On The Stability Of Group Formation: Managing The Conflict Within

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Abstract. This paper develops a positive analysis of stable group formation, highlighting the role of conflict management within groups. The analysis builds on a simple economic model that features a "winner-take-all" contest for control of some resource. When a group forms, members pool their efforts in that contest and, if successful, apply the resource to a joint production process. While reducing the severity of conflict over the contestable resource relative to the case of individual conflict, the formation of groups adds another layer of conflict—that is, one between the members of the winning group over the distribution of their joint product. The effectiveness of conflict management in enabling groups to resolve this second layer of conflict in more "peaceful" ways has some important implications for the equilibrium structure of groups as well as for the allocation of resources.

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1 Introduction

Appropriative behavior, driven largely by unrestricted self-interest of varying degrees, affects many different facets of economic life. The result is conflict of one form or another. In many forms, conflict often involves the actual use of violence or the threat of using violence—for example, in wars between nations, civil wars, organized criminal activity including gang warfare, strikes and lockouts, and ordinary crime. However, there are more refined forms of conflict as well, for example, where groups of individuals compete for economic advantage through lobbying, rent-seeking, and litigation. But, the costs of conflict extend well beyond the destructive consequences of the occasional use of force. Resources allocated directly to appropriative activities (to seize the property and wealth of others or to defend one's own property and wealth) represent lost opportunities in production. Furthermore, the greater sense of insecurity and uncertainty generates allocative distortions, for example, biasing the flow of resources away from any sort of investment that is vulnerable to theft or confiscation by others.

Economists have only recently begun to allow for the possibility of conflict and appropriation along side with production and trade in their study of economic interactions. Yet, even within a relatively short span of time, they have made some important progress in advancing our understanding of how conflict influences economic outcomes.[1] Nevertheless, focussing primarily on the implications of the darker side of self-interested behavior of individuals (or unitary actors), this research seems somewhat incomplete in its coverage. To be more precise, not much attention has been paid to the issues that arise when appropriative activities are carried out by individuals organized into groups.[2]

With an aim to start to close the gap, this paper extends the positive analysis of stable group formation in Garfinkel (2004) by considering the role of conflict management within the group. The analysis builds on a simple economic model that features a "winner-take-all" contest for control of some resource. Without the formation of groups, each individual participates in the contest independently. The winner, in turn, applies the resource in the production of a homogeneous consumption good. By contrast, when a

[1] For an insightful overview of this emerging literature, see Skaperdas (2003).

[2] There are, of course, some important exceptions, which are noted below.

group forms, members pool their efforts to secure the contestable resource. If successful in this effort, they then apply the resource to a joint production process. To highlight the role of conflict management at this level, the analysis abstracts from the potential gains that could be realized through joint production as might be modelled, for example, by increasing returns to scale in production.

Departing from the traditional theory of alliances that follows the pioneering work of Olson and Zeckhauser (1966), the analysis does not take the groups' membership as given.[3] Nor is "peace" assumed to prevail among group members. Instead, the distribution of their joint product is subject to another, separate conflict—that is, between the members of the group. Just as the possible emergence of conflict between individuals in their economic interactions can have important implications for the equilibrium distribution of resources and income, the possibility of conflict between individuals within groups should not be ignored.[4] Furthermore, the analysis does not appeal to the public-good nature of defense and appropriation to explain the emergence of groups—for example, the cost-saving advantages realized when taking defense measures against a common enemy [Sandler (1999)].[5]

The basis for group formation in this setting derives from the free-rider problem. Specifically, the individual's incentive to participate in the conflict over the contestable resource through her contributions to the group's collective effort is lower than when the individual acts alone. For the individual alone bears the full cost of her contribution, whereas the benefit possibly realized would have to be shared with the other members. Hence, relative to the case of individual conflict, the formation of groups can reduce the intensity of conflict, thereby improving overall welfare.[6] Applying the theory of endogenous coalition structures—in the spirit of Bloch (1996), Chwe (1994)

[3]See Sandler and Hartley (2001) who provide an updated survey of this literature.

[4]Analyses in the literature on collective rent seeking—e.g., Nitzan (1991)—similarly have two layers of conflict, but effectively treat them as one. For given the group's sharing rule, the two conflicts' outcomes are determined by each member's contribution to the group's effort in the inter-group conflict.

[5]While Skaperdas (1998) and Noh (2002) show, in different though related settings with three heterogeneous individuals, that a conflict technology having this sort of property is critical for the emergence of an alliance, Garfinkel (2004) suggests that such a technology might not even be necessary.

[6]This idea is implicit in Wärneryd (1998), though he considers a somewhat different issue, the endogenous formation of (two) jurisdictions.

and Ray and Vohra (1997, 1999), among others—Garfinkel (2004) shows that this effect alone may very well be sufficient to support the stability of group formation in equilibrium.[7]

However, the scope for group formation in this context can be limited in the sense that small groups tend to be more stable than large groups. This tendency can be attributed, at least in part, to the one feature of this setting that indirectly serves as the basis for group formation—namely, the conflict that emerges over the distribution of the joint product. Intra-group conflict means that each individual must devote some effort to secure a share of that product for herself, implying a diversion of scarce resources away from production. Following Grossman (2001), one can think of the members' efforts as an input into the process by which property rights are created or initial claims to property are transformed into property rights in what is essentially an anarchic environment. In the extreme case considered in Garfinkel (2004) where the distribution of the winning group's product is determined solely by these efforts, each individual's expected payoff is strictly decreasing in her group's size. For an increase in group size induces more aggressive competition between group members for a share of the product, implying a greater diversion of resources away from productive activities. The absence of conflict management within the group severely limits the appeal and thus the stability of larger groups in equilibrium.

Of course, one could argue that there are possible synergistic effects from organized economic activities and production that might alleviate the added instability that is associated with larger groups. In developing countries, for example, gains are realized in the pooling of resources and the sharing of risks as a form of community insurance. In developed countries, gains arise from the division of labor and specialization of production effected through the organization of the production process. However, the realization of such gains are predicated on the existence of some set of stable institutions—rules that govern joint or collective production and rules that govern the division of the gains themselves.[8] In other words, the possible advantages

[7] As discussed in more detail below, the equilibrium notion employed here, *farsighted stability* which is attributed specifically to Chwe (1994), expands the opportunities for "cooperation" among individuals who would behave otherwise in a noncooperative way.

[8] See Knight (1992) for a lucid discussion of the collective benefits of social institutions. Knight's central thesis, however, is that the key to understanding institutions and their

from joint production need not expand the role of group formation unless some mechanisms for managing intra-group conflict are already in place.

As such, the analysis focusses instead simply on conflict management, supposing that individuals within a group might be able to resolve the conflict that naturally arises over the distribution of the resources available to them or the product of their labor in more "civilized" ways involving less "social waste." Remaining agnostic about when institutions of conflict management emerge and how their effectiveness is determined, the analysis supposes that the actual distribution of the group's product among its members depends on a weighted average of (i) a "binding" agreement for the group members to share their joint product equally and (ii) the outcome of an intra-group contest, separate from that between groups.[9] The weight on the equal-sharing rule reflects the strength of the group's existing institutions to manage conflict, whether it be through the creation and enforcement of formal rules and laws or through social norms, without having to resort to appropriative activities. Thus, the more effective is the management of intra-group conflict, the fewer resources are diverted from production in the process by which each member is able to secure a share of the group's product for herself.[10]

While conflict management weakens the free-rider problem, it does not eliminate it. Thus, allowing for more peaceful and hence less costly mechanisms of managing conflict within the group expands the opportunities for group formation. Indeed, as long as the distribution of the group's joint product is determined partly by such mechanisms, the formation of the grand coalition is always a possible outcome. More generally, given the number of individuals in competition for the contestable resource, when mechanisms of conflict management are relatively more important in determining the distribution of the group's joint product, a greater variety of group structures, including those with larger groups, are possible in equilibrium. As such mechanisms become sufficiently effective, even structures with

evolution is to understand their distributional effects, the conflict such effects generate and how that conflict is resolved.

[9]The endogenous determination of conflict management within groups is left for future research as discussed below. [See Genicot and Skaperdas (2002) who model conflict management as an investment choice in a dynamic setting.]

[10]Of course, some institutions would seem to promote conflict (e.g., in the form of lobbying and other influence activities), thereby adding to "social waste."

larger groups become stable. However, larger is not unconditionally better. Provided that some intra-group conflict remains such that some resources must be diverted from production, aggregate expected payoffs eventually begin to decrease in group size. The grand coalition, in particular, is generally not the efficient outcome and thus there is no compelling reason to suppose that, among those which are stable, it is the most likely to emerge.

The paper is organized as follows: the next section presents the model of conflict which allows for the formation of multi-member groups. Treating the pre-conflict determination of the structure of groups as given, section 3 characterizes the allocation of resources and payoffs. Section 4 then studies the endogenous formation of groups, characterizing stable group structures, the role of less costly methods of conflict resolution within groups and their welfare implications. Section 5 offers some concluding remarks.

2 Analytical framework

Consider an environment populated by N identical, risk-neutral individuals, $\mathcal{I} = \{1, 2, \ldots, N\}$, who participate in a three-stage game. In the first stage, agents $i \in \mathcal{I}$ form coalitions or groups. A group is defined as any subset of the population, $\mathcal{A}_k \subseteq \mathcal{I}$, with membership $n_k \geq 1$, where $k = 1, 2, \ldots, A$ and A denotes the total number of groups. For future reference, let the structure of groups be indicated by $S = \{n_1, n_2, \ldots, n_A\}$, with the groups ordered such that $n_1 \geq n_2 \geq n_3 \cdots \geq n_A$. By definition, all individuals belong to a group. However, a group need not include more than one member. Moreover, this framework admits the possibility that everyone comes together to form a single group—the *grand coalition*: $\mathcal{A}_1 = \{1, 2, \ldots, N\}$.

