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Working Paper 2004-10

Finding out who the crooks are - Tax evasion with sequential auditing

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November 2004

Abstract

This paper investigates multi-item moral hazard with auditing contests. Although the presented model is widely applicable, we choose tax evasion as an exemplary application. We introduce a tax-evasion model where tax authority and taxpayer invest in detection and concealment. The taxpayers have multiple potential income sources and are heterogeneous with respect to their evasion scruples. The tax authority - unable to commit to an audit strategy - observes a tax declaration and chooses its auditing efforts. We show that a tax inspector prefers to audit source by source until he finds evidence for evasion to conduct a full-scale audit thereafter.

JEL-Classification: H26, D82, K42
Keywords: Moral Hazard, Auditing Rules, Contest, Tax Evasion

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1 Introduction

In the real world there are many examples for situations where principals ask agents to make statements which have consequences for later payoffs. A tax inspector may ask: How much rent did you get for a this certain property? A judge may ask a defendant in a murder trial where he had been during the night in question. An environmental inspector asks the firm owner if a certain production process complies with emission laws. Even in situations where no authority is involved such situations may occur. A potential buyer asks the antique seller if the chair is an original Louis XV. Quite often - like in the examples above - the payoff consequences are such that there are incentives to make an untruthful statement. In contract theory optimal direct revelation mechanisms are developed that prevent agents from cheating. Baron and Myerson (1982) is an example for the regulation of a monopolist, Chander and Wilde (1998) deal with the case of tax evasion. In the real world, however, the rules are not always incentive compatible. Agents do actually cheat. Therefore we observe institutions that try to verify whether statements are true. Fines have to be paid if a statement is found untruthful. In the literature agents only have the choice of being truthful or not. In reality they have another choice: They can invest resources in covering their untruthful statements. The property owner may accept a lower rent for payment in cash. The defendant may bribe someone to give him an alibi. The firm owner may pay someone to dump toxic waste. The antique dealer may use chemicals to artificially age the wood of the chair. This observation suggests that the outcome of an audit does not only depend on the auditing effort, but also on the concealment effort exerted by the agent. In these situations audit becomes a contest.

Often a principal asks an agent to make many payoff-relevant statements at once. The taxman asks for statements about more than one income source; the judge asks the defendant more than one question, the potential buyer wants to purchase more than just the single chair. This paper investigates how an auditor should proceed in a situation in of multiple-item moral hazard with auditing contests. For convenience we choose tax evasion as our application; but the model structure can be easily applied to any other situation described above.

2 Related literature

In the literature on tax evasion - and on moral hazard with audit in general - the detection technology is usually characterized by an audit probability. If an audit takes place a potential fraud is revealed with certainty.[1] This assumption is still widely used and was introduced by the early neoclassical tax evasion literature (Allingham and Sandmo, 1972; Yitzhaki, 1974).[2] In later contract theoretical contributions, where the tax authority has been introduced as a player, the tax inspector's strategy is the assignment of an audit probability to a received tax declaration (see e.g. Reinganum and Wilde (1985), Border and Sobel (1987), Mookherjee and Png (1989), Mookherjee and Png (1990), or Chander and Wilde (1998)). The audit costs increase with the audit probability. It is the purpose of this paper to investigate the audit process in a richer situation. We model the detection probability as the outcome of a contest, where the taxpayer invests in concealment, while the authority spends resources on detection. So the probability for the verification of earned income does not only depend on the authority's detection effort, but also on the effort the tax evader puts into concealment.[3]

As a second fundamental difference to the contract-theoretical tax-evasion literature we do

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[1] Exceptions are the imperfect auditing settings of Macho-Stadler and Perez-Castrillo (1997) and Boadway and Sato (2000).

[2] For a comprehensive survey of the neoclassical tax evasion literature see Cowell (1990a).

[3] The idea that the taxpayer can invest into concealment is explored in an optimal taxation framework by Cremer and Gahvari (1995). Yaniv (1999) interprets concealment investment as costly money laundering. The case of legal tax sheltering is examined by Cowell (1990b).

not allow the tax authority to commit beforehand to an audit strategy.[4] The reason that we prefer the non-commitment assumption is twofold. Firstly, we belief that in reality the tax authority does not really commit itself. An indication for this is the veil of secrecy surrounding the authority's audit strategies. Credible commitment, however, requires that the taxpayers know the audit strategies the authority will use. Furthermore, the taxpayers do not seem to believe commitment attempts. In a field experiment (Slemrod et al., 2001) taxpayers did not significantly change their reporting behaviour after receiving a letter telling them to be audited with certainty. Secondly, the optimal audit, fine and tax schemes, and the resulting reporting behaviour arising from the commitment models are not very realistic. Generally, optimal fines are very high (usually even maximal), taxes are regressive, and the revelation principle holds, which implies that there is no tax evasion in equilibrium.

The third distinct feature of our model is the introduction of heterogeneous taxpayers with multiple income sources. The taxpayers are assumed to differ in their behaviourally relevant attitudes towards tax evasion. These attitudes are captured by different moral costs of evasion.[5] The introduction of multiple income sources seems a natural improvement to the models, where income is homogeneous. So the declaration pattern over different income sources reveals some information to the authority about the likelihood of facing an evader. The tax inspector can get even more valuable information if he adopts a sequential auditing strategy. He can use the information gained from previous audits when deciding over detection efforts for sources that are not audited yet.

3 Plan of the paper

The paper is organized as follows. In the next section the setup of our model is described; some simplifying assumptions are introduced and discussed. In section 5 we derive the optimal concealment and audit efforts that will be used during the remainder of the paper. The optimal audit effort for a particular source is shown to depend on the believed probability that the income from this source is evaded. The taxpayer's decision over evasion and concealment effort depends on some source specific parameters. Section 6 deals with "ghosts" - crooks who entirely go underground. These people do not report any income regardless how much they may have earned.[6] We show that the sequential auditing strategy deals with ghosts more effectively. Compared to the simultaneous auditing a sequential auditing strategy imposes stronger restrictions on the environment in order to allow for ghost behaviour. We also show that sequential auditing leads to a higher expected payoff for the authority. Consequently, it is optimal for the authority to audit source by source at first to finally conduct a full-scale simultaneous audit whenever suspicion arises during the sequential audits.

Section 7 explores the case where the environment is not favourable enough for crooks to behave as ghosts. We characterize the arising hybrid equilibrium and compare the impact of different audit strategies. This is the situation most tax evaders will be in. We show for two sources that the intuition from the analysis of ghosts carries over. Section 8 shows that our model with sequential auditing can explain a phenomenon not well understood so far - moonlighting craftsmen. We show that it might be optimal for a crook to engage in both the black market economy and also in the official sector, even if going totally underground leads to higher payoffs than working only in the official sector does. Basically, the intuition behind this result is that sequential auditing may lead to an ex ante expected return from evasion that decreases with additional income sources allocated to the informal sector. The expected vengeance of an audit

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[4]The non-commitment assumption was inroduced by Reinganum and Wilde (1986) and Graetz et al. (1986) into the tax evasion literature. Khalil (1997) clarifies the effects of non-commitment in a monopoly-regulation framework.

[5]For models with similar considerations see Spicer and Lundstedt (1976), Gordon (1989), or Bordignon (1993).

[6]A more precise definition of ghost behaviour will be given later on.

if the authority gets suspicious during sequential checks is the reason for that. We conclude with a summary of our main results.

4 The model

In this section we develop our setup, introduce some notation and explain our simplifying assumptions. We begin with the timing.

4.1 Timing and some notation

There are N possible income sources. Income source i yields income Y_i if it is productive. Nature determines whether source i is productive or not. The probability that it is is denoted by λ_i. After observing the actual income generated by the possible sources the taxpayer has to file a tax return. He separately declares his income for the sources. The income declared for source i is denoted by d_i. The taxpayer has the possibility to invest some resources in order to reduce the verification probabilities p_i. His investment is measured by the sheltering effort e_i.

After having received the declaration d_i for every source the authority decides how many resources to invest in order to increase the probability that the true income is verified. The detection effort for source i is denoted by a_i. The tax inspector can exert different effort levels for different sources. Furthermore, he can audit sequentially if he prefers to do so. That means that the authority can decide which source(s) to examine first. Nature decides if the true income from the audited source(s) is verified. The probability of verification p_i depends on the efforts. Then, after observing the outcome of this audit(s) it can condition its effort for the next source(s) to be audited on this observation. At the time of the decision the authority has no knowledge about the concealment efforts exerted by the taxpayer.

4.2 Crooks and good citizens

We assume that there are some moral costs that are incurred by the taxpayer whenever he does not truthfully report an income source. Spicer and Becker (1980), Bayer and Reichl (1997), and Anderhub et al. (2001) show in experimental studies that different taxpayers may behave differently even if they face the same situation. Some people are always honest, some others do evade taxes. Social Psychology suggests that moral constraints are the source for heterogeneity. The attitude towards a certain criminal act is determined by its expected gain, internalized moral norms and peer group attitudes.[7] The attitude towards a crime determines the drive to commit the crime. Moral cost can be interpretd as the expected disutility arising from not conforming to internalised norms or to exhibit behaviour that conflicts with peer group attitudes. This justifies the inclusion of moral costs in the utility function of a taxpayer.[8]

For simplicity we assume that there are only two types of taxpayers. Crooks with low moral costs and law abiding citizens with moral costs that are high enough to force them to be honest. As in seminal reputation models (e.g. Kreps et al., 1982; Milgrom and Roberts, 1982a; Milgrom and Roberts, 1982b) we exogenously fix the behaviour for one type. The individuals with considerable scruples are always honest, since their evasion costs are assumed to be prohibitive. In this respect the sequential auditing setup is similar to a reputation model. But there is an important difference. The taxpayer has to decide over all his declarations before the authority starts to audit. So in the strict sense there is no room for reputation building although the structure of the belief formation is very similar.

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[7]The most influential theoretical framework is the "Theory of Planned Behaviour" established in Ajzen and Fishbein (1980) and extended in Ajzen (1991).

[8]This is admittedly a very crude black box simplification of a very complex psychological construct, but it will be sufficient to serve the purpose of this paper.

The moral cost are denoted by $\theta \in \{\theta_l, \theta_h\}$. Without loss of generality we normalize θ_l to 0. Realistically, we assume that the tax authority does not know the type of the taxpayer. Let β be the prior probability of facing a crook. We can interpret β as the fraction of crooks in the population. We assume β to be common knowledge.

