*)$! If $\beta^2 \lambda_m < 2$, however, the second term is larger with $(\lambda_r^*, \lambda_r^*)$ than with $(0, \lambda_m)$, and whether a citizen of country 1 prefers the own representative to be informed, given that also the other representative is informed, depends on the magnitude of the weight λ_{i1} ; the algebra shows that, for the median citizen of country 1, the critical level of λ_m is given by $\lambda_m = c/\beta^2$ (see the proof of Lemma 3).

So the reason why we obtain the relationship in (19) is that, from the median citizen's point of view, there is too little stabilization of the pollution shocks when both representatives have access to information. The reason for this, in turn, is twofold. First, for given representatives, there is a free-riding problem in the choice whether to make x_j sufficiently responsive to the signal. Second, anticipating this, the electorates of the two countries elect representatives who care less than the median citizens about the environment, thus making the free-riding problem even worse (cf. the discussion following Lemma 2). The outcome of this cutthroat competion between the countries becomes more extreme the larger is β , which is why we need $\beta^2 \lambda_m > c$ for the relationship in (19) to hold.

The following proposition summarizes the results.

Proposition 2. A majority of citizens in a country always want the own representative to have access to information, given that the other country's representative does not. If the other country's representative does have access to information, however, a majority of citizens want the own representative not to have access to information if $\lambda_m > c/\beta^2$ (where $c \approx .68$), and for $\lambda_m \ge 2/\beta^2$ all citizens agree on this.

Thus, if $\lambda_m > c/\beta^2$ and, at the outset of the game, in each country there was a referendum about whether the own representative should get access to an informative signal (and if the outcomes of the referenda then became commonly known), then this extended game would have two asymmetric equilibria: "no information" winning in country 1 but not in country 2, and vice versa.¹⁸ For the median voter in the country that *does* let its representative have access to information, however, the equilibrium outcome would be dominated by the outcome where $(\lambda_{r1}, \lambda_{r2}) = (\lambda_r^*, \lambda_r^*)$.

IV. Summary

Imagine that a society has a choice between putting a ban on lobbying or not, where "lobbying," if it is permitted, provides the political leadership with policy-relevant information prior to its decision on public policy. Will such lobbying make all citizens better off? If not, who will be the winners and who will be the losers?

This paper has investigated these questions within the framework of a relatively simple model of public policy under uncertainty and with two public goods: roads and pollution, where roads is chosen directly by an elected policymaker and the amount of pollution depends stochastically on

¹⁸There would also be a symmetric equilibrium in mixed strategies.

the amount of roads. In a one-country version of this model, it was shown that the majority of citizens are always better off from lobbying in the above sense. A minority, however, consisting of those citizens who care the least about the environment, are in favor of a ban on lobbying. The reason is that, if the policymaker makes her decision contingent on the information, ex ante uncertainty increases, and, for those citizens whose concern about the environment is relatively small, the increase in ex ante uncertainty carries a heavier weight than the reduction in ex post uncertainty.

In a two-country version of the model a majority of citizens of one of the countries can be against lobbying, provided that lobbying is allowed in the other country (or that the policymaker of that country has access to information through some other channel). The reason is that if lobbying were allowed in both countries, the two policymakers would face a free-riding problem in their decisions whether to make the amount of roads sufficiently responsive to their information. As a consequence, the two electorates delegate strategically the task of deciding on roads to policymakers who care less about the environment than do the median citizens. When this effect is sufficiently strong, the two policymakers care so little about the environment and make their decisions so unresponsive to the information that the median citizen of one of the countries (as well as all those citizens who care less about the environment than her) prefer the own policymaker not to have access to information. In short, the absence of an institution for information gathering — here informational lobbying — serves as a commitment device for a country and helps it avoid the free-riding problem.

Appendix

A. Proof of Lemma 1

To be able to invoke the median voter theorem one must show that Eu_i is single peaked in λ_r . Differentiate Eu_i in (9) with respect to λ_r :

$$\frac{\partial E u_i}{\partial \lambda_r} = -2\left(1 + 2k\beta + \lambda_i\beta^2\right) \left[\psi'(\lambda_r)\left[\psi(\lambda_r) - \psi(\lambda_i)\right] + \sigma^2 \rho^2 \varphi'(\lambda_i)\left[\varphi(\lambda_r) - \varphi(\lambda_i)\right]\right].$$
(A1)

It is easy to check that $\varphi' > 0$ and that ψ' has the same sign as $(\overline{y} - \beta \overline{x})$. By inspecting equation (A1) one sees that regardless of the sign of ψ' we have: $\frac{\partial E u_i}{\partial \lambda_r} > 0$ for any $\lambda_r < \lambda_i$, $\frac{\partial E u_i}{\partial \lambda_r} < 0$ for any $\lambda_r > \lambda_i$, and $\frac{\partial E u_i}{\partial \lambda_r} = 0$ for $\lambda_r = \lambda_i$. Hence, $E u_i$ is single peaked in λ_r , and the peak is at $\lambda_r = \lambda_i$. \Box