2.1 Stage 2: Conflict between groups

In the second stage, all individuals participate in a winner-take-all conflict/contest over a resource X, which is essential in the production of a homogeneous consumption good in the third stage. They participate either collectively with others or alone, as dictated by the structure of groups determined in stage one. For any given configuration of groups, each member $i \in \mathcal{A}_k$ chooses how much she will contribute to the group's appropriative effort, m_i.[11] The probability that group k wins the conflict and successfully secures the entire resource X is determined by

$$\mu_k = \frac{\sum_{i\in A_k} m_i}{\sum_{j=1}^{A}\sum_{i\in\mathcal{A}_j} m_i} \tag{1}$$

if $\sum_{j=1}^{A}\sum_{i\in\mathcal{A}_j} m_i > 0$; otherwise, $\mu_k = 1/A$ for all k.[12]

Members of any group k with $n_k > 1$, by assumption, have no special advantage over those individuals who choose to participate in the conflict on their own.[13] Nevertheless, this formulation captures one aspect of the public-good nature of appropriation/defense spending. In particular, appropriative efforts by different members of a given group are perfect substitutes for one another. Regardless of who provides any additional effort, all members enjoy the increased probability of securing the resource X it implies.

2.2 Stage 3: Joint production and conflict within the group

To fix ideas, suppose that group k, with $n_k > 1$, successfully captures the resource X. Individuals not belonging to that group, $i \in \mathcal{A}_{k'}$ where $k' \neq k$, receive nothing, implying that their second-stage efforts result only in a loss over the three stages $u_{ik'} = -m_i$.[14] This loss is simply the utility cost of

[11]Since production is not possible until the resource is secured, the cost of this effort, as specified below, can be interpreted as foregone leisure.

[12]This specification, first introduced by Tullock (1980) for individual rent-seeking, is the contest success function most commonly used in the conflict/contest, rent-seeking literature. As argued below, although it admits the possibility of a corner solution for all members of all groups, such a solution is not a possible equilibrium outcome. See Hirshleifer (1989) who discusses the properties of this specification and related ones. Note that, under the maintained assumption of risk-neutrality, a group's probability of winning and taking the entire prize X, μ_k, may be interpreted alternatively as its resource share.

[13]To allow for such effects, Skaperdas (1998) modifies (1) as follows:

$$\mu_k = \frac{(\sum_{i\in\mathcal{A}_k} m_i)^\gamma}{\sum_j^A(\sum_{i\in\mathcal{A}_j} m_i)^\gamma}$$

With $N = 3$, he finds that a stable alliance between two of the three agents is possible only when $\gamma > 1$ (i.e., under super-additivity). Noh (2002) obtains a similar result with a slightly different specification to allow for such effects.

[14]In other words, the second-stage conflict weakens individuals $i \in \mathcal{A}_{k'}$ sufficiently such that it is not possible for them to try to steal the product from the winning group in the third-stage conflict.

the individual's second-stage effort, which increases linearly in that effort.

Now consider the members of the winning group k. Each $i \in \mathcal{A}_k$ is identically endowed with a unit of labor, which she allocates to productive activities, l_i, and appropriative or security related activities, s_i, where

$$1 = l_i + s_i. \tag{2}$$

These activities using X, in turn, deliver goods for consumption at the end of the stage. Specifically, individuals $i \in \mathcal{A}_k$ collectively combine the resource X with a fraction of their labor endowment, $l_i = 1 - s_i$ in a joint (linear) production process to yield a homogeneous consumption good. Generally, for $n_k \geq 1$ using (2), the group's total product, Y_k, is specified as—

$$Y_k = \textstyle\sum_{i \in \mathcal{A}_k} [1 - s_i] X / n_k. \tag{3}$$

Although X might be considered a public good from the perspective of the second-stage (winner-take-all) conflict, at this stage X would be interpreted as a purely private good.[15]

For $n_k > 1$, each member must also devote some effort, s_i, towards securing a share of the final product. This latter activity, reflecting the conflict that emerges within the winning group, detracts from production. Assume that the *share* of final output, Y_k, enjoyed by agent $i \in \mathcal{A}_k$, σ_{ik}, depends on her security effort s_i, distinct from m_i, and on the effort by everyone else in her group, s_j for $j \neq i \in \mathcal{A}_k$. But her share need not depend entirely on those efforts. More formally, for $n_k \geq 1$,

$$\sigma_{ik} = \frac{1-\alpha}{n_k} + \frac{\alpha s_i}{\sum_{j \in \mathcal{A}_k} s_j}, \quad \alpha \in [0, 1] \tag{4}$$

[15]Specifying production as a joint process is common in the emerging literature on the effects of conflict on economic outcomes. [See, for example, the survey by Garfinkel and Skaperdas (2000).] Allowing for complementarities or increasing returns in production would provide another potential benefit of group formation. However, given the linear homogeneity of the technology as specified here, one need not suppose that there is any sort of interaction between group members in production. An alternative interpretation of the production technology (3) is that each member of the winning group takes an equal share of X at the beginning of the third stage and produces in isolation of the others. In this case, the share σ_{ik}, defined below in (4), would represent the fraction of her own product that member i defends and that which she captures from the other members of her group k.

if $\sum_{j \in \mathcal{A}_k} s_j > 0$; otherwise, $\sigma_{ik} = 1/n_k$ for all $i \in \mathcal{A}_k$.[16] Each member $i \in \mathcal{A}_k$, then, obtains a payoff of $u_{ik} = \sigma_{ik} Y_k - m_i$. Whether her group secures the resource or not, each individual alone bears the cost of her contribution to group's effort in the second-stage conflict. Even if the members could credibly agree to share the group's joint product equally ($\alpha = 0$)—as if the intra-group conflict could be resolved peacefully, implying that $s_i = 0$ for all $i \in \mathcal{A}_k$ and $\sigma_{ik} = 1/n_k$—the free-rider problem would remain relevant.

But, there is no presumption of "peace" here. While involved in a joint production process in the third stage, each member must devote some effort ($s_i > 0$) to secure a share of output given $\alpha > 0$. Nevertheless, the analysis does admit the possibility that (exogenously given) social institutions can mediate conflict within the group, and in doing so have implications for inter-group conflict.[17] In the context of this model, $1 - \alpha \geq 0$ measures the effectiveness of existing mechanisms of conflict management to determine the distribution of the group's product without having to rely on the members' current appropriative activities. Put differently, smaller values of α would reflect stronger social institutions to effect a less costly resolution of group conflict. But, in supposing that $\alpha > 0$ holds and, in addition, that some effort is required in the production process, the analysis of the model's third stage captures the fundamental trade-off between production and appropriation, first modelled half a century ago by Haavelmo (1954, pp. 91-98) and considered more recently by Garfinkel (1990), Hirshleifer (1991, 1995), Skaperdas (1992), and Grossman and Kim (1995), among others.[18]

[16] Note that, for $n_k = 1$, $\sigma_{ik} = 1$ regardless of the choice of s_i. This specification is similar to the sharing rule found in the collective rent seeking literature [e.g., Nitzan (1991)]. However, in contrast to the literature, the resolution of the intra-group conflict is determined independently of the members' relative choices of m_i.

[17] Related analyses which have considered more specifically the interrelations between democratic political institutions and international conflict, include Garfinkel (1994) and Hess and Orphanides (2001), among others.

[18] That such a trade-off does not emerge in the second stage might appear to be important for the central results of the analysis. What is important here, however, is that individuals do not fully internalize the benefits of their efforts in fighting over X relative to the costs they incur. In particular, the findings of this analysis would follow if it were alternatively based on a framework that is more in line with the collective rent-seeking literature such as that in Noh (2002), provided it was also modified so that individuals also valued leisure. Such a modification would drive a wedge between the individual's incentives and the collective interests of the group. By the same token, the qualitative results would remain intact if the analysis were based on a model in which there was no

3 The allocation of resources given the group structure

Treating the pre-conflict determination of the coalition structure S as given, the analysis now considers the allocation of resources in the second and third stages. Each individual aims to maximize her expected payoff which equals her expected consumption in the third stage net of the utility cost of her effort in securing the contestable resource in the second stage: $u^e_{ik} = \mu_k \sigma_{ik} Y_k - m_i$. In this multi-stage setting, the amount of resources available to anyone in the third stage will, of course, depend on second-stage choices. All individuals, when making their second-stage choices, will take this influence into account. In accordance with the equilibrium notion of subgame perfection, then, we solve the model backwards, starting with the third and final stage.

3.1 The outcome of the intra-group conflict

Given the outcome of the second-stage conflict over X, equations (3) and (4) imply that the payoff for each member i of the winning group k, $u_{ik} = \sigma_{ik} Y_k - m_i$ can be written as

$$u(s_i, m_i, n_k) = \left[\frac{1-\alpha}{n_k} + \frac{\alpha s_i}{\sum_{j \in \mathcal{A}_k} s_j}\right] \left[\sum_{j \in \mathcal{A}_k} (1 - s_j) \frac{X}{n_k}\right] - m_i \qquad (5)$$

Each individual $i \in \mathcal{A}_k$ chooses s_i to maximize this expression. Assume that group members make their third-stage choices simultaneously.

The conflict technology shown in (4) with $\alpha > 0$ generally implies that, if no appropriative effort were made by any member of the group, then anyone could capture all of αY_k with certainty by putting forth an infinitesimally small amount of effort. Since no one would leave such an opportunity unexploited even with α arbitrarily close to zero, the "peaceful" outcome where $s_i = 0$ for all $i \in \mathcal{A}_k$ cannot be an equilibrium outcome. As such, the

production in the third stage, as in the models of sequential conflict of Katz and Tokatlidu (1996) and Wärneryd (1998).

following condition must be satisfied at an optimum:

$$\frac{\alpha \sum_{j \neq i \in \mathcal{A}_k} s_j}{[\sum_{j \in \mathcal{A}_k} s_j]^2} \left[\sum_{j \in \mathcal{A}_k} (1 - s_j) \right] \geq \frac{1-\alpha}{n_k} + \frac{\alpha s_i}{\sum_{j \in \mathcal{A}_k} s_j} \tag{6}$$

with strict equality for $s_i < 1$. Given the symmetry of the group's membership, this condition implies that members choose the same labor allocation, $s_i = s$, resulting in an interior optimum: $s_i \in (0, 1)$ for all $i \in \mathcal{A}_k$. Using equations (3), (4) and (5), condition (6) as a strict equality implies the following Nash equilibrium of the third stage:

$$\hat{s}_k = \frac{\alpha(n_k - 1)}{1 + \alpha(n_k - 1)} \tag{7a}$$

$$\hat{u}(m_i, n_k) = \frac{X}{[1 + \alpha(n_k - 1)]n_k} - m_i, \tag{7b}$$

for all $i \in \mathcal{A}_k$. In this equilibrium for any $\alpha \in [0, 1]$, each member of the winning group enjoys an equal share of final output: $\sigma_{ik} = 1/n_k$, which is decreasing in the size of the group, n_k.[19] For $\alpha > 0$, given m_i, because a larger n_k implies a greater dilution of the prize X *and* a greater diversion of effort away from production towards security, the payoff is decreasing in the square of the size of the group. Note further that effective group conflict management (i.e., a smaller α) diminishes the latter effect. While s_k is increasing in α, $\hat{u}(m_i, n_k)$ is decreasing in α.