4.3 Pay-offs

In order to be able to specify the pay-offs we have to make assumptions about the objectives and risk preferences of the tax authority and the taxpayer. We assume both the authority and the taxpayer to be risk neutral. It is quite common to assume risk neutrality for the principal, which is the authority in our case. The assumption concerning the risk preferences of the taxpayer is not crucial for the remainder of the paper. Risk aversion would just increase the influence of the fine that is imposed in the case of the taxpayer is convicted for tax evasion.

The taxpayer maximizes income net of tax liability, resources invested in concealment, moral cost of evasion, and expected fines. The expected interim payoff after the gross incomes are realized and after both parties have made their decisions, but before nature decides which income sources are verifiable, can be written as

$$EU = \sum_{i=1}^{n} Y_i^a - T(d_i) - F(Y_i^a, d_i) \cdot p(e_i, a_i) - C(e_i) - \phi \cdot \theta \tag{1}$$

Here Y_i^a is the actual income from source i, $T(d_i)$ represents the tax liability for an income declaration d_i, while $F(Y_i^a, d_i)$ denotes the fine a taxpayer has to pay if his true income is verified after having declared an income of d_i. The fine includes the repayment of evaded taxes. By assuming the tax liabilities and potential fines to be additive over different income sources, we implicitly assume that the tax and penalty schemes are linear. The probability that the true income from an income source can be verified is denoted by $p(e_i, a_i)$. It depends on the detection and concealment efforts (a_i and e_i) that are exerted by the authority and the taxpayer. Obviously, $p(e_i, a_i)$ should increase with a_i and decrease with e_i. The concealment costs, which depend on the effort level, are given by $C(e_i)$. The moral cost incurred by the evasion of a certain income component is given by θ, while ϕ is an indicator variable for evasion.

Similarly the expected interim pay-off for the authority (before nature decides which income sources are verifiable) is given by:

$$ER = \sum_{i=1}^{N} T(d_i) + F(Y_i^a, d_i) \cdot p(e_i, a_i) - K(a_i) \tag{2}$$

We assume the authority to maximize enforced tax payments plus expected fines net of detection costs $K(e_i)$. The alternative assumption of a more selfish authority, which just wants to maximize expected fines net of detection costs, does not make a substantial difference for the model's implications.[9]

4.4 Simplifying assumptions

In order to keep the model tractable we have to make some simplifying assumptions about functional forms.

A1 The costs of influencing the verification probability in the favoured direction are increasing and convex for both players.

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[9]It does not matter for the results how much the tax authority - if at all - values paid taxes. This is true because the taxes are sunk at the moment the authority chooses its efforts. In models where the authority can credibly commit to announced audit schemes this distinction certainly matters.

The rationale for this assumption is the following. If one of the actors wants to shift the verification probability in his favoured direction he has to put in some costly effort. The bigger the shift intended the bigger is the effort required, and consequently the higher the costs are. In addition the actors should use the cheapest means of detection or covering first. That means it gets more expensive to achieve a further shift in probability if your effort increases. So the costs are convex.

A2 The marginal costs of influencing the probability do not depend on the effort the other player exerts.

This assumption takes away the strategic effect that the actors can influence with their efforts how hard it is for the opponent to shift the verification probability. We believe that this effect is relevant in reality. However, for our purpose - to investigate sequential auditing - we have to keep the contest simple to assure that the model remains tractable. The resulting additive structure of the detection probability seems to be a bit troublesome. However, the interpretation that an exogenous detection exists, which can be influenced to a certain degree by the two parties intuitively makes sense. If the tax authority does not exert any effort the taxpayer can reduce the detection probability to zero with a high effort. On the other hand, if the taxpayer does not exert any effort the authority can make sure with a high effort that the audit is successful. If the opposition in the contest exerts any effort though, it is not possible to reach certainty (of detection or getting away with evasion), even with a very high effort. We think that this restriction on the probability function is realistic.

Although these two assumptions are sufficient for the results, for simplicity we will impose specific restrictions on functional forms for probability and cost functions. The properties of the verification-probability function are the following. Subscripts denote partial derivatives:

$$
\begin{aligned}
p(e_i, a_i) &\in [0, 1] \\
p_e &< 0, \quad p_{ee} > 0, \\
p_a &> 0, \quad p_{aa} < 0 \\
p_{ae} &= 0.
\end{aligned}
\tag{3}
$$

A verification probability that is increasing and concave in a, but decreasing and convex in e together with linear cost satisfies the assumptions A1 and A2. So we define the effort cost for the taxpayer and the tax inspector as follows:

$$
\begin{aligned}
C(e_i) &= \frac{e_i}{\eta_i} \\
K(a_i) &= a_i
\end{aligned}
\tag{4}
$$

We introduce the parameter η to describe the concealment opportunity for the taxpayer. The higher η is, the cheaper it is for the taxpayer to hide a potential evasion.

A3 The marginal change in the detection probability approaches 0 for efforts a or e tending to infinity (i.e. $p_e \to 0$ if $e \to \infty$ and $p_a \to 0$ if $a \to \infty$), where the marginal probability changes for the first units of efforts are given by $p_e(0, a) = -\omega$ and $p_a(e, 0) = \tau$.

The first part is one of the commonly used Inada conditions to ensure the existence of an interior solution to the maximization problem at the upper end. Note that we have not imposed any conditions that prevent the optimal efforts to be zero. A zero effort of the authority can be seen as rubber-stamping a declaration. In reality such a behaviour is observed quite often. Furthermore, our notion of detection probability does not rule out a positive detection probability even with no effort exerted by the tax inspector. This reflects the fact that the detection of tax evaders may happen by chance or at least without too much active participation

of the tax authority.[10] The parameter τ - the gain of verification probability by the first most effective unit of effort - may be seen as a measure for the observability of an economic action. The corresponding parameter for the taxpayer is ω, which is the reduction in verification probability caused by the most effective concealment action of the taxpayer.

For many income sources the parameters ω and τ may be correlated. Income sources, where detection effort is (not) effective, gives rise to (not only) few opportunities to conceal. Income from dependent employment is an example for an income source where detection is effective while concealment is not.[11] However, there are counter examples. A craftsman engaged in the black economy may have few effective opportunities to conceal his activity, because the detection probability hinges crucially on the discretion of the trading partner while the authority has no cheap effective means of investigation.

A4 The taxpayer has no means of exerting any effort if there is no tax evasion to shelter. After an audit the authority learns whether the taxpayer put forward any sheltering effort or not. The probability that pure chance leads to the verification of the income is positive but smaller than one, i.e. $1 > p(0,0) > 0$.

This assumption makes sure that the tax inspector learns from an audit that a source yielded no income whenever this is the case. We argue that this is a reasonable assumption. Think of a flat owned by the taxpayer. It might be reasonable that it is possible for the taxpayer to shelter his income from letting it. But if he lives there himself, a tax inspector surely should learn from an audit that no income was created. In other words, we reduce the uncertainty a tax inspector may face while interpreting the results from auditing a source that was not earned. This will keep the updating process between audits tractable. The last part of the assumption makes sure that there is a certain uncertainty about income verification if both players do not invest into detection and coverage, respectively.

A5 The distribution of income generation is assumed to be dichotomous and independent for the different sources.
$$Y_i^a = \begin{cases} Y_i & \text{with probability} & \lambda_i \\ 0 & \text{with probability} & 1 - \lambda_i \end{cases}$$

As we will see, the audit effort decision for the tax inspector hinges crucially on the beliefs on the expected potential fine for a given declaration. If we allow for a continuous income distribution these beliefs become very complex. However, the additional complexity would not add any strategic elements to our setting. To allow for a continuous income distribution would also require an additional assumption about how the fine depends on the income.

A6 The declaration for a particular income source is a dichotomous choice (i.e. $d_i \in \{Y_i^a, 0\}$).

We allow the taxpayer only to declare his whole income from an income source or to declare nothing at all. We are aware that there are some income sources where this assumption is not appropriate (e.g. tips). But risk neutrality and the linear system we assumed always produces corner solutions for the declaration decision. By assuming a minor result of the model right away we do not have to deal separately with this issue for every case we consider.

5   Optimal efforts

Before we consider different audit and evasion strategies we determine the optimal detection and concealment efforts. We begin with the tax inspector. Whenever the tax authority decides to

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[10]Being denounced by an envious neighbour is a quite common fate evaders may have to face.

[11]Income from selling drugs is an example where detection is hard and concealment is easy.

audit a certain income component, for which it observed a declaration of zero, it faces the same auditing problem. It wants to maximize the expected fine by putting in some auditing effort. The effort which solves the maximization problem is the following (we omit the subscripts for the source here):

$$a^* = \arg\max_a \left[\mu(Y^a = Y \mid d = 0, H) \cdot p(e, a) \cdot F - a \right], \tag{5}$$

where $\mu(Y^a \mid d = 0, H)$ is the tax authority's belief - given a declaration of $d = 0$ and his information H - about the probability that the income was earned. The tax inspectors information H can be the prior information or some information that was gathered during previous audits. Then the first-order condition becomes

$$p_a \leq \frac{1}{\mu \cdot F}. \tag{6}$$

We also have to consider the case of a possible corner solution. In the case $\tau < 1/\mu F$ the optimal effort has to be 0. This is the case whenever the economic activity is too hard to observe and putting in effort never pays.

It follows from $p_{aa} < 0$ that the optimal effort a^* weakly increases with the fine F and the belief μ. Note that the optimal effort is independent of the effort the taxpayer might have exerted. If the authority believes with certainty that the income component was not earned after observing a declaration of zero (i.e. $\mu = 0$) the optimal detection effort is zero.

We turn to the taxpayer now. Suppose for instance that a taxpayer has earned the income component and decides not to declare this income. Then he faces the following maximization problem in order to choose the optimal hiding effort e^*:

$$e^* = \arg\max_e \left[Y - p(e, a) \cdot F - \frac{e}{\eta} \right]. \tag{7}$$

Then the first-order condition becomes

$$p_e \geq -\frac{1}{\eta \cdot F}. \tag{8}$$

We once again have to consider a possible corner solution. For $\omega > -1/\eta F$ the optimal effort is zero. This is the case whenever the concealment opportunity is very small and there are no effective and cheap means of coverage available. From the first-order condition and from $p_{ee} > 0$ follows that the optimal concealment effort e^* weakly increases with the fine F and with the concealment opportunity η. We summarize these findings in the following lemmas.[12]

Lemma 1 *If the tax inspector observes a declaration of 0 for an income component and chooses to audit, his optimal detection effort a^* has the following properties: $a^* \geq 0$, $da^*/de = 0$, $da^*/d\mu \geq 0$, $da^*/dF \geq 0$, $a^*(\mu \mid \mu = 0) = 0$.*

Lemma 2 *If the taxpayer earned an income from a source and decided to hide this income, his optimal concealment effort e^* has the following properties: $e^* \geq 0$, $de^*/da = 0$, $de^*/d\eta \geq 0$, $de^*/dF \geq 0$.*

The weak inequalities come from the fact that we did not rule out the corner solutions $a^* = 0$ and $e^* = 0$. If zero efforts are optimal a marginally increased incentive for concealment or detection does not necessarily lead to a positive effort becoming profitable.