B. Proof of Proposition 1

Differentiating Eu_i in (9) with respect to ρ^2 and evaluating at $\lambda_r = \lambda_m$ yield

$$\frac{\partial E u_i}{\partial \rho^2} |_{\lambda_r = \lambda_m} = -\left(1 + 2k\beta + \lambda_i \beta^2\right) \sigma^2 \varphi\left(\lambda_m\right) \left[\varphi\left(\lambda_m\right) - 2\varphi\left(\lambda_i\right)\right], \quad (A2)$$

which has the same sign as $(2\varphi(\lambda_i) - \varphi(\lambda_m))$. By using the definition of φ and by carrying out some algebra, one may show that $(2\varphi(\lambda_i) - \varphi(\lambda_m))$ in turn has the same sign as $(\lambda_i - \tilde{\lambda})$. \Box

C. Proof of Lemma 2

Differentiating Eu_{ij} with respect to λ_{rj} yields

$$\frac{\partial E u_{ij}}{\partial \lambda_{rj}} = 2\sigma^2 \rho^2 \left[-B_j^* \frac{\partial B_j^*}{\partial \lambda_{rj}} + \beta \lambda_{ij} \sum_{l=1}^2 \frac{\partial B_l^*}{\partial \lambda_{rj}} \left(1 - \beta \sum_{l=1}^2 B_l^* \right) \right]$$
$$= \frac{2\sigma^2 \rho^2 \beta^2 \left[-\lambda_{rj} \left(1 + \beta^2 \lambda_{rl} \right) + \lambda_{ij} \right]}{\left[1 + \beta^2 \left(\lambda_{r1} + \lambda_{r2} \right) \right]^3}.$$
(A3)

By inspecting (A3), it is easy to see that Eu_{ij} is single peaked in λ_{rj} with the peak at $\lambda_{rj} = \lambda_{ij}/(1 + \beta^2 \lambda_{rl})$. Having established this, let us look for the Nash equilibria of the game between the two median voters. First notice that for $\lambda_{ij} = \lambda_m > 0$ and $\lambda_{rj} = 0$, $\partial Eu_{mj}/\partial \lambda_{rj}$ is strictly positive for all $\lambda_{rl} \ge 0$. Hence, in any equilibrium we must have $\lambda_{r1}, \lambda_{r2} > 0$. In particular, we must have $\partial Eu_{mj}/\partial \lambda_{rj} = 0$ for the median voter of both countries. Subtracting one of these first-order conditions from the other and then rewriting yield $\lambda_{r1} = \lambda_{r2}$, so any equilibrium must be symmetric. Setting $\lambda_{r1} = \lambda_{r2} = \lambda_r^*$ in $\partial Eu_{mj}/\partial \lambda_{rj} = 0$ and then simplifying, we get the expression in (15) (there is also a second root which is negative and thus irrelevant). \Box

D. Proof of Lemma 3

Using (16) and (17), we can write $Eu_{m1}(\lambda_m, 0) < Eu_{m1}(\lambda_r^*, \lambda_r^*)$ as

$$\left(\sqrt{4\beta^2\lambda_m+1}-1\right)^2\left(\beta^2\lambda_m+1\right)<4\beta^4\lambda_m^2\left[4\left(\beta^2\lambda_m+1\right)-\left(4\beta^2\lambda_m+1\right)\right]=12\beta^4\lambda_m^2$$

Multiplying both sides by $\left(\sqrt{4\beta^2\lambda_m+1}+1\right)^2$ and then simplifying yield

$$12\beta^4 \lambda_m^2 \left(\sqrt{4\beta^2 \lambda_m + 1} + 1\right)^2 > \left(4\beta^2 \lambda_m\right)^2 \left(\beta^2 \lambda_m + 1\right),$$

or $\zeta \left(\beta^2 \lambda_m\right) \equiv 3 \left(\sqrt{4\beta^2 \lambda_m + 1} + 1\right)^2 - 4 \left(\beta^2 \lambda_m + 1\right) > 0$. This inequality must always hold, since $\zeta \left(0\right) > 0$ and $\zeta' \left(\beta^2 \lambda_m\right) > 0$ for all $\beta^2 \lambda_m > 0$. Next, using (16) and (18), we can write $Eu_{m1}\left(\lambda_r^*, \lambda_r^*\right) > Eu_{m1}\left(0, \lambda_m\right)$ as

$$\left(\beta^{2}\lambda_{m}+1\right)^{2}\left(\sqrt{4\beta^{2}\lambda_{m}+1}-1\right)^{2} < 4\left(\beta^{2}\lambda_{m}+1\right)^{2}-\left(4\beta^{2}\lambda_{m}+1\right)\left(\beta^{2}\lambda_{m}+2\right)\right] = 4\left(\beta^{2}\lambda_{m}\right)^{2}\left(2-\beta^{2}\lambda_{m}\right).$$

Multiplying both sides by $\left(\sqrt{4\beta^2\lambda_m+1}+1\right)^2$ and then simplifying yield

$$4\left(\beta^2\lambda_m+1\right)^2 < \left(2-\beta^2\lambda_m\right)\left(\sqrt{4\beta^2\lambda_m+1}+1\right)^2.$$

By multiplying out the squared terms and then simplifying further, one has equivalently $\beta^2 \lambda_m \left(\sqrt{4\beta^2 \lambda_m + 1} + 1\right) < 2$. It is easy to see that there is a unique strictly positive value of $\beta^2 \lambda_m$, say c, below which the inequality holds and above which it does not. The critical value c must thus be the unique positive root of $c \left(\sqrt{4c+1}+1\right) = 2$. Solving for c using Maple yields $c \approx .68$. \Box

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