3.2 The outcome of the inter-group conflict

Now consider the second-stage conflict between groups, again with the structure of groups fixed. Each individual i belonging to group k chooses m_i to maximize the *expected* value of (7b), given by

$$\hat{u}^e(m_i, n_k) = \frac{\mu_k X}{[1 + \alpha(n_k - 1)]n_k} - m_i \tag{8}$$

subject to the conflict technology, μ_k, as specified in (1). Individuals in all A groups make their decisions simultaneously. In (8), the first term repre-

[19] Of course, as indicated earlier, the specification for the conflict resolution technology (4) implies that for $i \in \mathcal{A}_k$, where $n_k = 1$ or $\alpha = 0$, $s_i = \bar{s} = 0$.

sents the product enjoyed by member i of group k, having won the conflict, weighted by the winning probability, μ_k. The second term represents the utility cost of fighting over the contestable resource; it is borne solely by the individual regardless of the outcome of that conflict.

Although the specification of the conflict technology (1) implies that $\sum_{j=1}^{A}\sum_{i\in\mathcal{A}_j} m_i > 0$, a fully interior solution is not guaranteed for all configurations of groups when $A > 2$. That is to say, the members of one or more groups might choose $m_i = 0$. But, the stability of a given configuration does require that all groups actively participate in the second-stage conflict.[20] In anticipation of our subsequent focus on stable groups and in the interest of brevity, the analysis to follow considers only such solutions. Accordingly, the individual's choice in the second-stage satisfies the following equality:

$$\frac{\sum_{i\notin\mathcal{A}_k} m_i}{[\sum_{j=1}^{A}\sum_{i\in\mathcal{A}_j} m_i]^2}\left[\frac{X}{[1+\alpha(n_k-1)]n_k}\right] = 1. \tag{9}$$

Maintaining focus on the case of within-group symmetry (i.e., when m_i equals a constant m_j for all $i \in \mathcal{A}_j$ $j = 1, 2, \ldots, A$), the condition shown in (9) implies

$$\frac{M - n_k m_k}{M^2}\left[\frac{X}{[1+\alpha(n_k-1)]n_k}\right] = 1 \tag{10}$$

where $M \equiv \sum_{j=1}^{A} n_j m_j$.[21] With this condition, one can find the equilibrium effort put forth by each individual belonging to group k of size n_k, given the structure of groups, S:

$$m(n_k, S) = [F - (A-1)(1+\alpha(n_k-1))n_k]\,\frac{(A-1)X}{n_k F^2} \tag{11}$$

for all k, where $F \equiv \sum_{j=1}^{A}[1+\alpha(n_j-1)]n_j$.[22]

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[20] See Lemma 2 in Appendix A.1, which follows the tables at the end of the paper.

[21] Note, however, since the probability of winning X depends on $\sum_{i\in\mathcal{A}_k} m_i$, not just m_i, only total effort by the group is uniquely determined; individual effort, m_i, is not. Although the focus here on the symmetric outcome may make the emergence of groups more likely, this focus seems most natural given the assumption that individual members of the group are identical.

[22] Specifically, rewrite (10) as $X(M - n_k m_k) = M^2[1+\alpha(n_k-1)]n_k$, and sum over all groups, $k = 1, 2, \ldots, A$ to obtain $AXM - XM = M^2\sum_{j=1}^{A}[1+\alpha(n_j-1)]n_j$. Simplifying and rearranging shows that, in equilibrium, $M = X(A-1)/F$, which with (10) yields

In the case where all groups are of equal size $n \geq 1$, $S \equiv \hat{S} = \{n, \ldots, n\}$,[23] the solution shown in (11) simplifies to $\hat{m} \equiv m(n, \hat{S}) = (N-n)X/N^2 n[1 + \alpha(n-1)]$. Under individual conflict where $S = \hat{S} = \bar{S}$,[24] this solution simplifies even further to $\bar{m} \equiv m(1, \bar{S}) = (N-1)X/N^2$. By contrast, when the grand coalition forms $n = N$, $m(n, \hat{S}) = 0$. As can easily be confirmed, under alternative, less extreme symmetric structures given $N(= An)$, $1 \leq n \leq N$, $m(n, \hat{S})$ is decreasing in n or equivalently increasing in A.

For any given structure of groups with $m_k > 0$ for all k, the solution for $m(n_k, S)$ reveals more generally that the equilibrium effort by the individual members of group k in the inter-group conflict is decreasing in the size of the group, n_k, as is the total effort by the group, $n_k m(n_k, S)$. Not surprisingly, then, the expected probability of winning the conflict in stage 2, given by $\mu_k = [F - (A-1)[1 + \alpha(n_k - 1)]n_k]/F$ for $A > 1$, is also decreasing in the group size, n_k.

Using this expression for μ_k, (8), and (11), the payoff expected by each individual member of group k at the end of stage one, $u^e(n_k, S)$, can be written as

$$u^e(n_k, S) = \frac{X[F - (A-1)[1 + \alpha(n_k - 1)]n_k][F - (A-1)[1 + \alpha(n_k - 1)]]}{[1 + \alpha(n_k - 1)]n_k F^2} \tag{12}$$

for $k = 1, 2, \ldots, A$, where as previously defined, $F \equiv \sum_{j=1}^{A} [1 + \alpha(n_j - 1)]n_j$ and $S = \{n_1, n_2 \ldots, n_A\}$.

Not surprisingly then, given any structure of groups where $m_k > 0$ for $k = 1, 2, \ldots, A$, individuals belonging to larger groups expect a smaller payoff than the payoff expected by those belonging to smaller groups:

$$u^e(n_1, S) \leq u^e(n_2, S) \leq \cdots \leq u^e(n_A, S) \tag{13}$$

where by assumption $n_1 \geq n_2 \geq \ldots \geq n_A$.[25] Of course, this ranking says

(11). From this solution, it follows that $m_k > 0$ for all k provided that $F > (A-1)[1 + \alpha(n_k - 1)]n_k$ holds for $n_k = n_1$. One necessary condition shown in Lemma 1 of Appendix A.1 is that the number of singleton groups must be strictly less than 2, or $n_{A-1} \geq 2$.

[23] Ignoring integer problems in this symmetric case, $A = N/n$ and $F = N[1 + \alpha(n-1)]$.

[24] In this case, $F = N$.

[25] One might conjecture that this ranking depends on the assumption, commonly made

nothing about an individual's incentive to move from one group to another, as it does not account for the effect of the hypothetical move on the efforts levels m by anyone in the stage-two conflict or others' incentive to move in response. Such incentives are considered more carefully in the analysis of the stable formation of groups that follows next.

4 Endogenous group formation and conflict management

Having characterized the allocation of resources in the second and third stages of the game given the structure of groups, the analysis now turns to the first stage of the game—namely, the formation of groups in equilibrium. In particular, defining an equilibrium of the first stage as an outcome where no individual can possibly increase her expected payoff, the analysis endogenizes the structure of groups, S.

4.1 Expected gains under symmetric group formation

As a preliminary step to that analysis, this subsection illustrates the gains that individuals might expect under a *symmetric, multi-member coalition structure*—i.e., where $n_k = n > 1$ for all k: $\hat{S} \equiv \{n, \ldots, n\}$. Using the expression for an individual's expected payoff given in (12), the expected gains under such a structure of groups relative to the outcome of individual conflict, $G^e(n) \equiv u^e(n, \hat{S}) - u^e(1, \bar{S})$, can be written as

$$G^e(n) = \frac{[(N - \alpha n)(n-1)]X}{N^2 n[1 + \alpha(n-1)]} \tag{14}$$

in the literature as well as in this paper, of linear costs of effort in the inter-group conflict. Analyzing a model of collective action (with effectively just one layer of conflict), Esteban and Ray (2001) show that, if instead these costs are increasing sufficiently quickly in effort (or money as the case may be), then the group's probability of winning the conflict would be increasing in its size. Nevertheless, they also find that, if the prize is purely private as in the present analysis, the expected payoff to each member would be decreasing in group size.

for $n > 1$.[26] Some straightforward calculations based on this expression establish the following:

Proposition 1 *Under a symmetric structure of coalitions, with $n_k = n > 1$ $\forall\ k = 1, 2, \ldots, A$, the gains expected by each individual, $G^e(n)$, are*
(a) *strictly positive for $n < N$,*
(b) *equal to 0 for $n = N$ when $\alpha = 1$,*
(c) *strictly positive for $n = N$ when $\alpha < 1$, and*
(d) *increasing in n, for $\alpha < \frac{N}{(n-1)^2N+n^2}$*

The potential for greater expected payoffs under group formation suggests that the cost-saving advantage to appropriative/defense activities by a group, which has been highlighted especially in the literature on alliances in international relations, might not be essential to the formation of multi-member groups. In the context of this simple model, the expected gains come in the form of a reduction in the severity of conflict over the contestable resource X for $1 < n < N$ regardless the extent to which the intra-group conflict can be resolved without having to devote effort specifically to that process ($\alpha \in [0, 1]$). No member of a group with $n > 1$ fully internalizes the benefits of her efforts in that conflict and so naturally devotes less effort to it. In the symmetric outcome, everyone else is doing just the same, so that the net effect on the winning probabilities in the conflict over X relative to the case of individual conflict is zero. Thus, as Proposition 1 indicates, there are potential gains under symmetric group formation, with $n < N$.