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[12]Note, that the second-order conditons are obviously fulfilled.

6 Auditing with ghosts

In the literature (e.g. Cowell and Gordon, 1995) taxpayers that fail to fill in a tax form are referred to as ghosts. In reality one may distinguish between non-filers and taxpayers that make a zero declaration. In our model, however, there is no strategic difference between the two different types of behaviour if we assume that the tax authority has at least the knowledge about the existence of the taxpayers. We assume this to be the case. We are aware that this assumption is problematic for countries (like the United Kingdom) where no system of registration exists. For countries with systems of registration (like e.g. Germany) the assumption seems reasonable.

In this section we look at the conditions to be fulfilled that behaving as a ghost with certainty occurs as an equilibrium strategy. We examine, what the authority might want to do against that and whether the possibility of sequential auditing - compared to simultaneous auditing - does help to deter taxpayers to behave as ghosts.

6.1 Ghosts with simultaneous auditing

Suppose there are N identical income sources with a hiding opportunity η, which yield income Y with probability λ each. Then the strategy a ghost will follow is characterized by

$$d_i^*(\theta_l) \;=\; 0 \quad \forall i \tag{9}$$

$$e_i^*(\theta_l) \;=\; \left\{ \begin{array}{lll} e^* & if & Y_i^a = Y \\ 0 & if & Y_i^a = 0 \end{array} \right., \tag{10}$$

where e^* solves the first-order condition (equation 8) if possible or is equal to 0 otherwise. Note that a ghost necessarily has to be a crook ($\theta = 0$), because we assumed that the moral evasion cost for the good citizens to be prohibitive. So an honest taxpayer with $\theta = \theta_h$ in the same situation always reports truthfully ($d_i = Y_i^a$) and consequently exerts no concealment effort:

$$d_i^*(\theta_h) \;=\; Y_i^a \quad \forall i \tag{11}$$

$$e_i^*(\theta_h) \;=\; 0 \quad \forall i \tag{12}$$

Consider the strategy of a tax inspector who simultaneously decides his detection efforts for all income sources. For this decision the tax inspector's beliefs are crucial. The tax authority has to assign a probability to every income source that tax evasion has taken place. The available information under simultaneous auditing is the prior probability that an income source is earned λ, the prior probability of facing a crook β, and the tax return (i.e. the vector of income declarations $\mathbf{d}$). Let us denote an observed declaration vector that contains only zeros as $\mathbf{d}_0$ and a declaration vector that does contain at least one element that is Y as $\mathbf{d}_Y$. Together $\mathbf{d}_0$ and $\mathbf{d}_Y$ contain all possible declaration patterns. In a Perfect Bayesian Equilibrium the beliefs have to be consistent with the strategy of the opponent. Obviously, in equilibrium the believed evasion probability for every income source has to be 0 if the tax inspector observes a declaration $\mathbf{d}_Y$, which is a tax form where at least one income of Y is declared. The reasoning goes like this: If we want to support a ghost equilibrium with consistent beliefs a tax inspector observing a single declared income component should update that this never can be the declaration of a crook, since a crook would behave as a ghost and would always submit a form $\mathbf{d}_0$ that contains only zeros. Consequently, the taxpayer he faces has to be of the honest type. The believed probability for the income sources to be evaded has to be zero for every potential income component. This equilibrium belief is denoted by

$$\mu_i^*(\mathbf{d}_Y) = 0 \quad \forall i.$$

What should the tax inspector think of a tax form that contains only zero declarations for all income sources? We first derive the updated probability that a $\mathbf{d}_0$ declaration comes from a crook. This probability should be the prior probability of facing a crook (i.e. β) normalized by

the probability that $\mathbf{d}_0$ is observed. The probability that an all-zero declaration comes from a poor honest taxpayer that didn't earn a single income source is given by $(1-\beta)(1-\lambda)^N$. Then the probability of facing a crook (denoted by $\rho^*(\mathbf{d}_0)$) after observing $\mathbf{d}_0$ is

$$\rho^*(\mathbf{d}_0) = \frac{\beta}{(1-\beta)(1-\lambda)^N + \beta}.$$

Consequently, the probability of facing an always evading crook, that earned income source i and evaded it, has to be

$$\mu_i^*(\mathbf{d}_0) = \lambda \cdot \rho^*(\mathbf{d}_0) \quad \forall i$$

Given these beliefs the tax authority will exert an effort for every income source that follows equation 6 where μ is given by $\mu_i^*(\mathbf{d}_0)$. The efforts will be $a_d^* = 0$ for an observation of $\mathbf{d}_Y$ and $a_0^* \geq 0$ if $\mathbf{d}_0$ is observed. A higher share of crooks in the population β, and a higher prior probability that the source is productive λ weakly increase a^*. The intuitive reason for this is that the prior probabilities for crooks and productive income sources increase the tax inspectors belief μ^* that the income source is earned and evaded given that he received a declaration of zero. This generates a higher incentive to audit (through the first-order condition). The effort in the possible equilibrium exerted by the tax inspector for all sources is described by:

$$a^* = \begin{cases} 0 & if \quad \mathbf{d} = \mathbf{d}_Y \\ a_0^*(\mu(\mathbf{d}_0)) & if \quad \mathbf{d} = \mathbf{d}_0 \end{cases}. \tag{13}$$

To find the parameter configurations that allow for a ghost equilibrium with simultaneous auditing we have to check if behaving as a ghost pays for the taxpayer (given the reaction of the tax authority). Let n denote the number of income sources (out of N possible) that were productive for the taxpayer. Then his payoff from going entirely underground if he is a crook can be written as:

$$U(\mathbf{d}_0, n) = n\left[Y - p(e^*, a_0^*)F - e^*/\eta\right]. \tag{14}$$

Proposition 1 *A ghost equilibrium (characterized by equations 9, 10, 11, 12, and 13) occurs only if ghost behaviour is optimal for the taxpayer when all income sources are earned. The condition is given by*

$$\frac{T - e^*/\eta}{F} \geq N\left[p(e^*, a_0^*) - p(e^*, 0)\right] + p(e^*, 0). \tag{15}$$

Proof. Note that the strategy of the tax authority a^* is always optimal for a given ghost behaviour and consistent beliefs. We have to check under what circumstances the taxpayer has no incentives to deviate from declaring only zeros given the auditing strategy of the taxpayer. Let us first determine the best deviation for the taxpayer. Declaring one of the n earned income sources and choosing optimal efforts pays $U_1 = (n-1)\left(Y - p(e^*, 0)F - e^*/\eta\right) + Y - T$. Declaring $j+1$ income source yields $U_{1+j} = (n-1-j)(Y - p(e^*, 0)F - e^*/\eta) + (1+j)(Y - T)$ with $j \in \{1, \cdots, n-1\}$. Then we find that

$$U_1 \gtreqless U_{1+j} \forall j \in \{1, \cdots, n-1\} \text{ if } p(e^*, 0) \lesseqgtr (T - e^*/\eta)/F. \tag{16}$$

Since U_{1+j} increases with j if $p(e^*, 0) > (T - e^*/\eta)/F$, we find that depending on $p(e^*, 0)$ the possible best deviation is either reporting truthfully for all sources or just declaring one income source. Comparing the equilibrium payoff from behaving as a ghost for n earned income sources (from 14) with the deviation payoff U_1 yields $U(\mathbf{d}_0, n) \geq U_1$ if $(T - e^*/\eta)/F \geq n\left[p(e^*, a_0^*) - p(e^*, 0)\right] + p(e^*, 0)$. In equilibrium this has to hold for all n. Since *rhs* of the previous inequality increases with n, while the left hand side is not influenced by n, the strongest condition on the parameters is given by $n = N$. The case where the best deviation is truthful behaviour is included in equation 15, because setting $n = 0$ reduces the condition to $p(e^*, 0) > (T - e^*/\eta)/F$,

which is just the condition for honesty being the best deviation from a possible ghost equilibrium. Replacing n by N in the inequality above gives the claimed condition. ∎

The condition for ghosts to exist is quite intuitive. On the left hand side of condition (15) we have the possible net gain per income source divided by the fine. Consequently, higher taxes T, higher concealment opportunities η and lower fines F promote ghost behaviour. On the other hand, the more effective the optimal audit effort a^* is compared to rubber-stamping ($a = 0$), the higher the gains from evading have to be for the taxpayer to be willing to act as a ghost. More potential income sources also have a deterrent effect. The intuition here is that more sources make it sweeter for a crook to declare just one of them truthfully in order to pretend to be one of the good citizens and to get away with the concealment of the other income components.

Loosely speaking, we will mainly find ghosts where not too many income opportunities exist, where the taxes are high while concealment is cheap, and where it doesn't make a big difference for the detection risk whether the authority investigates or not. Crooks may behave as ghosts if they have the opportunity to earn money with one off transactions that hardly leave any trails or checking possibilities. This result seems rather intuitive.[13] Our stylized model with multiple income sources gives a reasonable prediction on the influence that taxes, earning opportunities, fines, and source related audit efficiency may have on ghost behaviour.

6.2 Ghosts with sequential auditing

We now turn to the situation where the tax authority audits sequentially. The difference from the case with simultaneous auditing is that the tax inspector can use information gained from previous auditing to adjust his auditing effort. The main purpose of this section is to find out whether the additional information gained by sequential auditing is useful to prevent taxpayers from behaving as ghosts. To answer this question we derive a condition for a ghost equilibrium to appear when sequential auditing is possible. In order to compare the effectiveness of the two different audit regimes we check under which regime the environment has to be more favourable for the taxpayers to behave as ghosts.