The proposition also suggests, however, that the expected gains are limited, and more so the closer is α to 1. In the limiting case where $\alpha = 1$, such that resolving the conflict that arises within the group depends entirely on s_i, the expected gains $G^e(n)$ fall as the number of members in each group, n, rises above 1. As n increases and the second-stage conflict between groups weakens, the third-stage conflict over the distribution of the product within the group intensifies; from an ex ante perspective, the increased costs associated with the intensifying intra-group conflict exceed the decreased costs associated with the weakening inter-group conflict. As n approaches N,

[26] From (12) under the assumption that $n_k = n$ for all k, one can find $u^e(n, S) = [N(n-1)+n]X/N^2n[1+\alpha(n-1)]$. Similarly, $u^e(1, \bar{S})$ can be derived from (12) assuming $n_k = 1$ for all k: $u^e(1, \bar{S}) = X/N^2$. Note that the function shown in (14) is also defined for $n = 1$: $G^e(1) = 0$.

the expected gains from group formation go to zero. Of course, the actual outcome under group formation with $n = N$ will differ from that under individual conflict by virtue of the difference in the nature of the conflict in the two outcomes. But, since $\alpha = 1$ by assumption, the group has no means by which its members can resolve conflict without resorting to arms (s); therefore, when $\alpha = 1$ a move from individual conflict ($n = 1$) to the grand coalition ($n = N$) merely shifts the entire conflict from one level over X to another over Y, with no consequences in terms of expected payoffs. Still, for $n < N$, $G^e(n) > 0$ holds, so that the formation of symmetric groups on net enhances expected welfare. Garfinkel (2004) shows that, even in this very restrictive case, the expected gains arising from the free-rider problem alone are often sufficient to predict the emergence of groups in equilibrium.

But, a central issue here is how the effectiveness of managing conflict over the distribution of the group's joint product can enhance the benefits of group formation. Consider the other limiting case. If the members of a group could credibly agree to share the product equally without arming ($s = 0$), then $\alpha = 0$. The expected payoff under symmetric group formation, given by $u^e(n, S) = [N(n-1) + n]/N^2 n$ in this case, would be increasing monotonically in n, so that the expected gains under group formation, $G^e(n) = (n-1)X/Nn$, would also be increasing in $n \leq N$ and strictly positive when evaluated at $n = N$. For intermediate values of α, $\alpha \in (0, 1)$, as n, rises above 1, the expected gains rise and continue to rise until n reaches some threshold value above which the gains begin to fall. For n in excess of this threshold the expected gain is decreasing in n.[27] Nonetheless, for $\alpha < 1$, the expected gains at $n = N$ are strictly positive.

4.2 Deviations and stable group structures

There are, of course, potential gains from forming asymmetric groups too. The magnitude of these gains similarly would be influenced by the effectiveness of managing the conflict that emerges over the distribution of the group's joint product. But, now we turn to a more systematic analysis of equilibrium, taking into account the possible benefits that an individ-

[27] To give a specific example, for $\alpha = \frac{1}{4}$ and $N = 100$, the expected gains are increasing as we move from $n = 2$ ($A = 50$) to $n = 4$ ($A = 25$), but decrease as we increase n any further.

ual might realize by breaking away from her group as well as the potential benefits she and others can realize under group formation.

Recall that, under any given group structure, the payoff expected by each member of a group is smaller for larger groups [see equation (13)]. Thus, despite the positive gains to be realized under group formation, any individual could have an incentive to deviate—for example, to break from one multi-member group to join a smaller one or form her own stand-alone group. To fix ideas, the analysis defines an equilibrium as follows:

Definition. *A structure of groups, $S = \{n_1, n_2, \ldots, n_A\}$, is a stable, Nash equilibrium structure if* (i) *the payoff expected by each individual under that structure is at least as large as that under individual conflict and strictly larger for at least one individual, and* (ii) *any deviation from that structure by an individual eventually makes that individual worse off.*

This definition is related to the notion of *farsighted stability*, which follows the non-cooperative theory of coalition formation—including Bloch (1996) and Ray and Vohra (1997, 1999), among others—but is attributed specifically to Chwe (1994).[28] This equilibrium concept imposes certain internal consistency requirements, ruling out possible deviations. In particular, *stability* requires that the equilibrium be robust to deviations, which must be robust to further deviations themselves. The *farsightedness* of the equilibrium concept employed here requires the robustness of all subsequent deviations as well until a stable outcome is reached. Thus, individuals are envisioned as looking to the eventual (stable) payoff of a deviation from the structure of groups under consideration. Therefore, the evaluation of the potential gains from a given deviation must factor in the possibility of all subsequent deviations by other individuals and the resulting impact on expected payoffs. In the context of this model, although any individual would benefit, for example, by leaving her group to form her own group (with just one member) given the membership of the other groups and her former group, such deviations could ultimately induce a reversion to individual conflict, leaving everyone, including the original deviator, worse off. Accordingly, such deviations themselves would be deemed unprofitable and, thus,

[28] Also see Brams (1994) for related ideas.

would not pose a threat to the stability of group structure under consideration.[29] With such an emphasis on forward looking behavior, the equilibrium notion of farsighted stability tends to expand the opportunities for "cooperation" among individuals who would behave otherwise in a noncooperative way.[30]

For an open membership game in which no consent is required to join an already existing group, one must also verify that no individual has an incentive to leave her group to join another. From the discussion above, it should be clear that no individual would have an incentive to leave her group to join equal sized or larger group [see section 3.2]. However, there may be an incentive to join a smaller group. In fact, when the size of the largest group exceeds the smallest by 2 or more, each member of the largest group, $k = 1$, has an incentive, holding the rest of the structure of groups (including her own former group $k = 1$) fixed, to join one of the smaller groups. Ruling out such incentives requires that the largest group have, at most, one more member than any other group: $n_1 \leq n_j + 1$ for any $j = 2, \ldots, A$.[31]

Based on the discussion above, we have the following:

Proposition 2 (**Symmetric Groups.**) *Suppose that the number of individuals involved in the conflict over the contestable resource X, N, can be decomposed into the product of two integers, $A^* > 1$ and $n^* > 1$. Then, for all $\alpha \in [0, 1]$, the symmetric multi-member structure of coalitions with A^* groups each having n^* members, $\hat{S}^* = \{n^*, \ldots, n^*, \}$, is farsighted stable and a Nash equilibrium structure. Furthermore, when $\alpha \in [0, 1)$, the grand coalition ($A^* = 1$ and $n = N \geq 2$) is a possible equilibrium.*

Proof. See Appendix A.2 ■

[29] Of course, without having specified the dynamics that would take us from a potential deviation to the outcome involving individual conflict, invoking the notion of farsighted stability here might seem ad hoc at best. However, the analysis in connection with Lemmas 1 and 2 in Appendix A.1 is suggestive. Moreover, this stability notion has much theoretical appeal in its emphasis on internal consistency and on the importance of the eventual outcome over the immediate outcome. On these points, see Ray and Vohra (1997, 1999).

[30] As noted below, however, even when only deviations by individuals are ruled out, there are multiple equilibria in this context, and further refinements are possible.

[31] See Lemma 3 in Appendix A.1. Note that this result suggests that there are limits to the possible gains that can be realized from group-size asymmetry, identified by Katz and Tokatlidu (1996), once one allows for endogenous group formation. In their interesting analysis, Katz and Tokatlidu fix the number of groups to two.

The proposition establishes that, when $0 \leq \alpha \leq 1$, all multi-member, symmetric coalition structures with $A > 1$ are stable. In addition, the relevance of managing conflict in the distribution of the group's product ($\alpha < 1$) expands the opportunity for endogenous group formation simply by making the emergence of the grand coalition possible.[32] Thus, for $\alpha < 1$ and $N \geq 2$, there exists at least one multi-member equilibrium structure of groups—namely, the grand coalition. However, farsighted stability is not limited to symmetric group structures.

Proposition 3 (**Asymmetric Groups.**) *Given N, choose any A^*, where $1 < A^* < N$, and define $a \equiv N - A^*n^*$, where $1 \leq a < A^*$. The asymmetric multi-member structure of groups, with a groups having $n^* + 1$ members and $A^* - a$ groups having n^* members, $S^* = \{n^*+1, \ldots, n^*+1, n^* \ldots, n^*\}$, is farsighted stable and a Nash equilibrium structure provided n^* satisfies the inequality*

$$\frac{[F - (A^* - 1)(1 + \alpha n^*)(n^* + 1)][F - (A^* - 1)(1 + \alpha n^*)]}{(1 + \alpha n^*)(n^* + 1)F^2} > \frac{1}{(A^*n^* + a)^2}$$

and, in the case that $a = 1$, an additional inequality

$$\frac{A^*(n^* - 1)(1 + \alpha(n^* - 1)) + 1}{(A^*n^*(1 + \alpha(n - 1)) + 1)^2 n^*(1 + \alpha(n - 1))} < \frac{1}{(A^*n^* + 1)^2},$$

where $F = a(n^* + 1)(1 + \alpha n^*) + (A^* - a)n^*(1 + \alpha(n^* - 1))$.

Proof. See the appendix. ■

Under asymmetric group structures, although the expected gains are unevenly distributed, everyone must be at least as well off as they would be under individual conflict. This requirement along with the requirement that $n_1 \leq n_j + 1$ for any $j = 2, \ldots, A$, is embedded in the first inequality of the proposition. The inequality ensures further that, for $a = 2, \ldots, A^* - 1$ given N, a deviation by one individual originally belonging to a size–$n + 1$ group to form a stand-alone group would make it profitable for at least one other individual to follow her, which in turn would induce a reversion to individual conflict and thus prove to be unprofitable. Therefore, the first

[32] Recall that when $\alpha < 1$, $G^e(N) > 0$; by contrast, when $\alpha = 1$, $G^e(N) = 0$.

inequality alone is a sufficient condition for the farsighted stability of groups with $a \in (1, A)$. Matters may differ, however, for groups with $a = 1$. Nevertheless, the second inequality serves to rule out the profitability of individual deviations in this case.[33]

The first inequality imposes a lower bound on n, given A^* and a. This lower bound on n limits the degree of asymmetry between the size–n groups and the size–$n+1$ groups so as not to give too much of an advantage to the members of the smaller (size–n) groups in the contest for control of X. For example, when $n = 3$, the advantage enjoyed by the smaller groups over the larger groups ($n+1 = 4$) is relatively more mild than when $n = 2$ (and $n+1 = 3$). But, decreasing the number of larger sized groups ($n+1$) relative to the number of the smaller sized groups (n) (or equivalently decreasing a relative to A) puts members of the size–$n+1$ groups at a relatively greater disadvantage, implying a more severe lower bound on n is necessary to keep those individuals from deviating. When $a = 1$ given $N = An + 1$, this constraint is most binding. The second inequality shown in Proposition 2 similarly imposes a lower bound on n precisely when $a = 1$, though it is not always binding. Here, the degree of conflict management comes into play. In particular, when exchange within groups is more peaceful (α is smaller), an individual deviation to form a stand-along group is more likely to leave the expected payoffs of everyone else (belonging to one of the A groups of size n) above that obtained under individual conflict. In this case given N, the coalition structure with $a = 1$ cannot be supported in equilibrium. But as α increases, the second inequality is more likely to implied by the first inequality even when $a = 1$.

Tables 1 and 2 illustrate some of these tendencies, showing the stable multi-member structure of groups based on Propositions 2 and 3, by the equilibrium number of groups, A^* for varying degrees of conflict management, $\alpha = \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\}$.[34] Consider first the equilibrium structures when $\alpha = 1$. An inspection of the tables reveals that, given N and A^*, an equilibrium multi-member structure of groups need not exist—for $N < 4$, $N = 5$

[33] See the appendix for more details.

[34] Keep in mind that α is a negative indicator of the degree of conflict management. The minimum values of n, $\min n^*$, given A and $a \in [1, A)$, for which a stable multi-member structure of groups exists were calculated using Mathematica. Details are available upon request.

and 7.[35] But, as N increases, the conditions for stability, ruling out individual deviations only, weaken. Even when the distribution of the group's joint product is determined by s alone ($\alpha = 1$), there exists at least one stable, multi-member structure of groups for any $N \geq 8$, having $A^* = 2$ groups: for any even number $N \geq 8$, both groups have $n^* = N/2$ members; and, for any odd number $N > 8$, one group has $n^* = (N-1)/2$ members and the other has just one additional member, $n^* + 1 = (N+1)/2$ [see Table 1]. And, as clearly shown in Table 2, other group structures are also possible. Observe that, in general, for any given $A^* > 2$, group structures with fewer size–$n+1$ groups (or smaller a) are more likely to be farsighted stable when N is larger.

The possibility of a less costly resolution of intra-group conflict (i.e., $\alpha < 1$) expands the opportunities for group formation considerably. First, as noted earlier and shown in Table 1, the emergence of the grand coalition is always a possibility for $N \geq 2$ when $0 \leq \alpha < 1$, in contrast to when $\alpha = 1$. Second, observe from Table 1 how min n^*, satisfying the inequality in Proposition 3, depends positively on α.[36] In words, farsighted stability of endogenous group formation is limited by the extent to which the distribution of the winning group's product is determined by arms or equivalently by the weakness of the existing social institutions. Along similar lines, Table 2 shows a greater variety of possible equilibrium configurations of multi-member groups ($2 \leq A \leq 6$) for $\alpha = \frac{1}{4}$ than for $\alpha = 1$, given $4 \leq N \leq 25$. However, also note that for $\alpha = 0$, the second inequality constraint from Proposition 3 becomes binding. In this case, stable coalition formation with $a = 1$ cannot be supported in equilibrium.

[35] In a similar economic setting where the free-rider problem emerges, Esteban and Sákovics (2002) find no possibility for group formation. Their finding, based on just three players and no possibility of for intra-group "cooperation," is actually consistent with the finding of the present analysis where $N = 3$ and $\alpha = 1$. Furthermore, while they allow for increasing costs in the inter-group effort which would tend to make their result stronger, they do not employ the equilibrium notion of farsighted stability.

[36] Thus, the minimum number of individuals in competition for the contestable resource X ($N = A^* \times \min n^* + a$) required for the stability of a given coalition structure is smaller with smaller α. For example, a structure consisting of 5 groups, 3 of which have $a = n + 1$ members and 2 of which have $A - a = n$ members, requires at least $N = A^* \times \min n^* + a = 23$ when $\alpha = 1$; when $\alpha = \frac{1}{2}$, N must be at least 18; and when $\alpha = 0$, the minimum N is only 13.

4.3 Equilibrium structures and expected payoffs

Given the multiplicity of possible group structures, one would naturally wonder how they would be ranked by the participants.[37] Now for $\alpha = 1$, as considered in Garfinkel (2004), when $N < 9$ and there are multiple equilibrium structures, they are all symmetric. Hence, from Proposition 1, everyone would prefer the structure having the greatest number of groups, $A^* = N/n^*$ or equivalently the smallest number of group members, $n^* > 1$. Maintaining the assumption that $\alpha = 1$, in the case where both asymmetric and symmetric groups are possible, the ranking is not so obvious.

Table 3a reports the expected payoffs per individual in each group, under the alternative equilibrium group structures for $9 \leq N \leq 25$. It also reports the aggregate expected payoffs under each of those structures and under individual conflict. The table confirms that, under asymmetric group structures, the benefits of group formation relative to individual conflict are distributed unevenly.[38] When more than one stable multi-member coalition structure exists for a given N, the payoffs expected by members of the size–n groups increase unambiguously as we move to another asymmetric structure of groups with a larger number of groups, A^*. The same cannot be said for members of size–$n+1$ groups. While in some cases their expected payoffs increase as well, sometimes they fall but never below what would be expected under individual conflict as required by the definition of equilibrium. Furthermore, not surprisingly, as we move from an asymmetric structure of groups to a symmetric one with a greater number of groups (A^*), the payoffs expected by each member of a size–n group typically fall while those expected by each member of a size–$n+1$ group rise. Nonetheless, note that regardless of whether the group is symmetric or asymmetric, aggregate expected payoffs are unambiguously rising in the number of groups (A^*) for any given N. However, this tendency is unique for $\alpha = 1$.

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[37]One could sharpen the predictions of the analysis in terms of the number of groups and the size of groups given N by applying the equilibrium refinement introduced by Bernheim, Peleg and Whinston (1987), to account for deviations of groups of individuals as well as for individual deviations. There need not be any equilibrium at all, however. In any case, such an analysis is beyond the scope of the present paper.

[38]Note that, consistent with the previous discussion, for any given A^* and a, the asymmetry is smaller for larger n or equivalently larger N. Furthermore, given N, the expected payoffs tend to be more evenly distributed when the number of size–$n+1$ groups (a) is larger relative to A^*.

Table 3b similarly reports the expected payoffs per individual in each group, under the alternative equilibrium group structures, for $9 \leq N \leq 25$, when $\alpha = \frac{1}{4}$. In the case of symmetric groups for a given N, observe that increasing the number of groups and hence decreasing n need not imply an increase in expected payoffs [e.g., when $N = 12$, increasing A from 4 to 6 groups]. Nevertheless, at least for each N considered here, given that $\alpha \geq \frac{1}{4}$ holds, there exist structures, either symmetric or asymmetric and in some cases both, that yield higher expected payoffs to all individuals than the grand coalition. Thus, there would not seem to be a very compelling argument—certainly not one based on efficiency grounds alone—for the grand coalition to stand out as a focal point among all those group structures which are farsighted stable. Of course, numerical simulations can give only a partial picture at best. Table 3b, in particular, does not show all possible coalition structures given N. More importantly, it considers only one value of α.

Table 4 provides some additional perspective, considering a considerably larger N ($N = 100$) and a different variety of α's, $\alpha = \{0, \frac{1}{100}, \frac{1}{10}, 1\}$. Like Tables 3a and 3b, it shows the expected payoffs for stable coalitions structures only. Note that for $\alpha = 1$, aggregate payoffs are monotonically increasing in A (or equivalently decreasing in n). By contrast, for $\alpha = 0$ aggregate payoffs are monotonically decreasing in A (or equivalently decreasing in n). Hence, while one might predict the emergence of the equilibrium structure with $A = 50$ and $n = 2$ when conflict management is irrelevant in determining the distribution of the group's joint output ($\alpha = 1$), under the equally unpalatable assumption that such institutions alone can determine that distribution ($\alpha = 0$) one might naturally predict the emergence of the grand coalition ($A = 1$ and $n = 100$). However, the table also clearly indicates that, when conflict management is just a little less effective ($\alpha = 0.01$), the grand coalition is no longer an efficient outcome. Everyone's expected payoff could be increased if they were to break off into smaller equally sized groups, $A = \{2, 4, 5, 10, 20, 25, 50\}$. Decreasing the effectiveness of conflict management further (by setting $\alpha = 0.10$) implies that all of the other asymmetric group structures similarly dominate the grand coalition. Even though the grand coalition is a feasible outcome, it seems reasonable to conjecture that the other stable structures are more or at least just as likely to emerge in equilibrium.

5 Concluding remarks

This paper has investigated the stable formation of groups, showing that some form of conflict management *within* groups enhances the possibilities for group formation considerably, thereby weakening the conflict between groups. Most notably, the grand coalition can emerge for any number of individuals $N \geq 2$.

But what is especially striking is the finding that, unless the existing social institutions are sufficiently strong to render appropriative activities within groups unnecessary, the grand coalition is not likely to emerge as the "best" outcome.[39] That individuals in this model are assumed to be identical only underscores the importance of studying multilayered or sequential conflict and thus intra-group governance in economics.

Admittedly, insofar as the building of social institutions to manage conflict within the group is taken as exogenous and that the development of such institutions is costly, the analysis here would seem incomplete. Thus, an important and interesting extension, left for future research, would be to endogenize the management of conflict in a more dynamic setting.[40] Such an extension could reveal other fundamental tradeoffs that would, at the same time though perhaps to varying degrees, influence the link between economic performance and the degree of inter-group conflict. In particular, the development of institutions to manage conflict and their maintenance over time would divert scarce resources away from both production and appropriative activities between groups. Whereas the diversion away from production would hinder economic growth and development, the distraction from preparations for inter-group conflict could serve to promote peace and, in the process, enhance economic performance.

[39] Analyses finding a strong tendency for the emergence of the grand coalition—see, for example, Bloch, Sánchez-Pageés and Soubeyran (2002)—typically abstract from the difficulties of intra-group conflict, assuming instead that the members are able to make binding commitments to sharing rules.

[40] Genicot and Skaperdas (2002) have made some progress in this direction, though not in the context of a framework with endogenous group formation.