The most important change to the simultaneous auditing case is that the inspector adjusts his beliefs (about facing a crook) after every single audit result he receives. Note that we have to find the beliefs that belong to a ghost equilibrium, i.e. the taxpayer will always submit an all zero tax declaration if he is a crook. The believed probability of facing a crook before the first audit given a tax form that contains only zeros is the same as in the simultaneous case. Let us denote this belief as ρ_0. Now consider the belief ρ_1, which is the belief of facing a crook after observing the audit result for first income source. Three things can happen during the first audit: The authority may be able to find concealed income, may not be able to verify a certain income, or may definitely find no income. If the authority finds some evaded tax it knows with certainty that it faces a crook, ρ_1 has to be one. What can the tax inspector infer from not having succeeded in verifying the true income? Knowing that he would have been able to verify the income if it had been zero, he should infer that he faces an evader. In this case ρ_1 should also be one.[14] In the case that the audit verifies that no income is earned from the source in question the tax inspector cannot conclude with certainty whether he faces a crook or an honest citizen. He has to rely on the prior probability of facing a crook nomalised by the probability that for the *remaining* sources no income is declared. So the belief of facing a crook weakens, because the probability that the all-zero declaration comes from a poor citizen increases after an audit where no income was found.

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[13]Erard and Ho (2001) show econometrically that indeed the main characteristics of ghosts are earnings that are hardly observable.

[14]This "perfect" updating comes from assumption A4 and can be seen as the most favourable environment for sequential auditing. Nevertheless, even in a less favourable environment our main result that sequential auditing imposes a stricter condition on parameters to observe ghosts still goes through. But the analysis gets very complicated with "imperfect" updating.

Denote the belief of facing a crook before auditing income source $i+1$ - having audited sources 1 to i already - by $\rho_i(H)$, where H is the information gained by previous audits. Let H be one if there was an audit where a zero income could not be verified - i.e. proven tax evasion or the suspicion of an non-verifiable income - and zero otherwise. We can summarize the appropriate beliefs given an all-zero declaration and the history of audits:

$$\rho_i(H, \mathbf{d}_0) = \begin{cases} 1 & if \quad H = 1 \\ \frac{\beta}{(1-\beta)(1-\lambda)^{N-i}+\beta} & if \quad H = 0 \end{cases}.$$

So the belief relevant for the audit effort decision for source $i+1$ is given by

$$\mu_i(H, \mathbf{d}_0) = \lambda \cdot \rho_i(H, \mathbf{d}_0). \tag{17}$$

The optimal sequential audit efforts for an all-zero declaration and a given audit history is defined by equation 5, where the beliefs μ_i follow equation 17.

We have to be quite careful, when defining the beliefs and the resulting strategy the authority will adopt whenever it observes at least one declared income source. Naturally in the beginning the tax inspector - given the possible ghost equilibrium strategy of the taxpayer - should believe it is facing a honest taxpayer whenever he observes $\mathbf{d}_Y$. Then the authority has two possible ways to go ahead. It may rubber-stamp the form and close the case or it may keep the case open and update the beliefs according to whether chance leads to any surprising verification results. The first strategy is the same as simultaneously auditing with effort zero, the latter corresponds to sequential auditing with effort zero at the beginning. Given the possible equilibrium beliefs ex ante both strategies yield the same expected revenue for the tax authority, which is just the amount of taxes paid for the declared sources. It seems obvious that a tax inspector who does not expect any gains from waiting might prefer to close the case immediately. We do not explicitly model the waiting cost that might occur, but assume instead that the tax authority always audits simultaneously if the expected returns are equal.[15] It follows that

$$\begin{aligned} \mu_i(\mathbf{d}_Y) &= 0 \quad \forall i \\ a_i^*(\mathbf{d}_Y) &= 0 \quad \forall i. \end{aligned} \tag{18}$$

So far, we did not discuss the determinants for the optimal sequence of sources to be audited. In our simplified framework, where the sources are identical and not correlated, ex ante the tax authority is indifferent between audit paths. It has no relevance for the following results how we specify the sequence of audits.

In what follows we derive the payoff a taxpayer that evades all his income sources can expect. Since the sequence of income sources to be audited is not uniquely determined - every sequence and every randomisation over different sequences is possible, we present the expected payoff of a ghost in terms of beliefs over possible audit paths. We once more denote the number of earned income sources as n. Then the expected payoff for the ghost will be:

$$EU = n(Y - \frac{e^*}{\eta}) - F \sum_{k=1}^{n} E\left[p_k(e^*, a_k^*)\right].$$

The first term is the income net of concealment costs. The sum corresponds to the expected fine, where $E\left[p_k(e^*, a_k^*)\right]$ is the expected verification probability for the concealed income source k. We can simplify this expression if we express the expected verification probabilities $E\left[p_k(e^*, a_k^*)\right]$ in terms of an expected average verification probability $\bar{p}$:

$$EU(\mathbf{d}_0, n) = n\left[Y - \frac{e^*}{\eta} - F \cdot E\left[\bar{p}(n)\right]\right] \tag{19}$$

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[15] This assumption works in favour of ghost behaviour. If the tax inspector does not close the case the condition for profitable ghost behaviour becomes more restrictive. However, the assumption that tax inspector closes the case immedeatly is aquivalent to the assumption that there are waiting costs.

It will prove useful to establish a result about the behaviour of $E[\bar{p}]$ when the number of earned income sources varies. This is done in the following lemma.

Lemma 3 *The expected average verification probability for earned income sources $E[\bar{p}]$ increases weakly with n, the number of productive income sources.*

   Proof. See appendix. ∎

The intuition behind the result that the average expected detection probability increases with the number of productive income sources is quite simple: If a taxpayer earned more income sources and concealed them all, an authority that audits sequentially is more likely to find out earlier that it is facing a crook. Then the tax inspector will earlier step up the detection effort. So the average detection effort and the average expected detection probability increase.

   With this lemma in hand we are able to characterize the condition that has to hold for a ghost equilibrium in the case that the authority audits sequentially. Analogous to the simultaneous auditing case we get the following condition.

Proposition 2 *A ghost equilibrium under sequential auditing exists only if ghost behaviour is optimal for the taxpayer when all income sources are earned. The condition is given by*

$$\frac{T - e^*/\eta}{F} \geq (N-1)\left(p(e^*, a_\lambda^*) - p(e^*, 0)\right) + p(e^*, a_0^*) \tag{20}$$

where a_λ^* is the optimal effort for the authority when it believes to face a crook with certainty.

   Proof. The proof is basically along the same line as in the case of simultaneous auditing. Our assumption that an authority audits simultaneously whenever the ex ante expected pay-offs are equal to those from sequential auditing ensures that we have the same deviation pay-offs. The inequality characterizing the best deviation is given by (16) once again. Thus we have to compare U_1 with $EU(\mathbf{d}_0, n)$ from (19): $EU(\mathbf{d}_0, n) \geq U_1$ if $(T - e^*/\eta)/F \geq n(E[\bar{p}(n)] - p(e^*, 0)) + p(e^*, 0)$. Note that $E[\bar{p}(n)] \geq p(e^*, 0)$, since $p(e^*, 0)$ is a lower bound for p if the taxpayer exerts the optimal effort. Knowing this and that $\Delta E[\bar{p}]/\Delta n \geq 0$ (from the previous lemma) we can conclude that the *rhs* weakly increases with n while the *lhs* is constant. So the critical value for n is again $n = N$. If $n = N$ then the average audit probability does not depend on the audit path any more, because all possible audit paths become equivalent. The authority will start with the prior belief μ_0 and will update to $\mu_1 = \lambda$ after the first audit. Replacing $nE[\bar{p}(n)]$ by $(N-1)p(e^*, a_\lambda^*) + p(e^*, a_0^*)$ gives the claimed condition. ∎

   In principle the interpretation of the condition to be satisfied for ghost behaviour is the same as in the simultaneous auditing scenario. The main distinction is that the relevant measure for audit effectiveness is now the difference in detection risk between a rubber-stamping authority and an authority that invests in detection *while knowing that it faces a crook*.[16] This increased audit effectiveness reflects the additional information the authority can obtain by conducting its audits sequentially. Given the increased audit effectiveness under sequential auditing it is straightforward to show that the possibility to audit sequentially leads to a stricter condition that has to be satisfied in order to allow for ghost behaviour. So sequential auditing is an appropriate tool to reduce ghost behaviour. Some taxpayers who would choose to behave as ghosts under simultaneous auditing do not prefer to do so when auditing is sequential.

Proposition 3 *Sequential auditing reduces ghost behaviour by imposing a stronger condition on parameters in order to allow for a ghost equilibrium.*

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[16]Under simultaneous auditing the relevant measure for audit effectiveness was the difference between the verification probabilities for rubber-stamping and auditing with the prior belief.

Proof. The condition for sequential auditing is stronger whenever $(N-1)(p(e^*, a_\lambda^*) - p(e^*, 0)) + p(e^*, a_\lambda^*) > N\left[p(e^*, a_0^*) - p(e^*, 0)\right] + p(e^*, 0)$. Simplifying leads to $p(e^*, a_\lambda^*) > p(e^*, a_0^*)$ for $N > 0$, which implies $\lambda > \lambda\rho_0$. This is obviously true for $0 < \beta < 1$. ∎

The intuition behind the result is the following. By sequentially auditing the authority can learn from previous audits. Detecting an incorrectly declared income source tells the authority that it faces a crook. The case where an audit leads to the result that the actual income from a certain source cannot be verified makes the tax inspector suspicious. In both cases the authority will step up the audit effort for the remaining sources. The prospect of being heavily audited deters some taxpayers from behaving as ghosts.

6.3 The sequential auditing path

Implicitly, we assumed that the authority is willing to audit sequentially whenever condition (20) is satisfied. But this strategy is only credible if the ex ante expected payoff from sequential auditing exceeds the payoff from auditing all income sources at once. Suppose for instance that the tax authority chooses the rules of the game (sequential or simultaneous audits) after observing the tax form. Then we will discuss the case where the authority can newly decide after every audit how many of the remaining sources to audit in the next step.

6.3.1 Sequential versus simultaneous auditing

Suppose the authority has to decide after observing the income declaration whether to audit all sources at once or to audit just one income source at a time. The tax inspector will choose the latter strategy if his expected payoff from doing so is bigger than it is for simultaneous auditing. If the parameter setting allows for a ghost equilibrium for both auditing strategies - i.e. inequality (20) holds - then the interesting situation is a tax inspector observing an all-zero declaration.[17] Then it is possible to show that sequential auditing pays. This result is established in the following proposition.