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Table 1. Equilibrium group structures, $N > 1$

			$\min n^*$				
A^*	a	$A^* - a$	$\alpha = 1$	$\alpha = 3/4$	$\alpha = 1/2$	$\alpha = 1/4$	$\alpha = 0$
1	0	1	·	2	2	2	2
2	0	2	2	2	2	2	2
	1	1	4	3	2	2	·
3	0	3	2	2	2	2	2
	1	2	5	5	4	3	·
	2	1	3	2	2	2	1
4	0	4	2	2	2	2	2
	1	3	7	7	6	5	·
	2	2	5	4	4	3	2
	3	1	2	2	2	1	1
5	0	5	2	2	2	2	2
	1	4	9	9	8	7	·
	2	3	7	6	5	5	3
	3	2	4	4	3	3	2
	4	1	2	2	1	1	1
6	0	6	2	2	2	2	2
	1	5	11	10	10	8	·
	2	4	9	8	7	6	4
	3	3	6	6	5	4	3
	4	2	4	4	3	3	2
	5	1	2	2	1	1	1
7	0	7	2	2	2	2	2
	1	6	13	12	12	10	·
	2	5	11	10	9	8	5
	3	4	9	8	7	6	4
	4	3	6	6	5	4	3
	5	2	4	4	3	3	2
	6	1	2	1	1	1	1

Notes: A^* denotes the total number of groups, a of which have $n + 1$ members; the remaining $A^* - a$ have n members. Given A groups of which a are of size $n^* + 1$, the table indicates the minimum n that ensures the farsighted stability of the configuration under consideration. That configuration is a possible equilibrium structure when at least $N = A^* \times \min n^* + a$ individuals are competing for X.

Table 2. Equilibrium group sizes given $1 < A \leq 6$ for $N \leq 25$

		A				
	N	2	3	4	5	6
$\alpha = 1$	4	$(2, 2)$	·			
	5	·	·	·		
	6	$(3, 3)$	$(2, 2, 2)$	·	·	
	7	·	·	·	·	
	8	$(4, 4)$	·	$(2, 2, 2, 2)$	·	·
	9	$(5, 4)$	$(3, 3, 3)$	·	·	·
	10	$(5, 5)$	·	·	$(2, 2, 2, 2, 2)$	·
	11	$(6, 5)$	$(4, 4, 3)$	$(3, 3, 3, 2)$	·	·
	12	$(6, 6)$	$(4, 4, 4)$	$(3, 3, 3, 3)$	·	$(2, 2, 2, 2, 2, 2)$
	13	$(7, 6)$	·	·	·	·
	14	$(7, 7)$	$(5, 5, 4)$	·	$(3, 3, 3, 3, 2)$	·
	15	$(8, 7)$	$(5, 5, 5)$	$(4, 4, 4, 3)$	$(3, 3, 3, 3, 3)$	·
	16	$(8, 8)$	$(6, 5, 5)$	$(4, 4, 4, 4)$	·	·
	17	$(9, 8)$	$(6, 6, 5)$	·	·	$(3, 3, 3, 3, 3, 2)$
	18	$(9, 9)$	$(6, 6, 6)$	·	·	$(3, 3, 3, 3, 3, 3)$
	19	$(10, 9)$	$(7, 6, 6)$	$(5, 5, 5, 4)$	$(4, 4, 4, 4, 3)$	·
	20	$(10, 10)$	$(7, 7, 6)$	$(5, 5, 5, 5)$	$(4, 4, 4, 4, 4)$	·
	21	$(11, 10)$	$(7, 7, 7)$	·	·	·
	22	$(11, 11)$	$(8, 7, 7)$	$(6, 6, 5, 5)$	·	·
	23	$(12, 11)$	$(8, 8, 7)$	$(6, 6, 6, 5)$	$(5, 5, 5, 4, 4)$	$(4, 4, 4, 4, 4, 3)$
	24	$(12, 12)$	$(8, 8, 8)$	$(6, 6, 6, 6)$	$(5, 5, 5, 5, 4)$	$(4, 4, 4, 4, 4, 4)$
	25	$(13, 12)$	$(9, 8, 8)$	·	$(5, 5, 5, 5, 5)$	
$\alpha = \frac{1}{4}$	4	$(2, 2)$	·			
	5	(3,2)	·	·		
	6	$(3, 3)$	$(2, 2, 2)$	·	·	
	7	$(4, 3)$	·	$(2, 2, 2, 1)$	·	
	8	$(4, 4)$	$(3, 3, 2)$	$(2, 2, 2, 2)$	·	·
	9	$(5, 4)$	$(3, 3, 3)$	·	$(2, 2, 2, 2, 1)$	·
	10	$(5, 5)$	$(4, 3, 3)$	·	$(2, 2, 2, 2, 2)$	·
	11	$(6, 5)$	$(4, 4, 3)$	$(3, 3, 3, 2)$	·	$(2, 2, 2, 2, 2, 1)$
	12	$(6, 6)$	$(4, 4, 4)$	$(3, 3, 3, 3)$	·	$(2, 2, 2, 2, 2, 2)$
	13	$(7, 6)$	$(5, 4, 4)$	·	·	·
	14	$(7, 7)$	$(5, 5, 4)$	$(4, 4, 3, 3)$	$(3, 3, 3, 3, 2)$	·
	15	$(8, 7)$	$(5, 5, 5)$	$(4, 4, 4, 3)$	$(3, 3, 3, 3, 3)$	·
	16	$(8, 8)$	$(6, 5, 5)$	$(4, 4, 4, 4)$	·	·
	17	$(9, 8)$	$(6, 6, 5)$	·	·	$(3, 3, 3, 3, 3, 2)$
	18	$(9, 9)$	$(6, 6, 6)$	$(5, 5, 4, 4)$	$(4, 4, 4, 3, 3)$	$(3, 3, 3, 3, 3, 3)$
	19	$(10, 9)$	$(7, 6, 6)$	$(5, 5, 5, 4)$	$(4, 4, 4, 4, 3)$	·
	20	$(10, 10)$	$(7, 7, 6)$	$(5, 5, 5, 5)$	$(4, 4, 4, 4, 4)$	·
	21	$(11, 10)$	$(7, 7, 7)$	$(6, 5, 5, 5)$	·	·
	22	$(11, 11)$	$(8, 7, 7)$	$(6, 6, 5, 5)$	·	$(4, 4, 4, 4, 3, 3)$
	23	$(12, 11)$	$(8, 8, 7)$	$(6, 6, 6, 5)$	$(5, 5, 5, 4, 4)$	$(4, 4, 4, 4, 4, 3)$
	24	$(12, 12)$	$(8, 8, 8)$	$(6, 6, 6, 6)$	$(5, 5, 5, 5, 4)$	$(4, 4, 4, 4, 4, 4)$
	25	$(13, 12)$	$(9, 8, 8)$	$(7, 6, 6, 6)$	$(5, 5, 5, 5, 5)$	·

Table 3a. Expected payoffs under equilibrium structures, $\alpha = 1$

		A						
N		1	2	3	4	5	6	N
9	n	·	3.44	2.88	·	·	·	1.23
	$n+1$	·	1.37	2.88	·	·	·	1.23
	N	·	20.61	25.93	·	·	·	11.11
10	n	·	1.80	·	·	3.00	·	1.00
	$n+1$	·	1.80	·	·	3.00	·	1.00
	N	·	18.00	·	·	30.00	·	10.00
11	n	·	2.17	5.32	12.36	·	·	0.83
	$n+1$	·	1.03	1.10	1.02	·	·	0.83
	N	·	16.99	24.80	33.87	·	·	9.09
12	n	·	1.27	1.74	2.08	·	2.43	0.69
	$n+1$	·	1.27	1.74	2.08	·	2.43	0.69
	N	·	15.28	20.83	25.00	·	29.17	8.33
13	n	·	1.49	·	·	·	·	0.59
	$n+1$	·	0.79	·	·	·	·	0.59
	N	·	14.48	·	·	·	·	7.69
14	n	·	0.95	2.83	·	12.00	·	0.51
	$n+1$	·	0.95	0.82	·	0.78	·	0.51
	N	·	13.27	19.55	·	33.33	·	7.14
15	n	·	1.08	1.16	4.92	1.63	·	0.44
	$n+1$	·	0.63	1.16	0.78	1.63	·	0.44
	N	·	12.63	17.33	24.12	24.44	·	6.67
16	n	·	0.73	1.48	1.27	·	·	0.39
	$n+1$	·	0.73	0.39	1.27	·	·	0.39
	N	·	11.72	17.13	20.31	·	·	6.25
17	n	·	0.82	1.74	·	·	11.78	0.35
	$n+1$	·	0.51	0.63	·	·	0.63	0.35
	N	·	11.20	16.22	·	·	32.99	5.88
18	n	·	0.58	0.82	·	·	1.34	0.31
	$n+1$	·	0.58	0.82	·	·	1.34	0.31
	N	·	10.49	14.81	·	·	24.07	5.56
19	n	·	0.65	1.01	2.56	4.71	·	0.28
	$n+1$	·	0.42	0.34	0.59	0.60	·	0.28
	N	·	10.06	14.56	19.07	23.74	·	5.26

continued . . .

Table 3a. Expected payoffs ($\alpha = 1$) continued

		A						
N		1	2	3	4	5	6	N
20	n	·	0.47	1.17	0.85	1.00	·	0.25
	$n+1$	·	0.47	0.49	0.85	1.00	·	0.25
	N	·	9.50	13.89	17.00	20.00	·	5.00
21	n	·	0.52	0.62	·	·	·	0.23
	$n+1$	·	0.36	0.62	·	·	·	0.23
	N	·	9.14	12.93	·	·	·	4.76
22	n	·	0.39	0.74	1.35	·	·	0.21
	$n+1$	·	0.39	0.30	0.27	·	·	0.21
	N	·	8.68	12.68	16.78	·	·	4.55
23	n	·	0.43	0.84	1.55	2.14	4.57	0.19
	$n+1$	·	0.30	0.39	0.45	0.21	0.49	0.19
	N	·	8.37	12.17	15.86	20.28	23.50	4.35
24	n	·	0.33	0.48	0.61	2.42	0.82	0.17
	$n+1$	·	0.33	0.48	0.61	0.46	0.82	0.17
	N	·	7.99	11.46	14.58	18.79	19.79	4.17
25	n	·	0.36	0.56	·	0.67	·	0.16
	$n+1$	·	0.26	0.25	·	0.67	·	0.16
	N	·	7.72	11.23	·	16.80	·	4.00

Notes: The first column reports the individual and aggregate payoffs under the grand coalition when farsighted stable. The next five columns report the expected payoffs under the equilibrium multi-member coalition structures reported in Table 2. The last column (N) reports the analogous expected payoffs under individual conflict. The entries for n and $n+1$ report the payoffs expected by each member of groups having respectively n and $n+1$ members. For symmetric groups, the same payoff is indicated for both groups. The entry for N reports the expected payoffs summed over all individuals. [The individual payoffs might not sum to the aggregate payoffs due to rounding.] These calculations assume that $X = 100$.