Proposition 4 *If the taxpayer can behave as ghost under both audit rules then sequential auditing pays for the tax authority.*

Proof. The proof is in two steps. First we derive two sufficient conditions for our statement to be true, then we show that these conditions are necessarily satisfied. The proposition requires that the sum of the ex ante expected auditing pay-offs from sequential auditing is greater than the sum of the identical expected pay-offs from simultaneous auditing:

$$\sum_{i=0}^{N-1} E\left[\mu_i \cdot p(e^*, a^*(\mu_i)) \cdot F - a^*(\mu_i)\right] > N\left[\mu_0 \cdot p(e^*, a^*(\mu_0)) \cdot F - a^*(\mu_0)\right]. \tag{21}$$

The subscripts for the beliefs μ denote the number of audits that already have taken place (μ_0 is the prior belief). Note that E is the expectation operator. The inequality is certainly fulfilled if

$$E\left[\mu_i \cdot p(e^*, a^*(\mu_i)) \cdot F - a^*(\mu_i)\right] > \mu_0 \cdot p(e^*, a^*(\mu_0)) \cdot F - a^*(\mu_0) \ \forall i \in \{1, ..., N-1\}.$$

For $i = 0$ the expected pay-offs are identical, because the beliefs for the first audit are the same under both regimes. We know that under sequential auditing there are only two possible values for the beliefs at every stage. These are $\mu_{i,0}$ if no earned income source was audited before and

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[17] For a declaration with at least one positive declaration the authority is indifferent between both strategies; the expected equilibrium payoff is just the tax for the declared income components.

$\mu_{i,1} = \lambda$ otherwise. Denoting the ex ante belief that after i audits no earned income source will have been audited by ξ_i and eliminating the expectation operator leads to the condition

$$\xi_i \left(\mu_{i,0} \cdot p(e^*, a(\mu_{i,0})) \cdot F - a^*(\mu_{i,0}) \right) + (1 - \xi_i) \left(\lambda \cdot p(e^*, a(\lambda)) \cdot F - a^*(\lambda) \right)$$
$$> \mu_0 \cdot p(e^*, a^*(\mu_0)) \cdot F - a^*(\mu_0) \; \forall i \in \{1, ..., N-1\}.$$

The tree parts of the equation just depend on the beliefs. We can write:

$$\xi_i \cdot R(\mu_{i,0}) + (1 - \xi_i) R(\lambda) \;\; > \;\; R(\mu_0) \; \forall i \in \{1, ..., N-1\},$$
$$\text{with } R(x) \;\; = \;\; x \cdot p(e^*, a(x)) \cdot F - a^*(x)$$

This condition surely holds (applying Jensen's inequality) if R is convex in μ and $E[\mu_i] = \mu_0$. This is,

$$\frac{d^2}{d\mu^2} R(\mu) \;\; > \;\; 0 \tag{C1}$$
$$\xi_i \cdot \mu_{i,0} + (1 - \xi_i) \lambda \;\; = \;\; \mu_0 \; \forall i \in \{1, ..., N-1\} \tag{C2}$$

We examine (C1) first.

$$\frac{d^2}{d\mu^2} R(\mu) = F \cdot p' \left[a(\mu) \right] \cdot \left(2a'(\mu) + \mu \cdot a''(\mu) \right) + \tag{22}$$
$$+ F \cdot \mu \cdot a'(\mu)^2 \cdot p'' \left[a(\mu) \right] - a''(\mu)$$

Implicit differentiation of the first-order condition gives us the equilibrium change in a with respect to μ:

$$a'(\mu) = -\frac{d^2 R}{d\mu da} \bigg/ \frac{d^2 R}{da^2} = \frac{p'(a(\mu))}{\mu \cdot p''(a(\mu))}.$$

Substituting $a'(\mu)$ and the first-order condition $p'(a(\mu)) = 1/(F\mu)$ into (22) gives

$$\frac{d^2}{d\mu^2} R(\mu) = \frac{-F \cdot \mu^3}{p''(a(\mu))}.$$

Since $p''(a(\mu)) < 0$ by assumption, condition (C1) is satisfied.

Condition (C2) obviously has to hold. The ex ante expected belief after updating has to be equal to the prior. This is commonly true if the updating is done without errors. Since we assumed that the authority does not make any mistakes during the updating process (C2) is satisfied. The proof for our purpose can be found in the appendix. ∎

We briefly summarize what we have established so far in this section. If the authority can decide the rules of the game after observing the income declaration from a possible ghost (i.e. the ghost condition from proposition 2 holds and a declaration containing only zeros is observed) it will decide to audit sequentially. Sequential auditing is an equilibrium, because the payoff under this regime is higher than under simultaneous auditing. Furthermore, the possibility of auditing sequentially may deter some taxpayers from playing ghost, since the condition the parameters have to satisfy for ghost behaviour to be profitable is stronger.

6.3.2 Free auditing choice

In what follows we lift the restriction that the authority has to decide once and for all whether to audit sequentially or simultaneously. So suppose the tax inspector can decide after every audit how many sources he wants to audit next. If we keep the assumption that the authority will choose to audit sources together if this gives the same expected payoff as sequential auditing does, we get an auditing pattern in equilibrium that is widely observed in reality. Facing a potential ghost the tax inspector will audit source by source until he is sure that he is facing a crook. Then he will conduct a simultaneous full scale audit of the remaining sources. That such a procedure is indeed optimal for the tax authority is stated in the following proposition.

Proposition 5 *It is optimal for the authority facing a potential ghost to audit source by source as long as the belief that the remaining sources are productive and concealed is smaller than λ. If μ reaches λ it is optimal to audit all remaining sources simultaneously.*

Proof. See appendix. ■

The audit path that is described as optimal by the above proposition creates audit patterns that are widely observed in reality. The tax inspector picks a certain income source for audit. If he cannot find any concealed income during this audit he may switch to another potential income source to conduct checks with a reduced effort. But as soon as the inspector gets suspicious or even can prove evasion, all possible income sources are immediately checked with high effort. The assumption that the tax inspector audits simultaneously if he is indifferent between the two audit strategies is crucial for our result that in the case of suspicion simultaneous full-scale audits are conducted. However, there is no reason why the authority should conduct sequential full-scale audits. On the one hand there is nothing to learn about the taxpayer any more, since the tax inspector is certain to face a crook. And on the other hand sequential auditing could cause some additional costs that are not included in our framework. On might think of the possibility that the taxpayer may try to destroy evidence when he realizes that he will be subject to a full-scale audit.

7   Evasion of fractions of total income

In this section we explore what the taxpayers do if the environment is not favourable enough for ghost behaviour. Intuition suggests that in such a situation crooks may hide only some of their income components. This, however, does not increase the gain from evasion if the tax authority can anticipate, which income components are evaded. Consequently, a crook creates some additional uncertainty for the authority by randomly choosing the income components he evades. Such a behaviour decreases the authority's perceived probability that a particular income component is evaded. The lower probability causes a lower expected payoff from auditing and reduces the detection effort.

In the following two sections we characterize the equilibria that arise under the different audit rules if pure ghost behaviour does not pay. For simplicity we will only consider the case with two potential income sources. In order to compare the effectiveness of the two different audit rules we derive conditions on the parameters that allow for profitable evasion. It turns out that once again sequential auditing has the edge over simultaneous auditing. The environment necessary for profitable evasion has to be more favourable for the taxpayer under the sequential audit regime.

7.1   Simultaneous auditing

Recall the condition that ghost behaviour does pay under sequential auditing, which is given by (20). Then the condition that ghost behaviour does not pay for two potential income sources becomes

$$\frac{T - e^*/\eta}{F} < 2p(e^*, a_0^*) - p(e^*, 0).$$

Consider the following strategy of a taxpayer:

1. If both income components are earned he mixes between concealing both sources, concealing only the first source, and concealing only the second source.

2. If only one income component is earned he conceals it with certainty.

3. If no income is earned he truthfully declares zero for both sources.

Denote the mixing probabilities for the case that both sources are earned by $\alpha(0,0)$ for evading both sources, by $\alpha(0,y)$ for evading only the first source, and by $\alpha(y,0)$ for evading only the second source.

This is the only strategy with randomisation that guarantees profits from evasion. A mixed strategy that includes reporting truthfully, if some income is earned, necessarily leads to the same expected profit as being honest with certainty. Otherwise if evasion gives a higher payoff than being honest, then there is no reason why the taxpayer should choose to declare truthfully with a positive probability.

A tax man anticipating the taxpayer's mixing will have the following beliefs where the arguments for μ represent the observed declaration behaviour:

$$\mu(0,0) = \frac{\lambda^2 \cdot \beta \cdot \alpha(0,0) + (1-\lambda) \cdot \lambda \cdot \beta}{\lambda^2 \cdot \beta \cdot \alpha(0,0) + 2 \cdot (1-\lambda) \cdot \lambda \cdot \beta + (1-\lambda)^2}$$

$$\mu(0,y) = \frac{\lambda \cdot \beta \cdot \alpha(0,y)}{(1-\lambda)(1-\beta) + \lambda \cdot \beta \cdot \alpha(0,y))}$$

$$\mu(y,0) = \frac{\lambda \cdot \beta \cdot \alpha(y,0)}{(1-\lambda)(1-\beta) + \lambda \cdot \beta \cdot \alpha(y,0))}.$$

If the declaration for both sources is zero then $\mu(0,0)$ is the belief that one particular income sources is evaded. This belief is identical for both income sources. If one income source is declared the belief that a zero declaration for the other source comes from tax fraud is given by $\mu(0,y)$ and $\mu(y,0)$ where the source with a zero declaration is the one in question.

Denote the expected payoff from a pure strategy as $U_{ij}(d_i, d_j)$, where the subscripts denote the actual incomes from source i and j. The declaration behaviour is given by the arguments. To be willing to mix between evading both sources or just cheating for one source the taxpayer has to be indifferent between the expected pay-offs these pure strategies yield. Additionally, the payoff from these evasion strategies should not be smaller than the payoff from reporting truthfully. In equilibrium the following has to hold:

$$U_{yy}(0,0) = U_{yy}(y,0) = U_{yy}(0,y) \geq U_{yy}(y,y) \tag{23}$$

If only one source is earned in equilibrium the taxpayer prefers to evade it with certainty if:

$$U_{y0}(0,0) \geq U_{y0}(y,0) \text{ and } U_{0y}(0,0) \geq U_{0y}(y,0). \tag{24}$$

Combining (23) and (24) leads to the necessary condition that a crook uses the described mixed strategy in equilibrium. The condition is given in the following proposition. Here $p(\underline{\mu}(0,0))$ denotes the detection probability arising from the lowest possible belief that a source is evaded if an all-zero declaration is observed, while $p(\overline{\mu}(y,0))$ is the probability caused by the highest possible belief that one source is evaded if the other is declared.

Proposition 6 *For $p(\underline{\mu}(0,0)) \leq p(\overline{\mu}(y,0)) < (T - e^*/\eta)/F < 2p(e^*, a_0^*) - p(e^*, 0)$ under simultaneous auditing there exists a mixed strategy equilibrium where the taxpayers' expected payoff is higher than that from reporting truthfully.*

Proof. See the appendix. ∎

The condition above needs some explaining. The detection probabilities for evaded sources given a certain declaration pattern depend on the mixing probabilities. A crook chooses the mixing probabilities in order that the conditions (23) and (24) are satisfied. Whether this is possible depends on the parameters. For an equilibrium in mixed strategies where the taxpayer gets a profit from evasion the parameters have to be favourable enough that evasion pays $p(\underline{\mu}(0,0)) \leq p(\overline{\mu}(y,0)) < (T - e^*/\eta)/F$. But the environment should not be too favourable, because then - for $(T - e^*/\eta)/F > 2p(e^*, a_0^*) - p(e^*, 0)$ - to behave as a ghost with certainty becomes profitable. The environment is favourable for evasion if the taxes liabilities T are high, if

concealment is cheap (high η), and if the fines F are low. Additionally, a low earnings probability λ and a low proportion of crooks in the population β is beneficial for evasion.

The question arises what a taxpayer will do if neither ghost behaviour nor mixing lead to positive profits from evasion. In this case (i.e. $\max[p(\mu(y,0)), p(\mu(0,0))] > (T - e^*/\eta)/F$) it is possible to include the strategy to report truthfully in the mixing as well. This will further drive down the detection effort of the authority by reducing the beliefs that income is evaded. But, this will leave the taxpayer with no expected gain from tax evasion. To see this recall the indifference condition (23) for two earned income sources. Then $U_{yy}(0,0) = U_{yy}(y,0) = U_{yy}(0,y)$ implies

$$2p(\mu(0,0)) - p(\mu(y,0)) = \frac{T - e^*/\eta}{F} \tag{25}$$

The indifference condition for the case where one source is earned is given by $U_{y0}(0,0) = U_{y0}(y,0) = U_{0y}(0,0)$ which implies

$$p(\mu(y,0)) = \frac{T - e^*/\eta}{F}$$

By combining equations the condition becomes

$$p(\mu(0,0)) \cdot F = p(\mu(y,0)) \cdot F = T - e^*/\eta.$$

This implies that the expected payoff for the taxpayer is equivalent to the honesty payoff regardless how many income components are earned. The expected fine is always equal to the taxes saved net of concealment costs.[18] So we may obtain a hybrid equilibrium where a taxpayer who earned at least one source evades with positive probability although he does not expect any profit from evading. Why is reporting truthfully not an equilibrium? Reporting truthfully with certainty would lead to an authority rubber-stamping the tax declaration. But under the belief that the authority will rubber-stamp the declaration, the taxpayer prefers to evade. The less beneficial the environment is for evasion the lower the probability becomes that evasion takes place. Note that our assumption that the authority cannot commit to an audit strategy is crucial for this result.[19]

7.2 Mixing with sequential auditing

We now turn to the sequential auditing regime. We once again derive the conditions that have to be met for profitable tax evasion to take place. This is the case if mixing leads to a higher expected net payoff than being honest. The derivation of the conditions for this mixed strategy equilibrium is analogous to the case with simultaneous auditing. We just have to remember that we may have different beliefs for a tax authority observing an all-zero declaration before and after auditing the first source. The belief before auditing the first source of an all-zero declaration - denoted by $\mu_0(0,0)$ - will be the same as in the simultaneous case. The belief after the first audit will depend on the outcome of the first audit. Denote this belief as $\mu_{1,y}(0,0)$ if there was evasion and as $\mu_{1,0}(0,0)$ if there was no evasion. The first subscript gives the number of audits conducted, while the second gives the outcome of the audit if there was one. In the first case the perceived probability that the second source is earned and evaded increases, since it is now known that the taxpayer is a crook, while in the second case this probability decreases, because it is becoming more likely that the taxpayer might be an honest citizen. The beliefs in the case that one income source is declared are the same as in the simultaneous auditing

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[18]To satisfy the indifference condition it may be necessary that the taxpayer is totally honest with positive probability if both income sources are earned.

[19]Most authors seem to regard this as an unrealistic feature of moral hazard models without commitment, but introspection (my fare-evasion behaviour on commuter trains) suggests that mixing actually seems to happen.

scenario. Since it will turn out that $\mu(0, y) = \mu(y, 0)$ has to be true, we only give one of these beliefs. The relevant beliefs are given by:

$$\mu_0(0,0) = \frac{\lambda^2 \cdot \beta \cdot \alpha(0,0) + (1-\lambda) \cdot \lambda \cdot \beta}{\lambda^2 \cdot \beta \cdot \alpha(0,0) + 2 \cdot (1-\lambda) \cdot \lambda \cdot \beta + (1-\lambda)^2}$$

$$\mu_{1,y}(0,0) = \frac{\lambda \cdot \alpha(0,0)}{\lambda \cdot \alpha(0,0) + 1 - \lambda}$$

$$\mu_{1,0}(0,0) = \frac{\lambda \cdot \beta}{\lambda \cdot \beta + 1 - \lambda}$$

$$\mu(0,y) = \frac{\lambda \cdot \beta \cdot \alpha(0,y)}{(1-\lambda)(1-\beta) + \lambda \cdot \beta \cdot \alpha(0,y))}.$$

Recall the condition that ghost behaviour does not pay under sequential auditing:

$$\frac{T - e^*/\eta}{F} < p(e^*, a_\lambda^*) - p(e^*, 0) + p(e^*, a_0^*). \tag{26}$$

Using the same logic as in the simultaneous auditing scenario we can derive an analogous condition. Lower and upper bars once again denote lower and upper bounds for the beliefs depending on the mixing probabilities.

Proposition 7 *For*

$$\left[p\left(\underline{\mu}_0(0,0)\right) + p\left(\underline{\mu}_{1,y}(0,0)\right) \right] / 2 \leq p\left(\overline{\mu}(y,0)\right) <$$
$$< (T - e^*/\eta)/F < p(e^*, a_\lambda^*) - p(e^*, 0) + p(e^*, a_0^*) \tag{27}$$

under sequential auditing there exists a mixed strategy equilibrium where the taxpayer's expected payoff is higher than that from reporting truthfully.

Proof. See appendix. ∎

Note that for parameter configurations where neither ghost behaviour nor randomising over the evasion of different sources is profitable a crook will be honest with positive probability and have an expected payoff equal to the honesty payoff. The argument is the same as in the simultaneous case.

Comparing the conditions under the different auditing regimes shows that the requirements on the environment under sequential auditing (condition 27) are stricter than under simultaneous auditing $\left(p\left(\underline{\mu}(0,0)\right) \leq p\left(\overline{\mu}(y,0)\right)\right)$.[20] This means that for some parameter settings where a crook still makes profits from evasion under simultaneous audit he will not make any evasion profits if the authority audits sequentially. Sequential auditing therefore has the edge over simultaneous auditing once again.

8 Self-selection of moonlighters

In this section we argue that our model can explain a commonly observed pattern of self-selection into different income sources. The pattern in question is moonlighting. We think of people that are working in regular employment during the day while being active in the black market economy during evenings and weekends. Craftsmen are a prominent example. Why do these people not entirely engage in the black economy, or as an alternative just work long hours in the official sector? Standard explanations argue that small markets in the moonlighting sector drive the wages for workers or prices for firms down if the activity is increased. It is argued

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[20]This has to be the case since $p\left(\underline{\mu}_{1,y}(0,0)\right) > p\left(\underline{\mu}_0(0,0)\right) = p\left(\underline{\mu}(0,0)\right).$

that for this reason entirely going underground does not pay. See Cowell and Gordon (1995) for
the self-selection of firms, Cowell and Gordon (1990) for workers, or Gordon (1988) for a model
of black market transactions. We argue that there might be an additional incentive for people
splitting their activity between the black market and the official sector: a tax authority that is
auditing sequentially creates these incentives. If a taxpayer relies too heavily on black market
activity then a sequentially auditing tax authority learns too easily that the taxpayer is a crook,
which will lead to a full-scale audit. So it might be a profitable strategy to work in the official
sector during daytime - although evasion is not profitable there - just to engage in black-market
activity in the evenings where high evasion profits with a low detection risk are possible.

Suppose that the taxpayer has the possibility of receiving income from two income sources.
There are two different markets, the black market and the official sector. The taxpayer can
choose how many (of his two) sources to allocate to the different sectors. The black market
sector has the advantage that there tax evasion is profitable. Suppose that his profit from
moonlighting is greater than from honestly working in the official sector even if the taxpayer
allocates both sources to the underground economy.[21] In the official sector tax evasion never
pays, since the activity is too easy to observe by the authority (i.e. the observability parameters
τ, ω are prohibitive). The advantage in the official sector is that the gross income from the
sources is higher. This reflects the discount a customer demands for the contract enforcement
problems if a moonlighter is employed. Unlike other models we do not have to assume that the
gross income in the moonlighting sector decreases with the activity. This assumption made in
other models seems to be reasonable in the aggregate, but surely not on the individual level. A
painter "privately" decorating two flats on a weekend does not earn less money per flat than a
painter who just decorates one.

We will show that there are parameter configurations that make it optimal for the taxpayer
to divide his efforts between the two sectors even in the case where ghost behaviour pays for
someone who decided to devote his entire effort to the underground economy. In order to induce
such a self-selection choice the ex ante expected payoff from dividing the efforts (EU_m) has to
be higher than that from being a ghost in the black economy (EU_b) and has also to be higher
than the payoff from working entirely in the official sector (EU_o). The pay-offs are given by

$$EU_o = 2\lambda(Y_0 - T),$$

$$EU_m = \lambda(Y_o - T) + \lambda(Y_b - p(e^*, a_0^*) \cdot F - e^*/\eta),$$

and

$$EU_b = \lambda^2(2Y_b - (p(e^*, a_0^*) + p(e^*, a_\lambda^*)) \cdot F - 2e^*/\eta)$$
$$+ 2\lambda(1 - \lambda)(Y_b - (p(e^*, a_0^*) + p(e^*, a_1^*)) \cdot F/2 - e^*/\eta),$$

where Y_o and Y_b are the gross incomes in the official sector and in the black market, respectively.
The payoff if the craftsman only works in the official sector EU_o is twice the net income multiplied
by the earnings probability. The expected income from dividing efforts EU_m is the sum of the
expected incomes in the two sectors. The payoff for ghosts in the black market is given by
EU_b. The first part represents the case that both income components are earned (weighted
with probability λ^2 that this happens). Under sequential auditing one source will be audited
with the effort a_0^* corresponding to the prior while the other source is audited with effort a_λ^*.
The second part of EU_b corresponds to the two possible cases that just one source is earned
(with probability $\lambda(1 - \lambda)$ each). Depending on whether the authority audits the earned source
as the first or second the source, the effort will be a_0^* or a_1^*. Regardless of how the authority
mixes between audit sequences, ex ante on average the expected verification probability will be
$(p(e^*, a_0^*) + p(e^*, a_1^*))/2$.