Table 3b. Expected payoffs under equilibrium structures, $\alpha = 1/4$

		A						
N		1	2	3	4	5	6	N
9	n	3.70	7.54	5.76	·	40.50	·	1.23
	$n+1$	3.70	3.63	5.76	·	1.98	·	1.23
	N	33.33	48.32	51.85	·	56.36	·	11.11
10	n	3.08	4.50	7.90	·	4.80	·	1.00
	$n+1$	3.08	4.50	1.40	·	4.80	·	1.00
	N	30.77	45.00	52.98	·	48.00	·	10.00
11	n	2.60	5.26	9.56	16.27	·	39.64	0.83
	$n+1$	2.60	2.85	2.82	2.50	·	1.59	0.83
	N	28.57	43.38	51.22	55.00	·	55.56	9.09
12	n	2.22	3.40	3.97	4.17	·	3.89	0.69
	$n+1$	2.22	3.40	3.97	4.17	·	3.89	0.69
	N	26.67	40.74	47.62	50.00	·	46.67	8.33
13	n	1.92	3.88	5.08	·	·	·	0.59
	$n+1$	1.92	2.29	1.39	·	·	·	0.59
	N	25.00	39.28	47.62	·	·	·	7.69
14	n	1.68	2.65	5.99	7.38	15.49	·	0.51
	$n+1$	1.68	2.65	2.21	0.96	1.92	·	0.51
	N	23.53	37.14	46.03	51.97	53.98	·	7.14
15	n	1.48	2.98	2.89	8.61	3.26	·	0.44
	$n+1$	1.48	1.87	2.89	2.00	3.26	·	0.44
	N	22.22	35.86	43.33	49.86	48.89	·	6.67
16	n	1.32	2.13	3.55	2.90	·	·	0.39
	$n+1$	1.32	2.13	1.24	2.90	·	·	0.39
	N	21.05	34.09	42.95	46.43	·	·	6.25
17	n	1.18	2.36	4.10	·	·	15.00	0.35
	$n+1$	1.18	1.56	1.76	·	·	1.56	0.35
	N	20.00	32.96	41.59	·	·	53.33	5.88
18	n	1.06	1.75	2.19	4.62	7.11	2.67	0.31
	$n+1$	1.06	1.75	2.19	0.97	0.73	2.67	0.31
	N	19.05	31.48	39.51	46.64	51.43	48.15	5.56
19	n	0.96	1.92	2.62	5.30	8.08	·	0.28
	$n+1$	0.96	1.32	1.08	1.59	1.55	·	0.28
	N	18.18	30.42	39.00	44.98	49.08	·	5.26

continued ...

Table 3b. Expected payoffs ($\alpha = 1/4$) continued

		A						
N		1	2	3	4	5	6	N
20	n	0.87	1.46	2.98	2.13	2.29	·	0.25
	$n+1$	0.87	1.46	1.43	2.13	2.29	·	0.25
	N	17.39	29.23	37.85	42.50	45.71	·	5.00
21	n	0.79	1.59	1.72	2.68	·	·	0.23
	$n+1$	0.79	1.13	1.72	0.43	·	·	0.23
	N	16.67	28.35	36.19	42.72	·	·	4.76
22	n	0.73	1.24	2.01	3.16	·	6.94	0.21
	$n+1$	0.73	1.24	0.94	0.88	·	0.59	0.21
	N	16.00	27.27	35.66	42.08	·	51.09	4.55
23	n	0.67	1.34	2.26	3.58	4.37	7.75	0.19
	$n+1$	0.67	0.98	1.18	1.27	0.74	1.27	0.19
	N	15.38	26.50	34.68	40.76	46.10	48.58	4.35
24	n	0.62	1.06	1.39	1.62	4.91	1.88	0.17
	$n+1$	0.62	1.06	1.39	1.62	1.24	1.88	0.17
	N	14.81	25.56	33.33	38.89	44.38	45.24	4.17
25	n	0.57	1.15	1.59	1.97	1.68	·	0.16
	$n+1$	0.57	0.86	0.81	0.47	1.68	·	0.16
	N	14.29	24.86	32.82	38.85	42.00	·	4.00

Notes: The first column reports the individual and aggregate payoffs under the grand coalition when farsighted stable. The next five columns report the expected payoffs under the equilibrium multi-member coalition structures reported in Table 2. The last column (N) reports the analogous expected payoffs under individual conflict. The entries for n and $n+1$ report the payoffs expected by each member of groups having respectively n and $n+1$ members. For symmetric groups, the same payoff is indicated for both groups. The entry for N reports the expected payoffs summed over all individuals. [The individual payoffs might not sum to the aggregate payoffs due to rounding.] These calculations assume that $X = 100$.

Table 4. Expected Payoffs Under Equilibrium Structures for $N = 100$

		$\alpha = 0.00$		$\alpha = 0.01$		$\alpha = 0.10$		$\alpha = 1.00$	
A^*	(n, a)	N	n / $n+1$	N	n / $n+1$	N	n / $n+1$	N	n / $n+1$
50	(2,0)	51.00	0.51	50.50	0.50	46.36	0.46	‡25.50	0.26
			0.51		0.50		0.46		0.26
34	(2,32)	67.00	11.39	66.35	11.54	‡61.70	12.44	·	·
			0.22		0.21		0.12		·
25	(4,0)	76.00	0.76	73.79	0.74	†58.46	0.58	19.00	0.19
			0.76		0.74		0.58		0.19
20	(5,0)	81.00	0.81	77.88	0.78	57.86	0.58	16.20	0.16
			0.81		0.78		0.58		0.16
17	(5,15)	84.00	3.36	80.36	3.35	58.29	3.01	16.10	1.11
			0.56		0.52		0.31		0.06
15	(6,10)	86.00	2.29	81.84	2.26	·	·	·	·
			0.25		0.20		·		·
13	(7,9)	88.00	2.01	82.80	1.97	54.53	1.55	·	·
			0.44		0.39		0.15		·
12	(8,4)	89.00	1.34	83.20	1.28	·	·	·	·
			0.10		0.04		·		·
10	(10,0)	91.00	0.91	‡83.49	0.83	47.89	0.48	9.10	0.09
			0.91		0.83		0.48		0.09
8	(12,4)	93.00	1.24	†83.54	1.14	43.76	0.67	7.63	0.13
			0.64		0.55		0.23		0.03
7	(14,2)	94.00	1.07	83.05	0.96	40.64	0.50	6.67	0.09
			0.63		0.52		0.19		0.02
6	(16,4)	95.00	1.19	82.19	1.06	37.20	0.53	5.75	0.09
			0.84		0.71		0.30		0.04
5	(20,0)	96.00	0.96	80.67	0.81	33.10	0.33	4.80	0.05
			0.96		0.81		0.33		0.05
4	(25,0)	97.00	0.97	78.23	0.78	28.53	0.29	3.88	0.04
			0.97		0.78		0.29		0.04
3	(33,1)	·	·	·	·	23.17	0.24	2.94	0.03
			·		·		0.21		0.03
2	(50,0)	99.00	0.99	66.44	0.66	16.78	0.17	1.98	0.02
			0.99		0.66		0.17		0.02
1	(100,0)	‡100.00	1.00	50.25	0.50	9.17	0.09	1.00	0.01
			1.00		0.50		0.09		0.01

Notes: A^* denotes the total number of groups, a of which have $n + 1$ members; the remaining $A^* - a$ have n members. By construction, $a(n+1) + (A^* - a)n = 100$. These calculations assume $X = 100$. Under individual conflict ($A^* = 100$, $n = 1$), individual expected payoffs are 0.01, and aggregate expected payoffs are 1.00. [The individual payoffs might not sum to the aggregate payoffs due to rounding.] '‡' indicates the structure yielding the highest aggregate expected payoff and '†' indicates the symmetric structure yielding the highest aggregate payoff.

Mathematical appendix

A.1 Preliminary results for stability and equilibrium

Lemma 1 *Given any group structure with two or more stand-alone groups* $S = \{n_1, \dots, n_{A-2}, 1, 1\}$*, all members* $i \in \mathcal{A}_k$ *where* $n_k \geq 2$ *optimally choose not to participate in the second-stage conflict:* $m(n_k, S) = 0$.

Proof. The proof is only sketched here. By hypothesis, $n_{A-1} = n_A = 1$, implying that $F = \sum_{k=1}^{A-2} n_k[1+\alpha(n_k-1)]+2$. Even supposing that $n_k = n_1 \geq 2$ for $k < A-1$, such that F is equal to the largest possible value (given $n_{A-1} = n_A = 1$ and N), $F = (A-2)n_1[1+\alpha(n_1-1)]+2$, we would have $F - (A-1)n_1[1+\alpha(n_1-1)] \leq 0$, implying from (11) that members of group $k = 1$ choose not to participate: $m(n_1, S) = 0$. Then, F becomes $F' = \sum_{k=2}^{A-2} n_k[1+\alpha(n_k-1)]+2$. With repeated applications of this logic, given S, one can show sequentially that the remaining groups $k \geq 2$, for which $n_k \geq 2$ holds, have no incentive to participate in the conflict over X. ■

Lemma 2 *Stability of any given structure of groups,* $S = \{n_1, n_2, \dots, n_A\}$*, requires* $m_k > 0$*, for* $k = 1, 2, \dots, A$.

Proof. Suppose there exists a stable structure of groups, S', in which the members i of one group k have no incentive to participate in the conflict, $m(n_k, S') = 0$. By (11), this group must be $k = 1$. Each member $i \in \mathcal{A}_1$ obtains a payoff of just zero. Yet, given the membership of all other groups, any $i \in \mathcal{A}_1$ could secure a higher expected payoff by competing for X on her own. Suppose that just one member $i \in \mathcal{A}_1$ breaks away, yielding the new partition $S'' = S' \setminus \{n_1\} \bigcup \{n_1-1, 1\}$.[41] Hence, the original structure could not have been stable. Assuming that this deviation does not affect the participation decision of the remaining members of group $k = 1$ ($m(n_1-1, S'') = 0$), the new structure is not stable either.[42] Here, there are two cases to consider:

Case 1. $n_k = 1$, for $k \leq A$. If before the initial deviation, there had been one or more stand-alone groups, by Lemma 1, that deviation would in turn push all individuals remaining in a multi-member group (k for $n_k > 1$) out of the conflict $m(n_k, S'') = 0$.