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[21] This setting is the least favourable for any activity in the official sector; and therefore gives the strongest
result.

The condition $EU_m > EU_o$ - participating in both markets is better than just working in the official sector - reduces to

$$p(e^*, a_0^*) < \frac{T - e^*/\eta}{F} - \frac{Y_o - Y_g}{F}.$$

For participating in both markets to be better than to concentrate entirely on the underground $EU_m > EU_b$ has to hold. This reduces to

$$\lambda \cdot p(e^*, a_\lambda^*) + (1 - \lambda) \cdot p(e^*, a_1^*) > \frac{T - e^*/\eta}{F} - \frac{Y_o - Y_g}{F}.$$

Obviously, both inequalities for themselves do not conflict necessarily with the ghost condition for sequential auditing (20). The possibility to satisfy them simultaneously requires

$$p(e^*, a_0^*) < \lambda \cdot p(e^*, a_\lambda^*) + (1 - \lambda) \cdot p(e^*, a_1^*) \tag{28}$$

It is possible that this condition is satisfied. We know that $p(e^*, a_0^*)$ is larger than $p(e^*, a_1^*)$ and smaller than $p(e^*, a_\lambda^*)$. Whether the inequality is satisfied depends on the shape of the detection probability function and on the parameters λ and β. A high earnings probability λ and a small proportion of crooks in the population β makes it more likely that working in both worlds pays. To put it differently, as long as there are enough honest craftsmen and the earning opportunities are relatively secure, then a ruthless craftsman may choose to work in both the official sector and the black market.[22]

9 Conclusion

The tax declaration situation and the following examination of the tax form by the authorities create a highly complex strategic environment. In this paper this is modelled as a contest between taxpayer and authority where both invest in concealment and detection, respectively. We stressed the fact that income comes from different income sources. Since auditing one source may reveal valuable information about the taxpayer and his likely tax declaration behaviour for other sources, we allowed for sequential auditing. A comparison of the outcomes of the contest under this sequential audit rule with the outcome that occurs if the tax authority audits all sources at the same time was conducted. This helps to explain the rationale for a widely observed audit pattern: Tax inspectors sequentially conduct routine checks with a consecutive full-scale audit if suspicion of evasion arises from these checks. Furthermore we shed some light on the reasons why people moonlight in the black market sector and at the same time follow a normal job in the official sector.

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[22] This result generalizes for more income sources. It is possible (depending on the parameters) that the additional expected gross income from a source in the black market decreases. Then the wedge between gross earnings in the sectors makes it possible that working in both sectors may become optimal.

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A   Some proofs of lemmata and propositions in the main text

A.1   Proof of lemma 3

Proof. For the N income sources there are $N!$ possible audit paths. The taxpayer has beliefs about the probabilities that the different paths are followed by the tax inspector. These beliefs have to be consistent with the equilibrium strategy of the tax authority. Let the belief that a particular path is followed be ν_j with $\sum \nu_j = 1$. Then the expected average verification probability can be written as:

$$E[\bar{p}] = \sum_{j=1}^{N!} \frac{\nu_j}{n} \sum_{k=1}^{n} p_{k,j}$$

where $p_{k,j}$ is the (equilibrium) verification probability for income source k given that the audit path j is followed. Since the belief of the tax authority of facing a concealed income source if a previous audit was successful is λ, we know that for $n-1$ income components the verification probability has to be $p_\lambda = p(e^*, a_\lambda^*)$. The probability of the remaining source depends on the audit path. More precisely, this probability depends on the belief μ_r the tax inspector will have when he audits the first concealed income source. This belief will be

$$\mu_r = \frac{\beta}{(1-\beta)(1-\lambda)^{N-r} + \beta},$$

where r gives the number of audits before the first concealed income component is found. Note that r depends on the audit path and the number of earned income sources. Rewriting $E[\bar{p}]$ and

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denoting the probability associated with μ_r as $p_{r(j,n)}$ gives:

$$E[\bar{p}] = \sum_{j=1}^{N!} \nu_j \frac{(n-1)p_\lambda + p_{r(j,n)}}{n}.$$

If $\left[(n-1)p_\lambda + p_{r(j,n)}\right]/n$ increases with n for all paths j, then $E[\bar{p}]$ obviously also increases with n.

An additional income source that is earned can be audited before or after the critical income source r. If audited before the new r will be smaller. Otherwise r remains unchanged if n increases. It follows

$$r(j,n) \geq r(j, n+1) \quad \forall j, n < N.$$

From the observations that μ_r decreases with r and that $p(\cdot)$ weakly increases with μ due to a higher optimal detection effort (lemma 1), it follows

$$p_{r(j,n+1)} \geq p_{r(j,n)} \quad \forall j, n < N.$$

Using $p_{r(j,n)}$ as a lower bound for $p_{r(j,n+1)}$ we can write

$$\frac{\Delta}{\Delta n}\left(\frac{(n-1)p_\lambda + p_{r(j,n)}}{n}\right) = \frac{p_\lambda - p_{r(j,n)}}{n^2 + n}.$$

Since $\lambda > \mu_r$ and $p(x) \geq p(x')$ if $x > x'$ it follows that for the valid n $(0 < n < N)$ the *rhs* is positive or at least zero. This implies

$$\frac{\Delta E[\bar{p}]}{\Delta n} \geq 0.$$

This concludes the proof. ∎

A.2 Proof for correct updating (used to prove propositions 4 and 5)

Proof. The ex ante expected belief after i audits is given by:

$$E[\mu_i] = \mu_{i,0} \prod_{j=0}^{i-1}\left(1 - \mu_{j,0}\right) + \lambda\left(1 - \prod_{j=0}^{i-1}\left(1 - \mu_{j,0}\right)\right).$$

It is sufficient to show that $\Delta E[\mu_i]/\Delta i = 0$ for all i, since $E(\mu_0) = \mu_0$. The change of the expected beliefs for an increasing i is given by

$$\frac{\Delta E[\mu_i]}{\Delta i} = E[\mu_{i+1}] - E[\mu_i] =$$

$$= (\lambda - \mu_{i,0}) \prod_{j=0}^{i-1}\left(1 - \mu_{j,0}\right) - (\lambda - \mu_{i+1,0}) \prod_{j=0}^{i}\left(1 - \mu_{j,0}\right).$$

Multiplying the first term by $(1 - \mu_{j,0})/(1 - \mu_{j,0})$ and factoring gives

$$\left(\frac{\lambda - \mu_{i,0}}{1 - \mu_{i,0}} - \lambda + \mu_{i+1,0}\right) \prod_{j=0}^{i}\left(1 - \mu_{j,0}\right).$$

Using the values for $\mu_{i,0}$ and $\mu_{i+1,0}$ from (17) shows that the expression in the brackets is equal to zero. This concludes the proof. ∎

A.3 Proof of proposition 5

Proof. Let r be the number of sources already audited. Let $j \in \{1, ..., N-1-r\}$ be the number of sources next to be audited simultaneously. Then the expected continuation payoff will be

$$CR_{r,j} = j \cdot ER(\mu_r) + CR_{r+j},$$

where $ER(\mu_r)$ is the expected payoff from one of the income components that are audited together while CR_{r+j} gives the expected continuation payoff for all the remaining sources. The total continuation payoff from auditing the next j sources sequentially is denoted by $CR_{r,1}$ and can be written as

$$CR_{r,1} = \sum_{i=r}^{r+j-1} ER(\mu_i) + CR_{r+j}.$$

We have to show that

$$\begin{aligned} CR_{r,1} > CR_{r,j} \quad &\text{if} \quad \mu_r < \lambda \\ CR_{r,1} \leq CR_{r,j} \quad &\text{if} \quad \mu_r = \lambda \end{aligned} \quad \forall r \in \{0, ..., N-2\},\, j > 1,$$

We can write the difference as

$$\begin{aligned} CR_{r,1} - CR_{r,j} &= \left(\sum_{i=r}^{r+j-1} ER(\mu_i) + CR_{r+j} \right) - \left(j \cdot ER(\mu_r) + CR_{r+j} \right) \\ &= \sum_{i=r+1}^{r+j-1} ER(\mu_i) - (j-1) ER(\mu_r). \end{aligned}$$

Note that the CR_{r+j} depends on the audit history, but not on the audit strategy. So CR_{r+j} is the same for both audit strategies. We already see that for $\mu_r = \lambda$ the difference $CR_{r,1} - CR_{r,j}$ is zero, since $\mu_r = \lambda$ implies $\mu_i = \lambda$ for all $i \geq r$. On the other hand the difference is necessarily positive if

$$ER(\mu_i) > ER(\mu_r) \ \forall i \in \{r+1, ..., r+j\}.$$

We can eliminate the expectation operator by writing

$$\xi_{i/r-1} \cdot R(\mu_{i,0}) + \left(1 - \xi_{i/r-1} \right) R(\lambda) > R(\mu_r) \ \forall i \in \{r+1, ..., r+j\},$$

where $\xi_{i/r-1}$ this time denotes the belief that before the audit of source i no source that was productive will have been audited, given the history of audits up to $r-1$. Note that for $\mu_r = \lambda$ the beliefs $\xi_{i/r-1}$ are 0. For $\xi_{i/r-1} > 0$ the inequality above holds if:

$$\frac{d^2}{d\mu^2} R(\mu) \quad > \quad 0 \tag{C1}$$

$$\xi_{i/r-1} \cdot \mu_{i,0} + \left(1 - \xi_{i/r-1} \right) \lambda \quad = \quad \mu_r \ \forall i \in \{r+1, ..., r+j\}. \tag{C2'}$$

We have already proven convexity (C1), while condition C2' again expresses updating without errors. The observation that C2' is equivalent to C2, which was proven above, concludes the proof. ∎

A.4 Proof of proposition 6

Proof. $U_{yy}(y,0) = U_{yy}(0,y)$ implies that $p(\mu(0,y)) = p(\mu(y,0))$. Note that this has to be the case, since the probability is a monotonous function of the authority's effort and the optimal effort is a monotonous function of the belief. A further implication is that $\mu(0,y) = \mu(y,0)$ (short $\mu(y,0)$). It follows immediately that $\alpha(0,y) = \alpha(y,0)$.

$U_{yy}(0,0) = U_{yy}(y,0) = U_{yy}(0,y)$ implies

$$2p(\mu(0,0)) - p(\mu(y,0)) = \frac{T - e^*/\eta}{F} \tag{29}$$

For $\alpha(0,0) \to 1$ it follows $p(\mu(0,0)) \to p(e^*, a_0^*)$, since $\mu(0,0) \to \mu_0$. Then necessarily $\alpha(0,y) \to 0$ and $\mu(y,0) \to 0$, which implies $p(\mu(y,0)) \to p(e^*, 0)$. Our condition converges against the ghost condition with equality.

For $\alpha(0,0) \to 0$ we know $\mu(0,0) \to \lambda \cdot \beta / (2 \cdot \lambda \cdot \beta + 1 - \lambda) = \underline{\mu}(0,0) < \mu_0$ and $\alpha(0,y) \to 1/2$, which implies that $\mu(y,0) \to \lambda \cdot \beta / (2 \cdot (1-\lambda)(1-\beta) + \lambda \cdot \beta) = \overline{\mu}(y,0)$.

Note that $\mu(0,0)$ increases with $\alpha(0,0)$, while $\mu(y,0)$ decreases when $\alpha(0,0)$ increases. Since $p(\cdot)$ is increasing in μ a necessary condition for (29) to be satisfiable by appropriate mixing probabilities $\alpha(\cdot, \cdot)$ is

$$\min_{\alpha(\cdot,\cdot)}[2p(\mu(0,0)) - p(\mu(y,0))] < \frac{T - e^*/\eta}{F} < \max_{\alpha(\cdot,\cdot)}[2p(\mu(0,0)) - p(\mu(y,0))].$$

with the findings from above we can write:

$$2p(\underline{\mu}(0,0)) - p(\overline{\mu}(y,0)) < \frac{T - e^*/\eta}{F} < 2p(e^*, a_0^*) - p(e^*, 0). \tag{30}$$

Checking $U_{yy}(0,y) > U_{yy}(y,y)$ leads to $p(\mu(y,0)) < (T - e^*/\eta)/F$.[23] Taking the upper bound $p(\overline{\mu}(y,0)) < (T - e^*/\eta)/F$ and combining it with (30) leads to

$$p(\underline{\mu}(0,0)) \le p(\overline{\mu}(y,0)) < (T - e^*/\eta)/F < 2p(e^*, a_0^*) - p(e^*, 0)$$

as a sufficient condition to be satisfied. It remains to check that $U_{y0}(0,0) > U_{y0}(y,0)$ holds. Inspection shows that this reduces to the same condition as above $U_{yy}(0,y) < U_{yy}(y,y)$ (i.e. $p(\mu(y,0)) < (T - e^*/\eta)/F$). Since we checked for $U_{y0}(0,0) > U_{y0}(y,0)$ and $U_{yy}(0,0) = U_{yy}(0,y) > U_{yy}(y,y)$, evasion pays in this equilibrium. This concludes the proof. ■

A.5 Proof of proposition 7

Proof. The proof is along the same lines as the proof for the simultaneous case. The indifference and dominance conditions are the same as in the simultaneous case and are given by (23) and (24). The condition $U_{yy}(y,0) = U_{yy}(0,y)$ again implies that $\alpha(y,0) = \alpha(0,y)$.

$U_{yy}(0,0) = U_{yy}(y,0) = U_{yy}(0,y)$ implies

$$p(\mu_0(0,0)) + p(\mu_{1,y}(0,0)) - p(\mu(y,0)) = \frac{T - e^*/\eta}{F} \tag{31}$$

The *lhs* converges against $p(e^*, a_\lambda^*) - p(e^*, 0) + p(e^*, a_0^*)$ for $\alpha(0,0) \to 1$. Monotonicity of p in μ, $\mu_0(0,0)$ and $\mu_{1,y}(0,0)$ being decreasing in $\alpha(y,0)$, and $\mu(y,0)$ being increasing in $\alpha(y,0)$ makes sure that

$$p(\underline{\mu}_0(0,0)) + p(\underline{\mu}_{1,y}(0,0)) - p(\overline{\mu}(y,0)) < \frac{T - e^*/\eta}{F} \tag{32}$$

is a necessary condition for (31) to be satisfiable. Since $U_{yy}(0,y) > U_{yy}(y,y)$, we get

$$p(\mu(y,0)) < (T - e^*/\eta)/F.$$

$U_{yy}(0,0) > U_{yy}(y,y)$ gives

$$\left[p(\mu_0(0,0)) + p(\mu_{1,y}(0,0)) \right]/2 < (T - e^*/\eta)/F \tag{33}$$

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[23] Note, that we look for the strict inequality here to exclude the equilibria where tax evasion leads to the same expected payoff as honesty.

Using the relevant limits for the two inequalities above and combining them with (32) and (26) leads to the sufficient condition stated in the proposition. It remains to check that $U_{y0}(0,0) > U_{y0}(y,0)$ and $U_{0y}(0,0) > U_{0y}(0,y)$ does not lead to a stricter condition. If we denote the probability that the authority audits source one first with ψ then the two conditions imply[24]

$$\psi \cdot p(\mu_0(0,0)) + (1 - \psi) \cdot p(\mu_{1,0}(0,0)) \quad < \quad (T - e^*/\eta)/F \text{ and}$$
$$(1 - \psi) \cdot p(\mu_0(0,0)) + \psi \cdot p(\mu_{1,0}(0,0)) \quad < \quad (T - e^*/\eta)/F.$$

Certainly (33) is stricter, since $p(\mu_{1,y}(0,0)) > p(\mu_{1,0}(0,0))$. This concludes the proof. ∎

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[24] Note that we can choose ψ arbitrarily since the authority is indifferent about which source to audit first.

B   Supplement: Numerical example

B.1   Pure strategy equilibrium

In this section we present a numerical example to give a better flavour of how the abstract model actually works. Suppose that the probability of being caught for evasion is given by:

$$p(e, a) = .3 + \frac{\ln(.2 + a) - \ln(.2 + e)}{4}.$$

This function is designed to guarantee p to be between zero and one for relevant values of a and e.[25] Let the potential income per source be $y = 10$. The sources are productive with probability $\lambda = .3$. Let the linear tax rate be .5 (i.e. a liability per source of $T = 5$). The fine is $F = 7$. The effectiveness of the covering technology is fixed at $\eta = .33$. Assume that the proportion of crooks in the population is known to be $\beta = .3$. We fix the number of potential income sources at $N = 3$. Table 1 shows the expected pay-offs for ghosts and the deviation payoff, which is earned from pretending to be a good citizen by declaring at least one income source. The sequential ghost payoff is calculated under the assumption that the tax authority randomises with equal weights among the different audit paths. We see that for three earned income sources behaving as a ghost still pays under simultaneous auditing while sequential auditing prevents the taxpayer from ghost behaviour.

Earned sources	n=1	n=2	n=3
Simultaneous, ghost $U_{sim}(\mathbf{d}_0)$	7.96	15.93	23.89
Sequential, ghost $U_{seq}(\mathbf{d}_0)$	8.17	15.00	21.83
Deviation $U_{si/se}(\mathbf{d}_1)$	5.00	13.62	22.25

Table 1: Expected ghost and deviation payoff

And in fact, if we calculate the critical $\hat{N}$ from condition (20) for sequential auditing and from condition (15) for simultaneous auditing we see that $\hat{N}_{seq} = \text{Int}(2.79)$ and $\hat{N}_{sim} = \text{Int}(3.99)$. We know from our analytical analysis that for both auditing strategies a higher tax liability T facilitates ghost behaviour. Fixing the number of sources at $N = 3$ we can compute the critical T for ghost behaviour for both cases. We find that the minimal tax liabilities leading to ghost behaviour are $\hat{T}_{sim} = 3.36$ for simultaneous auditing and $\hat{T}_{seq} = 5.41$ for sequential auditing. To compare the revenue from different auditing regimes for the case that ghost behaviour always pays we let $T = 5.5$.

Earned sources	n=0	n=1	n=2	n=3	$E\Sigma_n$
Simultaneous $R_{sim}(\mathbf{d}_0)$	$-.27$	.61	1.50	2.39	.17
Sequential $R_{seq}(\mathbf{d}_0)$	$-.14$	.12	2.04	3.98	.22

Table 2: Revenue from auditing

Table 2 reports in the first four columns the revenues for given numbers of productive income sources (if a declaration of only zeros is observed). The last column weights these revenues with their likelihood and sums them up. This gives the relevant expected revenue from auditing with the respective strategies if an all-zero declaration is observed. Examining the overall revenue (including tax payments of honest taxpayers, fines, and detection costs) simultaneous auditing results in an expected collection of 71.8% of expected tax liabilities. The sequential auditing does slightly better with 72.4%. We see that sequential auditing pays.

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[25] This detection probability function gives values for the observability parameters $\tau = 5/4$ and $\omega = -5/4$.

B.2 Hybrid equilibrium

Return to the numerical example from above with all the parameters - except the tax liability - at their original values. We reduce the tax rate to .2 (i.e. the tax liability is $T = 2$). This ensures that for both audit strategies pure ghost behaviour does not pay for the case of two income sources. We fix $N = 2$. Applying condition 25 leads to the mixing probabilities: $\alpha_{yy}(0,0) = .16$ and $\alpha_{yy}(y,0) = \alpha_{yy}(0,y) = .42$ under simultaneous auditing. The expected payoff after earning both sources will be $EU_{yy} = 17.43$. Earning one source will lead to $EU_{y0} = 8.72$. The expected net revenue from auditing for the authority will be $ER = .93$. Over all the collection efficiency is 77% of the expected gross tax liability.

Sequential auditing is able to prevent the taxpayer from playing a mixed strategy equilibrium where a positive evasion gain is made, because the condition from proposition 7 is violated. He will play a mixed strategy equilibrium where he earns nothing from concealment. Consequently the pay-offs are $EU_{yy} = 16$ and $EU_{y0} = 8$. Although not having calculated the equilibrium probabilities and expected revenue, we know that the collection efficiency will be higher than under simultaneous auditing, since the new equilibrium includes honesty as a strategy. This will drive down both the audit effort and the ex ante expected concealment effort. This, combined with the observation that the payoff of the taxpayer is smaller, leads to the conclusion that the tax collection efficiency is higher under sequential auditing.