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[41] Under S'' assuming that all other alliances, $k \geq 2$, remain active in the stage-two conflict, $F'' = \sum_{k=2}^{A} n_k[1+\alpha(n_k-1)]+1$. Then, from (12), $u^e(1, S'') = [F'' - (A-1)]^2 X / F''^2 > 0$.

[42] Members of even smaller groups $k \geq 2$ might pull out of the conflict for an expected payoff of zero as well. That would not change the basic logic of the argument to follow. If instead $m(n_1-1, S'') > 0$, the new structure S'' would be stable and the proof would be complete.

Case 2. $n_A > 1$. Each of the remaining members of group $k = 1$ could obtain a higher payoff by competing for the contestable resource on her own as before. But then, from Lemma 1, a move by any one of them would result in another partition, $S''' = S'' \setminus \{n_1 - 1\} \bigcup \{n_1 - 2, 1\}$, such that anyone remaining in a multi-member group (k for $n_k > 1$) would pull out of the conflict $m(n_k, S''') = 0$.

In either case, all those individuals $i \in \mathcal{A}_k$ with $n_k > 1$ and thus $u^e(n_k, S) = 0$ would have an incentive to deviate from the existing structure of groups, S'' in case 1 and S''' in case 2. In the very least, each could leave her group to form a stand-alone group and, regardless of others' choices, expect a positive payoff equal to $u^e(1, \widetilde{S}) = X/\widetilde{N}^2$ where $\widetilde{S}$ consists of $\widetilde{N} \leq N$ singletons and $N - \widetilde{N} \geq 0$ individuals belonging to one or more multi-member groups. Given a zero payoff from non-participation, the incentive for any individual to move from her current multi-member group to form a single-member "group" would remain strictly positive.[43] ■

Lemma 3 *When the size of the largest group exceeds the smallest by 2 or more, any member of the largest has an incentive to join one of the smaller groups, holding the rest of the group structure (including the remainder of the largest group) fixed:* $u^e(n_j+1, S') > u^e(n_1, S)$, *where* $n_1 > n_j+1$ *and* $S' = S \setminus \{n_1, n_j\} \bigcup \{n_1-1, n_j+1\}$.

Proof. Using (12), it is necessary to verify that the following inequality holds:

$$\frac{[F' - (A-1)(n_j+1)(1+\alpha n_j)][F' - (A-1)(1+\alpha n_j)]}{(n_j+1)(1+\alpha n_j)F'^2} > \frac{[F - (A-1)[1+\alpha(n_1-1)]n_1][F - (A-1)[1+\alpha(n_1-1)]]}{[1+\alpha(n_1-1)]n_1 F^2} \tag{A.1}$$

for $n_1 > n_j + 1$, where

$$\begin{aligned} F &= [1+\alpha(n_1-1)]n_1 + [1+\alpha(n_j-1)]n_j + B \\ F' &= [1+\alpha(n_1-2)](n_1-1) + [1+\alpha n_j](n_j+1)) + B \\ B &= \textstyle\sum_{k\neq 1,j}[1+\alpha(n_k-1)]n_k. \end{aligned}$$

Assume that, under both group structures, $m_k > 0$ for all k. Some straightforward manipulations show that the inequality above will be satisfied if and only if the

[43] Given the focus on individual (uncoordinated) deviations in the analysis of equilibrium, it seems reasonable to conjecture a tendency towards individual conflict.

following condition is satisfied:

$$
\begin{aligned}
&(1+\alpha(n_1-1))n_1F^2\times\\
&\qquad\big[[F'-(A-1)(1+\alpha n_j)(n_j+1)][F'-(A-1)(1+\alpha n_j)]-\\
&\qquad[F-(A-1)(1+\alpha(n_1-1))n_1][F-(A-1)(1+\alpha(n_1-1))]\big]>\\
&\big[(n_j+1)(1+\alpha n_j)F^{'2}-n_1[1+\alpha(n_1-1)]F^2\big]\times\\
&\qquad\big[F-(A-1)(1+\alpha(n_1-1))n_1\big]\big[F-(A-1)(1+\alpha(n_1-1))\big]. \qquad (A.2)
\end{aligned}
$$

Note that if $n_1 = n_j + 1$, then $F = F'$ and the two sides of the expression are identical and equal to 0. However, the assumption that $n_1 > n_j + 1$ implies $F = F' + 2\alpha(n_1 - n_j - 1) > F'$, making the right-hand side of (A.2) negative. Thus, a sufficient condition for (A.1) to hold when $n_1 > n_j + 1$ is that the left-hand side of (A.2) be positive. Further manipulations show that the left-hand side of (A.2) is positive for $n_1 > n_j + 1$ if and only if,

$$
\begin{aligned}
&(n_1-n_j-1)(1+\alpha(n_1-1))n_1F^2\times\\
&\qquad\big[[(A-1)(\alpha(n_1+n_j)+1)-2\alpha][F'-(A-1)(1+\alpha n_j)]+\\
&\qquad\alpha(A-3)[F-(A-1)(1+\alpha(n_1-1))n_1]\big]>0. \qquad (A.3)
\end{aligned}
$$

When $A \geq 3$, the inequality clearly holds. When $A = 2$, there are just two groups and the term B vanishes from F and F'. In this case, one can verify that the condition above simplifies as follows:

$$
\begin{aligned}
&(n_1-n_j-1)(1+\alpha(n_1-1))n_1F^2\times\\
&\qquad\big[[(n_1-1)(1+\alpha(n_1-2))+\alpha n_j^2][\alpha(n_1+n_j-2)+1]+\\
&\qquad n_j[\alpha(n_1-1)+(1-\alpha)(1+\alpha(n_j-1))]\big]>0, \qquad (A.4)
\end{aligned}
$$

where $F = n_1(1 + \alpha(n_1 - 1)) + n_j(1 + \alpha(n_j - 1))$. Clearly this condition holds for $n_1 > n_j + 1$. Therefore, in equilibrium, the difference in the sizes of any two groups cannot be greater than 1; it must be 0 or 1. ■

A.2 Proof of Proposition 2: symmetric groups

Suppose $\alpha \in [0, 1]$, and consider symmetric groups $\hat{S}^*$, where $n^* \geq 2$. Since groups are all of the same size and $n^* \geq 2$, no individual would have an incentive to leave her group to join another [Lemma 3]. By Proposition 1, the expected payoffs under $\hat{S}^* \equiv \{n, \ldots, n\}$ for $A^* > 1$ and $n^* \geq 2$ are strictly greater than those under individual conflict, $\bar{S} \equiv \{1, 1, \ldots, 1\}$. Hence, any deviation which triggered a reversion to individual conflict would be considered unprofitable relative to $\hat{S}^*$. Now suppose an individual were to leave her group to form a stand-alone group.

Then there would be A^*+1 groups: A^*-1 groups of size n^*, the deviator's former group of size n^*-1, and the deviator's new single-member group. As one can verify using (11), given the membership of the deviator's former group ($n^*-1 \geq 1$), such a deviation from the symmetric structure would push the members of the other original A^*-1 groups to the corner $m=0$ and, by the logic in Lemma 2, would eventually trigger a reversion to individual conflict.

Now consider the grand coalition ($n=N$), supposing that $\alpha \in [0,1)$. By Proposition 1, the expected payoffs under the grand coalition are strictly greater than those under individual conflict. For $\alpha \in (0,1)$, one can easily confirm that a deviation from the grand coalition by just one individual would push everyone else to the corner solution ($m=0$), thereby triggering a reversion to individual conflict. Thus, such a deviation cannot be profitable. For $\alpha = 0$, such a deviation would leave each of the $N-1$ individuals remaining in the one large multi-member group with an expected payoff identical to that expected under individual conflict. Though not at a corner solution, each individual would have an incentive to deviate herself by forming her own "group" of just one member. If no one else were to do the same, she could obtain a much higher expected payoff. But, of course, this (second) deviation would push all others to the corner, eventually triggering a reversion to individual conflict. Since the payoff to individuals under the grand coalition exceeds that when just one individual deviates, and the individual who deviates next can do no worse from that point, it would be reasonable to expect the second deviation to follow the first, which would make deviating from the grand coalition unprofitable when $\alpha = 0$. Thus, for $\alpha \in [0,1)$, the grand coalition is a Nash equilibrium structure. ■

A.3 Proof of Proposition 3: asymmetric groups

By construction, under S^* each group is of size n or $n+1$, such that no individual has an incentive to join another group [Lemma 3]. The inequality ensures, in addition, that the expected payoff under that structure for any member belonging to a size–$n+1$ group is greater than that under individual conflict. Then, by (13), members of all groups would consider any deviation which triggered a reversion to individual conflict to be unprofitable relative to S^*. Thus, to verify that no individual (belonging to an $n+1$-member group) would have an incentive to form a group on her own, it is only necessary to show that the payoffs expected by another under that hypothetical deviation are less than that under individual conflict. For $a = 2, \ldots, A^*-1$ given N, such a deviation would result in a new partition with $A+1$ groups: A^*-a+1 groups with n^* members, $a-1$ groups with $n+1$ members and 1 stand-alone group, implying that $F - A(n+1)(1+\alpha n) = -(A-a)(1+2\alpha n) - 2\alpha n < 0$.

Thus, from (11), an individual deviation to form a stand-alone group would push the members of all the other $a-1$ groups with $n+1$ members to the corner ($m=0$) for a zero payoff. By the reasoning of Lemma 2, such a deviation would trigger a reversion to individual conflict, and would therefore be deemed unprofitable. For $a=1$, when an individual from the single group of size $n+1$ forms a group on her own, a new structure, again with $A+1$ groups, emerges: A groups of size n and one stand-alone group. In this case, such a deviation would not push anyone away from an interior optimum. However, as long as the second inequality shown in the proposition holds, everyone but the original deviator would be worse off than if there were no groups at all. Then, by the reasoning applied earlier, the initial deviation would induce a reversion to individual conflict, making everyone including the original deviator worse off relative to the initial group structure under consideration. ■